Bose condensation and turbulence

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Thermalization in Non-Abelian Plasmas

Classical-statistical field theory

$n(p) \sim \frac{1}{\alpha}$

over-populated

$\frac{1}{\alpha} \gg n \gg 1$

classical-particles

$n(p) \leqslant 1$

quantum

Classical statistical field theory

Kinetic regime

Effectively 2 to 2
Using 1/N resummation

Strong turbulence exponents

Wave turbulence exponents

What is a condensate?

In equilibrium: $$N = V \int \frac{d^3 k}{2\pi^3} \frac{1}{e^{\beta(\epsilon_k - \mu)} - 1}$$ Maximum at $\mu = \epsilon_0$

Condensation: $$N > N_{max}$$

Macroscopic occupation of the zero mode

Condensate fraction $\frac{N_0}{N}$

Particle distribution: $$n_k = \delta^3(k) n_0 + n'_k$$

In terms of 2point function $$F(x,y) = \{\phi(x), \phi(y)\}$$

$$n_k = F(k)\omega_k \Rightarrow \quad F(k=0) \sim V$$

condensate $$= \left(\frac{\int d^3 x \phi(x)}{V} \right)^2 = \frac{F(k=0)}{V}$$ Independent of the volume

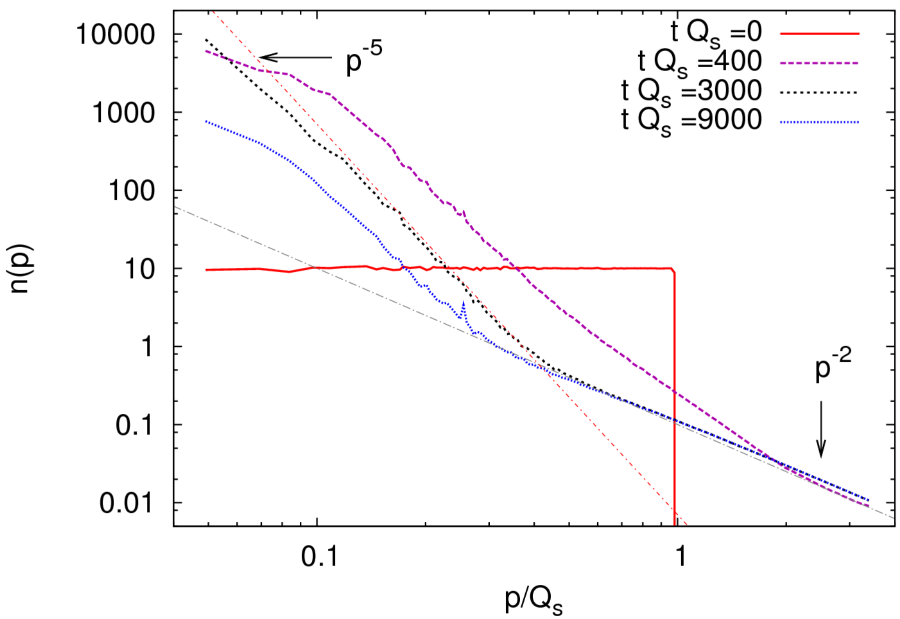
Condensation in bose gas

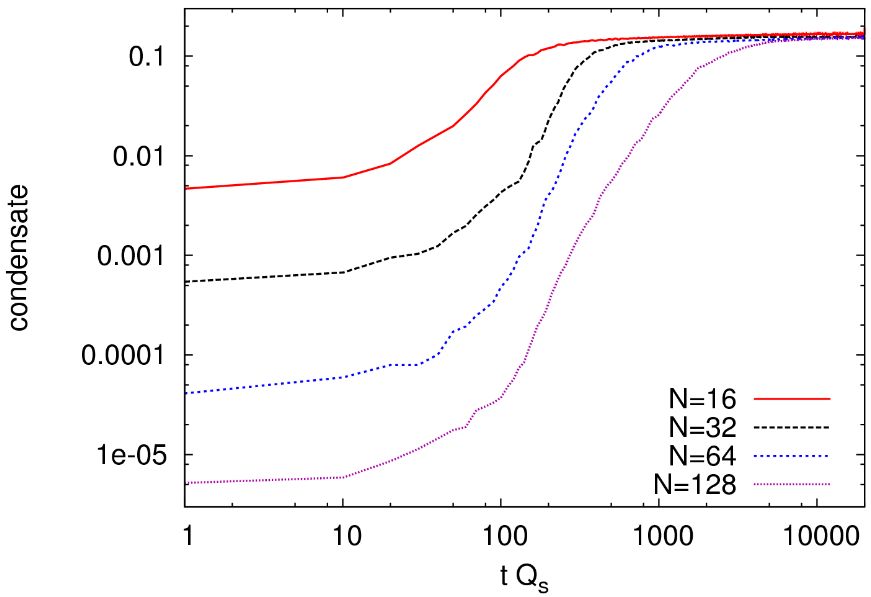
Non relativistic scalars described by complex field $\Psi(x,t)$

Gross-Pitaevski equation: $i\partial_t \Psi(x,t) = \left(-\frac{\partial_i^2}{2m} + g|\Psi(x,t)|^2 \right) \Psi(x,t)$

conserved particle number $n_{tot} = \int d^3x |\Psi(x,t)|^2$

occupation in zero mode: condensate = $\left| \frac{\int d^3x \Psi(x,t)}{V} \right|^2$



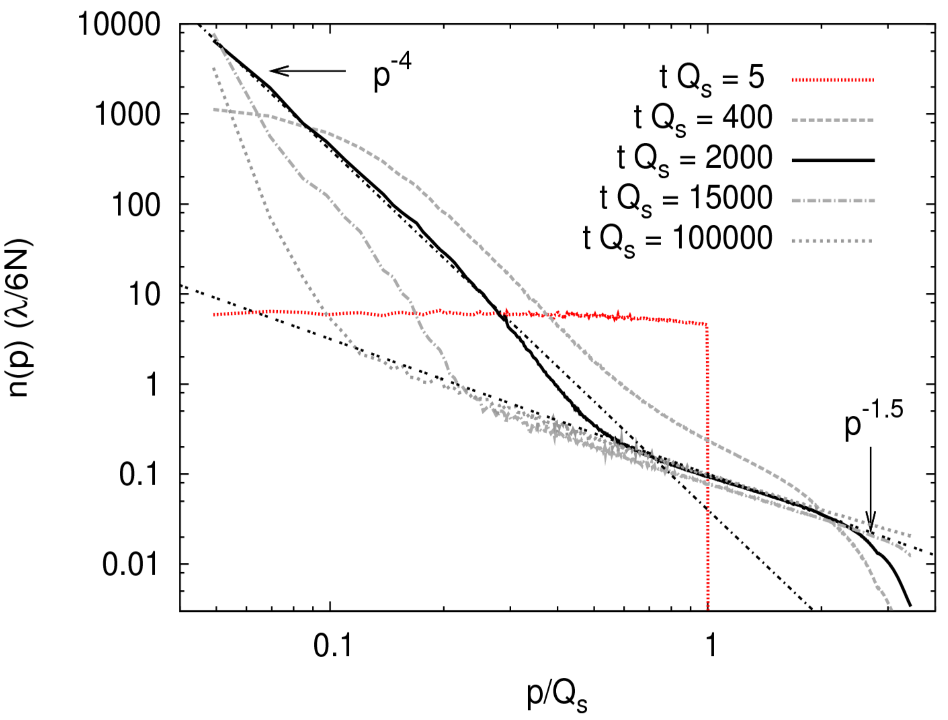


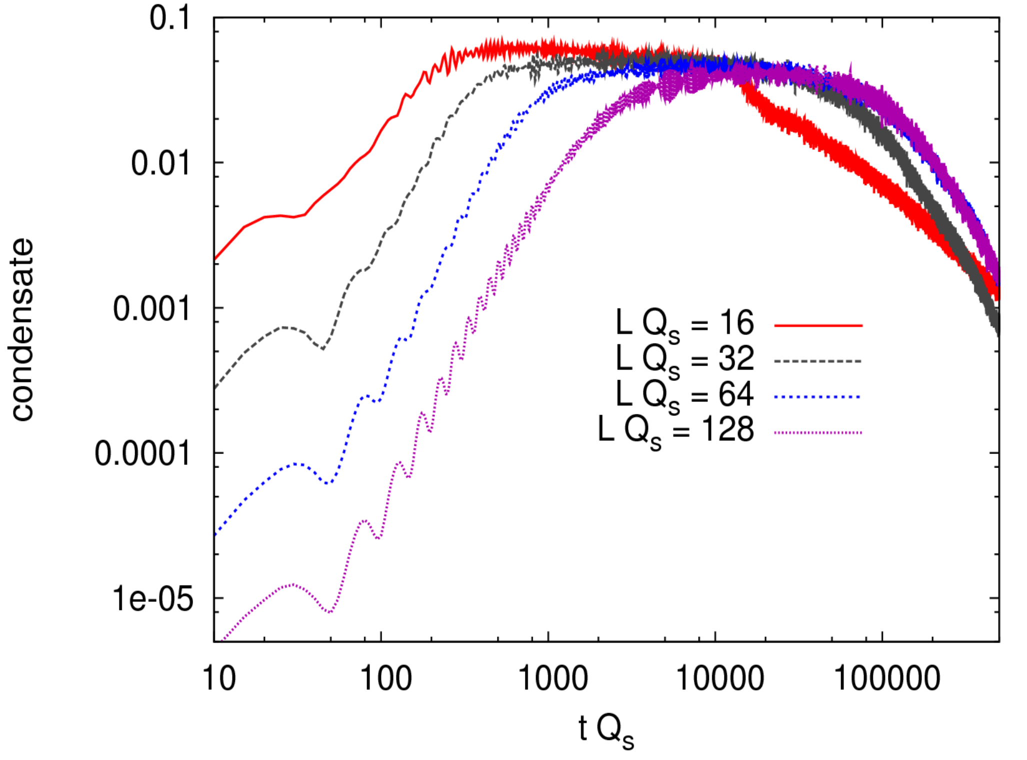
Non-equilibrium Bose condensation

O(4) massless relativistic scalars

Initial conditions: overpopulation

condensate $= \left\langle \left| \frac{\int d^3x\, \phi_a(x)}{V} \right|^2 \right\rangle_{ens}$





Turbulent cascade

Conserved charge $\partial_k n_k = 0$ → Stationary power law solution with k-independent flow

2->2 dominates: particle number effectively conserved

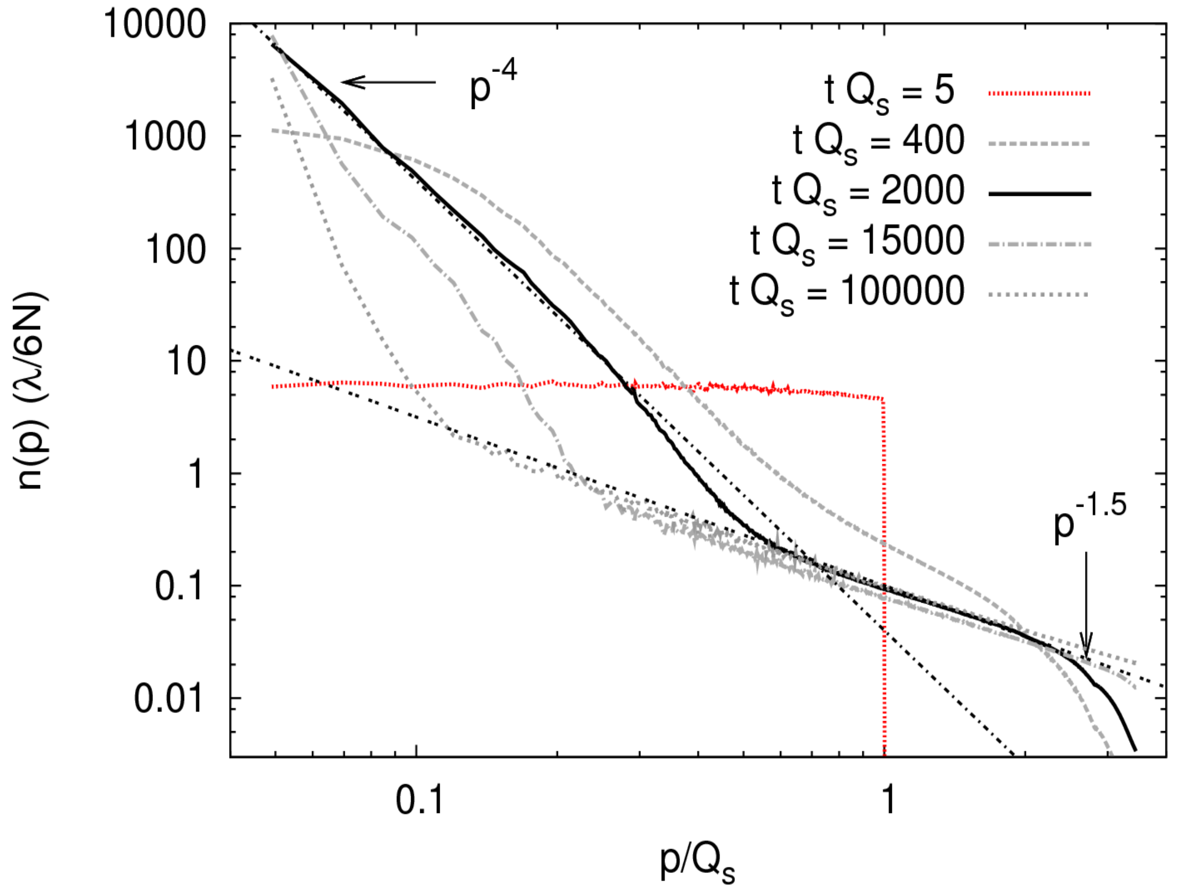
Dual cascade: particles to IR
energy to UV

Particle flow　　Energy flow

$$\kappa_{IR} = d+1 \text{ or } \kappa_{IR} = d+2$$

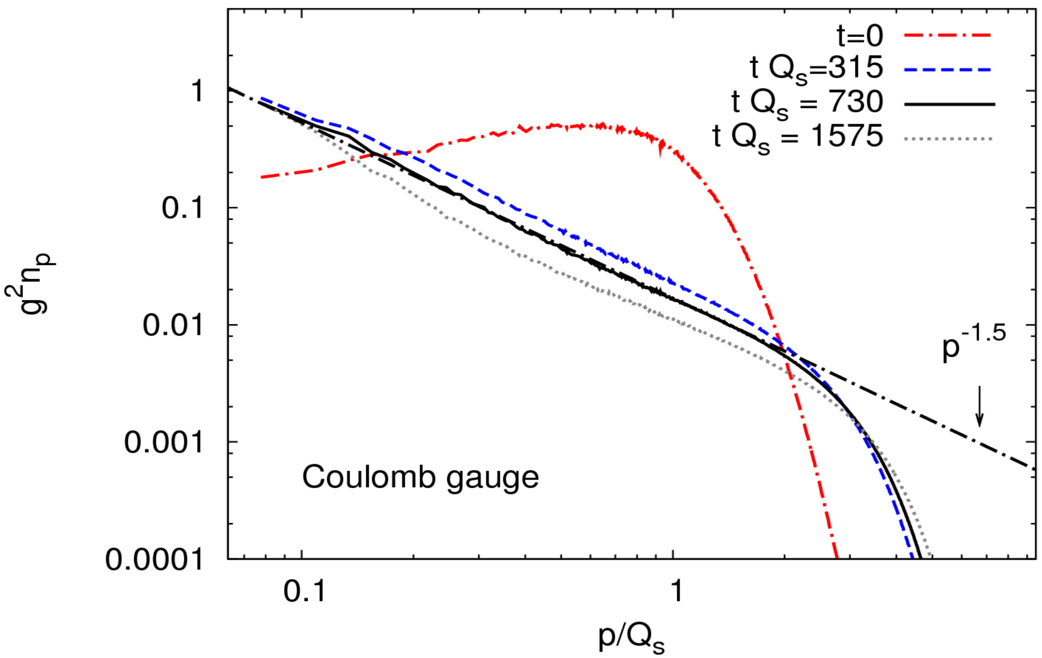
$$\kappa_{UV} = d-2 \text{ or } \kappa_{UV} = d-3/2$$

$$n_k \sim k^{-\kappa}$$



Gauge theory turbulence

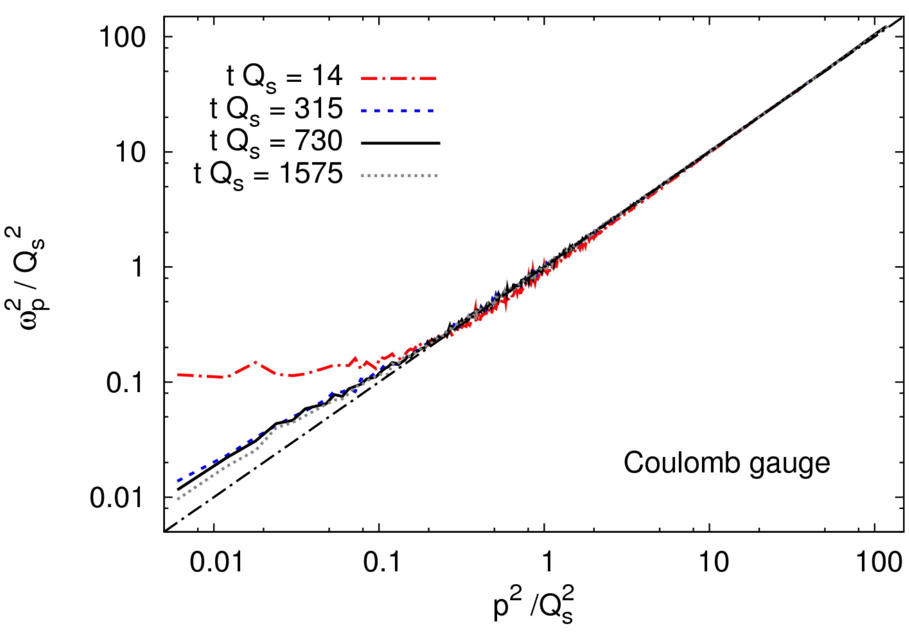
Pure SU(2) gauge theory   overpopulated initial condition



$$n_p \sim p^{-1.5}$$

same as scalar UV exponent

Dispersion



Kolmogorov Turbulence

In terms of corrleation functions

$\phi = \phi_a$ or A_μ^a

$$F(x,y) = \{\phi(x), \phi(y)\}$$

$$\rho(x,y) = [\phi(x), \phi(y)]$$

Stationarity condition:

(Collision integral vanishes)

$$\Sigma_\rho(p) F(p) - \Sigma_F(p) \rho(p) = 0$$

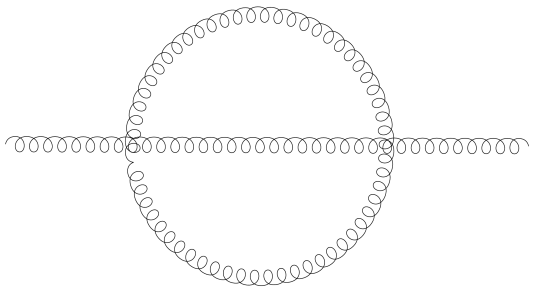
With self energy: $\Sigma(p)$

Scaling ansatz

$$F(s^z \omega, s p) = |s|^{-2-\kappa} F(\omega, p)$$

$$\rho(s^z \omega, s p) = |s|^{2-\eta} \rho(\omega, p)$$

Classicality condition $F(p) \gg \rho(p)$



$$\Sigma(p)=\int_{qkl} G(q)G(k)G(l)\,\delta^{(4)}(p+q+k+l)$$

Classical part of the stationarity condition:

$$0=\int_{\boldsymbol{p}\,qkl} V(p,q,k,l)^2\,\delta^{(4)}(p+q+k+l)\Big[F(p)F(q)F(k)\rho(l)+ F(p)F(q)\rho(k)F(l)+ F(p)\rho(q)F(k)F(l)+ \rho(p)F(q)F(k)F(l)\Big]$$

Zakharov transformation:

swapping momenta

$$l'=\xi p;\quad p'=\xi l;\quad k'=\xi k;\quad l'=\xi l$$

$$F(p)F(q)F(k)\rho(l)\Rightarrow\rho(p)F(q)F(k)F(l)$$

$$0=\int_{\boldsymbol{p}\,qkl} V(p,q,k,l)^2\,\delta^{(4)}(p+q+k+l)\,\rho(p)F(q)F(k)F(l)\left[1+\left|\frac{p_0}{q_0}\right|^{\Delta}\mathrm{sgn}\left(\frac{p_0}{q_0}\right)+\left|\frac{p_0}{k_0}\right|^{\Delta}\mathrm{sgn}\left(\frac{p_0}{k_0}\right)+\left|\frac{p_0}{l_0}\right|^{\Delta}\mathrm{sgn}\left(\frac{p_0}{l_0}\right)\right]$$

Solutions:

$$\Delta=-1$$

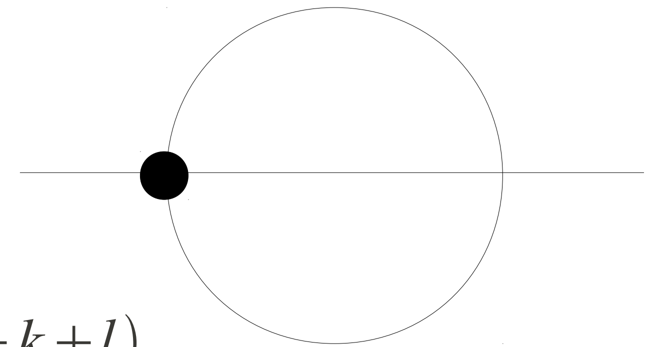
$$\Delta=0$$

On shell limit
2->2 dominates

$$\kappa=\frac{5}{3}\ \text{and}\ \kappa=\frac{4}{3}$$

IR resummation – Strong turbulence

1/N resummation: effective vertex



$$\Sigma(p) = \int_{kql} \lambda_{eff}(p+q) G(q) G(k) G(l) \delta^{(4)}(p+q+k+l)$$

$$\lambda_{eff}(p) = \frac{\lambda}{(1+\Pi^R(p))(1+\Pi^A(p))}$$

With one loop bubble:

$$\Pi(p) = \int_q G(p) G(p-q)$$

In the IR: $\Pi(p) \gg 1$

The vertex scales:

$$\lambda_{eff}(s\,p) = s^{2r} \lambda_{eff}(p) \text{ with } r = 3 + \kappa - d$$

In the UV: $\lambda_{eff} = \lambda$

$$\lambda(sp) = \lambda(p)$$

Strong turbulence in the IR:

$$\kappa = 4 \text{ or } 5 \text{ (in d=3)}$$

From 2PI to kinetic equations

Using Wigner coordinates

$$F_p(X) = \int d^4 s \exp(-i p_\mu s^\mu) F(X+s/2, X-s/2)$$

Gradient expansion, spatially homogeneous ensemble:

$$\partial_t \rho_p(X) = 0$$

$$2 p_0 \partial_t F_p(X) = \Sigma_p^\rho(X) F_p(X) - \Sigma_p^F(X) \rho_p(X)$$

Define:

$$F_p(X) = (n_p(X) + 1/2) \rho_p(X)$$

$$n_{eff}(t, \boldsymbol{p}) = \int_0^\infty \frac{dp_0}{2\pi} 2 p_0 \rho_p(X) n_p(X)$$

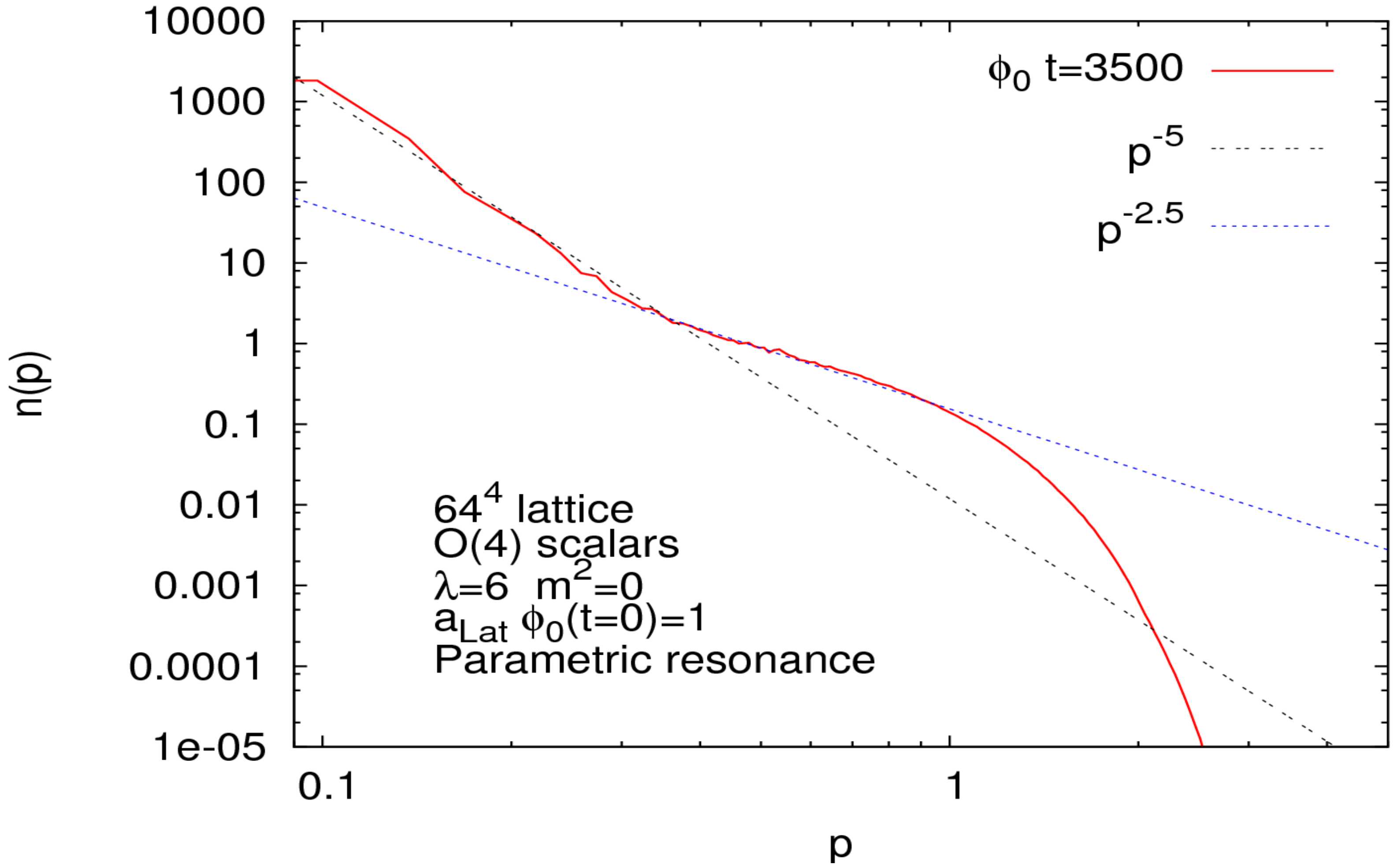
On-shell limit, only 2->2 contributes

$$\partial_t n_{eff}(t, \boldsymbol{p}) = \int d\Omega_{2\to 2} \left[(1+n_p)(1+n_l) n_q n_r - n_p n_l (1+n_q)(1+n_r) \right] \lambda_{eff}(p+l)$$

Effective kinetic description valid at $\lambda n \gg 1$

Turbulence in d=4

$$\kappa_{UV} = d - \frac{3}{2} \qquad \kappa_{IR} = d + 1$$



Conclusions

Scalar case well understood

Dual cascade
Condensation
Weak and strong exponents
from kinetic theory (with resummation)

Gauge theory

Gauge fixing necessary
UV exponent 3/2
condensation?

Time dependence of gauge theory exponent