Thermalization of boost-invariant plasma at strong coupling from AdS/CFT

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Outline

1. **Key question**
2. **AdS/CFT, hydrodynamics and nonequilibrium processes**
3. **Boost-invariant flow**
4. **The AdS/CFT method**
5. **Main results**
   - Nonequilibrium vs. hydrodynamic behaviour
   - Entropy
   - Characteristics of thermalization
6. **Conclusions**

Key question:

Understand the features of (early) thermalization for an evolving (*boost-invariant*) plasma system

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Thermalization

- At weak coupling the obvious definition would be to require thermal momentum distributions for quarks and gluons...
- At strong coupling, the picture of a gas of gluons is not really valid — alternatively require that observables such as 2-point functions/spatial Wilson loops/ entanglement entropy are the same as for a thermal system... explored in the AdS/CFT context
- This is very good for studying relaxation processes where the final state is some uniform static plasma system — this is not so for the plasma undergoing expansion
- For an expanding plasma fireball we need *local* equilibrium — bilocal probes get contaminated by collective flow
- We adopt an *operational* definition of thermalization — the point when plasma starts being describable by (viscous) hydrodynamics.

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- **Hydrodynamics** isolates long wavelength effective degrees of freedom of a theory
- The energy-momentum tensor $T_{\mu\nu}$ is expressed in terms of a local temperature T and flow velocity u^μ
- $T_{\mu\nu}$ is expressed as an expansion in the gradients of the flow velocities (shown here for $\mathcal{N}=4$ SYM)

$$T^{\mu\nu}_{rescaled} = \underbrace{(\pi T)^4(\eta^{\mu\nu}+4u^\mu u^\nu)}_{perfect\ \ fluid} - \underbrace{2(\pi T)^3\sigma^{\mu\nu}}_{viscosity} +$$

$$\underbrace{+(\pi T^2)\left(\log 2\, T^{\mu\nu}_{2a} + 2T^{\mu\nu}_{2b} + (2-\log 2)\left(\frac{1}{3}T^{\mu\nu}_{2c} + T^{\mu\nu}_{2d} + T^{\mu\nu}_{2e}\right)\right)}_{second\ \ order\ \ hydrodynamics}$$

- The coefficients of the various tensor structures are the transport coefficients. In a conformal theory these are pure numbers times powers of T.
- Full nonlinear hydrodynamic equations follow now from $\partial_\mu T^{\mu\nu}=0$
- The above form of $T_{\mu\nu}$ for $\mathcal{N}=4$ SYM at strong coupling is **not** an assumption but can be proven from AdS/CFT Minwalla et.al.

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Linearized hydrodynamics

- Look at small disturbances of the uniform static plasma...
- *If* $T_{\mu\nu}$ is described by (1^{st} order viscous) hydrodynamics then one can derive dispersion relation of long wavelength modes from hydrodynamic equations:
  shear modes:
  $$\omega_{shear} = -i \frac{\eta}{E+p} k^2$$
  sound modes:
  $$\omega_{sound} = \frac{1}{\sqrt{3}} k - i \frac{2}{3} \frac{\eta}{E+p} k^2$$
- If we were to include terms in $T_{\mu\nu}$ with more derivatives (higher order viscous hydrodynamics), we would get terms with higher powers of k in the dispersion relations...
- Hypothetical resummed *all-order* hydrodynamics would predict the full dispersion relation for these modes $\omega_{shear}(k)$, $\omega_{sound}(k)$

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What happens in the AdS/CFT description?

- The uniform static plasma system is described as a static planar black hole
- Small disturbances of the uniform static plasma $\equiv$ small perturbations of the black hole metric ($\equiv$ quasinormal modes (QNM))

$$g^{5D}_{\alpha\beta} = g^{5D,black\ hole}_{\alpha\beta} + \delta g^{5D}_{\alpha\beta}(z) e^{-i\omega t + ikx}$$

- Dispersion relation fixed by linearized Einstein's equations. Results for the sound channel

from Kovtun,Starinets hep-th/0506184

- This is equivalent to summing contributions from *all-order* viscous hydrodynamics
- But, **in addition**, there is an infinite set of higher QNM — effective degrees of freedom not contained in the hydrodynamic description at all!

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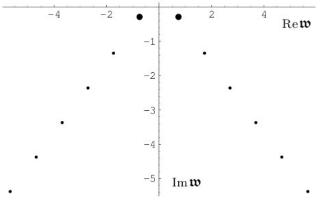

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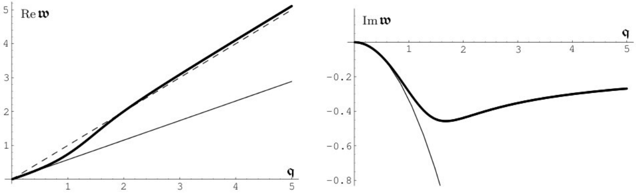

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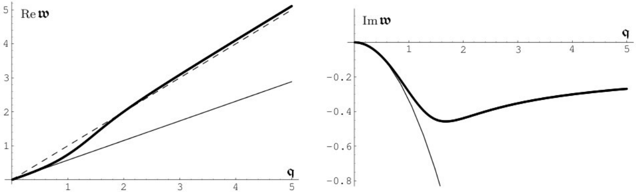

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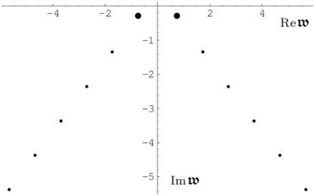

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- contain all-order viscous hydrodynamic modes (with specific values of all transport coefficients)
- **in addition** contain the dynamics of genuine nonhydrodynamical modes
- incorporate their interactions in a fully nonlinear (and unique) way

Consequence:
Einstein's equations can serve to study nonequilibrium processes in strongly coupled $\mathcal{N} = 4$ SYM and are an effective tool for exploring physics *beyond* hydrodynamics

Question:
In the case of boost-invariant plasma expansion can we unambigously determine
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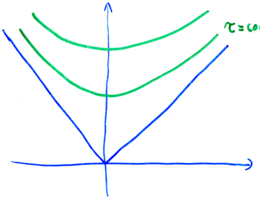

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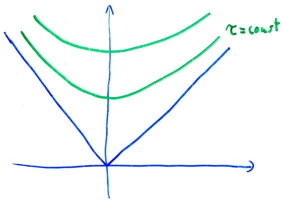

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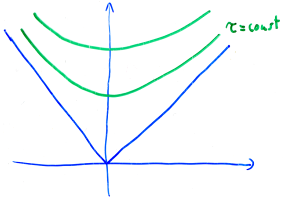

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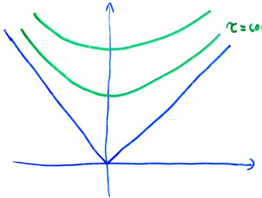

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Method: Describe the time dependent evolving strongly coupled plasma system through a dual 5D geometry — given e.g. by

$$ds^2 = \frac{g_{\mu\nu}(x^\rho, z) dx^\mu dx^\nu + dz^2}{z^2} \equiv g^{5D}_{\alpha\beta} dx^\alpha dx^\beta$$

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Results

- We have considered 20+8 initial conditions, each given by a choice of the metric coefficient $c(\tau = 0, u)$.
- We have chosen quite different looking profiles e.g.

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c_1(u) &= \cosh u \\
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where $F_{hydro}(w)$ is a *universal function* completely determined in terms of the hydrodynamic transport coefficients (shear viscosity, relaxation time and higher order ones). For strongly coupled $\mathcal{N} = 4$ plasma it becomes

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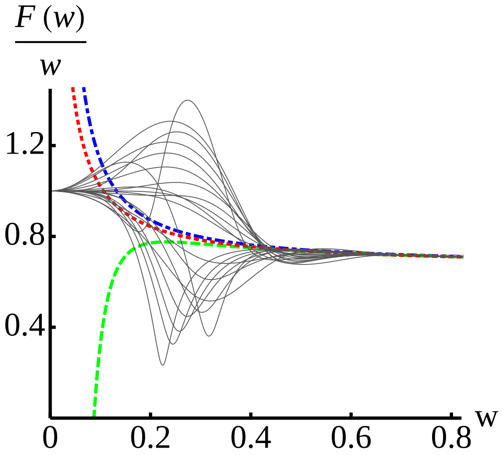

Nonequilibrium vs. hydrodynamic behaviour

- An observable sensitive to the details of the dissipative dynamics (e.g. hydrodynamics) is the pressure anisotropy

$$\Delta p_L \equiv 1 - \frac{p_L}{\varepsilon/3} = 12F(w) - 8$$

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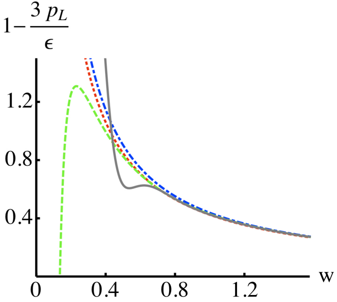

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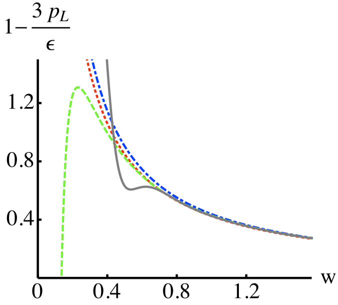

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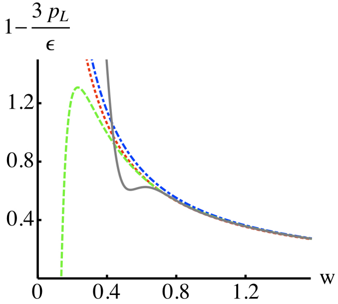

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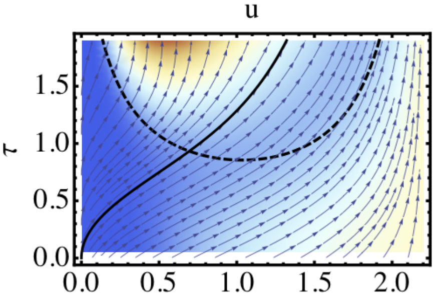

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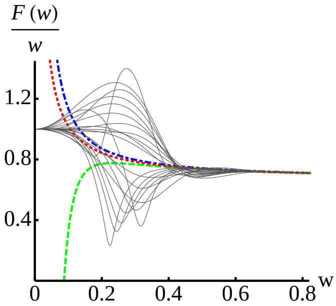

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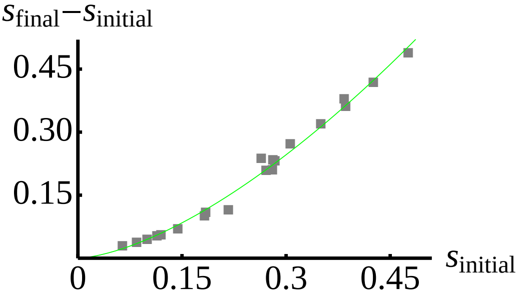

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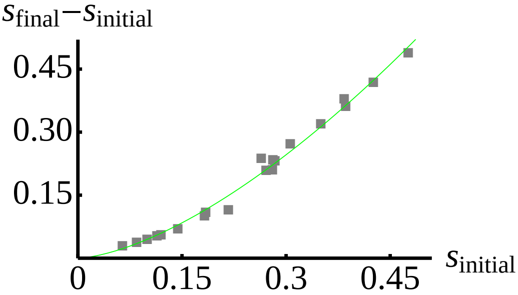

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- The initial entropy turns out to be a key characterization of the initial state
- There seems to be a lot of hidden regularity in the nonequilibrium dynamics
- We will show below that the initial entropy also characterizes the characteristics of the transition to hydrodynamics ($\equiv$ thermalization)
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A numerical criterion for thermalization

- We want to study systematically the properties of the plasma at the point when the dynamics becomes describable by viscous hydrodynamics...
- We adopted a numerical criterion for thermalization

$$\left\| \frac{\tau \frac{d}{d\tau} w}{F_{hydro}^{3^{rd}\ order}(w)} - 1 \right\| < 0.005$$

- We looked at the following features of thermalization:
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- N.B. sample initial conditions for hydrodynamics at RHIC ($\tau_0 = 0.25\, fm$, $T_0 = 500\, MeV$) assumed in [Broniowski, Chojnacki, Florkowski, Kisiel] correspond to $w = 0.63$
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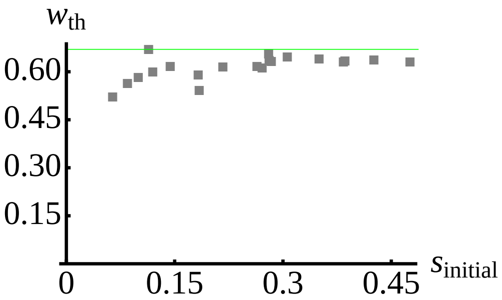

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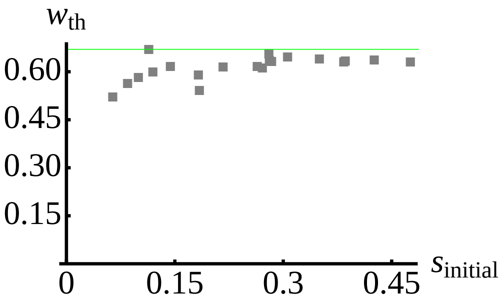

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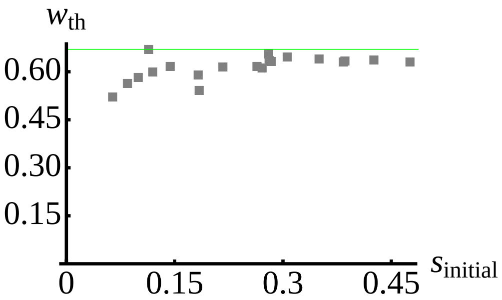

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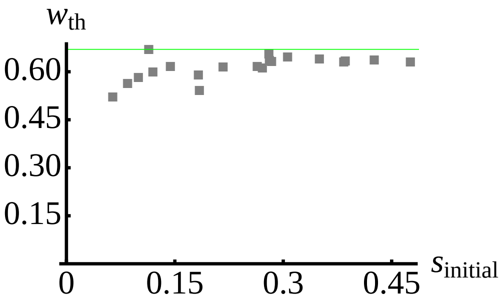

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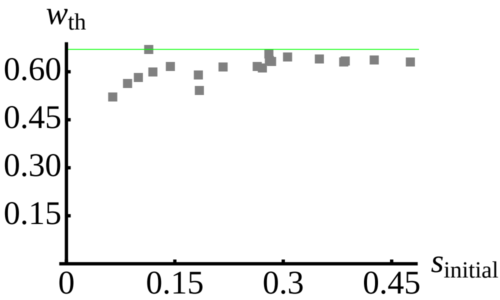

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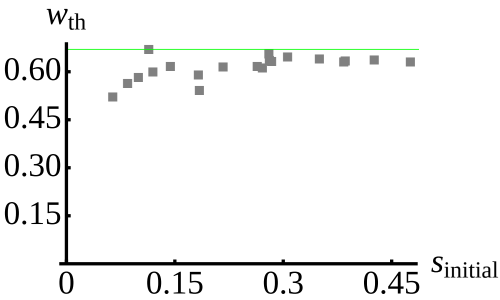

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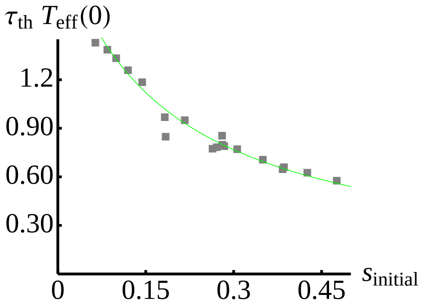

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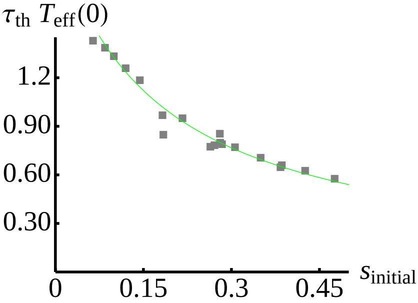

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Temperature at thermalization

- It is interesting to consider the ratio of the temperature at thermalization to the initial effective temperature
- This gives information on which part of the cooling process occurs in the far from equilibrium regime and which part occurs during the hydrodynamic evolution

- Note: for initial profiles with large $s_{initial}$, the energy density initially rises and only then falls $\longrightarrow$ even for $T_{th}/T_{eff}(0) \sim 1$ there is still sizable nonequilibrium evolution
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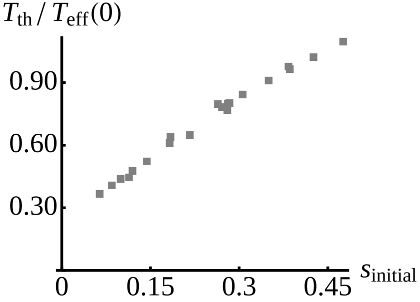

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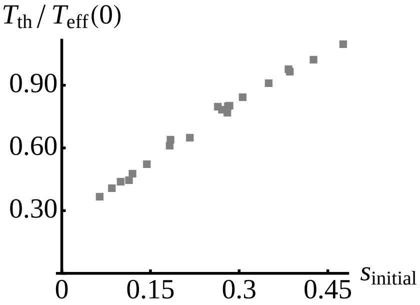

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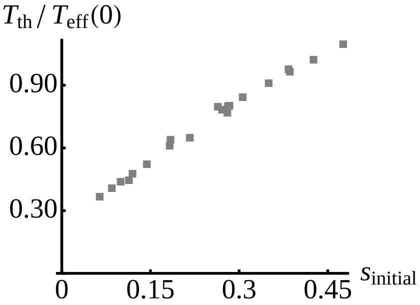

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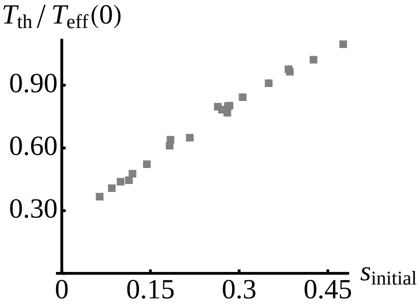

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- The AdS/CFT methods *do not* presuppose hydrodynamics so are applicable even to very out-of-equilibrium configurations
- Even though genuine nonequilibrium dynamics is very complicated, we observed surprising regularities
- Initial entropy seems to be a key physical characterization of the initial state determining the total entropy production and thermalization time and temperature
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