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Parity violation in two-nucleon systems

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Abstract Nuclear few-body systems become attractive avenues for the study of low-energy parity violation because experiments start to meet the precision requirements and theoretical calculations can be performed reliably. In this talk, an attempt of parametrizing low-energy parity-violating observables by the Danilov parameters will be introduced. Analyses of two-nucleon observables, based on the modern phenomenological potentials or the one of effective field theory, will be discussed.

1 Introduction

The Study of the strangeness-conserving ($\Delta S = 0$) hadronic weak interaction started shortly after the discovery of parity violation [1]. Although a lot of data have been accumulated during the past years, our understanding is still vague (see, e.g., Refs. [2, 3, 4, 5] for comprehensive reviews). The challenge is two-fold: Experimentally, one has to either meet the precision requirement, i.e., a signal-to-noise ratio $\sim 10^{-7}$ set by the relative strength of weak-to-strong interaction, or find particular systems, usually heavy, to amplify observables. Theoretically, a first-principle formulation of this interaction is still missing, and many related few- and many-body calculations are not reliable enough yet.

Nevertheless, such studies are still important in their own right for many reasons. Here we just list a few: (i) This is the only sector to probe the hadronic neutral weak interaction, which is among the very few pieces which have not been well-tested in the Standard Model. (ii) In contrast to the strong interaction, this concerns four-quark correlations at a much shorter distance, which provides additional information about quark dynamics at the nonperturbative regime. In fact, we have some evidence showing that the $\Delta I = 1$ (I refers to the isospin) component of this interaction is much smaller than the $\Delta I = 0, 2$ ones. This reminds us of a similar phenomenon, that is, the $\Delta I = 3/2$ suppression found in the $\Delta S = 1$ decay. (iii) It is needed for a better interpretation of several semi-leptonic processes under intensive experimental study.

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For example, the so-called nuclear "anapole" form factor or moments that contribute to parity-violating (PV) electron scattering on the proton or the nucleus, and atomic parity violation experiments.

2 Parity-violating nucleon-nucleon interaction

The progress of formulating the PV nucleon-nucleon (NN) interaction follows a similar historical track as the strong interaction, and proceeds in three major directions.

(I) The scattering amplitudes: A two-body interaction can be phenomenologically constrained by knowledge of energy-dependent scattering amplitudes, and the number of partial waves to be included increases with energy. In PV NN scattering, there are five lowest partial wave amplitudes: $v^{0,1,2}$ for the ${}^1S_0$–${}^3P_0(\Delta I = 0,\, 1,\, 2)$, u for the ${}^3S_1$–${}^1P_1$ ($\Delta I = 0$), and w for the ${}^3S_1$–${}^3P_1$ ($\Delta I = 1$) transition. It was first suggested by Danilov [6,7] to use these S-P amplitudes to describe low-energy PV processes, and generalized by Desplanques and Missimer [8] to analyze a wide range of pre-80's PV observables.

(II) The meson-exchange models: Considering the one-meson-exchange (OME) mechanism with intermediate mesons of mass $m \lesssim 1\,\mathrm{GeV}$ (this leaves $\pi^{\pm}$, ρ, and ω mesons as the only candidates; π^0 and η mesons are ruled out by CP conservation), one obtains the following PV NN interaction:

$$V^{\mathrm{PV}}_{\mathrm{OME}}(\boldsymbol{r}) = V^{\mathrm{PV}}_{\rho,\omega}(\boldsymbol{r}) + V^{\mathrm{PV}}_{\pi^{\pm}}(\boldsymbol{r})\,, \tag{1}$$

$$\begin{aligned} V^{\mathrm{PV}}_{\rho,\omega}(\boldsymbol{r}) = \frac{-1}{m_N}\Big\{ & g_\rho \left[h^0_\rho\,\tau_{\cdot} + h^1_\rho\,\tau^z_+ + h^2_\rho\,\tau^{zz}\right]\big(\boldsymbol{\sigma}_- \cdot \boldsymbol{u}_{\rho+}(\boldsymbol{r}) + i\,(1+\chi_\rho)\,\boldsymbol{\sigma}_\times \cdot \boldsymbol{u}_{\rho-}(\boldsymbol{r})\big) \\ & + g_\omega \left[h^0_\omega + h^1_\omega\,\tau^z_+\right]\big(\boldsymbol{\sigma}_- \cdot \boldsymbol{u}_{\omega+}(\boldsymbol{r}) + i\,(1+\chi_\omega)\,\boldsymbol{\sigma}_\times \cdot \boldsymbol{u}_{\omega-}(\boldsymbol{r})\big) \\ & - g_\rho\, h^1_\rho\,\tau^z_-\,\boldsymbol{\sigma}_+ \cdot \boldsymbol{u}_{\rho+}(\boldsymbol{r}) + g_\omega\, h^1_\omega\,\tau^z_-\,\boldsymbol{\sigma}_+ \cdot \boldsymbol{u}_{\omega+}(\boldsymbol{r}) \\ & + g_\rho\, h^{1'}_\rho\,\tau^z_\times\,\boldsymbol{\sigma}_+ \cdot \boldsymbol{u}_{\rho-}(\boldsymbol{r})\Big\}\,, \end{aligned} \tag{2}$$

$$V^{\mathrm{PV}}_{\pi^{\pm}}(\boldsymbol{r}) = \frac{1}{\sqrt{2}\,m_N}\,g_\pi\,h^1_\pi\,\tau^z_\times\,\boldsymbol{\sigma}_+ \cdot \boldsymbol{u}_{\pi-}(\boldsymbol{r})\,, \tag{3}$$

where m_N is the nucleon mass; the g_x's denote the parity-conserving (PC) x-meson-nucleon couplings, and the h^i_x's the PV ones with isospin change $\Delta I = i$; χ_ω and χ_ρ are the isoscalar and isovector strong tensor couplings, respectively; $\tau_{\cdot} \equiv \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$, $\tau^z_\pm \equiv (\tau^z_1 \pm \tau^z_2)/2$, $\tau^z_\times \equiv i\,(\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)^z/2$, and $\tau^{zz} \equiv (3\,\tau^z_1\,\tau^z_2 - \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)\,/(2\sqrt{6})$ are the isospin operators; $\boldsymbol{\sigma}_\pm \equiv \boldsymbol{\sigma}_1 \pm \boldsymbol{\sigma}_2$ and $\boldsymbol{\sigma}_\times \equiv i\,\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2$ are the spin operators. The spatial operator $\boldsymbol{u}_{x+}(\boldsymbol{r})$ ($\boldsymbol{u}_{x-}(\boldsymbol{r})$) is defined as the (anti-) commutator of $-i\,\boldsymbol{\nabla}$ with the Yukawa function $f_x(r)$

$$f_x(r) \equiv \frac{e^{-m_x\,r}}{4\,\pi\,r}\,. \tag{4}$$

The above form then becomes the standard in analyzing nuclear PV phenomena for many years after Desplanques, Donoghue, and Holstein (DDH) give their prediction for these meson-nucleon coupling constants, based on a quark model calculation [9]. However, there has not been a good agreement on h^i_x's between the experimental constraints and a rather liberal theoretical range a la DDH (two such discrepancies are pointed out in Refs. [10,11]). Also note that $V^{\mathrm{PV}}_{\mathrm{OME}}$ does not include contributions from, for example, two-pion (or multi-pion) exchange, therefore some of these meson-nucleon coupling constants should be realized in an effective sense.

(III) The effective field theory (EFT): Inspired by the success of applying EFT to formulate the PC NN interaction, Zhu et al. [12] recently gave the leading order (LO) PV NN interaction. This potential $V_{\rm EFT}^{\rm PV}$ is broken down in three components

$$V_{\rm EFT}^{\rm PV}(\boldsymbol{r}) = V_{1,\rm SR}^{\rm PV}(\boldsymbol{r}) + V_{-1,\rm LR}^{\rm PV}(\boldsymbol{r}) + V_{1,\rm MR}^{\rm PV}(\boldsymbol{r})\,. \qquad (5)$$

The short-range (SR) interaction takes the following form

$$\begin{aligned} V_{1,\rm SR}^{\rm PV}(\boldsymbol{r}) = \frac{2}{\Lambda_\chi^3} \Big\{ & \left[C_1 + (C_2 + C_4)\,\tau_+^z + C_3\,\tau. + C_5\,\tau^{zz}\right] \boldsymbol{\sigma}_- \cdot \boldsymbol{y}_{x+}(\boldsymbol{r}) \\ & + \left[\widetilde{C}_1 + (\widetilde{C}_2 + \widetilde{C}_4)\,\tau_+^z + \widetilde{C}_3\,\tau. + \widetilde{C}_5\,\tau^{zz}\right] \boldsymbol{\sigma}_\times \cdot \boldsymbol{y}_{x-}(\boldsymbol{r}) \\ & + (C_2 - C_4)\,\tau_-^z\,\boldsymbol{\sigma}_+ \cdot \boldsymbol{y}_{x+}(\boldsymbol{r}) + \widetilde{C}_6\,\tau_\times^z\,\boldsymbol{\sigma}_+ \cdot \boldsymbol{y}_{x-}(\boldsymbol{r}) \Big\}\,, \end{aligned} \qquad (6)$$

where Λ_χ is the scale of chiral symmetry breaking, which is related to the pion decay constant F_π by $\Lambda_\chi = 4\,\pi\,F_\pi \approx 1.161\,\mathrm{GeV}$. The spatial operators $\boldsymbol{y}_{m\pm}(\boldsymbol{r})$, as originally proposed in Ref. [12], have the properties that i) they are strongly peaked at $r = 0$ with a range parameter $1/m_x$, and ii) they approach $\delta(\boldsymbol{r})$ in the zero range (ZR) limit (i.e., $m_x \to \infty$). For example, a choice with

$$\boldsymbol{y}_{x\pm}(\boldsymbol{r}) = m_x^2\,\boldsymbol{u}_{x\pm}(\boldsymbol{r}) \to [-i\,\boldsymbol{\nabla}\,,\,\delta(r)/r^2]_\pm\,, \qquad (7)$$

would make this short range interaction resemble the heavy-meson-exchange potential $V_{\rho,\omega}^{\rm PV}(\boldsymbol{r})$. Such a softening of the contact interaction is mainly for the implementation in typical potential model calculations. In the strict contact form, $\boldsymbol{y}_{m+}(\boldsymbol{r})$ and $\boldsymbol{y}_{m-}(\boldsymbol{r})$ would yield the same matrix elements, so $V_{1,\rm SR}^{\rm PV}(\boldsymbol{r})$ has only five genuine low energy constants (LECs) [12,13,14]. The long-range (LR) interaction $V_{-1,\rm LR}^{\rm PV}(\boldsymbol{r})$ is due to one-pion exchange and has the exact same form as $V_{\pi\pm}^{\rm PV}(\boldsymbol{r})$. The medium-range (MR) interaction $V_{1,\rm MR}^{\rm PV}(\boldsymbol{r})$ is due to two-pion exchange which yields two isospin-spin operators similar to the SR $\widetilde{C}_2$ and $\widetilde{C}_6$ terms, and its complete form can be found in Refs. [12,15]. In a pionless EFT framework, these LR and MR interactions are effectively included in the SR interaction and contribute to $\widetilde{C}_6$ (one-pion) and $\widetilde{C}_2$ and $\widetilde{C}_6$ (two-pion), respectively.

3 Two-nucleon observables and Danilov parameters

To calculate PV observables, the first step is to determine the related wave functions of the initial and final states. The PC component $|\psi\rangle$ of a scattering or bound state is obtained by solving Lippmann-Schwinger or Schrödinger equation:

$$\begin{aligned} (E - H_0 + V^{\rm PV} \mp i\,\epsilon)\,|\psi\rangle^{(\pm)} &= 0\,, \\ (E_B + H_0 + V^{\rm PV})\,|\psi\rangle_B &= 0\,, \end{aligned}$$

and the PV admixture $|\widetilde{\psi}\rangle$ to the first-order Born approximation is obtained by solving the associated inhomogeneous differential equation:

$$(E - H_0 - V^{\rm PC} \mp i\,\epsilon)|\widetilde{\psi}\rangle^{(\pm)} = V^{\rm PV}\,|\psi\rangle^{(\pm)}\ ,$$
$$(E_B + H_0 + V^{\rm PC})|\widetilde{\psi}\rangle_B = -V^{\rm PV}\,|\psi\rangle_B\ .$$

Depending on the choice of $V^{\rm PC}$ and $V^{\rm PV}$, there exist three different calculation schemes:

1. Phenomenological: In this scheme, both $V^{\rm PC}$ and $V^{\rm PV}$ are taken from phenomenological models. It should be noted that most state-of-the-art nuclear few- and many-body calculations are performed in such fashions. However, as mentioned above, the theory-experiment agreement is still not satisfactory.
2. Hybrid EFT: This approach tries to combine the best of both worlds: high-quality wave functions calculated from phenomenological $V^{\rm PC}$ and the general $V^{\rm PV}$ derived in the spirit of EFT. Although it gains quite some success in various systems, the consistency of such an approach is questionable.
3. Pure EFT: In this scheme, $V^{\rm PC}$ and $V^{\rm PV}$ are both formulated in an EFT framework, so consistency is guaranteed. However, even in the spirit of EFT, there are a few variants. For example, depending on whether the momentum scale Q of a problem is much smaller than the pion mass m_π, one can choose to work with perturbative pions ("pionless" EFT) or still treat pions nonperturbatively (pionful EFT). The pionless EFT has the advantage of renormalization/regularization-scheme independence but a limited range of applicability ($Q/m_\pi \ll 1$). The pionful EFT, though with a larger range of applicability, has to deal with regulator dependence in cutting off the high momentum region.

As it is still far from clear at the moment which scheme yields the best theory-experiment agreement, in addition to reliable calculations in these frameworks, it would be nice if different calculations can be compared on the same footing. For this purpose, we shall introduce the Danilov parameters.

The five dimensionless Danilov parameters: $\bar{\lambda}_s^{pp,nn,np}$, $\bar{\lambda}_t$, and $\bar{\rho}_t$, are extracted from the associated zero-energy S–P amplitudes, $v^{pp,nn,np}$, u, and w, respectively (see Refs. [7,8,4,15] for details). Their usefulness lies in the fact that, at low energy, the energy dependence of S–P amplitudes can be completely factored out so these parameters remain constant. Therefore, within the applicable low-energy range, the Danilov parameters can be viewed as the effective LECs. Furthermore, because they are extracted from observables, they are physical and should be independent of models and theories. The only exception is the $\bar{\rho}_t$ parameter, because the contribution of one-pion exchange, whose energy dependence is different from the ones of MR and SR interactions (unless at very low energies), is usually subtracted from the ${}^3S_1$–${}^3P_1$ amplitude [8,15].

A detailed analysis of two-nucleon observables in terms of the Danilov parameters is given in Ref. [15]; here we use two examples to demonstrate its basic features. The calculation framework is hybrid: the Argonne v_{18} model [16] is employed for $V^{\rm PC}$ and the pionful EFT for $V^{\rm PV}$. Two sets of results will be presented: In the "bare" case, we use the m^2-weighted Yukawa propagator $m^2/(m^2+q^2)$ with $m = m_\rho \sim 770\,\text{MeV}$ to soften the contact interaction, and we do not introduce any cut-off factor to one-pion and two-pion exchanges. In the "mod" case, we introduce dipole cut-off factors $(\Lambda^2 - 4\,m^2)^2/(\Lambda^2 + q^2)^2$ with $\Lambda = 1.31\,\text{GeV}$ for the SR interaction, and $\Lambda = 1.72\,\text{GeV}$ for the MR and LR interactions. Such choices are mainly for an easy comparison with existing phenomenological calculations based on $V^{\rm PV}_{\rm OME}$ (primarily Refs. [11,17]). It should also be noted that the LECs are regulator and cut-off dependent in such calculations.

1. Longitudinal asymmetry A_L of $p\,p$ scattering at 13.6 and 45 MeV: The asymmetry factors are found to be

$$\begin{aligned} A^{pp}_L(13.6\,\mathrm{MeV}) &= (-2.471\,\widetilde{D}^{pp}_v - 1.984\,D^{pp}_v + 4.000\,\widetilde{C}^{2\pi}_2)\times 10^{-3}\,, \quad \text{(bare)} \\ &= (-1.614\,\widetilde{D}^{pp}_v - 1.402\,D^{pp}_v + 1.876\,\widetilde{C}^{2\pi}_2)\times 10^{-3}\,, \quad \text{(mod)} \end{aligned} \tag{8}$$

$$\begin{aligned} A^{pp}_L(45\,\mathrm{MeV}) &= (-4.377\,\widetilde{D}^{pp}_v - 3.781\,D^{pp}_v + 6.847\,\widetilde{C}^{2\pi}_2)\times 10^{-3}\,, \quad \text{(bare)} \\ &= (-2.797\,\widetilde{D}^{pp}_v - 2.652\,D^{pp}_v + 2.712\,\widetilde{C}^{2\pi}_2)\times 10^{-3}\,, \quad \text{(mod)} \end{aligned} \tag{9}$$

and the only relevant Danilov parameter $\bar{\lambda}^{pp}_s$ is found to be

$$\begin{aligned} \bar{\lambda}^{pp}_s &= 5.507\times 10^{-3}\,(\widetilde{D}^{pp}_v + 0.789\,D^{pp}_v - 1.655\,\widetilde{C}^{2\pi}_2)\times 10^{-3}\,, \quad \text{(bare)} \\ &= 3.628\times 10^{-3}\,(\widetilde{D}^{pp}_v + 0.849\,D^{pp}_v - 1.260\,\widetilde{C}^{2\pi}_2)\times 10^{-3}\,. \quad \text{(mod)} \end{aligned} \tag{10}$$

Recasting the asymmetry factors using the Danilov parameter yields

$$\begin{aligned} A^{pp}_L(13.6\,\mathrm{MeV}) &= -0.449\,\bar{\lambda}^{pp}_s + (-0.035\,D^{pp}_v - 0.088\,\widetilde{C}^{2\pi}_2)\times 10^{-3}\,, \quad \text{(bare)} \\ &= -0.445\,\bar{\lambda}^{pp}_s + (-0.032\,D^{pp}_v - 0.157\,\widetilde{C}^{2\pi}_2)\times 10^{-3}\,, \quad \text{(mod)} \end{aligned} \tag{11}$$

$$\begin{aligned} A^{pp}_L(45\,\mathrm{MeV}) &= -0.795\,\bar{\lambda}^{pp}_s + (-0.329\,D^{pp}_v - 0.395\,\widetilde{C}^{2\pi}_2)\times 10^{-3}\,, \quad \text{(bare)} \\ &= -0.771\,\bar{\lambda}^{pp}_s + (-0.276\,D^{pp}_v - 0.813\,\widetilde{C}^{2\pi}_2)\times 10^{-3}\,. \quad \text{(mod)} \end{aligned} \tag{12}$$

As one can obviously see, the asymmetry factors have almost model independent expressions in terms of the Danilov parameter, plus some remaining corrections. The larger correction for the 45 MeV case just reflects the fact that higher partial waves start to contribute so the S–P amplitudes become less dominant. Ignoring the corrections, the ratio of $A^{pp}_L(45\,\mathrm{MeV})/A^{pp}_L(13.6\,\mathrm{MeV}) \sim 1.75$ agrees very well with the experimental value $(-1.57\times 10^{-7})/(-0.93\times 10^{-7}) = 1.69$.

2. Photon asymmetry A_γ of radiative thermal neutron ($E \sim 0.025\,\mathrm{eV}$) capture by proton: The asymmetry factor is found to be

$$\begin{aligned} A^{np}_\gamma(\mathrm{th.}) &= (-0.288\,\widetilde{D}_w - 0.174\,D_w - 0.272\,\widetilde{C}^{\pi}_6 + 0.514\,\widetilde{C}^{2\pi}_6)\times 10^{-3}\,, \quad \text{(bare)} \\ &= (-0.187\,\widetilde{D}_w - 0.124\,D_w - 0.270\,\widetilde{C}^{\pi}_6 + 0.300\,\widetilde{C}^{2\pi}_6)\times 10^{-3}\,, \quad \text{(mod)} \end{aligned} \tag{13}$$

and the only relevant Danilov parameter $\bar{\rho}_t$ is found to be

$$\begin{aligned} \bar{\rho}_t &= 3.108\times 10^{-3}\,(\widetilde{D}_w + 0.604\,D_w - 1.771\,\widetilde{C}^{2\pi}_6)\,, \quad \text{(bare)} \\ &= 2.003\times 10^{-3}(\widetilde{D}^{'}_w + 0.664\,D^{'}_w - 1.586\,\widetilde{C}^{2\pi}_6)\,. \quad \text{(mod)} \end{aligned} \tag{14}$$

Recasting the asymmetry factor using the Danilov parameter gives

$$A_\gamma^{np} = -0.093\,\bar{\rho}_t - 0.106\,h_\pi^1 + (-0.003\,\widetilde{C}_6^{2\pi}) \times 10^{-3}\,, \qquad \text{(bare)}$$
$$= -0.093\,\bar{\rho}_t - 0.105\,h_\pi^1 + (-0.004\,\widetilde{C}_6^{2\pi}) \times 10^{-3}\,. \qquad \text{(mod)} \qquad (15)$$

One again sees how the Danilov parameter $\bar{\rho}_t$ along with h_π^1 can help to express $A_\gamma^{np}(\text{th.})$ in a model independent way, with almost no correction. A nontrivial point about this process is that it actually involves the deuteron–$^3P_1$ transition (a bound-free transition), and it comes with some surprise that this amplitude tracks with $^3S_1$–$^3P_1$ (a free-free transition) very well.

4 Conclusion

We here demonstrated that by using the Danilov parameters, low energy parity-violating observables can be cast in a model independent way. This has the great virtue that calculations of different setup, either phenomenological or effective-field-theory-like, can be compared and checked. Such analyses will be valuable to identify the candidate experiments that could best disentangle the nature of parity-violating nuclear interaction at low energy.

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