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Thermal Equilibration in Scalar Field Theory

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Resummation formula [GELIS, LAPPI, VENUGOPALAN (2008)]

NLO

$$T^{\mu\nu}_{\rm NLO} = \widehat{O}\; T^{\mu\nu}_{\rm LO}[\varphi_0]$$

$$\widehat{O} \approx \frac{1}{2}\int G(x,y)\,\frac{\delta}{\delta\varphi_0(x)}\,\frac{\delta}{\delta\varphi_0(y)}$$

Resummation formula [GELIS, LAPPI, VENUGOPALAN (2008)]

Resummation

$$T^{\mu\nu}_{\text{resum}} = e^{\widehat{O}}\; T^{\mu\nu}_{\text{LO}}[\varphi_0]$$

$$\widehat{O} \approx \frac{1}{2}\int G(x,y)\frac{\delta}{\delta\varphi_0(x)}\frac{\delta}{\delta\varphi_0(y)}$$

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Equivalent formulation

$$T^{\mu\nu}_{\text{resum}} = \int \big[Da(u)\big]\, e^{-\frac{1}{2}\iint \mathrm{d}^3\boldsymbol{u}\,\mathrm{d}^3\boldsymbol{v}\; a(\boldsymbol{u})\, G^{-1}(\boldsymbol{u},\boldsymbol{v})\, a(\boldsymbol{v})}\; T^{\mu\nu}_{\text{LO}}[\varphi_0 + a]$$

- Solve the EOM for the classical background field φ_0 with random gaussian fluctuations a on top of it.
- Semi-classical calculation that takes into account some quantum corrections.
- Monte Carlo Simulation.

Scalar field theory

Lagrangian of the theory

$$\mathcal{L}(\phi) = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \underbrace{\frac{g^2}{4!}\phi^4}_{V(\phi)} + J\phi$$

where

$$J \propto \theta(-x^0)$$

Why do we use this model?

- Scale invariance in $3+1$ dimensions
- Parametric resonance
- A lot simpler!

Form of the solution

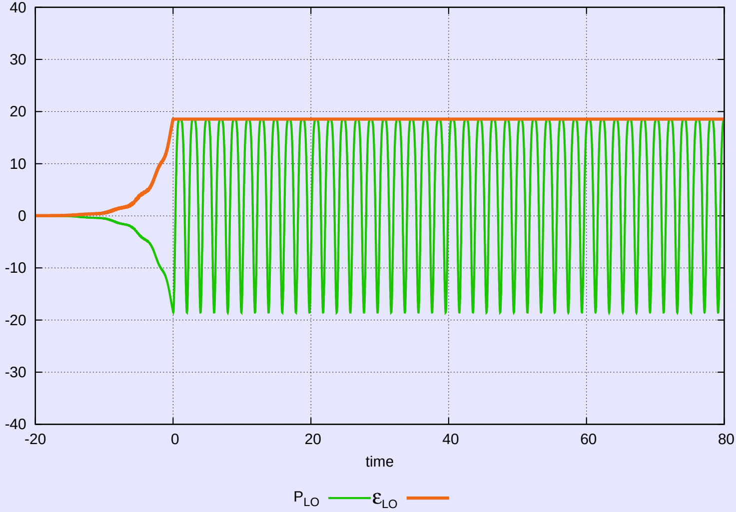
Initial condition of the EOM

$$\phi_{\text{init}}(t, \boldsymbol{x}) = \varphi_0(\boldsymbol{x}) + \sum_{\boldsymbol{k}} \Re\left[c_{\boldsymbol{k}}\, e^{i\omega_{\boldsymbol{k}} t}\, f_{\boldsymbol{k}}(\boldsymbol{x})\right]$$

with

$$\left[-\Delta + V''(\varphi_0)\right] f_{\boldsymbol{k}}(\boldsymbol{x}) = \omega_{\boldsymbol{k}}^2 f_{\boldsymbol{k}}(\boldsymbol{x})$$

$T^{\mu\nu}_{\mathrm{LO}}$ [Dusling, TE, Gelis, Venugopalan (2010)]



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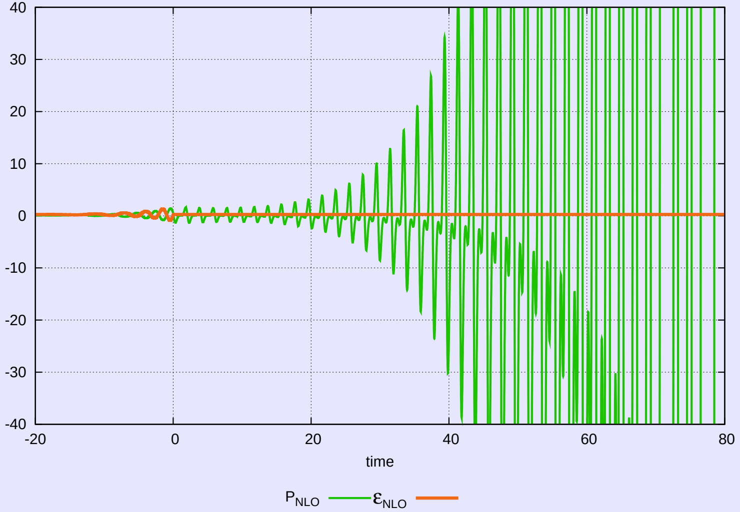

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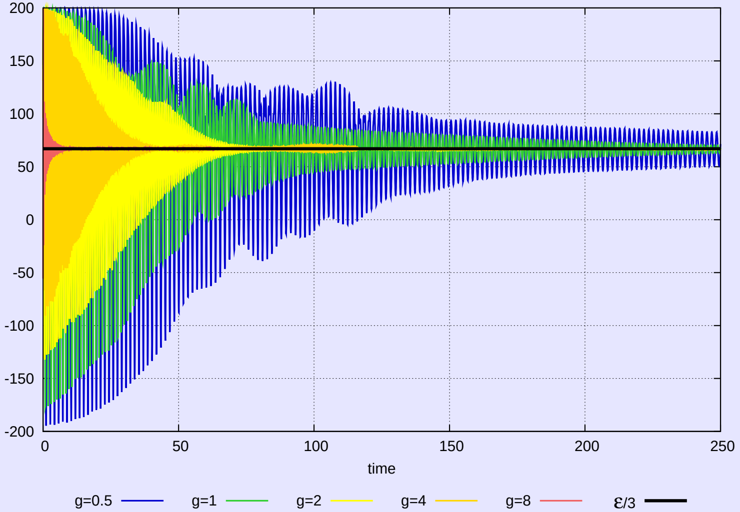
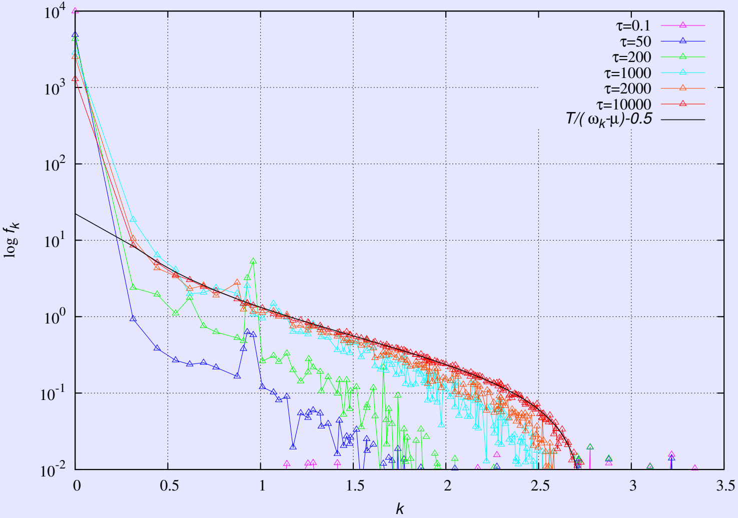
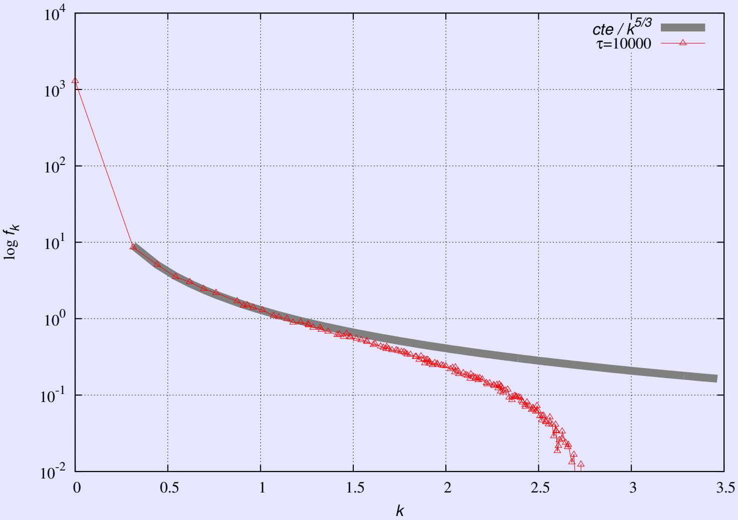


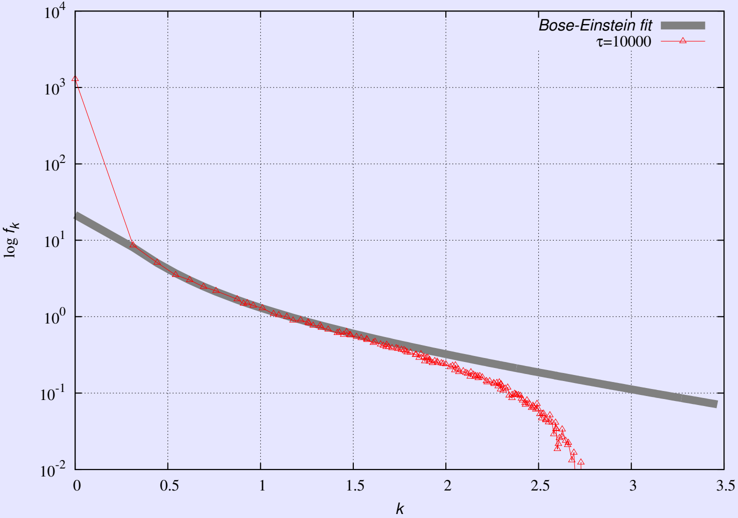
Occupation number, constant φ_0 [TE, GELIS (2011)]



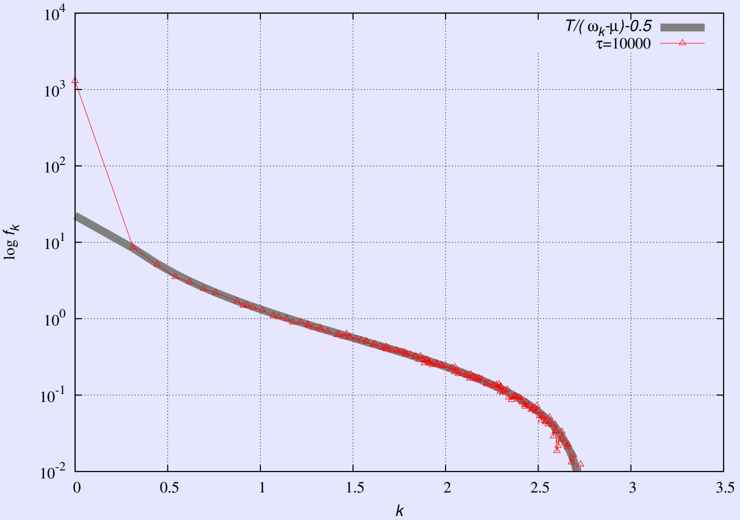
Kolmogorov scaling? (constant φ_0) [TE, GELIS (2011)]



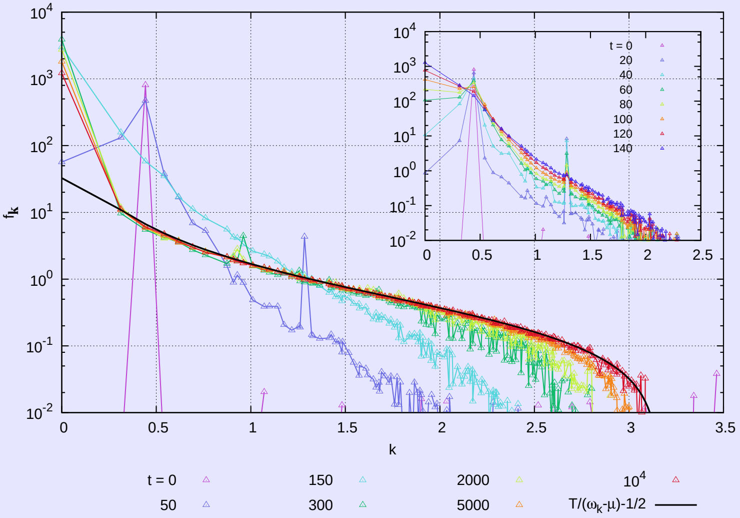
BOSE-EINSTEIN fit? (constant φ_0) [TE, GELIS (2011)]



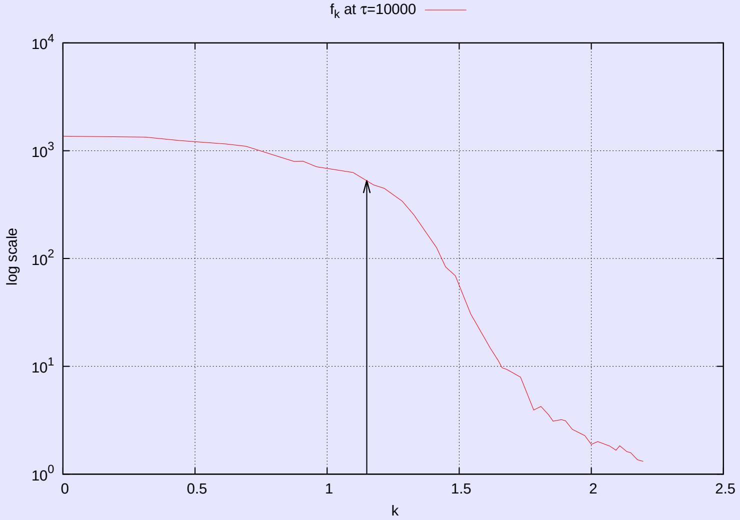
Classical fit, constant φ_0 [TE, GELIS (2011)]



Non-zero initial mode [TE, GELIS (2011)]



$\varphi_0(x, y) = M_0 \cos(k_x x + k_y y)$ [Work in progress]



Form of the solution

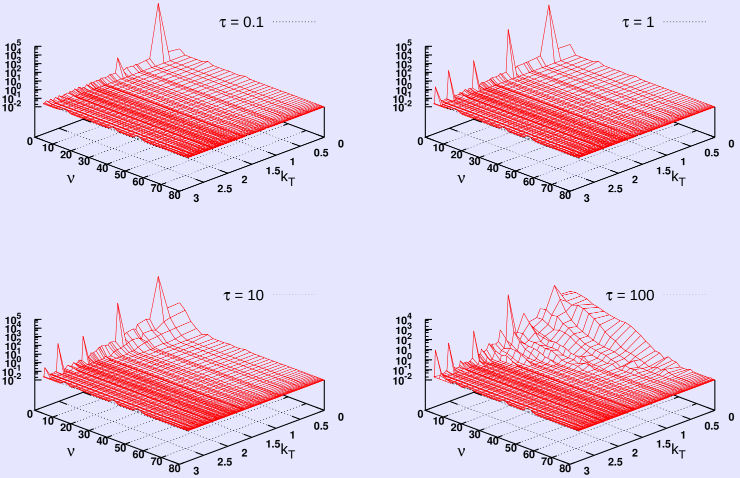
Initial condition of the EOM

$$\phi_{\text{init}}(\tau, \boldsymbol{x}_\perp, \eta) = \varphi_0(\boldsymbol{x}_\perp) + \sum_{\boldsymbol{k}_\perp, \nu} \Re\left[c_{\boldsymbol{k},\nu} H^{(2)}_{i\nu}\left(\omega_{\boldsymbol{k}_\perp} t\right)\, e^{i\nu\eta}\, f_{\boldsymbol{k}_\perp}(\boldsymbol{x}_\perp)\right]$$

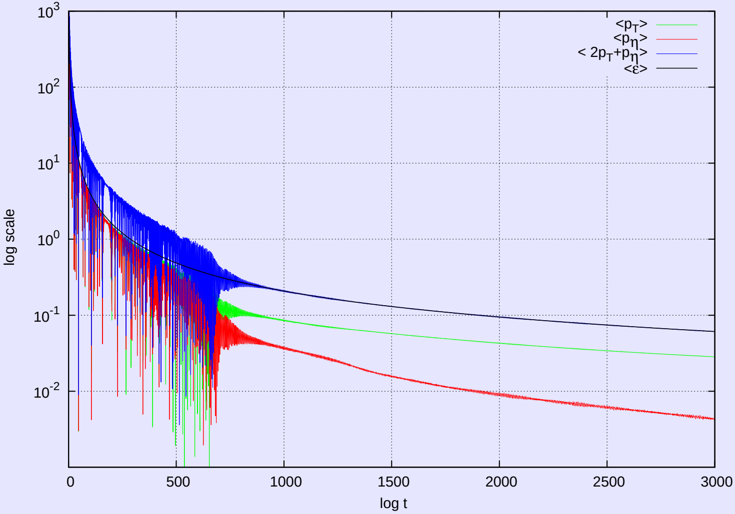
with

$$\left[-\Delta_\perp + V''(\varphi_0)\right] f_{\boldsymbol{k}_\perp}(\boldsymbol{x}_\perp) = \omega^2_{\boldsymbol{k}_\perp} f_{\boldsymbol{k}_\perp}(\boldsymbol{x}_\perp)$$

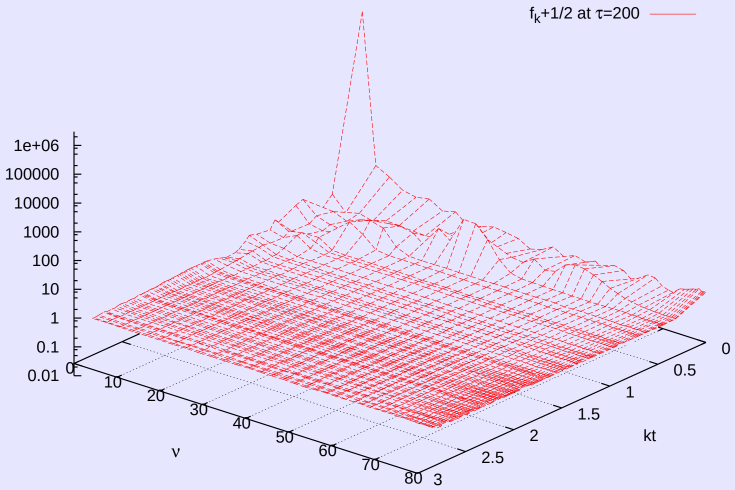
Bose-Einstein Condensate [Preliminary results]



$T^{\mu\nu}_{\text{resum}}$ [Preliminary results]



Lattice artifacts [Preliminary results]



Conclusion

Principal results for the fixed volume

- EOS $\epsilon = 3p$
- $p_x = p_y = p_z$
- $f_{\boldsymbol{k}} \propto \frac{T}{\omega_{\boldsymbol{k}} - \mu} - \frac{1}{2}$ at late times
- BOSE-EINSTEIN condensate

Principal results for the exanding volume

- BOSE-EINSTEIN condensate?
- EOS?
- $p_\eta \neq p_T$?

Backup slide