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T-odd Momentum Correlation in Radiative β Decay

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Abstract The triple-product correlations observable in ordinary neutron or nuclear beta-decay are all naively T violating and can connect, through an assumption of CPT invariance, to constraints on sources of CP violation beyond the Standard Model. They are also spin dependent. In this context the study of radiative beta-decay opens a new possibility, in that a triple-product correlation can be constructed from momenta alone. Consequently its measurement would constrain new spin-independent sources of CP violation. We will describe these in light of the size of the triple momentum correlation in the decay rate arising from electromagnetic final-state interactions in the Standard Model. Our expression for the corresponding T-odd asymmetry is exact in $\mathcal{O}(\alpha)$ up to terms of recoil order, and we evaluate it numerically under various kinematic conditions. We consider the pattern of the asymmetries in nuclear β decays and show that the asymmetry can be suppressed in particular cases, facilitating searches for new sources of CP violation in such processes.

Keywords Discrete symmetries · Weak decays · Neutrons

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1 Introduction

CP violation in flavor-changing processes is described in the Standard Model (SM) by a single-phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix, so that we test the SM mechanism through the comparison of different CP-violating observables. If we parametrize the CKM matrix in terms of a generalization of the Wolfenstein parametrization [1], valid to all orders in $\lambda \equiv V_{us}$ [2], we test the SM by ascertaining whether all observed CP-violating effects determine the same value of $(\bar{\rho}, \bar{\eta})$, which is the apex of the so-called unitarity triangle formed by the relationship $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$, as fixed by the unitarity of the CKM matrix. Noting updates from the summer of 2012, these tests are in complete agreement, where we refer to Ref. [3] for details and concrete illustrations. The first B-factory era, with key input from the Tevatron, established that both CP and flavor violation in flavor-changing processes is dominated by the CKM mechanism [4]. Nevertheless, the CKM mechanism cannot explain the value of the baryon asymmetry of the universe determined independently from the primordial ^{2}D abundance, as well as from studies of the power spectrum of the cosmic microwave background radiation. Thus we have not solved the problem the B-factories were supposed to solve: we have not yet understood the origin of the missing antimatter. To resolve this, we can push the empirical tests of the relationships predicated by the CKM mechanism to higher precision, as planned, e.g., through studies at future super-flavor factories [5]. We can also continue to make "null" tests, i.e., to measure quantities which may not be strictly zero, but which are inaccessibly small if calculated in the SM. Permanent electric dipole moments (EDMs) of nondegenerate systems, which violate T and P, are specific examples. So-called T-odd decay correlations, which can only be motion-reversal odd [6], serve as other examples of null tests; these observables, through CPT invariance, also probe sources of CP violation beyond the SM, though their appearance can also be mimicked by CP-conserving final-state interactions within the SM. In ordinary β-decay the possible T-odd correlations involve spin as well, and they can be linked, in specific new physics scenarios, to EDM limits [7], which are much more stringent, as studied recently in the context of the D term [8]. In this contribution we consider the study of a T-odd, P-odd correlation in *radiative* β decay; the appearance of an additional particle in the final state makes it possible to form a decay correlation from momenta alone, $\mathbf{p}_\gamma \cdot (\mathbf{p}_e \times \mathbf{p}_\nu)$, so that the particle spin does not enter. Note that $\mathbf{p}_\nu$ can be determined through momentum conservation if the recoil of the proton, or daughter nucleus is measured as well. With such an observable one can probe new, spin-independent sources of CP violation. In what follows, we discuss what physics can give rise to a triple momentum correlation, its possible size, and its connection to broader problems of interest, before detailing the computation of the electromagnetic final-state interactions in the SM which can mimic the effect. Barring other empirical constraints, it is the uncertainty in the latter which limits the size of the new physics window offered through this observable.

2 Anomalous interactions at low energies

Radiative corrections in gauge theories need not respect all the symmetries present in a massless Dirac theory; in particular, the axial vector current is no longer conserved and becomes anomalous. This physics is also manifest in effective theories of QCD at low energies, in which the pseudoscalar mesons, interpreted as the Nambu-Goldstone bosons of a spontaneously broken chiral symmetry, are the natural degrees of freedom. In this context the nonconservation of the axial current is captured through the inclusion of the Wess-Zumino-Witten (WZW) term, so that the chiral Lagrangian can then describe processes such as $K\bar{K} \to 3\pi$ and $\pi^0 \to \gamma\gamma$. If we study the gauge invariance of the WZW term in vector-like gauge theories such as QED, then the vector current is conserved [9]. Harvey, Hill, and Hill have observed, however, that the gauging of this term under the full electroweak gauge group $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ makes the baryon vector current anomalous and gives rise to "Chern-Simons" contact interactions, containing $\varepsilon^{\mu\nu\rho\sigma}$, at low energy [10,11]. Such structures can also be found in a chiral effective theory in terms of nucleons, pions, and a complete set of electroweak gauge fields; the requisite terms appear at $\mathrm{N}^2\mathrm{LO}$ in the chiral expansion [12]. That is,

$$\mathcal{L}^{(3)} = ... + \frac{c_5}{M^2} \bar{N} i \varepsilon^{\mu\nu\rho\sigma} \gamma_\sigma \tau^a \mathrm{Tr}(\tau^a \{\tilde{A}_\mu, [i\tilde{D}_\nu, i\tilde{D}_\rho]\}) N + ... , \qquad (1)$$

where we report the charged-current term only and note $\tilde{A}_\mu$ is a SU(2) matrix of axial-vector gauge fields, $\tilde{D}_\mu$ is a covariant derivative which contains a SU(2) matrix of vector gauge fields, N is a nucleon doublet, and M is nominally the nucleon mass. We refer to Ref. [12] for all details. Since the experimental value of M_W is large, we integrate out W^+ to find for neutron decay

$$-\frac{4c_5}{M^2} \frac{eG_F}{\sqrt{2}} \varepsilon^{\sigma\mu\nu\rho} \bar{p} \gamma_\sigma n \bar{\psi}_{eL} \gamma_\mu \psi_{\nu_e L} F_{\nu\rho} , \qquad (2)$$

where $2\psi_{eL} = (1 - \gamma_5)\psi_e$ and $F_{\nu\rho}$ is the electromagnetic field strength tensor. Thus the baryon weak vector current can mediate parity violation on its own, through the interference of the leading vector amplitude mediated by

$$\frac{G_F}{\sqrt{2}} g_V \bar{p} \gamma^\mu n \bar{\psi}_e \gamma_\mu (1 - \gamma_5) \psi_{\nu_e} \qquad (3)$$

with c_5 and its neutral current analogue. The T-odd momentum correlation probes the imaginary part of $g_V c_5$ interference. Existing constraints on c_5 are poor and come directly only from the measured branching ratio in neutron radiative β decay [13]. The best constraint on Im g_V comes from the recent D term measurement [14], to yield Im $g_V < 7 \times 10^{-4}$ at 68% CL. Thus a first limit on $\mathrm{Im}(g_V c_5)$ would limit $\mathrm{Im}(c_5)$. The coefficient c_5 need not be real in theories beyond the SM; contributions from hidden-sector Dirac fermions, e.g., via a primitive triangle graph, can mediate a contribution to $\mathrm{Im}(c_5)$. We suppose any hidden-sector fermions to be weakly coupled to known matter; nevertheless, the T-odd observable we consider probes CP phases associated with couplings to hidden-sector Dirac fermions. The notion that new matter could exist with QCD-like interactions is of long standing [15] and has recently been revisited [16,17]. We speculate that such particles could be possible constituents of dark-matter candidates in asymmetric dark matter models. In such models dark matter can possess a cosmic particle-antiparticle asymmetry just as baryons do, and the origin of dark matter can be connected to the cosmic baryon asymmetry (BAU), so that the dark-matter relic density would be set by the BAU and not by thermal freeze-out. The details of dark-matter and BAU formation, including their temporal sequence, are model dependent. In the current context, we find models in which a dark-matter asymmetry is formed and then transferred to the baryon sector particularly evocative [18]. Perhaps the hidden sector helps mediate baryogenesis. In this regard the appearance of new hidden-sector Dirac fermions and their complex phases could be helpful. We now turn to concrete prospects for their discovery in radiative β decay.

3 T-odd Correlation in Radiative β Decay

3.1 Probing Physics BSM

In $n(p_n) \to p(p_p)+e^-(l_e)+\overline{\nu}_e(l_\nu)+\gamma(k)$ decay the interference of the c_5 contribution with the leading $V-A$ terms [19,20] yields the following contribution to the decay rate

$$|\mathcal{M}|^2_{c_5} = 512 M^2 \frac{e^2 G_F^2}{2} \mathrm{Im}\,(c_5\, g_V) \frac{E_e}{l_e \cdot k} (\mathbf{l}_e \times \mathbf{k}) \cdot \mathbf{l}_\nu + \dots \,, \tag{4}$$

where we neglect corrections of radiative and recoil order. The pseudo-T-odd interference term is finite as $\omega \equiv k^0 \to 0$, so that its appearance is compatible with Low's theorem [21]. Defining $\xi \equiv (\mathbf{l}_e \times \mathbf{k}) \cdot \mathbf{l}_\nu$, we partition phase space into regions of definite sign, so that we form an asymmetry:

$$\mathcal{A}(\omega_{\min}) \equiv \frac{\Gamma_+(\omega_{\min}) - \Gamma_-(\omega_{\min})}{\Gamma_+(\omega_{\min}) + \Gamma_-(\omega_{\min})} \,, \tag{5}$$

where $\Gamma_\pm$ contains an integral of the spin-averaged $|\mathcal{M}|^2$ over the region of phase space with $\xi \gtrless 0$, respectively, neglecting corrections of recoil order. Note that $\omega^{\min}$ is the minimum detectable photon energy, where we refer to Ref. [20] for all details. Neglecting terms of $\mathcal{O}(c_5^2)$, and fixing $\omega^{\min}$, we find a branching ratio of $\mathrm{Br}(\omega^{\min} = 0.01\,\mathrm{MeV}) = 3.46 \times 10^{-3}$ with [22]

$$\mathcal{A}(\omega^{\min} = 0.01\,\mathrm{MeV}) = -5.65 \times 10^{-3} \mathrm{Im}\, \frac{c_5 g_V}{M^2} (\mathrm{MeV}^{-2}) \,, \tag{6}$$

where the asymmetry is dimensionless. For a larger $\omega^{\min}$, $\mathrm{Br}(\omega^{\min} = 0.3\,\mathrm{MeV}) = 8.62 \times 10^{-5}$ with

$$\mathcal{A}(\omega^{\min} = 0.3\,\mathrm{MeV}) = -5.36 \times 10^{-2} \mathrm{Im}\, \frac{c_5 g_V}{M^2} (\mathrm{MeV}^{-2}) \,. \tag{7}$$

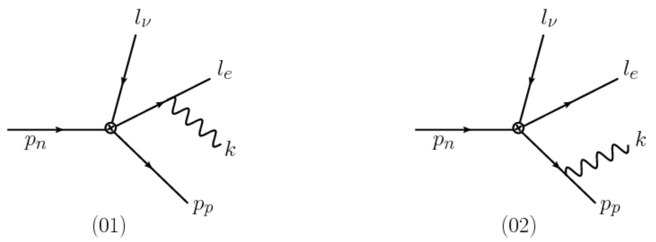

Fig. 1 Leading contributions to neutron radiative β decay; $\otimes$ denotes the effective weak vertex.

The experimental figure of merit is determined by $\mathcal{A}^2\mathrm{Br}$, so that the larger value of ω^{min} would be more efficacious. We note $|\mathrm{Im}(c_5/M^2)| < 11.5\,\mathrm{MeV}^{-2}$ at 68% CL [22] from the most recent measurement of the branching ratio for neutron radiative β decay [13].

3.2 SM Background

As we have noted, decay correlations can be motion-reversal-odd only, so that CP-conserving final-state interactions in the SM can induce T-odd decay correlations [23,24]. We note that a triple momentum correlation has been previously studied in $K^+ \to \pi^0 l^+ \nu_l \gamma$ decay [25,26]. There both electromagnetic and strong (π mediated) radiative corrections can mimic the T-odd effect, but the electromagnetic final-state interactions effects are orders of magnitude larger [27]. The small energy release associated with neutron and nuclear radiative β-decay imply that only electromagnetic radiative corrections can mimic the T-odd effect. The induced T-odd effects in this case have never been studied before, and we now describe our calculation [20,28].

A T-odd triple momentum correlation, i.e., a correlation linear in ξ, in the neutron radiative decay rate from SM physics can arise from the interference of the tree-level amplitude $\mathcal{M}_{\mathrm{tree}}$, as in Fig. 1, with the anti-Hermitian parts of the one-loop corrections to it, as in Fig. 2. Working in $\mathcal{O}(\alpha)$ and in leading recoil order, we have

$$|\mathcal{M}|^2 = |\mathcal{M}_{\mathrm{tree}}|^2 + \mathcal{M}_{\mathrm{tree}} \cdot \mathcal{M}^*_{\mathrm{loop}} + \mathcal{M}_{\mathrm{loop}} \cdot \mathcal{M}^*_{\mathrm{tree}} + \mathcal{O}(\alpha^2)\,, \tag{8}$$

where the T-odd contributions to $|\mathcal{M}|^2$ are given by

$$\overline{|\mathcal{M}|^2}_{\mathrm{T-odd}} \equiv \frac{1}{2}\sum_{\mathrm{spins}} |\mathcal{M}|^2_{\mathrm{T-odd}} = \frac{1}{2}\sum_{\mathrm{spins}} (2\mathrm{Re}(\mathcal{M}_{\mathrm{tree}} i \mathrm{Im}\mathcal{M}^*_{\mathrm{loop}}))\,. \tag{9}$$

To obtain $\mathrm{Im}\mathcal{M}_{\mathrm{loop}}$ we perform “Cutkosky cuts” [29], which means we simultaneously put intermediate particles in the loops on their mass shells in all physically allowed ways and then perform the relevant intermediate phase space integrals and spin sums. We have

$$\mathrm{Im}(\mathcal{M}_{\mathrm{loop}}) = \frac{1}{8\pi^2}\sum_n \int d\rho_n \sum_{s_n} \mathcal{M}_{fn}\mathcal{M}_{ni}\,, \tag{10}$$

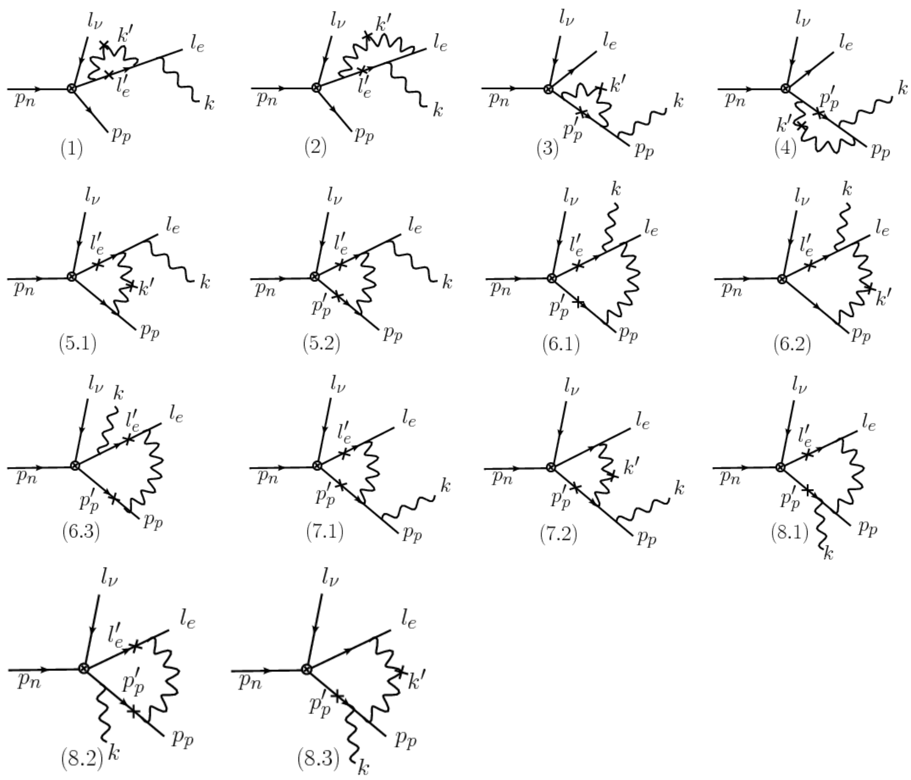

Fig. 2 All two-particle cut contributions to neutron radiative β decay in $\mathcal{O}(\alpha)$ up to corrections of recoil order. A "×" means that the intermediate particle has been put on its mass shell; two such symbols define the Cutkosky cut.

where $\sum_n$ refers to the summation over all possible cuts of the one-loop diagrams and $\int d\rho_n$ and $\sum_{s_n}$ refer to the intermediate phase space integration and spin sums, respectively, for a cut which yields state n. The 14 physical cut diagrams which appear are illustrated in Fig. 2. In doing the calculation, we find many cancellations: the interference of certain two-particle-cut diagrams with the tree-level graphs can sum to zero up to recoil-order terms. This is true, e.g., of the two-particle cuts which yield $\gamma - p$ scattering. Diagrams which yield $e - p$ scattering, 6.3 and 8.2, are individually infrared divergent and can be regulated by inserting a fictitious photon mass m_γ. The infrared divergence cancels, however, in the net contribution to the asymmetry, so that we can safely set m_γ to zero in the final result. Explicit computation reveals that contributions to the asymmetry from diagrams which yield $e - p$ and radiative $e - p$ scattering dominate the final numerical results. We refer the reader to Ref. [20] for all details. For neutron radiative β decay, for a $\gamma - e$ opening angle $\theta_{e\gamma}$ such that $\cos\theta_{e\gamma} \in [-0.9, 0.9]$, we compute the asymmetry induced by SM final-state interactions as a function of $\omega_{\min}$ and display the results in Table 1. The SM asymmetries are much smaller than those permitted by current

Table 1 $A_\xi^{\rm SM}$ for various decays

$\omega_{\rm min}$(MeV)	$A_\xi{}^{\rm SM}$ [n]	$A_\xi{}^{\rm SM}$ [^{19}Ne]	$A_\xi{}^{\rm SM}$ [^{6}He]
0.01	1.76×10^{-5}	-2.12×10^{-5}	7.00×10^{-5}
0.05	3.86×10^{-5}	-3.52×10^{-5}	1.14×10^{-4}
0.1	6.07×10^{-5}	-4.74×10^{-5}	1.52×10^{-4}
0.2	9.94×10^{-5}	-6.74×10^{-5}	2.13×10^{-4}
0.3	1.31×10^{-4}	-8.42×10^{-5}	2.63×10^{-4}
0.4	1.54×10^{-4}	-9.89×10^{-5}	3.07×10^{-4}
0.5	1.70×10^{-4}	-1.12×10^{-4}	3.45×10^{-4}
0.6	1.81×10^{-4}	-1.23×10^{-4}	3.79×10^{-4}
0.7	1.89×10^{-4}	-1.32×10^{-4}	4.07×10^{-4}

empirical constraints on c_5; they are accurate up to corrections of recoil order, so that $A_\xi{}^{\rm SM}$ should be accurate to our quoted precision.

Nuclear radiative β decay can also be studied [20, 28]. This possibility is of interest because the T-odd asymmetry driven by c_5 grows with the energy released in the decay. It is also possible to realize this without making $A_\xi^{\rm SM}$ larger because the observed quenching of the Gamow-Teller strength in nuclear decays can suppress $\mathcal{A}_\xi^{\rm SM}$ considerably [30]. The computation of the T-odd correlation induced by SM final-state interactions proceeds in a manner nearly identical to that of the neutron case. Superficially there appear to be many more graphs, but the additional ones all cancel in leading recoil order. We end up with the same sum of QED gauge-invariant combinations we found previously, though certain combinations are multiplied by the charge Z of the daughter. We present $A_\xi^{\rm SM}$ for radiative ^{19}Ne β-decay and ^{6}He β-decay in Table 1. The study of a Gamow-Teller decay, such as that of ^{6}He, for which $A_\xi^{\rm SM}$ is larger, might serve as a useful proof of principle experiment.

4 Summary

Harvey, Hill, and Hill suggest it is possible to study remnants of the baryon vector current anomaly in the Standard Model in low energy interactions. Such terms also appear in theories beyond the SM with $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ electroweak symmetry at low energies – and their couplings can be complex. This possibility can be probed through a triple-product momentum correlation in neutron and nuclear radiative β-decay.

The triple-product momentum correlation is P-odd and pseudo-T-odd but does not involve the nucleon spin; the constraints offered through its study in neutron and nuclear radiative β-decay are complementary but independent from those from EDMs.

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