Plasma Instabilities: Review

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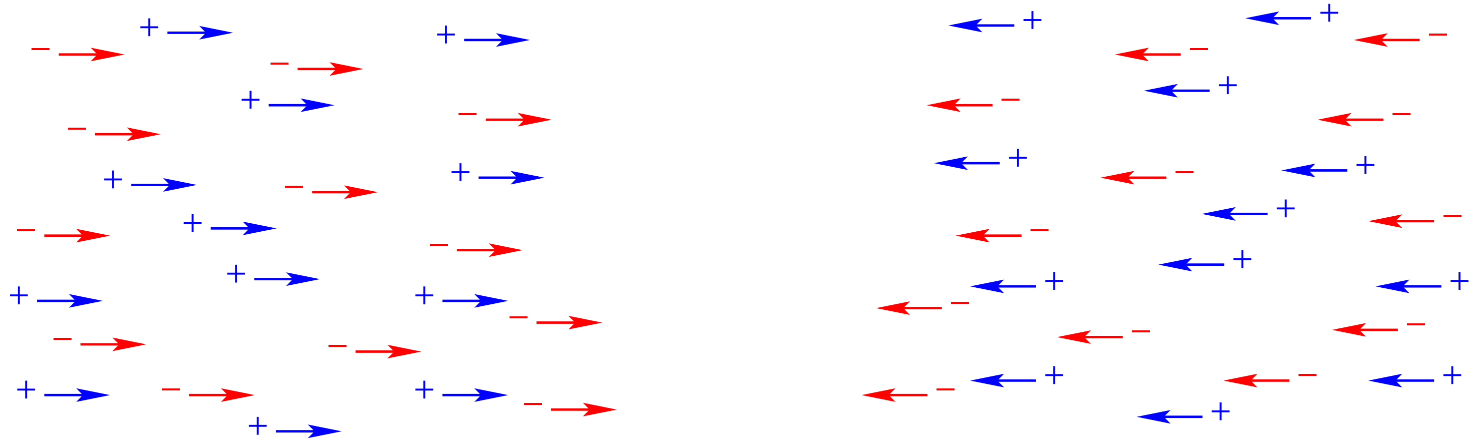
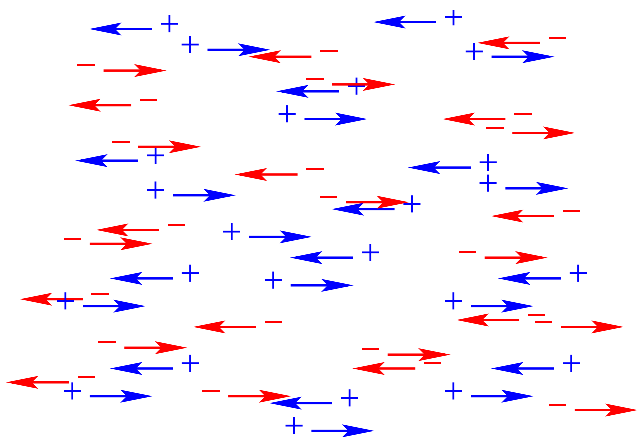
- Instabilities: general (Abelian) picture
- Requirements for instabilities to make sense
- Growth rate, dependence on anisotropy
- How big do they grow?
- What do they do?

Suppose two streams of plasma collide:



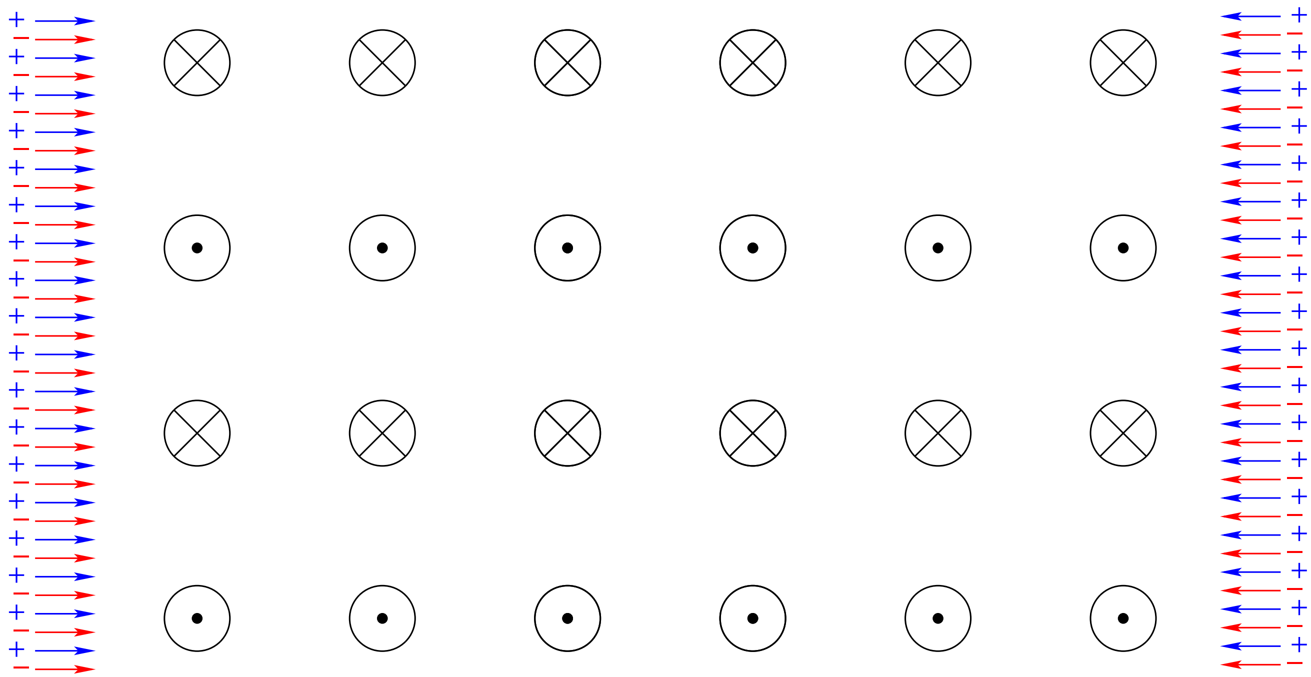
becomes



What happens?

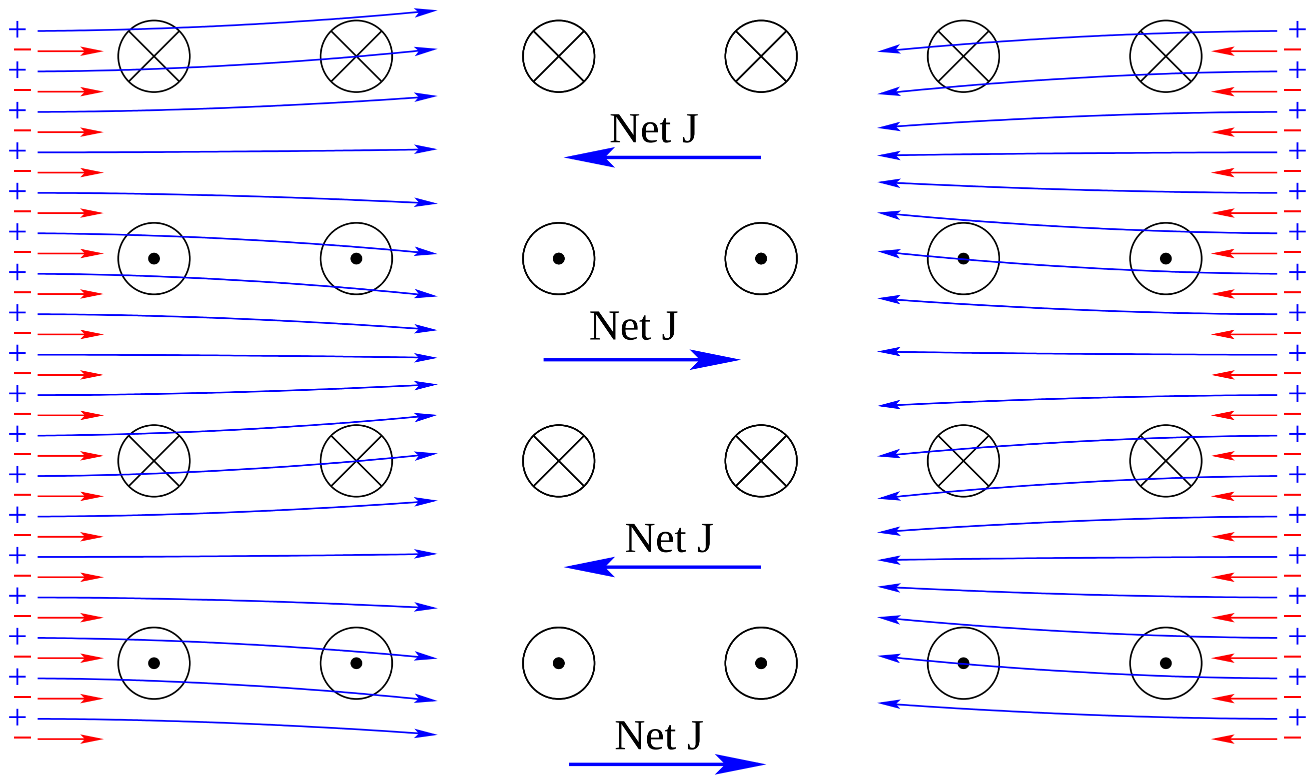
Magnetic field growth!

Consider the effects of a seed magnetic field $\hat{B} \cdot \hat{p} = 0$ and $\hat{k} \cdot \hat{p} = 0$



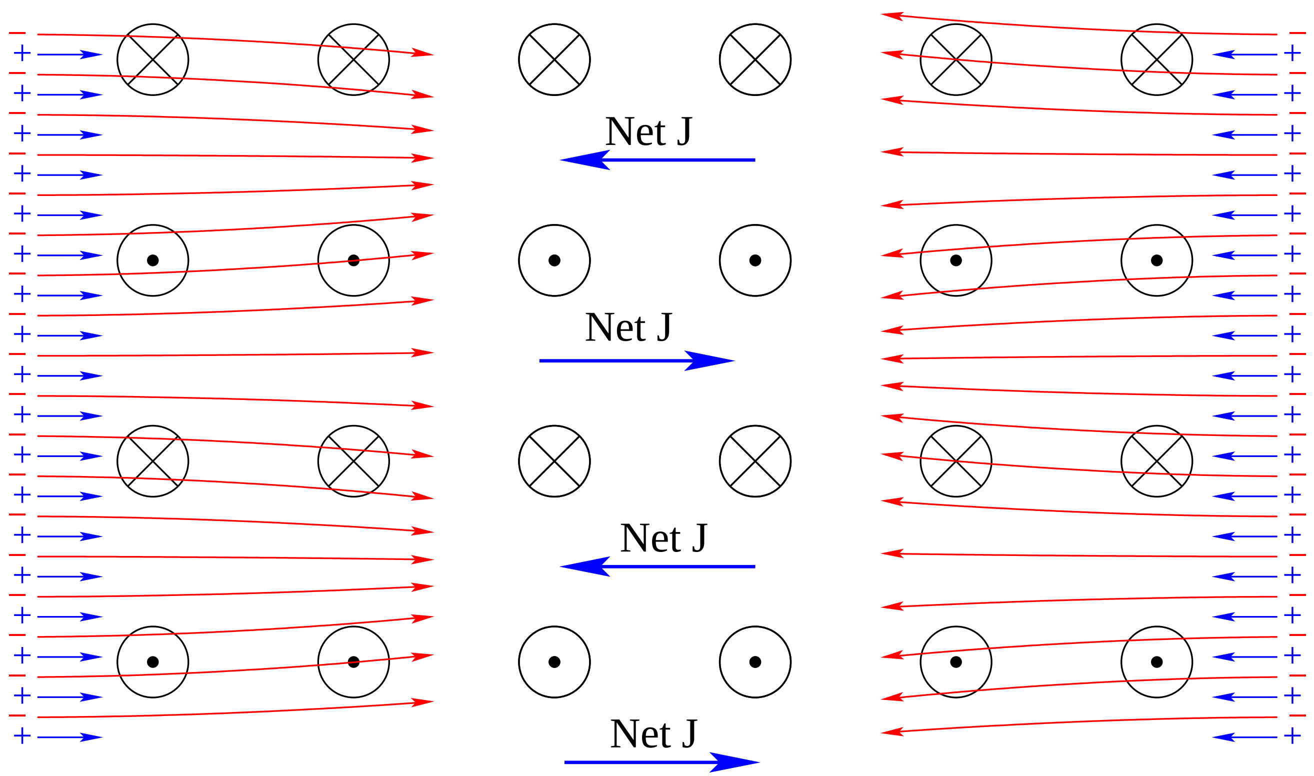
How do the particles deflect?

Positive charges:



No net ρ. Net current is induced as indicated.

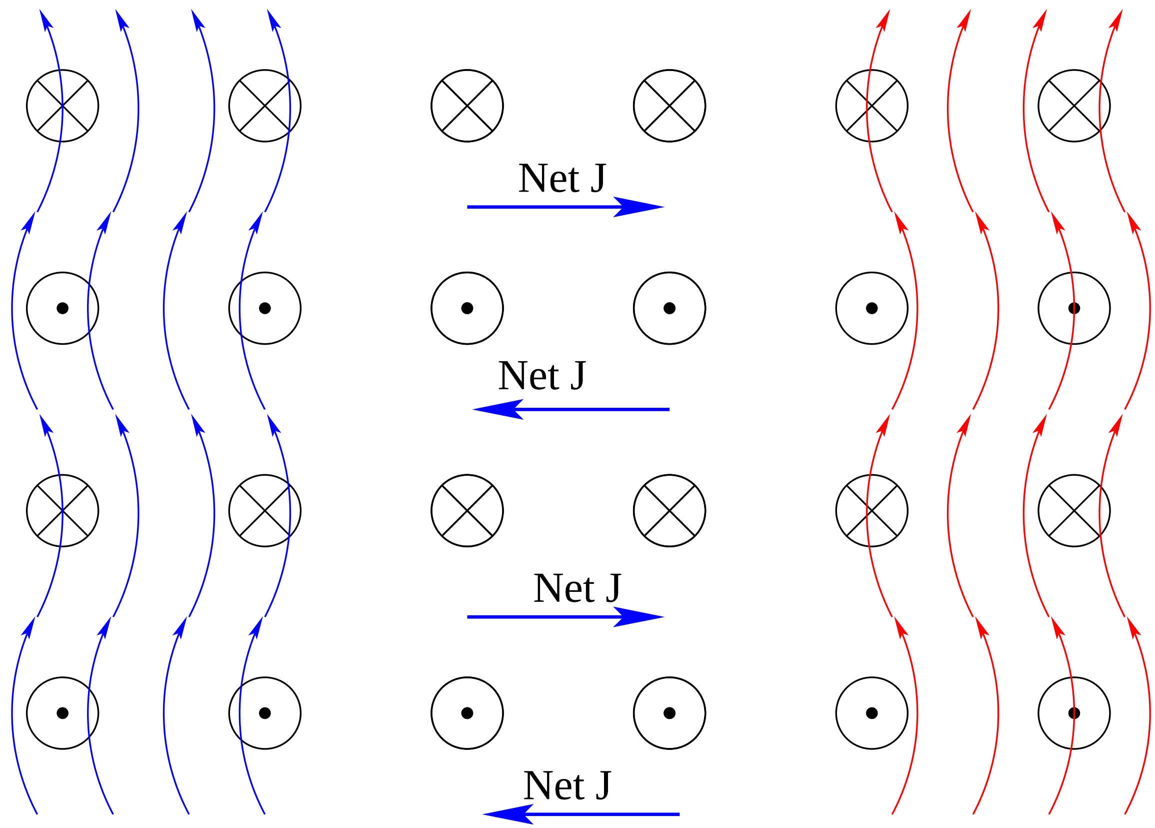
Negative charges: same-sign current contribution



Induced B *adds* to seed B. Exponential Weibel instability

Linearized analysis: B grows until bending angles become large.

Note: particles in other directions are stabilizing



Sum of J from two signs weakens seed magnetic field.

Isotropy: effects from different directions cancel!

What is B field growth rate?

Force, velocity, deflection:

$$F = eB\,; \qquad \Delta v = \frac{tF}{p} = \frac{eBt}{p}\,; \qquad \Delta y = \frac{\Delta v\, t}{2} = \frac{eB}{2p} t^2\,.$$

concentration of charges: $\sim \Delta y / \lambda_B$ or $k \Delta y$ k the B-wavevector

$$J \sim e n_{\rm chg}\, k \Delta y \sim e^2 B t^2 k \int \frac{d^3 p}{(2\pi)^3 2p} f(p)$$

Define the combination

$$e^2 \int \frac{d^3 p}{(2\pi)^3 p} f(p) \equiv m^2$$

Screening mass squared

From last slide:

$$J \sim (kB)m^2t^2 \,.$$

Current must compete with field terms in Ampere's law:

$$D \times B - D_t E = J$$

For current to *really matter*, need $J \sim D \times B \sim kB$.
This occurs when $m^2t^2 \sim 1$ or $t \sim 1/m$.
Hence growth rate estimate: $B \sim B_0 e^{\gamma t}$, $\gamma \sim m$.

Picture is self-consistent **IF** particles stay in same-sign B field for time scales $t\gamma > 1$.

Assumptions I Built In in Proceeding

Abelian fields? NO! Nonabelian works too! But:

- Classical field approximation: need $\alpha \ll 1$

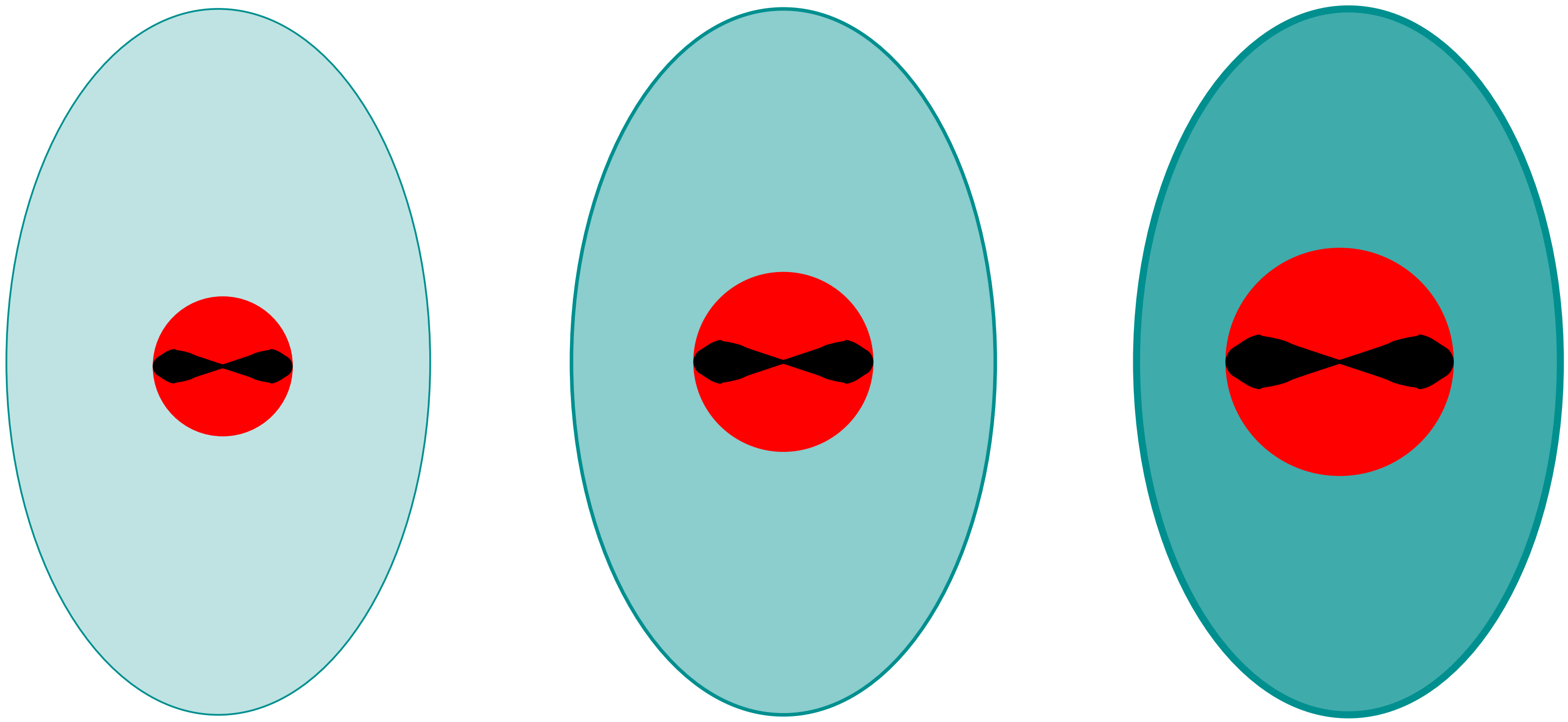
  I use e^2, g^2, α interchangeably

- Classical particle treatment: $\lambda_{p,\mathrm{deBroglie}} \gg k_{\mathrm{B}}^{-1}$ or $p \gg m, k_{\mathrm{inst}}$. Allows HL approx. Must be true in each direction!

Former: concepts make no sense at strong coupling (I think)
Latter: constraints on occupancies and level of anisotropy of excitations giving rise to instability

Dependence on Occupancy and Anisotropy

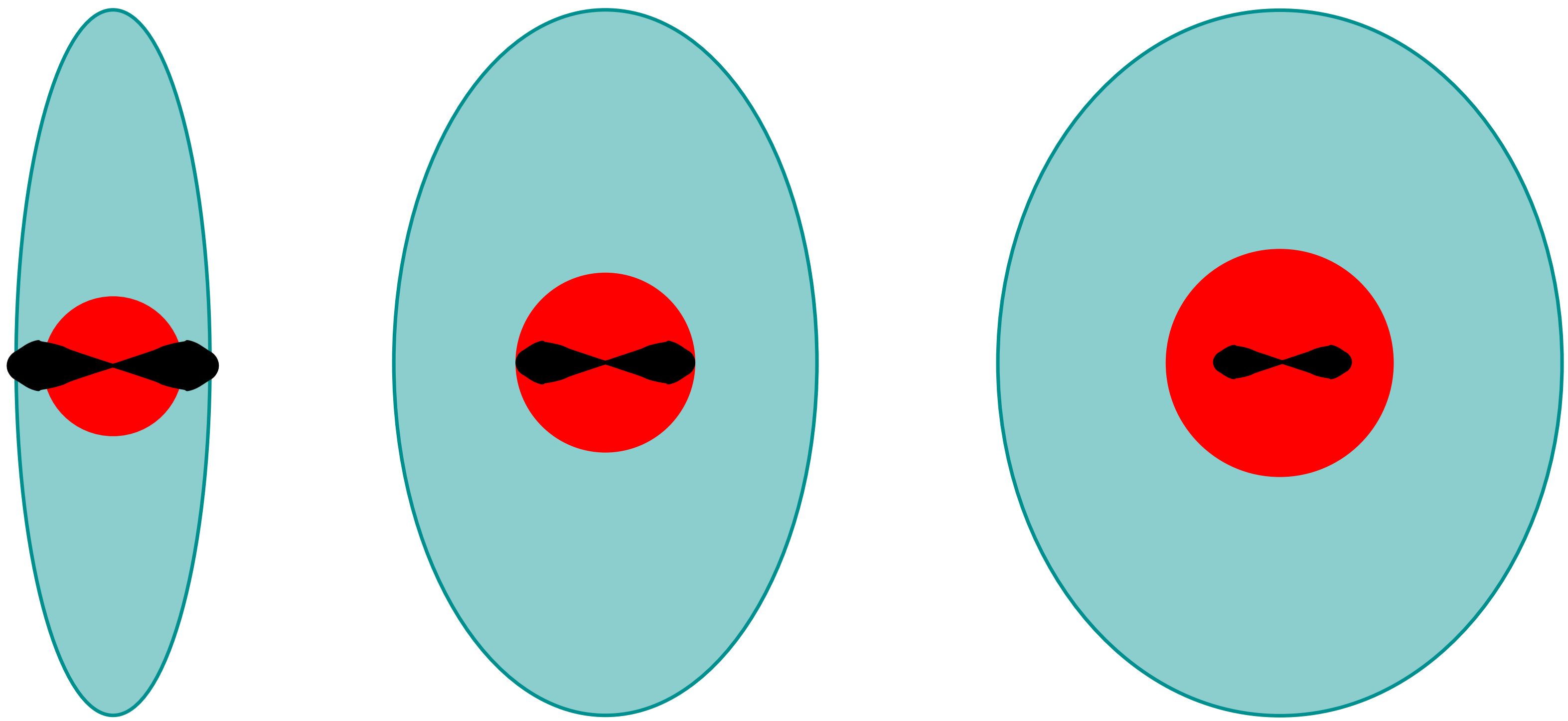
if typical momentum $p \sim Q$, $m^2 \sim \alpha Q^2 f$ (f typical occupancy):



Higher Occupancy: Larger m^2 (red), $k_{\rm inst}$ (black)

Dependence on Occupancy and Anisotropy

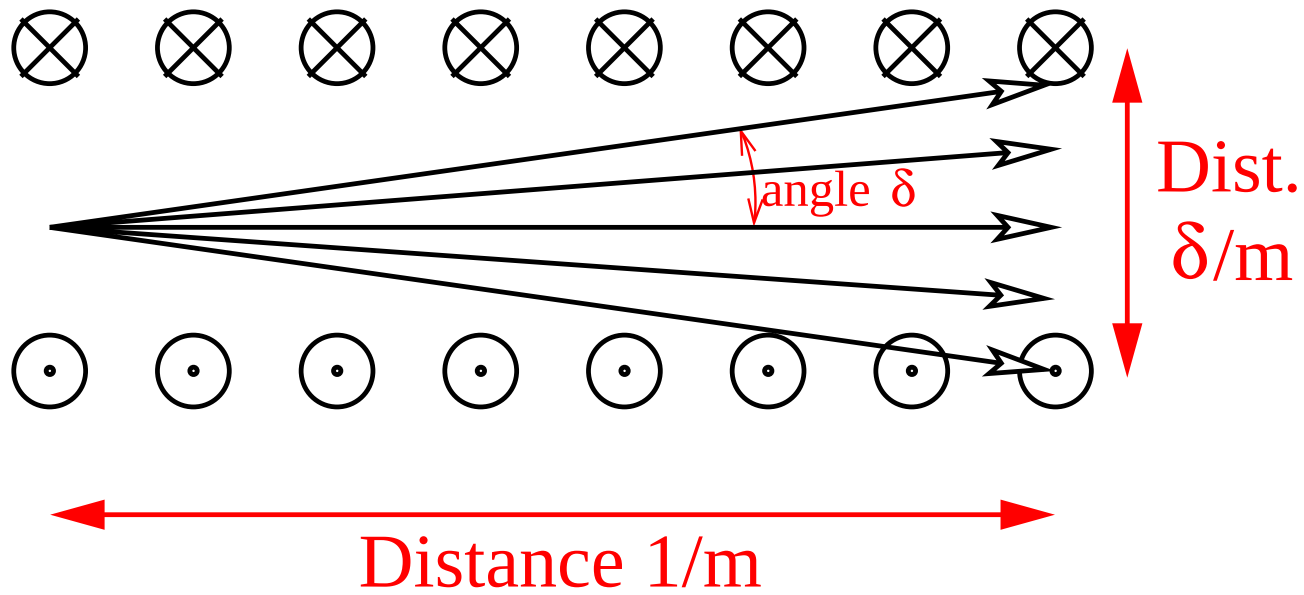
For high anisotropy, m^2 goes down, $k_{\rm inst}$ goes up!



Smaller m^2: less filled phase space, fewer part. Larger $k_{\rm inst}$: next!

High anisotropy and larger k_{inst}

Modes unstable whenever particles stay in same-sign B for $t > 1/m$. Narrow momentum distrib: time can be longer!



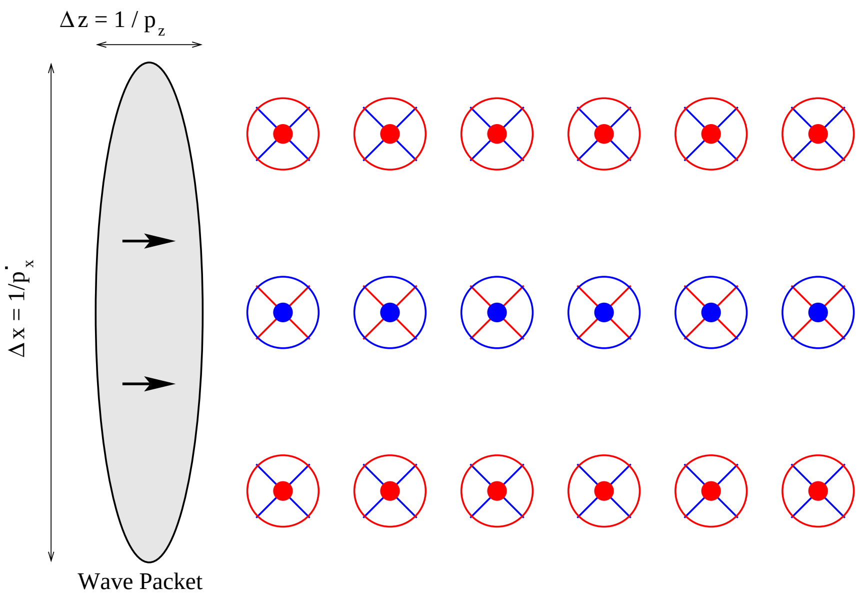
Time scale $t \sim \delta/k$ not $1/k$. So $k \sim m/\delta$ OK!

High anisotropy

$$m^2 \sim e^2 \int \frac{d^3p}{p} f(p) \sim \delta\alpha Q^2 f \quad \text{(less phase-space)}$$

But $k_{\rm inst} \sim m/\delta \sim \delta^{\frac{-1}{2}}$ larger.

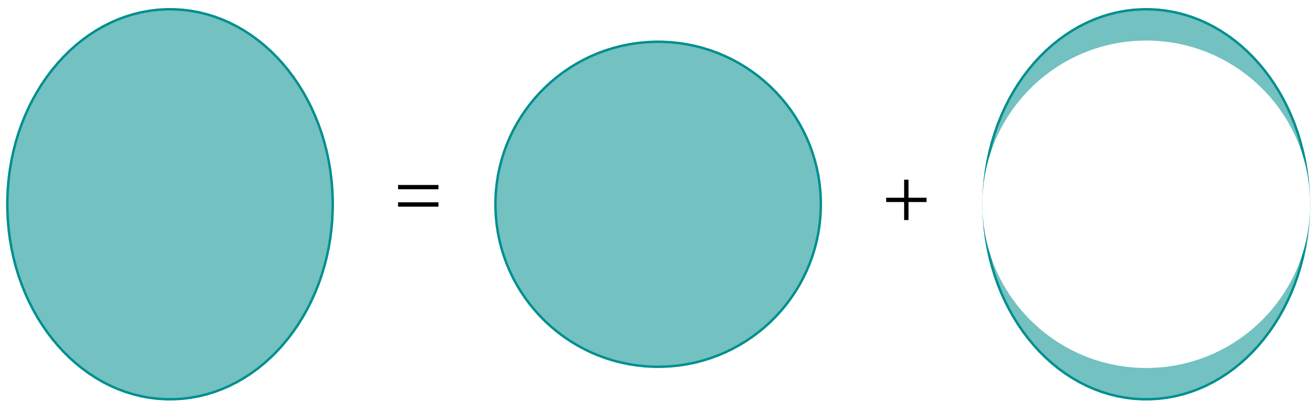
Treatment *inconsistent* if $k_{\rm inst} \sim m/\delta > \delta Q \sim p_z$



Wave packet does not fit! Alternate analysis in this regime: $k_{\rm inst} \sim \sqrt{mQ}$: more than δQ, less than m/δ.

Weak anisotropy

What if almost-isotropic, with a few ($\mathcal{O}(\epsilon)$) extras?



Isotropic part has no influence on what k's unstable!

Repeat treatment using ϵm^2 in place of m^2.

$k_{\rm inst} \sim \epsilon^{\frac{1}{2}} m$. But $\gamma \sim \epsilon^{\frac{3}{2}} m$.

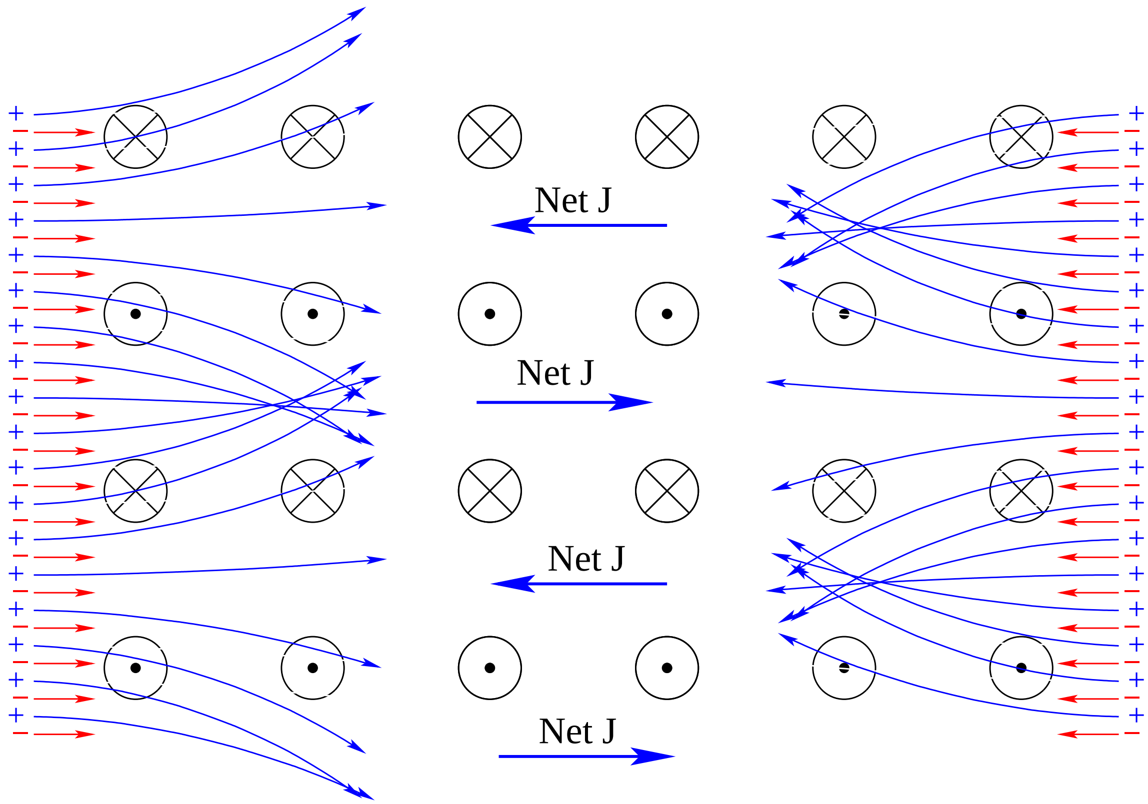
What limits B-field growth?

Many things *can* do the job:

- Large angle change: $\Delta y < 1/k$ or $B < \delta mp/g$ Robust
- Nielsen-Olesen instability: $B \lesssim k^2/g \sim m^2/(\delta^2 g)$ Robust
- Nonabelian Nonlinear Interactions:
  $B \lesssim k_\perp k_z/g \sim m^2/(\delta g)$ initial conditions?

Whichever works at smallest B-value is relevant.

Large angle change



Current stops building when particles bend too much.

$$\Delta y > 1/k \quad \Rightarrow \quad \Delta p > \delta p\,, \quad B > \delta mp/g$$

Nielsen-Olesen Instability

Uniform B field: circular orbits, quantized $p_\perp$: ($\perp$ to B that is)

$$p_\perp^2 = (1+2n)eB\,.$$

Energies of allowed excitations *FOR SPIN-1*:

$$E^2 = p_\perp^2 + p_z^2 + 2e\vec{s}\cdot\vec{B} = p_z^2 + (2n+1\pm 2)eB$$

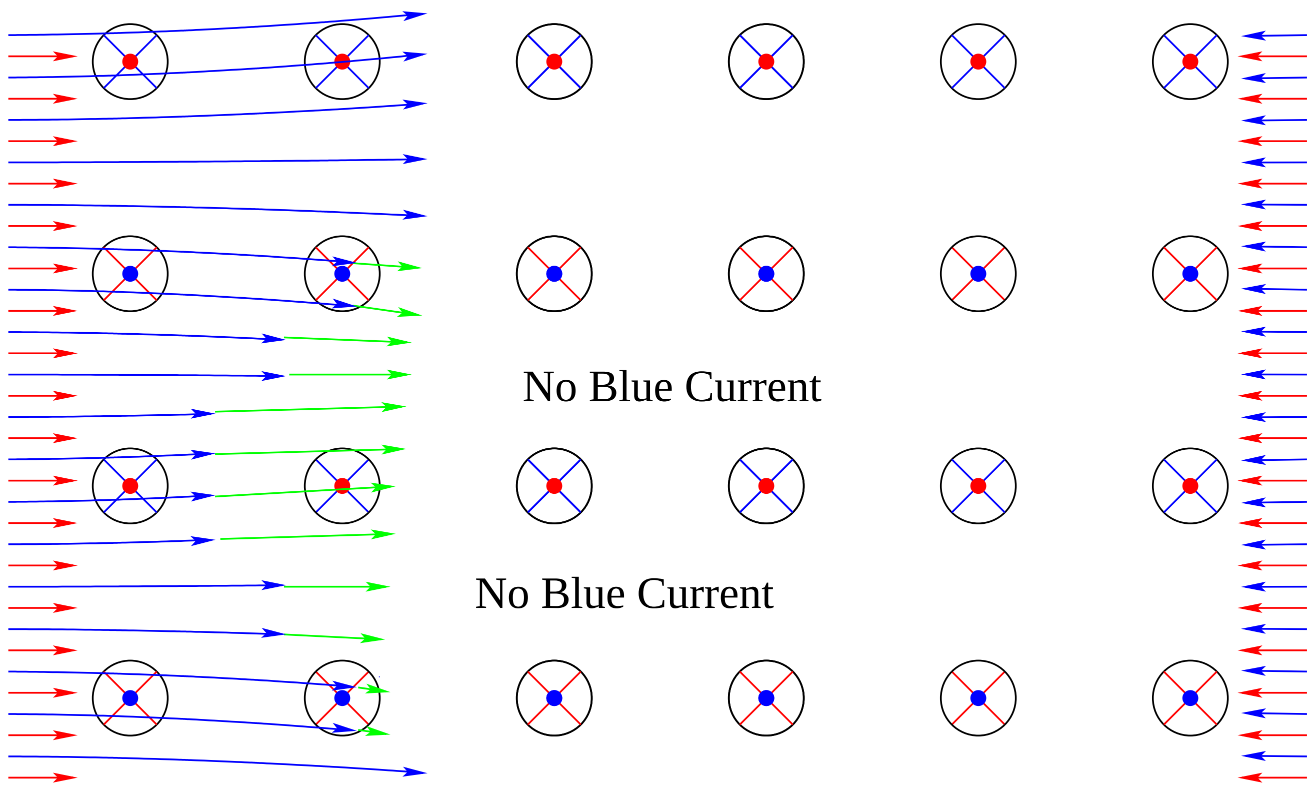
One mode has negative E^2; exp growth, $\gamma_{\text{N-O}} \sim \sqrt{gB}$

Uniform B approx is OK if $p_\perp^2$ increments $2eB > k_{\rm inst}^2$. So $k_{\rm inst} < \sqrt{eB}$ to avoid instability

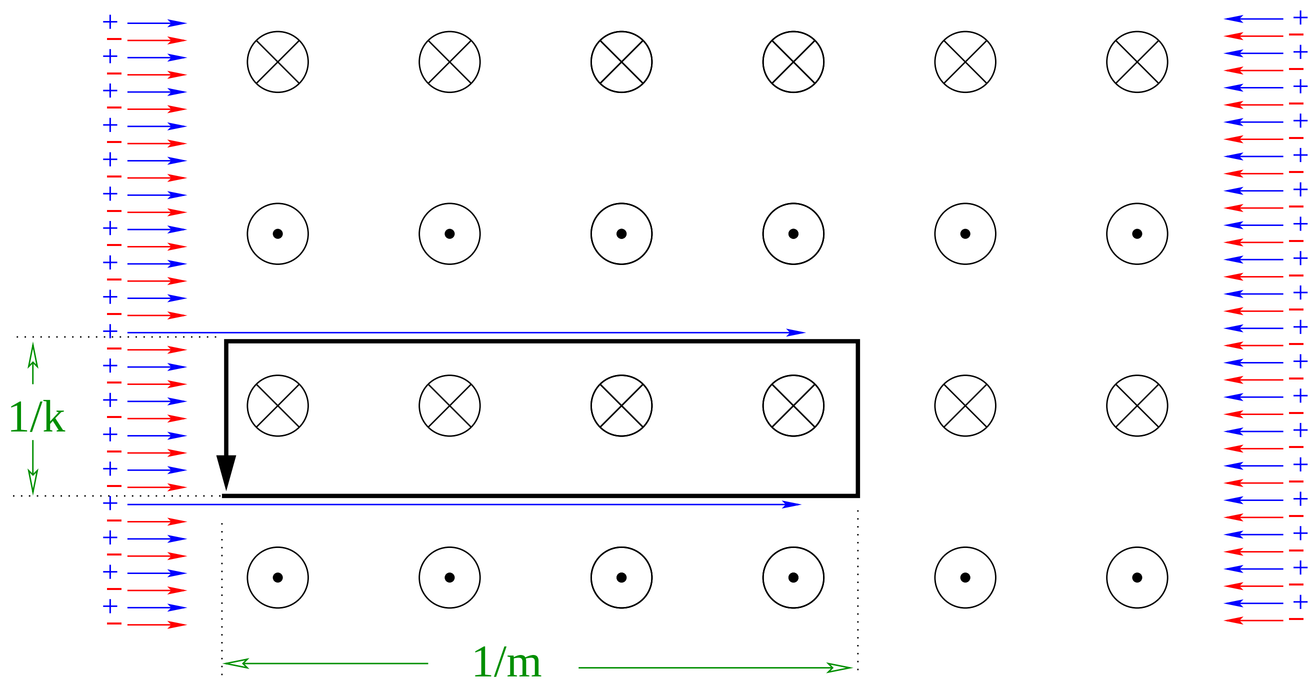
Nonabelian Nonlinearities

Suppose modes grow with many colors and k's.

One color acts to rotate J's due to another color:



Requirement this happens: $A \sim \nabla$ in covariant deriv, $A \sim m/g$ or $B \sim m^2/(g\delta)$. Gauge-invariant version:



Color randomization when Wilson loop shown has $\mathcal{O}(1)$ phase.

What do plasma instabilities do?

Main thing: angle-change.

$\Delta\theta \ll 1$ for self-consistency (we saw)

Many small independent "kicks": describe with $\hat{q}$

$$\hat{q} \equiv \frac{dp_\perp^2}{dt} \sim \frac{(\Delta p)^2}{t_{\rm coh}} = F^2 t_{\rm coh} \sim \alpha B^2 t_{\rm coh}$$

Now $B \sim m^2/g\delta$, $t_{\rm coh} \sim 1/m$

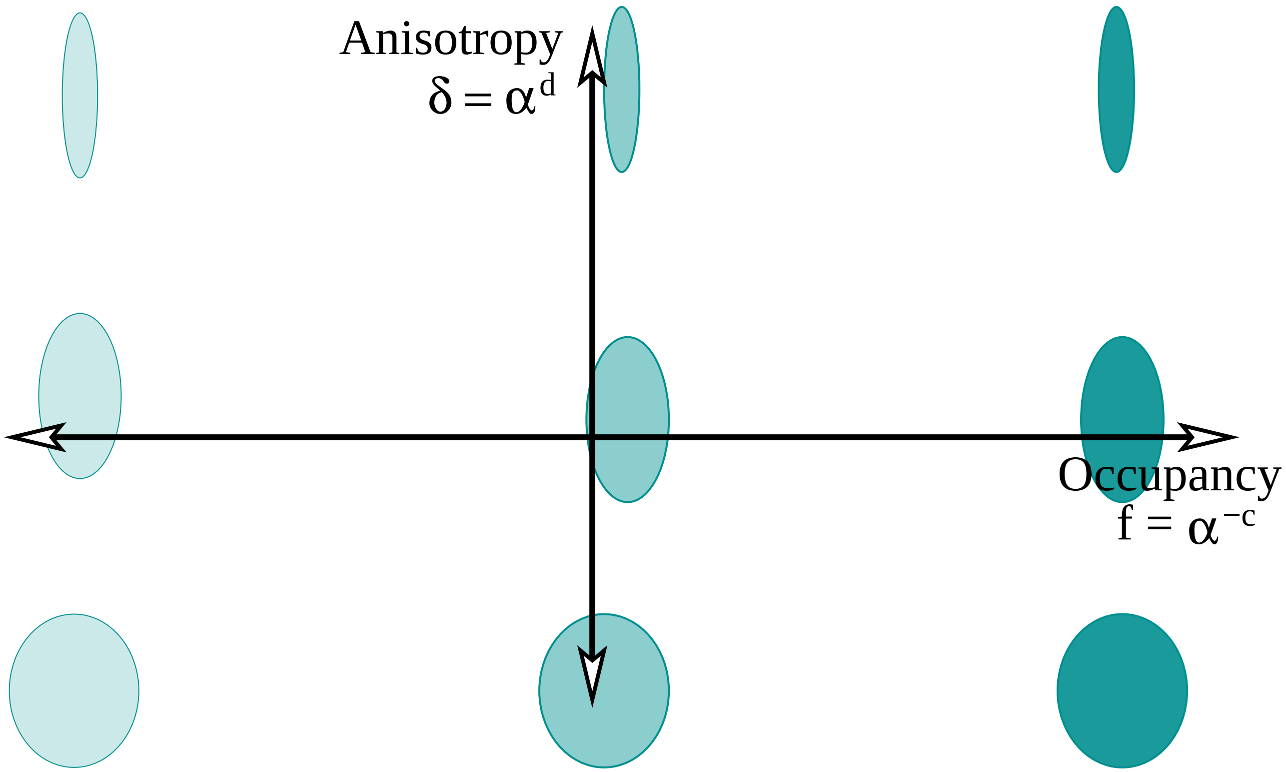
$$\hat{q} \sim \frac{m^3}{\delta^2}$$

Thermal-like, $\mathcal{O}(1)$ anisotropic: g^3T^3 (elastic: g^4T^3)

Enhanced by $1/\delta^2$ when large anisotropy.

Where are plasma instabilities important?

Assume **ONE** typical momentum scale, *eg*, Q_s

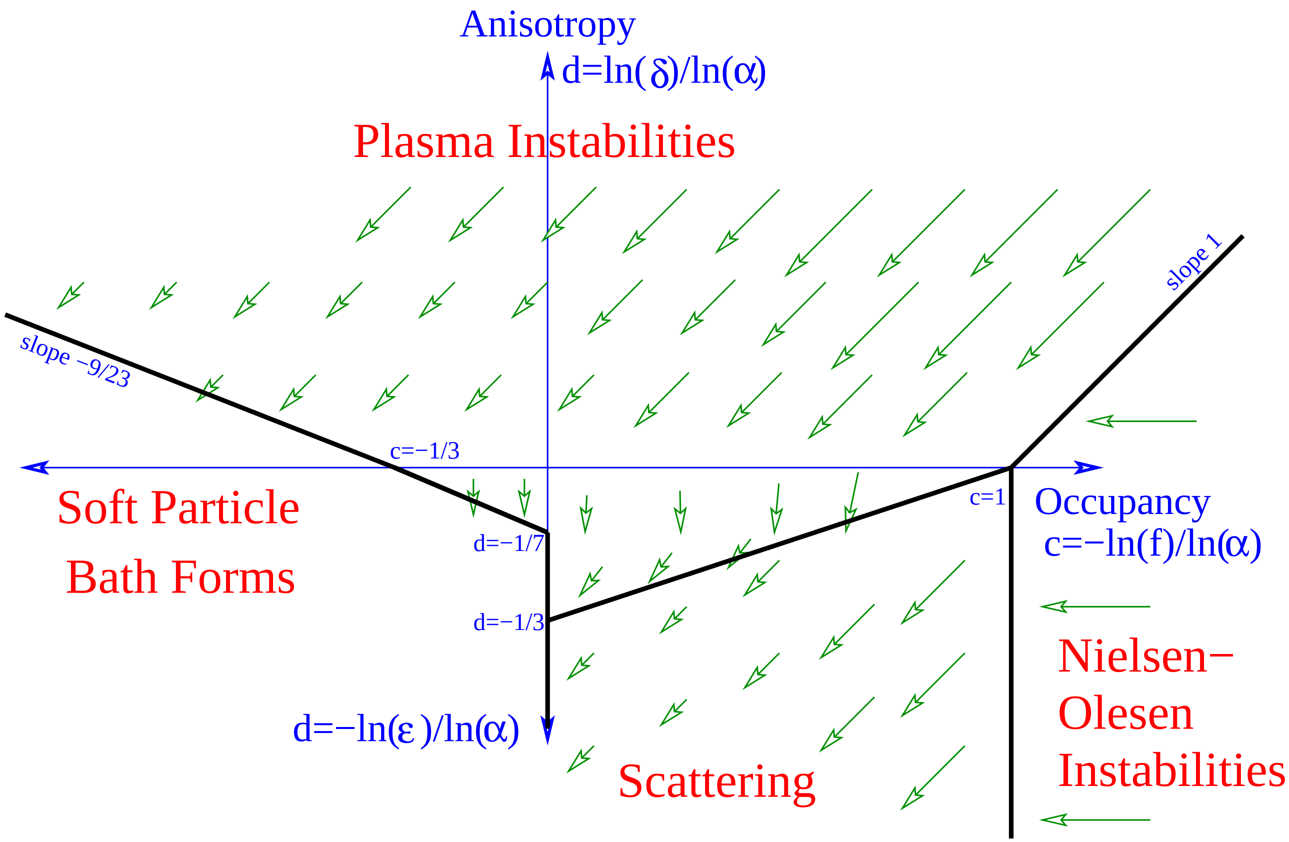


Occupancy-anisotropy plane.

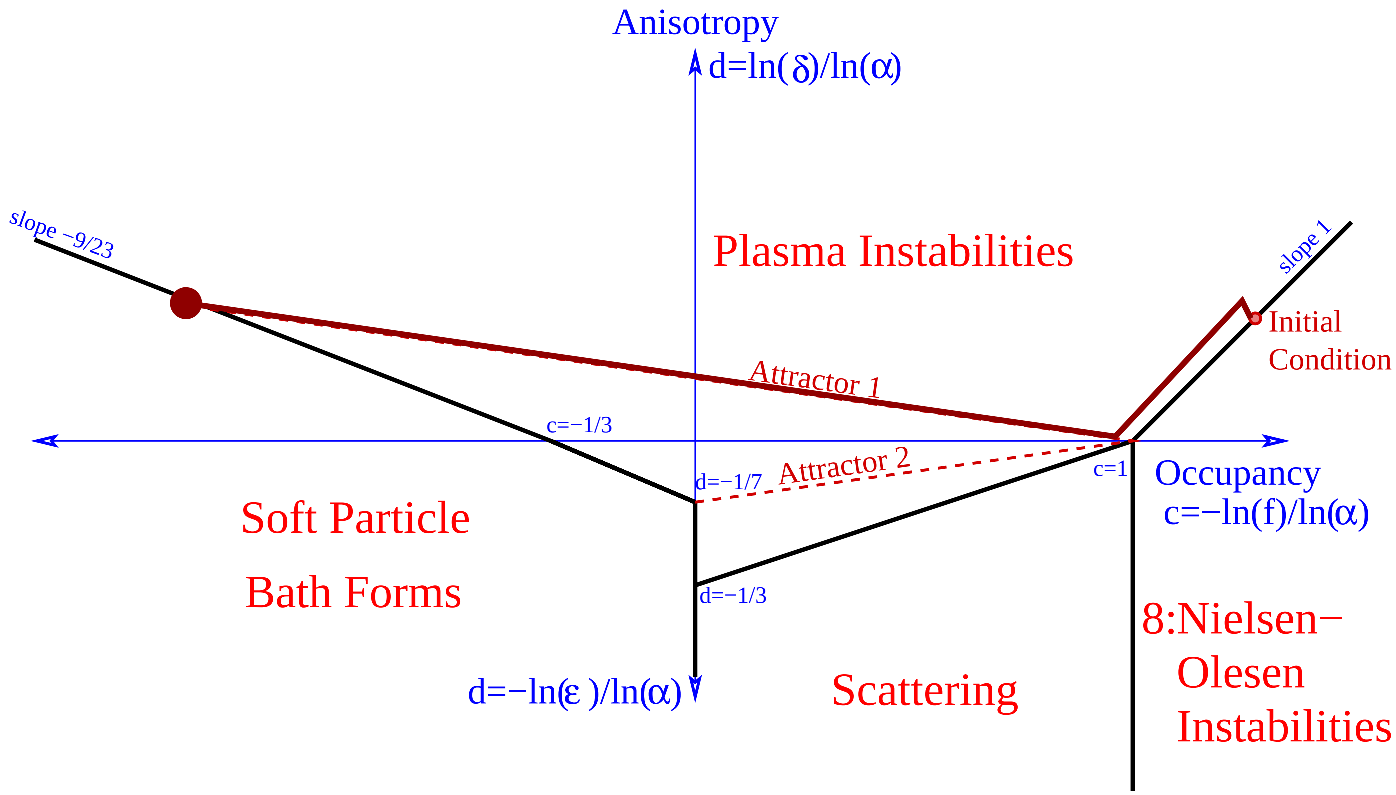
$f \sim \alpha^{-c}\Theta(p - Q_s)\Theta(|p_z - \alpha^d Q_s|)$

Where are plasma instabilities important?

Consider feedback of instab. on hard modes, radiation, merging: Kukela Moore I



Application: Heavy Ions Kurkela Moore II



Thermal bath dominates: $t \sim \alpha_s^{\frac{-12}{5}} Q_s^{-1}$

equilibration: $t \sim \alpha_s^{\frac{-5}{2}} Q_s^{-1}$.

Conclusions

- Plasma instabilities *generic* in anisotropic, $\alpha \ll 1$ plasmas
- Especially important in situations of high anisotropy
- More phenomena can limit growth in a nonabelian than in an abelian context
- Should play a pivotal role in the equilibration process in heavy ion collisions (in the toy case of $\alpha_\mathrm{s} \ll 1$)
- To make quantitative predictions we need to understand the weak anisotropy case.