Factorization in high energy nucleus-nucleus collisions

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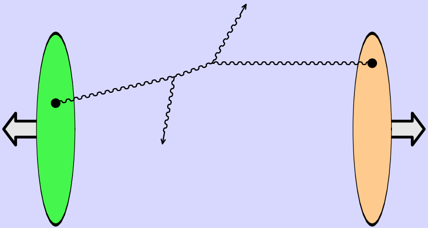
Multiple scatterings and gluon recombination

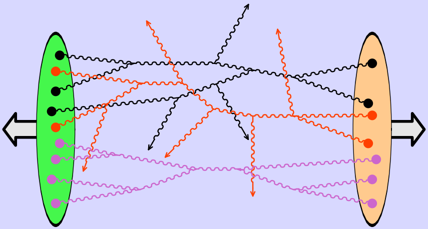
- Main difficulty: How to treat collisions involving a large number of partons?

Multiple scatterings and gluon recombination



- Dilute regime : one parton in each projectile interact
  - ▷ large Q^2, no small-x effects
  - ▷ single parton distributions + DGLAP evolution

Multiple scatterings and gluon recombination



- Dense regime : multiparton processes become crucial
  - ▷ gluon recombinations are important (saturation)
  - ▷ multi-parton distributions + JIMWLK evolution
  - ▷ new techniques are required (Color Glass Condensate):

$$\mathcal{L} = -\frac{1}{4}F^2 + J \cdot A$$

(gluons only, field A for $k^+ < \Lambda$, classical source J for $k^+ > \Lambda$)

Color Glass Condensate = effective theory of small x gluons

[McLerran, Venugopalan (1994), Jalilian-Marian, Kovner, Leonidov, Weigert (1997), Iancu, Leonidov, McLerran (2001)]

- The fast partons ($k^+ > \Lambda^+$) are frozen by time dilation
  ▷ described as static color sources on the light-cone :

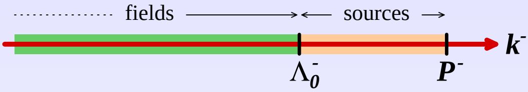
$$J^\mu = \delta^{\mu+}\rho(x^-, \vec{\boldsymbol{x}}_\perp) \qquad\qquad (0 < x^- < 1/\Lambda^+)$$

- The color sources ρ are random, and described by a probability distribution $W_{\Lambda^+}[\rho]$

- Slow partons ($k^+ < \Lambda^+$) may evolve during the collision
  ▷ treated as standard gauge fields
  ▷ eikonal coupling to the current J^μ : $J_\mu A^\mu$

$$\mathcal{S} = \underbrace{-\frac{1}{4}\int F_{\mu\nu}F^{\mu\nu}}_{\mathcal{S}_{\mathrm{YM}}} + \int \underbrace{J^\mu A_\mu}_{\text{fast partons}}$$

Renormalization group evolution, JIMWLK equation



- The cutoff between the sources and the fields is not physical, and should not enter in observables
- Loop corrections contain logs of the cutoff
- These logs can be cancelled by letting the distribution of the sources depend on the cutoff

$$\Lambda \frac{\partial W[\rho]}{\partial \Lambda} = \mathcal{H}\left(\rho, \frac{\delta}{\delta \rho}\right) W[\rho] \qquad \text{(JIMWLK equation)}$$

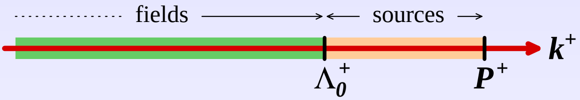
- Originally, proven in situations involving only one nucleus

  What about nucleus-nucleus collisions?

  Do the logs mix the sources of the two nuclei?

Power counting

- CGC effective theory with cutoff at the scale Λ_0^+ :



$$\mathcal{S} = \underbrace{-\frac{1}{4}\int F_{\mu\nu}F^{\mu\nu}}_{\mathcal{S}_{\rm YM}} + \int \underbrace{\left(J_1^\mu + J_2^\mu\right)}_{\text{fast partons}} A_\mu$$

- Expansion in g^2 in the saturated regime:

$$\langle T^{\mu\nu}\rangle \sim \frac{Q_s^4}{g^2}\left[c_0 + c_1\,g^2 + c_2\,g^4 + \cdots\right]$$

Leading Order

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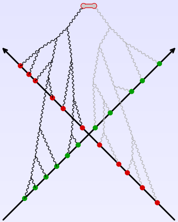
$$T^{\mu\nu}_{\rm LO} = \sum_{\rm trees}$$ 

Energy-momentum tensor at LO :

$$T^{\mu\nu}_{\rm LO} = \frac{1}{4} g^{\mu\nu}\, \mathcal{F}^{\lambda\sigma}\mathcal{F}_{\lambda\sigma} - \mathcal{F}^{\mu\lambda}\mathcal{F}^{\nu}{}_{\lambda}$$

$$\underbrace{\left[\mathcal{D}_\mu, \mathcal{F}^{\mu\nu}\right] = J_1^\nu + J_2^\nu}_{\rm Yang-Mills\ \ equation} \quad , \quad \lim_{t\to-\infty} \mathcal{A}^\mu(t,\vec{\boldsymbol{x}}) = 0$$

Next to Leading Order [FG, Venugopalan (2006)]

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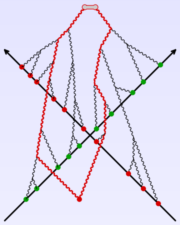
$$T^{\mu\nu}_{\rm NLO} = \sum_{\rm trees}$$



Energy-momentum tensor at NLO :

$$T^{\mu\nu}_{\rm NLO} = \left[\frac{1}{2}\int_{\vec{u},\vec{v}\in\Sigma}\int_{k}\left[a_k\,\mathbb{T}\right]_u\left[a_k^*\,\mathbb{T}\right]_v + \int_{\vec{u}\in\Sigma}\left[\alpha\,\mathbb{T}\right]_u\right] T^{\mu\nu}_{\rm LO}$$

$\Sigma =$ initial Cauchy surface , $\quad \mathbb{T} \sim \delta/\delta\mathcal{A}_{\rm init}$

- does not include virtual quarks loops
- a_k and α are calculable analytically

Leading Logs [FG, Lappi, Venugopalan (2008)]

Logs of Λ^+ and Λ^-

$$\frac{1}{2}\int\!\!\int_{\vec{u},\vec{v}\in\Sigma}\!\!\int_{\boldsymbol{k}} \left[a_{\boldsymbol{k}}\,\mathbb{T}\right]_{\boldsymbol{u}}\left[a^*_{\boldsymbol{k}}\,\mathbb{T}\right]_{\boldsymbol{v}} + \int_{\vec{u}\in\Sigma} \left[\alpha\,\mathbb{T}\right]_{\boldsymbol{u}} =$$

$$= \ln\left(\Lambda^+\right)\,\mathcal{H}_1 + \ln\left(\Lambda^-\right)\,\mathcal{H}_2 + \text{terms w/o logs}$$

$\mathcal{H}_{1,2}$ = JIMWLK Hamiltonian

- Roughly speaking, the mapping is:

$$\left[a_{\boldsymbol{k}}\,\mathbb{T}\right]_{\boldsymbol{u}} \longrightarrow \int d^2\vec{\boldsymbol{x}}_\perp\,\frac{\boldsymbol{u}^i_\perp-\boldsymbol{x}^i_\perp}{(\boldsymbol{u}_\perp-\boldsymbol{x}_\perp)^2}\,\left[\Omega(\boldsymbol{x}_\perp)-\Omega(\boldsymbol{u}_\perp)\right]_{ab}\,\nabla^b_{\boldsymbol{x}}$$

- No mixing between the logs of Λ^+ and Λ^-
- Ensures the factorizability of these logs into JIMWLK-evolved distributions $W[\rho_{1,2}]$

Factorization of the Leading Logs of 1/x

- One can factorize all the powers of $\alpha_s \log(1/x_{1,2})$

Energy-momentum tensor at Leading Log accuracy

$$\left\langle T^{\mu\nu}\right\rangle_{\text{LLog}} = \int \left[D\rho_1\, D\rho_2\right]\, W_1\left[\rho_1\right]\, W_2\left[\rho_2\right]\, \underbrace{T^{\mu\nu}_{\text{LO}}[\rho_{1,2}]}_{\text{for fixed } \rho_{1,2}}$$

- The factor $T^{\mu\nu}_{\text{LO}}$ under the integral does not depend on y: the rapidity dependence comes entirely from the distributions $W_{1,2}$

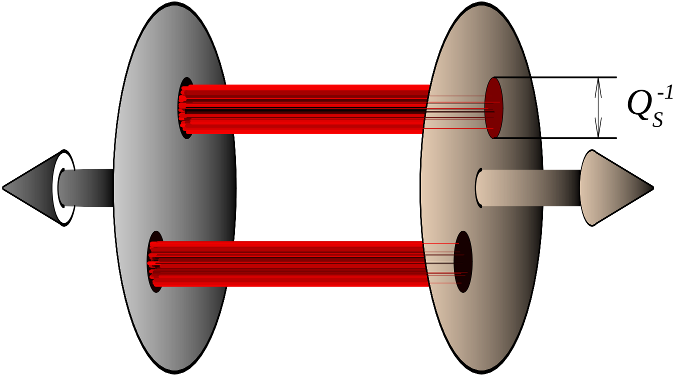
Multi-point correlations at Leading Log

- The previous factorization can be extended to multi-point correlations :

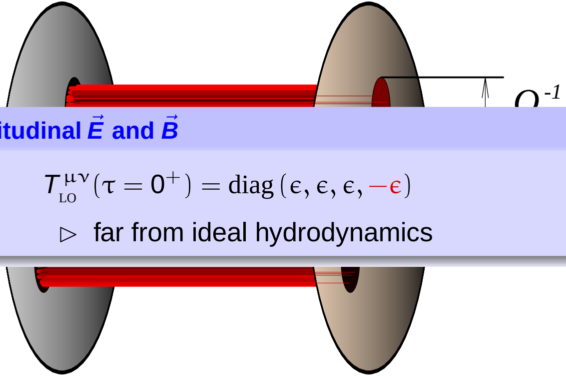
$$\left\langle T^{\mu_1\nu_1}(x_1)\cdots T^{\mu_n\nu_n}(x_n)\right\rangle_{\rm LLog} =$$
$$= \int \left[D\rho_1\, D\rho_2\right]\, W_1\left[\rho_1\right]\, W_2\left[\rho_2\right]\, T^{\mu_1\nu_1}_{\rm LO}(x_1)\cdots T^{\mu_n\nu_n}_{\rm LO}(x_n)$$

- Note: at Leading Log accuracy, all the rapidity correlations come from the evolution of the distributions $W[\rho_{1,2}]$
  - ▷ they are a property of the pre-collision initial state
- Long range ($\Delta y \sim \alpha_s^{-1}$) correlations in rapidity
- Caveat : for this formula to be true, all the separations $(x_i - x_j)^2$ must be space-like

Energy momentum tensor at LO



Energy momentum tensor at LO



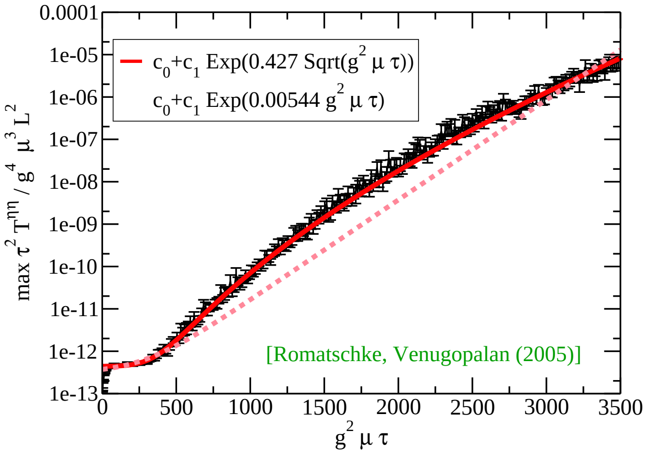
$T^{\mu\nu}$ for longitudinal $\vec{E}$ and $\vec{B}$

$$T^{\mu\nu}_{\rm LO}(\tau=0^+) = \mathrm{diag}\,(\epsilon,\epsilon,\epsilon,-\epsilon)$$

▷ far from ideal hydrodynamics

Weibel instabilities for small perturbations

[Mrowczynski (1988), Romatschke, Strickland (2003), Arnold, Lenaghan, Moore (2003), Rebhan, Romatschke, Strickland (2005), Arnold, Lenaghan, Moore, Yaffe (2005), Romatschke, Rebhan (2006),...]



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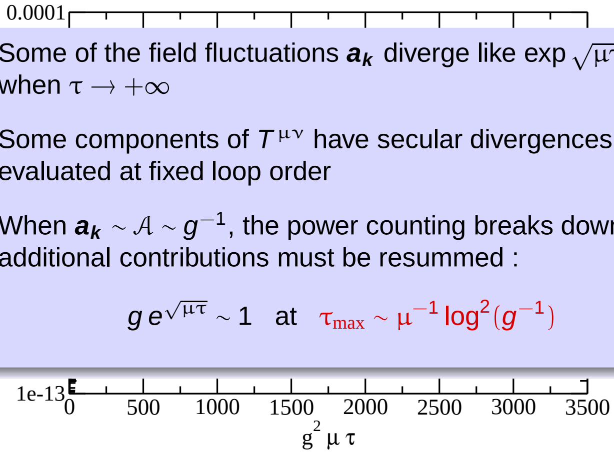

- Some of the field fluctuations $\boldsymbol{a_k}$ diverge like $\exp\sqrt{\mu\tau}$ when $\tau \to +\infty$
- Some components of $T^{\mu\nu}$ have secular divergences when evaluated at fixed loop order
- When $\boldsymbol{a_k} \sim \mathcal{A} \sim g^{-1}$, the power counting breaks down and additional contributions must be resummed :

$$g\, e^{\sqrt{\mu\tau}} \sim 1 \quad \text{at} \quad \tau_{\text{max}} \sim \mu^{-1} \log^2(g^{-1})$$

Improved power counting

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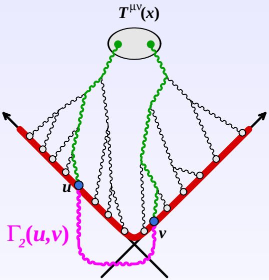
$$\text{Loop} \sim g^2 \qquad , \qquad \mathbb{T}_{\boldsymbol{u}} \sim e^{\sqrt{\mu\tau}}$$



- 1 loop : $(g e^{\sqrt{\mu\tau}})^2$

Improved power counting

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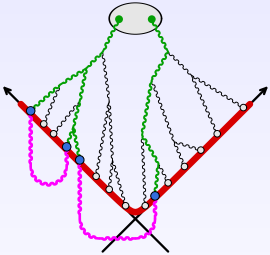
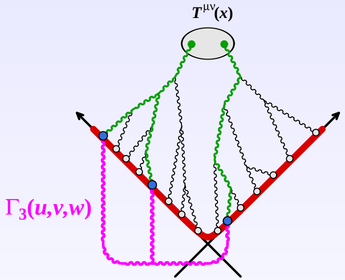


- 1 loop : $\left(g e^{\sqrt{\mu\tau}}\right)^2$
- 2 disconnected loops : $\left(g e^{\sqrt{\mu\tau}}\right)^4$

Improved power counting

$$\text{Loop} \sim g^2 \qquad , \qquad \mathbb{T}_{\boldsymbol{u}} \sim e^{\sqrt{\mu\tau}}$$



- 1 loop : $(g e^{\sqrt{\mu\tau}})^2$
- 2 disconnected loops : $(g e^{\sqrt{\mu\tau}})^4$
- 2 nested loops : $g(g e^{\sqrt{\mu\tau}})^3$ ▷ subleading

Leading terms at τ_{max}

- All disjoint loops to all orders
  ▷ exponentiation of the 1-loop result

Resummation of the leading secular terms

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$$T^{\mu\nu}_{\rm resummed} = \exp\left[\frac{1}{2}\int_{\boldsymbol{u},\boldsymbol{v}\in\Sigma}\underbrace{\int_{\boldsymbol{k}}[\boldsymbol{a}_{\boldsymbol{k}}\mathbb{T}]_{\boldsymbol{u}}[\boldsymbol{a}^*_{\boldsymbol{k}}\mathbb{T}]_{\boldsymbol{v}}}_{\mathcal{G}(\boldsymbol{u},\boldsymbol{v})} + \int_{\boldsymbol{u}\in\Sigma}[\alpha\mathbb{T}]_{\boldsymbol{u}}\right] T^{\mu\nu}_{\rm LO}[\mathcal{A}_{\rm init}]$$

Resummation of the leading secular terms

$$T^{\mu\nu}_{\text{resummed}} = \exp\left[\frac{1}{2}\int_{\boldsymbol{u},\boldsymbol{v}\in\Sigma} \underbrace{\int_{\boldsymbol{k}} [\boldsymbol{a}_{\boldsymbol{k}}\mathbb{T}]_{\boldsymbol{u}}[\boldsymbol{a}^*_{\boldsymbol{k}}\mathbb{T}]_{\boldsymbol{v}}}_{\mathcal{G}(\boldsymbol{u},\boldsymbol{v})} + \int_{\boldsymbol{u}\in\Sigma}[\alpha\mathbb{T}]_{\boldsymbol{u}}\right] T^{\mu\nu}_{\text{LO}}[\mathcal{A}_{\text{init}}]$$

$$= \int [D\chi]\ \exp\left[-\frac{1}{2}\int_{\boldsymbol{u},\boldsymbol{v}\in\Sigma} \chi(\boldsymbol{u})\mathcal{G}^{-1}(\boldsymbol{u},\boldsymbol{v})\chi(\boldsymbol{v})\right] T^{\mu\nu}_{\text{LO}}[\mathcal{A}_{\text{init}}+\chi+\alpha]$$

- The evolution remains classical, but we must average over a Gaussian ensemble of initial conditions
- Note : the constant shift α can be absorbed into a redefinition of $\mathcal{A}_{\text{init}}$