



Quest for the QCD phase diagram in extreme environments



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Typical Extreme's



High Temperature

up to $T \sim \Lambda_{\text{QCD}} \sim 200\text{MeV}$

Relativistic Heavy-Ion Collision

High Baryon Density

up to $\rho_{\text{B}} \sim (\Lambda_{\text{QCD}})^3 \sim 1\text{fm}^{-3}$

Relativistic Heavy-Ion Collision, Neutron Star

Strong Magnetic Field

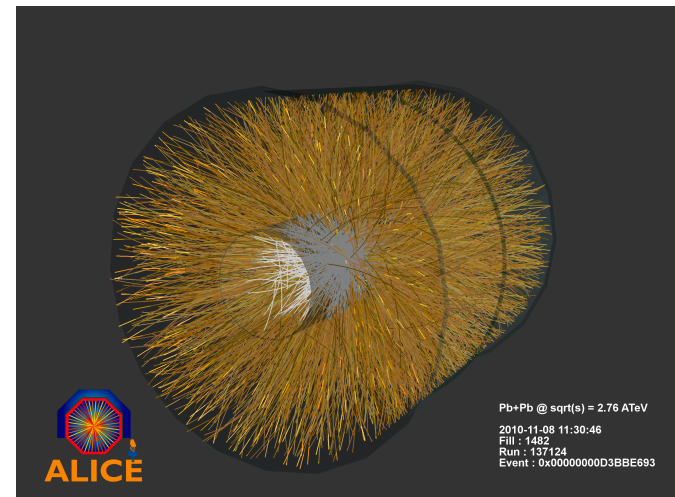
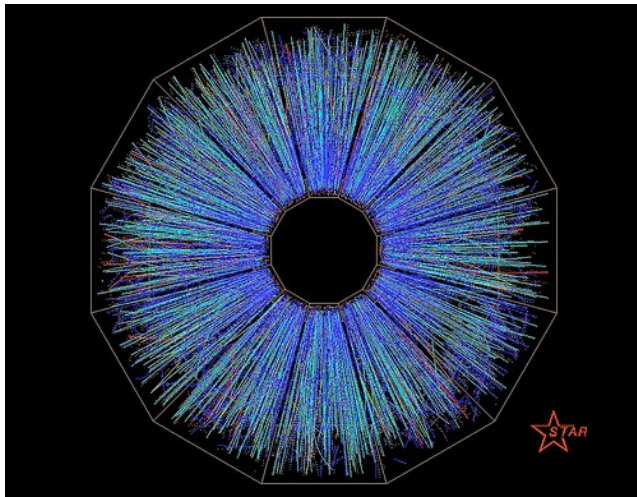
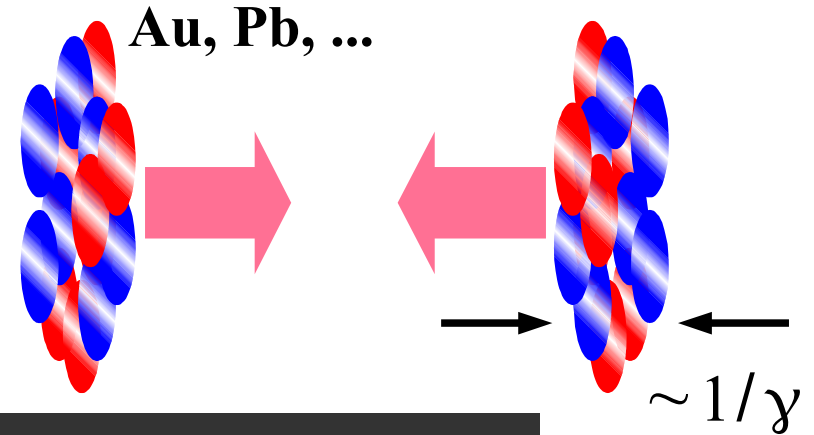
up to $eB \sim (\Lambda_{\text{QCD}})^2 \sim 10^{18}$ gauss

Relativistic Heavy-Ion Collision, Neutron Star

Relativistic Heavy-Ion Collision

LHC: $\sqrt{s_{NN}} = 2.7 \text{ TeV} \rightarrow \gamma \sim 1400$

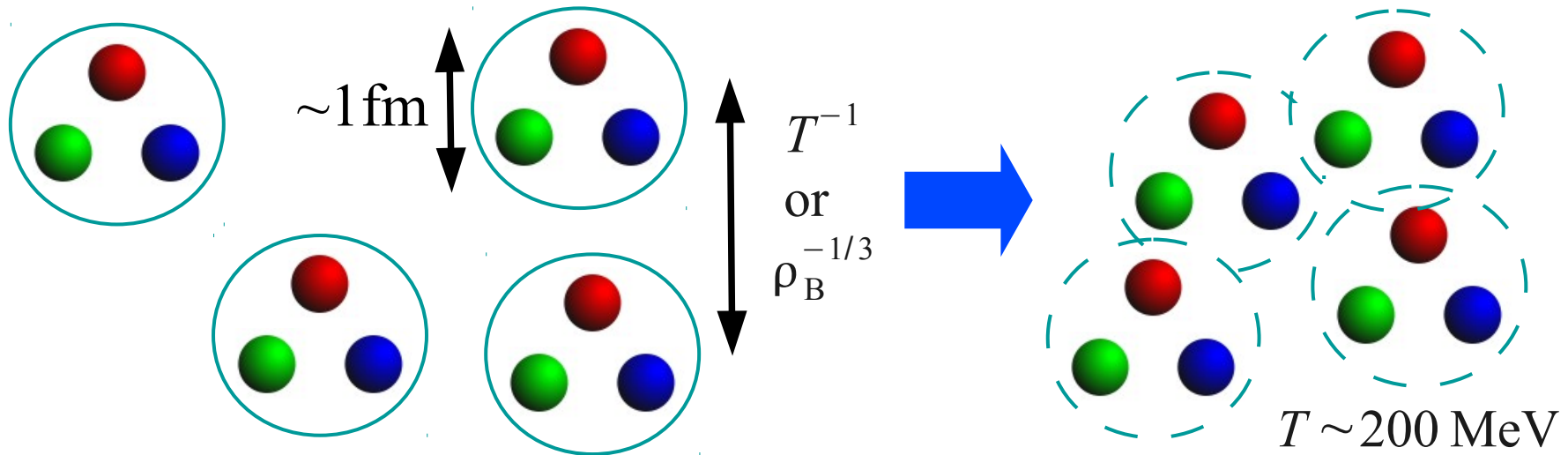
RHIC: $\sqrt{s_{NN}} = 200 \text{ GeV} \rightarrow \gamma \sim 100$



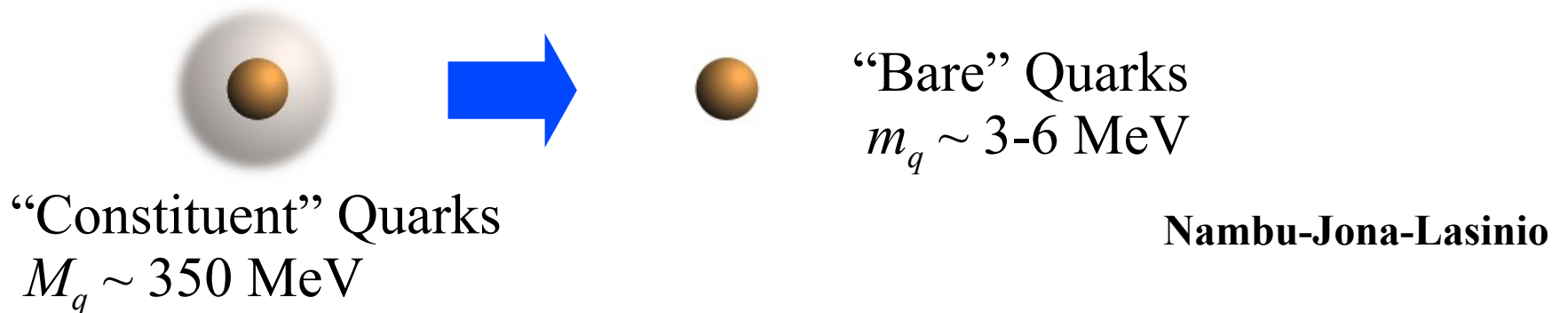
Thermalization achieved (elliptic flow by a hydro-model)
Initial temperature $> 200 \text{ MeV}$ (distribution of thermal photon)

Two Major Phase Transitions in QCD

Quark Deconfinement Transition (Center Symmetry)



Chiral Phase Transition (Chiral Symmetry)



Symmetry Entanglement



$$m_q = 0$$

Quark Mass

$$m_q = \infty$$



~~Center Symmetry~~

Center Symmetry

Physical
World?

Chiral Symmetry

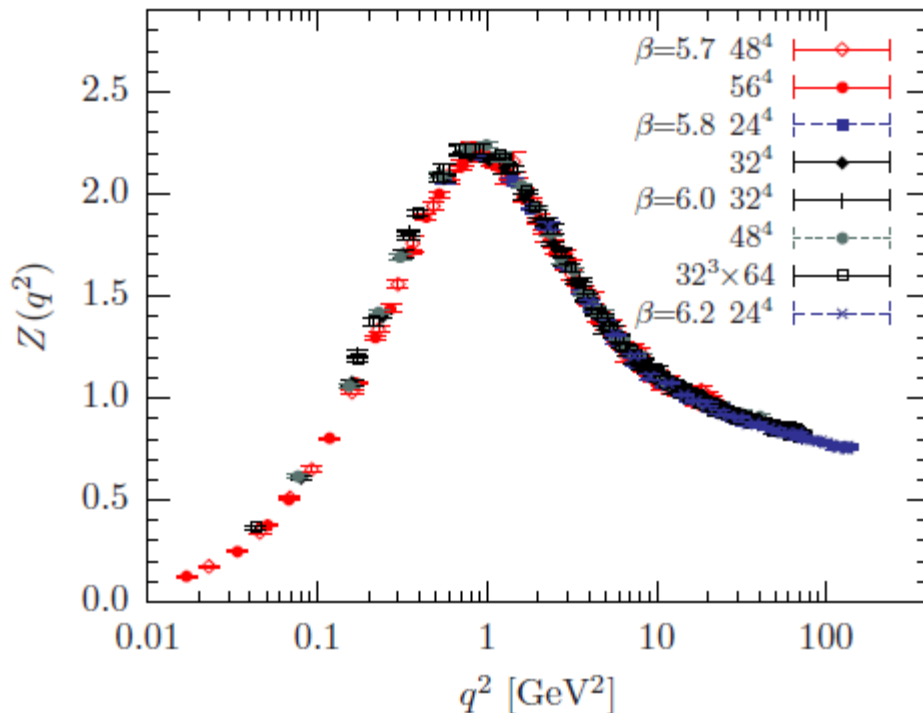
~~Chiral Symmetry~~

Understanding “Deconfinement”

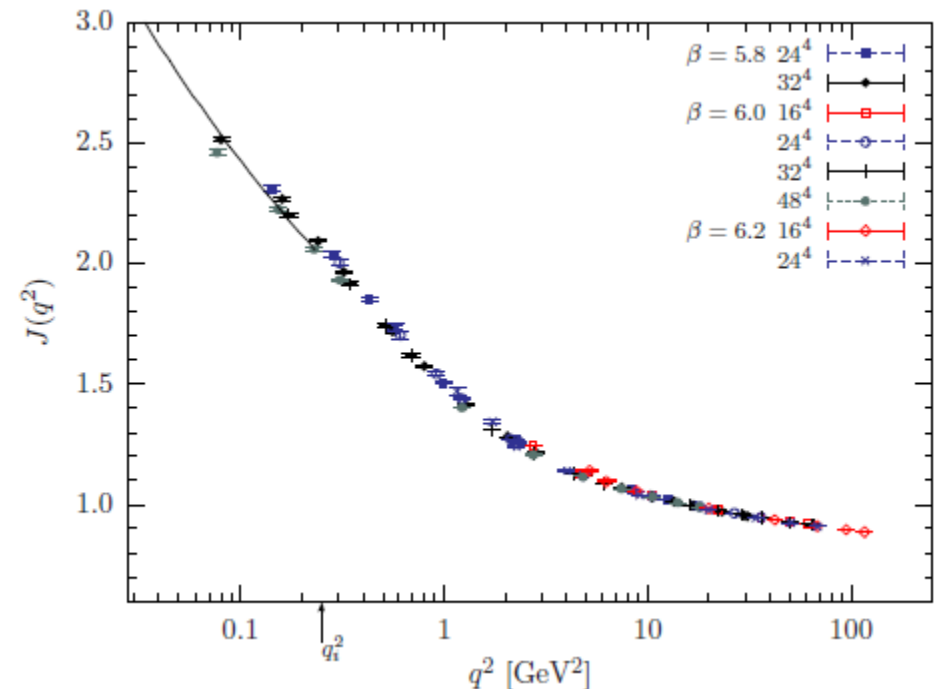


Confinement understood from the non-perturbative propagators of gluons and ghosts in the Landau gauge

Ilgenfrits-Muller-Preussker-Sternbeck-Schiller-Bogolubsky (2007)



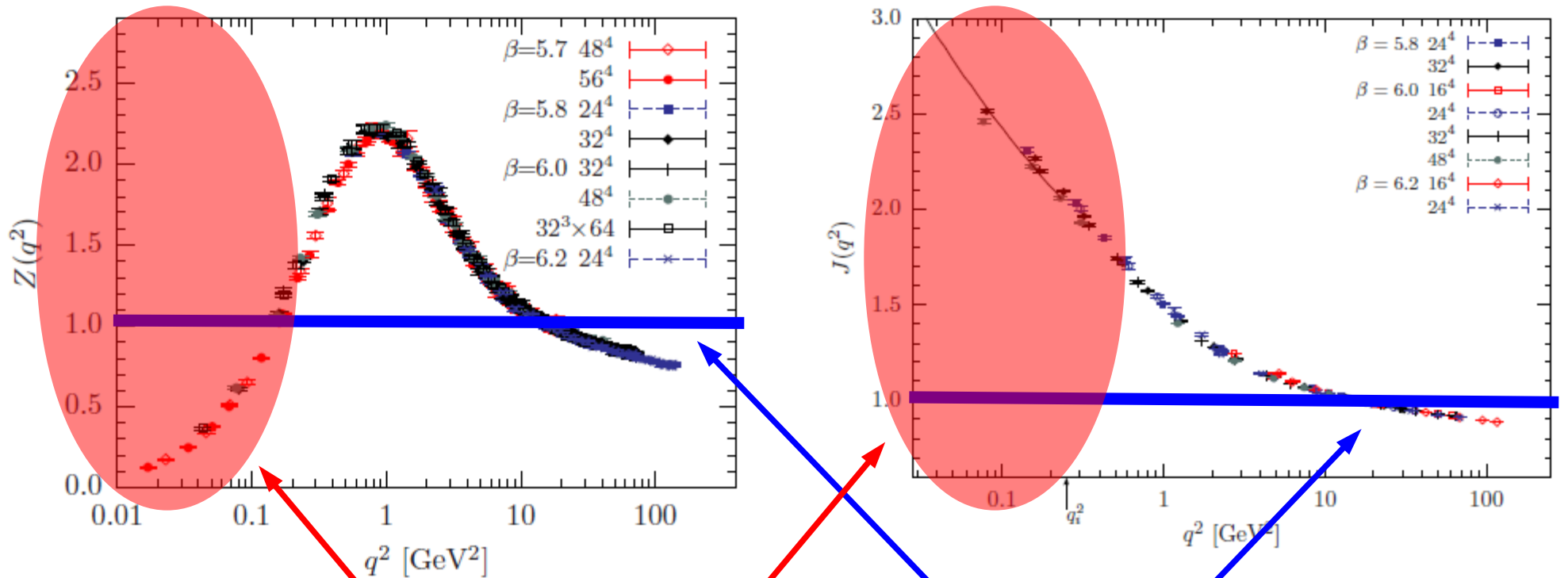
Gluons IR suppressed



Ghosts IR enhanced (confinement)

Gribov's scenario

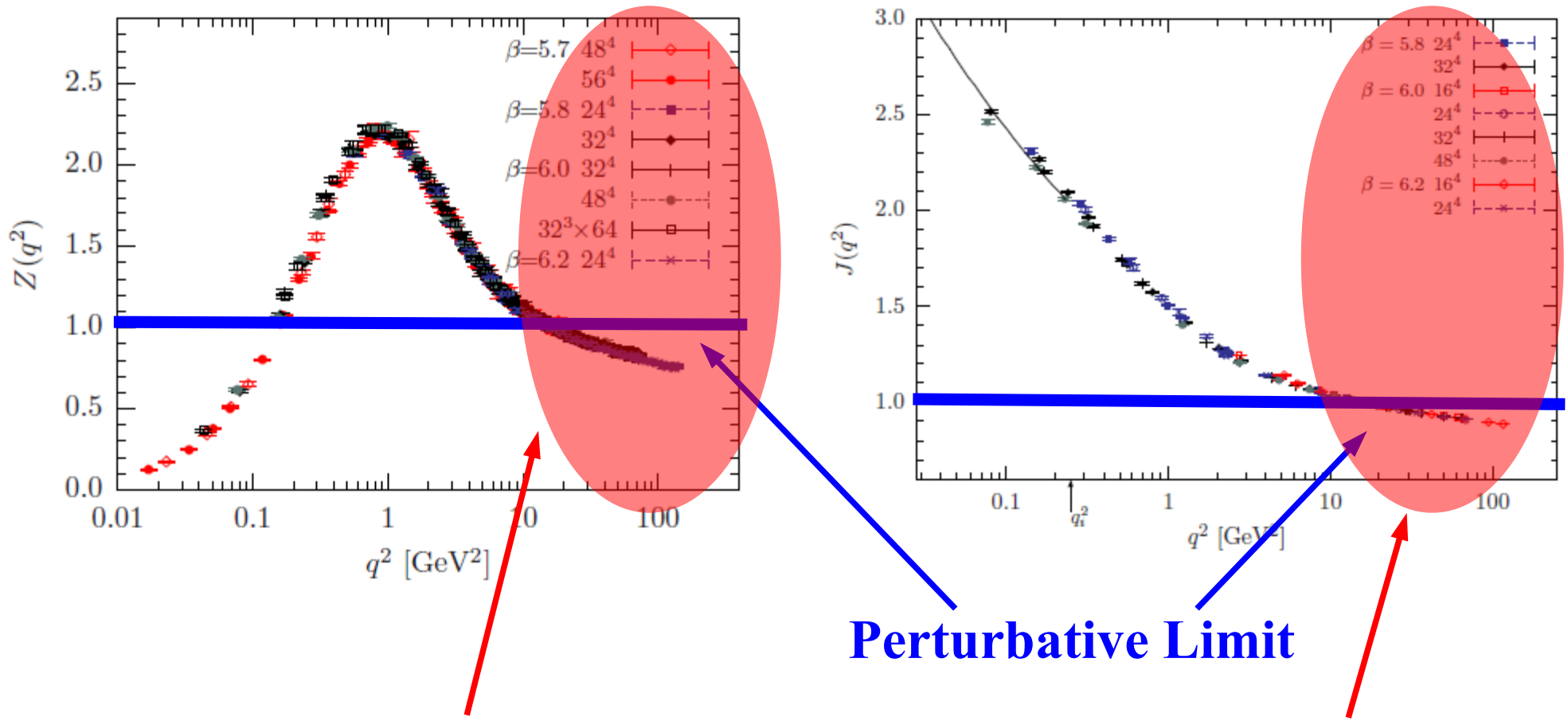
Confinement at Low Temperature



Ghost Dominance → Color Confinement

Perturbative Limit

Deconfinement at High Temperature



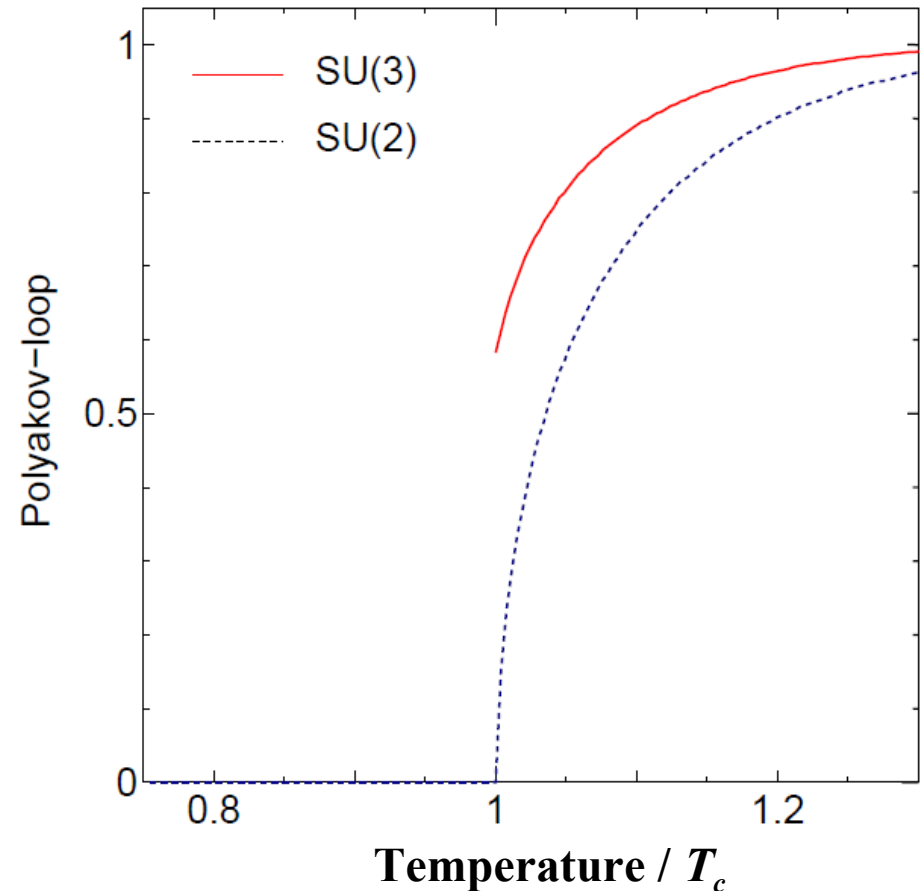
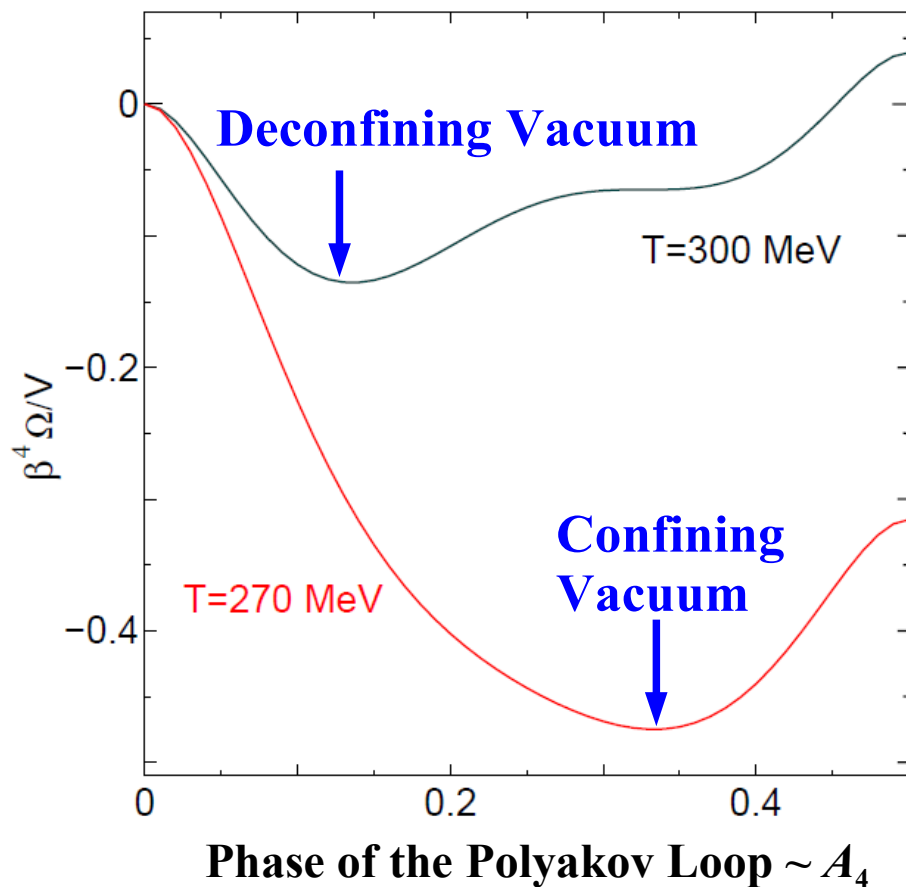
Perturbative Limit

**All Excitations with $p \sim 2\pi T \rightarrow$ Perturbative Limit
Two Transverse Gluons (unphysical ones canceled)**

Phase Transition from Propagators

$$\ln Z = -\frac{1}{2} \text{tr} \ln D_{\text{gluon}}^{-1}(A_4) + \text{tr} \ln D_{\text{ghost}}^{-1}(A_4) + \dots$$

Balance between gluons and ghosts



Confinement \rightarrow Deconfinement

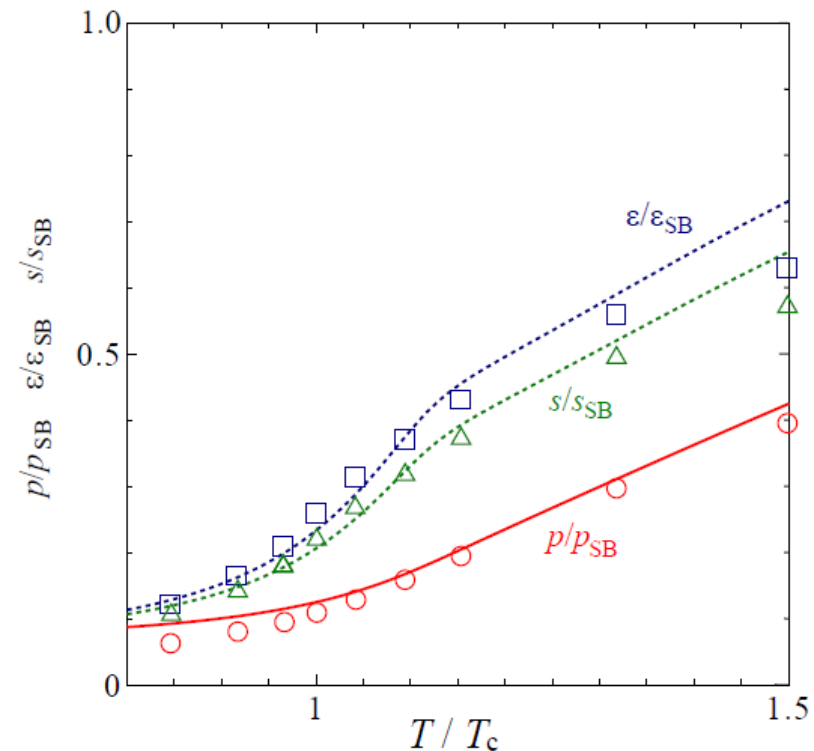
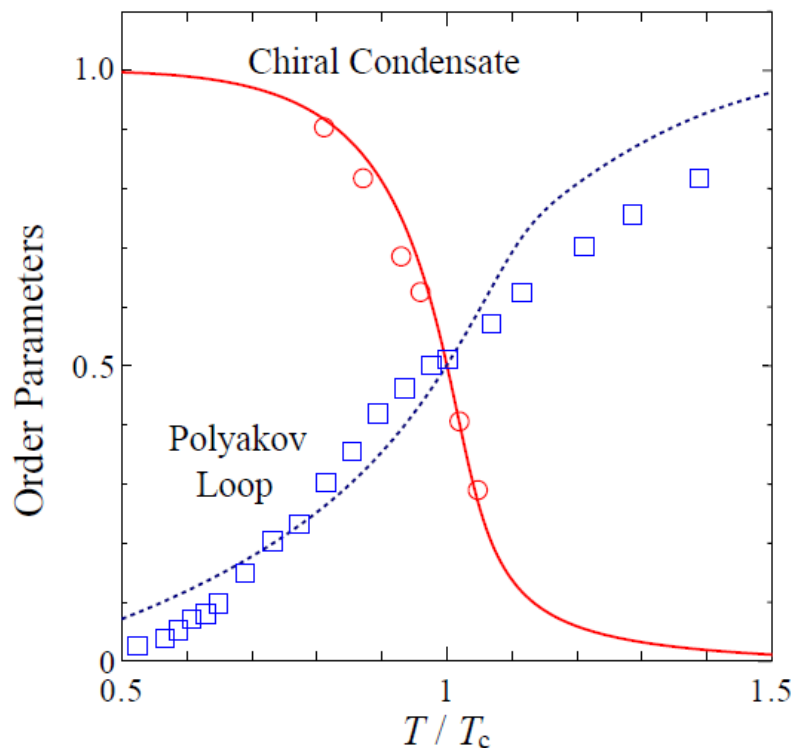
Braun-Gies-Pawlowski, KF-Kashiwa

Dynamical Quarks at Finite m_q

$$\ln Z_F = \text{tr} \ln (1 + e^{-(E-\mu)/T}) + \text{tr} \ln (1 + e^{-(E+\mu)/T}) \quad \text{Partition func. for quarks}$$

➔ $\text{tr} \ln (1 + L e^{-(E-\mu)/T}) + \text{tr} \ln (1 + L^\dagger e^{-(E+\mu)/T})$

Coupling through the covariant derivative



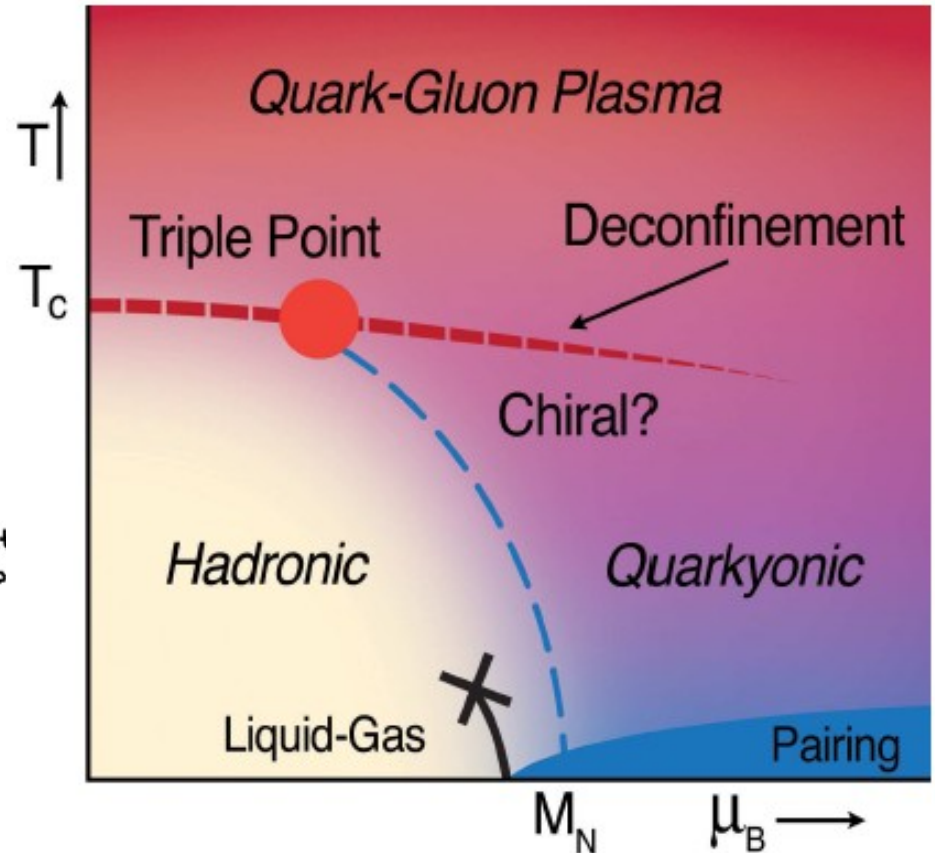
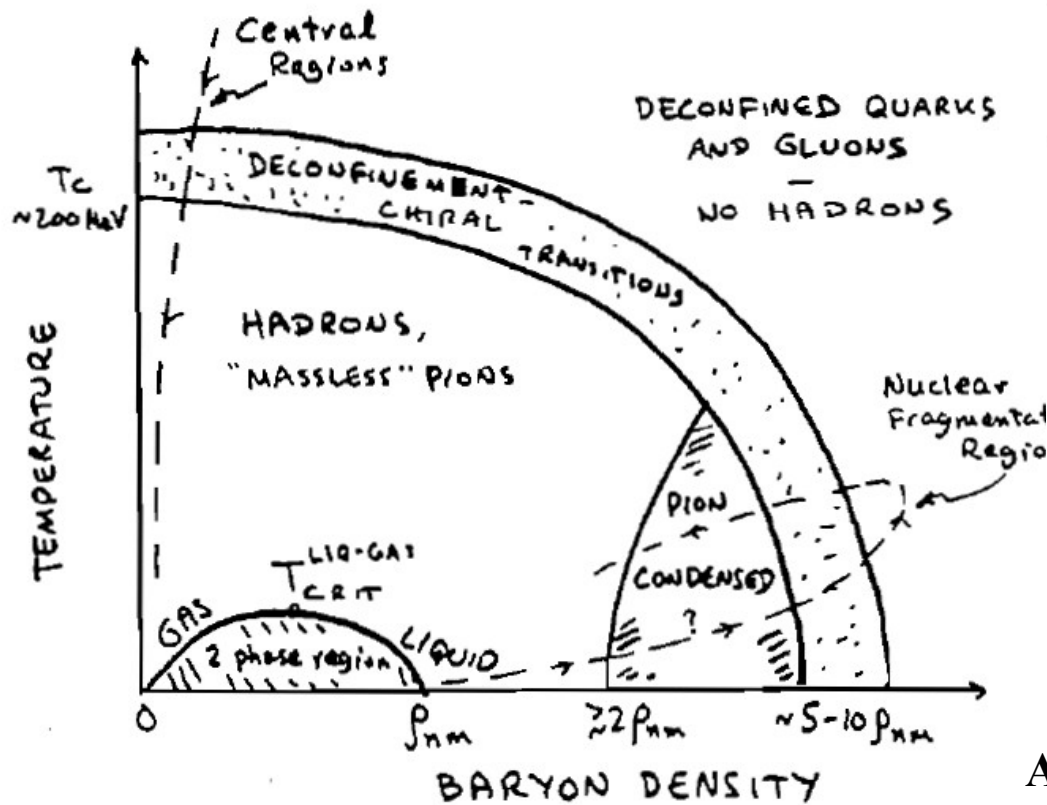
Phase Diagrams including Density

Prototype in 1983

and

Update in 2009

PHASE DIAGRAM OF NUCLEAR MATTER.



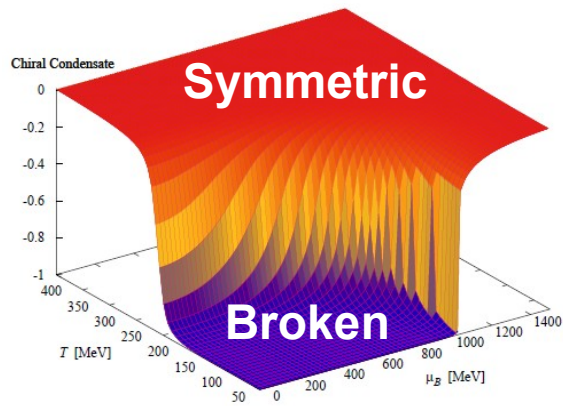
Andronic-Blaschke-Braun-Cleymans-Munzinger-KF-McLerran-Oeschler-Pisarski-Redlich-Sasaki-Satz-Stachel (2009)

Modern Picture

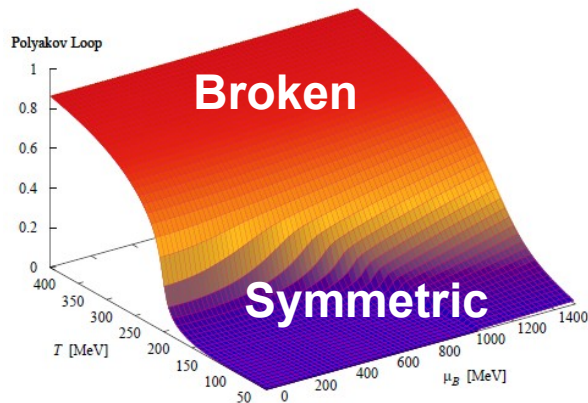


Typical Model Results

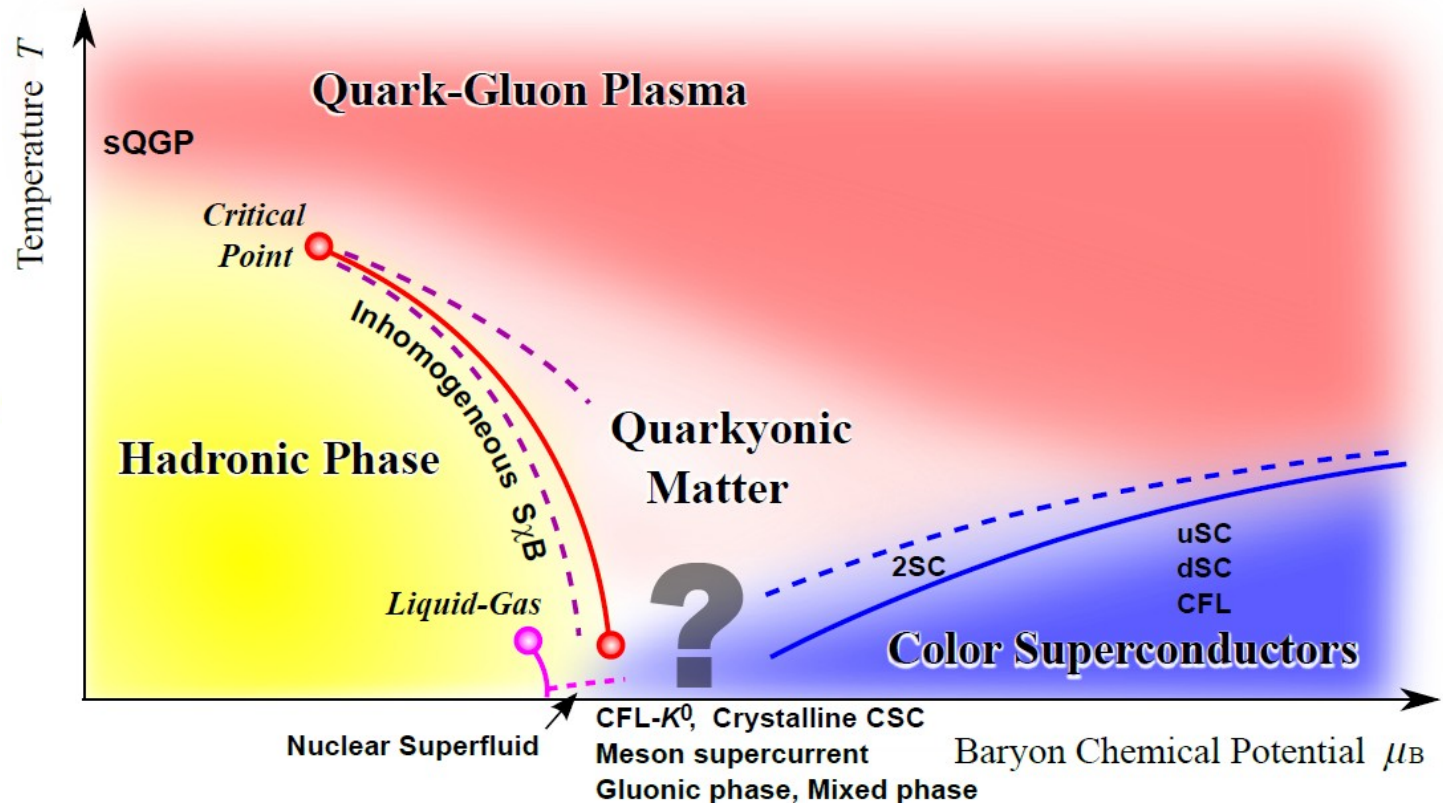
Chiral Symmetry



Center Symmetry



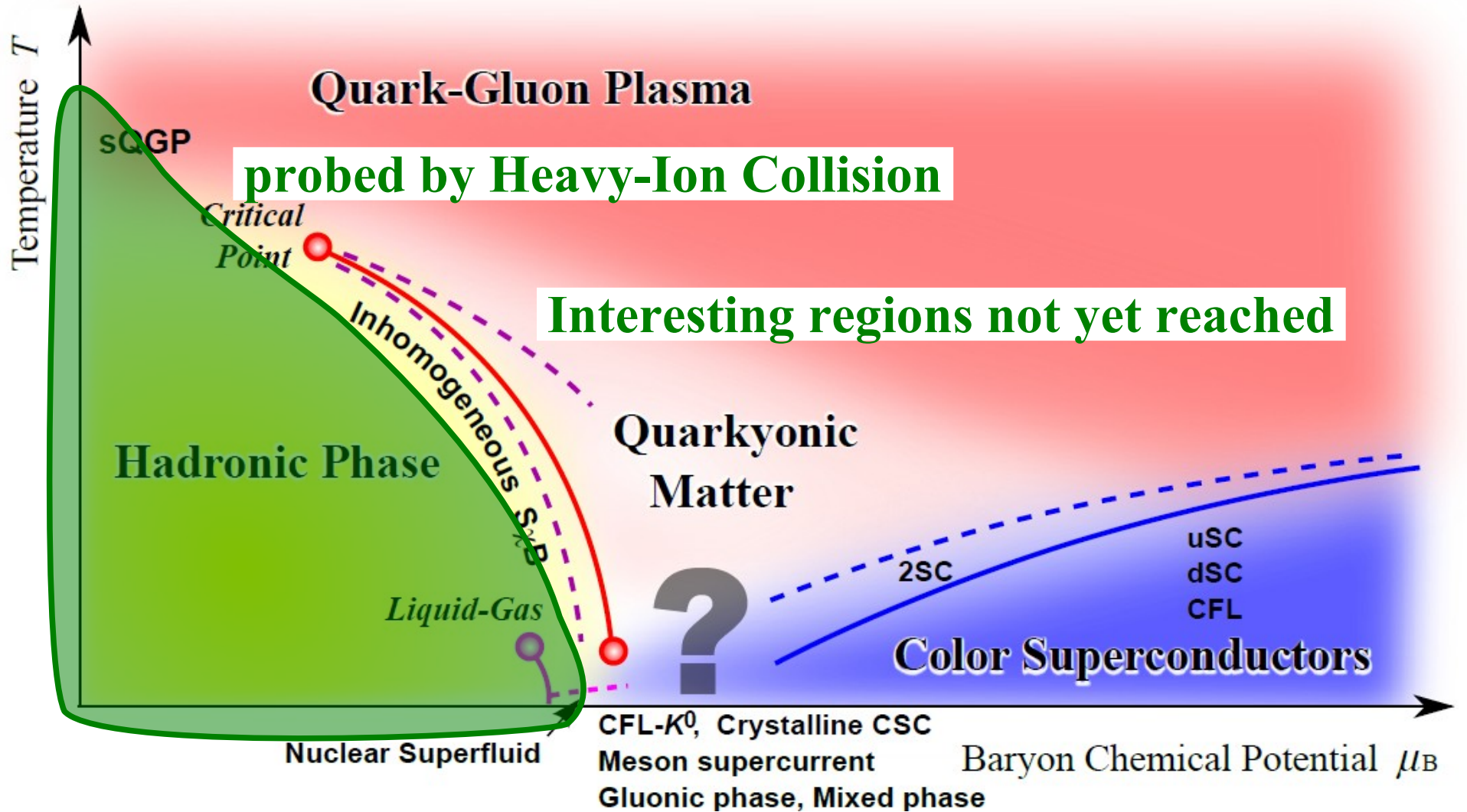
Conjectured Phase Structure



KF (2008)

KF-Hatsuda (2010)

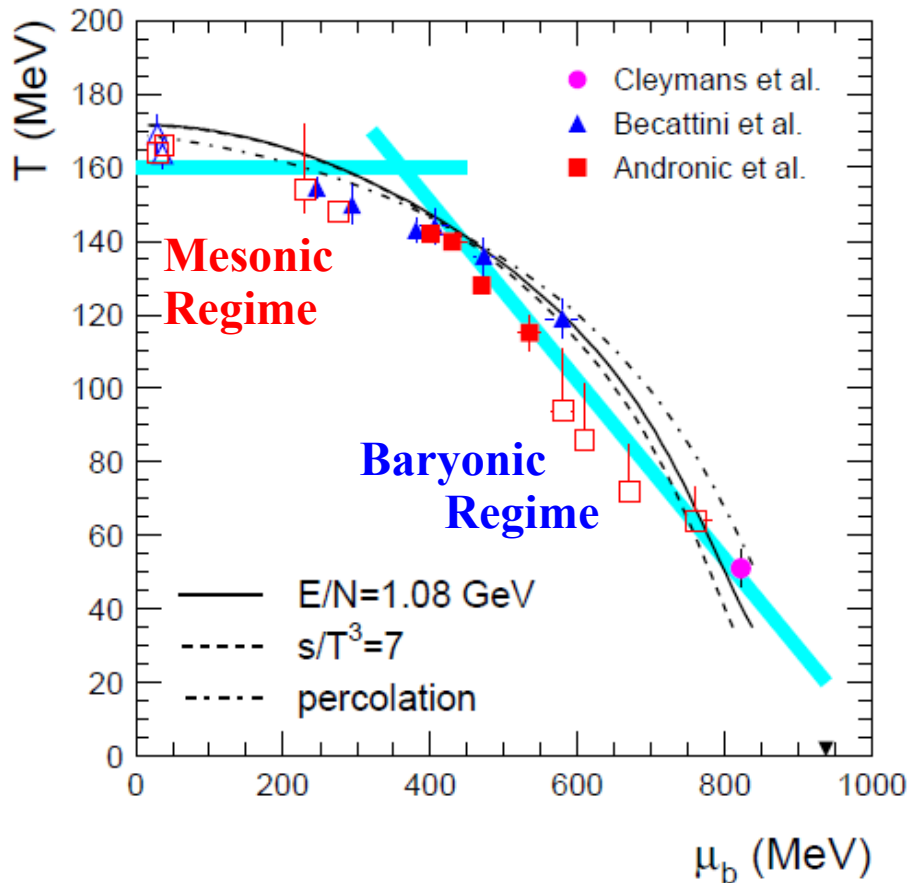
Experimental Confirmation



KF-Hatsuda (2010)

Interpretation of Data

Freeze-out points are located by the particle yields
 Two regimes in **meson-dominance** and **baryon-dominance**



Mesonic Hagedorn Transition

$$Z \sim \int dm \rho(m) e^{-m/T}$$

$$\rho(m) \sim e^{m/T_H}$$

$$T_c = T_H$$

Baryonic Hagedorn Transition

$$Z \sim \int dm \rho_B(m) e^{-(m_B - \mu_B)/T}$$

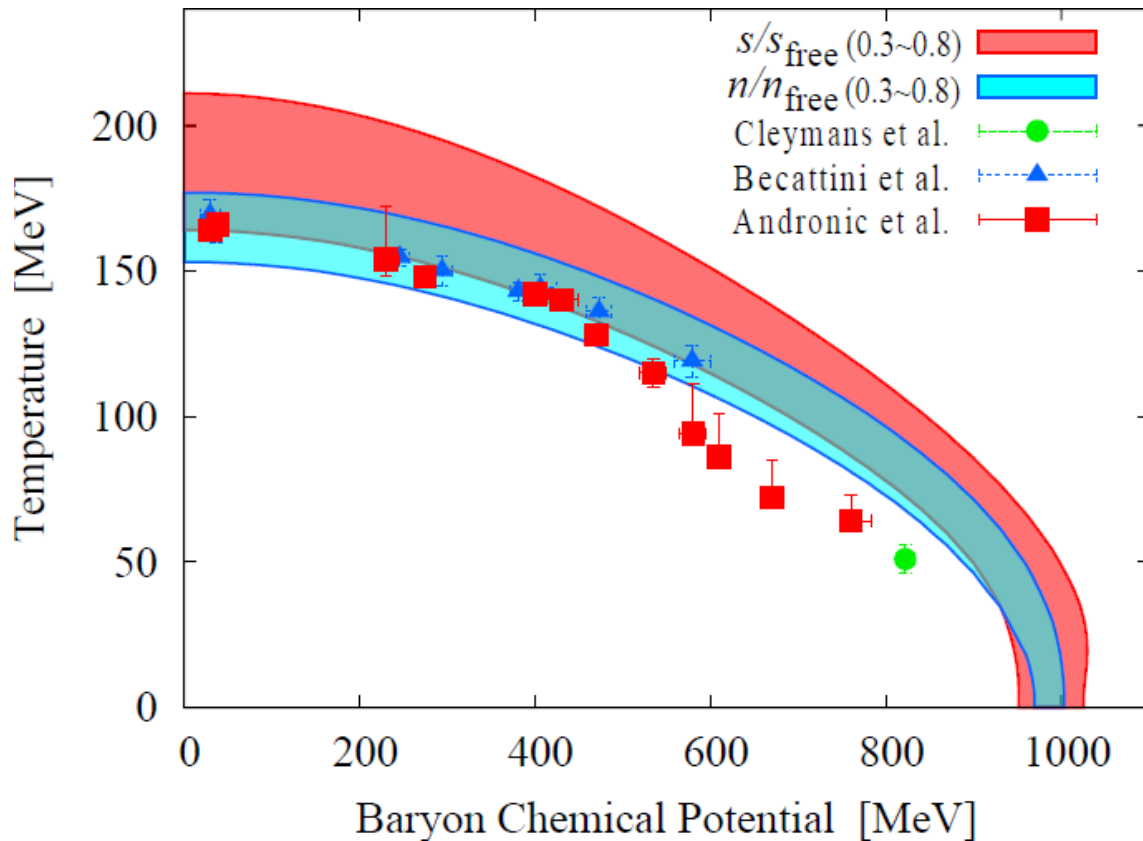
$$\rho(m) \sim e^{m_B/T_B}$$

$$T_c = (1 - \mu_B/m_B) T_B$$

Thermodynamics

Statistical Model Interpretation

KF (2010)

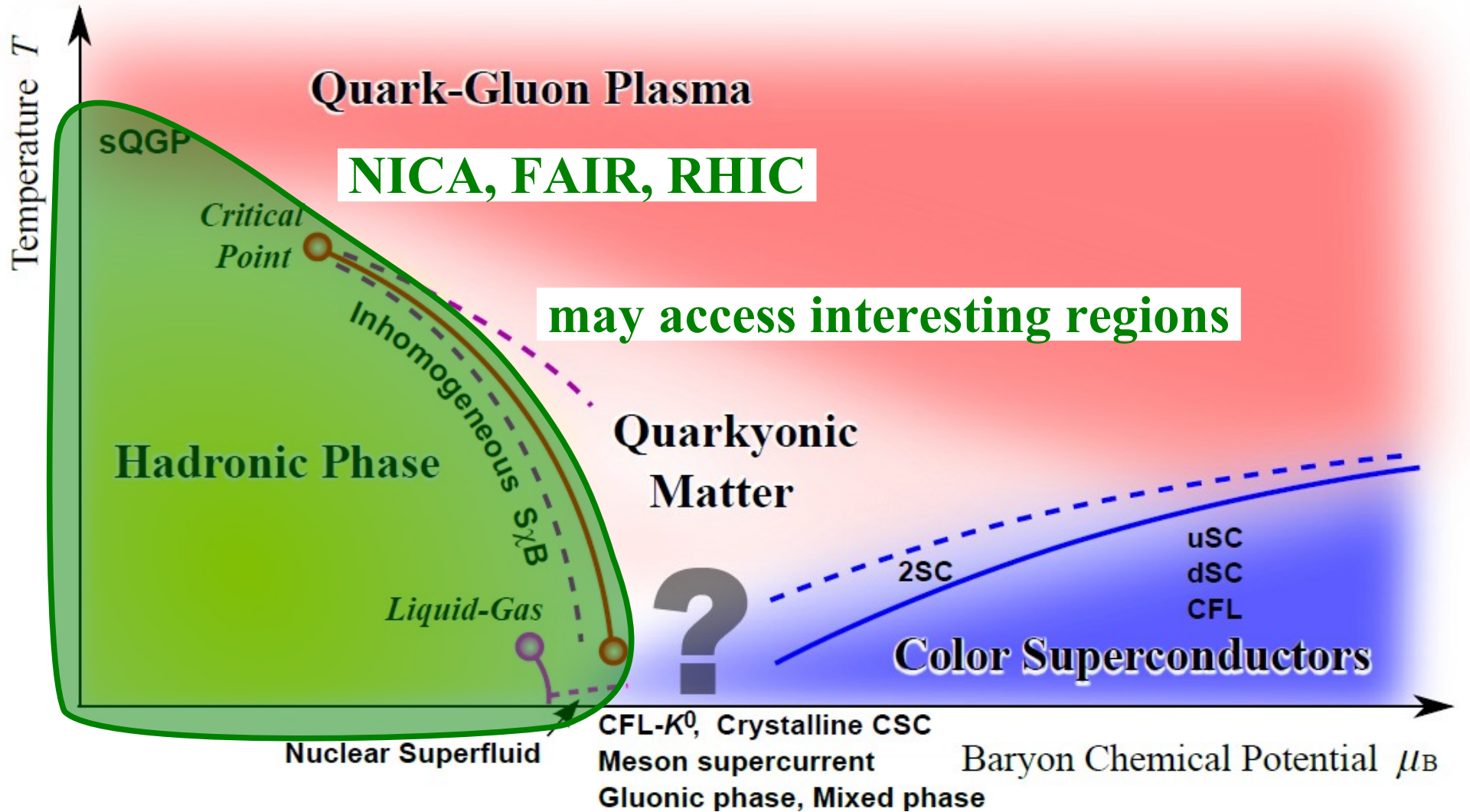


Gluon Deconfinement
~ Increasing entropy

Quark Deconfinement
~ Increasing density

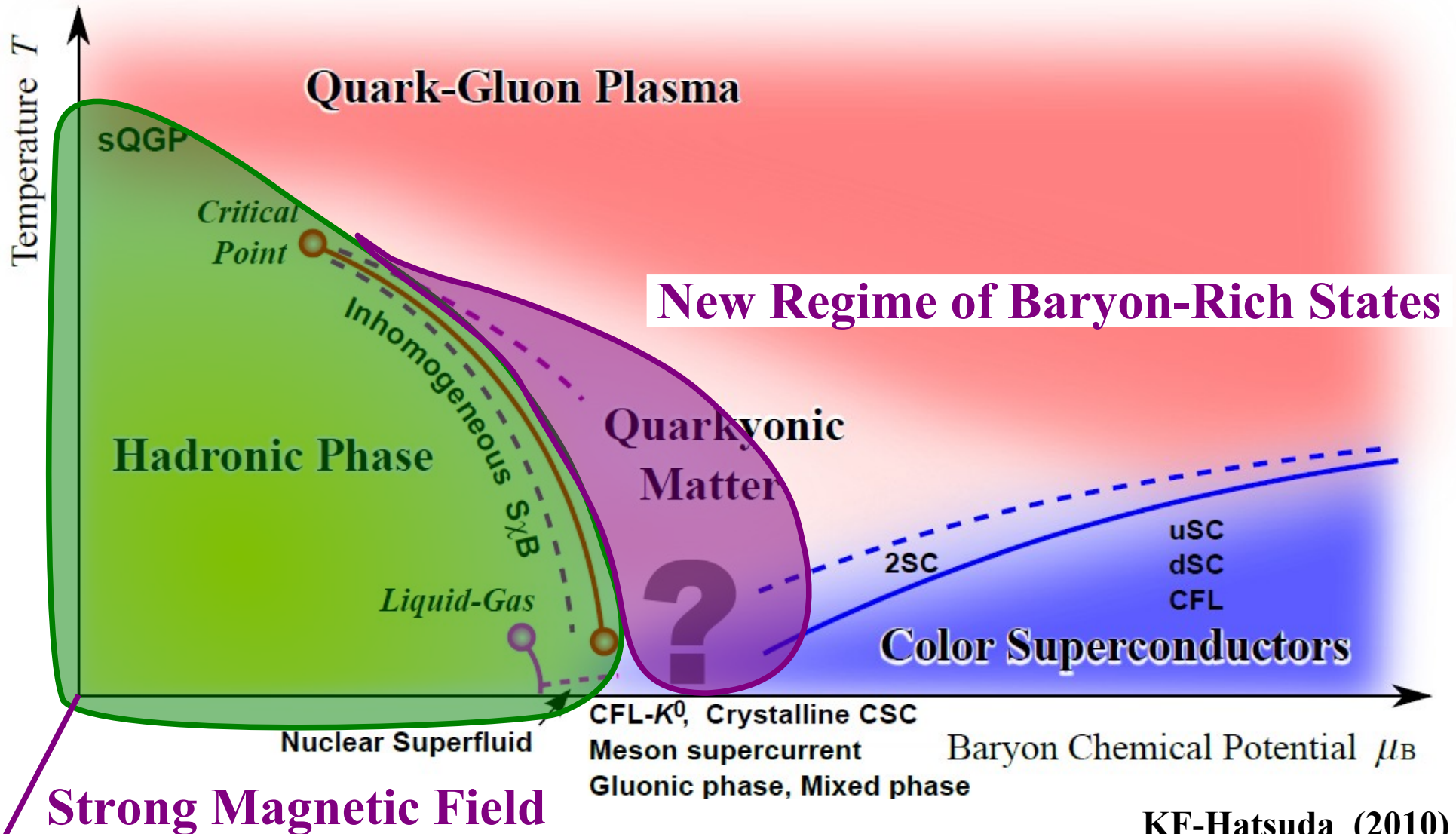
Thermodynamic quantities
taken over by (quasi-)gluons
and (quasi-)quarks
(beyond the Hagedorn limit)

Experimental Challenges



KF-Hatsuda (2010)

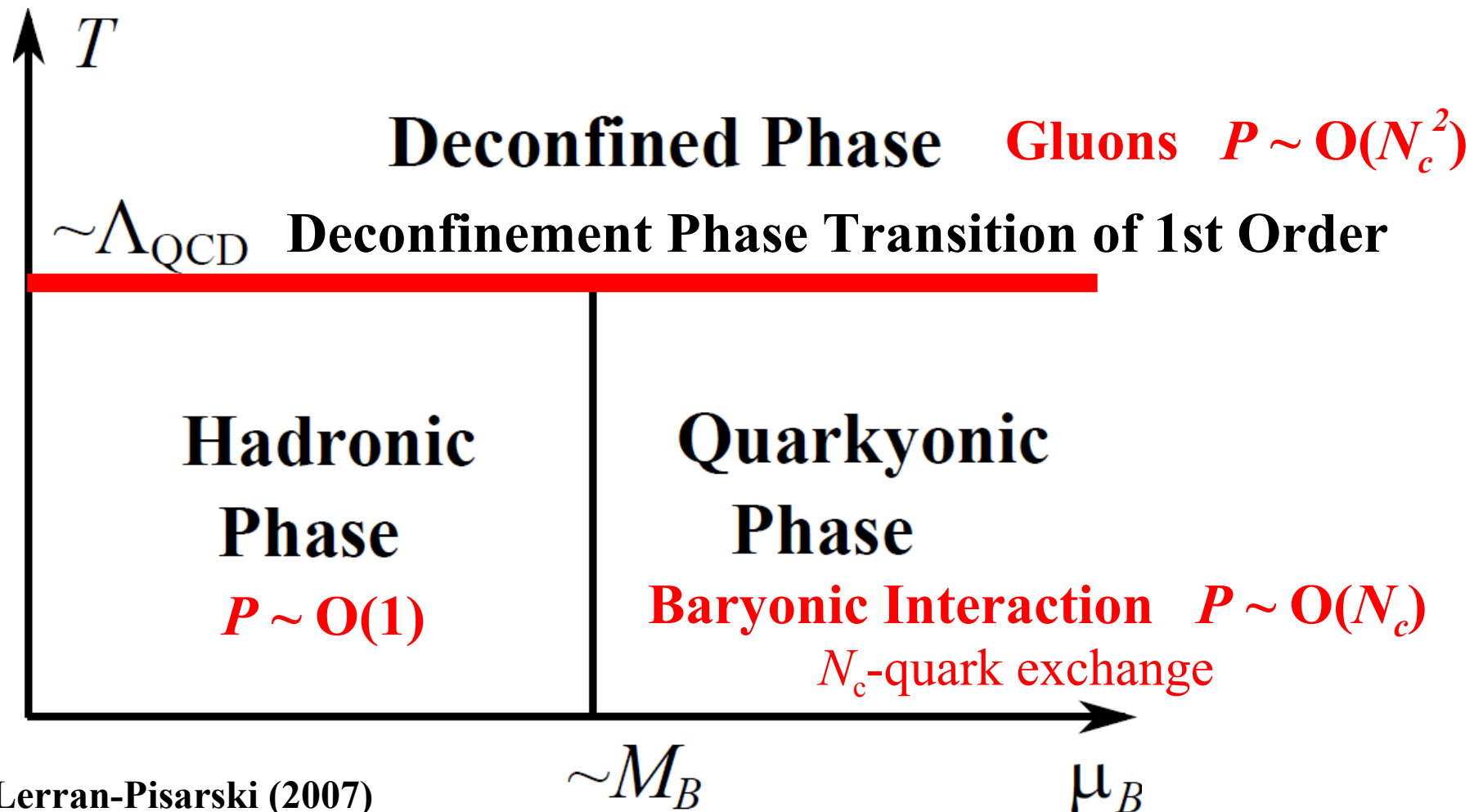
Theoretical Speculations



KF-Hatsuda (2010)

New Regime at Large μ_q and N_c

Phase Diagram of Large- N_c QCD

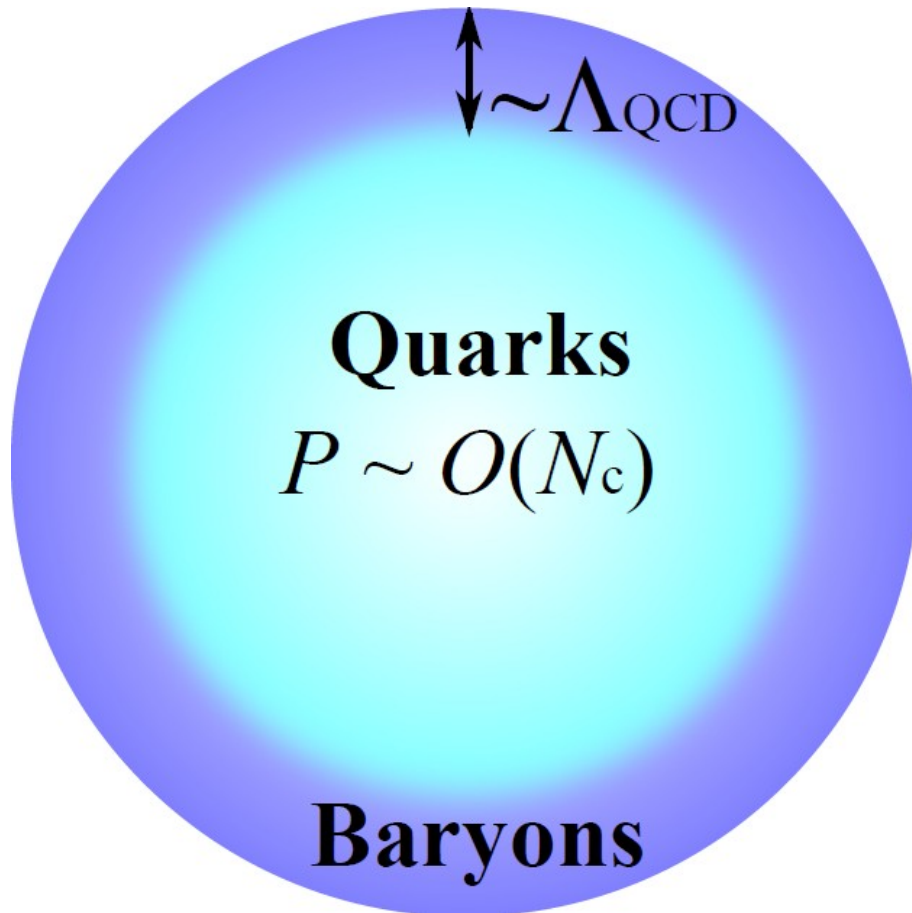


McLerran-Pisarski (2007)

Quarkyonic Matter



Structure of the Fermi Sphere



**Ground state of
large- N_c quark matter
at $\mu_q \gg \Lambda_{\text{QCD}}$**

McLerran, Pisarski
Hidaka, Kojo

**Interacting Baryon Crystal
 \sim Quasi-quark Gas**

Quarkyonic Chiral Spiral ($\mu_q \gg \Lambda_{QCD}$)



Choose one direction z with $p_z \sim \mu_q$ ($p_x, p_y \sim \Lambda_{QCD}$)
(1+1)D system effectively

$$\begin{aligned} & \bar{\psi} (i \gamma^z \partial_z + \mu \gamma^0) \psi \\ & = \bar{\psi}' (i \gamma^z \partial_z) \psi' \quad \psi = e^{i \gamma^0 \gamma^z \mu z} \psi' \end{aligned}$$

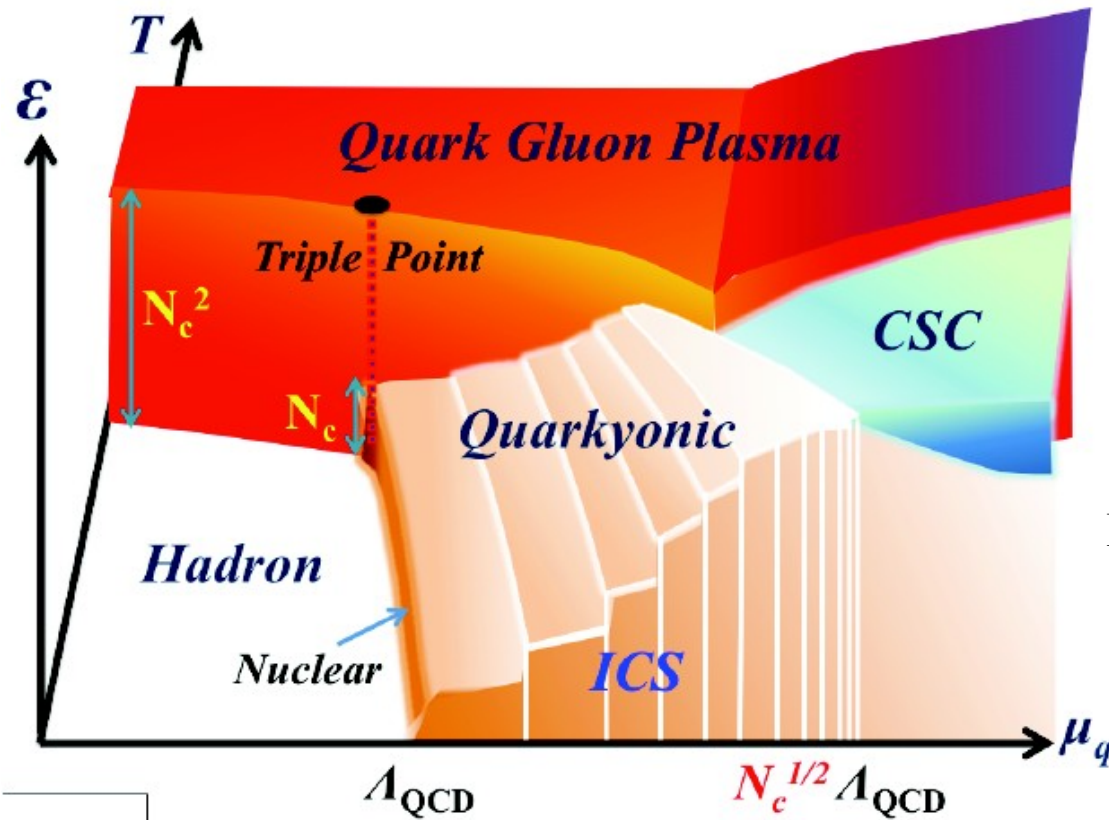
$\langle \bar{\psi}' \psi' \rangle =$ Homogeneous condensate at zero density

$$\langle \bar{\psi} \psi \rangle = \langle \bar{\psi}' \psi' \rangle \cos(2\mu z)$$

$$\langle \bar{\psi} \gamma^0 \gamma^z \psi \rangle = \langle \bar{\psi}' \psi' \rangle \sin(2\mu z)$$

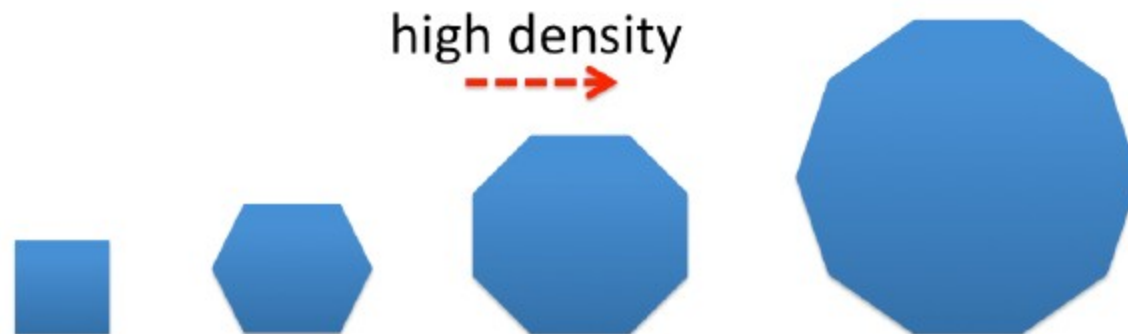
This quasi-(1+1)D system forms “one patch”

Interweaving Chiral Spirals

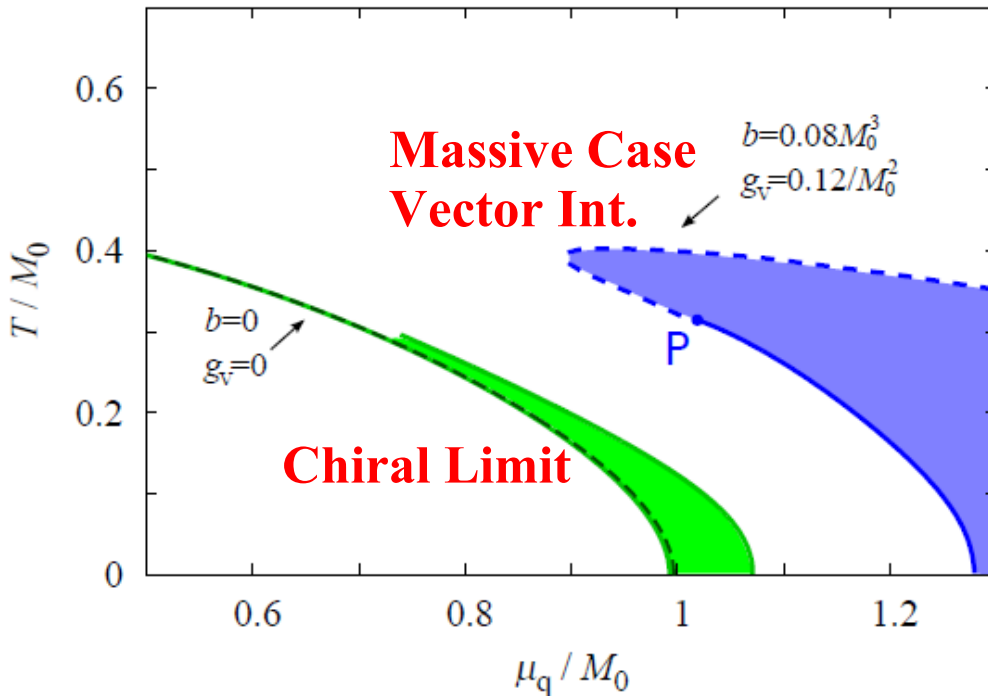


As the Fermi sphere enlarges, the patch number increases, forming a chiral quasi-crystal.

Kojo-Hidaka-KF-McLerran-Pisarski (2011)



Some Generic Features



$$E_p = \sqrt{p_x^2 + p_y^2 + (\sqrt{p_z^2 + M^2} - q)^2}$$

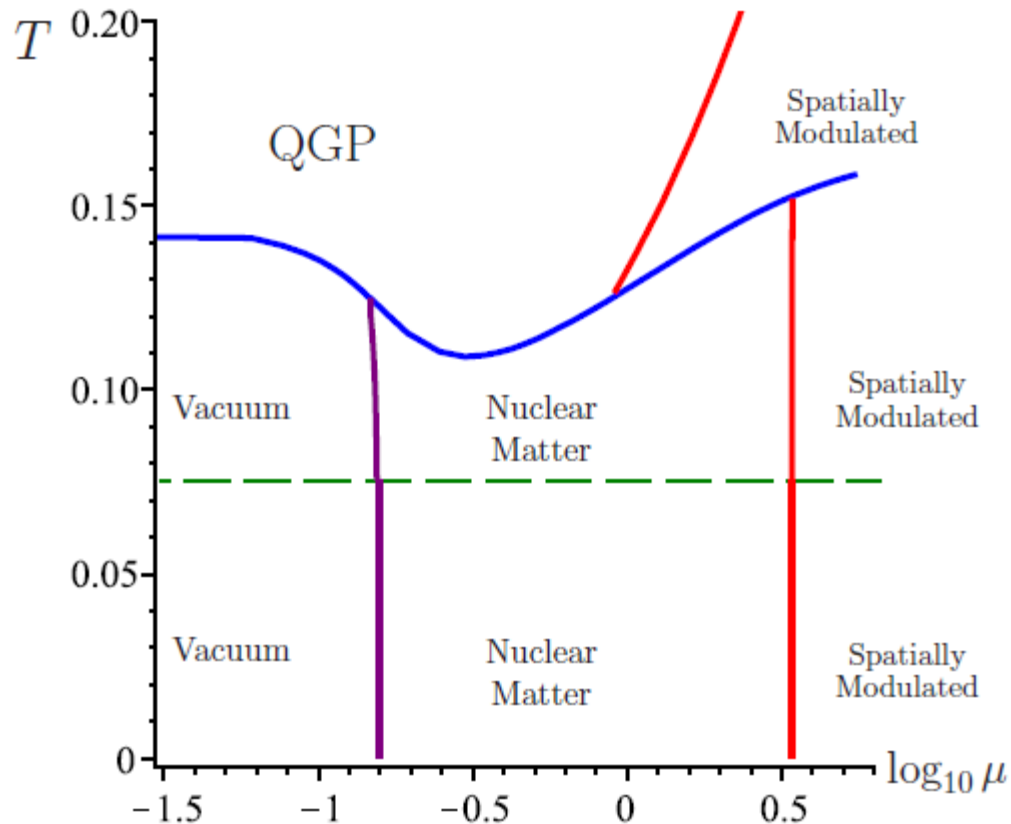
Effect of the dynamical mass M significantly canceled by q

**Even when N_c and μ_q are not infinitely large,
the chiral spiral is favored near the phase boundary
of chiral symmetry**

Nakano-Tatsumi (2003), KF (2012)

Holographic Evidence

State-of-the-art phase diagram in holographic model



Instability to inhomogeneous states is seen (in a different way from QCD...)

Nakamura-Ooguri-Park, Chuang-Dai-Kawamoto-Lin-Yeh (2010)

Density Effect ~ Magnetic Field Effect



Energy dispersion relation in B

$$\omega^2 = p_z^2 + \underline{2|eB|(n + 1/2)} + m^2 - 2s e B$$

Transverse motion = Harmonic Oscillator

Fermions ($s=1/2$) have zero mode – dominant at large B
Quasi-(1+1)D system is realized along the B direction.

Very strong B + Any $\mu_q \rightarrow$ Chiral Spiral

Basar-Dunne-Kharzeev

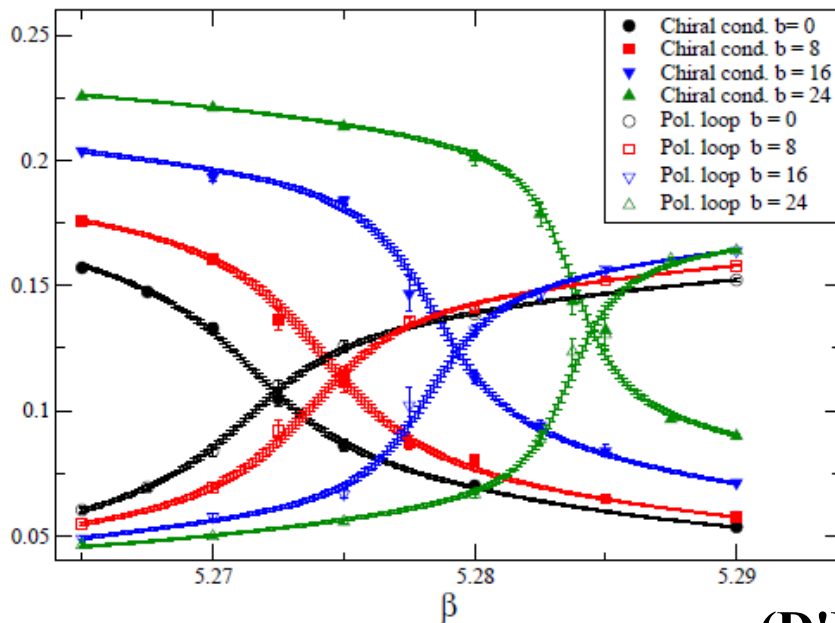
Very strong B + Attractive Int.

\rightarrow Cooper Instability \rightarrow Magnetic Catalysis

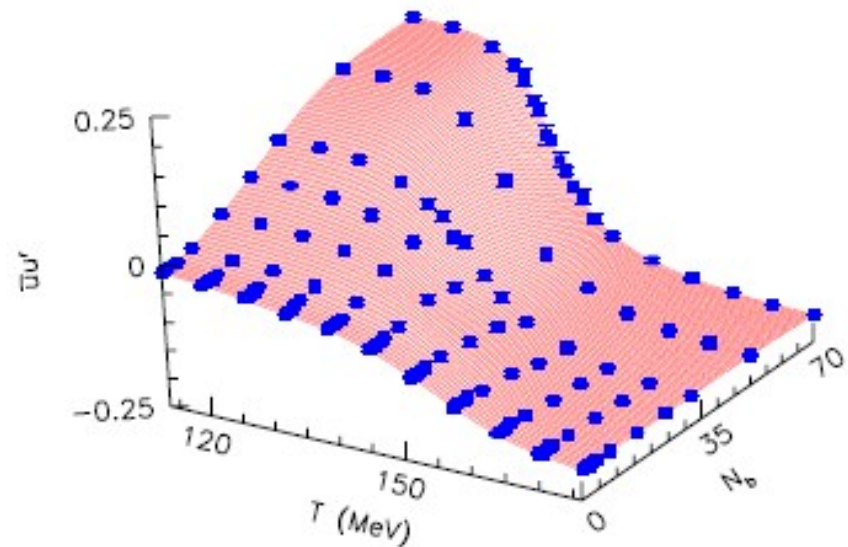
Klimenko, Gysynin-Miransky-Shovkovy

B Effect on the Phase Diagram

QCD phase transitions affected by *B*



(D'Elia et al)



(Fodor et al)

Monte-Carlo simulation is possible (no sign problem)

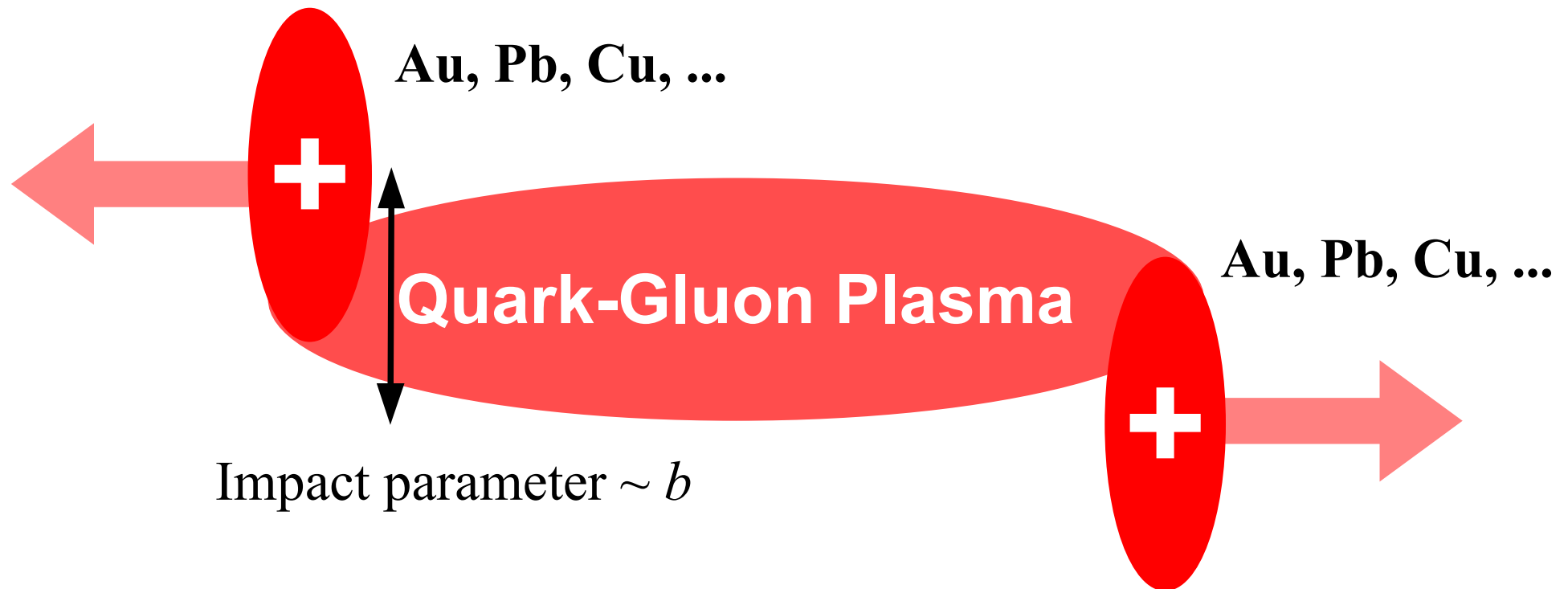
T_c increases or decreases?

We can learn lessons for the high-density QCD!

Reality in Experiments

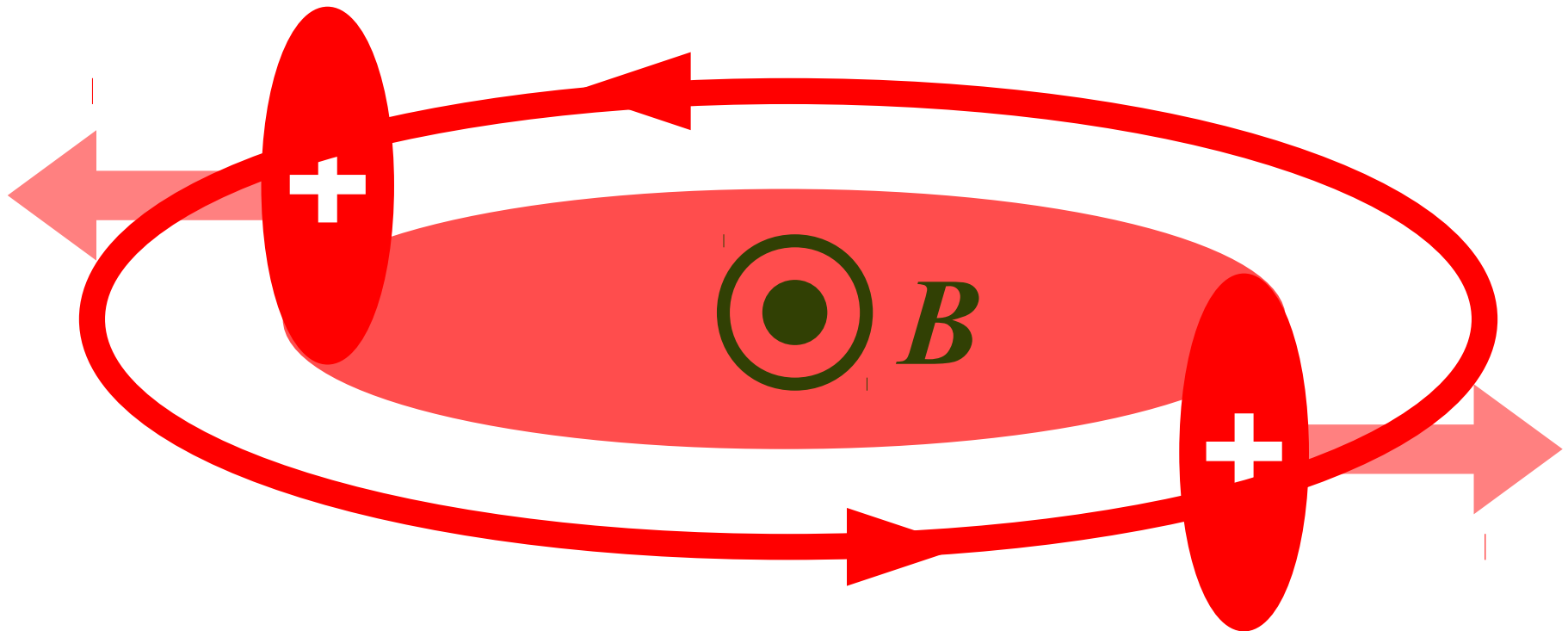
Relativistic Heavy-Ion Collision

Moving almost at the speed of light



Origin of the Magnetic Field

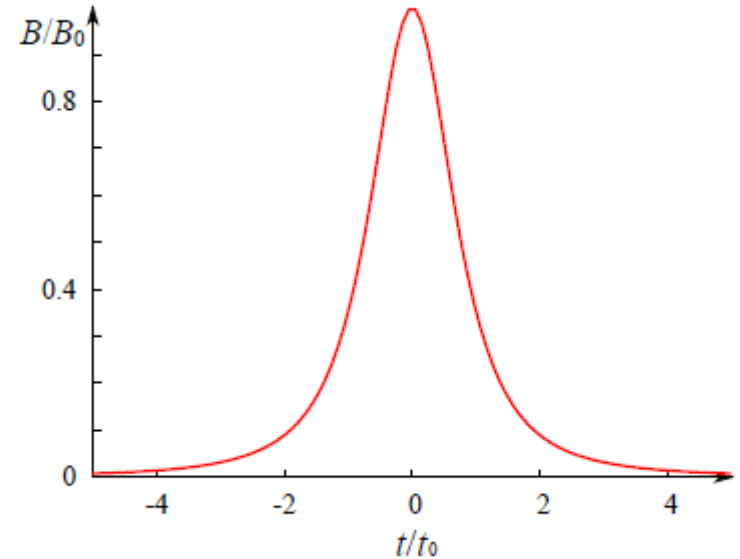
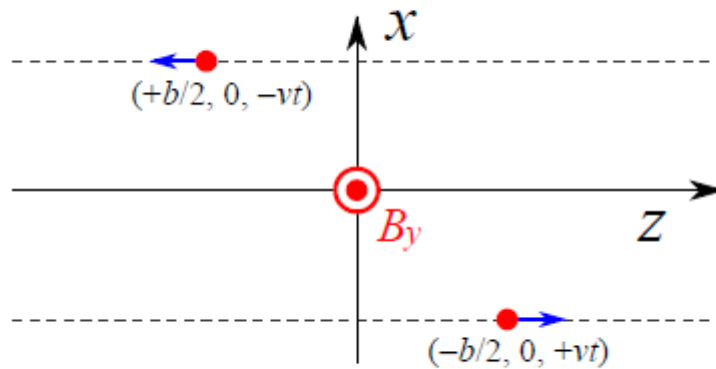
Strong B generated due to Electrodynamics



on top of the Quark-Gluon Plasma

Point-charge Approximation

Lienard-Wiechert potential



$$eB(t) = \frac{eB_0}{[1 + (t/t_0)^2]^{3/2}}$$

$$eB_0 = (47.6 \text{ MeV})^2 \left(\frac{1 \text{ fm}}{b} \right)^2 Z \sinh(Y) , \quad t_0 = \frac{b}{2 \sinh(Y)}$$

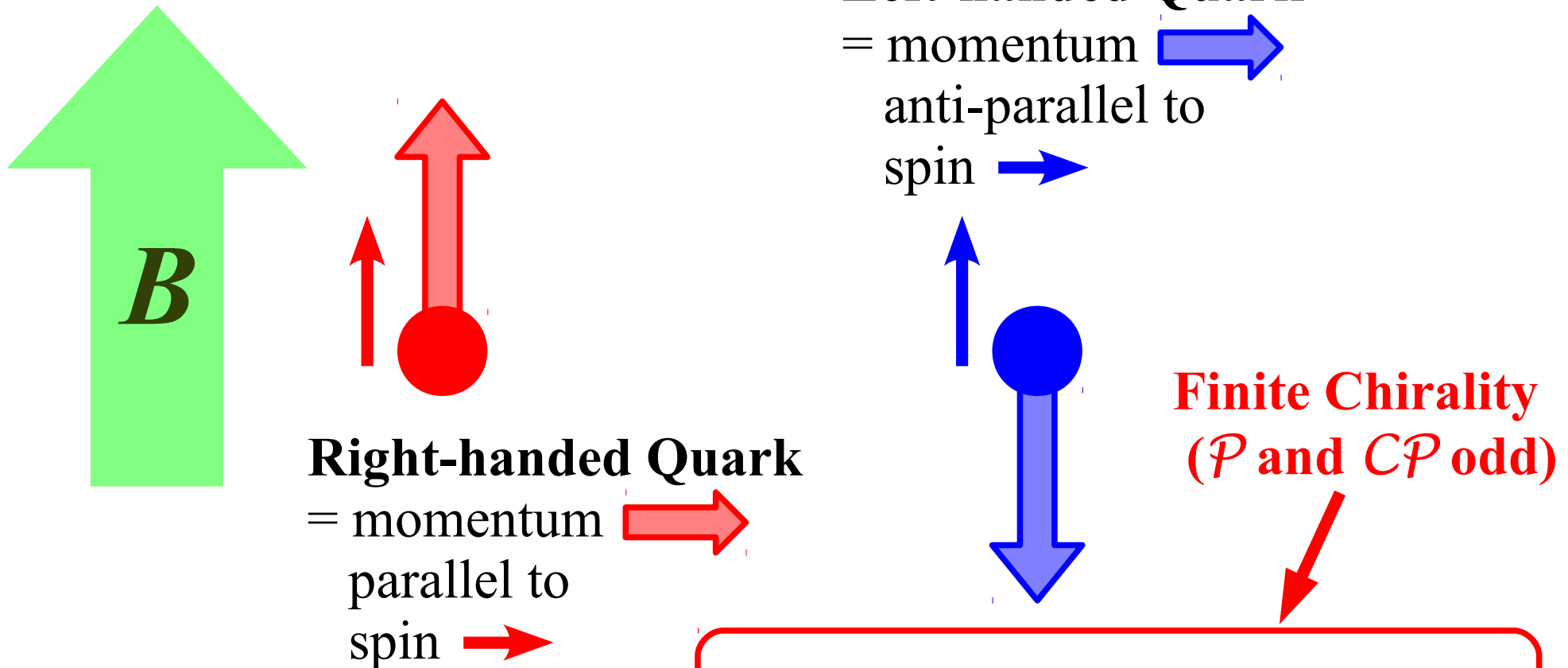
$$\sim 10^{18} \text{ gauss}$$

Interesting phenomena expected!

Discussed by Rafelski, Mueller, ... (~1976)

Chiral Magnetic Effect

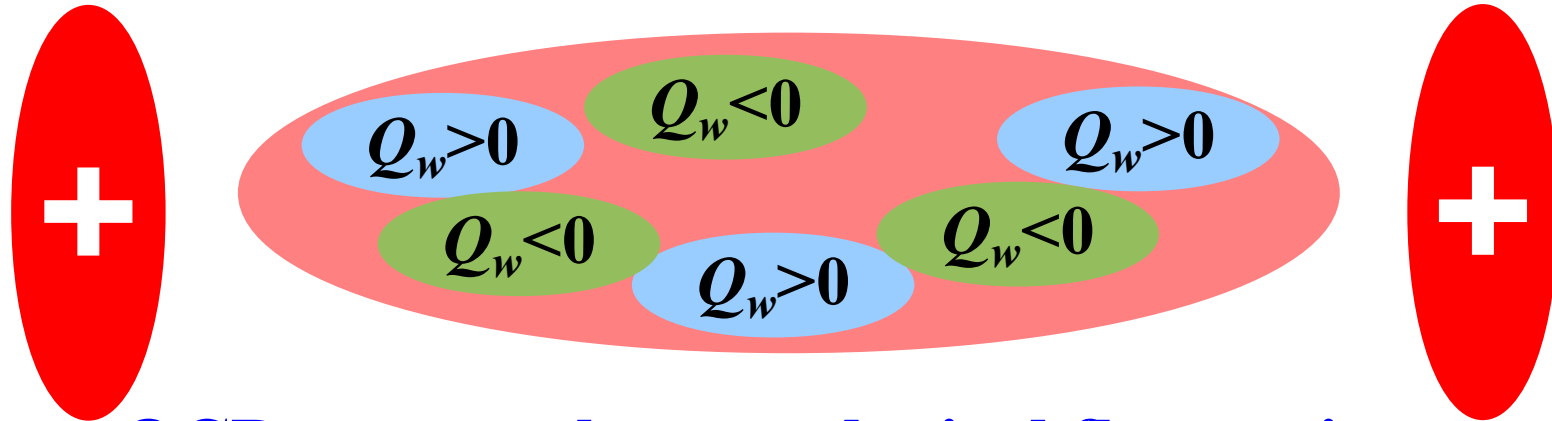
Classical Picture



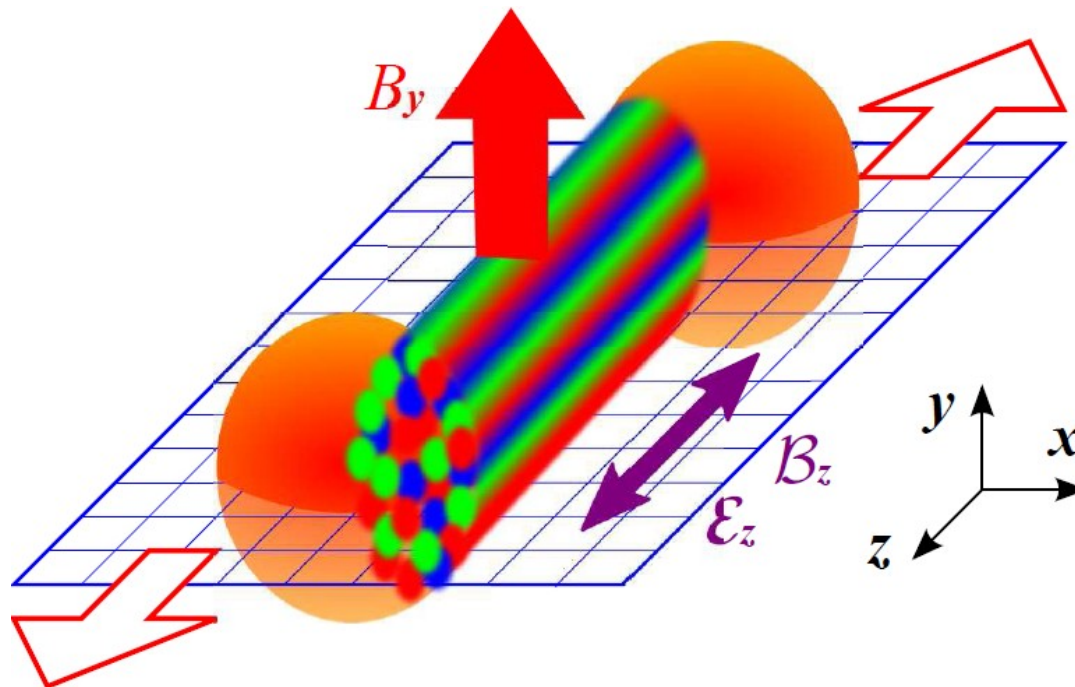
Kharzeev-McLerran-Warringa (2007)

$$J \neq 0 \quad \text{if} \quad N_5 = N_R - N_L \neq 0$$

Local Parity Violation (LPV)



QCD vacuum has topological fluctuations



**Initial condition of HIC
(Color Glass Condensate)
accommodates topological
fluctuations**

Current Generation through Anomaly



Chiral Magnetic Effect → Charge Separation

$$\mathbf{j} = N_c \sum_{\text{flavor}} \frac{q_f^2 \mu_5}{2\pi^2} \mathbf{B}$$

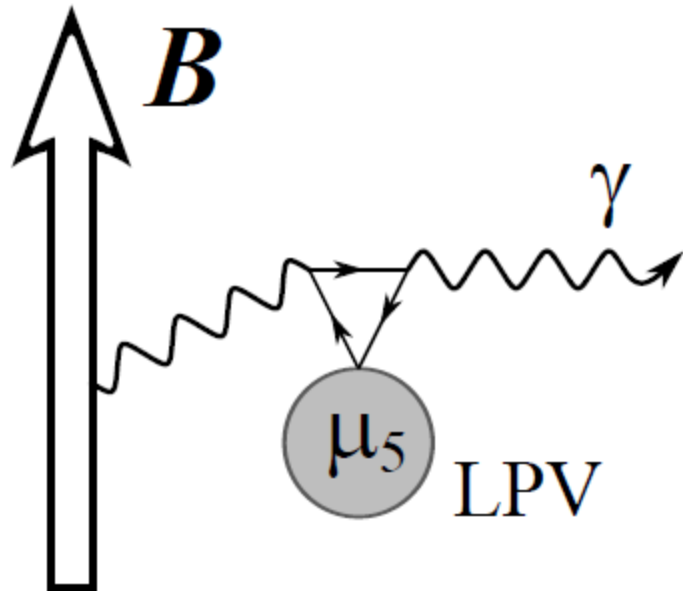
Vilenkin (1980), Metlitski-Zhitnitsky, KF-Kharzeev-Warringa



Photon Production from the CME



$$\Gamma_p = \int N_c \sum_f \frac{q_f^2 B_z(x)}{2\pi^2} \cdot \frac{\partial_t \theta(x)}{2N_f} \cdot A_z(x)$$



Reverse process of the Primakoff effect

$\gamma + \gamma^*$ (background) \rightarrow (neutral meson)



$(\text{LPV}) + \gamma^* \rightarrow \gamma$

KF-Mameda (2012)

Soft photon production is under active discussions

Summary

QCD phase diagram – Chiral and Center Symmetry

- *High Temperature* – Phase transitions well understood from the zero- T properties of confinement.
- *High Baryon Density* – Inhomogeneous states favored near the phase boundary of homogeneous states.
- *Strong B Field* – Effects on the phase diagram not yet understood. Anomalous phenomena (\mathcal{P} and $C\mathcal{P}$ odd effects)

Experimental efforts focused on the baryon-rich matter and the visible effects of the strong B :

- Systematic fluctuation measurements to confirm the local parity violation / critical point / inhomogeneity