“RS-A_4 suppression” of Flavor and CP violation

Avihay Kadosh

Center For Theoretical Physics

University of Groningen

Based on: 1) A.Kadosh and E.Pallante, JHEP08(2010)115

2) A.Kadosh and E.Pallante, JHEP06(2011)121

Outline

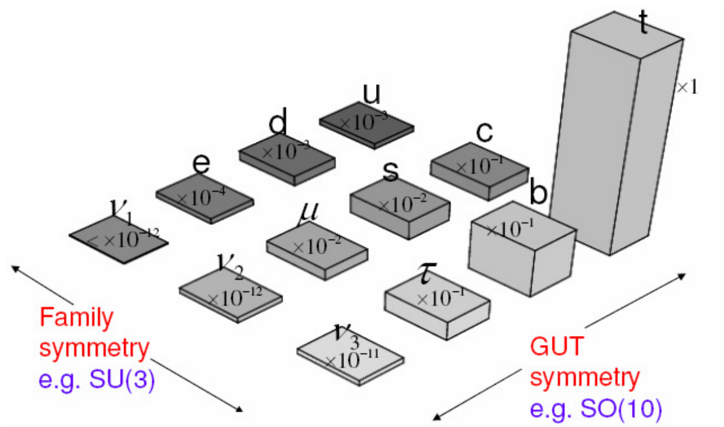
- Motivation
- The RS-A_4 setup
- Main Features
- Phenomenology and comparison with flavor anarchy
- Conclusions

Motivation

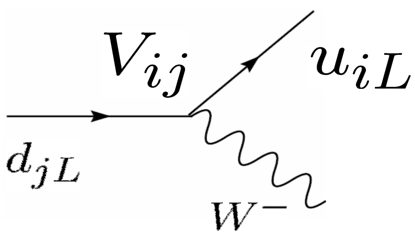
Gauge Hierarchy M_{Pl} vs. TeV $\mathrm{O}(10^{16})$

Find a unified framework to account for the masses and mixing patterns of quarks and leptons.

Fermion mass hierarchy



Smallness and hierarchy of quark mixing angles



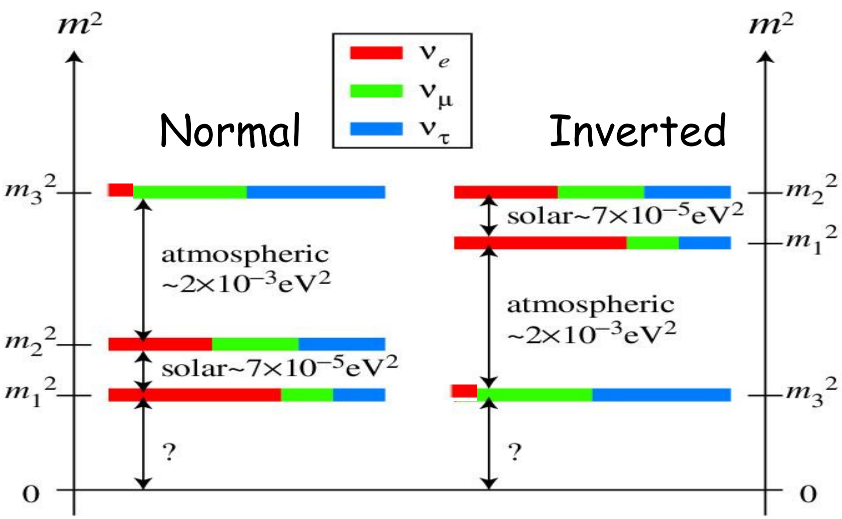
$$V_{ij} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ -\lambda & 1 & \lambda^2 \\ \lambda^3 & -\lambda^2 & 1 \end{pmatrix}$$

$$\lambda_{CKM} \simeq 0.2257$$

Largeness of neutrino mixing angles and smallness of neutrino masses

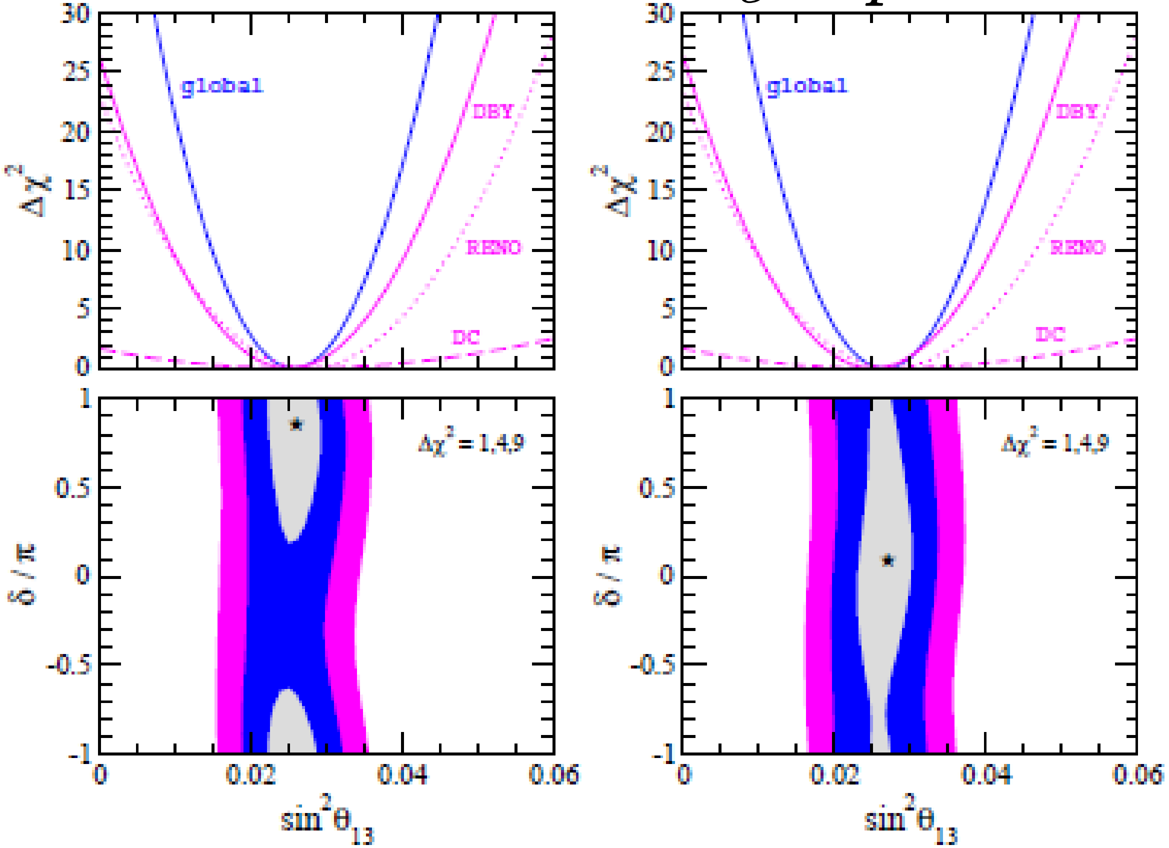
TBM
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$
+H.O ?

See-saw mechanism is the most elegant solution ⇨ $m_\nu = -m_D^T M_R^{-1} m_D$



parameter	best fit $\pm1\sigma$	2σ	3σ
Δm^2_{21} $[10^{-5}\text{eV}^2]$	7.62 ± 0.19	7.27–8.01	7.12–8.20
Δm^2_{31} $[10^{-3}\text{eV}^2]$	$2.53^{+0.08}_{-0.10}$ $-(2.40^{+0.10}_{-0.07})$	$2.34 - 2.69$ $-(2.25 - 2.59)$	$2.26 - 2.77$ $-(2.15 - 2.68)$
$\sin^2\theta_{12}$	$0.320^{+0.016}_{-0.017}$	0.29–0.35	0.27–0.37
$\sin^2\theta_{23}$	$0.49^{+0.08}_{-0.05}$ $0.53^{+0.05}_{-0.07}$	0.41–0.62 0.42–0.62	0.39–0.64
$\sin^2\theta_{13}$	$0.026^{+0.003}_{-0.004}$ $0.027^{+0.003}_{-0.004}$	0.019–0.033 0.020–0.034	0.015–0.036 0.016–0.037
δ	$\left(0.83^{+0.54}_{-0.64}\right)\pi$ 0.07π [a]	$0 - 2\pi$	$0 - 2\pi$

DAYA BAY & RENO $\bar{\nu}_e + p \to e^+ + n$

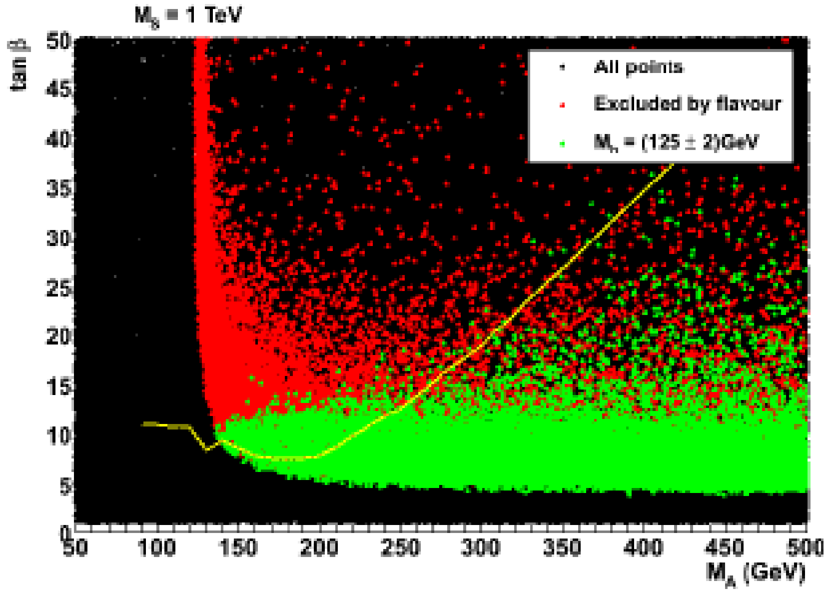


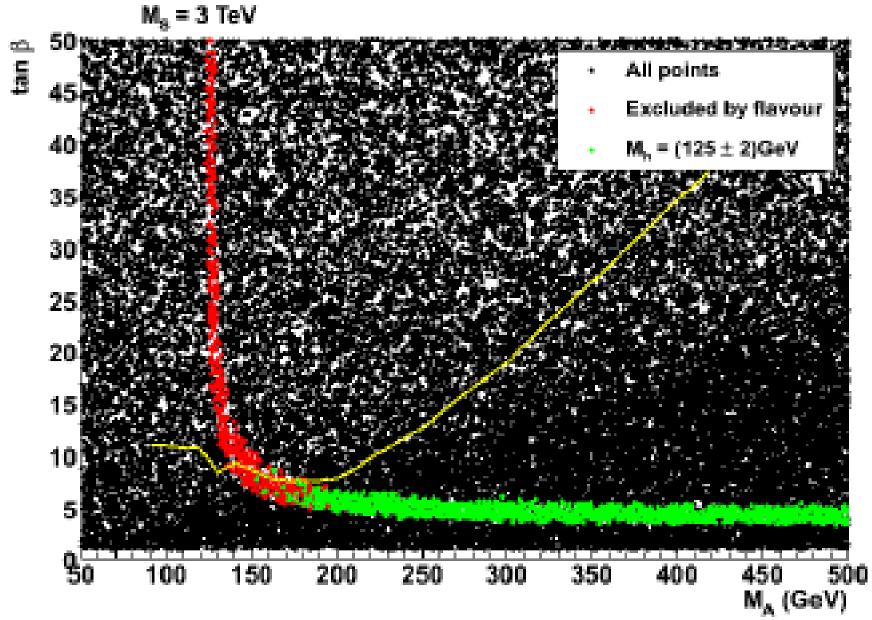
$$\sin^2(\theta_{13}) = 0.026^{+0.003}_{-0.004}(1\sigma)(NH)$$

$$\theta_{13} \simeq \frac{\theta_C}{\sqrt{2}} \approx 9^\circ$$

Valle et al.- arXiv:1205.4018 [hep-ph]

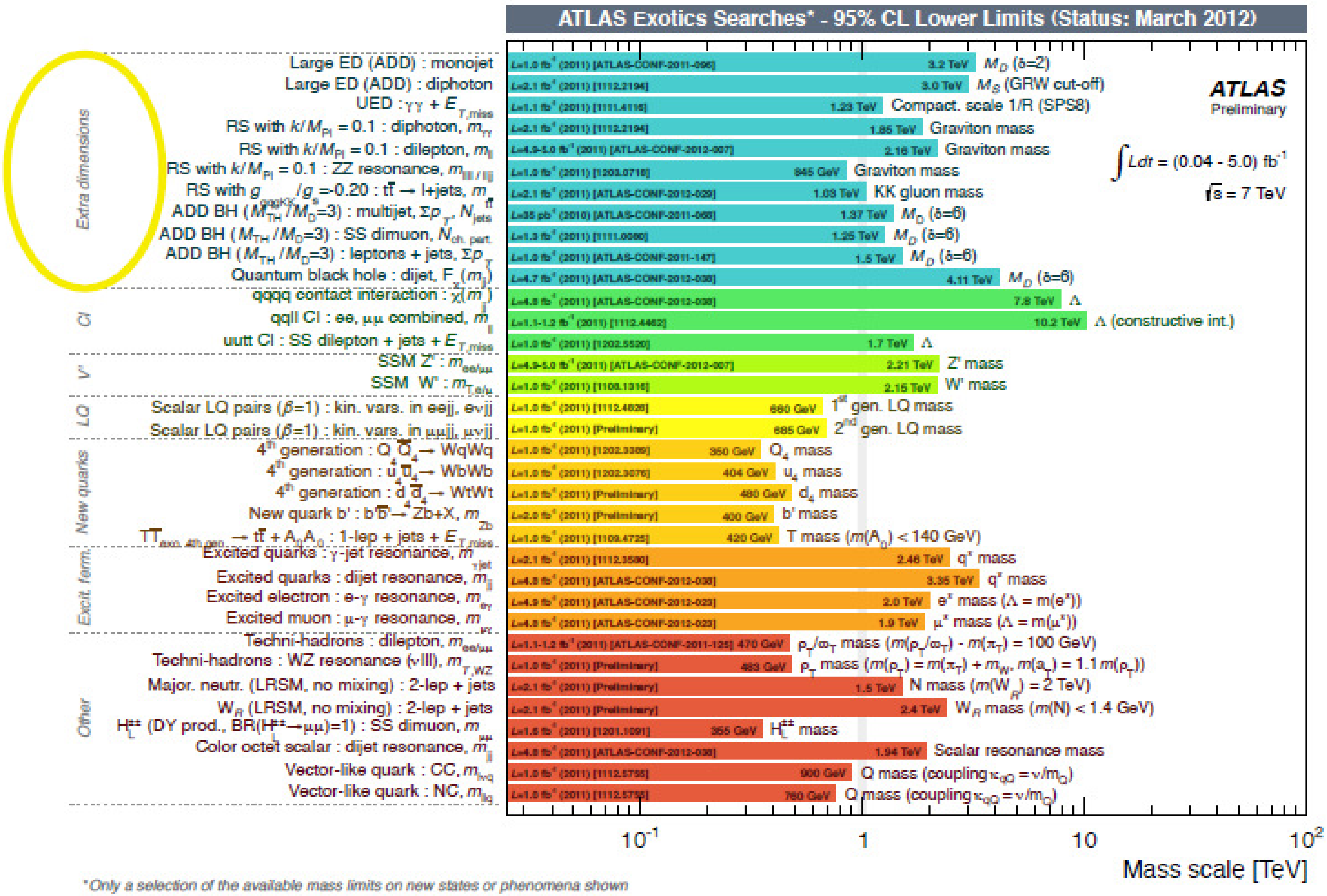
SUSY example: pMSSM maximal mixing scenario (N. Mahmoudi, Moriond 2012)





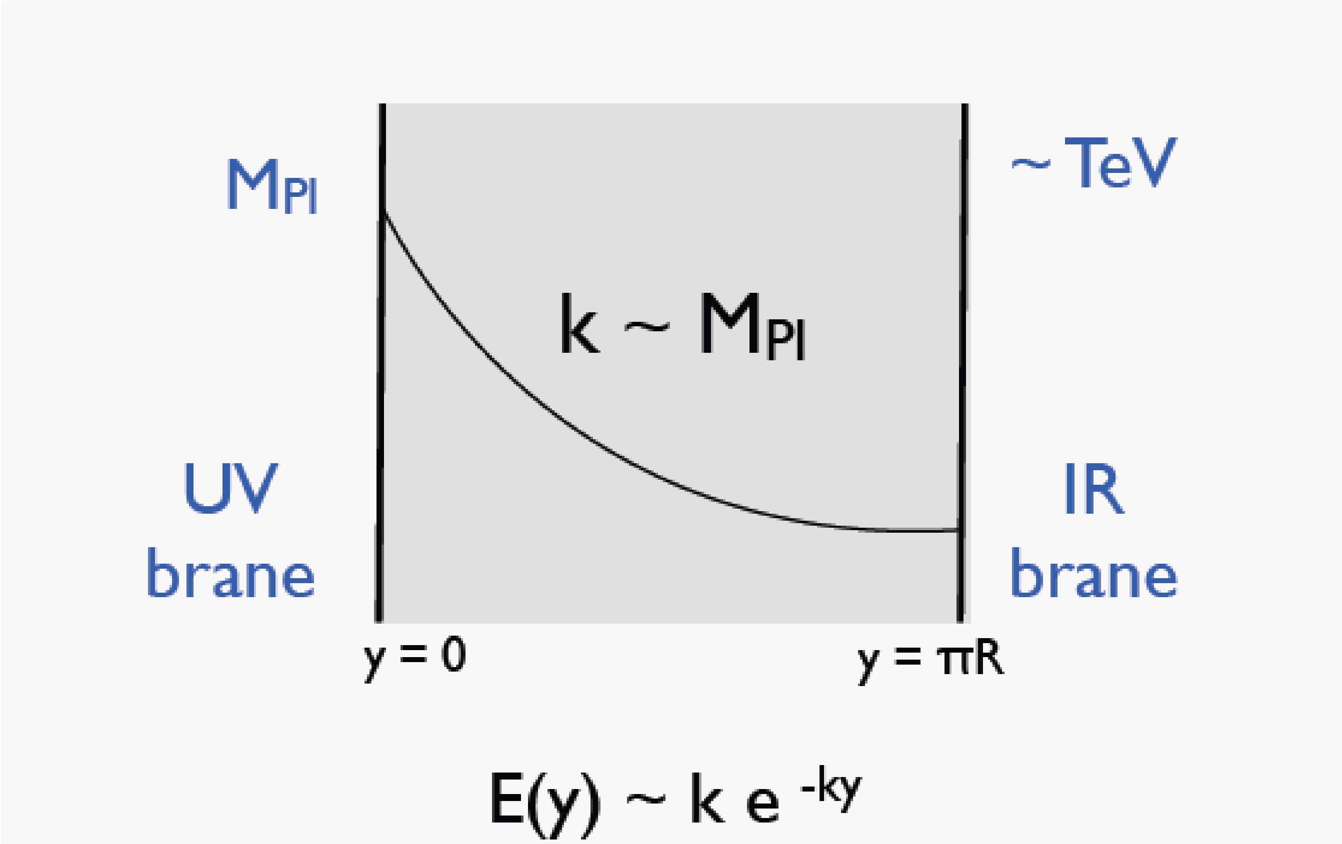
yellow line: CMS limit with 4.6/fb
Flavor constraints from: $b \rightarrow s\gamma$, $B \rightarrow \tau\nu$ and new LHCb limit on $B_s \rightarrow \mu+\mu-$
Assumed discovery of Higgs at (125 ± 2) GeV

LHC constraints on Warped Xdims



(Randall & Sundrum, 1999)

AdS_5 (S_1/Z_2)

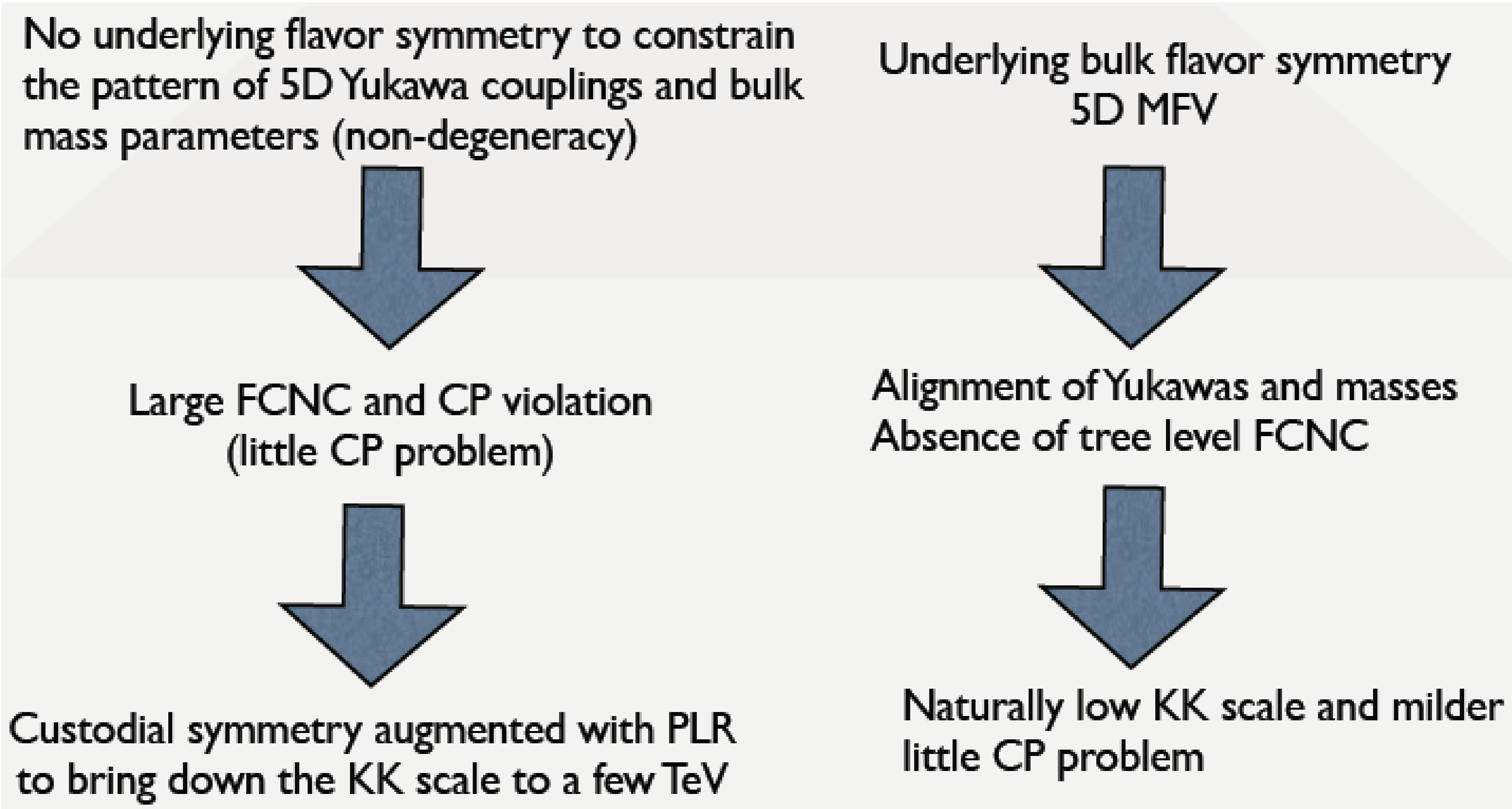


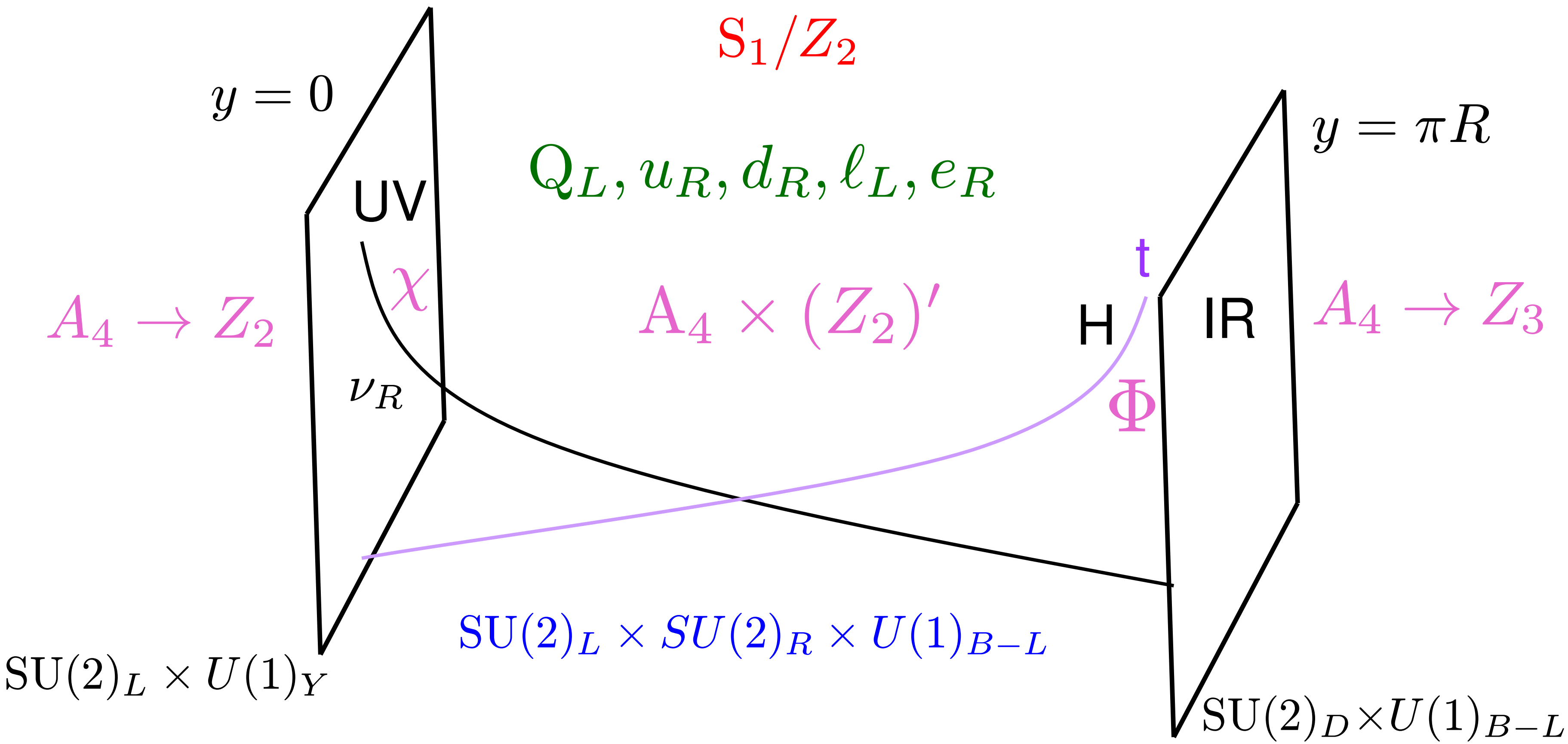
Addressing the "Gauge Hierarchy Problem"

Two scales: $k_{UV} \equiv k \sim M_{Pl}$

$k_{IR} \equiv k\, e^{-\pi k R}$ $kR \simeq 11$ [KK scale]

Anarchy vs. Structure





$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dy^\nu + dy^2$$

$$\Phi(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} \Phi^{(n)}(x^\mu) f_n(y)\,,$$

KK decomposition

$$\chi_0(c_{f_i}, y) = \sqrt{\frac{2k(1/2 - c_{f_i})}{e^{\pi k R(1-2c_{f_i})} - 1}}\, e^{(2-c_{f_i})k|y|}$$

Fermion zero modes

The Yukawa Lagrangian

$$\mathcal{L}_{5D}^{Yuk} = \underbrace{\mathcal{L}_{LO}}_{\text{UV/IR}} + \underbrace{\mathcal{L}_{NLO}}_{\text{Cross brane}}$$

UV/IR:

$\Lambda_{5D}^{-1/2}\bar{\ell}_L H \nu_R$

$(\Lambda_{5D}^{-1/2}\chi, M)\bar{\nu}_R^c \nu_R$

$\Lambda_{5D}^{-2}\bar{Q}_L(\bar{\ell}_L)\Phi H(u_R, d_R, e_R)$

Cross brane:

$\Lambda_{5D}^{-3/2}\bar{\ell}_L H\chi\nu_R$

$\Lambda_{5D}^{-7/2}\bar{Q}_L(\ell_L)\Phi\chi H(u_R, d_R, (e_R))$

A_4 Assignments

$\mathrm{SU}(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times A_4$

scalars $\Phi \sim (1,1,1,0)\,(\underline{\mathbf{3}})$, $\chi \sim (1,1,1,0)\,(\underline{\mathbf{3}})$, $H\,(1,2,2,0)\,(\underline{\mathbf{1}})$

Quarks:

$$Q_L \sim \left(3,2,1,\frac{1}{3}\right)(\underline{\mathbf{3}})$$

$$u_R \oplus u'_R \oplus u''_R \sim \left(3,1,2,\frac{1}{3}\right)(\underline{\mathbf{1}} \oplus \underline{\mathbf{1}}' \oplus \underline{\mathbf{1}}'')$$

$$d_R \oplus d'_R \oplus d''_R \sim \left(3,1,2,\frac{1}{3}\right)(\underline{\mathbf{1}} \oplus \underline{\mathbf{1}}' \oplus \underline{\mathbf{1}}'')$$

Leptons:

$$\ell_L \sim (1,2,1,-1)\,(\underline{\mathbf{3}})$$

$$\nu_R \sim (1,1,2,0)\,(\underline{\mathbf{3}})$$

$$e_R \oplus e'_R \oplus e''_R \sim (1,1,2,-1)\,(\underline{\mathbf{1}} \oplus \underline{\mathbf{1}}' \oplus \underline{\mathbf{1}}'')$$

LO Results

$$m_\nu = -m_D^T M_R^{-1} m_D$$

SSB of A_4 $\langle \Phi \rangle = (v_\Phi, v_\Phi, v_\Phi)$

$A_4 \to Z_3$

$$V_L^{u,d,e} = U(\omega) = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}$$

$\omega = e^{2\pi i/3}$

$$V_R^{u,d,e} = 1_{3\times 3}$$

$$V_{CKM} = V_L^{u\dagger} V_L^d = 1$$

No quark mixing at LO!

$\langle \chi \rangle = (0, v_\chi, 0)$

$A_4 \to Z_2$

$$V_L^\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$V_R^\nu = V_L^{\nu\,T}$$

$$V_L^{e\dagger} V_L^\nu = U(\omega)^\dagger V_L^\nu = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{\omega^2}{\sqrt{6}} & \frac{\omega^2}{\sqrt{3}} & -\frac{e^{-i\pi/6}}{\sqrt{2}} \\ -\frac{\omega}{\sqrt{6}} & \frac{\omega}{\sqrt{3}} & \frac{e^{-5i\pi/6}}{\sqrt{2}} \end{pmatrix}$$

TBM neutrino mixing

$(J_{CP}^\nu = 0)$

NLO Corrections to the CKM Matrix

- Cross brane flavon interactions induce deviations of the CKM matrix from unity

$$M_q+\Delta M_q = U(\omega)\sqrt{3}\begin{pmatrix} \tilde{y}_{q_1}v+(x_1^q+y_1^q)/3 & (x_2^q+y_2^q)/3 & (x_3^q+y_3^q)/3 \\ (x_1^q+\omega y_1^q)/3 & \tilde{y}'_{q_2}v+(x_2^q+\omega y_2^q)/3 & (x_3^q+\omega y_3^q)/3 \\ (x_1^q+\omega^2 y_1^q)/3 & (x_2^q+\omega^2 y_2^q)/3 & \tilde{y}''_{q_3}v+(x_3^q+\omega^2 y_3^q)/3 \end{pmatrix}$$

q=u,d

12 complex parameters $(x_i^q, y_i^q) = f_\chi^q C_\chi(\tilde{x}_i^q, \tilde{y}_i^q) \simeq 0.05(\tilde{x}_i^q, \tilde{y}_i^q)$

•Parameterizing V_{CKM} in terms of λ : $\mathcal{O}(x_i^{u,d}, y_i^{u,d})$

$$\Longrightarrow V_{CKM} = \begin{pmatrix} 1+\mathcal{O}(\lambda^2) & a\lambda & b\lambda^3 \\ -a^*\lambda & 1+\mathcal{O}(\lambda^2) & c\lambda^2 \\ -b^*\lambda^3 & -c^*\lambda^2 & 1+\mathcal{O}(\lambda^2) \end{pmatrix}$$

$|V_{ub}| \neq |V_{td}|$ and phase structure $\Longrightarrow$ $\mathcal{O}\left((x_i^{u,d})^2, (y_i^{u,d})^2\right)$

NLO Corrections to the PMNS Matrix

- Cross brane flavon interactions induce deviations of the PMNS matrix from TBM

$$M_\ell + \Delta M_\ell = U(\omega)\sqrt{3}\begin{pmatrix} \tilde{y}_e v + (x_1^\ell + y_1^\ell)/3 & (x_2^\ell + y_2^\ell)/3 & (x_3^\ell + y_3^\ell)/3 \\ (x_1^\ell + \omega y_1^\ell)/3 & \tilde{y}_\mu v + (x_2^\ell + \omega y_2^\ell)/3 & (x_3^\ell + \omega y_3^\ell)/3 \\ (x_1^\ell + \omega^2 y_1^\ell)/3 & (x_2^\ell + \omega^2 y_2^\ell)/3 & \tilde{y}_\tau v + (x_3^\ell + \omega^2 y_3^\ell)/3 \end{pmatrix}$$

12 complex parameters

$$V_L^{\ell(NLO)} = U(\omega)\begin{pmatrix} 1 & \lambda_\ell(\tilde{x}_2^\ell + \tilde{y}_2^\ell) & \lambda_\ell(\tilde{x}_3^\ell + \tilde{y}_3^\ell) \\ -\lambda_\ell(\tilde{x}_2^{\ell *} + \tilde{y}_2^{\ell *}) & 1 & \lambda_\ell(\tilde{x}_3^\ell + \omega\tilde{y}_3^\ell) \\ -\lambda_\ell(\tilde{x}_3^{\ell *} + \tilde{y}_3^{\ell *}) & -\lambda_\ell(\tilde{x}_3^{\ell *} + \omega^2\tilde{y}_3^{\ell *}) & 1 \end{pmatrix}$$

$$\Delta V_L^{\nu(NLO)} = \begin{pmatrix} \epsilon_{11}^\nu & 0 & \epsilon_\nu \\ 0 & \epsilon_{22}^\nu & 0 \\ \epsilon_\nu & 0 & \epsilon_{11}^{\nu *} \end{pmatrix} \Rightarrow V_{PMNS} = (V_L^{\ell\dagger} V_L^\nu)^{(NLO)}$$

$$|\epsilon_{11,22}^\nu| \approx |\epsilon_\nu|^2$$

$$\delta \equiv 2\arctan\epsilon_{11}$$

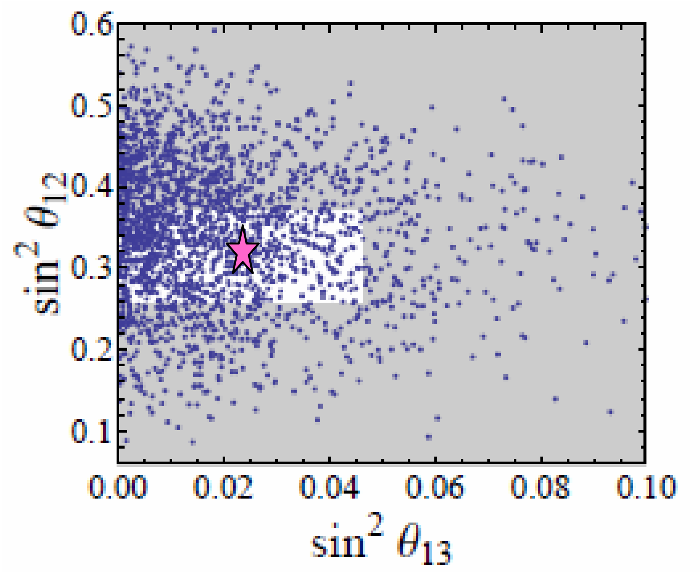
- Parameterizing V_{PMNS} in terms of $\lambda_\ell = f_\chi^\ell C_\chi \simeq 0.05$, $\epsilon_\nu \simeq 0.08$

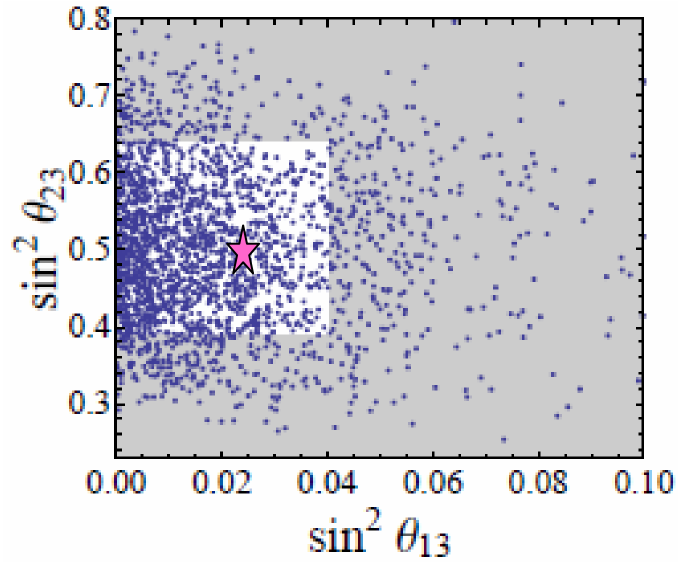
NLO Corrections to the PMNS matrix-(cont.)

$$\theta_{13}^{NLO} \simeq \frac{e^{i\delta}-1}{\sqrt{6}} - \frac{\epsilon_\nu}{\sqrt{3}} + \frac{1-\omega e^{i\delta}}{\sqrt{6}}(\tilde{x}_2^\ell + \tilde{y}_2^\ell + \omega \tilde{x}_3^\ell + \omega \tilde{y}_3^\ell)\lambda_\ell$$

$$\theta_{23}^{NLO} \simeq \frac{\omega e^{i\delta}-1}{\sqrt{6}} - \frac{\epsilon_\nu}{\sqrt{3}} + \frac{e^{i\delta}-1}{\sqrt{6}}(\tilde{x}_2^{\ell *} + \tilde{y}_2^{\ell *})\lambda_\ell + \frac{1-\omega^2 e^{i\delta}}{\sqrt{6}}(\tilde{x}_3^{\ell *} + \omega^2 \tilde{y}_3^{\ell *})\lambda_\ell$$

$$\theta_{12}^{NLO} \simeq \frac{1}{\sqrt{3}} - \frac{\omega^2}{\sqrt{3}}(\tilde{x}_2^\ell + \tilde{y}_2^\ell)\lambda_\ell - \frac{\omega}{\sqrt{3}}(\tilde{x}_3^\ell + \tilde{y}_3^\ell)\lambda_\ell$$



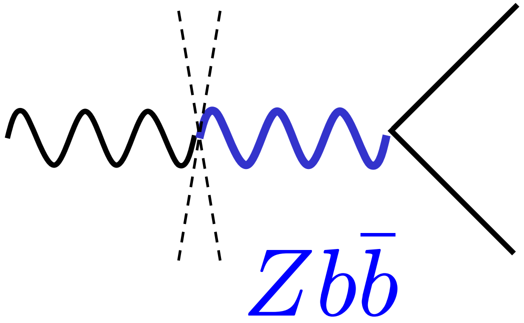


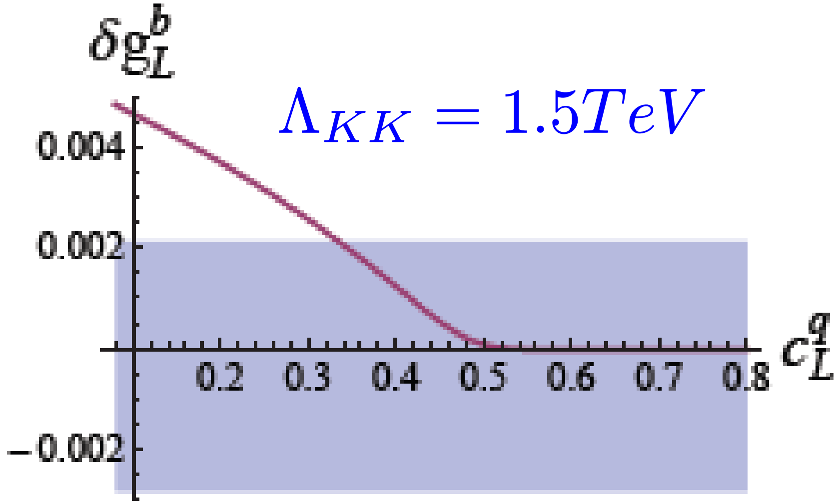
Current Neutrino oscillation data (Including Daya Bay and RENO) can still be explained with natural O(1) parameters!!!

Main Features of RS-A_4 setup

Degenerate C_L!!!

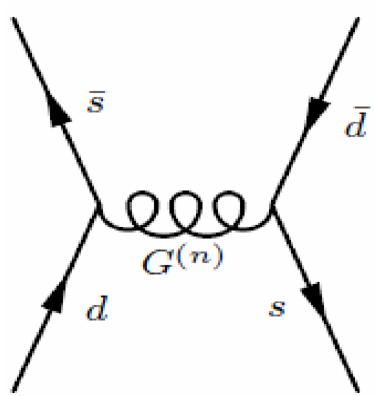
EWPM





$$\epsilon_K^{NP} = 0$$

vs.



$M_{KK} \gtrsim 10\,TeV$

(In flavor anarchy)

Neutron EDM at 1-loop and HMFCNC $\left(C_{2,4}^{K,D}\right)$ at tree level, strongly suppressed.

Phenomenology-Dipole Operators

As a first step we work in the mass insertion approximation

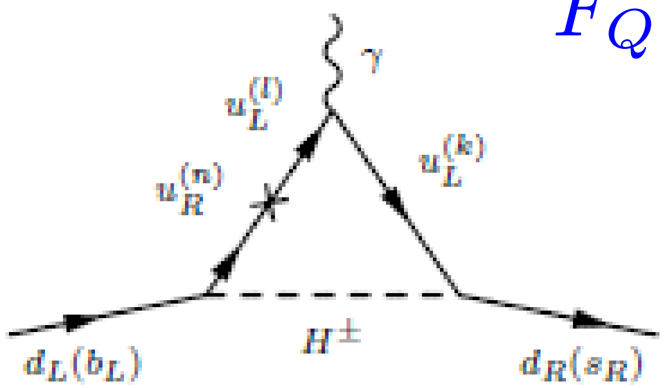
-Flavor part of amplitude in terms of spurions $F_Q, F_{u,d}, \hat{Y}_{u,d}$

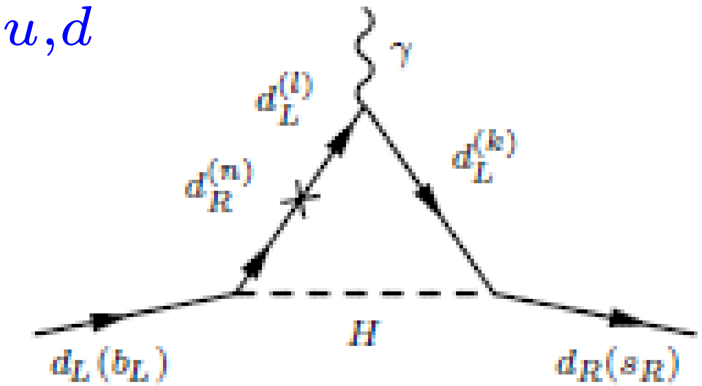
(Agashe, Soni, Perez 2004)

-IR Higgs vs. Bulk Higgs couplings $r_{nm}^{H\Phi(\chi)}(c_{Q_i}, c_{u_j,d_j}, \beta)$

$$O_7^\gamma = \bar{q}_L^i \sigma^{\mu\nu} F_{\mu\nu} q_R^j \qquad i = j = d\ (EDM)$$

$$F_Q Y_{u,d} Y_{u,d}^\dagger Y_d F_{u,d}$$





Dipole Operators (cont.)

$$(C_{7\gamma(8g)}^{d-type})_{ij} = \frac{m_{d_i} A^{1L} f_Q^2}{v^2 M_{KK}} \Big[V_R^{d\dagger} \mathrm{d}iag(f_{d,s,b}^2) (\hat{r}_{00}^d)^{-1} \tilde{r}_{01}^d \, \tilde{r}_{11}^d (\hat{r}_{00}^d)^{-1} V_R^d$$
$$\times \mathrm{d}iag(m_{d,s,b}^2) V_R^{d\dagger} \, (\hat{r}_{00}^d)^{-1} \hat{r}_{10}^d \, V_R^d \, \Big]_{ij}$$

$$\tilde{r}_{01}^{u,d} \tilde{r}_{11}^{u,d} = \hat{r}_{01}^{u,d} \hat{r}_{11}^{u,d} + \hat{r}_{01-+}^{u,d} \hat{r}_{1-+1}^{u,d}$$ → Overlap corrections

Various levels of Approximation

$(V_R^d)_{LO} = 1_{3\times 3}$ → EDM=0

$(V_R^d)_{NLO}$ + Degenerate Overlaps → EDM=0

$(V_R^d)_{NLO}$ + Non degenerate Overlaps

→ $EDM \sim \mathcal{O}\left((m_d/m_s)^2 f_\chi^{u,d} \Delta r\right) \approx 10^{-29} e \cdot cm$

Dipole Operators (cont.)

Main drawback of spurion analysis → Failure to account for the explicit coupling to the various types (BC) of KK modes.

Second step- diag. of the 1 gen. KK mass matrix

$$(\mathcal{A}_{ij})_D^{overlap} = \frac{\left(\sum_n (\hat{Y}_{KK}^{d_i})_{1n}^{mass} (\hat{Y}_{KK}^{d_j})_{n1}^{mass}\right)\Big|_{overlap}}{M_{KK}^{(n)d_j}}$$

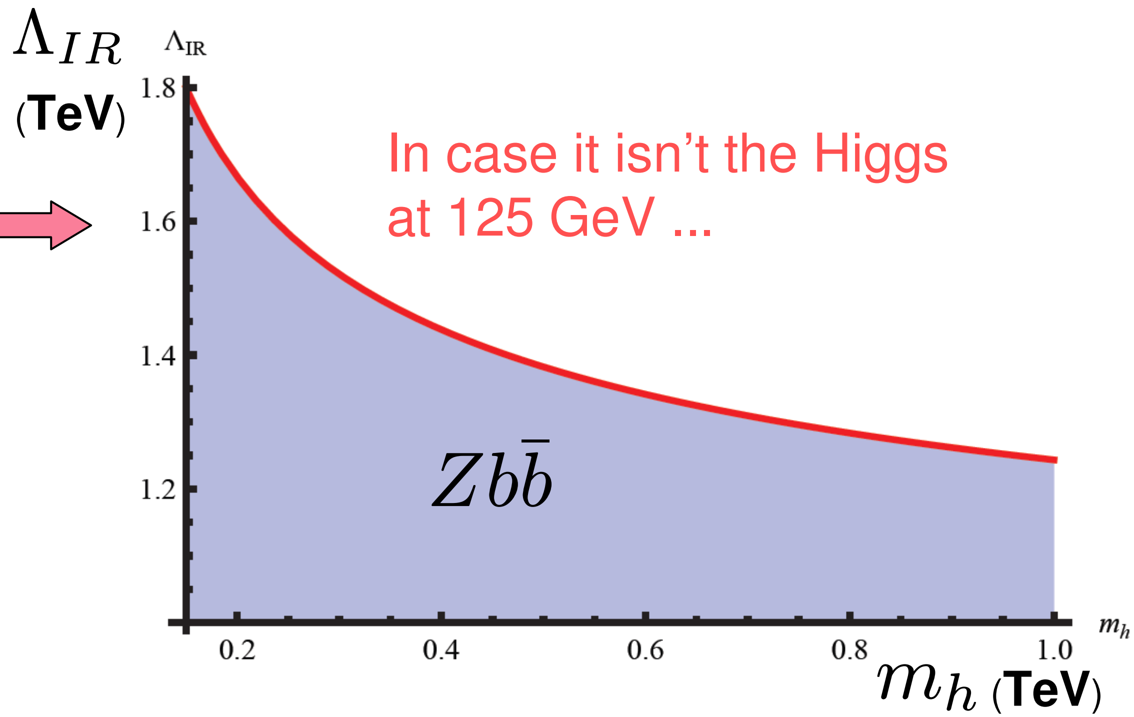
Generational mixing effects → Mass insertion approx.

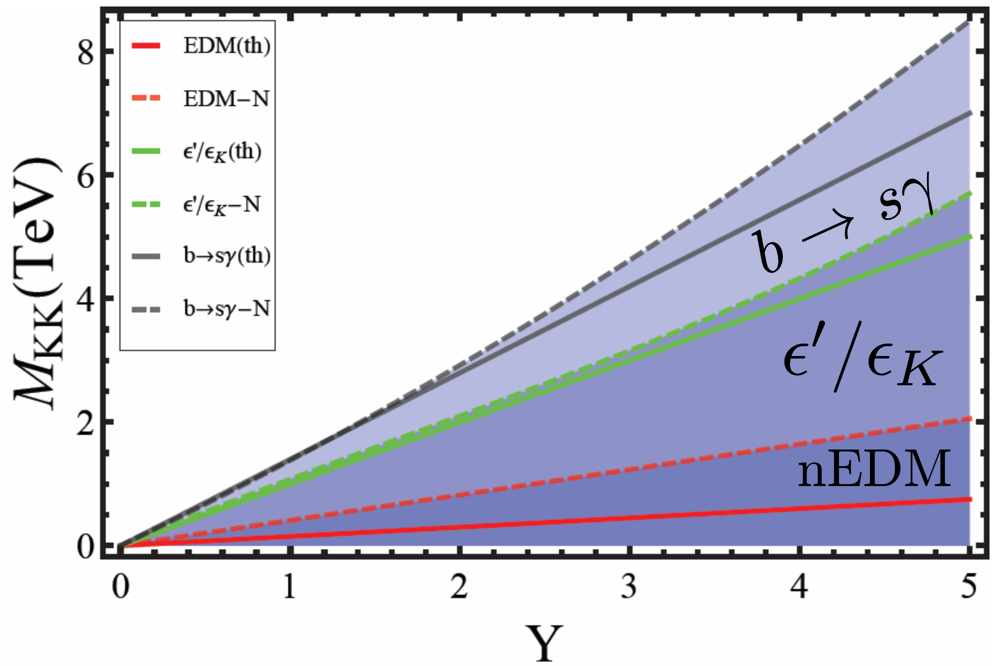
(Agashe-Azatov-Zhu 2009, Gedalia-Isidori-Perez 2009)

Third step- Approximate analytical and numerical diag. of 3 gen. zero $+1^{st}$ KK mass matrices. (12x12)

The constraint of Zbb with running of Higgs mass

(Casagrande, Neubert et al. , 2010)





Most significant constraints from dipole operators

Conclusions

$RS\text{-}A_4$ → Vacuum Alignment, Flavor Hierarchy
EW-Planck Hierarchy, CKM, TBM+…,
Neutrino masses. Naturalness!
EWPM constrain bulk masses!

Significant Relaxation of Pheno. constraints compared to flavor anarchy, due to degeneracy of C_L!!!
New constraints from LHCb, MEG, ATLAS, CMS,…

Possible extensions-

P_{LR} extended Custodial Symm. → ZMA remains the same!

Larger (other…) flavor symmetries

"Soft Wall" , Radion Phenomenology,…

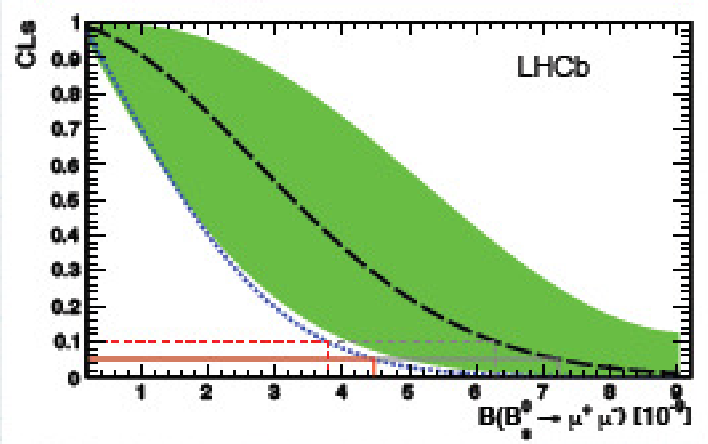
Questions

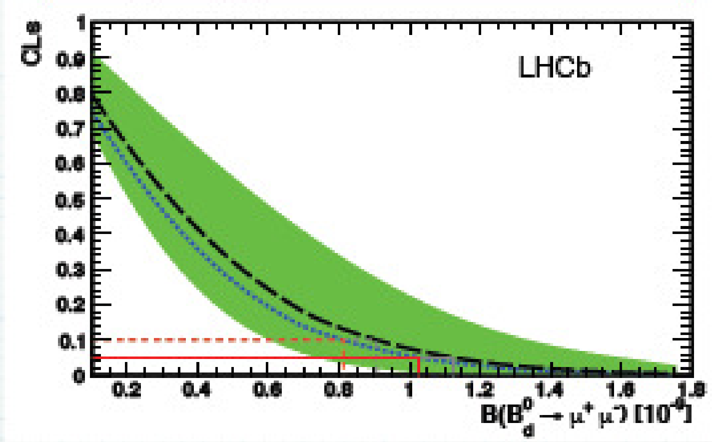
(. . .)

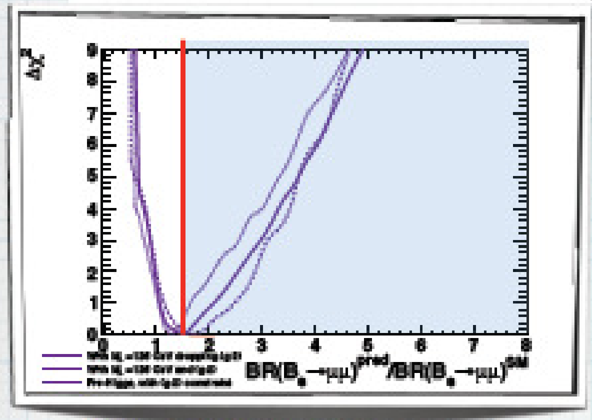
$B_{(s)} \to \mu\mu$

$B(Bs \to \mu\mu) < 4.5\ 10^{-9}$ at 95% CL

$B(B \to \mu\mu) < 10.3\ 10^{-10}$ at 95% CL







mode	limit	at 90% C.L.	at 95% C.L.
$B^0_s \to \mu^+\mu^-$	expected bg+SM	6.3×10^{-9}	7.2×10^{-9}
	expected bg only	2.8×10^{-9}	3.4×10^{-9}
	observed	3.8×10^{-9}	4.5×10^{-9}
$B^0 \to \mu^+\mu^-$	expected	9.1×10^{-10}	11.3×10^{-10}
	observed	8.1×10^{-10}	10.3×10^{-10}

best limit!

1-CLb = 0.18

1-CLb =0.60

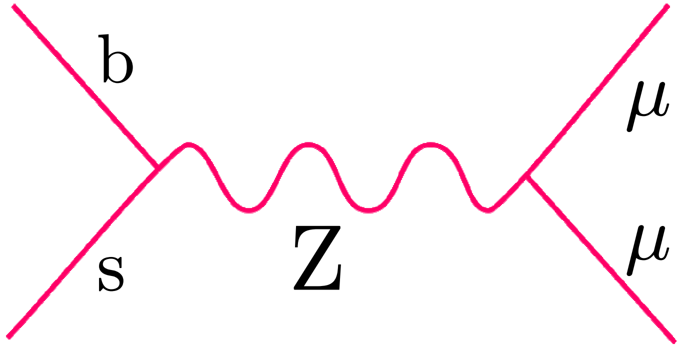
BR estimation:

$\mathcal{B}(Bs \to \mu\mu) = (0.8^{+1.8}_{-1.3})\ 10^{-9}$

simultaneous unbinned LL fit to the mass to the 8 BDT bins

expected BR from minimum of the LL and error from ΔLL=0.5, coverage BR [0,SM] 82%

RS-A$_4$ contributions to $B_{s,d} \to \mu^+\mu^-$



$$\frac{BR(B_s \to \mu^+ \mu^-)_{NP}}{BR(B_s \to \mu^+ \mu^-)_{SM}} = \frac{|(Y_Z^B)^{V-A}|^2}{Y(M_W^2/m_t^2)}$$

$$BR(B_s \to \mu^+\mu^-)_{SM} \simeq 3.2 \pm 0.2 \times 10^{-9}$$

(1-loop penguins and boxes are extremely suppressed)

$$(Y_Z^B)^{V-A} = \frac{(-V_{tb}^* V_{ts})^{-1}}{4M_Z^2 g_{SM}^2} \underbrace{(\Delta_L^{\ell\ell}(Z) - \Delta_R^{\ell\ell}(Z))}_{\text{SM like}} \underbrace{(\Delta_L^{sb}(Z) - \Delta_R^{sb}(Z))}_{\text{Extremely suppressed}}$$

$$g_{SM}^2 \equiv \frac{G_F \alpha}{2\sqrt{2}\pi \sin^2 \theta_W}$$

SM like

Extremely suppressed
Double layer KK mixing

$$\mathrm{BR}(\mathrm{B}_s \to \mu^+\mu^-)_{RS-A_4} \approx 3 \times 10^{-11}$$

$$\mathrm{BR}(\mathrm{B}_d \to \mu^+\mu^-)_{RS-A_4} \approx 2 \times 10^{-13}$$

$$\mathcal{L}^{4D} \supset \hat{Y}^{u,d}_{ij} h^{(\,,*)}_{0(4D)} \Big[\psi^{0\dagger}_{Q_i} f^{-1}_{Q_i} \psi^{0}_{u_j,d_j} f^{-1}_{u_j,d_j} r^{H}_{00}(c_{Q_i}, c_{u_j,d_j}, \beta) + \sum_n \psi^{0\dagger}_{Q_i} f^{-1}_{Q_i} \psi^{n}_{u_j,d_j} r^{H}_{0n}(c_{Q_i}, c_{u_j,d_j}, \beta)$$
$$+ \sum_n \psi^{0\dagger}_{Q_i} f^{-1}_{Q_i} \psi^{n-+}_{u_j,d_j} r^{H}_{0n-+}(c_{Q_i}, c_{d_j,u_j}, \beta) + \sum_n \psi^{n\dagger}_{Q_i} \psi^{0}_{u_j,d_j} f^{-1}_{u_j,d_j} r^{H}_{n0}(c_{Q_i}, c_{u_j,d_j}, \beta)$$
$$+ \sum_{n,m} \psi^{n\dagger}_{Q_i} \psi^{m}_{u_j,d_j} r^{H}_{nm}(c_{Q_i}, c_{u_j,d_j}, \beta) + \sum_{n,m} \psi^{n--}_{Q_i} (\psi^{m--}_{u_j,d_j})^{\dagger} r^{H}_{n^-m^-}(c_{Q_i}, c_{u_j,d_j}, \beta)$$
$$+ \sum_{n,m} \psi^{n\dagger}_{Q_i} \psi^{m-+}_{u_j,d_j} r^{H}_{nm-+}(c_{Q_i}, c_{d_j,u_j}, \beta) + \sum_{n,m} \psi^{n--}_{Q_i} (\psi^{m+-}_{u_j,d_j})^{\dagger} r^{H}_{n^-m^{+-}}(c_{Q_i}, c_{d_j,u_j}, \beta) \Big]$$

$$\hat{Y}^{u(d)}_{ij} = \frac{1}{v} r^{-1}_{00}(c_{Q_i}, c_{u_j(d_j)}, \beta) \left(F^{-1}_{Q} V^{u(d)}_{L} \mathrm{diag}(m_{u,c,t(d,s,b)}) V^{u(d)\dagger}_{R} F^{-1}_{u(d)} \right)_{ij}$$

$$(C^{d-type}_{7\gamma(8g)})_{ij} = \frac{A_{1L}}{v^2 M_{KK}} \Big[(V^{d}_{L})^{\dagger}_{i\ell} (\hat{r}^{d}_{0n})_{\ell\ell_1} (\hat{r}^{d}_{00})^{-1}_{\ell\ell_1} \left(V^{d}_{L} \mathrm{diag}(m_{d,s,b}) (V^{d}_{R})^{\dagger} \mathrm{diag}(f^{2}_{d,s,b}) \right)_{\ell\ell_1}$$
$$\times (\hat{r}^{d}_{nm})_{\ell_2\ell_1} (\hat{r}^{d}_{00})^{-1}_{\ell_2\ell_1} \left(V^{d}_{R} \mathrm{diag}(m_{d,s,b}) V^{d\dagger}_{L} \mathrm{diag}(f^{2}_{Q1,Q2,Q3}) \right)_{\ell_1\ell_2} (\hat{r}^{d}_{m0})_{\ell_2\ell_3}$$
$$\times (\hat{r}^{d}_{00})^{-1}_{\ell_2\ell_3} \left(V^{d}_{L} \mathrm{diag}(m_{d,s,b}) (V^{d}_{R})^{\dagger} \right)_{\ell_2\ell_3} (V^{d}_{R})_{\ell_3 j} \Big] ,$$

A_4 Simplifications

$$\hat{r}^{u,d}_{00,10,01} = diag(r_{00,10,01}(c^L_q, c_{u_i,d_i}, \beta)) \qquad \hat{r}^{u,d}_{11} = diag(r_{11}(c_{u_i,d_i}, c^L_q, \beta))$$

$$\hat{r}^{u,d}_{01-+} = diag(r_{01-+}(c^L_q, c_{d_j,u_j}, \beta)) \quad \hat{r}^{u,d}_{1-+1} = diag(r_{1-+1}(c_{d_i,u_i}, c^L_q, \beta))$$

$$(C^{d-type}_{7\gamma(8g)})_{ij} = \frac{m_{d_i} A^{1L} f^2_Q}{v^2 M_{KK}} \Big[V^{d\dagger}_R \mathrm{d}iag(f^2_{d,s,b})(\hat{r}^d_{00})^{-1} \tilde{r}^d_{01}\, \tilde{r}^d_{11} (\hat{r}^d_{00})^{-1} V^d_R\, \mathrm{d}iag(m_{d,s,b}) \times \mathrm{d}iag(m_{d,s,b}) V^{d\dagger}_R\, (\hat{r}^d_{00})^{-1} \hat{r}^d_{10}\, V^d_R \Big]_{ij},$$

$$B^{u,d}_P = \max\left((\hat{r}^{u,d}_{00})^{-3} (\hat{r}^{u,d}_{01} \hat{r}^{u,d}_{11} + \hat{r}^{u,d}_{01-+} \hat{r}^{u,d}_{1-+1})\, \hat{r}^{u,d}_{10} \right)$$

Near degeneracy
Of overlap corrections

$$(C^{d-type}_7)_{ij} = \frac{A^{1L} m_{d_i} m_{d_j} B^d_P}{v^2 M_{KK}} \left[V^{d\dagger}_R \mathrm{d}iag(f^2_{d_1,d_2,d_3}) V^d_R \mathrm{d}iag(m_{d,s,b}) V^{d\dagger}_L \mathrm{d}iag(f^2_{Q_1,Q_2,Q_3}) V^d_L \right]_{ij}$$

$$= \frac{A^{1L} f^2_Q m_{d_i} m^2_{d_j} B^d_P}{v^2 M_{KK}} \sum_{n=1}^{3} (V^d_R)^*_{ni} (V^d_R)_{nj} f^2_{d_n}$$

1 generation KK Yukawa matrices

$$\mathrm{Y}^{d}_{KK} = \begin{pmatrix} \bar{Q}_L^{d(0)} \\ \bar{d}_L^{(1^{--})} \\ \bar{Q}_L^{d(1)} \\ \bar{\tilde{d}}_L^{(1^{+-})} \end{pmatrix}^T \begin{pmatrix} \breve{y}_d f_q^{-1} f_d^{-1} r_{00} & 0 & \breve{y}_d f_q^{-1} r_{01} & \breve{y}_u f_q^{-1} r_{101} \\ 0 & \breve{y}_d^* r_{22} & 0 & 0 \\ \breve{y}_d f_d^{-1} r_{10} & 0 & \breve{y}_d r_{11} & \breve{y}_u r_{111} \\ 0 & \breve{y}_u^* r_{222} & 0 & 0 \end{pmatrix} \begin{pmatrix} d_R^{(0)} \\ Q_R^{d(1^{--})} \\ d_R^{(1)} \\ \tilde{d}_R^{(1^{-+})} \end{pmatrix}$$

$$\hat{Y}^{d(h_-)}_{KK} = \begin{pmatrix} \bar{Q}_L^{d(0)} \\ \bar{d}_L^{(1^{--})} \\ \bar{Q}_L^{d(1)} \\ \bar{\tilde{d}}_L^{(1^{+-})} \end{pmatrix}^T \begin{pmatrix} -\breve{y}_u f_q^{-1} f_u^{-1} r_{00} & 0 & -\breve{y}_u f_q^{-1} r_{01} & -\breve{y}_d f_q^{-1} r_{101} \\ 0 & \breve{y}_d^* r_{22} & 0 & 0 \\ -\breve{y}_u f_u^{-1} r_{10} & 0 & -\breve{y}_u r_{11} & -\breve{y}_d r_{111} \\ 0 & \breve{y}_u^* r_{222} & 0 & 0 \end{pmatrix} \begin{pmatrix} u_R^{(0)} \\ Q_R^{u(1^{--})} \\ u_R^{(1)} \\ \tilde{u}_R^{(1^{-+})} \end{pmatrix}$$

$$\hat{Y}^{d(h_+)}_{KK} = \begin{pmatrix} \bar{Q}_L^{u(0)} \\ \bar{u}_L^{(1^{--})} \\ \bar{Q}_L^{u(1)} \\ \bar{\tilde{u}}_L^{(1^{+-})} \end{pmatrix}^T \begin{pmatrix} \breve{y}_d f_q^{-1} f_d^{-1} r_{00} & 0 & \breve{y}_d f_q^{-1} r_{01} & \breve{y}_u f_q^{-1} r_{101} \\ 0 & -\breve{y}_u^* r_{22} & 0 & 0 \\ \breve{y}_d f_d^{-1} r_{10} & 0 & \breve{y}_d r_{11} & \breve{y}_u r_{111} \\ 0 & -\breve{y}_d^* r_{222} & 0 & 0 \end{pmatrix} \begin{pmatrix} d_R^{(0)} \\ Q_R^{d(1^{--})} \\ d_R^{(1)} \\ \tilde{d}_R^{(1^{-+})} \end{pmatrix}$$

4X4 one gen. KK mass matrix

$$\frac{\hat{\mathrm{M}}_d^{KK}}{(M_{KK})} = \begin{pmatrix} \bar{Q}_L^{d(0)} \\ \bar{d}_L^{(1--)} \\ \bar{Q}_L^{d(1)} \\ \bar{\tilde{d}}_L^{(1+-)} \end{pmatrix}^T \begin{pmatrix} \breve{y}_d f_q^{-1} f_d^{-1} r_{00} x & 0 & \breve{y}_d f_q^{-1} r_{01} x & \breve{y}_u f_q^{-1} r_{101} x \\ 0 & \breve{y}_d^* r_{22} x & 1 & 0 \\ \breve{y}_d f_d^{-1} r_{10} x & 1 & \breve{y}_d r_{11} x & \breve{y}_u r_{111} x \\ 0 & \breve{y}_u^* r_{222} x & 0 & 1 \end{pmatrix} \begin{pmatrix} d_R^{(0)} \\ Q_R^{d(1--)} \\ d_R^{(1)} \\ \tilde{d}_R^{(1-+)} \end{pmatrix}$$

$$\breve{y}_{u,d} \equiv (\hat{y}_{u,d}^{LO})_{11} v_\Phi^{4D} e^{k\pi R}/k$$

12X12 three gen. KK mass matrix

$$\hat{\mathrm{M}}_{Full}^{D} = M_{KK} \begin{pmatrix} \hat{\mathrm{M}}_d^{KK}/M_{KK} & x\hat{Y}_{KK}^{s}(\hat{y}_{12}^{LO}, f_s) & x\hat{Y}_{KK}^{b}(\hat{y}_{13}^{LO}, f_b) \\ x\hat{Y}_{KK}^{d}(\hat{y}_{21}^{LO}, f_d) & \hat{\mathrm{M}}_s^{KK}/M_{KK} & x\hat{Y}_{KK}^{b}(\hat{y}_{23}^{LO}, f_b) \\ x\hat{Y}_{KK}^{d}(\hat{y}_{31}^{LO}, f_d) & x\hat{Y}_{KK}^{s}(\hat{y}_{32}^{LO}, f_s) & \hat{\mathrm{M}}_b^{KK}/M_{KK} \end{pmatrix}$$

$O_L^{s\,KK}$

One example of KK diag. matrix

$$
\begin{pmatrix}
1 & 0.94 f_Q^{-1} r_{01} \breve{y}_s x & \frac{f_Q^{-1}}{\sqrt{2}} r_{101} \breve{y}_c x & \frac{f_Q^{-1}}{\sqrt{2}} r_{101} \breve{y}_c x \\
-0.94 f_Q^{-1} r_{01} \breve{y}_s x & 1 & (6.06 r_{11} + 4.72 r_{22}) e^{i\theta_c} \breve{y}_s^* x & -(6.06 r_{11} + 4.72 r_{22}) e^{i\theta_c} \breve{y}_s^* x \\
\mathcal{O}(x^2) & (8.6(r_{11} + 8.1 r_{22}) \breve{y}_s & -\frac{e^{i\theta_c}}{\sqrt{2}} + \frac{(r_{111}^2 - r_{222}^2)\breve{y}_c + r_{11}^2 (|\breve{y}_s|^2/\breve{y}_c^*)}{4\sqrt{2}(r_{111}+r_{222})} x & \frac{e^{i\theta_c}}{\sqrt{2}} + \frac{(r_{111}^2 - r_{222}^2)\breve{y}_c + r_{11}^2 (|\breve{y}_s|^2/\breve{y}_c^*)}{4\sqrt{2}(r_{111}+r_{222})} x \\
-f_Q^{-1} r_{101} \breve{y}_c^* x & \mathcal{O}(x^2) & \frac{1}{\sqrt{2}} + \frac{(r_{111}^2 - r_{222}^2)\breve{y}_c + r_{11}^2 (|\breve{y}_s|^2/|\breve{y}_c|)}{4\sqrt{2}(r_{111}+r_{222})} x & \frac{1}{\sqrt{2}} - \frac{(r_{111}^2 - r_{222}^2)\breve{y}_c + r_{11}^2 (|\breve{y}_s|^2/|\breve{y}_c|)}{4\sqrt{2}(r_{111}+r_{222})} x
\end{pmatrix}
$$

12X12 additional A_4 rotation

$$
\hat{\mathrm{O}}_{L,R}^{(U,D)A_4} = \begin{pmatrix}
(V_{L,R}^{u,d})_{11} \times \tilde{\mathbb{1}}_{4\times4} & (V_{L,R}^{u,d})_{12} \times \tilde{\mathbb{1}}_{4\times4} & (V_{L,R}^{u,d})_{13} \times \tilde{\mathbb{1}}_{4\times4} \\
(V_{L,R}^{u,d})_{21} \times \tilde{\mathbb{1}}_{4\times4} & (V_{L,R}^{u,d})_{22} \times \tilde{\mathbb{1}}_{4\times4} & (V_{L,R}^{u,d})_{23} \times \tilde{\mathbb{1}}_{4\times4} \\
(V_{L,R}^{u,d})_{31} \times \tilde{\mathbb{1}}_{4\times4} & (V_{L,R}^{u,d})_{32} \times \tilde{\mathbb{1}}_{4\times4} & (V_{L,R}^{u,d})_{33} \times \tilde{\mathbb{1}}_{4\times4}
\end{pmatrix}
$$

Dynamical Completion Issues

- Vacuum Alignment – We will have to make sure that the scalar potential doesn't ruin the specific VEV structure we are interested in.
- Most importantly in any flavor model one should explain the origin of quark and lepton masses and their hierarchy (FN, GUT's, WED, UED etc.…).
- Ultimately, a dynamical origin for the A_4 symmetry should be supplemented.
- One of the possibilities is obtaining A_4 via compactification of a 6 dimensional flat space on an orbifold T_2/Z_2. The various fields reside on the four orbifold fixed points (Branes).

(Feruglio and Altarelli)

Off diagonal CKM elements

$$V_{us} = -V_{cd}^* \simeq \left((\tilde{x}_2^d + \tilde{y}_2^d) f_\chi^s - (\tilde{x}_2^u + \tilde{y}_2^u) f_\chi^c\right)$$

$$V_{cb} = -V_{ts}^* \simeq \left((\tilde{x}_3^d + \omega \tilde{y}_3^d) f_\chi^b - (\tilde{x}_3^u + \omega \tilde{y}_3^u) f_\chi^t\right)$$

$$V_{ub} = -V_{td}^* \simeq \left((\tilde{x}_3^d + \tilde{y}_3^d) f_\chi^b - (\tilde{x}_3^u + \tilde{y}_3^u) f_\chi^t\right)$$

$$\mathrm{V}_{CKM} = \begin{pmatrix} 1 & V_{us} & V_{ub} \\ -V_{us}^* & 1 & V_{cb} \\ -V_{ub}^* & -V_{cb}^* & 1 \end{pmatrix}$$

$$\mathcal{H} = \begin{pmatrix} H & \tilde{H} \end{pmatrix} = \begin{pmatrix} h_0^* & h_+ \\ -h_+^* & h_0 \end{pmatrix} \qquad h_0(x,y) = v_H(\beta_H, y) + \sum_n h_0^{(n)}(x)\phi_n(y)$$

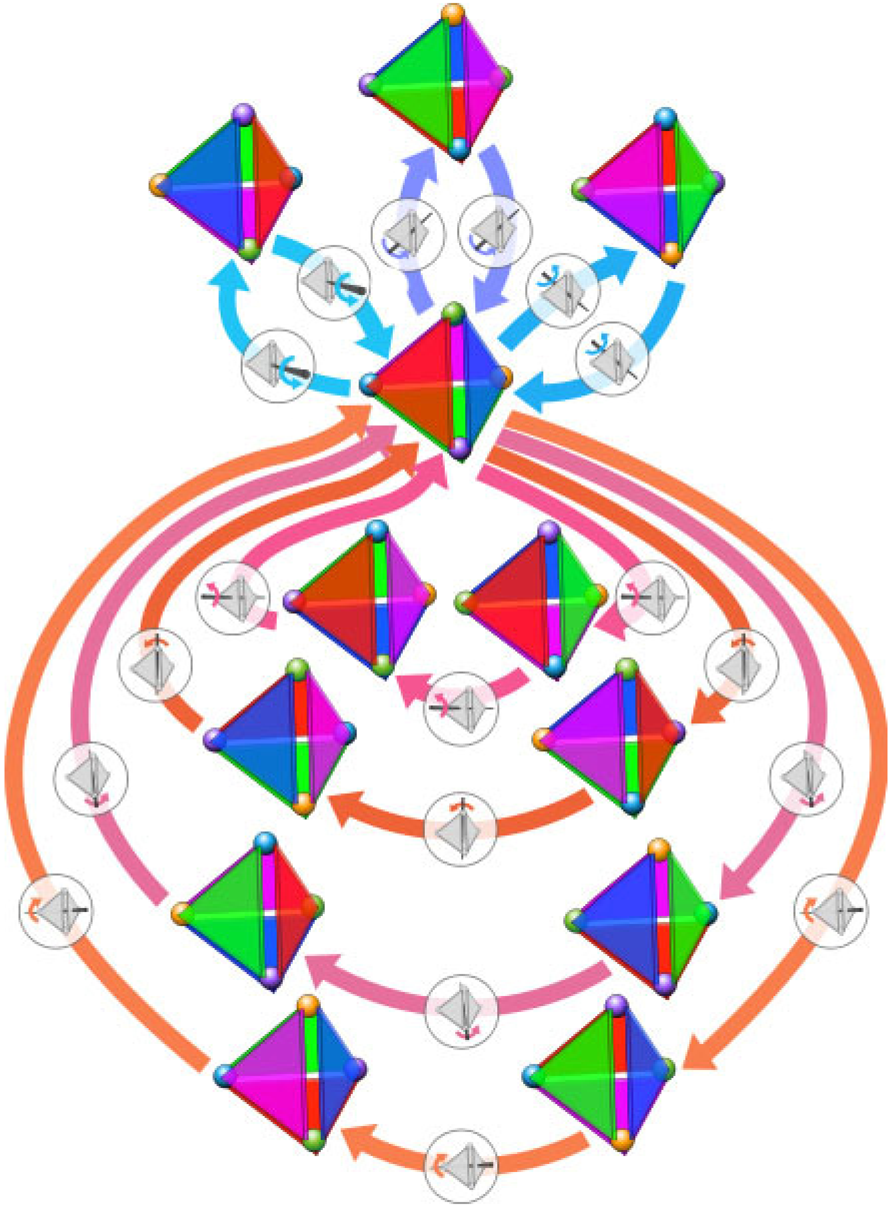
ZMA RH diag. Matrices

$$V_R^q = \begin{pmatrix} 1 & \Delta_1^q & \Delta_2^q \\ -(\Delta_1^q)^* & 1 & \Delta_3^q \\ -(\Delta_2^q)^* & -(\Delta_3^q)^* & 1 \end{pmatrix}$$

$$\Delta_1^q = \frac{m_{q_1}}{m_{q_2}} \left[f_X^{q_1} \left((\tilde{x}_1^q)^* + \omega^2 (\tilde{y}_1^q)^* \right) + f_X^{q_2} \left(\tilde{x}_2^q + \tilde{y}_2^q \right) \right]$$

$$\Delta_2^q = \frac{m_{q_1}}{m_{q_3}} \left[f_X^{q_1} \left((\tilde{x}_1^q)^* + \omega (\tilde{y}_1^q)^* \right) + f_X^{q_3} \left(\tilde{x}_3^q + \tilde{y}_3^q \right) \right]$$

$$\Delta_3^q = \frac{m_{q_2}}{m_{q_3}} \left[f_X^{q_2} \left((\tilde{x}_2^q)^* + \omega (\tilde{y}_2^q)^* \right) + f_X^{q_3} \left(\tilde{x}_3^q + \omega \tilde{y}_3^q \right) \right]$$



The Tetrahedral Group A_4

- A(4) is the group of even permutations of 4 objects
- It is also isomorphic to the symmetry group of a regular tetrahedron, and is a subgroup of SO(3)
- Other extensions include:

  $T' \quad \Delta(27) \quad \Sigma(81)$

- Will be used to explain proximity of mixing in the lepton sector to TBM, and proximity of mixing in the quark sector to unity.
- Differs from other types of flavor models: “Anarchic”, continuous flavor groups, GUT’s, (SUSY),…

Some A(4) Basic properties:

•A(4) has one real triplet, $\underline{3}$ and three "singlets": $\underline{1}$, $\underline{1}'$ and $\underline{1}''$

$$\underline{3} \otimes \underline{3} = \underline{3}_s \oplus \underline{3}_a \oplus \underline{1} \oplus \underline{1}' \oplus \underline{1}'', \quad \text{and} \quad \underline{1}' \otimes \underline{1}' = \underline{1}''$$

$$(\underline{3} \otimes \underline{3})_{\underline{3}_s} = (x_2y_3 + x_3y_2\,,\, x_3y_1 + x_1y_3\,,\, x_1y_2 + x_2y_1),$$

$$(\underline{3} \otimes \underline{3})_{\underline{3}_a} = (x_2y_3 - x_3y_2\,,\, x_3y_1 - x_1y_3\,,\, x_1y_2 - x_2y_1),$$

$$(\underline{3} \otimes \underline{3})_{\underline{1}} = x_1y_1 + x_2y_2 + x_3y_3,$$

$$(\underline{3} \otimes \underline{3})_{\underline{1}'} = x_1y_1 + \omega\, x_2y_2 + \omega^2\, x_3y_3, \qquad \omega = e^{i2\pi/3}$$

$$(\underline{3} \otimes \underline{3})_{\underline{1}''} = x_1y_1 + \omega^2\, x_2y_2 + \omega\, x_3y_3,$$

A simple A(4) Model

(Ma, Feruglio, Altarelli, Babu, Volkas…)

•We assign the SM fermions to the following representations:

$$Q_L \sim \left(3, 2, \tfrac{1}{3}\right) (\underline{\mathbf{3}}) \qquad \ell_L \sim (1, 2, -1) (\underline{\mathbf{3}})$$

Under A(4)

SU(3)xSU(2)xU(1)

$$u_R \oplus u'_R \oplus u''_R \sim \left(3, 1, \tfrac{4}{3}\right) (\underline{1} \oplus \underline{1}' \oplus \underline{1}'') \qquad \nu_R \sim (1, 1, 0) (\underline{\mathbf{3}})$$

$$d_R \oplus d'_R \oplus d''_R \sim \left(3, 1, -\tfrac{2}{3}\right) (\underline{1} \oplus \underline{1}' \oplus \underline{1}'') \qquad e_R \oplus e'_R \oplus e''_R \sim (1, 1, -2) (\underline{1} \oplus \underline{1}' \oplus \underline{1}'')$$

•The scalar sector of this model will be given by:

$$\Phi \sim (1, 2, -1) (\underline{\mathbf{3}}), \quad \phi \sim (1, 2, -1) (\underline{\mathbf{1}}), \quad \chi \sim (1, 1, 0) (\underline{\mathbf{3}}).$$

•We will also need an additional U(1) symmetry which will be explicitly broken to Z_2 under which ϕ, χ and Q_L are odd and the rest of the fields are even.

• The Yukawa Lagrangian is:

$$\mathcal{L}_{\rm Yuk} = \lambda_u\ (\overline{Q}_L\Phi)_{\underline{1}}\, u_R + \lambda'_u(\overline{Q}_L\Phi)_{\underline{1}'}\, u''_R + \lambda''_u(\overline{Q}_L\Phi)_{\underline{1}''}\, u'_R +$$

$$+\ \lambda_d(\overline{Q}_L\tilde{\Phi})_{\underline{1}}\, d_R + \lambda'_d(\overline{Q}_L\tilde{\Phi})_{\underline{1}'}\, d''_R + \lambda''_d(\overline{Q}_L\tilde{\Phi})_{\underline{1}''}\, d'_R +$$

$$+\ \lambda_\nu(\bar{\ell}_L\nu_R)_{\underline{1}}\,\phi + M[\overline{\nu}_R(\nu_R)^c]_{\underline{1}} + \lambda_\chi[\overline{\nu}_R(\nu_R)^c]_{\underline{3}_s}\cdot\chi +$$

$$+\ \lambda_e(\bar{\ell}_L\tilde{\Phi})_{\underline{1}}\, e_R + \lambda'_e(\bar{\ell}_L\tilde{\Phi})_{\underline{1}'}\, e''_R + \lambda''_e(\bar{\ell}_L\tilde{\Phi})_{\underline{1}''}\, e'_R + h.c.$$

• And the resulting mass matrix in each sector, f=(u,d,e):

$$\left(\overline{f}_{1L}, \overline{f}_{2L}, \overline{f}_{3L}\right)\begin{pmatrix} \lambda v_1 & \lambda' v_1 & \lambda'' v_1 \\ \lambda v_2 & \omega\lambda' v_2 & \omega^2\lambda'' v_2 \\ \lambda v_3 & \omega^2\lambda' v_3 & \omega\lambda'' v_3 \end{pmatrix}\begin{pmatrix} f_R \\ f''_R \\ f'_R \end{pmatrix} + h.c.$$

The Neutrino Sector

- From the Yukawa Lagrangian we get that the Dirac and the bare Majorana mass matrices are proportional to the identity:

$$M_\nu^D = \lambda_\nu v_\phi 1 \equiv m_\nu^D 1 \quad \text{and} \quad M_{\nu Bare}^{Maj.} = M1$$

The required non-trivial structure is supplied by the Yukawa coupling to the field, χ , which turns out to be:

$$\lambda_\chi \left(\overline{\nu}_{1R}, \overline{\nu}_{2R}, \overline{\nu}_{3R} \right) \begin{pmatrix} 0 & \chi_3 & \chi_2 \\ \chi_3 & 0 & \chi_1 \\ \chi_2 & \chi_1 & 0 \end{pmatrix} \begin{pmatrix} (\nu_{1R})^c \\ (\nu_{2R})^c \\ (\nu_{3R})^c \end{pmatrix}$$

•Inserting the VEV of χ the resulting 6x6 mass matrix is:

$$\begin{pmatrix} 0 & 0 & 0 & m_\nu^D & 0 & 0 \\ 0 & 0 & 0 & 0 & m_\nu^D & 0 \\ 0 & 0 & 0 & 0 & 0 & m_\nu^D \\ m_\nu^D & 0 & 0 & M & 0 & M_\chi \\ 0 & m_\nu^D & 0 & 0 & M & 0 \\ 0 & 0 & m_\nu^D & M_\chi & 0 & M \end{pmatrix} \qquad M_\chi \equiv \lambda_\chi v_\chi$$

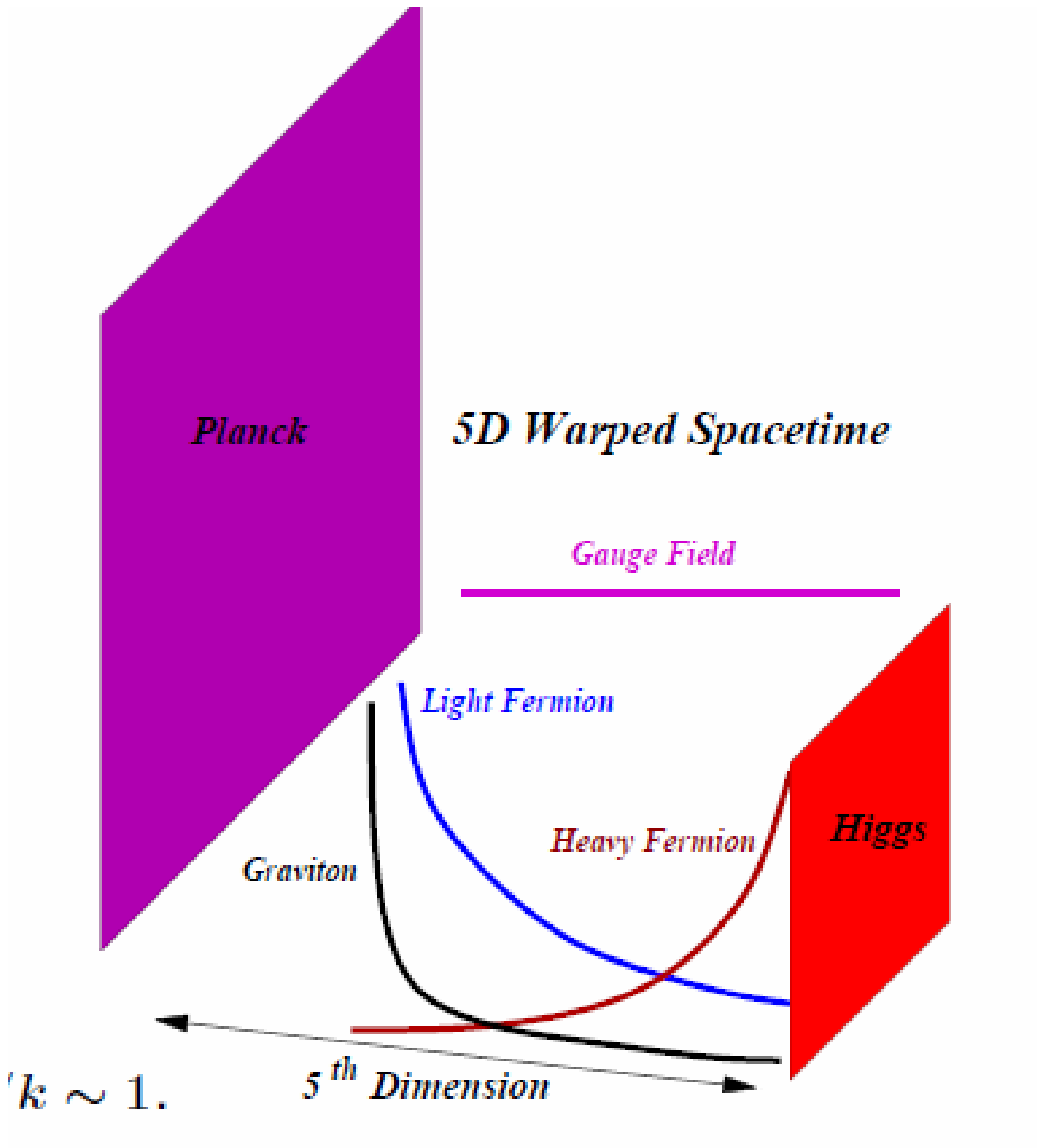
- In the see-saw limit, $|M|, |M_\chi| \gg m_\nu^D$ the effective 3x3 mass matrix for the light neutrinos is given by:

$$M_L = -M_\nu^D M_R^{-1} (M_\nu^D)^T = -\frac{(m_\nu^D)^2}{M} \begin{pmatrix} \frac{M^2}{M^2 - M_\chi^2} & 0 & -\frac{M M_\chi}{M^2 - M_\chi^2} \\ 0 & 1 & 0 \\ -\frac{M M_\chi}{M^2 - M_\chi^2} & 0 & \frac{M^2}{M^2 - M_\chi^2} \end{pmatrix}$$

•The diagonalization matrix turns out to be : $V_L^\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & 1 \end{pmatrix}$

•So the MNSP matrix at this order is Tri-bi-maximal:

$$V_{MNSP} = V_L^{e\dagger} V_L^\nu = U(\omega)^\dagger V_L^\nu = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{\omega}{\sqrt{6}} & \frac{\omega}{\sqrt{3}} & -\frac{e^{i\pi/6}}{\sqrt{2}} \\ -\frac{\omega^2}{\sqrt{6}} & \frac{\omega^2}{\sqrt{3}} & \frac{e^{-i\pi/6}}{\sqrt{2}} \end{pmatrix}$$


Planck
5D Warped Spacetime
Gauge Field
Light Fermion
Heavy Fermion
Higgs
Graviton
5 th Dimension
'k ∼ 1.