## Standard model at the LHC:

## QCD

SSP 2012, Groningen

## Eric Laenen

## No LHC without understanding QCD

QCD at the LHC operates in new regime: high energy, large multiplicities. It produces interesting needles

- Higgs, top, SM, BSM, ...
and enormous amounts of hay
- Jets, b's, underlying events, multi-parton scattering

How to separate?

- Develop/use the best theoretical tools available
- Much interaction among theorists and experimenters

LHC poses a daunting challenge to QCD theorists

## QCD predictions at the LHC



Factorization theorem
Collins, Soper, Sterman; Bodwin
pdf's initial state spin etc averaging

$$
\begin{aligned}
\langle O\rangle= & \frac{1}{2 s} \int d x_{1} f\left(x_{1}\right) \int d x_{2} f\left(x_{2}\right) \frac{1}{\left(2 s_{1}+1\right)\left(2 s_{2}+1\right) n_{1} n_{2}} \\
& \times \sum_{n} \int d \phi_{n-2} O\left(p_{1}, \ldots, p_{n}\right)|A(n)|^{2}+\sum_{p=1} c_{p}\left(\frac{\Lambda}{Q}\right)^{p} \\
& \text { phase space integral observable amplitude } \quad \text { power corrections }
\end{aligned}
$$

## QCD predictions, simplified



- Each input has uncertainty
- Tools have intrinsic accuracy


## QCD predictions, less simplified



## This talk

Flavor of state-of-the-art in

- LO,NLO,NNLO
- Resummation
- Monte Carlo
with some recent applications and present challenges


## Higher order QCD

## LO, NLO cross sections

Multi-differential hadronic NLO cross section

$$
\frac{d \sigma^{p p \rightarrow X}}{d^{3} p_{1} \ldots d^{3} p_{n}}=\sum_{a, b} \int d x_{1} d x_{2} f_{a}\left(x_{1}, \mu_{F}\right) f_{b}\left(x_{2}, \mu_{F}\right)
$$

$$
\times \hat{\sigma}_{a b}\left(p_{a}+p_{b} \rightarrow p_{X}, \alpha_{s}\left(\mu_{R}\right), \mu_{R}, \mu_{F}\right)+\mathcal{O}\left(\frac{\Lambda^{2}}{Q^{2}}\right)
$$

Multi-differential parton-level NLO cross section

Renormalization and Factorization scale

For NNLO, add "N" in all the right places..

## Parton distribution functions

$$
f_{i / P}\left(x, \mu_{F}\right)
$$

## Probability for parton of type " i " inside proton $P$ to interact having momentum fraction " $x$ "

$\checkmark$ There are 13.
$\checkmark$ Universal, and crucial for accurate predictions at LHC
$\checkmark$ Sophisticated approaches, various groups, publicly available
$\checkmark$ No time to do justice..

## Status of Higher Order

| Order | $2 \rightarrow 1$ | $2 \rightarrow 2$ | $2 \rightarrow 3$ | $2 \rightarrow 4$ | $2 \rightarrow 5$ | $2 \rightarrow 6$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | LO |  |  |  |  |  |
| $\alpha_{s}$ | NLO | LO |  |  |  |  |
| $\alpha_{s}^{2}$ | NNLO | NLO | LO |  |  |  |
| $\alpha_{s}^{3}$ | NNNLO | NNLO | NLO | LO |  |  |
| $\alpha_{s}^{4}$ |  |  |  | NLO | LO |  |
| $\alpha_{s}^{5}$ |  |  |  |  | NLO | LO |

- LO well-understood, now more efficient than ever
- NLO: a flood of new developments
- NNLO $2 \rightarrow 2$ starting now..
$\checkmark$ Show now also start including NLO QCD + EW
NNNLO: for $\mathrm{F}_{2}(\mathrm{x}, \mathrm{Q})$, from same (Nikhef) foundry as NNLO splitting functions


## Need for computer algebra!!

FORM Vermaseren

Result of a brute force calculation (actually only a small part of it):


## NLO: all it took was a wish...

- First composed in Les Houches in 2005, added to in 2007,2009 by Joey Huston
- List of "doable" calculations needed for LHC
- They have now all been done
- (except tttt)

..and a revolution in calculational methods

Nonlinear progress in $\mathrm{W}+\mathrm{n}$ jets
D. Maitre, Talk at SM@LHC workshop, NBI

## Recent progress

- Number of jets in addition to the vector boson


다 LO
든 NLO
₹MC@NLO

- POWHEG
$\geqslant \mathrm{HEJ}$

Berger, Bern, Dixon, Febres-Cordero, Forde

## Leading order

The problem here is not handling divergences, but handling complexity. For $g g \rightarrow \mathrm{ng}$ number of diagrams grows factorially

| n | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| diagrams | 4 | 25 | 220 | 2485 | 34300 | 559405 | $1.05 \mathrm{E}+07$ |

Nowadays routinely handled:

- Madgraph/Madevent (helicity amplitudes), Sherpa/Amegic++, Helac/ Phegas, Alpgen (recursion), Comphep (matrix elements)
$\checkmark$ after many inventions


## LO scattering: know your quantum numbers

$$
\mathcal{A}_{n}(1, . ., n)=g^{n-2} \sum_{\sigma} \overbrace{\text { Color-ordered amplitudes }}^{\text {Color-decomposition }}
$$

For $n=6: 34300 \rightarrow 50 \mid$ color-ordered amplitudes
Specify all helicities $\rightarrow$ can use efficient spinor techniques

$$
\begin{aligned}
& A\left(1^{ \pm}, 2^{\oplus}, \ldots, n^{ \pm}\right) \bar{u}_{-}=\langle p-| \\
&\langle p q\rangle \equiv\langle p-\mid q+\rangle, \quad[p q] \equiv\langle p+\mid q-\rangle
\end{aligned}
$$

MHV amplitude

$$
A_{n}^{\text {tree }}\left(1^{+}, \ldots, j^{-}, \ldots, k^{-}, \ldots, n^{+}\right)=i(\sqrt{2})^{n-2} \frac{\langle j k\rangle^{4}}{\langle 12\rangle \ldots\langle n 1\rangle}
$$

But what about more minuses?
From twistor approach (witten): use these as building blocks


## Post-twistor recursion



Construct helicity amplitude by sewing togetherMHV building blocks using I propagator.

Six diagrams only! (Was 220 in this case..)

## Pre-twistor recursion (Leiden, I980's and 90's)

Berends, Giele


- Define "currents": one gluon off-shell, n off-shell. Obey "obvious" recursion.
- Analytically elegant, numerically efficient, was important for top-quark discovery (VECBOS: Berends, Kuij, Tausk, Giele)

What is faster, the old or the new?

## Speed

| Final | BG |  | BCF |  | CSW |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| State | CO | CD | CO | CD | CO | CD |
| $2 g$ | 0.24 | 0.28 | 0.28 | 0.33 | 0.31 | 0.26 |
| $3 g$ | 0.45 | 0.48 | 0.42 | 0.51 | 0.57 | 0.55 |
| $4 g$ | 1.20 | 1.04 | 0.84 | 1.32 | 1.63 | 1.75 |
| $5 g$ | 3.78 | 2.69 | 2.59 | 7.26 | 5.95 | 5.96 |
| $6 g$ | 14.2 | 7.19 | 11.9 | 59.1 | 27.8 | 30.6 |
| $7 g$ | 58.5 | 23.7 | 73.6 | 646 | 146 | 195 |
| $8 g$ | 276 | 82.1 | 597 | 8690 | 919 | 1890 |
| $9 g$ | 1450 | 270 | 5900 | 127000 | 6310 | 29700 |
| $10 g$ | 7960 | 864 | 64000 | - | 48900 | - |

Dinsdale, Ternick, Weinzier

| $n$ | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Berends-Giele | 0.00005 | 0.00023 | 0.0009 | 0.003 | 0.011 | 0.030 | 0.09 | 0.27 | 0.7 |
| Scalar | 0.00008 | 0.00046 | 0.0018 | 0.006 | 0.019 | 0.057 | 0.16 | 0.4 | 1 |
| MHV | 0.00001 | 0.00040 | 0.0042 | 0.033 | 0.24 | 1.77 | 13 | 81 | - |
| BCF | 0.00001 | 0.00007 | 0.0003 | 0.001 | 0.006 | 0.037 | 0.19 | 0.97 | 5.5 |

The old is still faster (except for MHV)...

## LO, NLO, etc



## Combine with PDF's, put in MC integrator, apply cuts etc

Calculate in $D=4-2 \varepsilon$ dimensions

NLO $\hat{\sigma}^{(1)}$


Cancel IR poles $I / \varepsilon^{2}$ before anything else

NNLO $\hat{\sigma}^{(2)}$


Cancel IR poles $I / \varepsilon^{4}$ etc before anything else; hard!

## Handling intermediate IR divergences

$d \sigma_{N L O}=\int_{d \Phi_{n+1}}\left(d \sigma_{N L O}^{R}-d \sigma_{N L O}^{S}\right)+\left[\int_{d \Phi_{n}} d \sigma_{N L O}^{V}+\int_{d \Phi_{n}}\left(\int_{d \Phi_{1}} d \sigma_{N L O}^{S}\right)\right]$

Subtraction schemes (ElIs, Ross, Terano): find clever $d \sigma_{N L O}^{S}$

- Dipole (catani, seymour): emitter + spectator + soft-collinear parton
$\checkmark$ Color-charge operators; collinear split + half of soft
- Antenna (Kosower): 2 radiators + soft-collinear parton
$\checkmark$ Color-ordering; soft + half collinear splits


- FKS (Frixione, Kunszt, Signer): isolate each singularity with projectors. Use plus prescriptions to define IR-finite part.


## Why go beyond LO?

## Precision!

- Accurate prediction of production rates
- Better modelling of distribution shapes due to extra partons
- Self-diagnostics of PT
- New channels open up beyond LO, not necessarily small



## NLO benefit

## Preliminary results for $\mathrm{W}+5$ jets

- First 2 --> 6(7) calculation at NLO for the LHC



## One-loop revolution



Since boxes, triangles etc are standard basis: find coefficients
Many ideas based on unitarity: construct a function from its poles and branch cuts

- poles: lower \# of external lines

- branch cuts: lower \# of loops
$\frac{1}{p^{2}+i \epsilon} \longrightarrow-i 2 \pi \delta_{+}\left(p^{2}\right)$

Replace propagator by cut one, as in phase space

## Generalized unitarity



$\mathcal{M}=\sum_{i} a_{i}(4)$ Boxes $_{i}+\sum_{i} b_{i}(4)$ Triangles $_{i}+\sum_{i} c_{i}(4)$ Bubbles $_{i}+\sum_{i} d_{i}(4)$ Tadpoles $_{i}+$ Rational term

## Basis set of integrals

$\checkmark$ Calculate cuts of $M$ on left
$\checkmark$ Cut on right only in Boxes etc
$\checkmark$ Match, find coefficients a,b,c,d
$\checkmark$ Use other methods to find Rational term

## More one-loop ideas

$\checkmark$ Numerical loop integration Nagy, Soper
$\checkmark$ Numerical solution to BG recursions Ellis, Giele, Zanderighi

$$
I_{\mu_{1} \mu_{2} \ldots \mu_{n}}=\int \frac{d^{D} l}{(2 \pi)^{D}} \frac{l_{\mu_{1}} \cdots l_{\mu_{n}}}{D_{0} D_{1} \cdots D_{N}} \quad \text { Tensor integral }
$$

$\checkmark$ Better Passarino-Veltman Denner, Dittmaier; Binoth, Guillet, Pilon, Heinrich, Schubert

$$
\int d^{D} l \frac{1}{D_{1} D_{2} D_{3}}=\int d^{D} l \int_{0}^{1} d x \int_{0}^{1-x} d y \frac{1}{\left[x D_{1}+y D_{2}+(1-x-y) D_{3}\right]^{3}}
$$

$\checkmark$ Sector decomposition of multiple Feynman parameter integrals plus contour deformation

Anastasiou, Beerli, Daleo
$\checkmark$ Algebraic inversion of coefficients of scalar integrals at integrand level, using unitarity conditions, and partial fractioning ( $p$ p $\rightarrow \mathrm{VVV}$ )

Ossola, Papadopoulos, Pittau Ellis, Giele, Kunszt; Melnikov

Lively marketplace of ideas, now settling on a few vendors

## Web tools: MadGraph, aMC@NLO



## Generate processes online using MadGraph 5

To improve our web services we request that you register. Registration is quick and free. You may register for a password by clicking here. Please note the correct reference for MadGraph 5, JHEP 1106(2011)128, arXiv:1106.0522 [hep-ph]. You can still use MadGraph 4 here.

Code can be generated either by:

```
I. Fill the form:
\begin{tabular}{|c|c|c|}
\hline Model: & 5 M - 8 & Model descriptions \\
\hline Input Process: & & Examples/format \\
\hline
\end{tabular}
p and j definitions: p-j=du scd~u~s~c~g ;
sum over leptons: 1+ - e+, mu+ ta+; l- =e-, mu- ta-; vi = ve, vm, vi;vl~ = ve~, vm~, vt~ ;
Submit
```


## Web tools: MadGraph, aMC@NLO

## aMC@NLO web page

The project

## Home <br> People <br> Contact

News

## MC Tools

(registration needed)

Online MC
generation
My DataBase
Codes Download
Compare with
MadLoop
Event samples DB
Communication

Citations
Publications
Talks \& Seminars
Resources
Useful links
File Sharing
sun
Sul.....

MadLoop results for process $u g>t b \sim d$, in the 5 light flavours SM, QED power :

Status: Completed!

- PS point \# 1:

```
Born = +3.47867369141388E-07
    co = -4.62998838291617E-07
    c
    C-2 = -4.57313341661980E-08
```

    \# PDG E \(P_{x} \quad P_{y}\)
    \(12+1.00000000000000 \mathrm{E}+03+0.00000000000000 \mathrm{E}+00+0.00000000000000 \mathrm{E}+00+1.0\)
    \(221+1.00000000000000 \mathrm{E}+03+0.00000000000000 \mathrm{E}+00+0.00000000000000 \mathrm{E}+00-1.0\)
    \(36+9.06726648404084 \mathrm{E}+02+4.02305316456112 \mathrm{E}+01-8.88902778955366 \mathrm{E}+02+2.4\)
    \(4-5+4.46909530065018 \mathrm{E}+02-3.07136746737082 \mathrm{E}+02+3.08528664834847 \mathrm{E}+02-1.0\)
    \(51+6.46363821530899 \mathrm{E}+02+2.66906215091471 \mathrm{E}+02+5.80374114120519 \mathrm{E}+02+9.8\)
    - PS point \# 2:

$$
\text { Born }=+1.55330837443265 \mathrm{E}-06
$$

## NLO, does it work?

PP $\rightarrow 4$ jets

## W+n jets



Bern et al
[Blackhat]


Yes

## Two-loop methods

Laporta algorithm: reduce all tensor integrals to basis of scalar integrals.
Mellin-Barnes transform: $\frac{1}{(A+B)^{\nu}}=\frac{1}{\Gamma(\nu)} \frac{1}{2 \pi i} \int_{C} d z \frac{A^{z}}{B^{\nu+z}} \Gamma(-z) \Gamma(\nu+z)$

- Inverse Feynman parameter trick. Can do FP integrals now easily. Contour integrals automatized (AMBRE) Czakon

Sector decomposition:

- Hack up multi-dimensional FP parameter space, such that I singularity per region.



## NNLO

## Why/when NLO not enough?

$\checkmark$ When uncertainties at NLO are still large
$\checkmark$ For extracting precise values from data
$\checkmark$ When NLO corrections are large


We have now for hadron colliders:
van Neerven, Harlander, Kilgore, Anastasiou, Melnikov, Ravindran,
Smith, Dixon, Petriello
$\checkmark$ Inclusive $\mathrm{W}, \mathrm{Z}$ and H cross sections, and rapidity distributions
$\checkmark$ Fully differential V and H production Anastasiou, Melnikov, Petriello; Catani, Grazzini
$\checkmark$ Spacelike, and diagonal time-like, splitting functions


## Excitement: NNLO top cross section finally here

## (at least for qq->QQ)

Baernreuther, Mitov, Czakon [spring 2012]


$$
\sigma_{\text {tot }}^{\text {res }}=7.067_{-0.232(3.3 \%)}^{+0.143(2.0 \%)}[\text { scales }]_{-0.122(1.7 \%)}^{+0.186(2.6 \%)}[\mathrm{pdf}]
$$

## Best prediction at NNLO+NNLL

$\checkmark$ Uncertainty now only a few \% at NNLO.
$\checkmark$ First NNLO calculation with initial hadrons and full color structure

## All order QCD: resummation

$\checkmark$ Effects of soft gluons can be summed to all orders
$\checkmark$ Many ways to derive exponential form
$\checkmark$ Algebraic: PT is exponent of "webs", after eikonal approximation Gatheral; Frenkel, Taylor; Sterman
$\checkmark$ Webs: subset of diagrams, with modified color factors
$\hat{O}=1+\alpha_{s}\left(L^{2}+L+1\right)+\alpha_{s}^{2}\left(L^{4}+L^{3}+L^{2}+L+1\right)+\ldots$


+ suppressed terms

$\checkmark$ For Higgs/Drell-Yan inclusive cross section:

$$
\hat{\sigma}_{i}(N)=C\left(\alpha_{s}\right) \times \exp \left[\int_{0}^{1} d z \frac{z^{N-1}-1}{1-z}\left\{2 \int_{\mu_{F}^{2}}^{(1-z)^{2} Q^{2}} \frac{d \mu^{2}}{\mu^{2}} A_{i}\left(\alpha_{s}\left(\mu^{2}\right)\right)+D_{i}\left(\alpha_{s}(1-z) Q^{2}\right)\right\}\right]
$$

Sterman; Catani, Trentadue, Grazzini, de Florian, Forte, Ridolfi, Vogelsang, Kidonakis, EL, Magnea, Stavenga, White, Ridolfi, Moch, Vogt, Eynck, Ravindran, Becher, Neubert, Ji, Idilbi,...

## WEBS FOR QCD AMPLITUDES

Gardi, EL, Stavenga, White
Mitov, Sterman, Sung

- Found purely exponential structure

$$
\sum \mathcal{F}(D) C(D)=\exp \left[\sum_{d, d^{\prime}} \mathcal{F}(d) R_{d d^{\prime}} C\left(d^{\prime}\right)\right]
$$

$$
\sum_{d^{\prime}} R_{d d^{\prime}}=0
$$

Eigenvalues 0 or 1

## Projector matrix

- multi-parton webs are "closed sets" of diagrams, with modified color factors

(3a)

(3b)

(3c)


$$
\frac{1}{6}[C(3 a)-C(3 b)-C(3 c)+C(3 d)] \times[M(3 a)-2 M(3 b)-2 M(3 c)+M(3 d)]
$$

## Resummation

$\checkmark$ State of the art NNLL
$\checkmark$ Different approaches, healthy competition
$\checkmark$ Much progress in analytical understanding:

- Approaching all order knowledge of IR structure of QCD amplitudes


## Monte Carlo simulation of QCD

## Monte Carlo

Great advances in Monte Carlo in recent years

- Multipurpose: PYTHIA 8 (C++), HERWIG++, SHERPA
- Matrix element based: Alpgen, Madgraph/Madevent, Comphep, Helac
- NLO combined with parton showers:MC@NLO, POWHEG,
- Renaissance after many theorists entered field recently, many new ideas $\checkmark$..but it don't really count when there ain't code.


## Matrix element generators

Good description when no two partons are too collinear, or one is soft


Calculate full tree-level matrix element using

- Diagrams
$\checkmark$ Helicity amplitudes (MadEvent, Amegic++), squared amplitudes (Comphep)
- Leiden recursion relations (Alpgen, Helac/Phegas)
- Always fixed number of partons
$\checkmark$ MadEvent: $2 \rightarrow 6$, HELAC, Alpgen,Amegic++ $2 \rightarrow 8$


## Parton showers

Approximate description of matrix element when radiation is mostly collinear and soft


Set up as iterative Markov process


Sudakov form factor: probability of no emission between two emissions (virtual graphs)

$$
\Delta_{i}\left(t_{1}, t_{2}\right)=\exp \left[-\sum_{(j k)} \int_{t_{2}}^{t_{1}} \frac{d t}{t} \frac{\alpha_{S}(t)}{2 \pi} \int d z P_{i, j k}(z) \int \frac{d \varphi}{2 \pi}\right]
$$

Number of partons per event not fixed

## How to combine best of both?

## Matching ME to PS

CKKW Catani, Krauss, Kuhn, Webber
$\checkmark$ Small (large) angles: use PS (ME)
$\checkmark$ Define matching angle (scale)
$\checkmark$ Above: dress up ME with Sudakov form factors
$\checkmark$ Below: use PS, but don't allow pT above matching scale

## MLM Mangano

$\checkmark$ Generate N -jet with ME.
$\checkmark$ Define matching angle (scale) and start PS
$\checkmark$ Demand hard jets have original parton, otherwise reject
$\checkmark$ For each N the shower cannot increase N

## Matching NLO to PS

Match to avoid double counting again

- emission from NLO and PS should be counted once
- virtual part of NLO and Sudakov should not overlap
- two main approaches:

Frixione, Webber; Nason
$\checkmark$ MC@NLO exact to NLO, no overshoot, some negative weights
Nason; Frixione, Oleari
$\checkmark$ POWHEG insists on having positive weights, exponentiates complete real matrix element.

Match both to NLO and ME: Vincia

Giele, Kosower, Skands

Effective theory based: GenEvA

Bauer, Tackmann, Thaler


## MC@NLO and tit

## Frixione, Nason, Webber

- First process in MC@NLO with final state colored partons, multiple color flows
- Interpolates well between NLO and parton showers

Combine the best parts of MC and Feynman-diagram approach


## Automatic NLO + PS

- Is well-underway
- POWHEG Box
- aMC@NLO
- Mashups:
- POWHEG + Madgraph4
- Sherpa + MC@NLO


## Single top at NLO

s-channel:
timelike W

(1)

(2)
t-channel: spacelike W

(3)

Important process for LHC

- Allows measurement of $\mathrm{V}_{\text {tb }}$ per channel

Easier check of chiral structure of Wtb vertex than ..
Infer the b-density

(a) $\mathrm{p}_{\mathrm{T}}^{\mathrm{t}}[\mathrm{GeV}]$

Included in MC@NLO \& POWHEG

Frixione, EL, Motylinski, Webber

Harris,EL,Phaf,Sullivan, Weinzierl; Cao, Schwienhorst, Yuan; Zhu; Campbell, Ellis, Tramontano

## Challenges, an incomplete (wish)list

- Near: Complete full NLO automation
$\checkmark$ fast, accurate, flexible, easy code
$\checkmark$ matched to LL parton showers
- Far-ish: begin NNLO wishlist, leading to automation
$\checkmark$ also NLO QCD + NLO EW corrections
- Near-ish: NLL parton showers, and matching
- Near: Full NNLL resummation for QCD processes
$\checkmark$ including next-to-eikonal corrections


## Conclusions

- Remarkable developments in perturbative QCD
$\checkmark$ higher order calculations, Monte Carlo's, and their combinations
$\checkmark$ with some remarkable engineering
- Due to young and experienced researchers alike

QCD theorists are up to the LHC challenge

