Towards thermalization in heavy-ion collisions: CGC meets the 2PI formalism

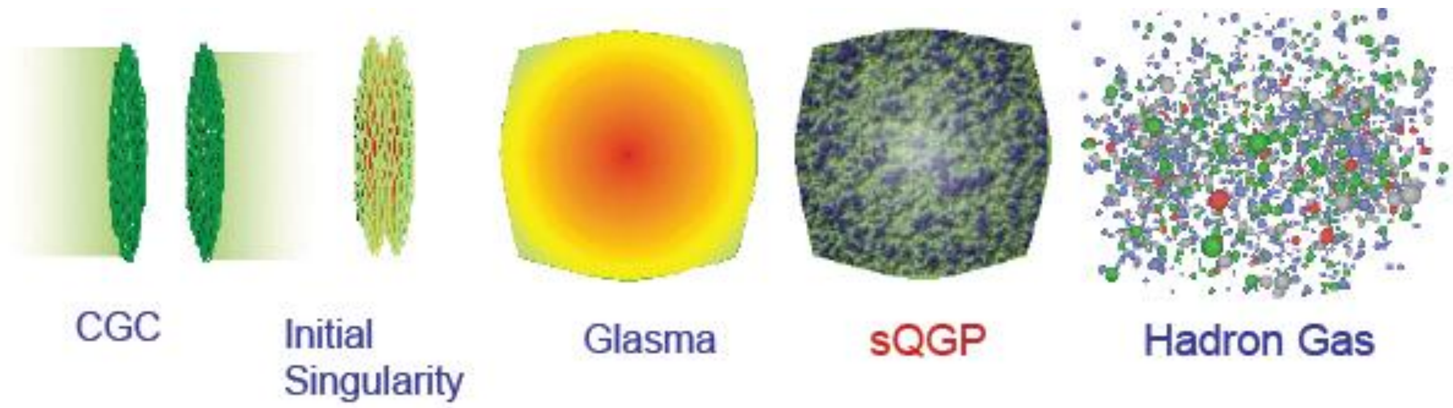
Yoshitaka Hatta

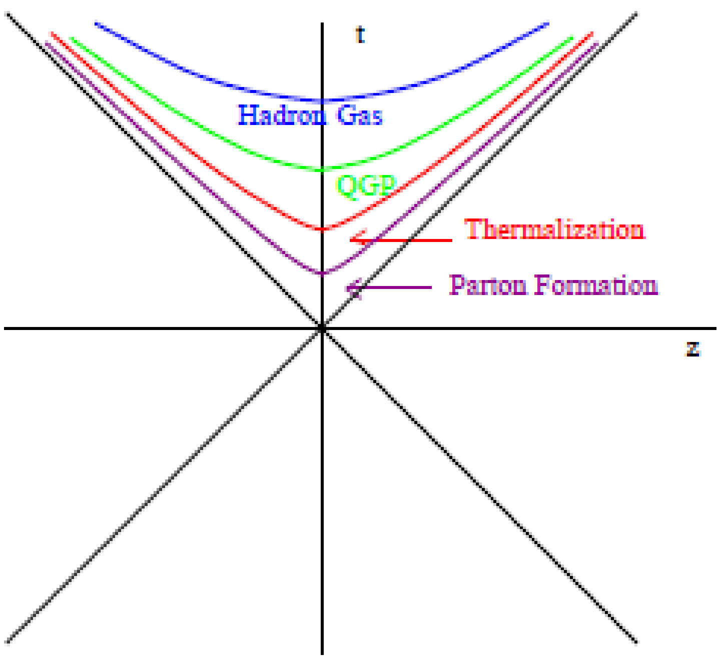
(U. Tsukuba)

with Akihiro Nishiyama

arXiv: 1108.0818 (Nucl. Phys. A, in press)

Early stage of HIC





Hydro works at late times

Earliest times Color Glass Condensate (CGC)

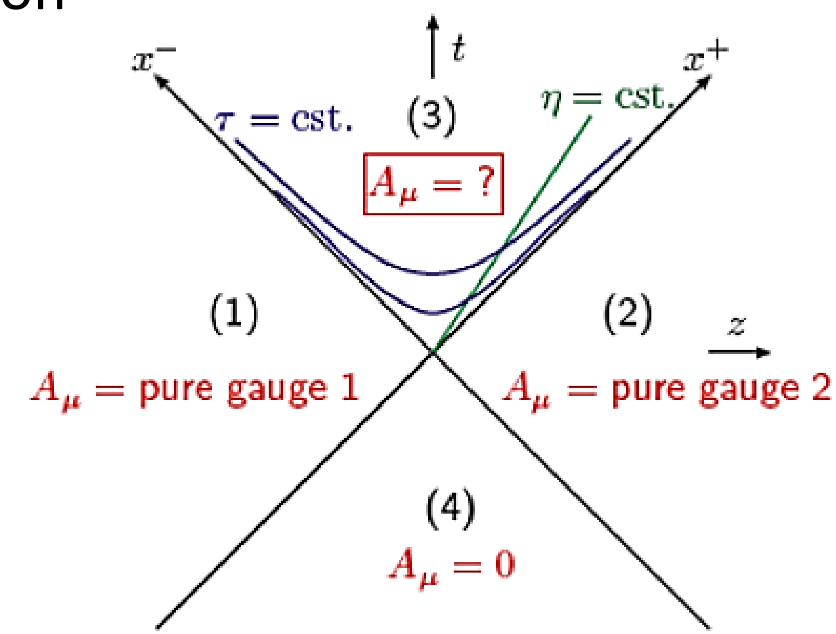
Intermediate times difficult.
Very fast equilibration ?

Gluodynamics in the $\tau - \eta$ coordinates

Proper time $\tau = \sqrt{t^2 - (x^3)^2}\,,$ Rapidity $\eta = \tanh^{-1}\frac{x^3}{t}$

Solve the classical Yang—Mills equation in the gauge $A^\tau = 0$

$$-\frac{1}{\tau}\partial_\tau\left(\frac{1}{\tau}\partial_\tau A_\eta\right) + \frac{1}{\tau^2}D_i F_{i\eta} = 0\,,$$
$$-\frac{1}{\tau}\partial_\tau\left(\tau\partial_\tau A_i\right) + \left(\frac{1}{\tau^2}D_\eta F_{\eta i} + D_j F_{ji}\right) = 0$$



with the initial condition

$$A_i = \mathcal{A}_i^1 + \mathcal{A}_i^2\,, \qquad A_\eta = 0\,,$$
$$\tau\partial_\tau A_i = 0\,, \qquad \frac{1}{\tau}\partial_\tau A_\eta^a = -g f_{abc}\mathcal{A}_i^{1b}\mathcal{A}_i^{2c}$$

Kovner, McLerran, Weigert (1995)

Classical statistical approach

Classical YM eq. alone is insufficient for the problem of thermalization.
Need quantum fluctuations   Romatschuke, Venugopalan (2006);

State of the art :   (Classical Yang-Mills on a lattice)
+ (Gaussian averaging of Quantum fluctuations)

- vanishing 1-point function $\langle A \rangle = 0$
  the initial condition characterized by the 2-point function $\langle AA \rangle$

  Berges, Scheffler, Sexty, (2008)

- background CGC field $\langle A \rangle_J \neq 0$ + fluctuations $\langle \delta A \delta A \rangle$.
  Integrate over J at the end

  Dusling, Gelis, Venugopalan, (2011)

2PI formalism

Berges, AIP conf. proc. 739, 3 (2005)

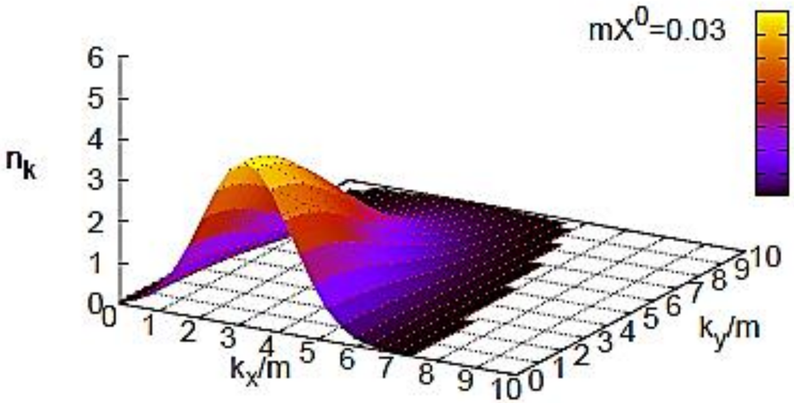
- First principle calculation in field theories out of equilibrium (Kadanoff-Baym).
- Based on the CJT effective action

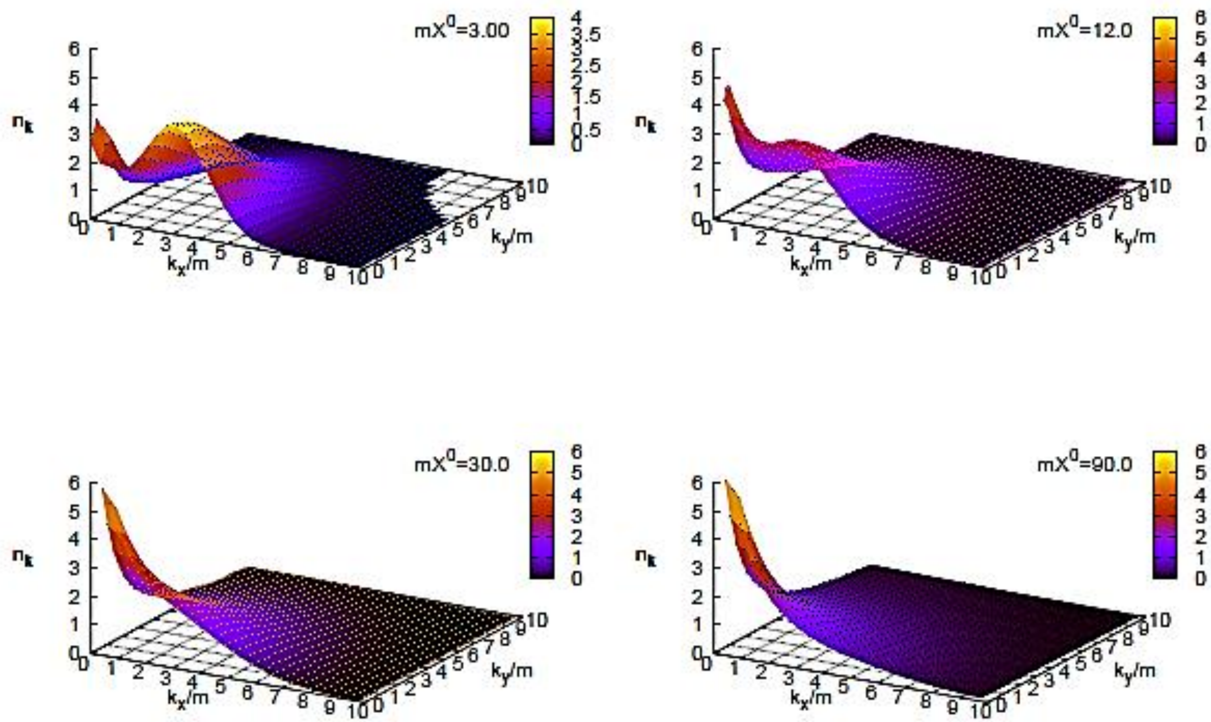
$$\Gamma[A,G] = S_{YM}[A] + \frac{i}{2}\mathrm{tr}\ln G^{-1} + \frac{i}{2}\mathrm{tr}\, G_0^{-1}[A]G + \Gamma_2[A,G]$$

Classical field   Quantum fluctuation $G = \langle \delta A \delta A \rangle$ 2PI diagrams

- Achieves quantum thermal equilibrium (Bose-Einstein distribution) starting from far-from-equilibrium initial conditions

`tsunami' distribution





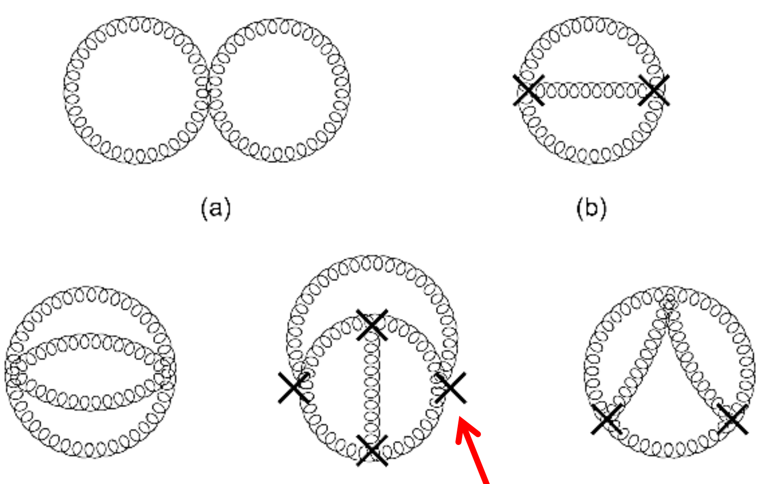
(2+1)D gluodynamics

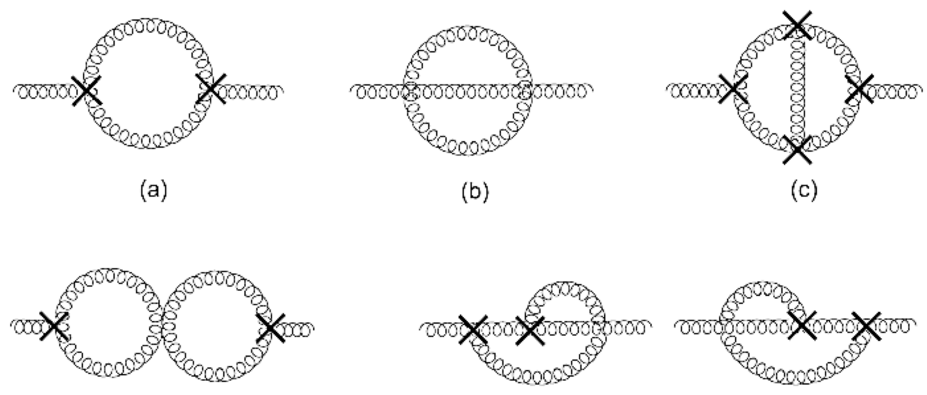
Nishiyama, Ohnishi, arXiv: 1011.4750

Bose-Einstein distribution

2PI effective action

$$\Gamma[A,G] = S_{YM}[A] + \frac{i}{2}\mathrm{tr}\ln G^{-1} + \frac{i}{2}\mathrm{tr}\, G_0^{-1}[A]G + \Gamma_2[A,G]$$





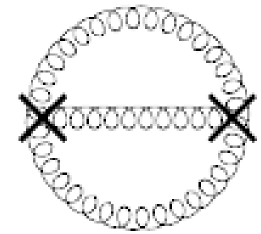
$D = \partial - igA$

$$G(x,x') = \langle \mathrm{T}_C\{\delta A(x)\delta A(x')\}\rangle = \mathcal{F} - \frac{i}{2}(\theta_C(\tau-\tau') - \theta_C(\tau'-\tau))\rho$$

``Statistical function" ``Spectral function"

Equation for the classical field

$$\frac{\delta\Gamma}{\delta A} = 0$$



$$\left(\partial_\tau^2 A_I + \frac{1}{\tau}\partial_\tau A_I - \delta_{I\zeta}\frac{A_I}{\tau^2} - D_J F_{JI}\right)^a$$

$$= g f_{abc}\left(D^{be}_{xI}\mathcal{F}^{ec}_{JJ}(x,y) + D^{be}_{xJ}\left(\mathcal{F}^{ec}_{JI}(x,y) - 2\mathcal{F}^{ec}_{IJ}(x,y)\right)\right)_{y=x}$$

$$+\frac{ig^2}{2}C_{ab,cd}\int_{\tau_0}^{\tau} d^4y\, V^y_{lmn,LMN}\Big[\rho^{bl}_{IL}(x,y)\mathcal{F}^{cm}_{JM}(x,y)\mathcal{F}^{dn}_{JN}(x,y)$$

$$+(\mathcal{F}\rho\mathcal{F}) + (\mathcal{F}\mathcal{F}\rho) - \frac{1}{4}(\rho\rho\rho)\Big]$$

Rescale $\quad \zeta \equiv \tau\eta \qquad A_\eta = \tau A_\zeta\,, \qquad a_\eta = \tau a_\zeta\,, \qquad \partial_\eta = \tau\partial_\zeta$

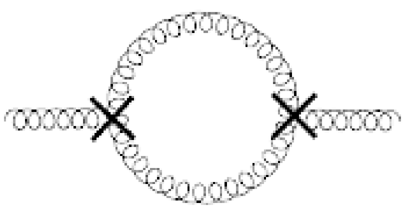
In the new coordinates $x^I = (\zeta, x_\perp)$

Feynman rules are formally the same as in the flat metric case.

Equation for the fluctuation

$$\frac{\delta\Gamma}{\delta G} = 0 \;\Rightarrow$$

$$\left[\left(\partial_\tau^2 + \frac{1}{\tau}\partial_\tau - \frac{1}{\tau^2}\delta_{I\zeta}\right)\delta_{IJ} - \left(D^2\delta_{IJ} - D_I D_J - 2igF_{IJ}\right)\right]^{ab} \mathcal{F}^{bc}_{JK}(x,y)$$
$$+g^2\left(C_{ad,be}\mathcal{F}^{de}_{IJ}(x,x) + \frac{1}{2}C_{ab,de}\mathcal{F}^{de}_{MM}(x,x)\delta_{IJ}\right)\mathcal{F}^{bc}_{JK}(x,y)$$
$$= -\int_{\tau_0}^{\tau} d^4z\,\Pi_\rho(x,z)^{ab}_{IJ}\mathcal{F}^{bc}_{JK}(z,y) + \int_{\tau_0}^{\tau'} d^4z\,\Pi_{\mathcal{F}}(x,z)^{ab}_{IJ}\rho^{bc}_{JK}(z,y)\,,$$



$$\left[\left(\partial_\tau^2 + \frac{1}{\tau}\partial_\tau - \frac{1}{\tau^2}\delta_{I\zeta}\right)\delta_{IJ} - \left(D^2\delta_{IJ} - D_I D_J - 2igF_{IJ}\right)\right]^{ab} \rho^{bc}_{JK}(x,y)$$
$$+g^2\left(C_{ad,be}\mathcal{F}^{de}_{IJ}(x,x) + \frac{1}{2}C_{ab,de}\mathcal{F}^{de}_{MM}(x,x)\delta_{IJ}\right)\rho^{bc}_{JK}(x,y)$$
$$= -\int_{\tau'}^{\tau} d^4z\,\Pi_\rho(x,z)^{ab}_{IJ}\rho^{bc}_{JK}(z,y)\,,$$

$$\Pi_{\mathcal{F}}(x,y)^{ab}_{IJ} = \frac{1}{2}\overrightarrow{V}^{x}_{alm,ILM}\left(\mathcal{F}^{ll'}_{LL'}\mathcal{F}^{mm'}_{MM'} - \frac{1}{4}\rho^{ll'}_{LL'}\rho^{mm'}_{MM'}\right)_{xy}\overleftarrow{V}^{y}_{bl'm',JL'M'}$$

$$\Pi_\rho(x,y)^{ab}_{IJ} = \frac{1}{2}\overrightarrow{V}^{x}_{alm,ILM}\left(\mathcal{F}^{ll'}_{LL'}\rho^{mm'}_{MM'} + \rho^{ll'}_{LL'}\mathcal{F}^{mm'}_{MM'}\right)_{xy}\overleftarrow{V}^{y}_{bl'm',JL'M'}$$

Remarks

- Transform covariantly under the residual (τ -independent) gauge transformation

$$A_I \to U A_I U^\dagger + \frac{i}{g} U \partial_I U^\dagger \qquad a_I^a \to (U a_I U^\dagger)^a = U^{ab} a_I^b$$

- Challenging to solve numerically if the background is inhomogeneous.

Berges , Roth (2010)

- The initial condition for $\mathcal{F}$ nontrivial

Fukushima, Gelis, McLerran (2007)

Dusling, Gelis, Venugopalan (2011)

Comparison with the previous work

Son (1996) ; Dusling, Epelbaum, Gelis, Venugopalan, (2010)

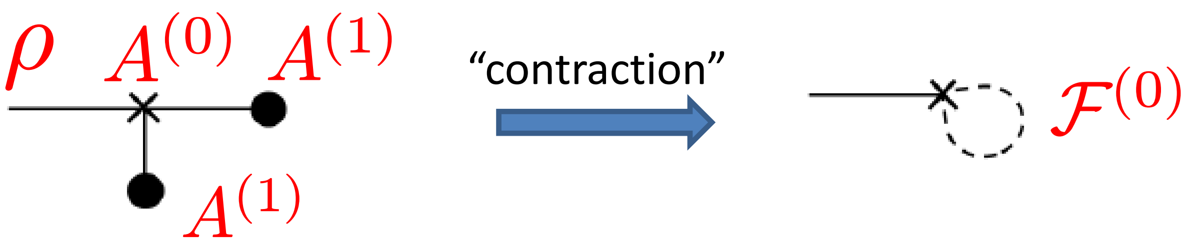
Solve the YM equation perturbatively $$A = A^{(0)} + A^{(1)} + A^{(2)} + \cdots$$

exact classical solution ($A^{(0)}$) linearized around the solution ($A^{(1)}$)

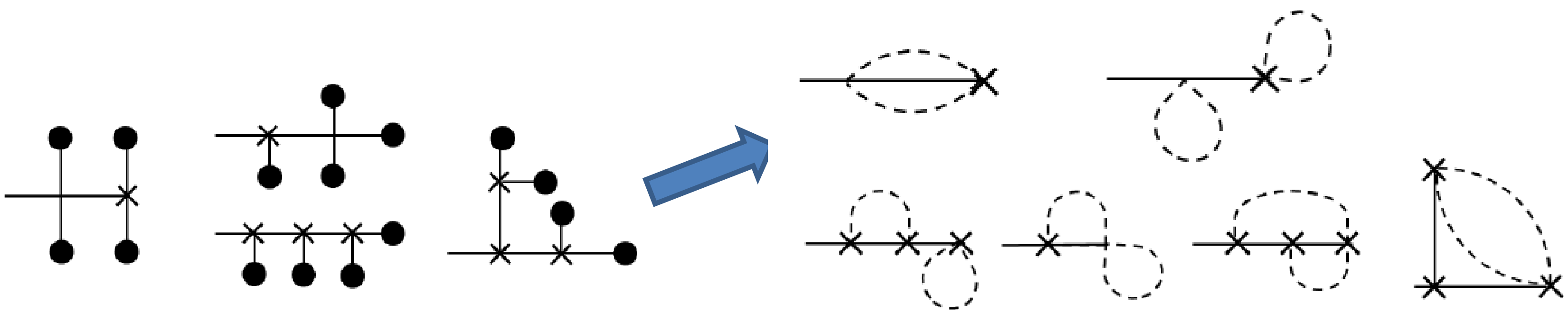
with the initial condition $A(\tau_0) = A^{(0)}(\tau_0) + A^{(1)}(\tau_0)$

identify $\mathcal{F}^{(0)}(x, y) \equiv A^{(1)}(x)A^{(1)}(y)$

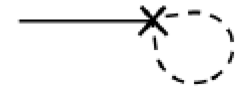
$A^{(2)} \sim$



$A^{(4)} \sim$



Diagrammatic interpretation

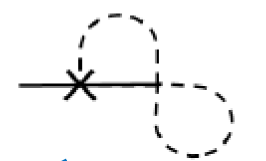


$$\left(\partial_\tau^2 A_I + \frac{1}{\tau}\partial_\tau A_I - \delta_{I\zeta}\frac{A_I}{\tau^2} - D_J F_{JI}\right)^a$$

$$= g f_{abc}\left(D^{bc}_{xI}\mathcal{F}^{ec}_{JJ}(x,y) + D^{bc}_{xJ}\left(\mathcal{F}^{ec}_{JI}(x,y) - 2\mathcal{F}^{ec}_{IJ}(x,y)\right)\right)_{y=x}$$

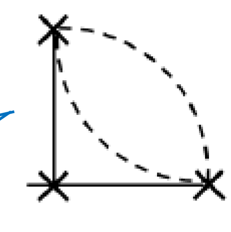
$$+\frac{ig^2}{2}C_{ab,cd}\int_{\tau_0}^{\tau} d^4y\, V^y_{lmn,LMN}\Big[\rho^{bl}_{IL}(x,y)\mathcal{F}^{cm}_{JM}(x,y)\mathcal{F}^{dn}_{JN}(x,y)$$

$$+(\mathcal{F}\rho\mathcal{F}) + (\mathcal{F}\mathcal{F}\rho) - \frac{1}{4}(\rho\rho\rho)\Big]$$

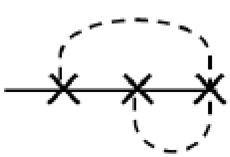


$$\left[\left(\partial_\tau^2 + \frac{1}{\tau}\partial_\tau - \frac{1}{\tau^2}\delta_{I\zeta}\right)\delta_{IJ} - (D^2\delta_{IJ} - D_I D_J - 2igF_{IJ})\right]^{ab}\mathcal{F}^{bc}_{JK}(x,y)$$

$$+g^2\left(C_{ad,be}\mathcal{F}^{de}_{IJ}(x,x) + \frac{1}{2}C_{ab,de}\mathcal{F}^{de}_{MM}(x,x)\delta_{IJ}\right)\mathcal{F}^{bc}_{JK}(x,y)$$

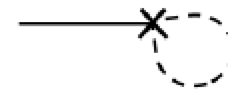


$$= -\int_{\tau_0}^{\tau} d^4z\,\Pi_\rho(x,z)^{ab}_{IJ}\mathcal{F}^{bc}_{JK}(z,y) + \int_{\tau_0}^{\tau'} d^4z\,\Pi_{\mathcal{F}}(x,z)^{ab}_{IJ}\rho^{bc}_{JK}(z,y)\,,$$

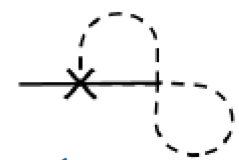


$$\Pi_{\mathcal{F}}(x,y)^{ab}_{IJ} = \frac{1}{2}\overrightarrow{V}^x_{alm,ILM}\left(\mathcal{F}^{ll'}_{LL'}\mathcal{F}^{mm'}_{MM'} - \frac{1}{4}\rho^{ll'}_{LL'}\rho^{mm'}_{MM'}\right)_{xy}\overleftarrow{V}^y_{bl'm',JL'M'}$$

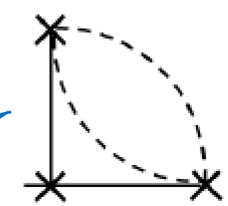
Diagrammatic interpretation

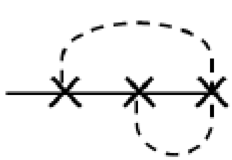


$$\left(\partial_\tau^2 A_I + \frac{1}{\tau}\partial_\tau A_I - \delta_{I\zeta}\frac{A_I}{\tau^2} - D_J F_{JI}\right)^a$$
$$= g f_{abc}\left(D^{be}_{xI}\mathcal{F}^{ec}_{JJ}(x,y) + D^{be}_{xJ}\left(\mathcal{F}^{ec}_{JI}(x,y) - 2\mathcal{F}^{ec}_{IJ}(x,y)\right)\right)_{y=x}$$
$$+\frac{ig^2}{2}C_{ab,cd}\int_{\tau_0}^{\tau} d^4y\, V^y_{lmn,LMN}\Big[\rho^{bl}_{IL}(x,y)\mathcal{F}^{cm}_{JM}(x,y)\mathcal{F}^{dn}_{JN}(x,y)$$
$$+(\mathcal{F}\rho\mathcal{F}) + (\mathcal{F}\mathcal{F}\rho) - \frac{1}{4}(\rho\rho\rho)\Big]$$



$$\left[\left(\partial_\tau^2 + \frac{1}{\tau}\partial_\tau - \frac{1}{\tau^2}\delta_{I\zeta}\right)\delta_{IJ} - \left(D^2\delta_{IJ} - D_I D_J - 2igF_{IJ}\right)\right]^{ab}\mathcal{F}^{bc}_{JK}(x,y)$$
$$+g^2\left(C_{ad,be}\mathcal{F}^{de}_{IJ}(x,x) + \frac{1}{2}C_{ab,de}\mathcal{F}^{de}_{MM}(x,x)\delta_{IJ}\right)\mathcal{F}^{bc}_{JK}(x,y)$$
$$= -\int_{\tau_0}^{\tau} d^4z\,\Pi_\rho(x,z)^{ab}_{IJ}\mathcal{F}^{bc}_{JK}(z,y) + \int_{\tau_0}^{\tau'} d^4z\,\Pi_{\mathcal{F}}(x,z)^{ab}_{IJ}\rho^{bc}_{JK}(z,y)\,,$$





$$\Pi_{\mathcal{F}}(x,y)^{ab}_{IJ} = \frac{1}{2}\overrightarrow{V}^x_{alm,ILM}\left(\mathcal{F}^{ll'}_{LL'}\mathcal{F}^{mm'}_{MM'} - \frac{1}{4}\rho^{ll'}_{LL'}\rho^{mm'}_{MM'}\right)_{xy}\overleftarrow{V}^y_{bl'm',JL'M'}$$

Classical approximation

$$\mathcal{F}\mathcal{F} \gg \rho\rho \to 0$$

In free scalar theory at equilibrium,

$$\mathcal{F}(t-t',p) = \frac{\cos(t-t')p}{p}\left(n(p)+\frac{1}{2}\right) \qquad \rho(t-t',p) = \frac{\sin(t-t')p}{p}$$

Valid at low momentum $\quad p \ll T$

Classical thermal equilibrium (Rayleigh-Jeans)

$$\Pi_\rho \mathcal{F} - \rho \Pi_\mathcal{F} = 0 \quad \Rightarrow \quad n(p) = \frac{T}{p} - \frac{1}{2}$$

Classical statistical : all loops, but systematically neglect $\rho\rho$ terms at each order

2PI : fixed loop, but keep all terms.
essential for quantum equilibration : Bose-Einstein, Boltzmann (exponential)

Problems (and qualms)

- Fixed order 2PI : Gauge dependence is of higher order (Arrizabalaga-Smit), Ward identities OK for bare theory (Reinosa-Serreau). Survives renormalization?
- $\langle A \rangle = 0$ or $\langle A \rangle \neq 0$?
- Breaks down at the height of instability?
- Numerically (very) expensive when $\langle A \rangle \neq 0$