Light hypernuclei within chiral EFT

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Outline

- Baryon-Baryon (BB) interactions in chiral effective field theory (EFT)
- Numerical approach:
  - Jacobi no-core shell model (J-NCSM) for S=-1
  - Similarity Renormalization Group (SRG)
- Results:
  - Λ separation energy in ${}^{4}_{\Lambda}\mathrm{He}$, ${}^{5}_{\Lambda}\mathrm{He}$, ${}^{7}_{\Lambda}\mathrm{Li}$ with NLO13 & NLO19 potentials
  - CSB results: $({}^{4}_{\Lambda}\mathrm{He}, {}^{4}_{\Lambda}\mathrm{H})$, $({}^{7}_{\Lambda}\mathrm{Be}, {}^{7}_{\Lambda}\mathrm{Li}^{*})$, $({}^{8}_{\Lambda}\mathrm{Be}, {}^{8}_{\Lambda}\mathrm{Li})$
- Summary

BB interactions in χEFT

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	2B force	3B force	4B force	
LO (Q^0)	$\pi(K,\eta)$ Weinberg '90	—	—	2 NN, 5 YN LECs (short range parameters)
NLO (Q^2)	Ordonez, van Kolck '92	—	—	+7 NN, +23 YN LECs
N²LO (Q^3)	Ordonez, van Kolck '92	van Kolck '94; Epelbaum et al. '02		**+2 NNN LECs, +5 ΛNN LECs**
N³LO (Q^4)	... Kaiser '00 - '02	[parameter-free] ... Bernard, Epelbaum, HK, Meißner,'08, '11	[parameter-free] ... Epelbaum '06	+15 NN LECs
N⁴LO (Q^5)	... Entem, Kaiser, Machleidt, Nosyk '15 Epelbaum, HK, Meißner '15	... Girlanda, Kievsky, Viviani '11 HK, Gasparyan, Epelbaum '12,'13	...	+5 NN LECs

(adapted from H. Krebs CD workshop, 18th November 2021)

- LECs are determined via a fit to experiment:
  - ~5000 NN + Nd scattering data + ^{2}H, ^{3}H/^{3}He → NN forces up to N^{4}LO+, 3NF up to N^{2}LO

    (P. Reinert et al EPJA (2018), P. Maris et al PRC 103(2021))
  - ~36 YN data + $^3_\Lambda$H → YN forces up to NLO (NLO13, NLO19) and N^{2}LO (YNN forces contribute)

    (J. Haidenbauer et al NPA 915(2013), EPJA 56(2019), HYP2022 talk+ proceeding)

Jacobi-NCSM approach

diagonalize the A-body translationally invariant hypernuclear Hamiltonian

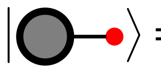
$$H = T_{rel} + V^{NN} + V^{YN} + V^{NNN} + {\color{red}V^{YNN}} + \Delta M$$

$\Lambda N \leftrightarrow \Sigma N$ (for ΔM)

in a finite A-particle harmonic oscillator (HO) basis

- basis states for $S = -1$ systems:

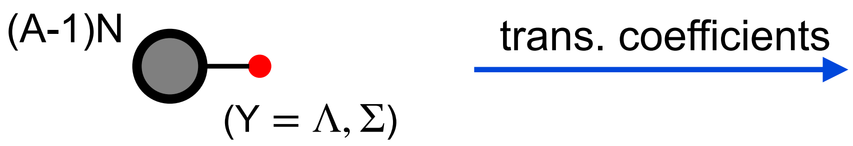
(A-1)N

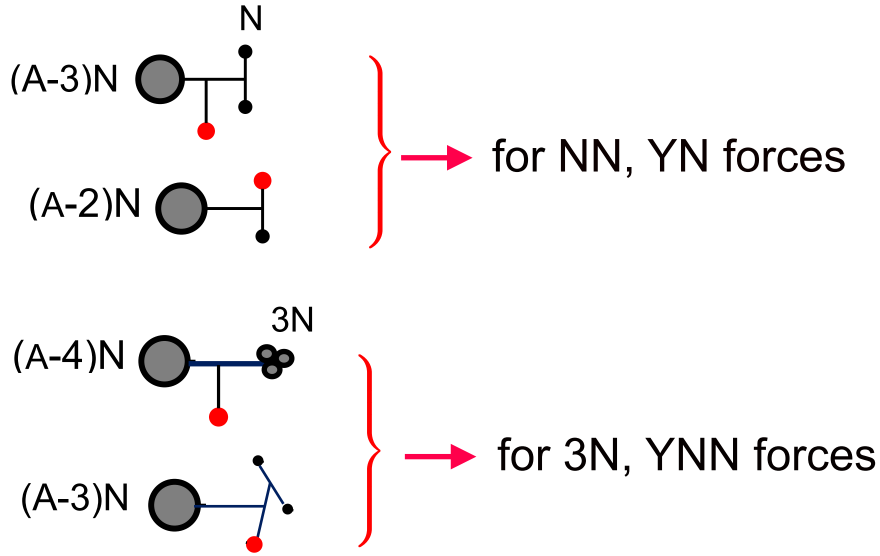


$$\Big| \bullet\!-\!{\color{red}\bullet} \Big\rangle = |\,\mathcal{N} J T, \underbrace{\mathcal{N}_{A-1} J_{A-1} T_{A-1},}_{\color{red}antisym.(A-1)N}\ \underbrace{n_Y l_Y I_Y\, t_Y;}_{\color{red}\Lambda(\Sigma)\ state}\ (J_{A-1}(l_Y s_Y) I_Y)\, J, (T_{A-1} t_Y) T \rangle$$

$\color{red}\Lambda(\Sigma)$

- intermediate bases for evaluating Hamiltonian:





- basis truncation: $\mathcal{N} = \mathcal{N}_{A-1} + 2n_\lambda + \lambda \leq \mathcal{N}_{max}$ $\quad \Rightarrow E_b = E_b(\omega, \mathcal{N}_{max})$

→ extrapolate in ω- and $\mathcal{N}$-spaces to obtain converged results

(HL, J. Haidenbauer, U. Meißner, A. Nogga EPJA (2020))

Similarity Renormalization Group (SRG)

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Idea: continuously apply unitary transformation to H to suppress off-diagonal matrix elements

→ **observables (binding energies) are conserved due to unitarity of transformation**

F.J. Wegner NPB 90 (2000). S.K. Bogner, R.J. Furnstahl, R.J. Perry PRC 75 (2007)

$$\frac{dV(s)}{ds} = \left[\left[T_{rel}, V(s)\right], H(s)\right], \qquad H(s) = T_{rel} + V(s) + \Delta M$$

$$s = 0 \to \infty \qquad V(s) = V_{12}(s) + V_{13}(s) + V_{23}(s) + V_{123}(s), \quad V_{123} \equiv V_{NNN}\ (V_{YNN})$$

- separate SRG flow equations for 2-body and 3-body interactions: **(S.K. Bogner et al PRC75 (2007), K. Hebeler PRC85 (2012))**

$$\frac{dV^{NN}(s)}{ds} = [[T^{NN}, V^{NN}], T^{NN} + V^{NN}]$$

$$\frac{dV^{YN}(s)}{ds} = [[T^{YN}, V^{YN}], T^{YN} + V^{YN} + \Delta M]$$

$$\frac{dV_{123}}{ds} = [[T_{12}, V_{12}], V_{31} + V_{23} + V_{123}] + [[T_{31}, V_{31}], V_{12} + V_{23} + V_{123}] + [[T_{23}, V_{23}], V_{12} + V_{31} + V_{123}] + [[T_{rel}, V_{123}], H_s]$$

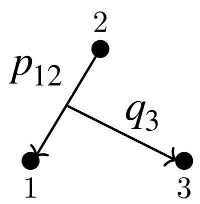
Eqs.(1)

→ **SRG-induced 3BFs are generated even if** $V_{123}^{\text{bare}} = 0$

- Eqs.(1) are solved by projecting on a 3N (YNN) Jacobi-momentum basis:

$$|p_{12}\,\alpha_{12}\rangle \equiv |p_{12}, (l_{12}s_{12})J_{12}\,(t_1t_2)t_{12}\,m_{t12}\rangle; \quad ((-1)^{l_{12}+S_{12}+t_{12}} = -1)$$

$$|p_{12}\,q_3\,\alpha\,J\,T; \alpha_{12}\,I_3t_3\rangle \equiv |p_{12}\,q_3, ((l_{12}s_{12})J_{12}\ (l_3s_3)I_3)J\ ((t_1t_2)T_{12}\,t_3)T\rangle$$



A=3-5 hypernuclei with SRG-induced YNN

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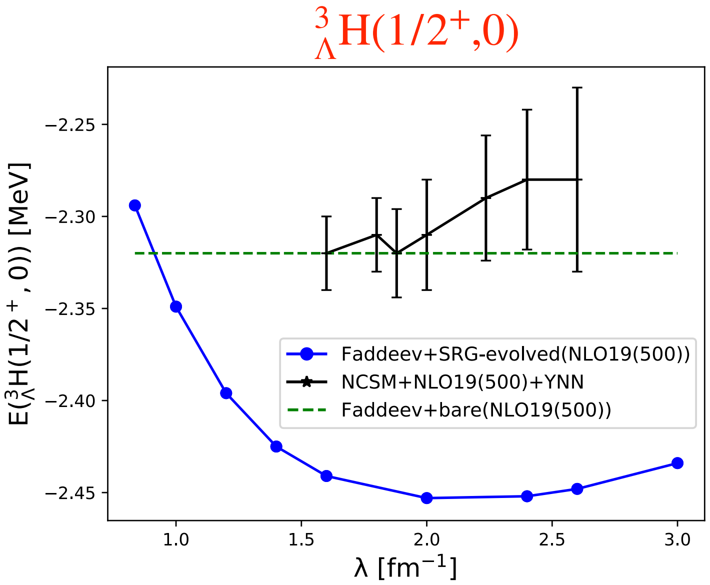
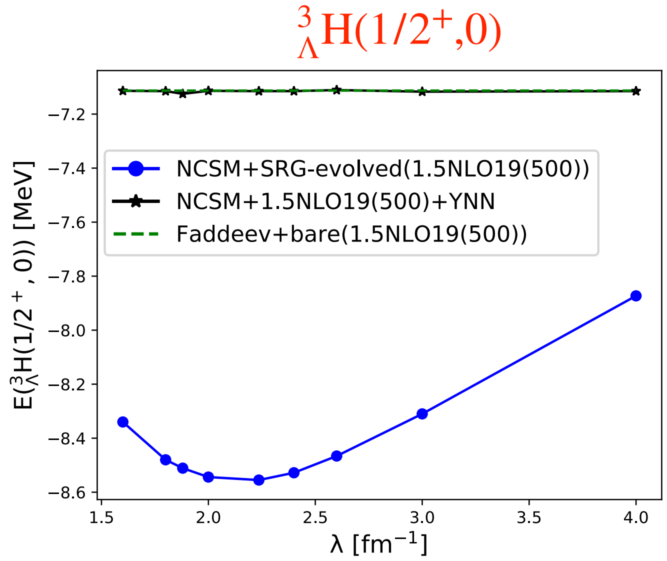
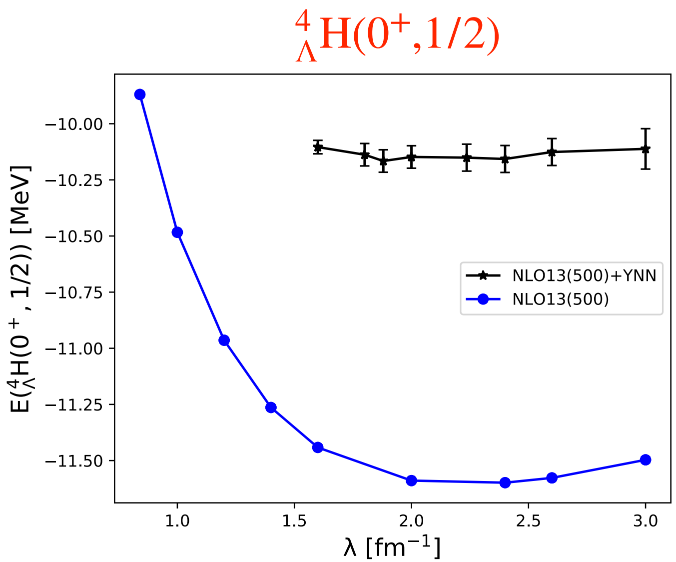
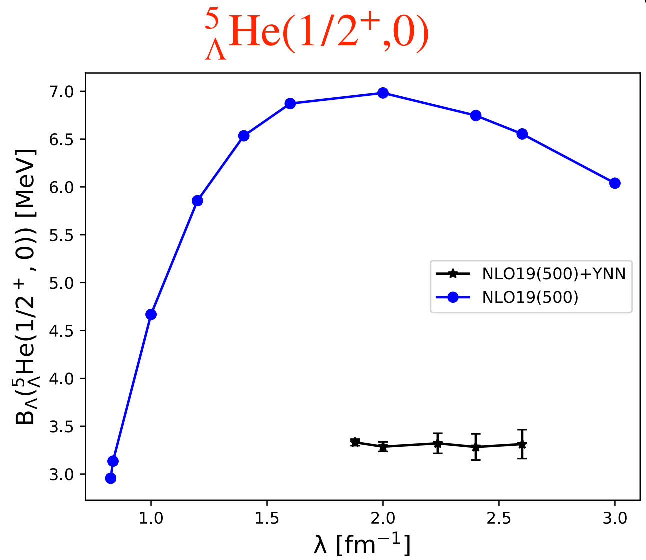




NN:SMS N^{4}LO+(450)

3N: N^{2}LO(450)





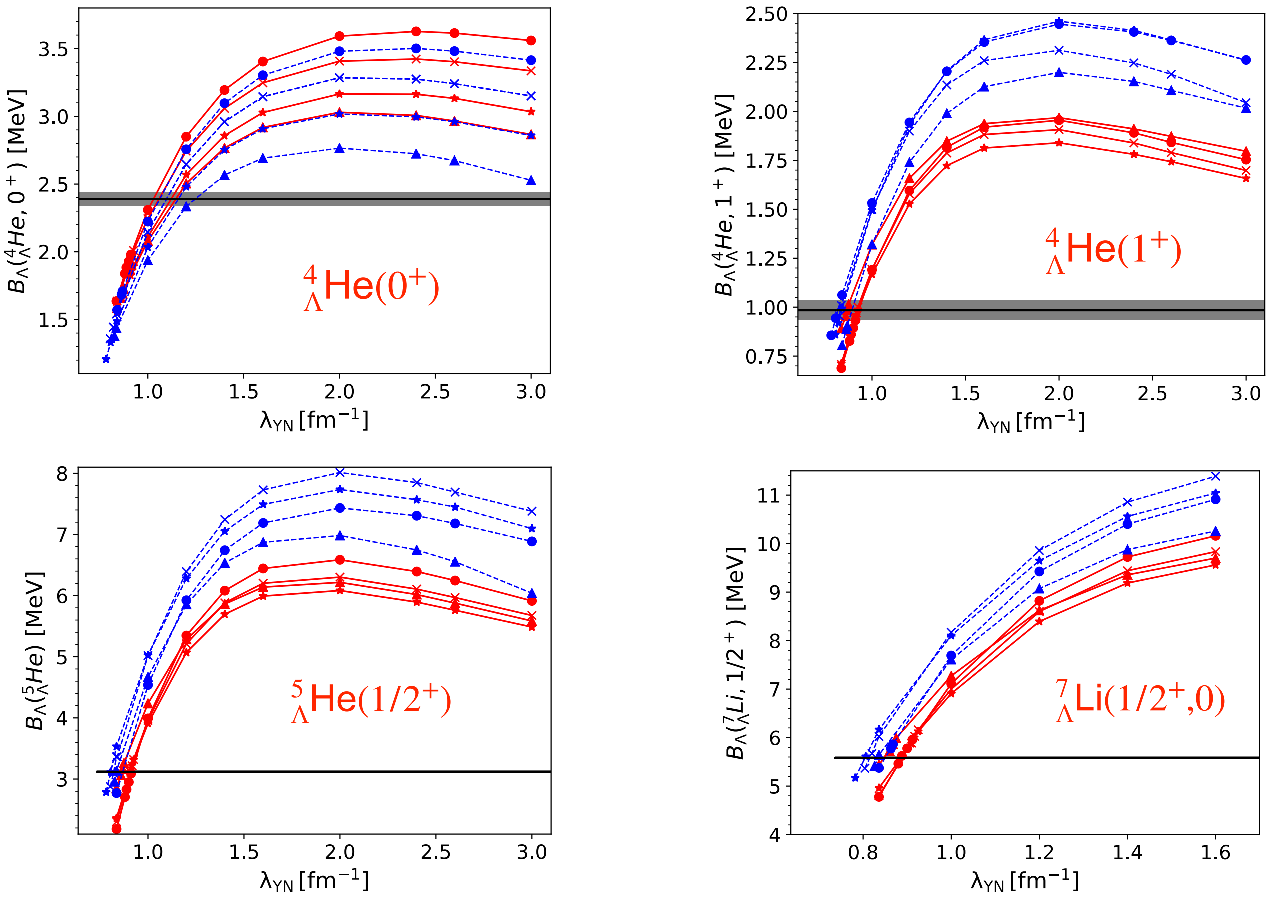
→ **contributions of SRG-induced YNNN forces to $B_\Lambda({}^4_\Lambda\mathrm{H},\ {}^5_\Lambda\mathrm{He})$ are negligible**

(R. Wirth, R. Roth PRL117 (2016), PRC100 (2019))

Impact of YN interactions on $B_\Lambda(A \leq 7)$

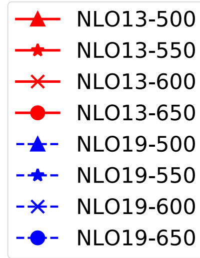
- NLO13 and NLO19 are almost phase equivalent
- NLO13 characterised by a stronger $\Lambda N - \Sigma N$ transition potential (especially in $^3S_1$)

  → **manifest in higher-body observables** **(J.Haidenbauer et al., NPA 915 2019))**



NN: SMS N^{4}LO+(450)

YN: NLO



(HL, J. Haidenbauer, U. Meißner, A. Nogga EPJA (2020))

- B_Λ(NLO19) > B_Λ(NLO13) → possible contribution of chiral YNN force

Impact of YN interactions on $B_\Lambda(A \leq 7)$

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- NLO13 and NLO19 are almost phase equivalent
- NLO13 characterised by a stronger $\Lambda N - \Sigma N$ transition potential (especially in $^3S_1$)

→ **manifest in higher-body observables** **(J.Haidenbauer et al., NPA 915 2019))**

	$^4_\Lambda$H 0^+	$^4_\Lambda$H 1^+	$^5_\Lambda$He $1/2^+$	$^7_\Lambda$Li $(1/2^+, 0)$
NLO13(500)	$\mathbf{1.551 \pm 0.007}$	0.823 ± 0.003	2.22 ± 0.06	5.28 ± 0.68
NLO19(500)	$\mathbf{1.514 \pm 0.007}$	1.27 ± 0.009	3.32 ± 0.03	$\mathbf{6.04 \pm 0.30}$
Exp	$2.16 \pm 0.08^{(1)}$	$1.07 \pm 0.08^{(1)}$	$3.12 \pm 0.02^{(1)}$	$\mathbf{5.85 \pm 0.13(10)^{(2)}}$ $5.58 \pm 0.03^{(1)}$

NN:SMS N^{4}LO+(450)

+3N: N^{2}LO(450)

+SRG-induced YNN

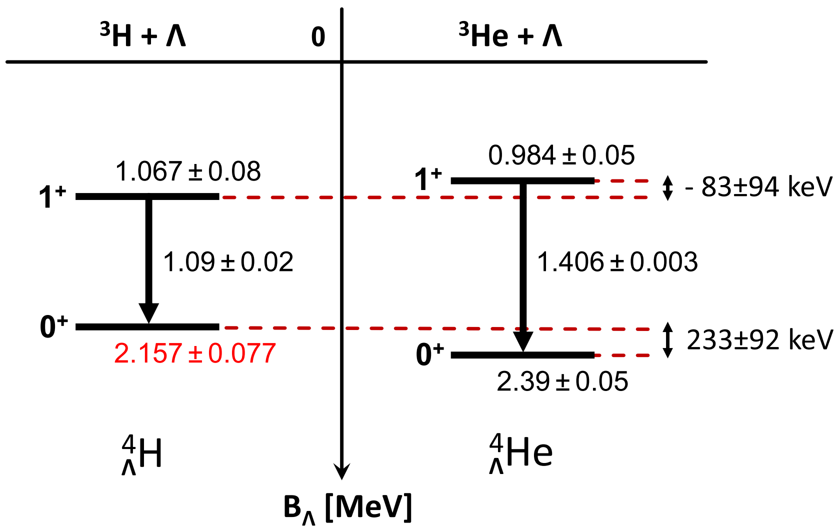
$^{(2)}$**M. Agnello PLB 681(2009)**

$^{(1)}$**M. Juric NPB 52(1973)**

- $^4_\Lambda$H(1^+), $^5_\Lambda$He, $^7_\Lambda$Li are fairly well described by **NLO19(500)**; NLO13 underbinds these systems
- YNN contributes at N2LO. Using decuplet saturation scheme → YNN is promoted to NLO (**2LECs**)

→ use $B_\Lambda(^4_\Lambda\text{H}/^4_\Lambda\text{He}(0^+,1^+))$ **or** $B_\Lambda(^4_\Lambda\text{H/He}(0^+),\ ^5_\Lambda\text{He}(1/2^+))$ to fix the additional **2LECs** (work in progress)

CSB in A=4 doublet: ${}^4_\Lambda\mathrm{H}$, ${}^4_\Lambda\mathrm{He}$



(Schulz et al, (2016); Yamamoto et al, (2015))

$$\Delta E(1^+) = B_\Lambda({}^4_\Lambda\mathrm{He}, 1^+) - B_\Lambda({}^4_\Lambda\mathrm{H}, 1^+) = -83 \pm 94 \text{ keV}$$

$$\Delta E(0^+) = B_\Lambda({}^4_\Lambda\mathrm{He}, 0^+) - B_\Lambda({}^4_\Lambda\mathrm{H}, 0^+) = 233 \pm 92 \text{ keV}$$

→ **Coulomb contribution almost cancels in B_Λ**

(Bodmer et al, 1985)

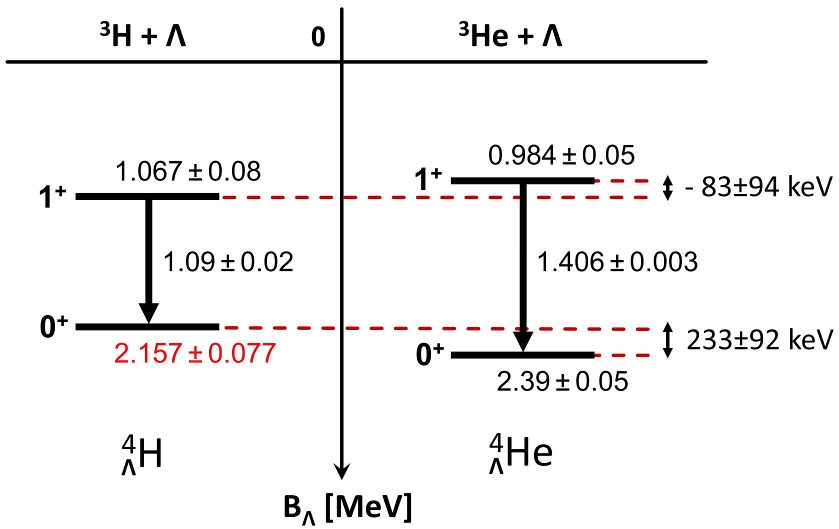
- **2 additional LECs (at LO)** contributing to CSB are adjusted to $\Delta E(0^+, 1^+)$

(fm//keV)	$a_s^{\Lambda p}$	$a_s^{\Lambda n}$	δa_s	$a_t^{\Lambda p}$	$a_t^{\Lambda n}$	δa_t	$\Delta E(0^+)$	$\Delta E(1^+)$
NLO19(**500**) no CSB	**-2.91**	**-2.91**	**0**	**-1.42**	**-1.41**	**-0.01**	**34**	**10**
CSB1(**500**)	**-2.65**	**-3.20**	**0.55**	**-1.58**	**-1.47**	**-0.11**	**249**	**-75**
CSB1(550)	-2.64	-3.21	0.57	-1.52	-1.41	-0.11	252	-72
CSB1(600)	-2.63	-3.23	0.6	-1.47	-1.36	-0.09	243	-67
CSB1(650)	-2.62	-3.23	0.61	-1.46	-1.37	-0.09	250	-69

(J. Haidenbauer, U-G. Meißner and A. Nogga FBS 62(2021))

→
- **CSB in singlet (${}^1S_0$) is much larger than in triplet** (${}^3S_1$)
- predictions for A=4 are **independent of cutoff**, same results for NLO13
- predictions for CSB in A=7,8 multiplets ?

CSB in A=4 doublet: ${}^4_\Lambda\mathrm{H}$, ${}^4_\Lambda\mathrm{He}$



(Schulz et al, (2016); Yamamoto et al, (2015);
[(1)]**Star collaboration (2022))**

$$\Delta E(1^+) = B_\Lambda({}^4_\Lambda\mathrm{He}, 1^+) - B_\Lambda({}^4_\Lambda\mathrm{H}, 1^+)$$
$$= -83 \pm 94 \text{ keV}$$
$$= -160 \pm 140 \pm 100^{(1)} \text{ keV}$$

$$\Delta E(0^+) = B_\Lambda({}^4_\Lambda\mathrm{He}, 0^+) - B_\Lambda({}^4_\Lambda\mathrm{H}, 0^+)$$
$$= 233 \pm 92 \text{ keV}$$
$$= 160 \pm 140 \pm 100^{(1)} \text{ keV}$$

what could be consequence on CSB in A=7,8?

- **2 additional LECs (at LO)** contributing to CSB are adjusted to $\Delta E(0^+, 1^+)$

(fm//keV)	$a_s^{\Lambda p}$	$a_s^{\Lambda n}$	δa_s	$a_t^{\Lambda p}$	$a_t^{\Lambda n}$	δa_t	$\Delta E(0^+)$	$\Delta E(1^+)$
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- **CSB in singlet ($^1S_0$) is much larger than in triplet** ($^3S_1$)
- predictions for A=4 are **independent of cutoff**, same results for NLO13
- predictions for CSB in A=7,8 multiplets ?

CSB in A=7 isotriplet: ${}^{7}_{\Lambda}$He, ${}^{7}_{\Lambda}$Li*, ${}^{7}_{\Lambda}$Be

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	NLO19(500)	NLO13(500)	Exp.[2] emulsion	Exp.[2] counter
${}^{7}_{\Lambda}$Be	5.54 ± 0.22	4.30 ± 0.47	5.16 ± 0.08	**?**
${}^{7}_{\Lambda}$Li*	$\mathbf{5.64 \pm 0.28}$	4.42 ± 0.58	5.26 ± 0.03	$\mathbf{5.53 \pm 0.13}$
${}^{7}_{\Lambda}$He	$\mathbf{5.64 \pm 0.27}$	4.39 ± 0.54		$\mathbf{5.55 \pm 0.1}$

Separation energies in A=7 isotriplet

NN:SMS N^4LO+(450)

+3N: N^2LO(450)

+SRG-induced YNN

	YN	ΔT	ΔNN	ΔYN $^1S_0$	ΔYN $^3S_1$	ΔYN total	$\Delta E_{\Lambda}^{pert}$
$({}^{7}_{\Lambda}\text{Be}, {}^{7}_{\Lambda}\text{Li}^*)$	NLO13	6.8	-24	-1.0	0	0	-17.2(30)
	CSB1	**7.8**	**-24**	**-49.3**	**25.5**	**-24**	**-40.2(30)**
	NLO19	5.8	-40	-0.6	0	0	-34.2(30)
	CSB1	**5.8**	**-41**	**-43.1**	**42.1**	**-0.3**	**-35.2(30)**
	Gal[1]	3	-70			50	-17
	Exp[2]						-100 ± 90

(HL, J. Haidenbauer, U-G. Meißner and A. Nogga in preparation)

[1] **A. Gal PLB 744 (2015)**

[2] **E. Botta et al., NPA 960 (2017)**

→ • **NLO19(500)** predicts rather accurately separation energies in A=7 isotriplet

• **NLO13 & NLO19 CSB results for A=7 are comparable to experiment**

CSB in A=8 doublet: ${}^{8}_{\Lambda}\mathrm{Be}$, ${}^{8}_{\Lambda}\mathrm{Li}$

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	λ_{YN}	${}^{8}_{\Lambda}\mathrm{Be}$	${}^{8}_{\Lambda}\mathrm{Li}$
NLO13	0.765	5.56 ± 0.25	5.57 ± 0.30
NLO19	0.823	7.15 ± 0.10	7.17 ± 0.10
Hiyama et al.		6.72	6.80
Exp. emulsion		6.84 ± 0.05	6.80 ± 0.03
Exp. **counter**		**?**	**?**

Separation energies in A=8 doublet, computed at λ that reproduces $B_{\Lambda}({}^{5}_{\Lambda}\mathrm{He})$

YN	ΔT	ΔNN	ΔYN			$\Delta E_{\Lambda}^{pert}$
			${}^{1}S_{0}$	${}^{3}S_{1}$	total	
NLO13	12.2	8	-2.1	0	-4.0	16.2(50)
CSB1	**11.9**	**7**	**99.8**	**55.5**	**158.8**	**177.7(50)**
NLO19	6.6	-11	-0.9	0	-1.9	-6.3(50)
CSB1	**6.3**	**-11**	**62**	**79.1**	**147.3**	**142.6(50)**
Hiyama[1]						160
Gal[2]	11	-81			119	49
Exp[3]						40 ± 60

NN:SMS N^{4}LO+(450)

+3N: N^{2}LO(450)

+SRG-induced YNN

[1] **E. Hiyama et al., PRC 80 (2009)**

[2] **A. Gal PLB 744 (2015)**

[3] **E. Botta et al., NPA 960 (2017)**

- **CSB1** fits lead to a larger CSB in A=8 doublet as compared to experiment

→ experimental CSB result for A=8 could be **larger than** 40 ± 60 **keV**?

CSB estimated for A=4 could **still be too large** or have **different spin-dependence**?

Fitting LECs to new Star measurement

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$$\Delta E(1^+) = B_\Lambda({}^4_\Lambda\mathrm{He}, 1^+) - B_\Lambda({}^4_\Lambda\mathrm{H}, 1^+)$$
$$= -83 \pm 94 \text{ keV} \Rightarrow \text{(CSB1)}$$
$$= -160 \pm 140 \pm 100 \text{ keV} \Rightarrow \text{(CSB1A)}$$

$$\Delta E(0^+) = B_\Lambda({}^4_\Lambda\mathrm{He}, 0^+) - B_\Lambda({}^4_\Lambda\mathrm{H}, 0^+)$$
$$= 233 \pm 92 \text{ keV} \Rightarrow \text{(CSB1)}$$
$$= 160 \pm 140 \pm 100 \text{ keV} \Rightarrow \text{(CSB1A)}$$

	NLO19(500)	CSB1	CSB1A
$a_s^{\Lambda p}$	-2.91	-2.65	-2.58
$a_s^{\Lambda n}$	-2.91	-3.20	-3.29
$\boldsymbol{\delta a_s}$	**0**	**0.55**	**0.71**
$a_t^{\Lambda p}$	-1.42	-1.57	-1.52
$a_t^{\Lambda n}$	-1.41	-1.45	-1.49
$\boldsymbol{\delta a_t}$	**-0.01**	**-0.12**	**-0.03**

	${}^4_\Lambda\mathrm{He} - {}^4_\Lambda\mathrm{H}$ 0^+	${}^4_\Lambda\mathrm{He} - {}^4_\Lambda\mathrm{H}$ 1^+	${}^7_\Lambda\mathrm{Be} - {}^7_\Lambda\mathrm{Li}^*$	${}^7_\Lambda\mathrm{Li}^* - {}^7_\Lambda\mathrm{He}$	${}^8_\Lambda\mathrm{Be} - {}^8_\Lambda\mathrm{Li}$
NLO19	-7.5	-10.5	-34.3	-14.3	-11
CSB1	209.5	-70.5	-26.3	-3.3	135
CSB1A	129.5	-134.5	-83.3	-62.3	74
Exp			-100 ± 90	-20 ± 230	40 ± 60

NN : $\mathrm{N^4LO^+}(450)$; $\lambda_N = 1.6 \text{ fm}^{-1}$

YN : NLO19(500); $\lambda_{YN} = 0.823 \text{ fm}^{-1}$

$B_\Lambda({}^5_\Lambda\mathrm{He}, \mathrm{NLO19}) = 3.35 \pm 0.03$ MeV

(HL, J. Haidenbauer, U-G. Meißner and A. Nogga in preparation)

→ **CSB1A** fit predicts reasonable CSB in both A=7 and A=8 systems

Summary

study ${}^{4}_{\Lambda}\mathrm{H}(0^+,1^+)$, ${}^{5}_{\Lambda}\mathrm{He}$, ${}^{7}_{\Lambda}\mathrm{Li}$ **hypernuclei using chiral 2B & 3N interactions + SRG-induced YNN**

- NLO19 potential reproduces fairly well experimental values for ${}^{4}_{\Lambda}\mathrm{H}(1^+)$, ${}^{5}_{\Lambda}\mathrm{He}$ and ${}^{7}_{\Lambda}\mathrm{Li}$
- NLO13 underbinds A=4-7 hypernuclei

→ difference in predictions of NLO13 & NLO19 will be removed by appropriate chiral YNN force

study CSB in A=7 isotriplet and A=8 doublet using χ 2BFs + 3BFs:

- **CSB1** fit reproduces experimental results for A=4 & 7 systems
  but lead to a somewhat larger than the experimental CSB for the ${}^{8}_{\Lambda}\mathrm{Be}$, ${}^{8}_{\Lambda}\mathrm{Li}$ doublet
- **CSB1A** fit yields reasonable CSB for A=7 & 8 systems

Thank you for the attention!

		${}^{8}_{\Lambda}$Be	${}^{8}_{\Lambda}$Li	${}^{5}_{\Lambda}$He
NLO13	YNN_SRG		5.75 ± 1.08	2.22 ± 0.06
	$\lambda = 0.765$	5.56 ± 0.25	5.57 ± 0.30	2.22 ± 0.04
NLO19	YNN_SRG		7.33 ± 1.15	3.32 ± 0.03
	$\lambda = 0.823$	7.15 ± 0.10	7.17 ± 0.10	3.35 ± 0.02
Experiment [4]		6.84 ± 0.05	6.80 ± 0.03	3.12 ± 0.02