The spectrum of initial fluctuations in the little bang

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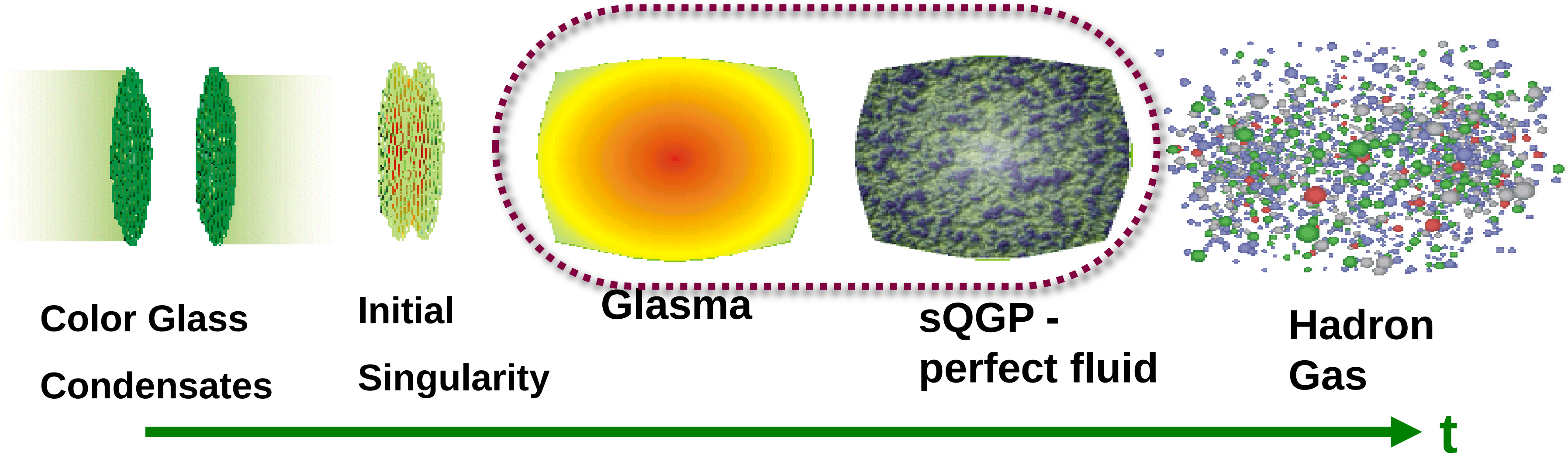
EMMI Workshop on Thermalization, Heidelberg, December 11-12, 2011

Quantum fluctuations in the Glasma



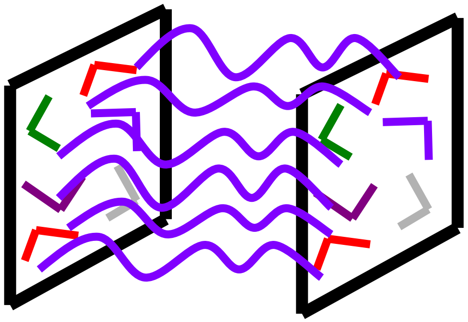
Two kinds of important quantum fluctuations:

a) **Before the collision: p_η=0 modes – factorized into the wavefunction (Francois' talk)**

a) **After the collision $p_\eta \neq 0$; hold the key to early time dynamics**

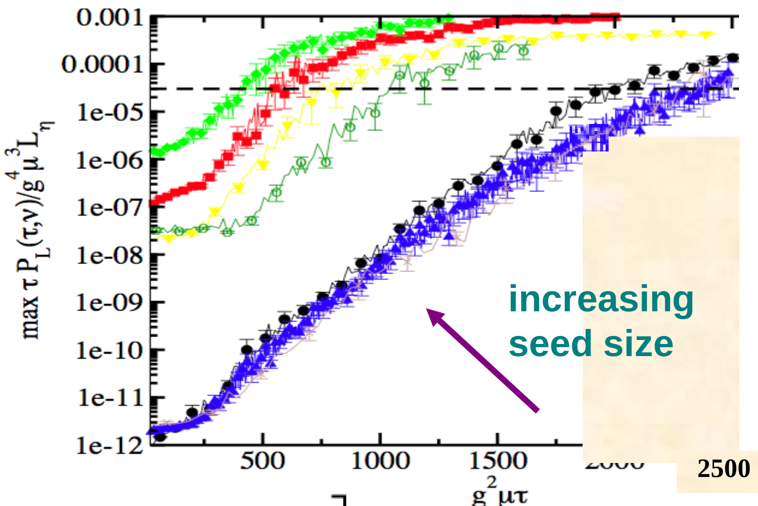
From Glasma to Plasma

Romatschke,RV
Fukushima,Gelis,McLerran



Quant. fluct.
grow exponentially
after collision

As large as classical field at 1/Qs !



$$T^{\mu\nu}_{\mathrm{N^{n-1}LO}} = \left[\int_\Sigma d^3u_1 \cdots d^3u_n \, \Gamma_n(u_1 \cdots u_n) \cdot \mathbf{T}_{u_1} \cdots \mathbf{T}_{u_n}\right] T^{\mu\nu}_{\mathrm{LO}}$$

Gelis,Lappi,RV

For $p_\eta \neq 0$ modes: $$\mathbf{T}_u \mathcal{A}(x) \sim \frac{\delta \mathcal{A}(x)}{\delta \mathcal{A}(0,y)} \sim \exp\left(\sqrt{Q_s \tau}\right)$$

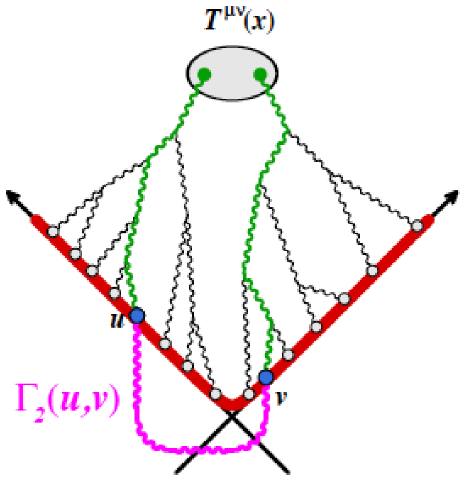
- **Requires resummation of ``secular" divergences to all orders in pert. theory**

$$\left[g \exp\left(\sqrt{Q_S \tau}\right)\right]^n$$

Quantum fluctuations: power counting

Dusling,Gelis,RV, arXiv1106.3297 (2011)

NLO:

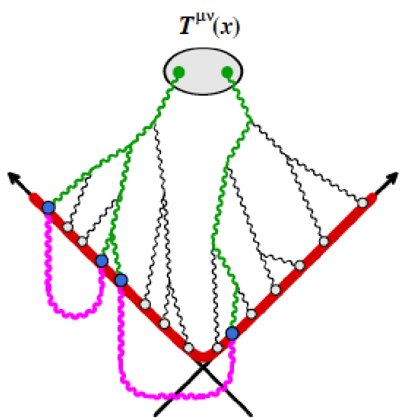


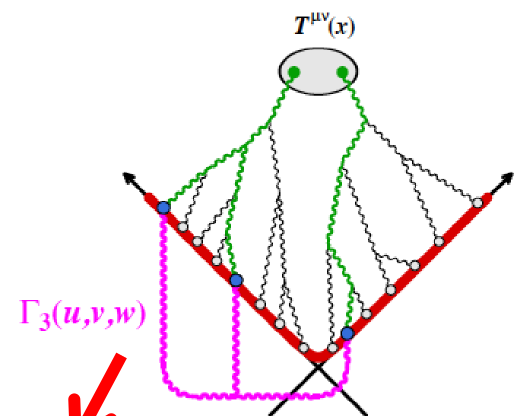
$$\Gamma_2^{\mu\nu}(u_1,u_2) = \int \frac{d^3k}{(2\pi)^3 2E_k}\, a^{\mu}_{-k}(u_1)\, a^{\nu}_{+k}(u_2)$$

$$\left[\frac{\delta^2 S_{\rm YM}}{\delta A^\mu A^\nu}\right]_{\rm A=A_{cl}} a^{\nu}_{\pm k} = 0$$

$$\lim_{x^0\to-\infty} a^{\mu}_{\pm k,\lambda a}(x) = \epsilon^\mu(k)\, T^a\, e^{\pm i k\cdot x}$$

Higher orders:





Suppressed by g relative to term on left in the power counting

Resum leading contributions: $$T^{\mu\nu}_{\rm resummed}(x) = \exp\left[\frac{1}{2}\int_\Sigma d^3u\, d^3v\, \Gamma_2(u,v)\cdot \mathbf{T}_u \mathbf{T}_v\right] T^{\mu\nu}_{\rm LO}$$

Spectrum of initial fluctuations

Dusling,Gelis,RV, arXiv1106.3297 (2011)

$$T^{\mu\nu}_{\rm resummed}(x) = \int \mathcal{D}\alpha \, F_0[\alpha] \, T^{\mu\nu}_{\rm LO}[A_{\rm cl.} + \alpha](x)$$

$$F_0[\alpha] \propto \exp\left[-\frac{1}{2}\int_\Sigma d^3u \, d^3v \, \alpha(u) \, {\Gamma_2}^{-1} \alpha(v)\right]$$

Initial spectrum of fluctuations

$$\langle\langle T^{\mu\nu} \rangle\rangle_{\rm LLx+Linst.} = \int [D\rho_1][D\rho_2] \, W_{\rm Y_{beam}-Y}[\rho_1] \, W_{\rm Y_{beam}+Y}[\rho_2] \times \int [da(u)] \, F_{\rm init}[a] \, T^{\mu\nu}_{\rm LO}[A_{\rm cl}(\rho_1, \rho_2) + a]$$

W's determined from solution of JIMWLK equation –recent progress!

Rummukainen, Weigert (2003)
Dumitru,Jalilian-Marian,Lappi,Schenke,RV, arXiv:1108.4764
Iancu,Triantafyllopolous, arXiv:1112.1104

Computing small fluctuations in the Glasma

1) Construct τ-independent inner product on initial Cauchy surface at $\tau=0^+$

2) Solve small fluctuation equations in Glasma background at $\tau=0^+$

3) Determine physical solutions

Gaussian random variable

$$\langle c_{\nu k} c_{\mu l}\rangle = 0$$

$$\langle c_{\nu k} c^*_{\mu l}\rangle = 2\pi\delta(\nu-\mu)\delta_{kl}$$

$$A(\tau,\eta,x_\perp) = A_{\rm cl.}(\tau,x_\perp) + \frac{1}{2}\int \frac{d\nu}{2\pi} d\mu_K\, c_{\nu K}\, e^{i\nu\eta}\,\chi_K(x_\perp)\, H^{(2)}_{i\nu}(\lambda_K\tau) + c.c$$

$$[D^2 + V''(A_{\rm cl.})]\chi_K(x_\perp) = \lambda_K^2\,\chi_K(x_\perp)$$

4) Well defined algorithm – numerical computations feasible

Hydrodynamics from quantum fluctuations

Dusling,Epelbaum,Gelis,RV (2011)

Previously, in inflationary context:
Son, Khlebnikov+Tkachev (1996)

"Toy" example: scalar Φ^4 theory

Gaussian random variable

$$\langle c_{\nu k} c_{\mu l} \rangle = 0$$

$$\langle c_{\nu k} c^*_{\mu l} \rangle = 2\pi \delta(\nu - \mu)\delta_{kl}$$

$$\phi(\tau, \eta, x_\perp) = \phi_{\rm cl.}(\tau, x_\perp) + \frac{1}{2}\int \frac{d\nu}{2\pi}\, d\mu_k\; c_{\nu k}\; e^{i\nu\eta}\chi_k(x_\perp) H_{i\nu}(\lambda_k \tau) + c.c$$

Satisfies the equation

$$[-\partial_\perp^2 + V''(\phi_0)]\chi_k = \lambda_k^2\, \chi_k$$

- These quantum modes satisfy the properties of high lying quantum eigenstates of classically chaotic systems conjectured by Berry and argued by Srednicki (and others) as essential for thermalization of a quantum fluid

- For a scalar theory, phase decoherence of trajectories appears to lead to hydrodynamic behavior (see talk by Thomas Epelbaum)

Some open questions

- What is the relation of our power counting to the 2PI framework of Berges et al. (talk by Yoshitaka Hatta) ? Can one include "sub-leading" contributions by preserving the formal structure of the spectrum of fluctuations

- When does this framework break down? Is there a smooth matching to kinetic theory?