Noname manuscript No.
(will be inserted by the editor)

Quest for the QCD phase diagram in extreme environments

Kenji Fukushima

Received: date / Accepted: date

Abstract We review the state-of-the-art status of the research on the phase diagram of QCD matter out of quarks and gluons. Our discussions particularly include the extreme environments such as the high temperature, the high baryon density, and the strong magnetic field.

Keywords Quantum Chromodynamics · Phase Diagram · Relativistic Heavy-Ion Collision · Quantum anomaly

1 Introduction

It is one of the biggest challenges to clarify the whole phase diagram of matter out of quarks and gluons as a function of the temperature T, the baryon or quark chemical potential μ_q (see Fig. 1), and other external parameters such as the magnetic field B. This was originally an academic theory question, and is now a question of great pragmatic interest not only for theorists but also for experimentalists. The relativistic heavy-ion collision experiments can create a quark-gluon plasma at sufficiently high T and μ_q of order of Λ_{QCD}, and furthermore, it may be accompanied by a gigantic magnetic field that is again of order of Λ_{QCD} if the heavy ions collide non-centrally.

Quantum Chromodynamics (QCD) is the fundamental theory to describe the interactions of quarks and gluons. The QCD framework has been completely established for long time, but it does not necessarily mean that we already know the answer from the theory. Needless to say, the definition of a theory does not automatically come with the answer. Since QCD is a compli-

K. Fukushima
Department of Physics, Keio University,Kanagawa 223-8522, Japan
Tel.: +81-45-566-1691
Fax: +81-45-566-1672
E-mail: fuku@rk.phys.keio.ac.jp

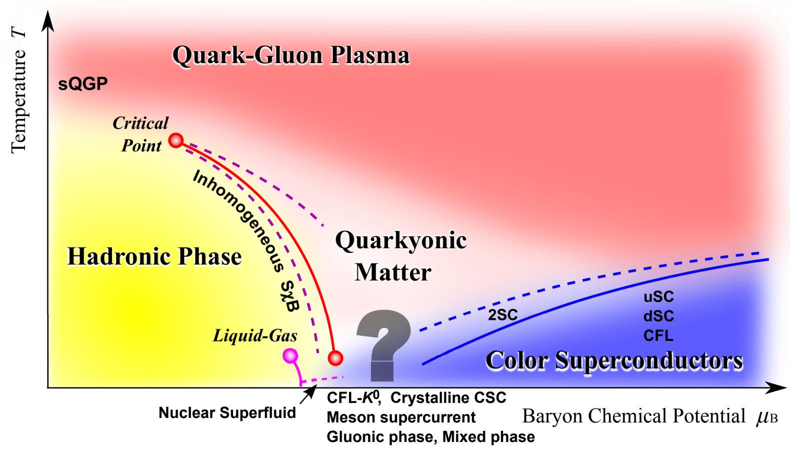

Fig. 1 Conjectured phase diagram taken from Ref. [1]. The solid curve near the edge of the hadronic phase represents the first-order chiral phase transition, which ends at the critical point(s).

cated non-linear theory, it accommodates various aspects of the quantum field theory, and it still poses very intriguing problems even today.

The T-effect is more straightforward than others. The typical momentum of all physical excitations is characterized by T, so that the strong coupling constant α_s runs smaller as T increases. After all, due to the asymptotic freedom, the system turns to a weakly-coupled quark-gluon plasma where color degrees of freedom are released and chiral symmetry is restored. Therefore, the finite-T business already reaches the level of quantitative precision science.

It is increasingly difficult to apply any theoretical approaches to investigate QCD matter at finite baryon density. The Monte-Carlo simulation is possible only for $\mu_q/T \ll 1$ where no interesting phenomenon occurs yet. Because the first-principle calculation based on QCD is unavailable, the second best would be the model study, i.e. the analysis using some effective model that shares the same properties as QCD (for example, global symmetries). There are countless model studies and the results are sometimes not quite consistent with each other. One should be very careful of model artifacts and should try to extract robust essence out from the results.

The so-called QCD critical point might suffer from such potential danger of model artifacts. The first-order phase transition driven by finite-μ_q effect was conjectured within the framework of the chiral effective models such as the Nambu–Jona-Lasinio (NJL) model and the Quark-Meson (QM) model. It is a natural consequence from the first-order phase boundary that an exact second-order phase transition point appears at the terminal of the first-order phase boundary. It is, however, a logical possibility that there is no first-order boundary at all but only smooth crossover everywhere. In contrast to the critical point, inhomogeneous states are energetically favored in such a way that is fairly model-independent.

Deconfinement at high baryon density is a subtle issue. Unlike the finite-T case, not all particles but only quarks are directly modified by μ_q, that is, quark excitations must have a momentum above μ_q, but it is not the case for gluons. Thus, we cannot say that the running coupling constant should be small even when μ_q is infinitely large. To understand the possibility of deconfinement at high density, we should take account of back-reactions from the quark sector to the gluon sector. The gluon properties are influenced by μ_q only through the polarization effects of quarks and the quark one-loop diagram would lead to the screening mass $\sim g\mu_q$ to the longitudinal gluons. Then, one might want to say that the long-ranged force is screened by the quark polarization and the quark confinement is lost. Such kind of argument seems to justify the perturbative calculations for large μ_q. There is, however, no convincing analysis of the deconfinement mechanism along the line of this intuitive argument. In fact, as mentioned, it is only the longitudinal part that would acquire the non-zero screen mass and the transverse gluons are only dynamically screened. Besides, in the Landau gauge for example, it is known that the ghost enhancement in the deep infrared region is responsible for confinement, which has no direct coupling to quarks, either. This implies that the quark confinement may well persist in the high-density medium.

The same subtlety lies in the case with strong B because the B-effect would affect gluons only through the quark polarization diagrams too. The polarization tensor has a different structure from the finite-μ_q situation, and in this case only one of the two transverse (to the B direction) gluons gets massive. In a similar sense to the previous discussions, it is non-trivial whether the quark deconfinement could be induced by B or not. So far, nothing (including the lattice-QCD simulation) hints any tendency to deconfinement due to B. In other words, if we can achieve some theoretical understanding of confinement/deconfinement under strong B, it would help us with understanding the state of matter at high baryon density.

In what follows, we will pick up and look over several topical issues at high T, high μ_q, and strong B, in order.

2 High temperature

The finite-T effects have been successfully investigated in both theoretical and experimental ways. The lattice-QCD simulation provides us with reliable data and we can say with enough confidence that we already know the facts. The quark deconfinement and the chiral symmetry restoration both occur smoothly as a function of T and there is no true phase transition. Therefore, it is rather pointless to care too much about the precise value of the critical temperature T_c. Sometimes one can define the so-called pseudo-critical temperature as a pragmatic measure to refer to the location of crossover. The pseudo-critical temperature is, however, convention dependent and could be easily changed

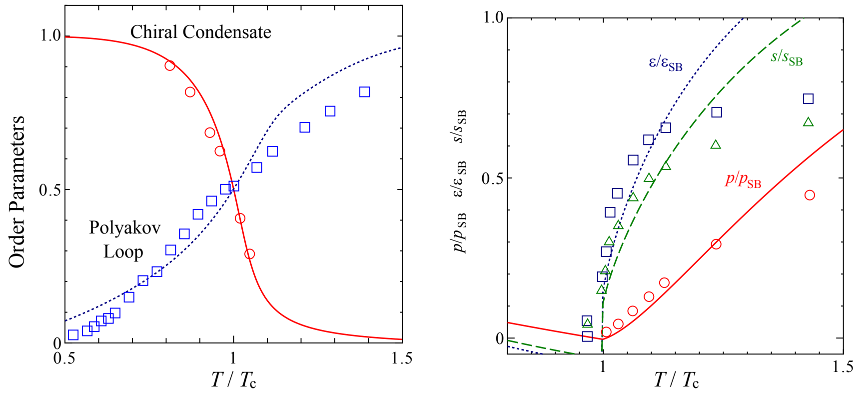

Fig. 2 (Left) Behavior of the order parameters for quark deconfinement (Polyakov loop) and chiral symmetry restoration (chiral condensate) as a function of the normalized temperature. The potential to determine the order parameters is constructed by means of the gluon and the ghost propagators. (Right) Behavior of the thermodynamic quantities corresponding to the calculated order parameters. Figures taken from Ref. [2].

by $\sim 20\%$ with a different prescription. Some people mention on whether the deconfinement and the chiral symmetry restoration should occur simultaneously or there should be any discrepancy in respective T_c's. Such a question is not well-defined, though. A fair statement should be that the chiral phase transition is continuous but shows nearly critical behavior, and at the same time, the quark deconfinement takes place smoothly and simultaneously.

One may think that there is nothing more to investigate theoretically, for the lattice-QCD results are already final. Knowing the facts is not sufficient to know the underlying physics. One promising way to make use of the gluon and ghost propagators in a certain gauge [2]. Figure 2 shows the calculation results using the full gluon and ghost propagators. One may still wonder that the full propagators are also merely the facts. We would object this. The propagators are far more fundamental than bulk quantities like the pressure, the entropy density, and the internal energy density. The most important is that the usage of the propagators to construct the thermodynamics would open an opportunity to implement the screening effect due to the quark polarization in a diagrammatic way.

3 High baryon density

Our emphasis is that the quark polarization and the associated screening effect make the finite-μ_q study complicated. Instead of solving this cumbersome problem, one can go to another direction – take a special limit in which the screening effect is just negligible. Such a strategy is of no use to give any concrete answer to the real world but enable us to give some definite and model-

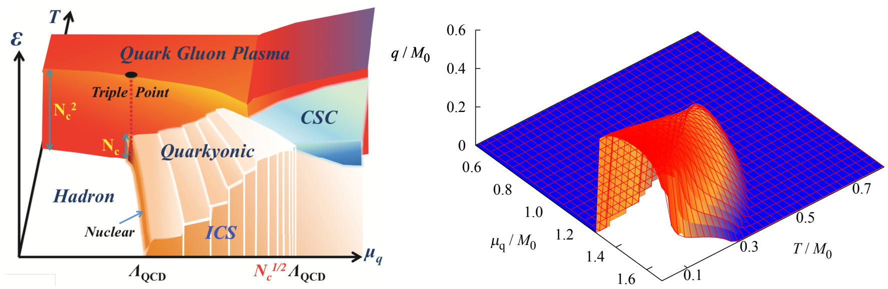

Fig. 3 (Left) Conjectured phase diagram including the quarkyonic chiral spirals and the successive phase transitions associated with the number of patches on the Fermi surface. The figure is taken from Ref. [4]. (Right) Island-like shape of the inhomogeneous state seen in the behavior of the wave-number that characterizes the spatial modulation. The figure is taken from Ref. [5].

independent answer. The information on the real world may be accessed as a result of the extrapolation from the limiting world.

It is known that the quark loop effects are suppressed by $1/N_c$ with N_c being the number of colors as compared to the gluon loop contributions. This is simply because there are $N_c^2 - 1$ gluons, while the number of quarks is N_c in the color fundamental representation. In the large-N_c limit, therefore, we can completely forget about the quark polarization effect, so that the quark confinement should not be lost obviously! This is a good news, and at the same time, a bad news.

As long as confinement persists, we have to deal with colorless objects such as mesons and baryons, and quasi-quarks are not the correct degrees of freedom. To make the situation even worse, in the large-N_c limit, baryons interact very strongly. Therefore, the price to pay for dropping the quark polarization is to encounter composite confined objects with strong interactions among them. This sounds like the law of conservation of the difficulty.

An interesting observation was made in Ref. [3]. They performed the order estimate of the pressure P of this confined baryonic matter and found $P \sim O(N_c)$ because of the interaction energy between baryons. [Note that there are N_c ways to exchange a quark between two baryons, leading to the interaction energy of order of N_c. This argument can be extended to multi-baryon interactions.] It is interesting that $P \sim O(N_c)$ is also implied from a weakly-interacting quark gas. Then, based on this observation, it would be conceivable to conjecture some sort of correspondence between strongly-interaction baryonic matter and weakly-interacting quark matter. Roughly speaking, this is a sort of duality and such a dual state is called the quarkyonic (= quark + baryonic) matter.

What is the physics implication of the quarkyonic matter to the QCD phase diagram? Figure 3 shows one example. At high density the quarkyonic matter is realized, and it appears accompanied by inhomogeneous chiral condensates. Such inhomogeneity is concluded from the (1+1)-dimensional nature of quarks sitting on the Fermi surface. On the Fermi surface the momentum of quarks is of order of μ_q. Supposing that $p_z \sim \mu_q$, the transverse momenta p_x and p_y are of order of $\Lambda_{\text{QCD}} \ll \mu_q$. Note that p_z could be arbitrary since there is no preferred direction. Thus, we can imagine that a patch of the size of Λ_{QCD} is dominated by the (1+1)-dimensional dynamics and the whole Fermi surface should be covered by those patches. The surface are is proportional to μ_q^2, and the patch size is of order of Λ_{QCD}^2, so that the number of the patches should increase with increasing μ_q. The step-like structure in the left of Fig. 3 represents the successive phase transitions associated with the change of the patch number.

One might have had an impression that the spatially inhomogeneous state is peculiar to the large-N_c limit. This is not so, and the inhomogeneous state should be favored by not only the low-dimensional nature but also by a physical mechanism that is known as the Overhauser effect. If the chiral symmetry is broken to lead to a large value of the constituent quark mass, the baryon or quark density is suppressed by the mass, and it costs the energy accordingly. So, if there is any way that allows for a large quark mass and a large quark density at the same time, it should lower the system energy most efficiently. This is actually possible by the introduction of the spatially inhomogeneous chiral condensate. The right of Fig. 3 shows the wave-number of such spatial inhomogeneity as a function of T and μ_q obtained in some (generic) model setup, which looks like an island floating in the intermediate density region [5]. Note that the calculation assumes a vector interaction and no QCD critical point exists in the conventional sense. We would stress that the inhomogeneous state is more robust than the hypothetical critical point, and we should think more seriously about the possible experimental signature for the inhomogeneous state.

4 Strong magnetic field

The phase diagram on the phase of T and B is not clear yet. Some time ago, mainly in the model studies, it was believed that the quark deconfinement and the chiral symmetry restoration should become separate. The argument is the following: It is generally true that the chiral symmetry breaking is enhanced by the B-effect and the extreme example of this is the magnetic catalysis. At zero temperature where the magnetic catalysis works the best, the chiral condensate should be an increasing function of increasing B. This suggests that T_c inherent in chiral restoration should be pushed upward by B, while deconfinement is not much affected due to the lack of direct coupling between B and gluons.

A first surprise came from the lattice-QCD data showing that simultaneous crossover of deconfinement and chiral symmetry restoration holds for any B [6]. This result of deconfinement significantly affected by B implies that the quark polarization effect is more important than naively expected. Also, we have to suspect that we miss something in the study of high-μ_q matter because such strong locking between deconfinement and chiral restoration is not quite implemented there. Motivated by the lattice-QCD data, some improvements on the effective models have been proposed, but they are ad hoc and do not seem to be under theoretical control.

A second surprise came later from the lattice-QCD data again [7]. Contrary to the magnetic catalysis, they reported the decreasing behavior of T_c with increasing B. Because such exotic behavior is seen only in the case with sufficiently light quarks, it is unlikely that the dynamics of deconfinement has anything to do with this decreasing behavior. The most reasonable mechanism that can account for this should be the effect of fluctuations of massless Nambu-Goldstone bosons. In fact B explicitly breaks isospin symmetry and makes $\pi^{\pm}$ as heavy as the energy scale eB, but π^0 still remains massless. The important point is that π^0 is a neutral meson and insensitive to B as long as it is to be regarded as a point particle, but for very strong $eB \gg \Lambda_{\mathrm{QCD}}^2$, the dispersion relation of π^0 is substantially deformed and this change would cause an infrared singularity that prohibits the chiral symmetry breaking.

Another interesting phenomenon in quark matter under strong B is the manifestation of the quantum anomaly. At finite μ_q with strong B, for example, an axial-vector current is generated via quantum processes, which should have effects on the chiral spiral structure we addressed in the previous section.

Indeed, for the purpose to probe the QCD phase diagram in the relativistic heavy-ion collision experiment, the quantum anomaly involving B could provide us with a new signature for chiral symmetry restoration. Actually, for those who are working on chiral symmetry in QCD matter, it is a source of headache how to confirm the chiral symmetry restoration by real experiment. There is no clear signature for the chiral dynamics. The in-medium modification of vector meson properties should be relevant to chiral symmetry, but the interpretation requires theoretical models.

The B-induced anomalous processes such as the chiral magnetic effect for example could give a more direct evidence for the realization of chiral symmetry. The story is, however, not such straightforward and the correct physical interpretation needs careful deliberations [8].

In the future the B-effect on quark matter deserves more theoretical investigations. There are many interesting issues and it is not our intention to try to enumerate all of them. Let us quickly look over several topics randomly. It is a very intriguing question whether quark deconfinement happens or not solely by the B-effect at zero temperature and zero baryon density. What if a finite T and/or μ_q is introduced? Related to this, one might have a chance to evade the notorious sign problem at finite μ_q if the imposed B is strong enough to make the complete dimensional reduction to (1+1)-dimensional space-time. This should be examined by the distribution of the eigenvalues of the Dirac operator in the presence of B and μ_q. The eigenvalue distribution would give us a deeper insight to the chiral symmetry breaking through the Banks-Casher relation, and it should be interesting to formulate the magnetic catalysis as a result of the accumulation of the eigenvalues associated with the Landau quantization. If B is strong enough, quark matter is reduced to a (1+1)-dimensional fermionic system, and it should be regarded as the Tomonaga-Luttinger liquid where not quasi-fermion but bosonic density should be the physical degrees of freedom. The Hamiltonian is then written in terms of two density operators of the left-handed and the right-handed sectors, and they oscillate and change to each other alternately, that is nothing but the chiral magnetic wave. We are now working on them and several works are about to be finalized.

Acknowledgements The talk is based on the papers written in collaboration with T. Hatsuda, T. Hayata, Y. Hidaka, K. Kashiwa, T. Kojo, K. Mameda, L. McLerran, and R. Pisarski. The author thanks them for fruitful discussions.

References

1. K. Fukushima and T. Hatsuda, "The phase diagram of dense QCD," Rept. Prog. Phys. **74**, 014001 (2011).
2. K. Fukushima and K. Kashiwa, "Polyakov loop and QCD thermodynamics from the gluon and ghost propagators," arXiv:1206.0685 [hep-ph].
3. L. McLerran and R. D. Pisarski, "Phases of cold, dense quarks at large N(c)," Nucl. Phys. A **796**, 83 (2007).
4. T. Kojo, Y. Hidaka, K. Fukushima, L. D. McLerran and R. D. Pisarski, "Interweaving Chiral Spirals," Nucl. Phys. A **875**, 94 (2012).
5. K. Fukushima, arXiv:1204.0594 [hep-ph] (to appear in Phys. Rev. D).
6. M. D'Elia, S. Mukherjee and F. Sanfilippo, "QCD Phase Transition in a Strong Magnetic Background," Phys. Rev. D **82**, 051501 (2010).
7. G. S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S. D. Katz, S. Krieg, A. Schafer and K. K. Szabo, "The QCD phase diagram for external magnetic fields," JHEP **1202**, 044 (2012).
8. K. Fukushima and K. Mameda, arXiv:1206.3128 [hep-ph].