Noname manuscript No.
(will be inserted by the editor)

The muon magnetic moment and new physics

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1 Introduction

The LHC era has had its first spectacular success with the discovery of a new particle compatible with a Standard Model (SM) Higgs boson. The LHC promises great progress in understanding the nature of electroweak symmetry breaking (EWSB). Additional, non-LHC observables are nevertheless important, for they can provide complementary information on EWSB, and they can unravel the existence of physics beyond the SM invisible at the LHC and possibly unrelated to EWSB.

The muon magnetic moment a_μ has a special role because it is sensitive to a large class of models related and unrelated to EWSB and because it combines several properties in a unique way: it is a flavour- and CP-conserving, chirality-flipping and loop-induced quantity. In contrast, many high-energy collider observables at the LHC and a future linear collider are chirality-conserving, and many other low-energy precision observables such as electric dipole moments or processes such as $\mu \to e\gamma$ are CP- or flavour-violating. These properties might be the reason why there is a significant deviation between the experimental and the SM value of a_μ,

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (28.7 \pm 8.0) \times 10^{10}, \tag{1}$$

while there is no significant deviation in electroweak precision observables and searches for electric dipole moments and charged lepton-flavour violation have been unsuccesful in finding a non-zero result.

In these proceedings we will first briefly review the current status and future prospects of a_μ^{exp} and a_μ^{SM} and we will explain the general structure and model-dependence of contributions from new physics. Then we will discuss concrete

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examples of contributions from and constraints on new physics models, with particular emphasis on the complementarity to recent LHC results.

2 Current status and future prospects

The uncertainty quoted in Eq. (1) is the quadratic sum of various experimental and theoretical errors. The experimental value of the muon magnetic moment has been determined by a series of measurements at BNL [1] with a final statistical and systematic uncertainty of $(\pm 5.4_{\mathrm{stat}} \pm 3.3_{\mathrm{syst}}) \times 10^{-10}$, which is dominated by statistics. The importance of this result has motivated two new experiments. One is already under construction at Fermilab [**?**], using the same technique as used at BNL, where high-energy muons at the "magic relativistic γ" are used, for which electric focusing fields in the ring do not perturb the muon spin precession. The second is planned at J-PARC [**?**], using ultra-cold muons with smaller γ but no electric focusing field. Both of these complementary experiments aim to reduce the statistical uncertainty by more than a factor 4.

The precision of the SM theory prediction is currently even higher than the experimental one. The remaining theory error is dominated by the hadronic vacuum polarization (HVP) contributions. These can be related to the cross section for $e^+e^- \to$ hadrons, and the increasingly precise experimental data for this cross section lead to consistent recent evaluations by several groups [2–4] (for recent overviews see [4,5]). The error used in Eq. (1) is taken from Ref. [2] and is $\pm 4.2 \times 10^{-10}$. Earlier discrepancies between these e^+e^--based results and alternative ones using data from τ-decays have been dramatically reduced [2,6,7].

A subdominant part of the SM theory error is due to the hadronic light-by-light scattering (HLbL) contributions. Here progress is very difficult since hadronic dynamics is relevant in kinematical regimes where neither perturbation theory nor established low-energy effective theories are valid. In spite of using different approaches, the results of various groups agree within the quoted errors, see in particular Refs. [4,8]. The result quoted in Eq. (1) is based on the evaluations of Refs. [2,8], and has the theory error $\pm 2.6 \times 10^{-10}$.

3 New physics contributions in general

General contributions from new physics to a_μ are best understood by using a relation between a_μ and m_μ, the muon mass. Both a_μ and m_μ correspond to quantum field operators which flip chirality, i.e. convert a left-handed into a right-handed muon. For this reason, contributions of new physics at some scale Λ to both quantities, a_μ(N.P.) and δm_μ(N.P.), are linked as

$$a_\mu(\text{N.P.}) = \mathcal{O}(1) \times \left(\frac{m_\mu}{\Lambda}\right)^2 \times \left(\frac{\delta m_\mu(\text{N.P.})}{m_\mu}\right). \tag{2}$$

As discussed in [9], this relation is model-independent, but the *value* of the constant $C = \delta m_\mu(\mathrm{N.P.})/m_\mu$ is highly model-dependent. It is important that the $\mathcal{O}(1)$ factors do not contain any coupling constants of $1/16\pi^2$ factors — those are contained in the constant C. A first consequence of this relation is that new physics can explain the currently observed deviation (1) only if Λ is at the TeV scale or smaller (assuming no fine-tuning in the muon mass, $|C| < 1$).

Second, the relation illustrates how widely different contributions to a_μ are possible.

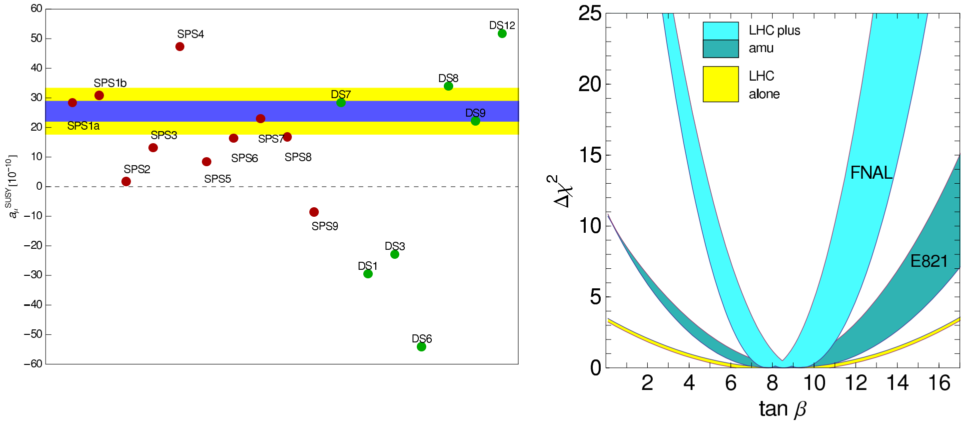
- For models with new weakly interacting particles (e.g. Z', W', little Higgs or universal extra dimension models [10,11]) one typically obtains perturbative contributions to the muon mass $C = \mathcal{O}(\alpha/4\pi)$. Hence for weak-scale masses these models predict very small contributions to a_μ and might be challenged by the future more precise a_μ measurement. Models of this kind can only explain a significant contribution to a_μ if the new particles interact with muons but are otherwise hidden from searches. An example is the model with a new gauge boson associated to a gauged lepton number $L_\mu - L_\tau$ [12], where a gauge boson mass of $\mathcal{O}(100\ \mathrm{GeV})$ is viable.
- For supersymmetric (SUSY) models one obtains an additional factor $\tan\beta$, the ratio of the two Higgs vacuum expectation values [13]. A numerical approximation for the SUSY contributions is given by

  $$a_\mu^{\mathrm{SUSY}} \approx 13 \times 10^{-10} \left(\frac{100\,\mathrm{GeV}}{M_{\mathrm{SUSY}}}\right)^2 \tan\beta\ \mathrm{sign}(\mu), \tag{3}$$

  where M_{SUSY} denotes the common superpartner mass scale and μ the Higgsino mass parameter. It agrees with the generic result (2) for $C = \mathcal{O}(\tan\beta\times\alpha/4\pi)$ and is exactly valid if all SUSY masses are equal to M_{SUSY}. The formula shows that the observed deviation could be explained e.g. for relevant SUSY masses of roughly 200 GeV and $\tan\beta \sim 10$ or SUSY masses of 500 GeV and $\tan\beta \sim 50$. However, the SUSY prediction for a_μ depends strongly on the detailed scenario, and if SUSY exists a_μ will help to measure the SUSY parameters.
- Models with large $C \simeq 1$ are of interest since there the muon mass is essentially given by new physics loop effects. Some examples of such radiative muon mass generation models are given in [9]. For examples within SUSY see e.g. [14,15]. In such models a_μ can be large even for particle masses at the TeV scale.

4 Supersymmetry and a_μ

As discussed above, supersymmetry with moderate to large $\tan\beta$ and masses in the 200–500 GeV range can easily explain the currently observed deviation (1). We now discuss the supersymmetry contributions in more detail. At the one-loop level, the diagrams involve either charginos and sneutrinos, or neutralinos



(a) (b)

Fig. 1 (a) SUSY contributions to a_μ for the SPS benchmark points (red), and for the "degenerate solutions" from Ref. [22]. (b) Possible future $\tan\beta$ determination assuming that a slightly modified MSSM point SPS1a (see text) is realized. The bands show the $\Delta\chi^2$ parabolas from LHC-data alone (yellow) [24], including the a_μ with current precision (dark blue) and with prospective precision (light blue). The width of the blue curves results from the expected LHC-uncertainty of the parameters (mainly smuon and chargino masses) [24].

and smuons. The relevant parameters are thus the soft mass parameters for the 2nd generation sleptons, the gaugino masses M_2, M_1, and the Higgsino mass parameter μ. Strongly interacting particles, squarks and gluinos, and their masses are irrelevant.

If all the relevant mass parameters are equal, the approximation (3) is valid, and the dominant contribution is from the chargino–sneutrino diagrams. If μ is very large, the bino-like neutralino contribution is approximately linear in μ and can dominate. If there is a large mass splitting between the left- and right-handed smuon, even the sign can be opposite to Eq. (3), see the discussions in [16,13].

At the two-loop level various contributions are possible with potentially relevant impact. Photonic two-loop corrections always decrease the one-loop result slightly [19], and two-loop diagrams with either a sfermion (stop, sbottom, . . .) loop or a chargino loop can be large even if the one-loop contributions are suppressed [17,18]. For large $\tan\beta$, two-loop $(\tan\beta)^2$-enhanced effects become important [20].

Within supersymmetry the contributions to a_μ are therefore very model-dependent, and a_μ places important constraints on how supersymmetry can be realized. Fig. 1 illustrates this. The left plot shows the values for the so-called SPS benchmark points [21]. These span a wide range and can be positive or negative, due to the factor sign(μ) in Eq. 3. The discriminating power of the current (yellow band) and an improved (blue band) measurement is evident from Fig. 1(a). One might think that if SUSY exists, the LHC-experiments will find it and measure its parameters. The green points illustrate that this is not the case. They correspond to "degenerate solutions" of Ref. [22] —

different SUSY parameter points which cannot be distinguished at the LHC alone (see also Ref. [23] for the LHC inverse problem). They have very different a_μ predictions, in particular different signs for μ, and hence a_μ can resolve such LHC degeneracies.

The right plot of Fig. 1 illustrates that the SUSY parameter $\tan\beta$ can be measured more precisely by combining LHC-data with a_μ. It is based on the assumption that SUSY is realized, found at the LHC and the origin of the observed a_μ deviation (1). To fix an example, we use a slightly modified SPS1a benchmark point with $\tan\beta$ scaled down to $\tan\beta = 8.5$ such that $a_\mu^{\rm SUSY}$ is equal to an assumed deviation $\varDelta a_\mu = 255 \times 10^{-11}$.[1] Ref. [24] has shown that then mass measurements at the LHC alone are sufficient to determine $\tan\beta$ to a precision of ± 4.5 only. The corresponding $\varDelta\chi^2$ parabola is shown in yellow in the plot. In such a situation one can study the SUSY prediction for a_μ as a function of $\tan\beta$ (all other parameters are known from the global fit to LHC data) and compare it to the measured value, in particular after an improved measurement. The plot compares the LHC $\varDelta\chi^2$ parabola with the ones obtained from including a_μ, $\varDelta\chi^2 = [(a_\mu^{\rm SUSY}(\tan\beta) - \varDelta a_\mu)/\delta a_\mu]^2$ with the errors $\delta a_\mu = 80 \times 10^{-11}$ (dark blue) and 34×10^{-11} (light blue). As can be seen from the Figure, using today's precision for a_μ would already improve the determination of $\tan\beta$, but the improvement will be even more impressive after a future more precise a_μ measurement.

To date, the LHC has not found indications for supersymmetric particles, so a tension is rising between four pieces of experimental information and theoretical prejudice:

- If supersymmetry is the origin of the deviation in a_μ, the supersymmetric particles cannot be too heavy, in particular the smuons and charginos/neutralinos.
- The negative results of the LHC searches for supersymmetric particles imply lower limits, in particular on squark and gluino masses.
- The constraint that a SM-like Higgs boson mass is around 126 GeV requires either very large loop corrections from large logarithms or non-minimal tree-level contributions from additional non-minimal particle content.
- The requirement of small fine-tuning between supersymmetry-breaking parameters and the Z-boson mass prefers certain particles, in particular stops, gluinos and Higgsinos to be rather light.

Apart from fine-tuning, it is of course possible to accommodate all experimental data in the minimal supersymmetric standard model, which has enough free parameters [25]. However, the Constrained MSSM (CMSSM) cannot simultaneously describe all data [26], while slight extensions such as the Non-universal Higgs mass model (NUHM) or a model with gauge-mediated supersymmetry breaking and extra vector-like matter [27] are marginally consistent with all data.

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[1] The actual SPS1a point is ruled out by LHC, however for our purposes only the weakly interacting particles are relevant, and these are not excluded. The following conclusions are neither very sensitive to the actual $\tan\beta$ value nor to the actual value of the deviation $\varDelta a_\mu$.

Models inspired by naturalness, where the spectrum is such that fine-tuning is minimized while squarks and gluinos evade LHC bounds, can explain the observed Higgs boson mass but completely fail to explain a_μ [28].

An interesting possibility is provided by supersymmetric scenarios that realize radiative generation of the muon mass. Since the muon mass at tree level is given by the product of a Yukawa coupling and the vacuum expectation value of the Higgs doublet H_d, there are two kinds of such scenarios. First, one can postulate that the muon Yukawa coupling is zero but chiral invariance is broken by soft supersymmetry-breaking A-terms. Then, the moun mass, and $a_\mu^{\rm SUSY}$, arises at the one-loop level and $a_\mu^{\rm SUSY}$ can be large even for TeV-scale smuon masses [14,15]. Second, one can postulate that the vacuum expectation value $\langle H_d \rangle$ is very small or zero [29,30]. Then, the muon mass and $a_\mu^{\rm SUSY}$ arise at the one-loop level from loop-induced couplings to the other Higgs doublet.

5 Conclusions

In spite of tremendous progress at the LHC, a_μ is still a very important constraint on physics beyond the SM. The increasing difficulty to explain the a_μ deviation and satisfy LHC bounds and Higgs mass constraints highlights this. It is conceivable that the observed deviation (1) is real but not due to new physics at the electroweak scale, but e.g. due to new very light particles, as suggested e.g. in [31]. In such a case, the resolution of the EWSB puzzle would be the task of the LHC and a possible future linear collider, while the new light particles could be probed by dedicated low-energy precision experiments such as the next generation a_μ measurements.

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