# Preheating and Thermalization in the Early Universe

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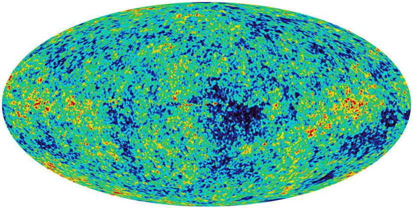
# Outline

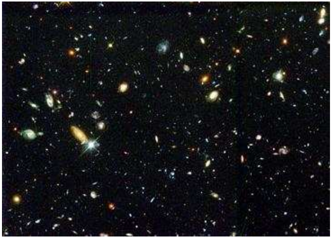
1. Introduction
2. Overview of Preheating
3. Turbulent equilibration
    - Turbulence in the simplest $\lambda\phi^4$ model
    - Kinetic theory of the turbulence
    - Role of zero-mode
    - Driven and decaying turbulence during reheating
4. Thermalization

- ``Inflation'', a period of accelerated universe expansion, solves successfully a number of problems of classical cosmology. (Universe age, flatness, homogeneity, etc.)
- It has unanticipated amazing predictive power:



CMBR anisotropy
379,000 years after

$\Downarrow$



Large-scale structure
13.7 billions years after

During Inflation the Universe is empty,  in a vacuum state.
How vacuum was turned into radiation ?



Particle physicist

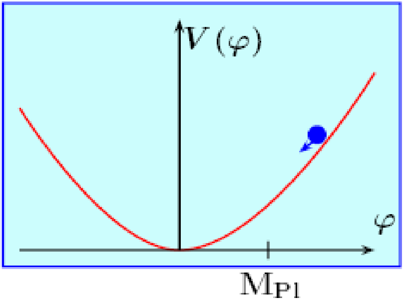
Cosmologist

# Chaotic Inflation

Equation of motion

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{dV}{d\varphi} = 0$$

If $H \gg m$ the field rolls down slowly




$\varphi > M_{Pl}$ Inflation

$\varphi < M_{Pl}$ Reheating

# Initial linear stage

During Inflation the Universe is “empty”. But small fluctuations obey

$$\ddot{u}_k + [k^2 + m_{\text{eff}}^2(\tau)]\, u_k = 0$$

and it is not possible to keep fluctuations in vacuum if $m_{\text{eff}}$ is time dependent

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The source for $m_{\text{eff}} = m_{\text{eff}}(\tau)$ is time-dependence of classical backgrounds:

- Expansion of space-time, $a(\tau)$
- Evolution of the inflaton field, $\phi(\tau)$

# Coupling to the inflaton

| Scalar X | Fermion $\psi$ |
|---|---|
| $m_{\text{eff}}^2 = m_X^2 + g^2\phi^2(t)$ | $m_{\text{eff}} = m_\psi + g\phi(t)$ |

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Relevant parameter:

$$g^2 \;\rightarrow\; q \equiv \frac{g^2\phi^2}{4m_\phi^2}$$

Note: $q$ can be very large since

$$\frac{\phi^2}{m_\phi^2} \approx 10^{12}$$

*Kofman, Linde & Starobinsky (94)*

# Bose versus Fermi :

Scalar X

$$m_{\text{eff}}^2 = m_X^2 + g^2\phi^2(t)$$

Fermion $\psi$

$$m_{\text{eff}} = m_\psi + g\phi(t)$$

Bose stimulation.
Occupation numbers grow,
$n = e^{\mu t}$

Pauli blocking.
Occupation numbers
$n < 1$

Explosive decay of the inflaton

Particles are massless at
$\phi(t) = -m_\psi / g$

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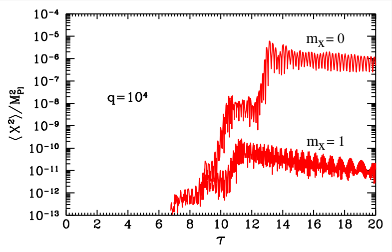


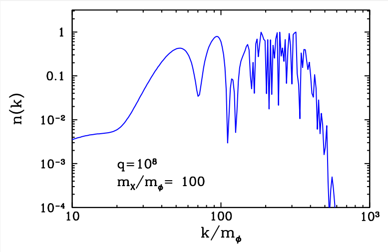

# Bose versus Fermi :

Scalar X

$$m^2_{\text{eff}} = m^2_X + g^2\phi^2(t)$$

Fermion $\psi$

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Bose stimulation.
Occupation numbers grow,
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Explosive decay of the inflaton

$\Downarrow$

Quantum to classical transition

*Khlebnikov & I.T. (96)*

Non-thermal phase transitions

*Kofman, Linde & Starobinsky (96)*

*I.T. (96)*

Pauli blocking.
Occupation numbers
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$\Downarrow$

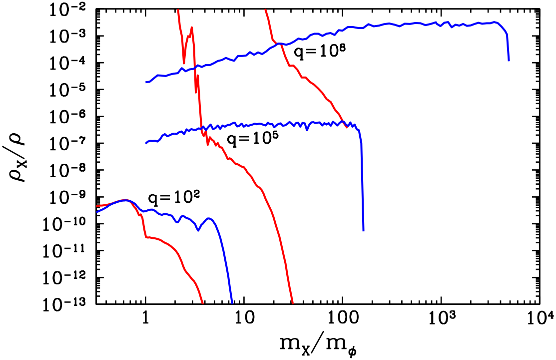
Intensive creation of superheavy fermions

*Giudice, Peloso, Riotto, & I. T. (99)*

Trans-Planckian features in CMB

*Chung, Kolb, Riotto, & I. T. (00)*

# Matter creation: Bose versus Fermi




$\phi \propto \sin(mt)$

Particle production at the end of inflation in the model with inflaton potential

$$V(\phi) = m^2\phi^2$$

Red lines: fraction of produced Bosons
Blue lines: fraction of produced Fermions

# Matter creation: end of linear stage

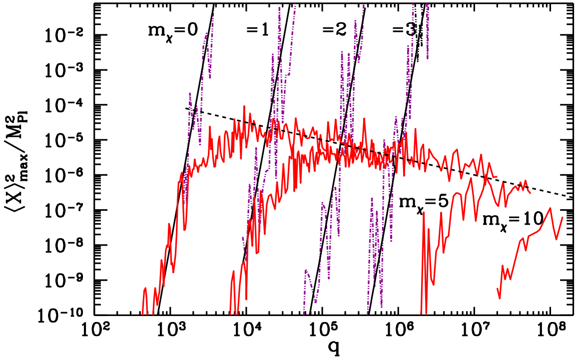
In the linear approximation in Minkowski space-time occupation numbers grow exponentially without limit.

To solve the problem one needs to understand the non-linear dynamics.

- At large occupation numbers it is possible to map quantum evolution into classical:
  **quantum density matrix ⇒ classical density matrix**

  Khlebnikov, I.T. (1996)
- This allows numerical modelling on the lattice
  - starting from vacuum
  - through "parametric resonance", then through preheating
  - and down to physical effects in question

# When particle production ends?




Maximum filed variance in the model with inflaton potential

$$V(\phi) = m^2\phi^2$$

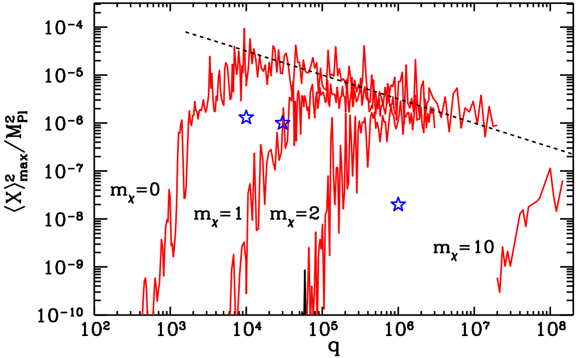
and interaction

$$g^2 X^2 \phi^2$$

.

Dotted lines: linear problem.
Solid lines: Hartree approximation.

# When particle production ends?




Maximum filed variance in the model with inflaton potential

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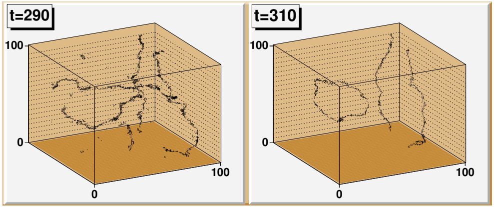
Solid lines: Hartree approximation.
Stars: complete non-linear problem.

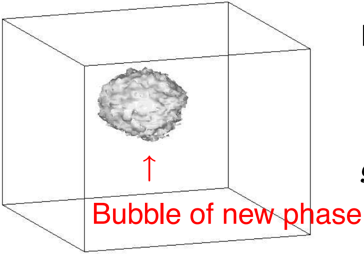
# Non-thermal Phase Transitions




String formation

$$V(\phi) = \lambda(\phi_1^2 + \phi_2^2 - v^2)^2$$

$v \sim 10^{16}$ GeV




First order phase transition

$$V(\phi, X) = \lambda(\phi^2 - v^2)^2 + g^2\phi^2X^2$$

$g^2/\lambda = 200$

*Khlebnikov, Kofman, Linde & I.T. (98)*

# Thermalization after Inflation

Questions:

- How system approaches equilibrium?
- When? What is thermalization temperature?

Are of general interest and important for practical applications. It influences:

- Inflationary predictions
- Baryogenesis
- Abundance of gravitino and dark matter relics
- Primordial fluctuations

# Approach:

- Lattice simulations (as a guidance)

Various quantities can be measured as functions of time:

- Zero mode, $\phi_0 = \langle \phi \rangle$
- Variance, $\langle \phi^2 \rangle$ - $\phi_0^2$
- Particle number, $n_k = \langle a^\dagger(k) a(k) \rangle$
- Correlators, $\langle aa \rangle$, $\langle a^\dagger a^\dagger aa \rangle$, $\langle \pi^2 \rangle$, ...

- Kinetic theory (weak wave turbulence)

Compare to lattice results and extrapolate.

*Felder & Kofman (2001)*

*Micha & I.T. (2004)*

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  (weak wave turbulence)

Compare to lattice results and extrapolate.

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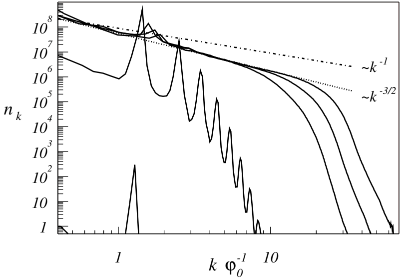
Consider simplest $\lambda\varphi^4$ model.

In conformal frame, $\phi = \varphi/a$, and rescaled coordinates, $x^\mu \to \sqrt{\lambda}\varphi(0)\, x^\mu$, the equation of motion takes very simple form

$$\Box\phi + \phi^3 = 0$$

# Turbulent spectra

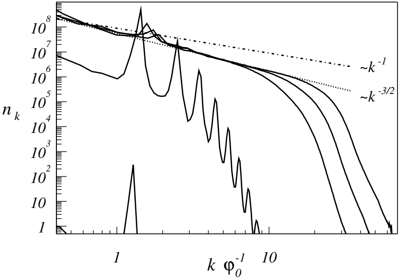



Re-scale the field and coordinates by the current amplitude of the zero mode

$$\Box\phi + \phi^3 = 0$$

Here $x^\mu \to x^\mu\, \phi_0$ and therefore $k \to k\,/\phi_0$

# Turbulent spectra

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Here $x^\mu \to x^\mu\, \phi_0$ and therefore $k \to k\,/\phi_0$

Let $n \sim k^{-s}$. Theory of a stationary wave turbulence predicts

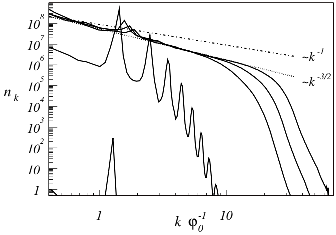
- $s = \frac{5}{3}$ for 4-particle interaction
- $s = \frac{3}{2}$ for 3-particle interaction

# Turbulence

**Variety of turbulent behavior/regimes/definitions:**

- Weak wave turbulence
- Strong turbulence

- Driven turbulence
- Decaying (free) turbulence

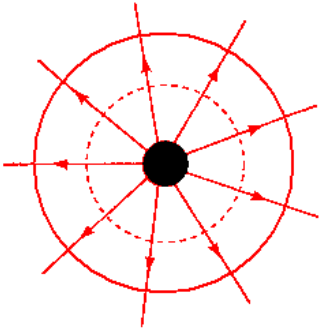


# Kolmogorov Turbulence

We have a source of energy (or particles) located at $k = k_i$ and a sink located at $k = k_f$. Energy conserves

$$\partial_t(\omega_k n_k) + \nabla_k \cdot j_k = 0,$$

In statiory state energy flux is constant through any surface



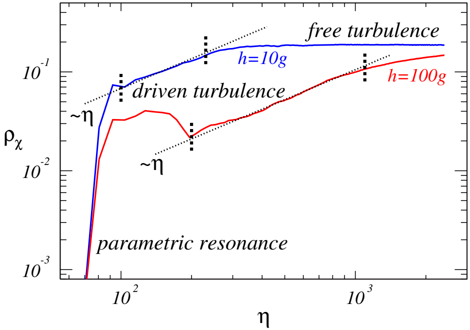
$$\frac{dE}{dt} = \text{const}$$

Pumped energy should grow linearly with time

$$E = \text{const} \cdot t$$

# Three major epochs of reheating

$$V(\chi, X) = \frac{1}{4}\phi^4 + \frac{g}{2}\phi^2\chi^2 + \frac{h}{4}\chi^4$$




- Parametric resonance
- Driven turbulence
- Free turbulence

At large $h$ and/or $g$ the parametric resonance stops early.

*Micha & I.T. (2004)*

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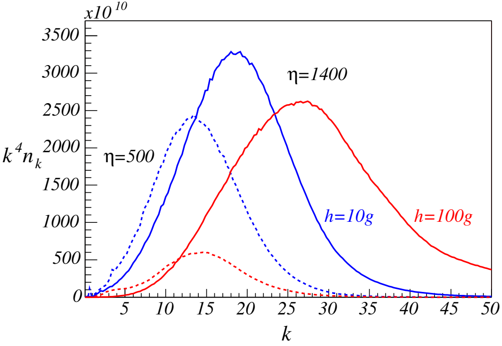


- Parametric resonance
- Driven turbulence
- Free turbulence

But distributions evolve faster at late times.

*Micha & I.T. (2004)*

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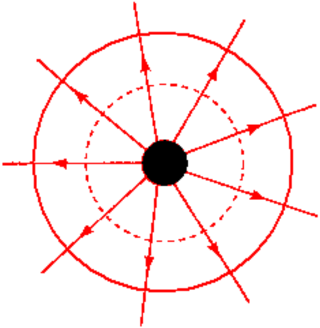
$$\partial_t(\omega_k\, n_k) + \nabla_k \cdot j_k = 0,$$

or

$$\omega_k I_k[n] + \nabla_k \cdot j_k = 0.$$

In statiory state energy flux is constant through any surface



$$j_k = C\ k^{-d+1+\delta}$$

$$\nabla_k \cdot j_k = \delta C\ k^{-d+\delta}$$

In stationary state $\delta \longrightarrow 0$ and

$$I_k[n] \rightarrow k^{-(d+\alpha)} \equiv k^{-\nu}$$

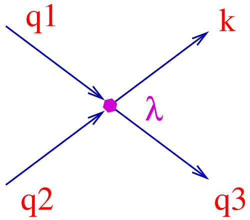
where $\omega(k) = k^{\alpha}$

# Kinetic Theory

Kinetic equation $\dot{n}_k = I_k[n]$

Collision integral $I_k[n] = \int d\Omega(k, q_i)\, F(k, q_i)$

Example:




$$d\Omega(k, q_i) = \frac{(2\pi)^4 |M|^2}{2\omega_k} \delta^4(k_\mu, q_{i\mu}) \prod_{i=1}^{3} \frac{d^3 q_i}{2\omega_i (2\pi)^3}$$

For classical waves ($n \gg 1$)

$$F(k, q_i) = (n_k + n_{q_1}) n_{q_2} n_{q_3} - n_k n_{q_1} (n_{q_2} + n_{q_3})$$

In full quantum problem

$$F(k, q_i) = (1 + n_k)\,(1 + n_{q_1}) n_{q_2} n_{q_3} - n_k n_{q_1} (1 + n_{q_2})\,(1 + n_{q_3})$$

# Scaling

Collision integral $I_k[n] = \int d\Omega(k, q_i)\, F(k, q_i)$

Rescaling of $n$:

$$F(\zeta n) = \zeta^{m-1} F(n)\,,$$

where $m$ is the number of particles which participate in the process.

Rescaling of momenta:

$$d\Omega(\xi k, \xi q_i) = \xi^{\mu}\, d\Omega(k, q_i)\,,$$

where $\mu$ depends upon theory and number of dimensions, e.g. $\mu = 1$ for a relativistic theory with dimensionless couplings.

# Scaling

## Application to stationary turbulence

Let $n(q) \propto q^{-s}$. Then

$$F(\xi k, \xi q_i) = \xi^{-s\,(m-1)}\, F(k, q_i)\ .$$

This gives

$$I_{\xi k}[n] = \xi^{-\nu} I_k[n]\ ,$$

where

- $\nu = s\,(m-1) - \mu$.

With the condition of scale independent flux

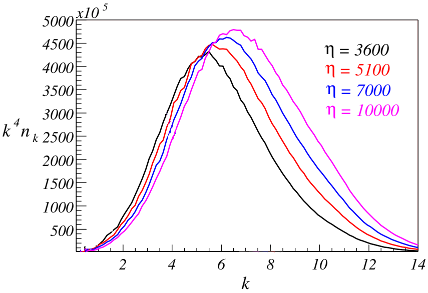
- $\nu = d + \alpha$

we find

$$s = \frac{d + \alpha + \mu}{m - 1}$$

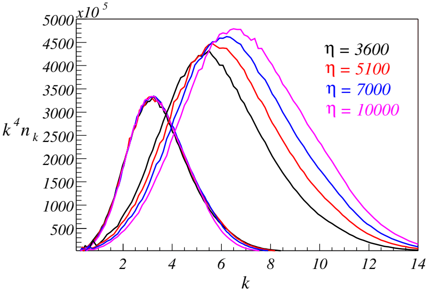
- $s = \frac{5}{3}$ for 4-particle interaction
- $s = \frac{3}{2}$ for 3-particle interaction

# Self-similar evolution




Particle numbers on the lattice
in the regime of free turbulence

# Self-similar evolution

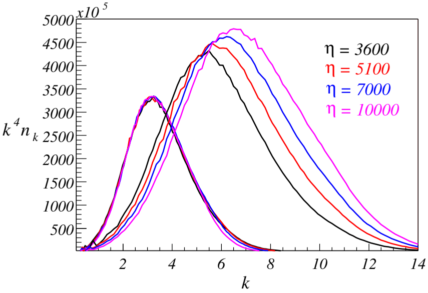



Particle numbers on the lattice in the regime of free turbulence evolve self-similarly

$$n(k,\eta) = \tau^{-q}\, n_0(k\tau^{-p})$$

with $p = \frac{1}{5}$

# Self-similar evolution

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**Theory:**

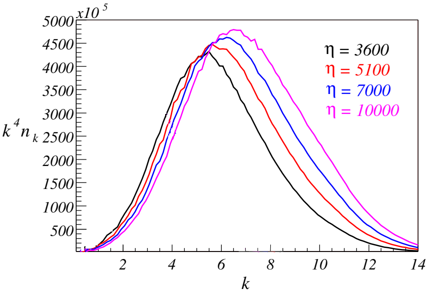
Free turbulence

- $p = \frac{1}{7}$ for 4-particle interactions
- $p = \frac{1}{5}$ for 3-particle interactions

Driven turbulence

- $p = \frac{3}{7}$ for 4-particle interactions
- $p = \frac{2}{5}$ for 3-particle interactions

# Self-similar evolution

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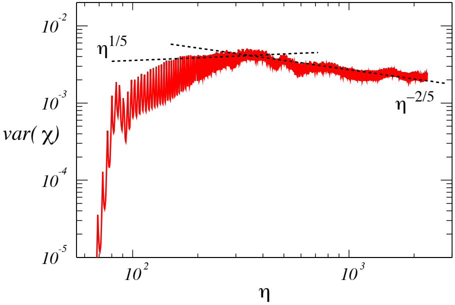
**Thermalization:**

Position of the peak moves as

$$k(\tau) = k_0\, \tau^p$$

Thermalization will occur when $k_{\mathrm{max}}^4 \sim T^4 \sim$ (initial energy).

# Field variance




$$var(\chi,\eta) = \tau^{v}\, var(\chi,0)$$

Time dependence of the variance of $\chi$ field in the model $h = 10g$

## Theory:

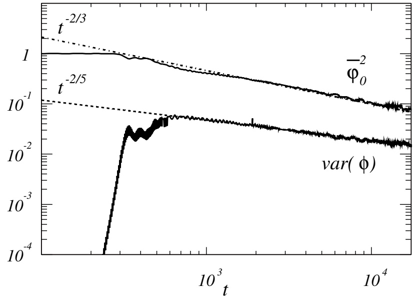
Driven turbulence

- $v = +\frac{1}{7}$ for 4-particle interactions
- $v = +\frac{1}{5}$ for 3-particle interactions

Free turbulence

- $v = -\frac{2}{7}$ for 4-particle interactions
- $v = -\frac{2}{5}$ for 3-particle interactions

# Amplitude of the zero mode.




$$\phi_0^2(\eta) = \tau^{-z}$$

**Theory:**

Free turbulence

- $z = \frac{2}{5}$ for 4-particle interactions
- $z = \frac{2}{3}$ for 3-particle interactions

# Summary

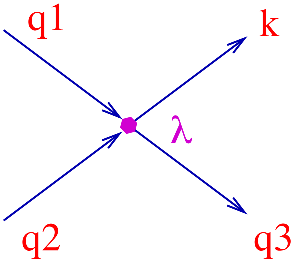
All scaling exponents agree with predictions for 3-particle interactions, which for $k$-independent matrix elements are

$$
\begin{aligned}
p &= 1/(2m-1) \\
s &= d - m/(m-1) \\
v &= 2/(2m-1) \\
z &= 2/(d(m-1)-m)
\end{aligned}
$$

with $d = 3$ and $m = 3$

# Bose-condensate dominates

How 3-particle interactions can appear in $\lambda\phi^4$-theory?



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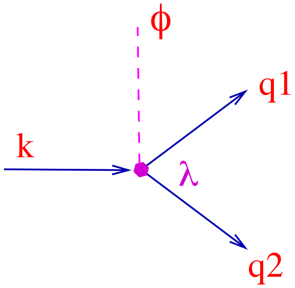



3-particle collision integral can be obtained from the 4-particle one with the substitution

$$\frac{n_p}{\omega_p} \to \frac{n_p}{\omega_p} + (2\pi)^3 \delta^{(3)}(\vec{p})\bar{\phi}_0^2$$

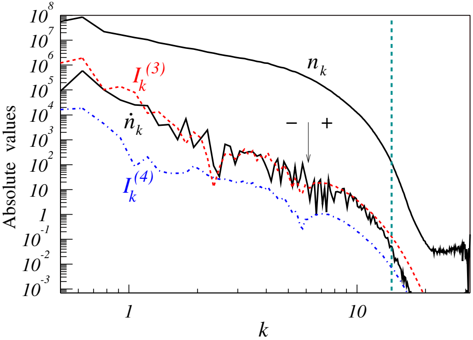
# Bose-condensation on a lattice

- The condensate quickly recovers to the original value after being set to zero "by hands".
- This prohibited direct check of scaling laws for $m = 4$.
- For a dedicated lattice studies of Bose-condensation see

*Damle, Najumdar, and Sachdev (1996)*

*Khlebnikov & I.T. (1999)*

# Test of kinetic description




Collision integrals and $\dot{n}(k)$ at $\eta = 5000$.

$I_k^{(3)}$ agrees with $\dot{n}(k)$ to the left of the vertical dashed line

Red line: 3-particle collision integral, $I_k^{(3)}$

Blue line: 4-particle collision integral, $I_k^{(4)}$

# Conclusions

- We identify three different stages of the Universe reheating
  - ``Parametric resonance.'' Fast exponential growth of energy in fluctuations, but only a small fraction of energy is transferred during this stage.
  - Driven turbulence. Linear growth. Major mechanism of energy transfer.
  - Free turbulence. Long stage of thermalization.
- Turbulent evolution is self-similar.
- Bose-condensate of zero mode governs evolution.
- Explicit expressions for particle occupation numbers.
- Estimates for reheating time and temperature.