Glasma Simulations, Turbulence, and Kolmogorov Spectrum

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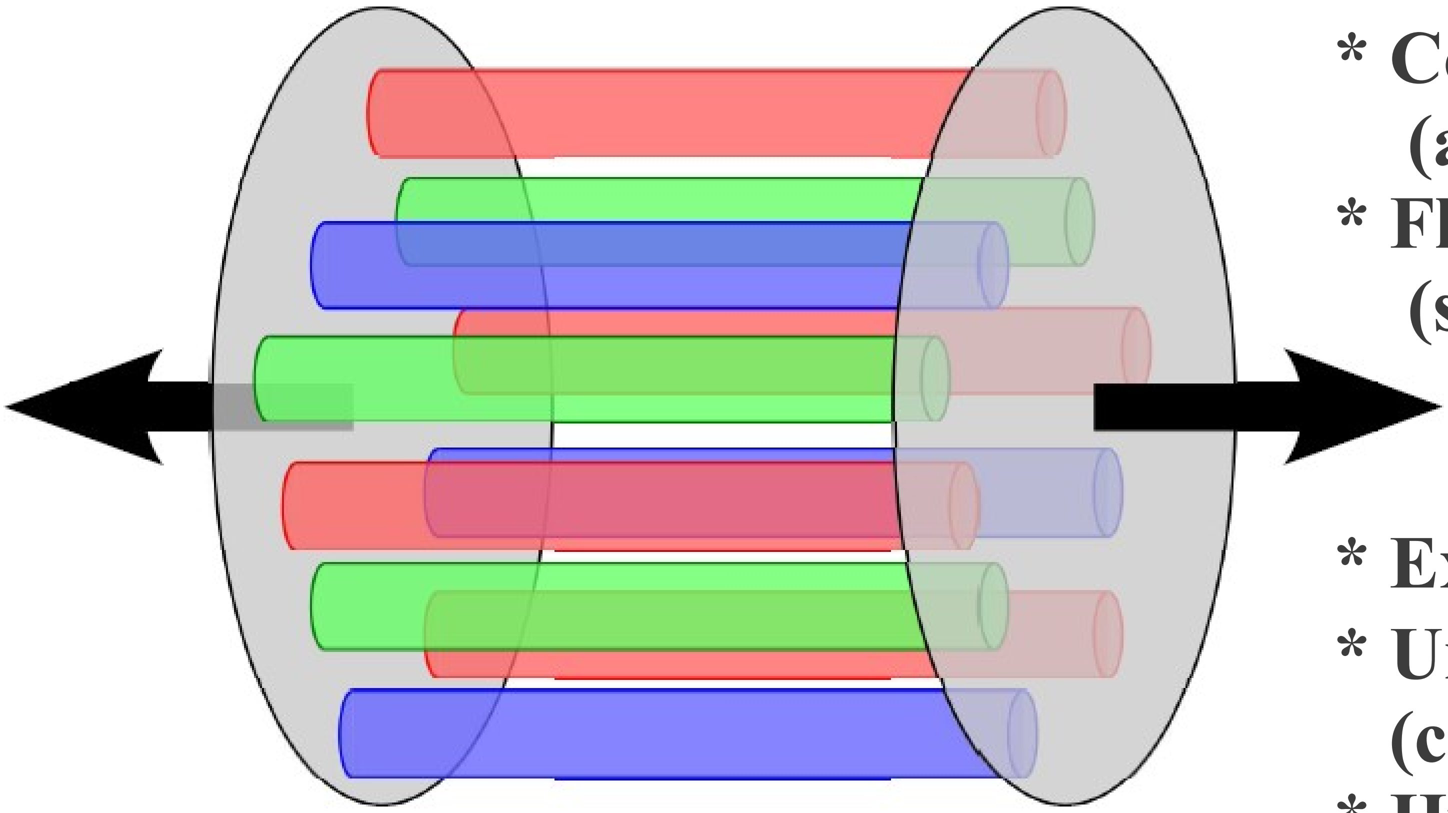
Glasma = (Color) Glass + Plasma

Intuitive Picture of Glasma



- * Boost Invariance
- * Coherent Fields (amp. ~ $1/g$)
- * Flux Tube (size ~ $1/Q_s$)

- * Expanding
- * Unstable in η (cascade to UV)
- * Hydro Input?

Formulations

Initial Condition

MV Model

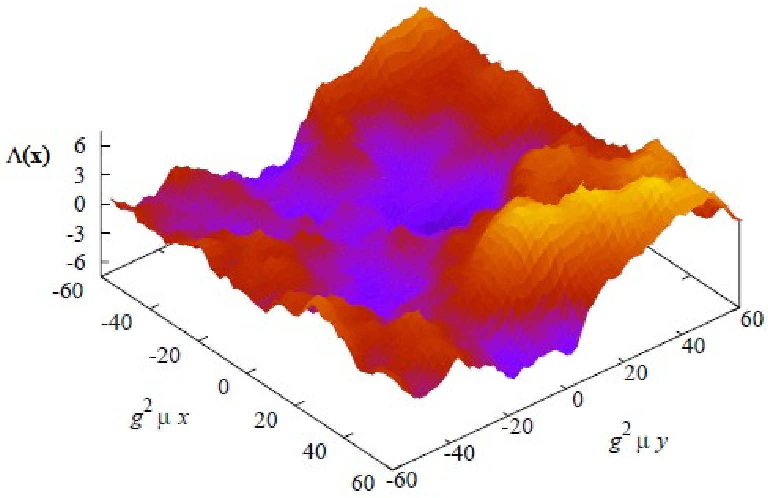
Color source distribution is Gaussian
(No spatial correlation in transverse direction)

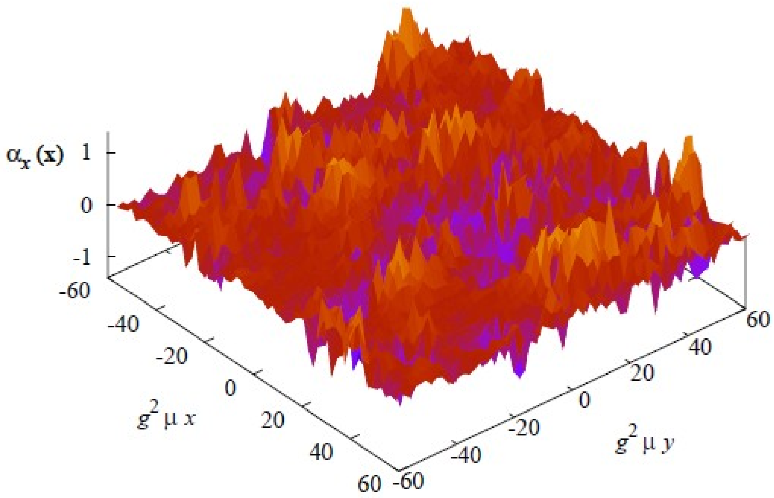
Solve the Poisson Eq

$$\partial_\perp^2 \Lambda^{(m)}(\boldsymbol{x}_\perp) = -\rho^{(m)}(\boldsymbol{x}_\perp)$$

Gauge Configuration

$$e^{-ig\Lambda(\boldsymbol{x}_\perp)} e^{ig\Lambda(\boldsymbol{x}_\perp + \hat{i})} = \exp[-ig\alpha_i(\boldsymbol{x}_\perp)]$$





Formulations

Time Evolution

$$E^i = \tau \partial_\tau A_i , \qquad E^\eta = \tau^{-1} \partial_\tau A_\eta$$

$$\partial_\tau E^i = \tau^{-1} D_\eta F_{\eta i} + \tau D_j F_{ji}$$

$$\partial_\tau E^\eta = \tau^{-1} D_j F_{j\eta}$$

Classical Equations of Motion in the Bjorken Coordinates

(in the "temporal" gauge $A_\tau = 0$)

Ensemble Average

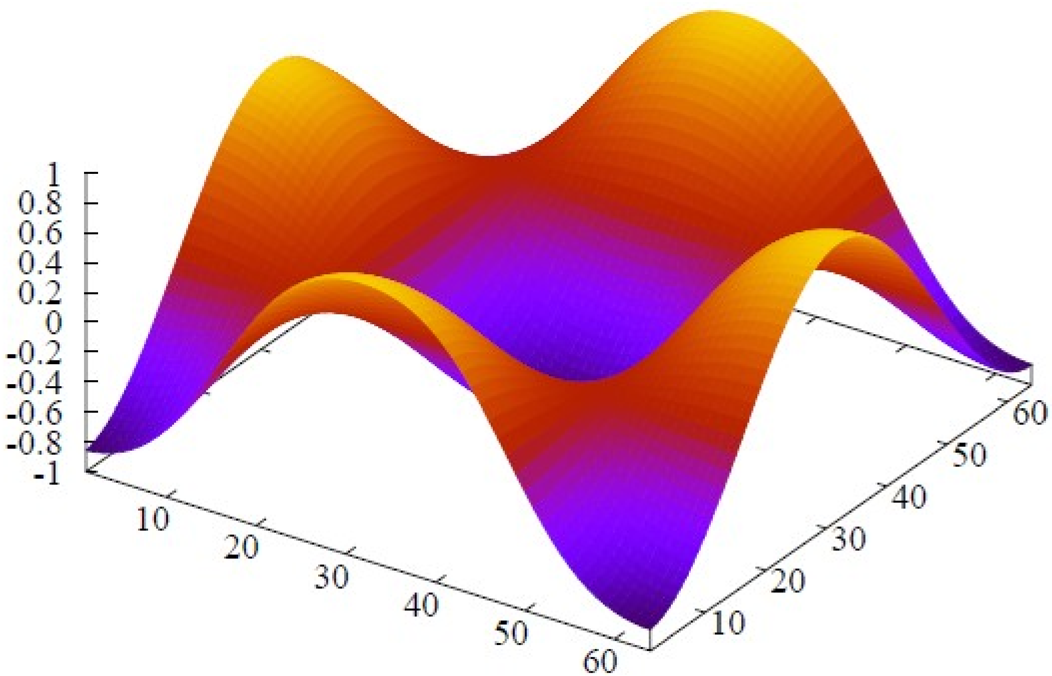
$$\langle\langle O[A] \rangle\rangle_{\rho_t , \rho_p} \sim \int D\rho_t \, D\rho_p W_x[\rho_t] W_{x'}[\rho_p] \, O[A[\rho_t , \rho_p]]$$

Stress Tensor (Energy, Pressures)

Quantum fluctuations partially included in the initial state

Example of Simulations

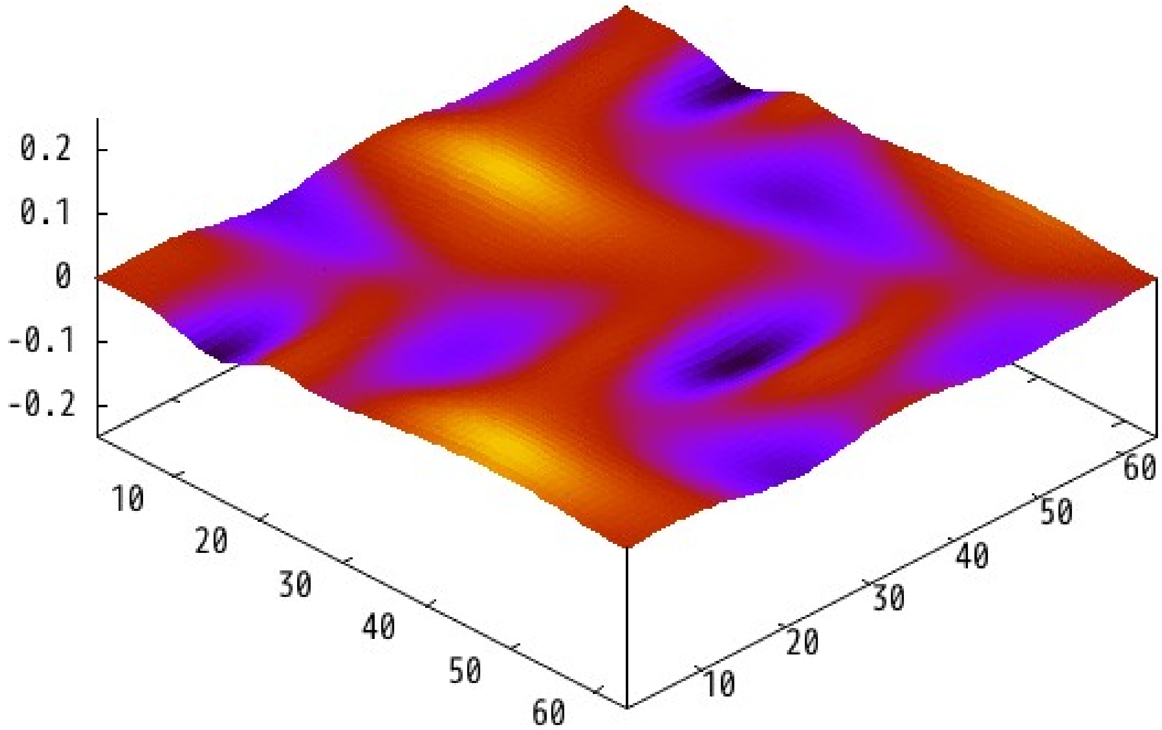
Demonstration with smooth source (one flux-tube)



$$\mathrm{tr}\left[\tau^1 U_1^{(1)}(x\,,y)\right]$$

Example of Simulations

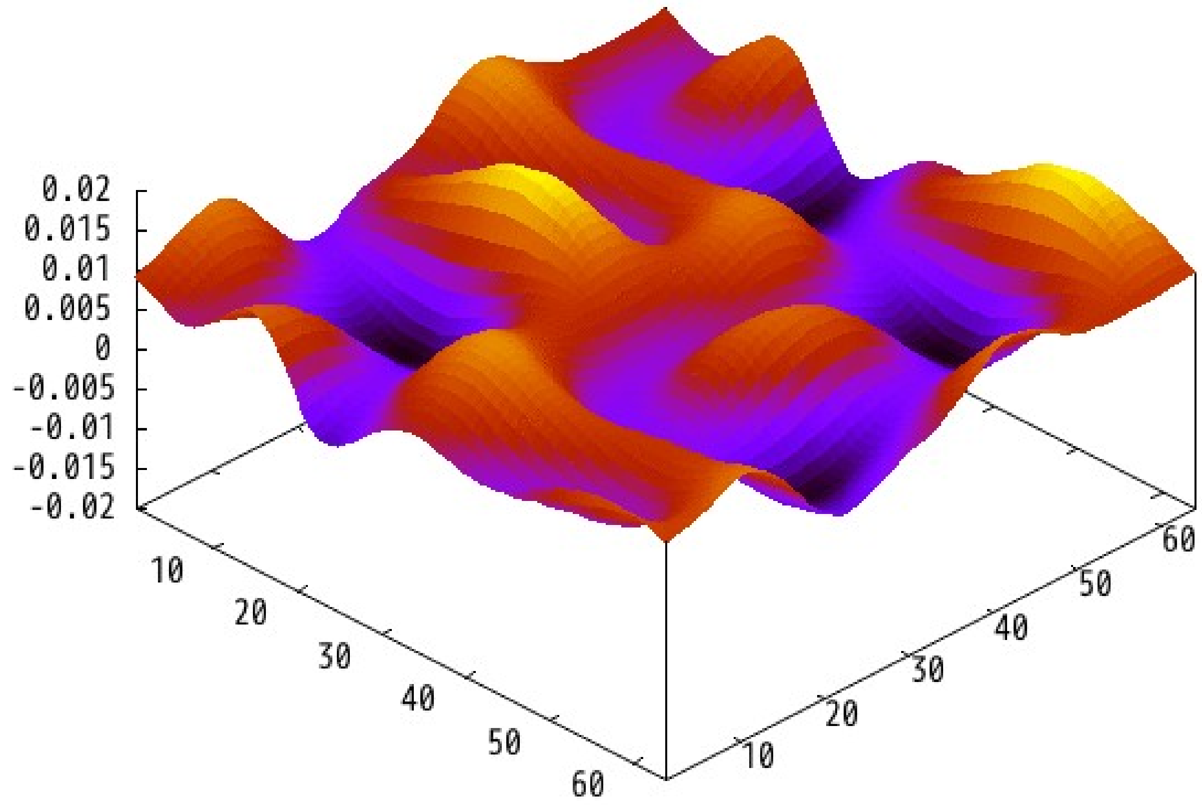
E_1



True electric field $\sim E_i/\tau$

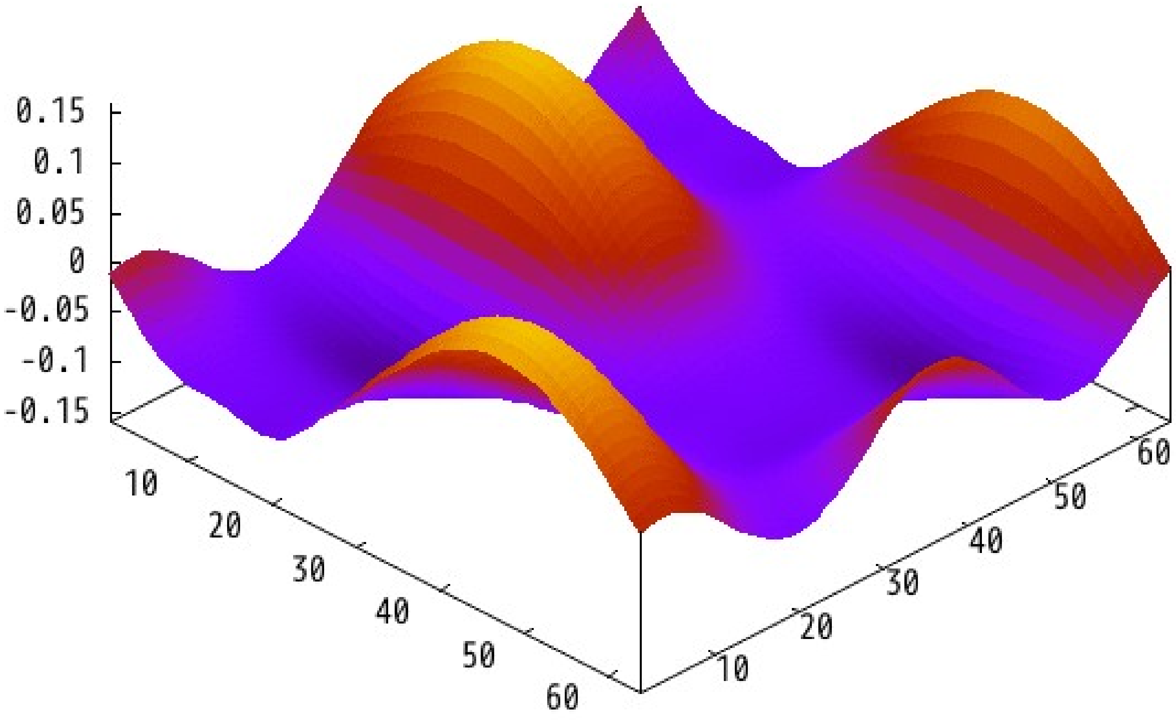
Example of Simulations

E_η



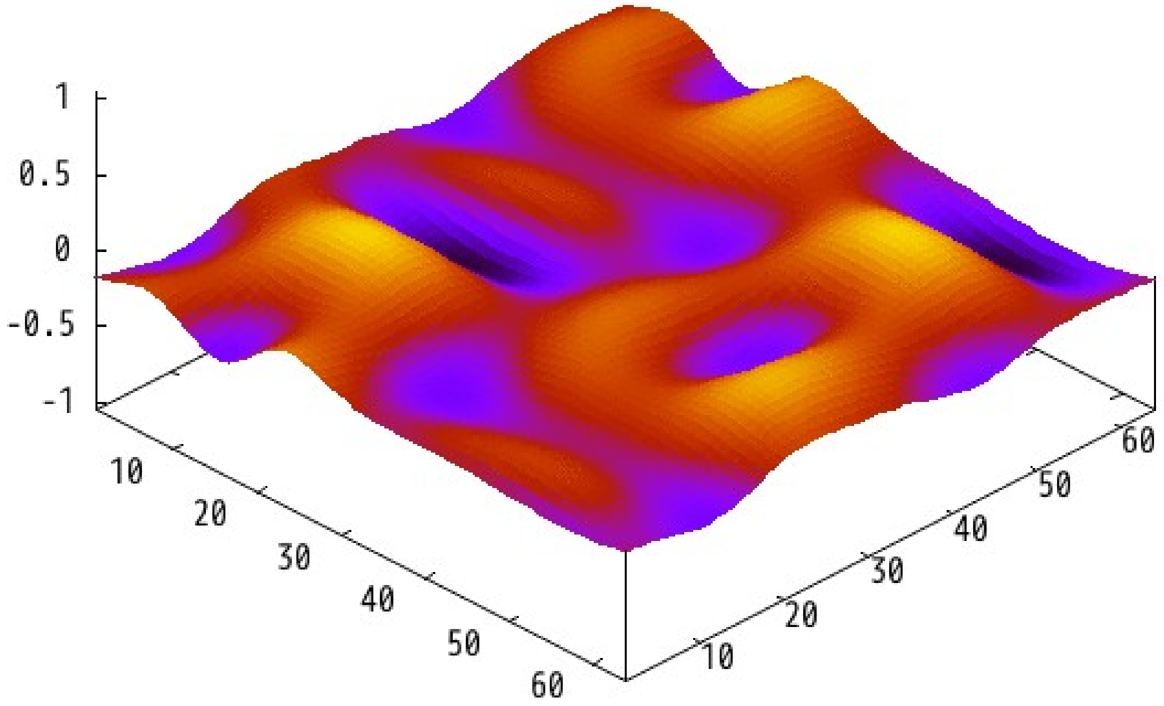
Example of Simulations

***U*₁**



Example of Simulations

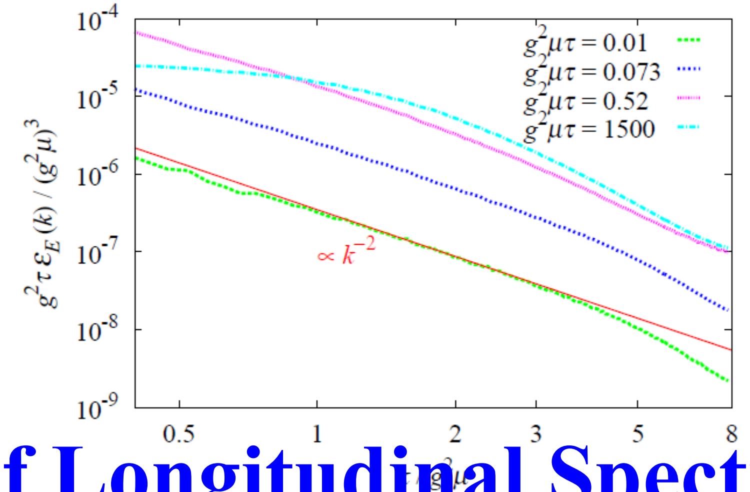
U_η



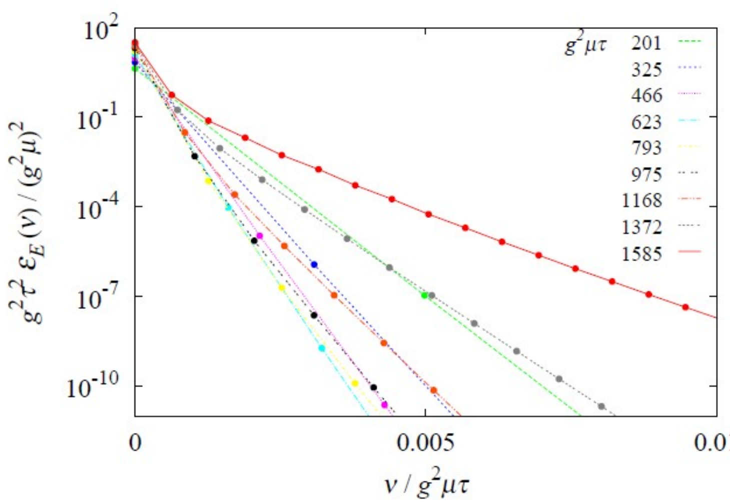
A_η , E_i fast modes (multiple time scales)

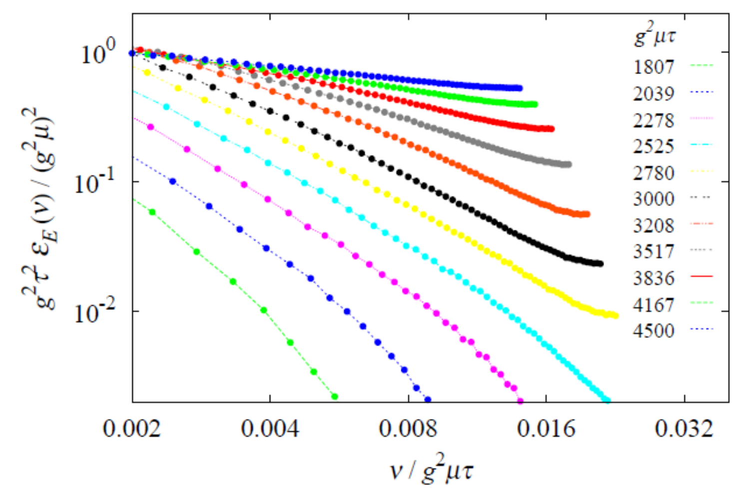
Real Simulations

Initial Transverse Spectrum



Evolution of Longitudinal Spectrum

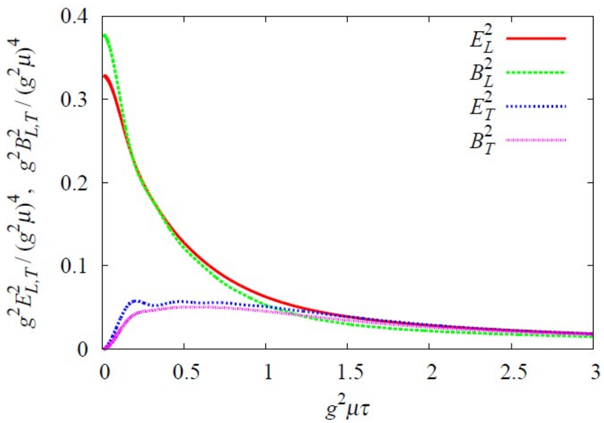


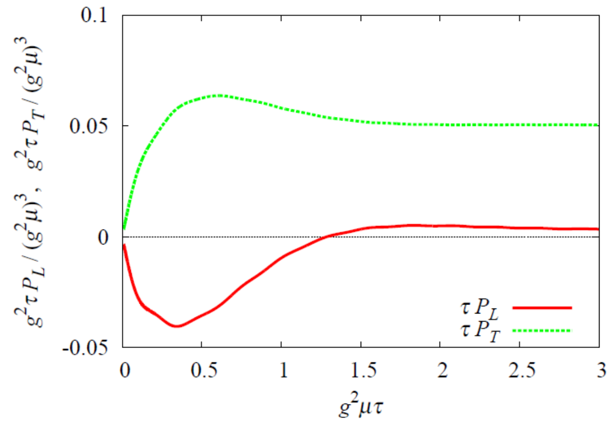


Fukushima-Gelis

Some Known Facts about Glasma Simulation

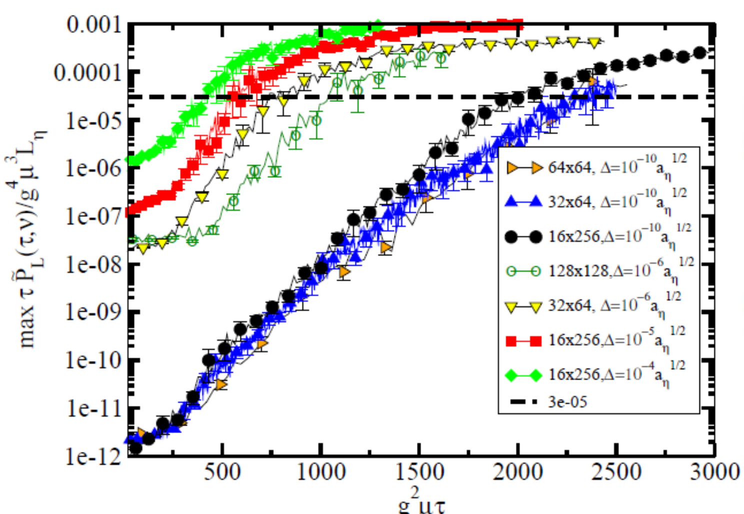
Free streaming after $1/Q_s$

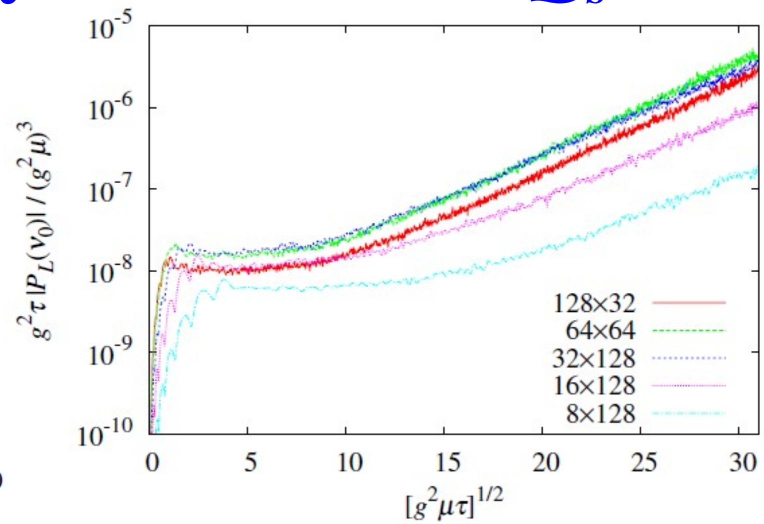




$$P_L \simeq 0$$

Glasma instability at $100\sim1000/Q_s$





Instability ends when non-Abelianized
Then spectrum shows asymptotic scaling
(Tomorrow...)

Romatschke-Venugopalan

Fukushima-Gelis

Questions

Physics Problems

1. What would be the role of the color flux-tube structure in the thermalization?
2. Why is the time scale of the Glasma instability such long; relevant time scale?
3. What is the very initial rise? Sensitive to the initial condition; deterministic?

Technical Problems

1. Why not continuum but lattice?
2. Implementation of the JIMWLK improvement

Color Flux-Tube Structure

In the MV model no color flux-tube is included. (No "serious" simulation with color flux-tube structure exists so far to the best of my knowledge.)

If the flux-tube is implemented, the Nielsen-Olesen instability should be seen. (Fujii, Itakura, Iwazaki)

$$\omega^2 = p_z^2 + 2|g\,B|(n+1/2) + m^2 - 2\,s\,g\,B$$

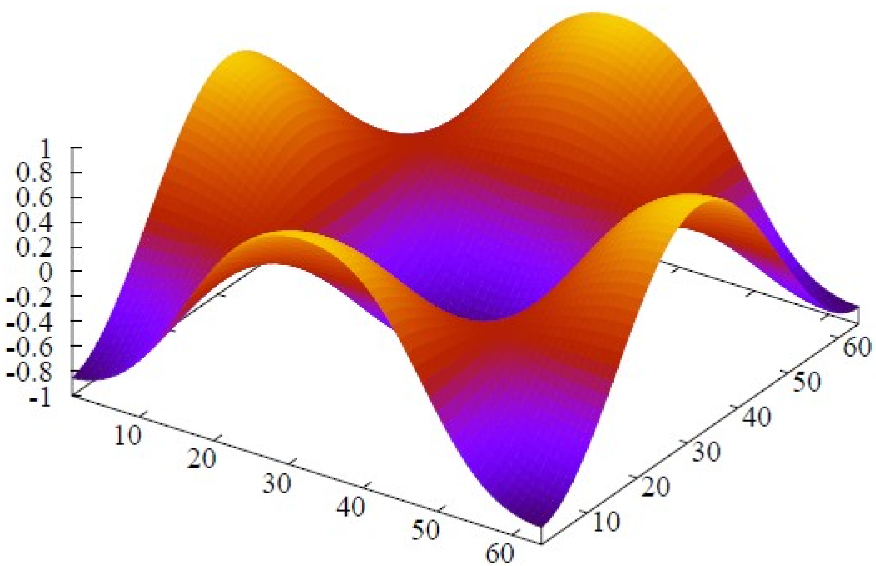
(Impact on CME)

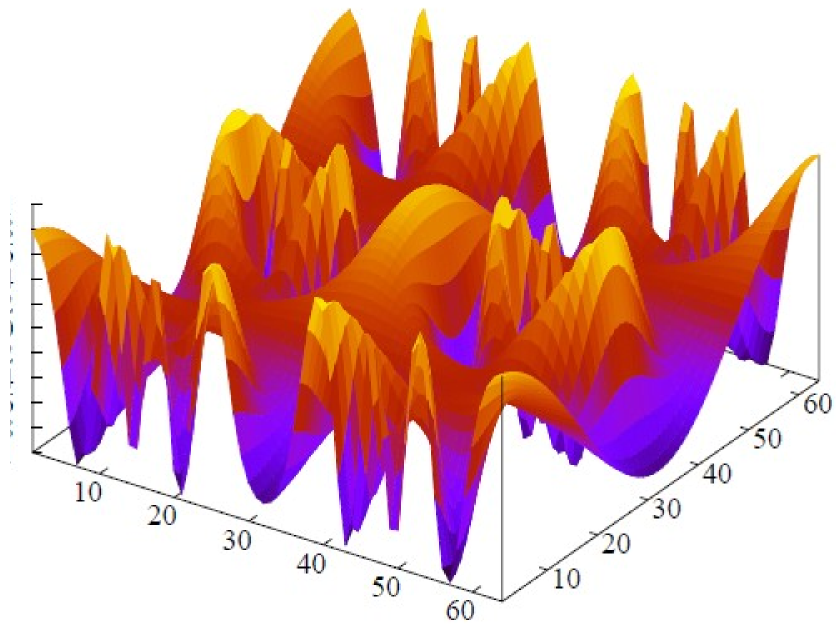
Transverse correlation from the JIMWLK evolution.

Solving the Langevin equation... (Weigert)

Large Amplitude in SU(2)

Gauge fields as angle variables

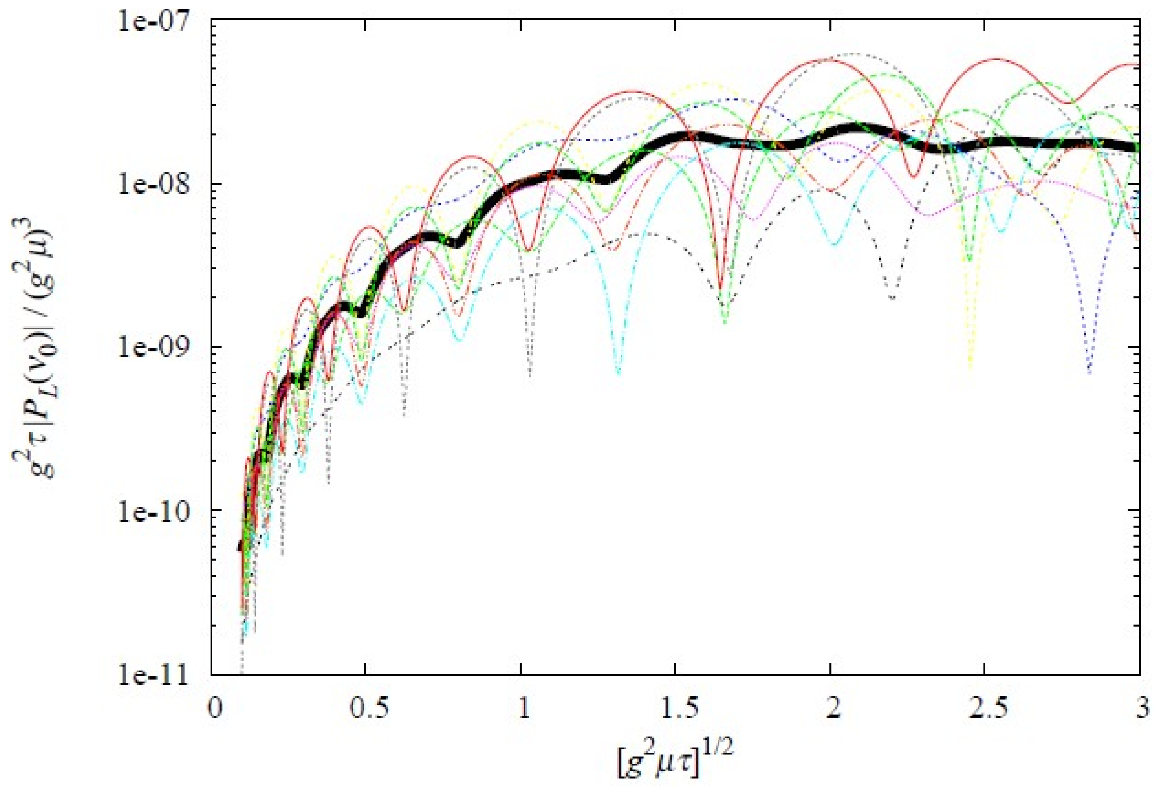




No problem if we use the continuum variables...
Why we stick to the link variables...?

Initial Rise

Still weak but seen till ~1/*Qs*



Implication? Wrong choice of initial configuration? What is the initial spectrum? Only quantum?

When Turbulent?

Reynolds number

$$R = \frac{U_0 L}{\nu}$$

L – typical size of the system
U – typical velocity of the system
ν – viscosity

Typical values of Reynolds number[16][17]

- Ciliate ~ 1 x 10^{-1}
- Smallest Fish ~ 1
- Blood flow in brain ~ 1 × 10^{2}
- Blood flow in aorta ~ 1 × 10^{3}

Onset of turbulent flow ~ 2.3 × 10^{3} to 5.0 × 10^{4} for pipe flow to 10^{6} for boundary layers

- Typical pitch in Major League Baseball ~ 2 × 10^{5}
- Person swimming ~ 4 × 10^{6}
- Fastest Fish ~ 1 x 10^{6}
- Blue Whale ~ 3 × 10^{8}
- A large ship (RMS Queen Elizabeth 2) ~ 5 × 10^{9}

From Wikipedia

When Turbulent?

Typically a flow becomes turbulent for $R \sim 10^3$

Reynolds number in a turbulent flow

$$R = \frac{u(l)\,l}{\nu}$$

l – typical scale of eddy size
u – typical size of eddy velocity
ν – viscosity

Larger Eddies $\rightarrow$ Smaller Eddies
$\rightarrow$ Kolmogrov (Smallest) Scale (stabilized by viscosity)

Reynolds Number

In many cases people would think as follows:

$$R \sim \frac{\text{(Inertial Term)}}{\text{(Viscous Term)}}$$

ex)

$$\frac{\partial s}{\partial \tau} = \frac{s}{\tau} + \frac{1}{\tau^2} \frac{\frac{4}{3}\eta + \zeta}{T} = -\frac{s}{\tau}\left(1 - R^{-1}\right) \qquad R(\tau) = \frac{s}{\frac{4}{3}\eta + \zeta} T\tau$$

More useful to think *R* in turbulence as:

$$R \sim \frac{\text{(Time Scale of Molecular Motions)}}{\text{(Time Scale of Turbulent Spreading)}} \quad \text{for fixed box size}$$

Turbulence is much for efficient for (heat) transport

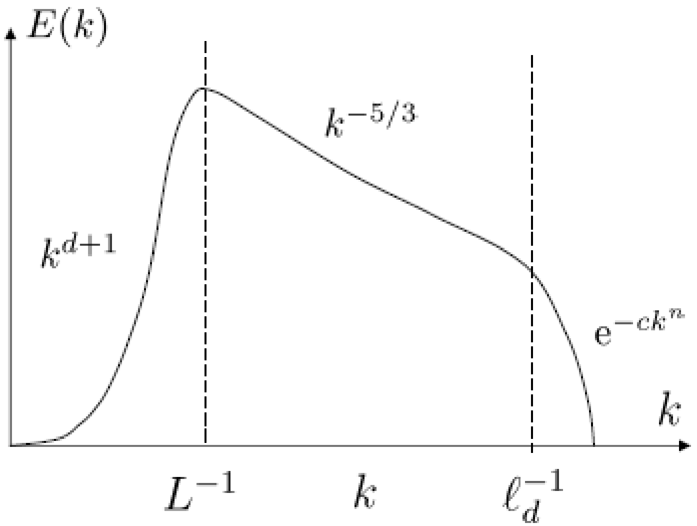
Question

What is the effective theory of QCD
in the limit of $R \to \infty$?

Hint: Scaling Analysis

Kolmogorov Scalings

Energy spectrum



l_d : Kolmogorov Smallest Scale

This is only one typical output from scaling.

Kolmogorov Hypothesis

Scaling functions of ν and ε (in a homogeneous isotropic turbulence)

$$\text{Kolmogorov Length Scale} \sim \eta = \nu^{3/4} \varepsilon^{-1/4}$$
$$\text{Kolmogorov Time Scale} \sim \sigma = \nu^{1/2} \varepsilon^{-1/2}$$

Only dependence on ε (in an inertial region up to the Kolmogorov length scale)

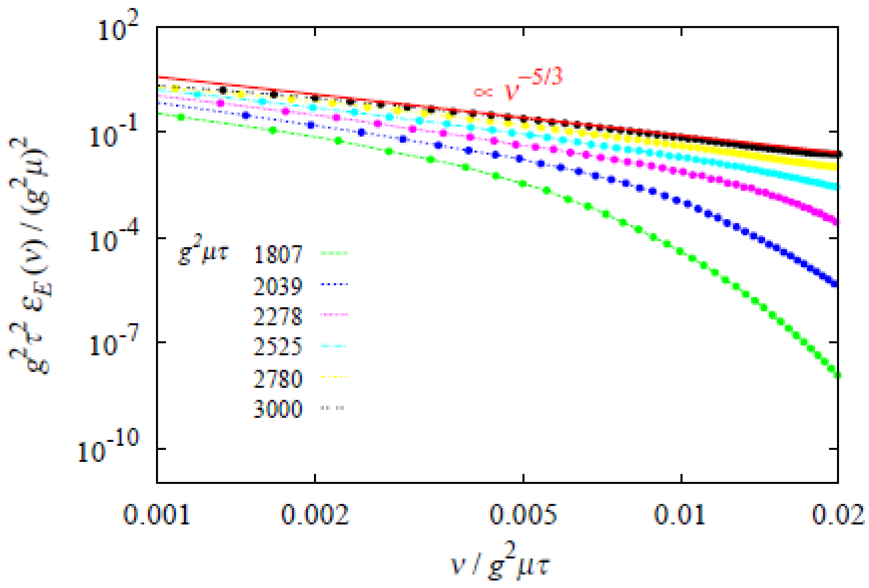
Velocity p-point Function

$$S_2(r) = C \varepsilon^{2/3} r^{2/3}$$
$$S_p(r) = C_p \varepsilon^{p/3} r^{p/3}$$

Dimensional Analysis

$$\varepsilon_E = \int \frac{\mathrm{d}\nu}{2\pi} \varepsilon_E(\nu)$$

$$\varepsilon_E(\nu) \equiv \left\langle \mathrm{tr}\left[E^{\eta a}(-\nu) E^{\eta a}(\nu) + \tau^{-2} \left(E^{ia}(-\nu) E^{ia}(\nu) \right) \right] \right\rangle$$



$$[\nu / c\tau] = l^{-1}$$

$$[V_\perp (c\tau)^2 \varepsilon_E(\nu)] = l^3 t^{-2}$$

$$[\psi] = l^2 t^{-3}$$

$$\tau^2 \varepsilon_E(\nu) \propto (\nu/\tau)^{-5/3}$$

Safe from gauge ambiguity $A_\tau = 0$ (gauge) $A_\eta \propto \tau^2 = 0$ at $\tau = 0$

Questions

The true question is not whether it is Kolmogorov or not, but the true question is whether the Kolmogorov hypothesis holds or not...

Diagrammatic derivations
(Wyld-Shut'ko theory, Hopf theory, etc...)