Thermalization in a parton kinetic description including Bremsstrahlung

C. Greiner, EMMI Rapid Reaction Task Force, Heidelberg, december 2011

in collaboration with:

I.Bouras, A. El, O. Fochler, F. Reining, J. Uphoff, C. Wesp, Zhe Xu

- early thermalization, elliptic flow and viscosity

QCD thermalization using parton cascade

VNI/BMS: K.Geiger and B.Müller, NPB 369, 600 (1992)

S.A.Bass, B.Müller and D.K.Srivastava, PLB 551, 277(2003)

ZPC: B. Zhang, Comput. Phys.Commun. 109, 193 (1998)

MPC: D.Molnar and M.Gyulassy, PRC 62, 054907 (2000)

AMPT: B. Zhang, C.M. Ko, B.A. Li, and Z.W. Lin, PRC 61, 067901 (2000)

BAMPS: Z. Xu and C. Greiner, PRC 71, 064901 (2005); 76, 024911 (2007)

BAMPS: Boltzmann Approach of MultiParton Scatterings

A transport algorithm solving the Boltzmann-Equations for on-shell partons with pQCD interactions

$$p^{\mu}\partial_{\mu}f(x,p) = C_{gg\to gg}(x,p) + C_{gg\leftrightarrow ggg}(x,p)$$

(Z)MPC, VNI/BMS, AMPT

new development ggg → gg,
radiative „corrections“

Elastic scatterings are ineffective in thermalization !

Inelastic interactions are needed !

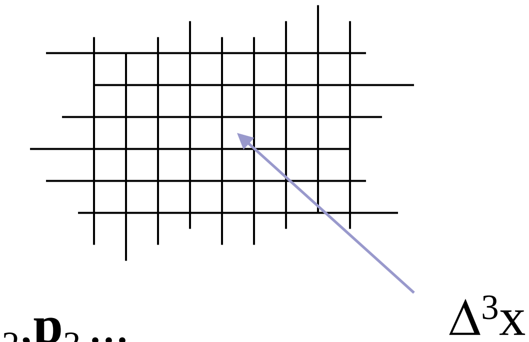
Xiong, Shuryak, PRC 49, 2203 (1994)
Dumitru, Gyulassy, PLB 494, 215 (2000)
Serreau, Schiff, JHEP 0111, 039 (2001)
Baier, Mueller, Schiff, Son, PLB 502, 51 (2001)

BAMPS:
Z. Xu and C. Greiner, PRC 71, 064901 (2005);
Z. Xu and C. Greiner, PRC 76, 024911 (2007)

Stochastic algorithm

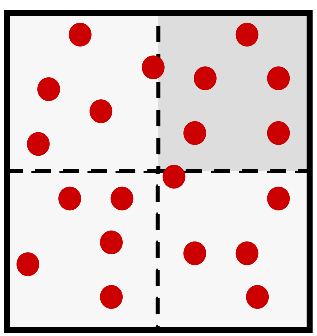
P.Danielewicz, G.F.Bertsch, Nucl. Phys. A 533, 712(1991)
A.Lang et al., J. Comp. Phys. 106, 391(1993)

cell configuration in space →



for particles in Δ^3x with momentum $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3 \ldots$

collision probability:



$$\mathit{for}\ 2 \leftrightarrow 2 \quad P_{22} = v_{rel}\sigma_{22}\frac{\Delta t}{\Delta^3 x}$$

$$\mathit{for}\ 2 \rightarrow 3 \quad P_{23} = v_{rel}\sigma_{23}\frac{\Delta t}{\Delta^3 x}$$

$$\mathit{for}\ 3 \rightarrow 2 \quad P_{32} = \frac{I_{32}}{8E_1E_2E_3}\frac{\Delta t}{(\Delta^3 x)^2}$$

$$I_{32} = \frac{1}{2}\int \frac{d^3 p_{1'}}{(2\pi)^3 2E_{1'}} \frac{d^3 p_{2'}}{(2\pi)^3 2E_{2'}} \left| M_{123\to 1'2'} \right|^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 + p_3 - p_{1'} - p_{2'})$$

screened partonic interactions in leading order pQCD

$$\left|M_{gg\to gg}\right|^2 = \frac{9g^4}{2}\frac{s^2}{(q_\perp^2+m_D^2)^2},$$

elastic part

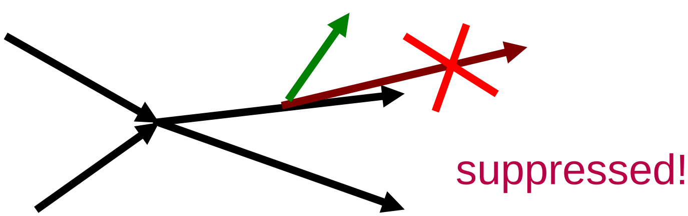
$$\left|M_{gg\to ggg}\right|^2 = \left(\frac{9g^4}{2}\frac{s^2}{(q_\perp^2+m_D^2)^2}\right)\left(\frac{12g^2q_\perp^2}{k_\perp^2\left((\vec{k}_\perp-\vec{q}_\perp)^2+m_D^2\right)}\right)\Theta_{LPM}(k_\perp\Lambda_g-\cosh y)$$

radiative part

J.F.Gunion, G.F.Bertsch, PRD 25, 746(1982)
T.S.Biro at el., PRC 48, 1275 (1993)
S.M.Wong, NPA 607, 442 (1996)

screening mass: $m_D^2 = m_D^2(x,t) = 16\pi\alpha_s\int\frac{d^3p}{(2\pi)^3}\frac{1}{p}(3f_g+n_f f_q),$

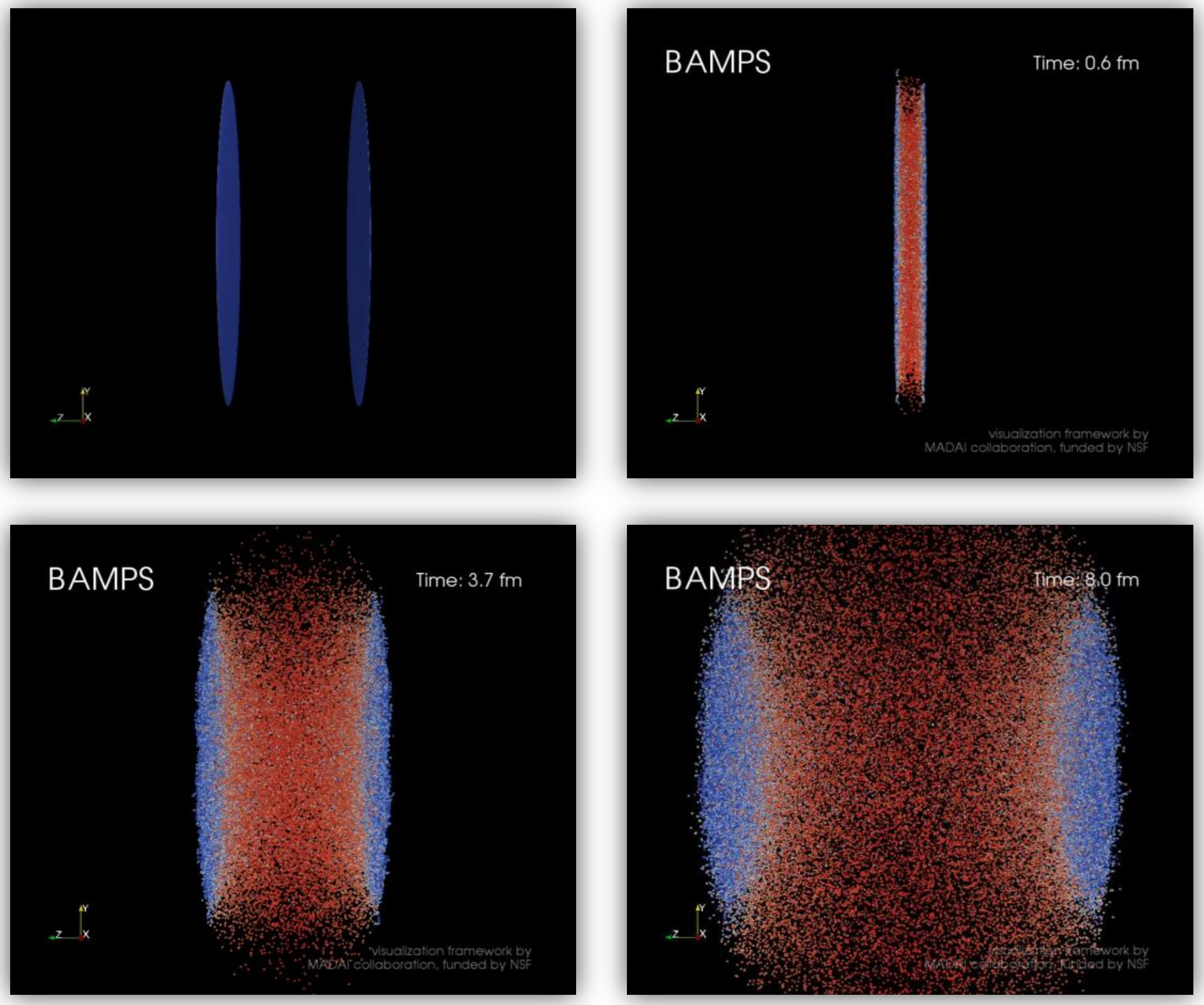
LPM suppression: the formation time $\Delta\tau \approx \frac{1}{k_\perp}\cosh y < \Lambda_g$



Λ_g: mean free path

Heavy-ion collision at LHC

BAMPS simulation of QGP phase at LHC at s_{NN} = 2.76 TeV



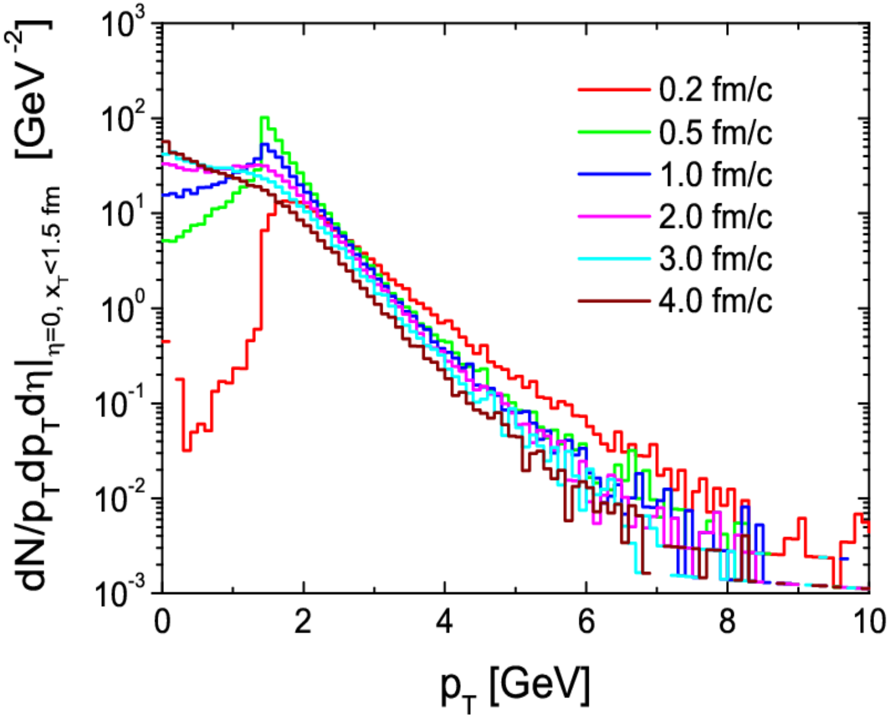
Jan Uphoff

Visualization framework courtesy MADAI collaboration, funded by the NSF under grant# NSF-PHY-09-41373

p_T spectra

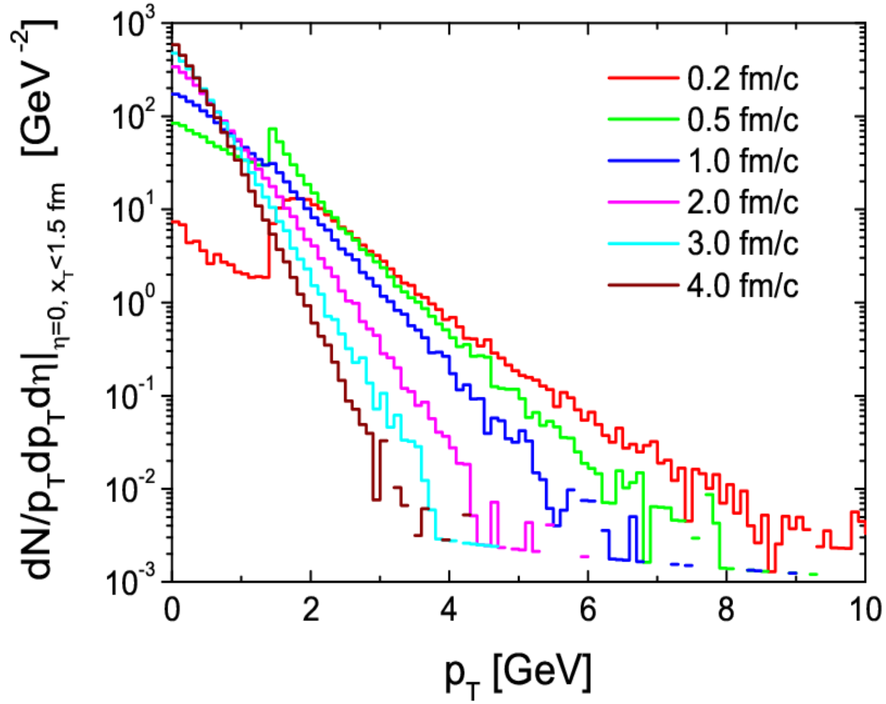
at collision center: x_T<1.5 fm, Δz < 0.4 t fm of a central Au+Au at $s^{1/2}$=200 GeV
Initial conditions: minijets p_T>1.4 GeV; coupling α_s=0.3

simulation pQCD, **only 2-2**



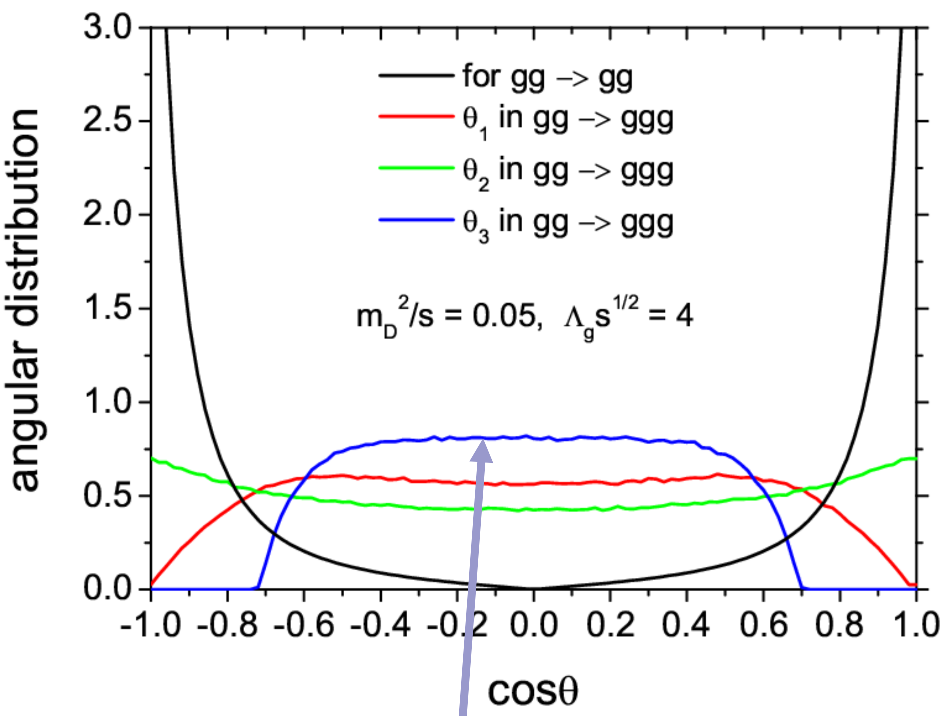
2-2: NO thermalization

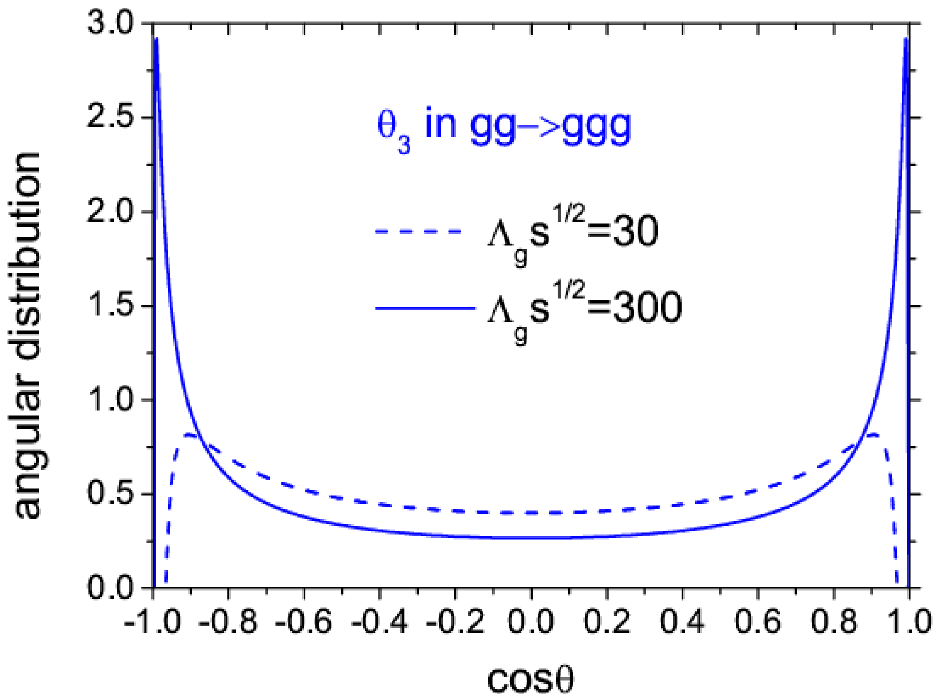
simulation pQCD 2-2 + 2-3 + 3-2



3-2 + 2-3: thermalization!
Hydrodynamic behavior!

distribution of collision angles



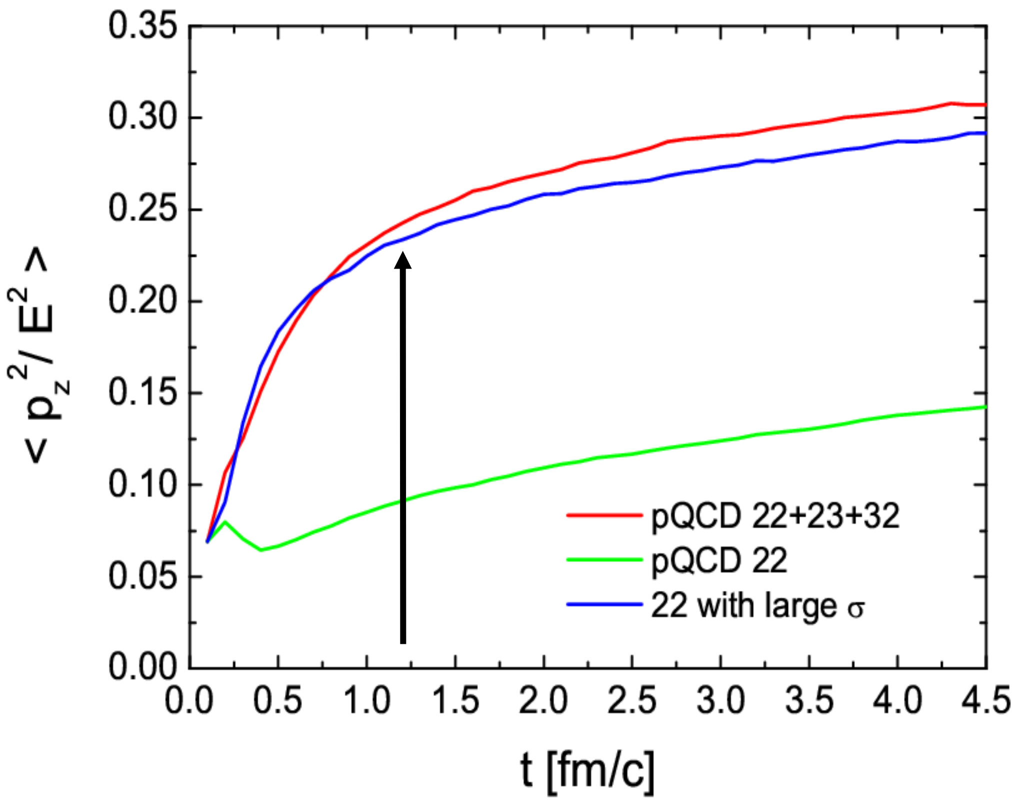


at RHIC energies

$gg \longrightarrow gg$: small-angle scatterings

$gg \longleftrightarrow ggg$: large-angle bremsstrahlung

time scale of thermalization



Theoretical Result !

$$\tau \approx 1\,\mathrm{fm/c}$$

$$< \frac{p_Z^2}{E^2} > (t) = < \frac{p_Z^2}{E^2} >_{eq} + \left(< \frac{p_Z^2}{E^2} > (t_0) - < \frac{p_Z^2}{E^2} >_{eq} \right) \exp\left(-\frac{t - t_0}{\tau} \right)$$

τ = time scale of kinetic equilibration.

Transport Rates

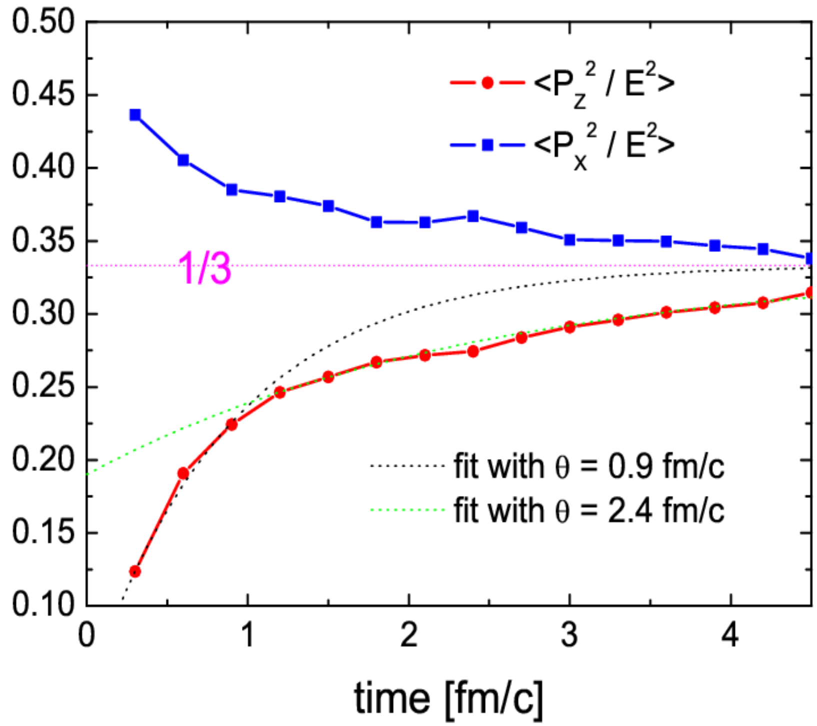
$$\frac{1}{\tau} = R^{tr}_{drift} + R^{tr}_{gg->gg} + R^{tr}_{gg->ggg} + R^{tr}_{ggg->gg}$$

with $$R_i^{tr} = \frac{\int \frac{d^3p}{(2\pi)^3} v_z^2 C_i - \left\langle v_z^2 \right\rangle \int \frac{d^3p}{(2\pi)^3} C_i}{n\,(\frac{1}{3} - \left\langle v_z^2 \right\rangle)},$$

$$i = gg \to gg, gg \to ggg, ggg \to gg$$

- Transport rate is describing kinetic equilibration.
- Transport collision rates have an indirect relationship to the collision-angle distribution.

Z. Xu and CG, PRC 76, 024911 (2007)



$$Q = P_Z^2 / E^2,$$

$$\overline{Q} = \frac{\int \frac{d^3p}{(2\pi)^3} f(p,x,t) Q}{\int \frac{d^3p}{(2\pi)^3} f(p,x,t)}$$

$$\dot{\overline{Q}}(t) = \frac{1}{n} \int \frac{d^3p}{(2\pi)^3} \frac{\partial f}{\partial t} Q - \frac{1}{n} \overline{Q}(t) \int \frac{d^3p}{(2\pi)^3} \frac{\partial f}{\partial t}$$

$$\frac{\partial f}{\partial t} = -\frac{\vec{P}}{E} \vec{\nabla} f + I_{22} + I_{23} + I_{32}$$

$$< \frac{p_Z^2}{E^2} > (t) = < \frac{p_Z^2}{E^2} >_{eq} + \left(< \frac{p_Z^2}{E^2} > (t_0) - < \frac{p_Z^2}{E^2} >_{eq} \right) \exp\left(- \frac{t - t_0}{\theta} \right) \quad \Rightarrow \quad \dot{\overline{Q}}(t) = C_{drift} + C_{22} + C_{23} + C_{32}$$

θ(t) gives the timescale of kinetic equilibration.

$$\frac{\dot{\overline{Q}}(t)}{\overline{Q}_{eq} - \overline{Q}(t)} = \boxed{\frac{1}{\theta} = R_{drift}^{tr.} + R_{22}^{tr.} + R_{23}^{tr.} + R_{32}^{tr.},}$$

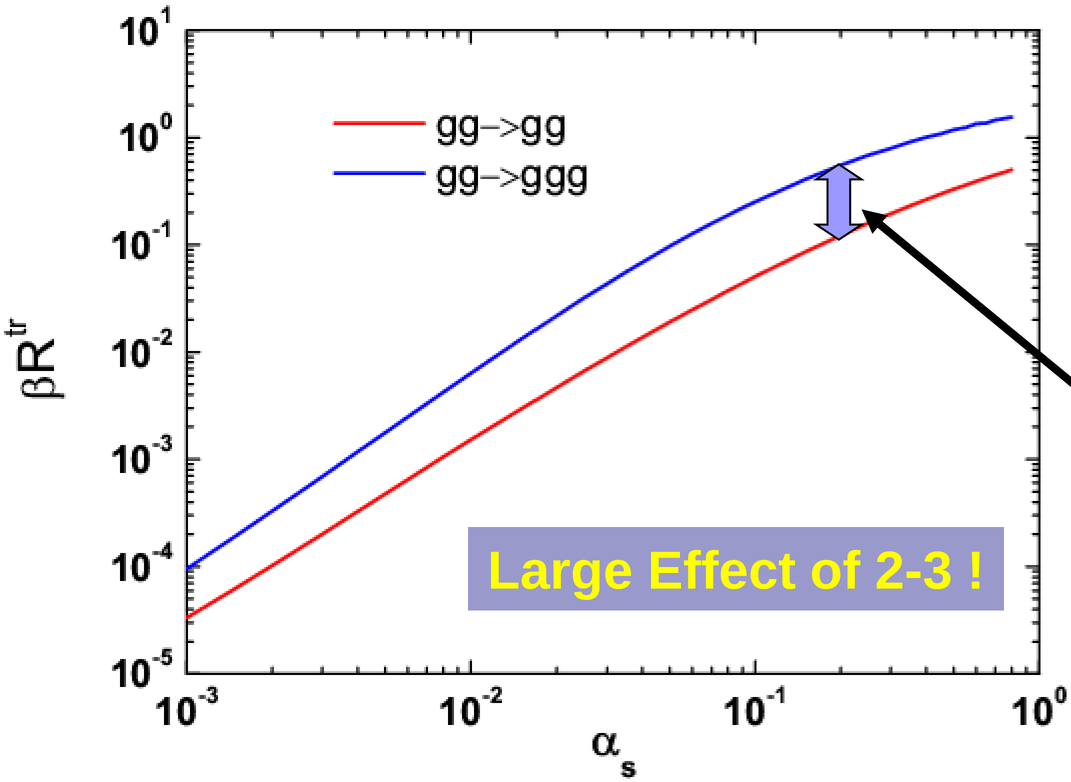
Transport Rates

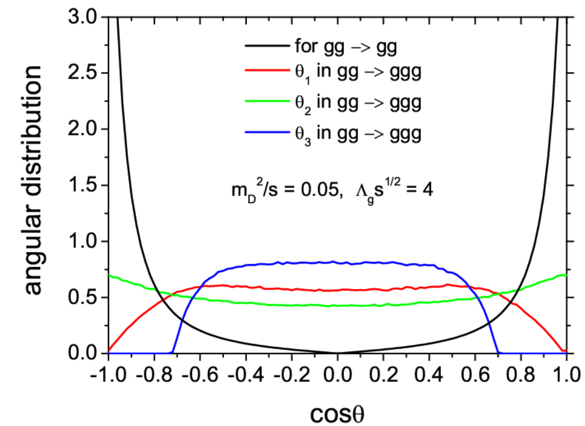
for $\alpha_s = 0.3$, T = 400 MeV

$\langle \sigma_{gg->gg} \rangle = 0.82\,\mathrm{mb}$

$\langle \sigma_{gg->ggg} \rangle = 0.57\,\mathrm{mb}$

$$\langle \sigma_{gg->gg} \rangle > \langle \sigma_{gg->ggg} \rangle$$





$$gg \to gg: \quad \beta R^{tr}_{22} \sim \alpha_s^2 (\ln \alpha_s)^2$$

$$gg \to ggg: \quad \beta R^{tr}_{23} \sim \alpha_s^2 (\ln \alpha_s)^2 \quad \text{for } \alpha_s > 0.01$$

$$\beta R^{tr}_{23} \sim \alpha_s^3 (\ln \alpha_s)^2 \quad \text{for } \alpha_s < 0.01$$

bottom-up scenario of thermalization

R.Baier, A.H.Mueller, D.Schiff and D.T.Son, PLB502(2001)51

- $Q_s^{-1} << t << \alpha^{-3/2} Q_s^{-1}$ Hard gluons with momenta about Q_s are freed and phase space occupation becomes of order 1.

- $\alpha^{-3/2} Q_s^{-1} << t << \alpha^{-5/2} Q_s^{-1}$

  (h+h –> h+h+s)

Hard gluons still outnumber soft ones, but soft gluons give most of the Debye screening.

- $\alpha^{-5/2} Q_s^{-1} << t << \alpha^{-13/5} Q_s^{-1}$

  (h+h –> h+h+s; s+s –> s+s; h+s –> sh+sh+s)

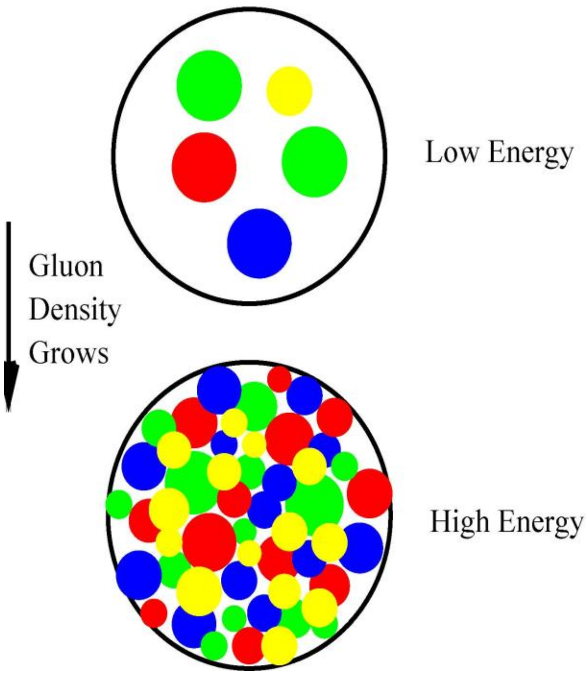
Soft gluons strongly outnumber hard gluons.
Hard gluons loose their entire energy to the thermal bath.

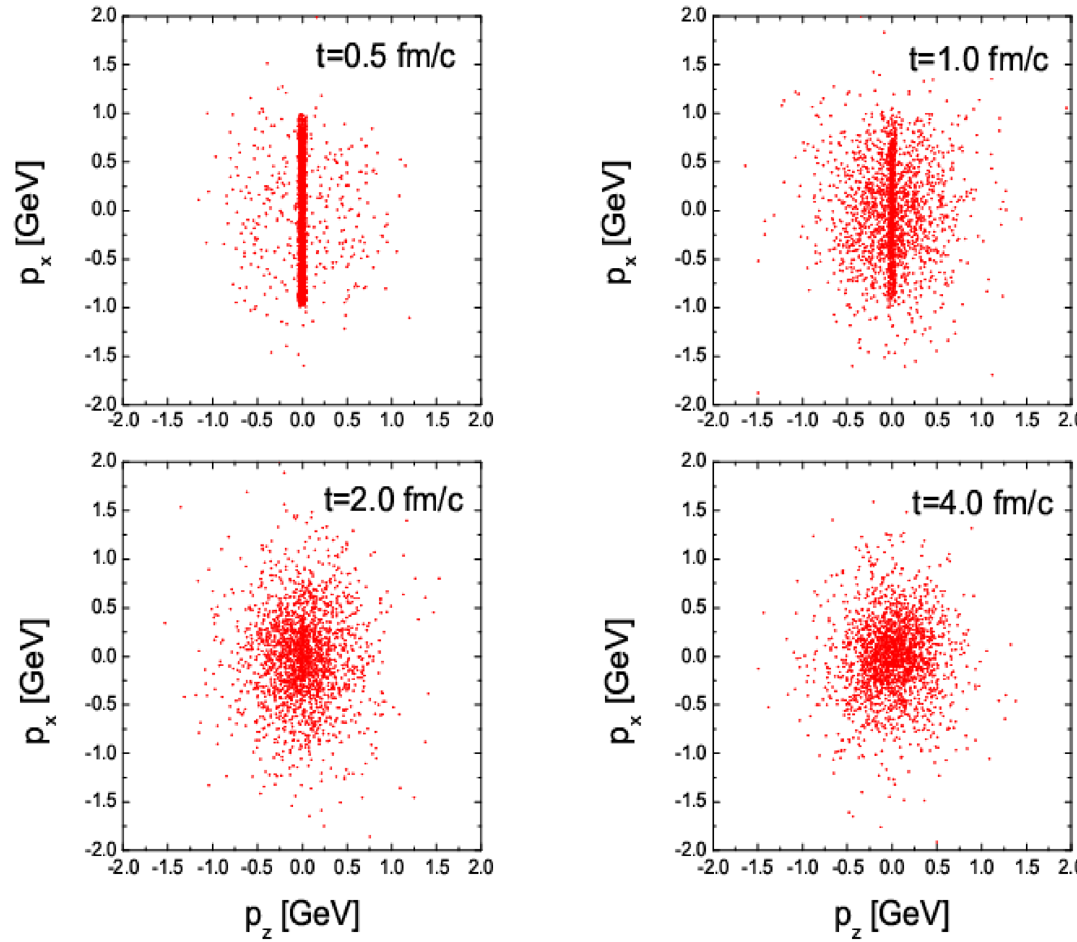
- After $\alpha^{-13/5} Q_s^{-1}$ the system is thermalized: $T \sim t^{-1/3}$, $T_0 \sim \alpha^{2/5} Q_s$

Initial condition with Color Glass Condensate

$$\boxed{f(x,p) \sim \frac{c}{\alpha_s N_c} \frac{1}{\tau_0} \delta(p_z) \theta(Q_s^2 - p_t^2)}$$

$$c = 0.4$$
$$\alpha_s = 0.3$$
$$\tau_0 = 0.4\mathrm{fm/c}$$
$$Q_s = 1\mathrm{GeV}$$



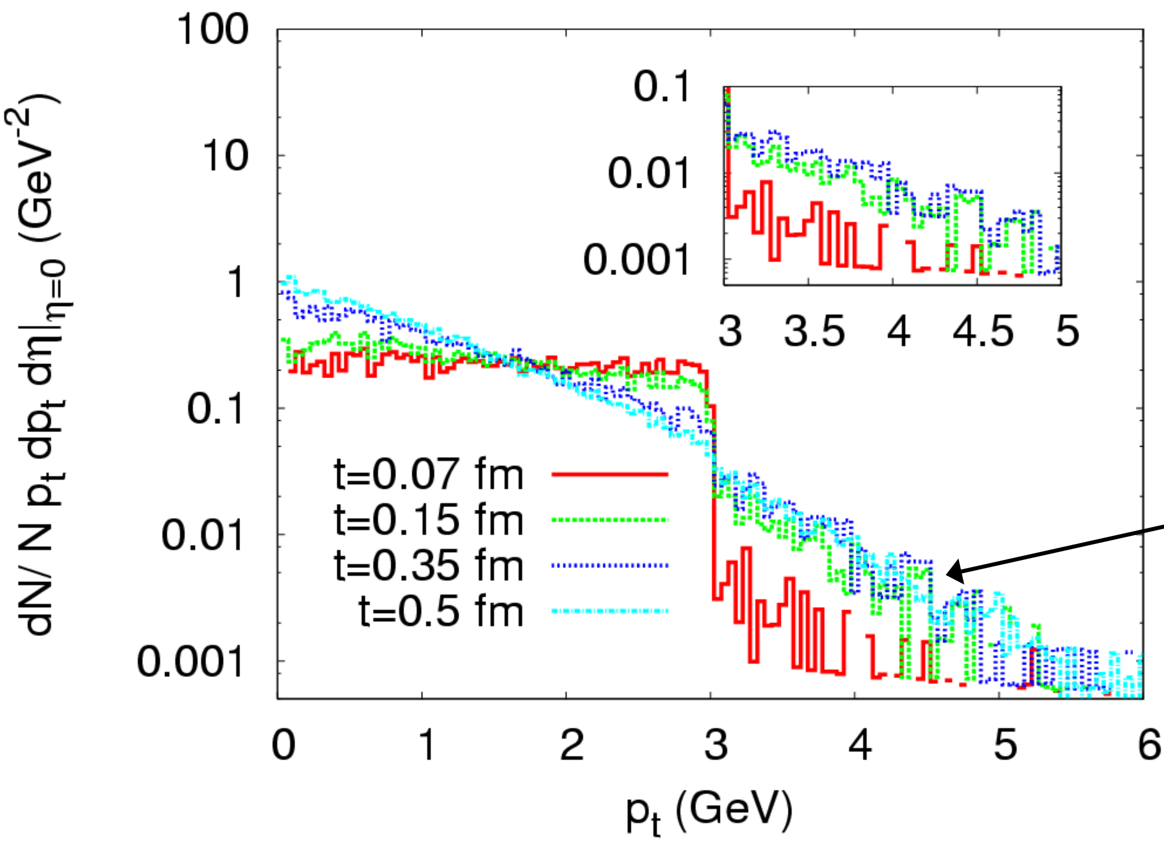


η: [-0.05:0.05] and $x_t < 1.5$ fm

p_T spectra

Initial conditions: Color Glass Condensate Q_s=3 GeV; coupling α_s=0.3

A.El, Z. Xu and CG, Nucl.Phys.A806:287,2008.



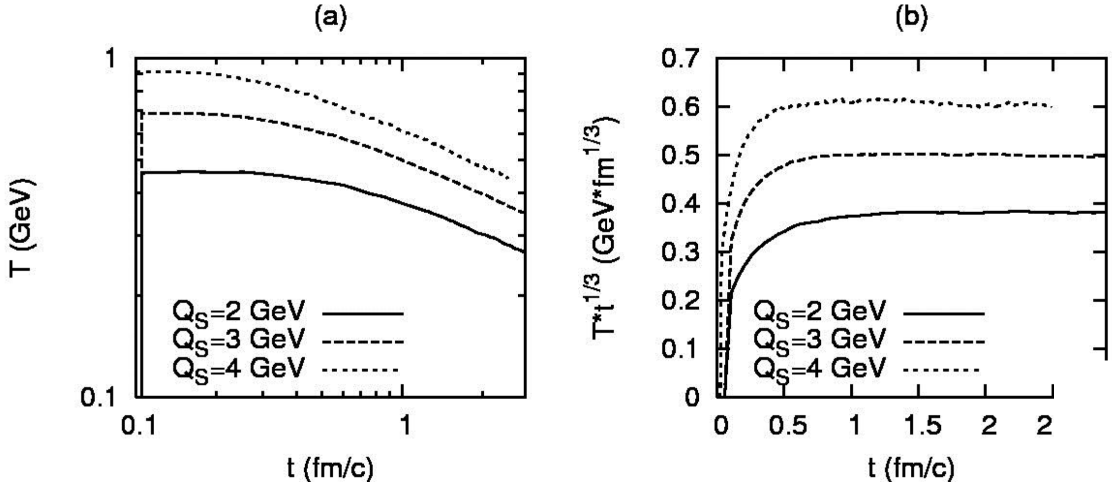
ggg $\longrightarrow$ gg !
This 3-2 is missing in the **Bottom-Up scenario** (Baier, Dohkshitzer, Mueller, Son (2001)).

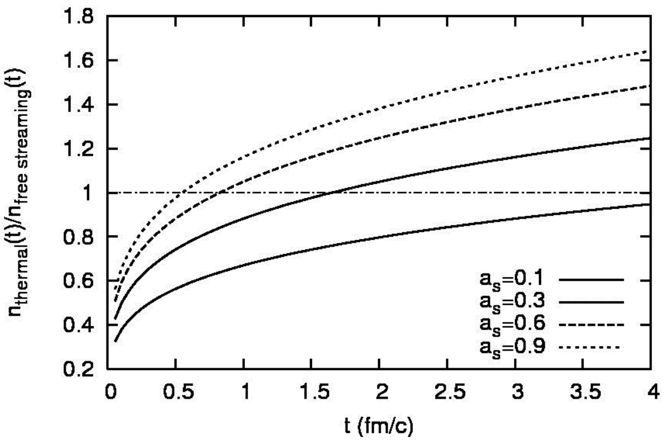
Bottom up is not working as advocated:
no tremendous soft gluon production,
soft modes do not thermalize before the hard modes

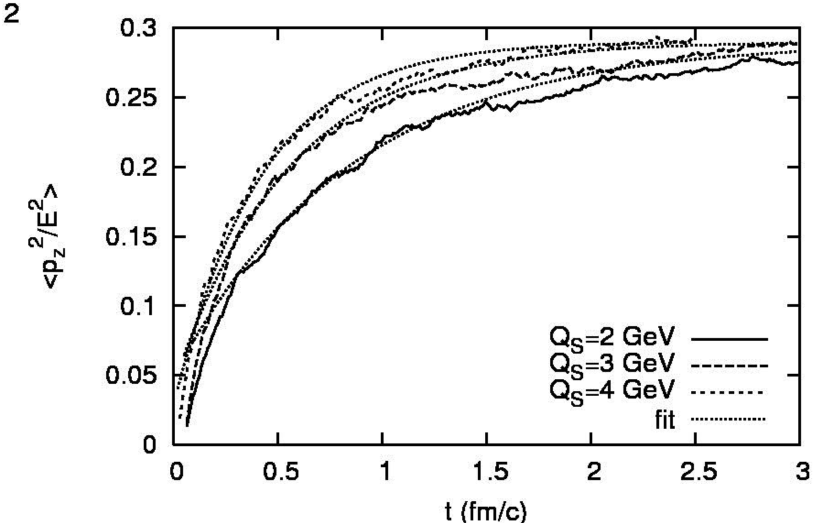
α_s=0.3

Not the full Bottom-Up story...

Andrej El





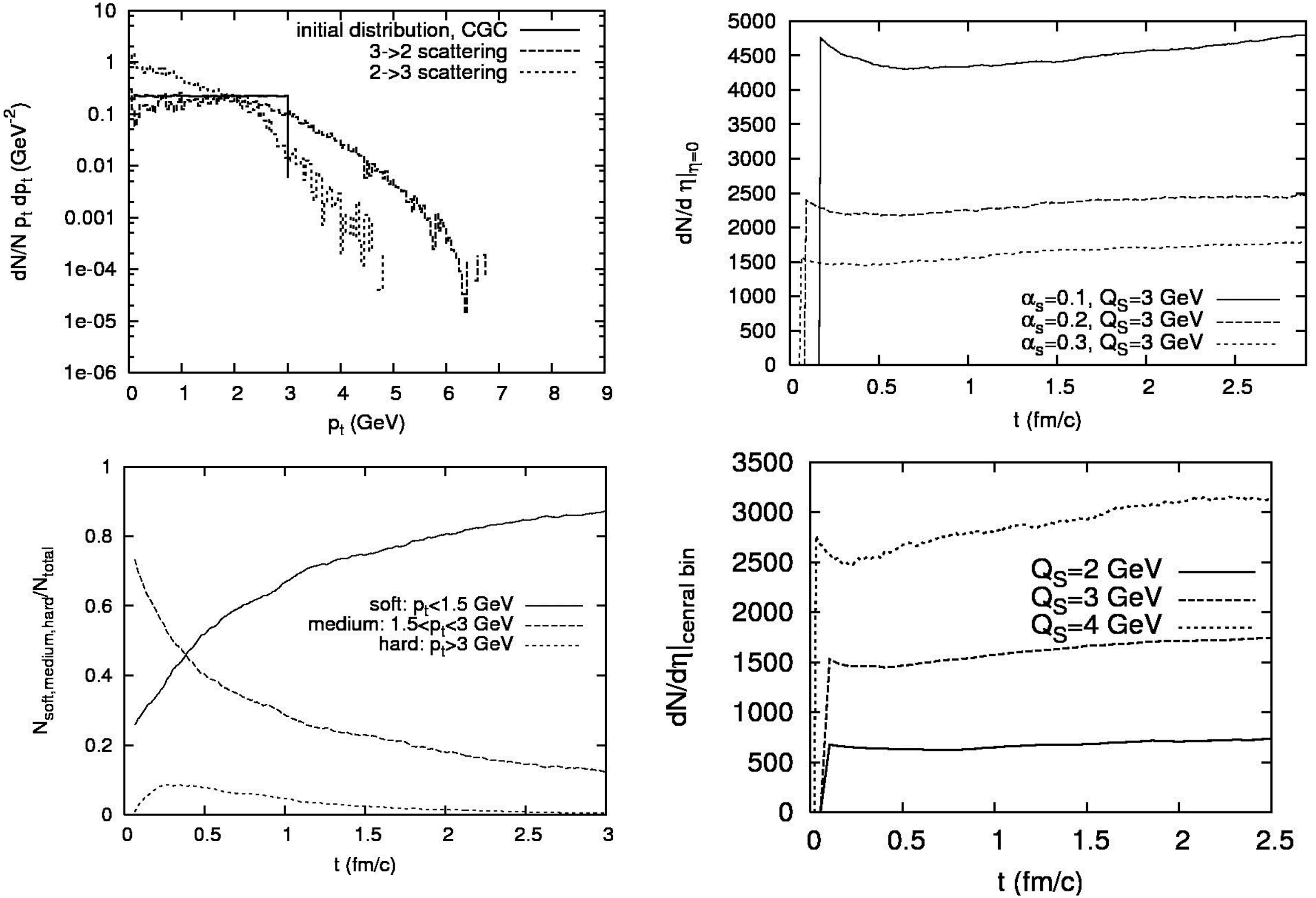


$$\tau_{eq} \sim \frac{1}{Q_s}$$

Evolution of temperature: $T_{initial}$= 2/9 Q_s

Temperature increases initially due to ggg->gg.

Evolution of Particle Number in bottom-up scenario

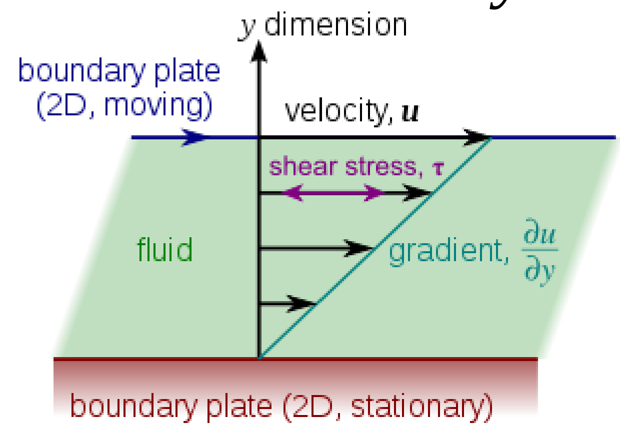


→ Particle number decreases in the very first moment

→ **No net soft gluon production at early times!**

Shear Viscosity η

$$\tau_{xy} \equiv -\eta \frac{\partial u_x}{\partial y}$$



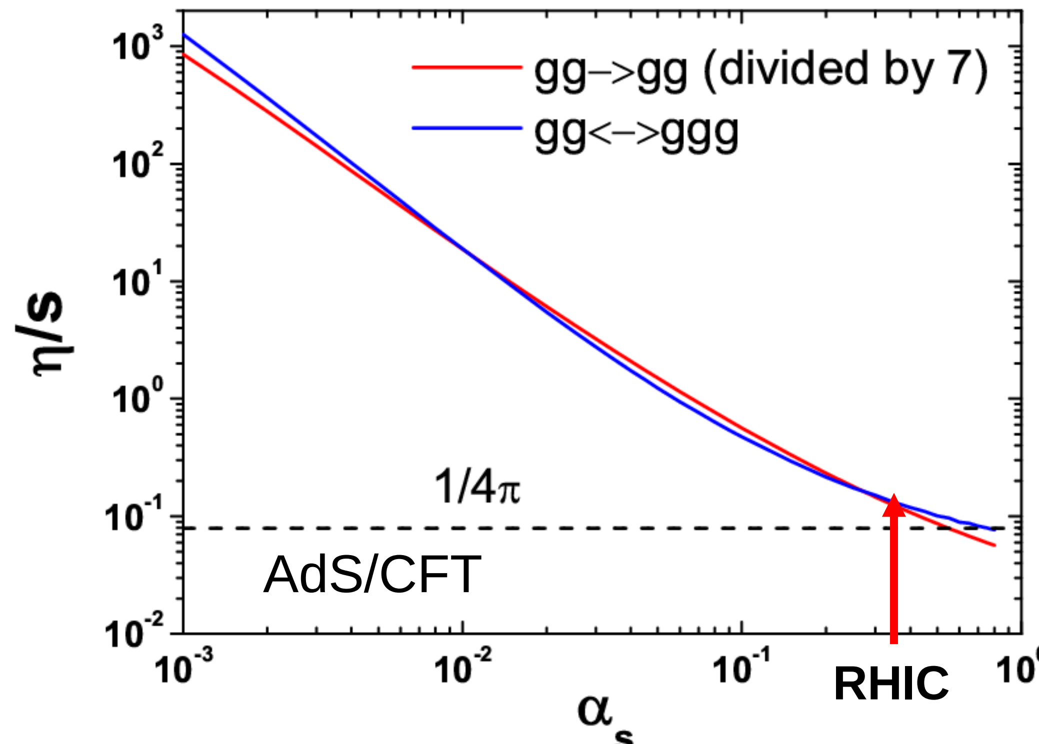
Navier-Stokes approximation $\eta = \dfrac{T_{xx} + T_{yy} - 2T_{zz}}{2(3\partial_z u_z - \vec{\nabla}\cdot\vec{u})}$

$$\eta \cong \frac{1}{5} n \frac{\left\langle E\left(\frac{1}{3} - \frac{p_z^2}{E^2}\right)\right\rangle}{\frac{1}{3} - \left\langle \frac{p_z^2}{E^2} \right\rangle} \frac{1}{\sum R^{tr} + \frac{3}{2} R_{23} - R_{32}}$$

relation: η <-> R^{tr}

Z. Xu and CG,

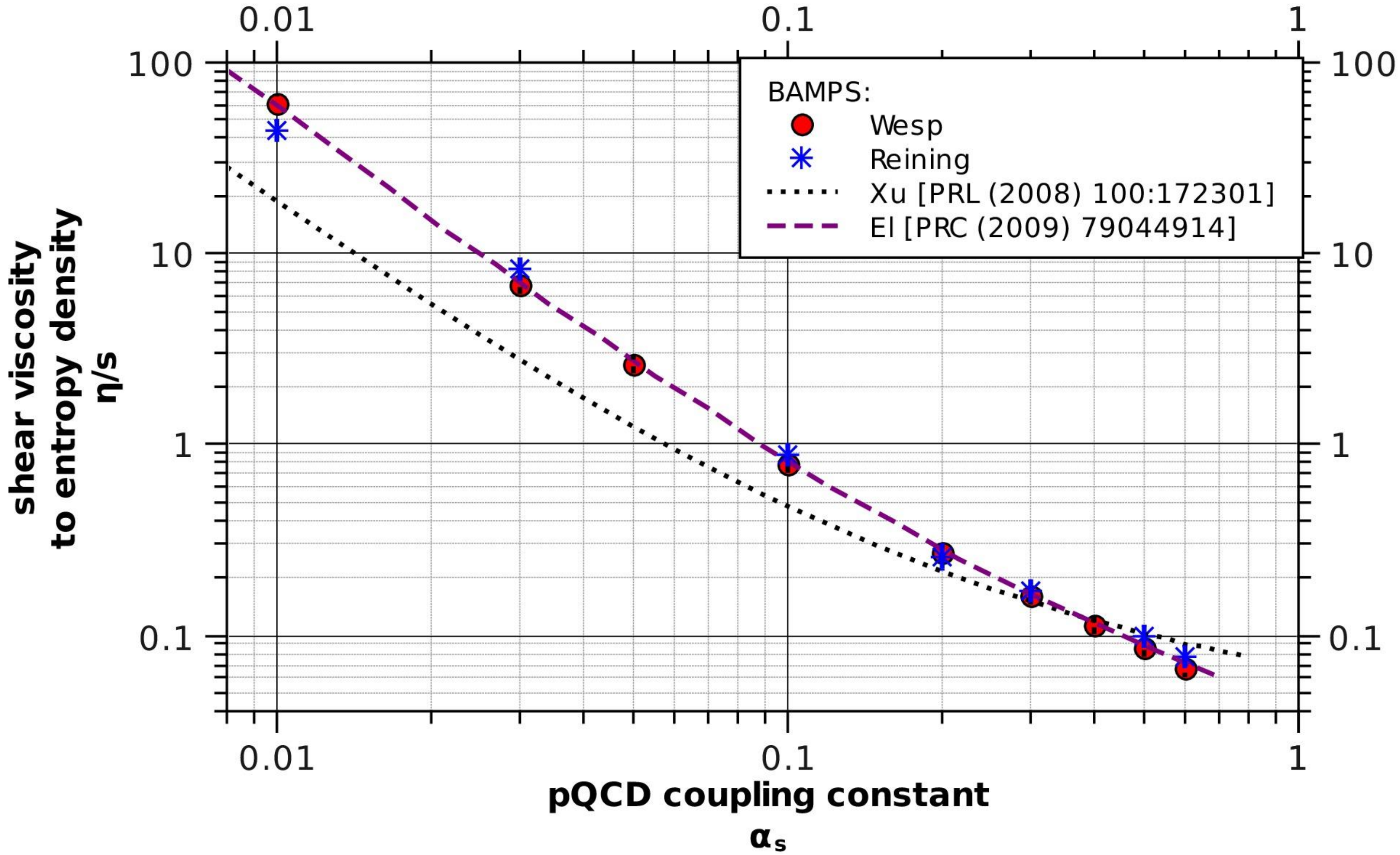
Phys.Rev.Lett.100:172301,2008.



numerical extraction of viscosity

C. Wesp

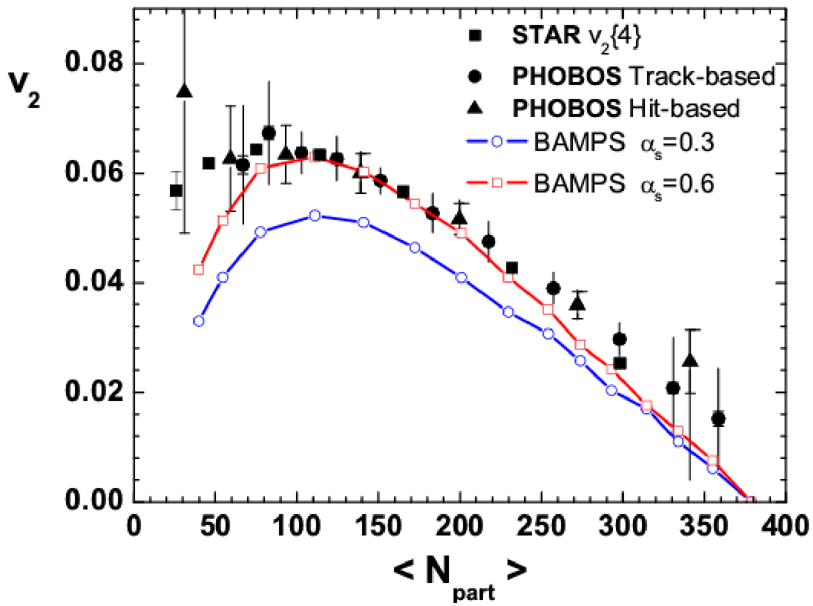
Green-Kubo relation: $$\eta = \frac{1}{10T}\int_0^\infty dt \int_V d^3r \left\langle \pi^{ij}(r,t)\pi^{ij}(0,0)\right\rangle$$

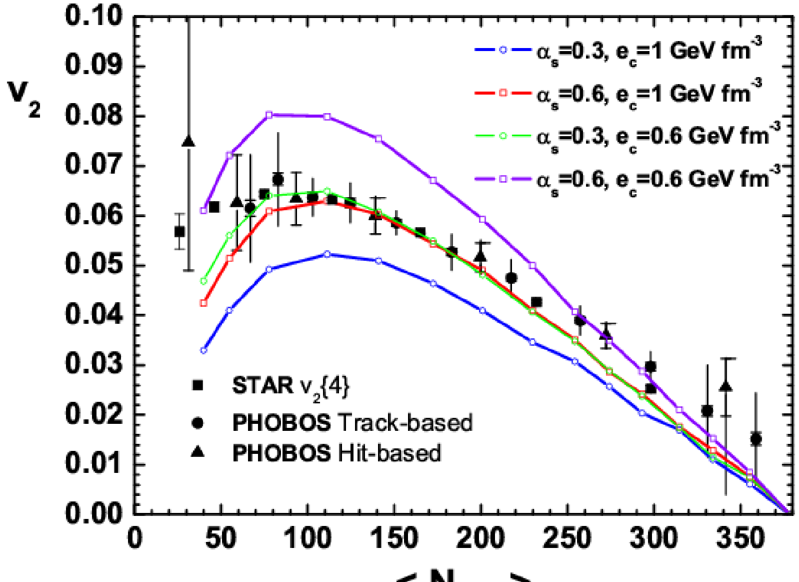


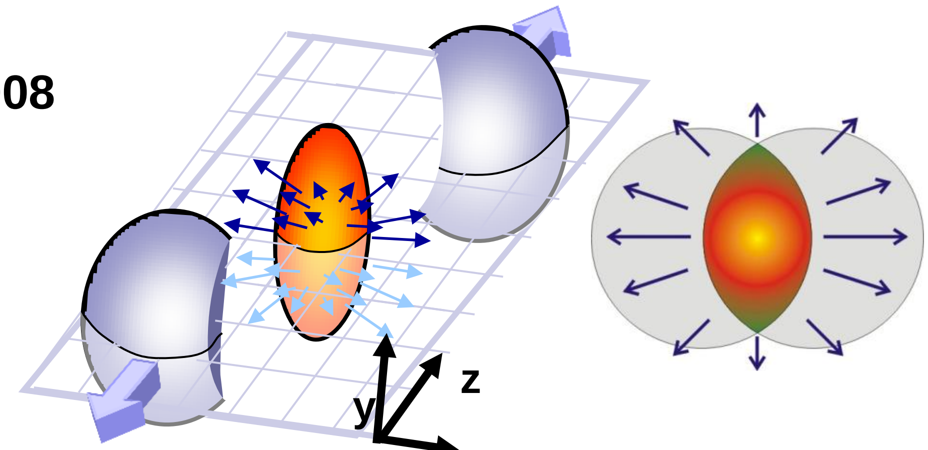
Elliptic Flow and Shear Viscosity in 2-3 at RHIC

2-3 Parton cascade BAMPS

Z. Xu, CG, H. Stöcker, PRL 101:082302,2008

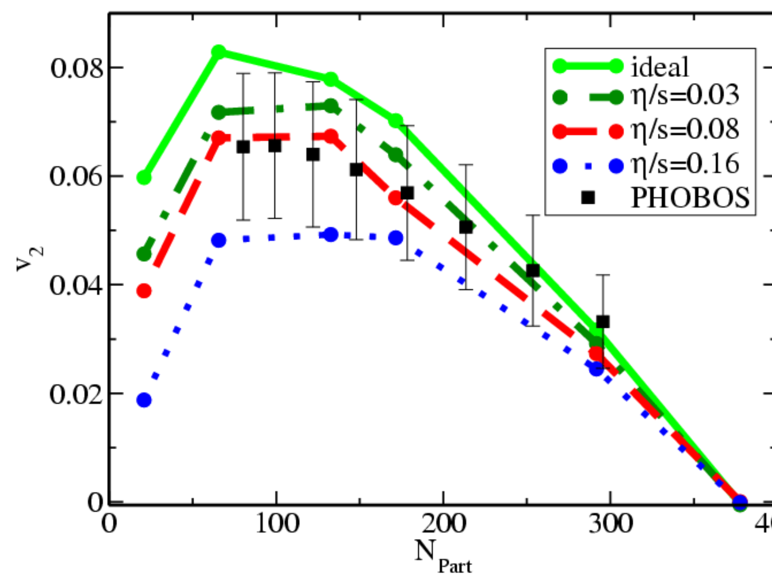






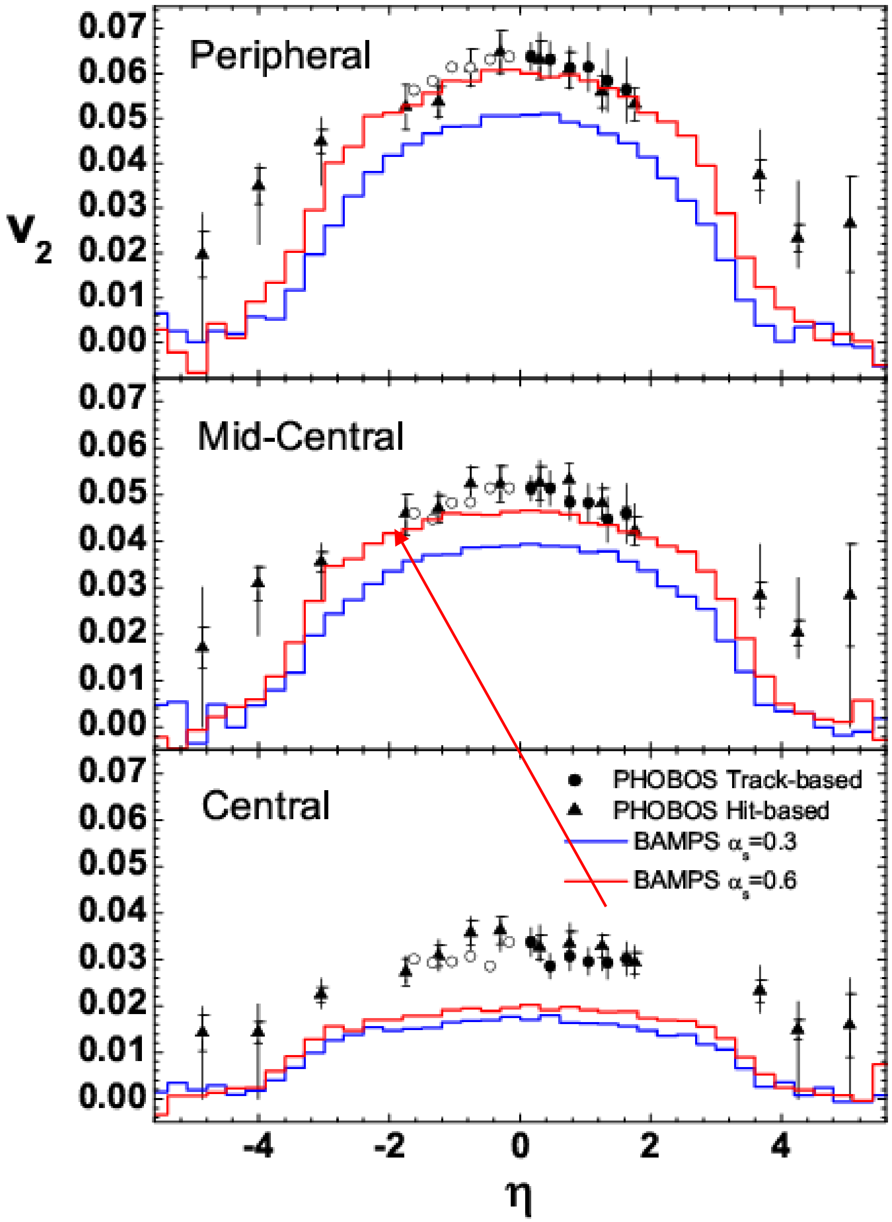
viscous hydro.

Romatschke, PRL 99, 172301,2007

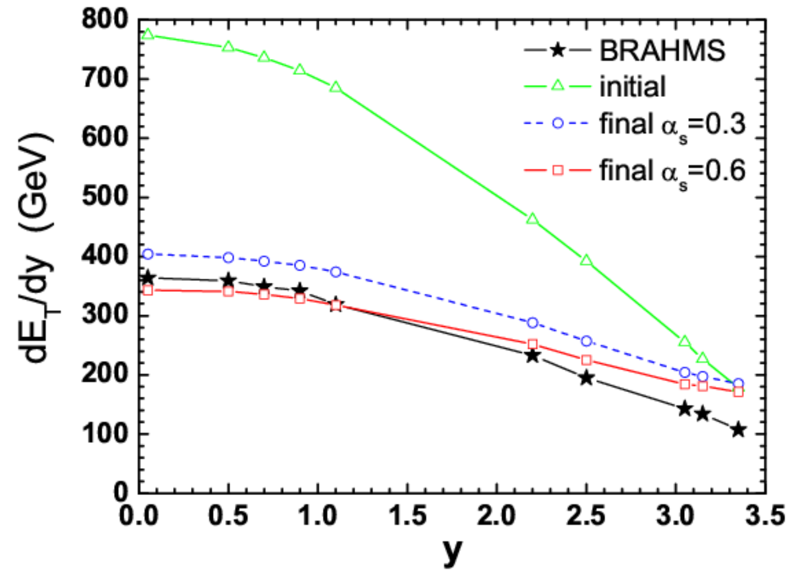


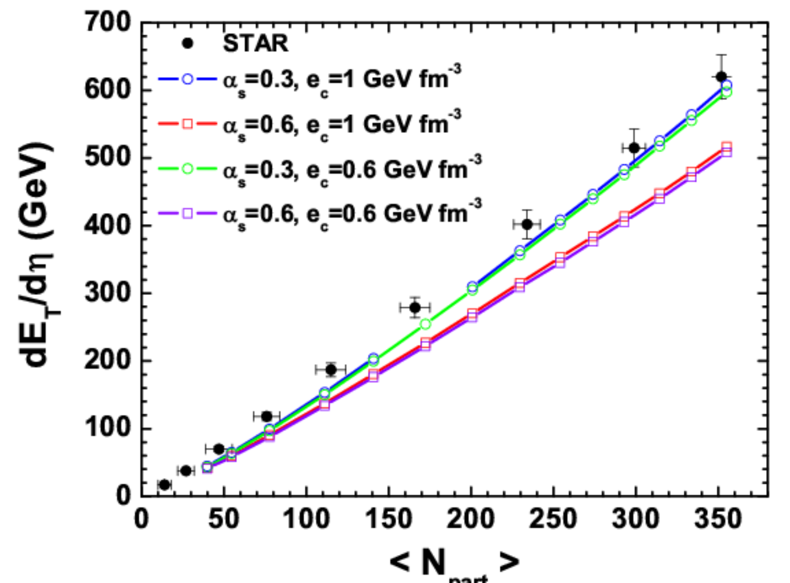
η/s at RHIC: 0.08-0.2

Rapidity Dependence of v_2: Importance of 2-3! BAMPS



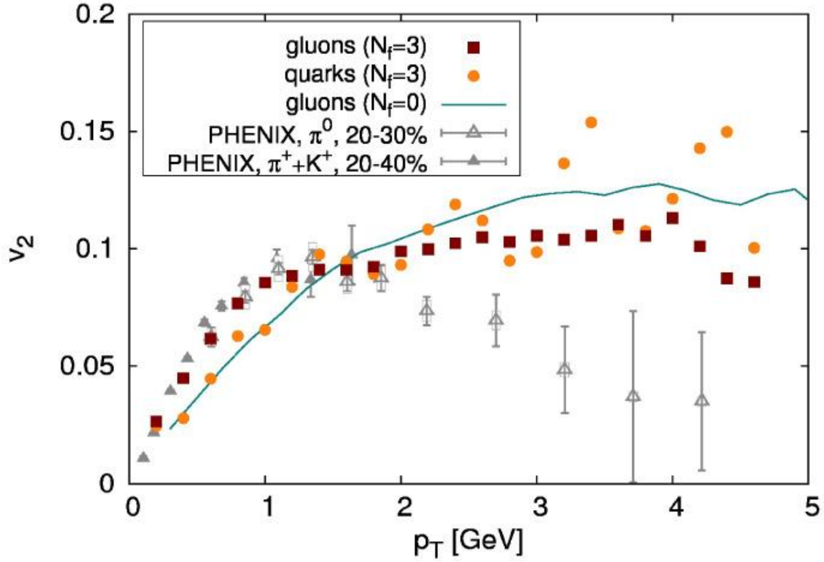
evolution of transverse energy



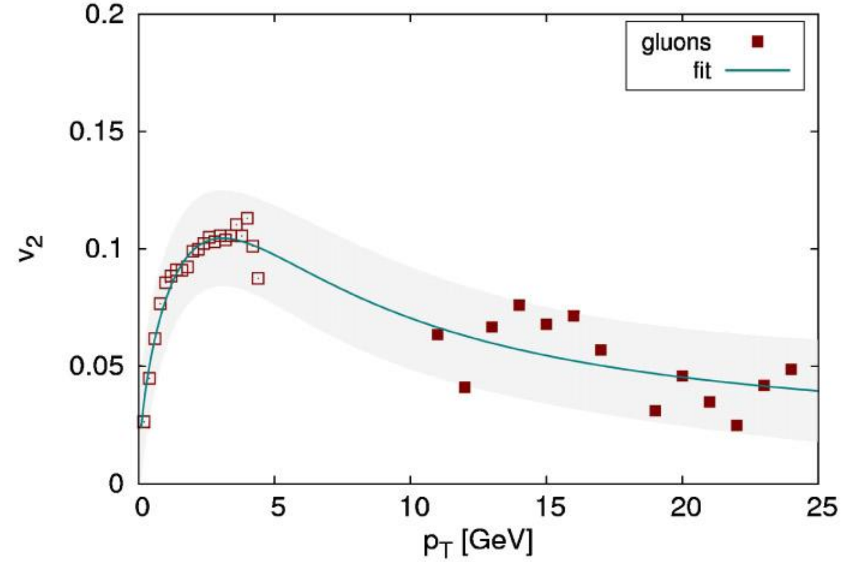


Elliptic flow at RHIC

Differential $v_2(p_T)$ at RHIC from BAMPS



High-p_T v_2 from BAMPS

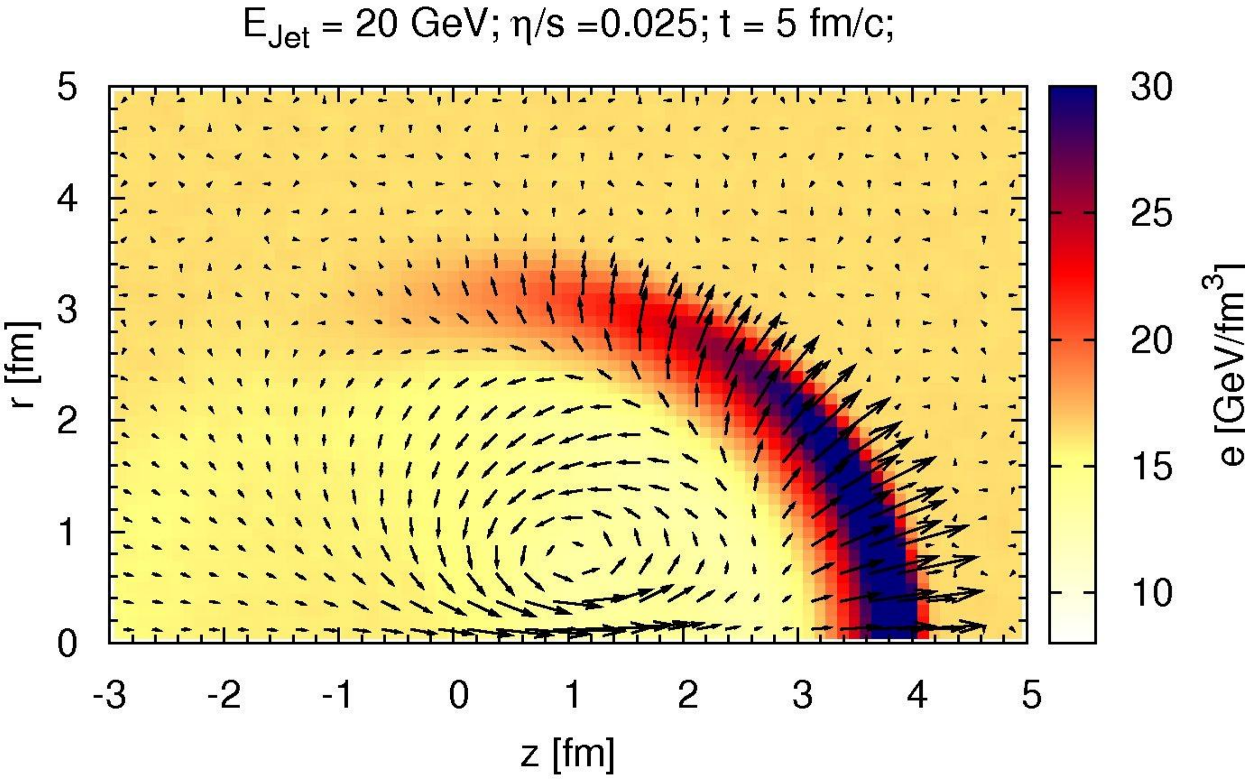


- Differential elliptic flow of gluons and quarks is (almost) the same
- NCQ scaling the experimental data, the magnitude of quark $v_2(p_T)$ is ok, but peak shifts $\Rightarrow$ hadronization mechanisms?
- Qualitative features of high-p_T v_2 agree with PHENIX π^0 data
  - fitted using $v_2(p_T) = \left(a + \frac{1}{p_T^n}\right) \frac{(p_T/\lambda)^m}{1+(p_T/\lambda)^m}$

Stoped Jet in BAMPS:

$E_{jet} = 20\,\mathrm{GeV}$

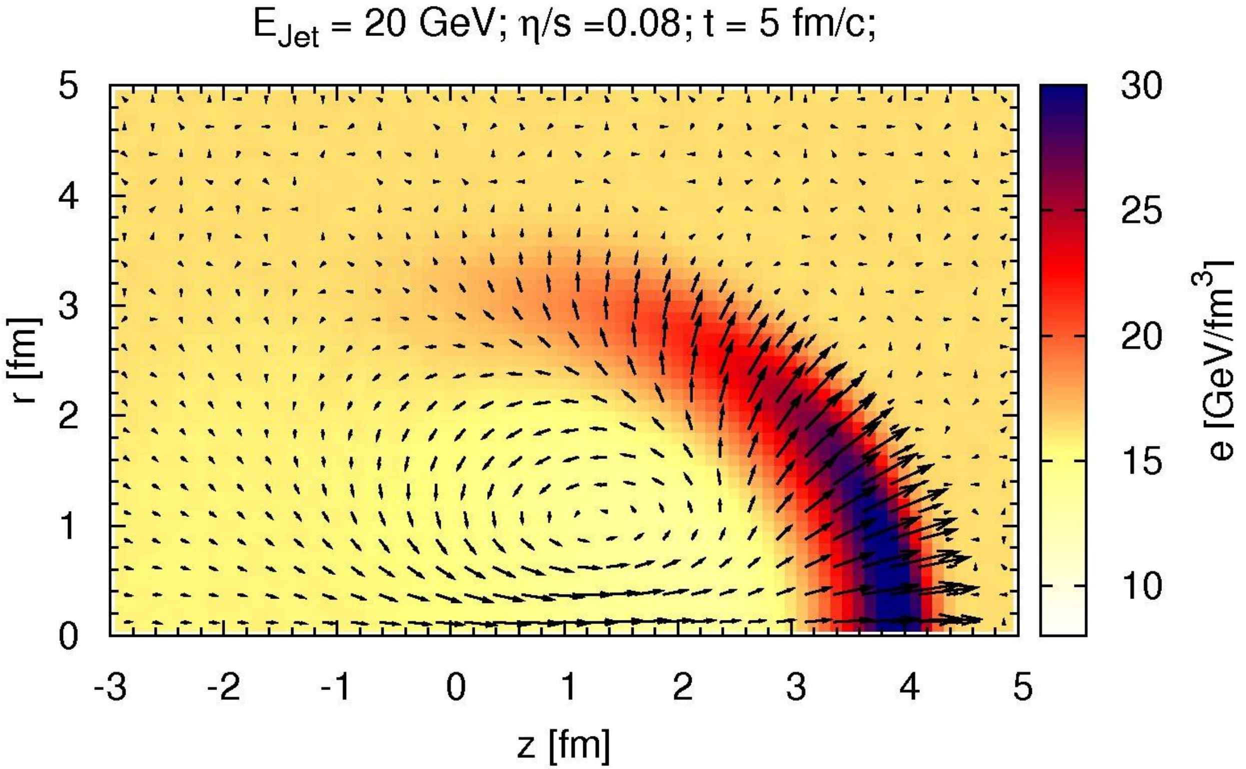
$\eta / s = 0.025$



Stoped Jet in BAMPS:

$$E_{jet} = 20\,\mathrm{GeV}$$

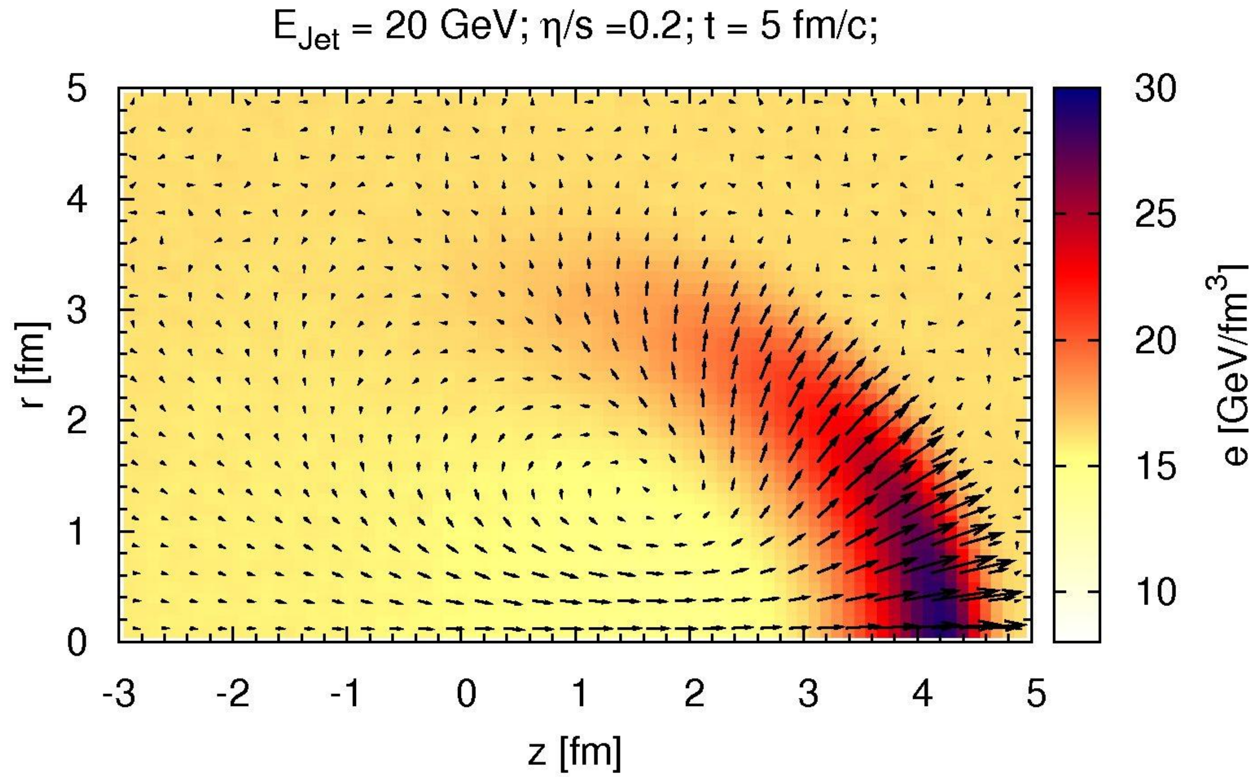
$$\eta / s = 0.08$$



Stoped Jet in BAMPS:

$E_{jet} = 20\,\mathrm{GeV}$

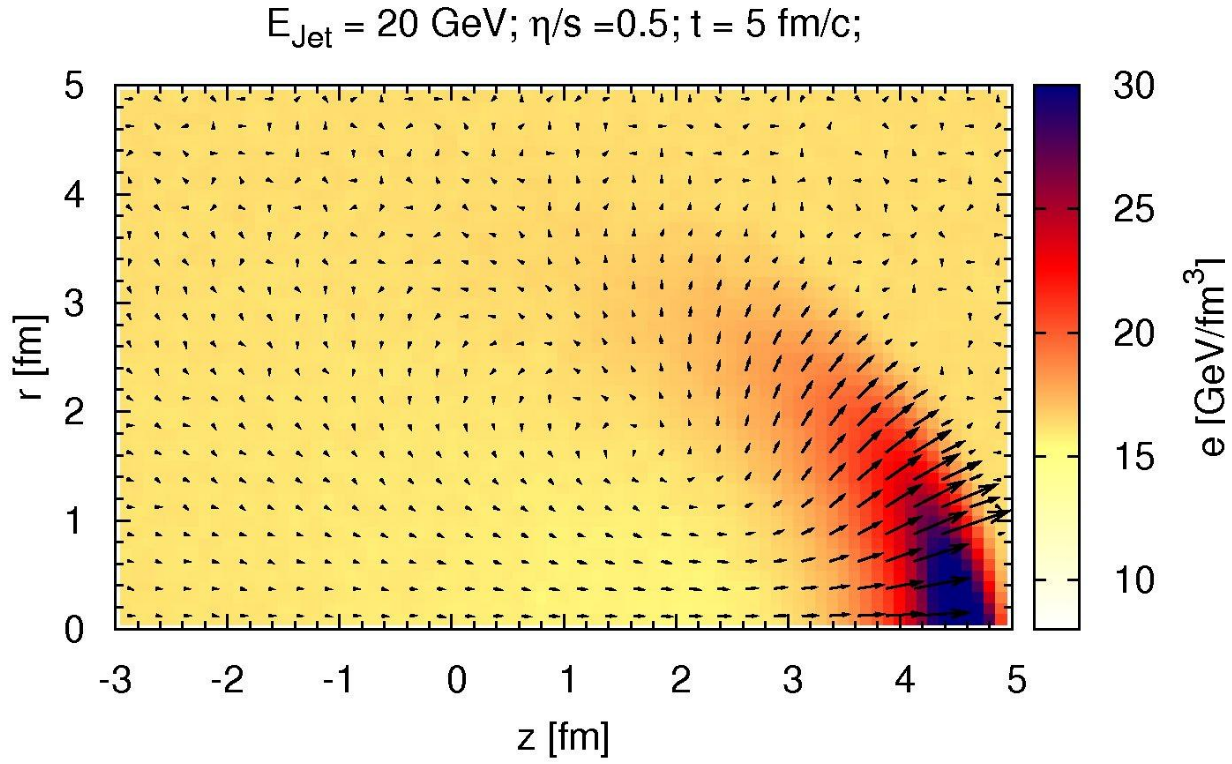
$\eta / s = 0.2$



Stoped Jet in BAMPS:

$$E_{jet} = 20\,\text{GeV}$$

$$\eta / s = 0.5$$



Hydro vs BAMPS

A. El, A. Muronga, Z. Xu, C. Greiner, *arXiv:1007.0705v1 ,in press NPA (2010)*

Bjorken scenario of boost-invariant 1D expansion

$$\dot{\pi} = -\frac{\pi}{\tau_\pi} + \frac{8}{27}\frac{e}{\tau} - \frac{1}{2}\frac{\pi}{\tau} + \frac{1}{2}\pi\frac{\dot{T}}{T} + \frac{1}{2}\pi\frac{\dot{e}}{e} + \frac{3}{2}\frac{\pi^2}{e\tau} + \frac{3}{2}\frac{\pi^2}{e}\frac{\dot{T}}{T} + \frac{3}{2}\frac{\pi^2}{e}\frac{\dot{e}}{e} - 4\frac{\pi^2}{e\tau}$$

Second-order (Israel-Stewart)

Third-order
(*AE, Z. Xu, C.Greiner PRC81:041901,2010*)

$$\dot{e} = -\frac{4}{3}\frac{e}{\tau} + \frac{\pi}{\tau}$$

$$\dot{n} + \frac{n}{\tau} = \int C[f]d\Gamma = R_{23} - R_{32} = \frac{1}{2}n^2(1-\lambda)\sigma_{23}$$

$$\lambda = n/n_{eq} = n/d_g T^3$$

one-component system with particle emission and absorption

Hydro vs BAMPS in 1D

$$\frac{p_L}{p_T} = \frac{p - \pi}{p + \pi/2}$$

A. El, Z. Xu, C. Greiner, PRC 81 (2010) 041901

$$\dot{\pi} = -\frac{\pi}{\tau_\pi} - \frac{4}{3}\frac{\pi}{\tau} + \frac{8}{27}\frac{e}{\tau} - x\frac{\pi^2}{e\tau}$$

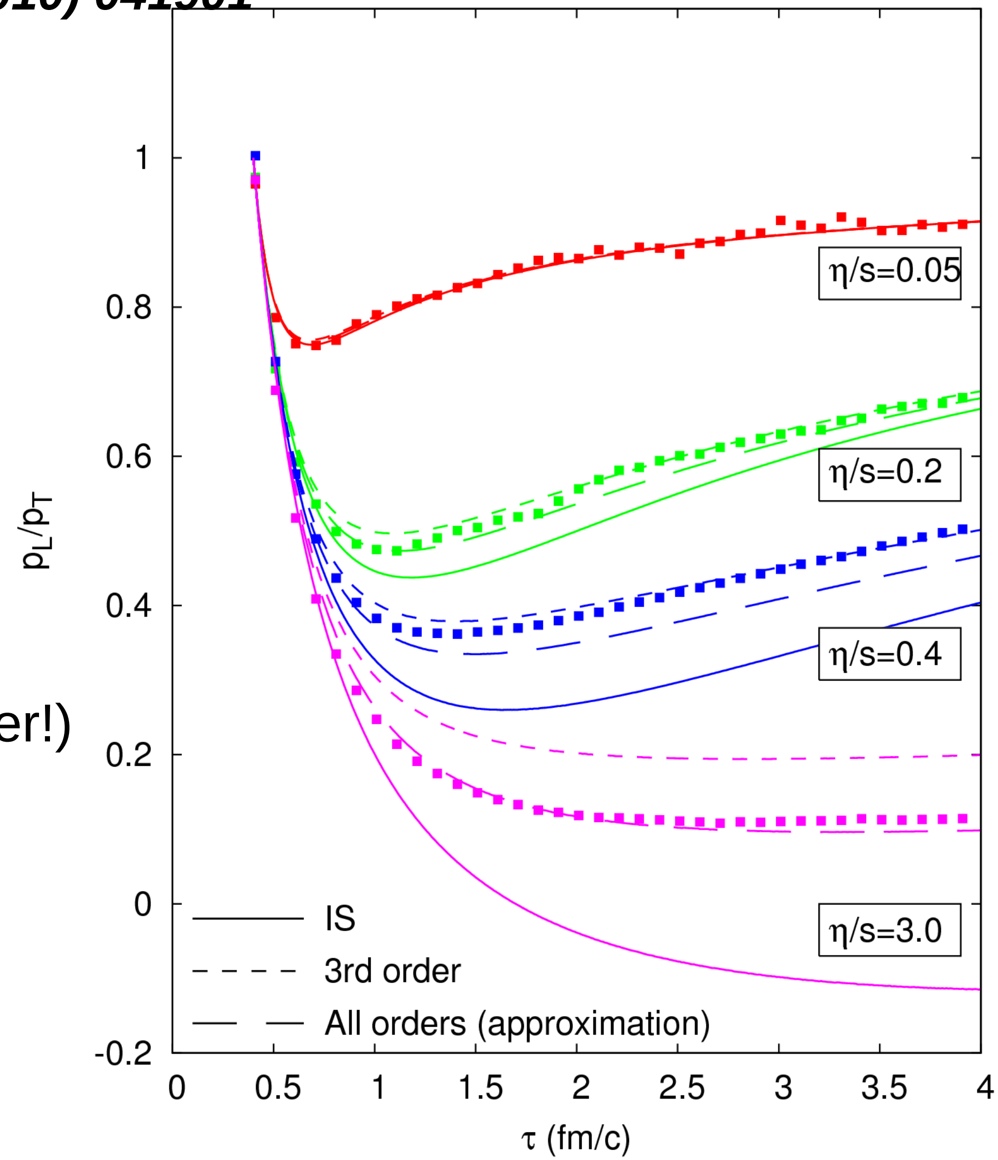
x=0: Israel-Stewart

x=3: third-order rel. diss. hydro

x=5/3: approximative 'all-orders' Eq.

> Resummation works at strong dissipation (large Knudsen number!)

> Inclusion of third order terms reduces deviations, esp. at late times.



Dissipative effects on pT spectra

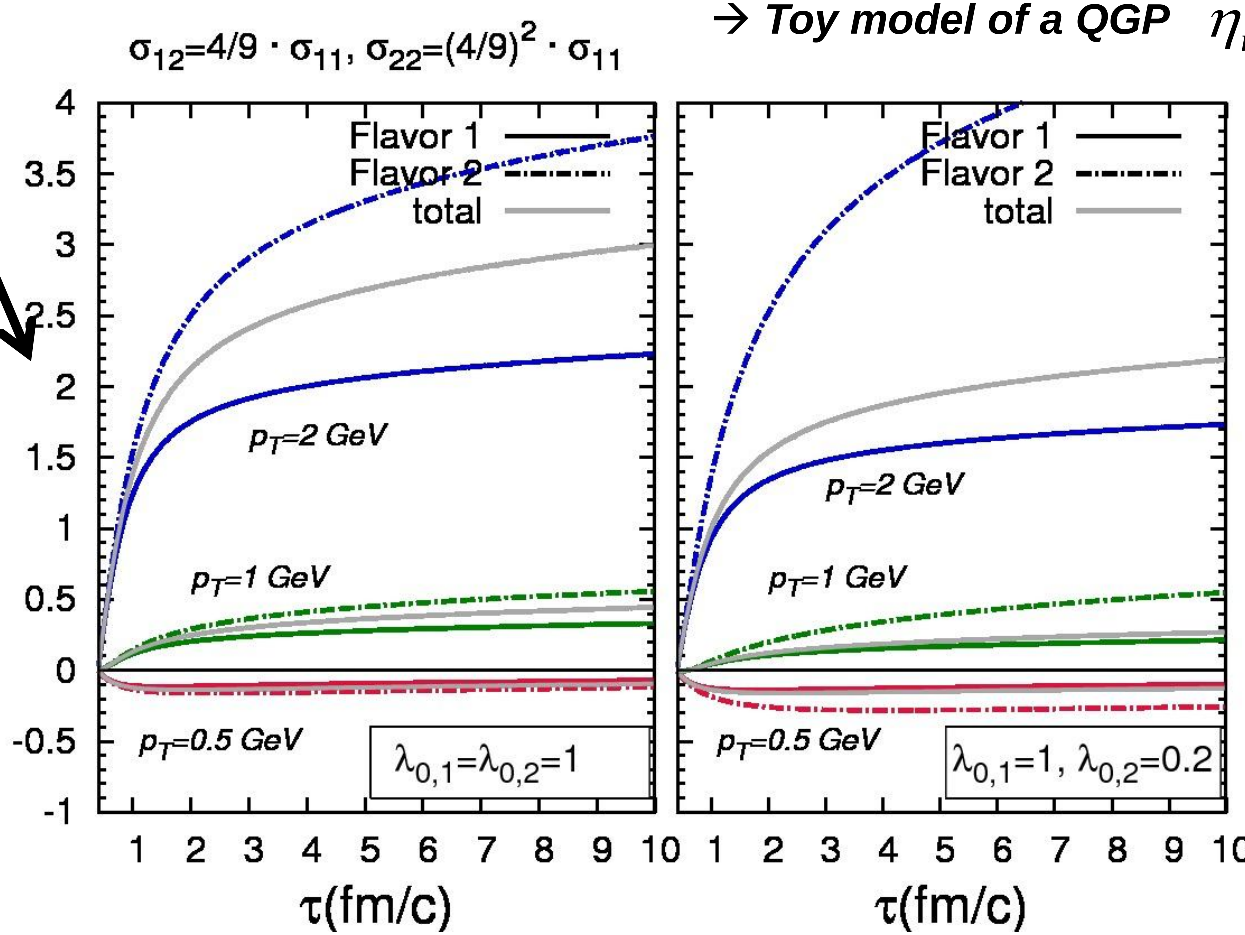
$$\Delta_{diss}(p_T) \equiv \frac{\delta N_i - \delta N_{i,EQ}}{\delta N_{i,EQ}}$$

$$\sigma_{11} = \frac{0.9}{T^2}, \sigma_{12} = \frac{4}{9}\sigma_{11}, \sigma_{22} = \left(\frac{4}{9}\right)^2 \sigma_{11}$$

1=Gluons, 2=Quarks+Antiquarks (3 Flavours)
4/9=Casimir factors

→ ***Toy model of a QGP*** $\eta_{mixture} / s \approx 0.2$

RHIC initial conditions (gauged to meet dN/dy and dET/dy)



Summary

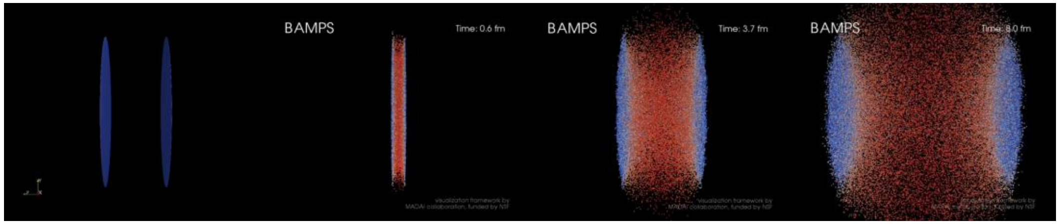
Inelastic/radiative pQCD interactions (23 + 32) explain:

fast thermalization

large collective flow

small shear viscosity of QCD matter at RHIC

semirealistic jet-quenching of gluons



Future/ongoing analysis and developments:

light and heavy quarks

jet-quenching (Mach Cones, ridge, fluctuations)

hadronisation and afterburning (UrQMD) needed to determine

how imperfect the QGP at RHIC and LHC can be

… and dependence on initial conditions

dissipative hydrodynamics

BAMPS Group

Zhe Xu and Carsten Greiner:
Heads and Supervisors + Administrator

Oliver Fochler:
Investigation of Jet-Quenching, Code maintanance of BAMPS,

Andrej El:
Comparison of viscous hydro to BAMPS

Jan Uphoff:
Including heavy quarks

Felix Reining and **Christian Wesp:**
Extracting shear viscosity from BAMPS using different Ansatz

Ioannis Bouras:
Investigation of shocks and Mach Cones in BAMPS,
Comparison to viscous hydro

F. Lauciello, B. Linnik, A. Meistrenko and **F. Senzel:**
Master students

Initial conditions

Gluons:

- **PYTHIA**
  scaling to heavy-ion collisions with Glauber model (considering shadowing) and energy conservation

- **Minijets**
  (low p_T cut-off at 1.4 GeV)

- **Color glass condensate**

Heavy quarks:

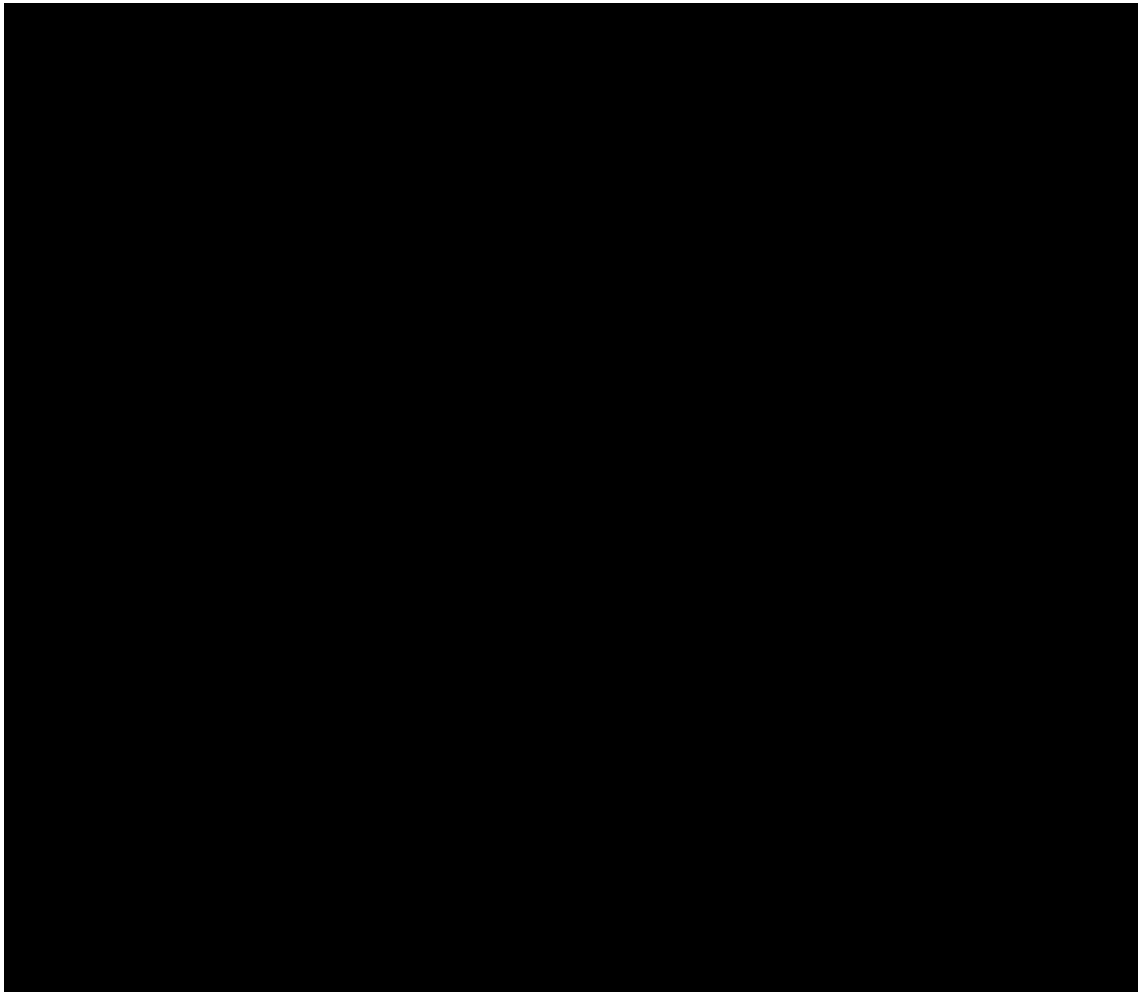
- **PYTHIA**
  Monte Carlo Event Generator for nucleon-nucleon collisions

- **NLO pQCD**
  Distributions from R. Vogt

- **MC@NLO**
  Next-to-leading order matrix elements

Heavy-ion collision at LHC

BAMPS simulation of QGP phase at LHC at s_{NN} = 2.76 TeV



Visualization framework courtesy MADAI collaboration, funded by the NSF under grant# NSF-PHY-09-41373

Weak or strong ….

Validity of kinetic transport - relation to shear viscosity

$$\frac{\eta}{s} = \frac{4}{15} T \lambda_{\mathrm{mfp}}$$

Semiclassical kinetic theory?

$$\lambda_{\mathrm{mfp}} \gtrsim d_{\mathrm{gluon}}$$

$$d_{\mathrm{gluon}} = n^{-1/3} = \pi^{2/3}/((16)^{1/3} T)$$

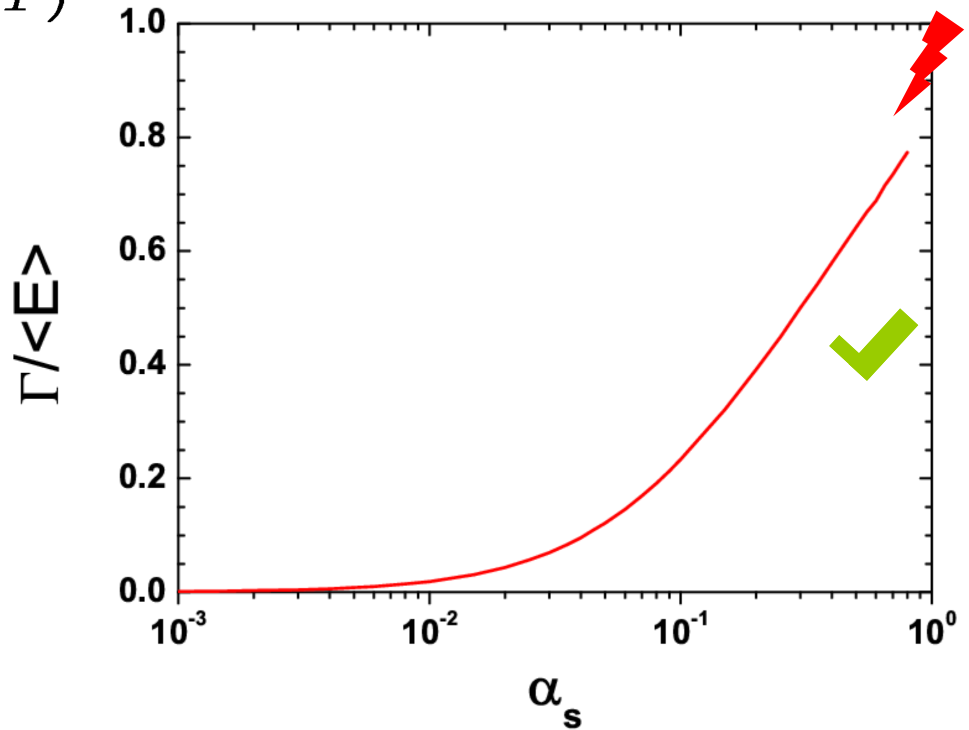
$$\Rightarrow \frac{\eta}{s} \gtrsim 0.22$$

Quantum mechanics: quasiparticles?

$$\Gamma \lesssim E \sim (3)T$$

$$\Gamma \sim \lambda_{\mathrm{mfp}}^{-1} \lesssim (3)T$$

$$\Rightarrow \frac{\eta}{s} \gtrsim \frac{4}{15} \cdot \frac{1}{(3)} \approx \frac{1}{4\pi}$$



Important scales for kinetic transport & simulations

particle density: $n(t) = (d)^{-3} \sim T^3$

cross section: $\sigma_{\mathrm{tot}} \; (\sigma_{\mathrm{transport}})$

mean free path: $\lambda_{\mathrm{mfp}} = 1/(n\,\sigma_{\mathrm{tot}})$

Simulations solve Boltzmann equation: $\lambda_{\mathrm{mfp}} \lesssim \sqrt{\dfrac{\sigma_{\mathrm{tot}}}{\pi}}$

$\rightarrow$ test particles and other schemes

Semiclassical kinetic theory: $\boxed{\lambda_{\mathrm{mfp}} \gtrsim d}$

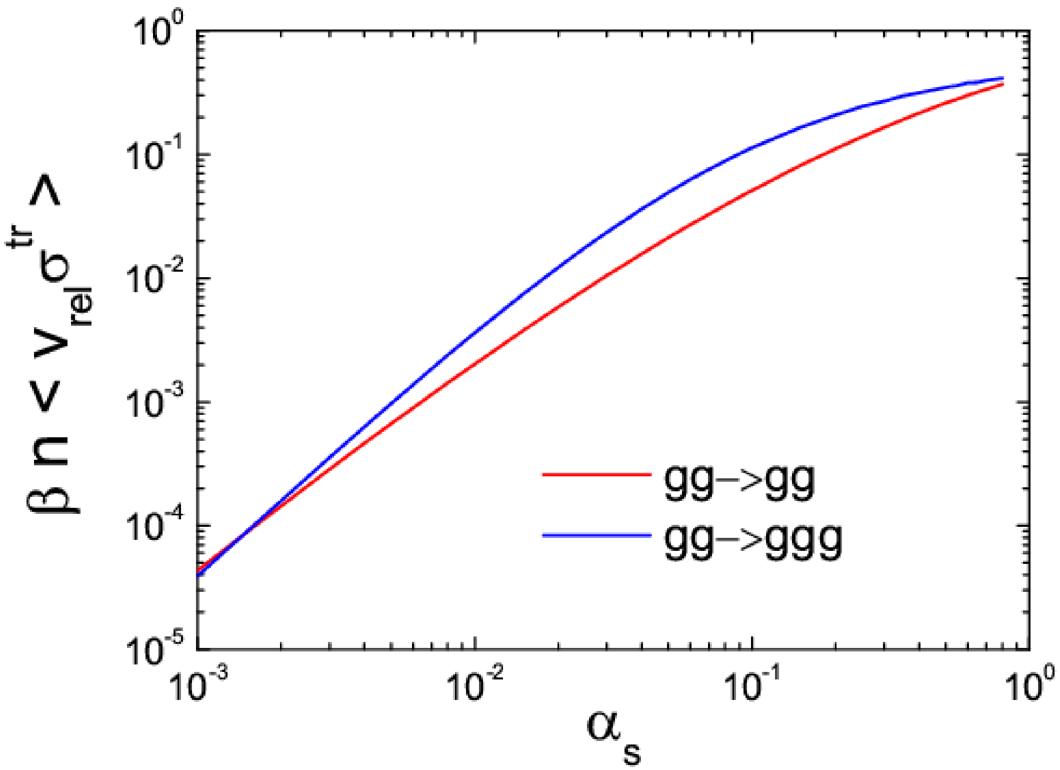
$$\sigma_{\mathrm{tot}}^{\mathrm{eff}} = 45\,\mathrm{mb} = \mathrm{const.}$$

$$\left.\begin{array}{r} \Rightarrow \lambda_{\mathrm{mfp}} = 1/(n\,\sigma_{\mathrm{tot}}^{\mathrm{eff}}) \sim 1/n \\ d = n^{-1/3} \end{array}\right\} \xrightarrow{n \to \infty} \boxed{\lambda_{\mathrm{mfp}} \ll d}$$

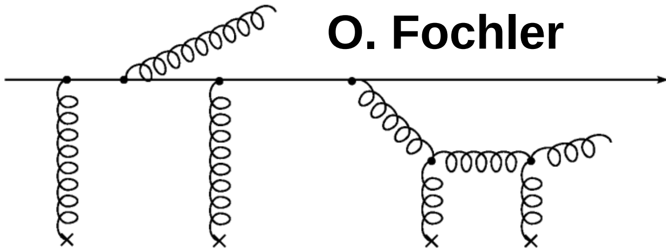
(Quantum mechanics: $\Gamma \simeq n\,\sigma_{\mathrm{tot}}\,\bar{v} \leftrightarrow \bar{E} \sim 3T \Rightarrow \Gamma \gg \bar{E}$)

$$\sigma_{\mathrm{pQCD}} \sim (\alpha_s)^i \log()1/T^2 \Rightarrow \boxed{\lambda_{\mathrm{mfp}} \gtrsim d}$$

transportcrosssection $\sigma^{tr} = \int d\theta \frac{d\sigma}{d\theta} \sin^2\theta$



$\left\langle \sigma^{tr}_{gg->ggg} \right\rangle \geq \left\langle \sigma^{tr}_{gg->gg} \right\rangle$ BUT, this is **not** the full story !



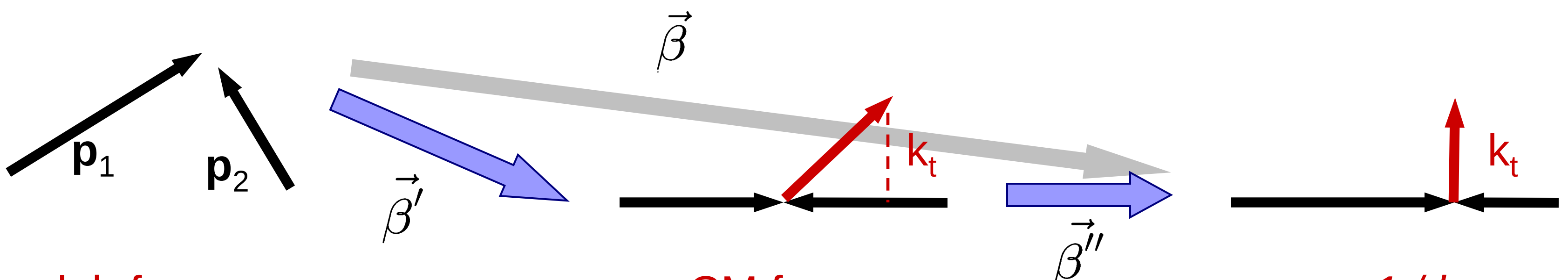
O. Fochler

LPM-effect

- transport model: incoherent treatment of gg→ggg processes

➔ parent gluon must not scatter during formation time of emitted gluon
  - discard all possible interference effects (Bethe-Heitler regime)

$$\boxed{|M_{gg\to ggg}|^2 \to |M_{gg\to ggg}|^2\,\Theta\,(\lambda - \tau)}$$



lab frame
$\lambda = 1/ < n\sigma >$

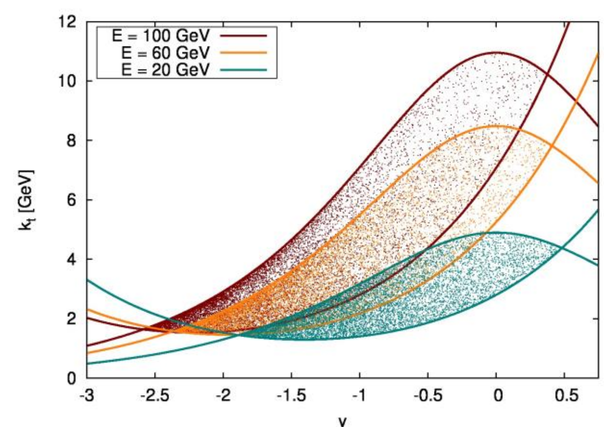
CM frame

$\beta'' = \tanh y$

$\tau = 1 / k_t$

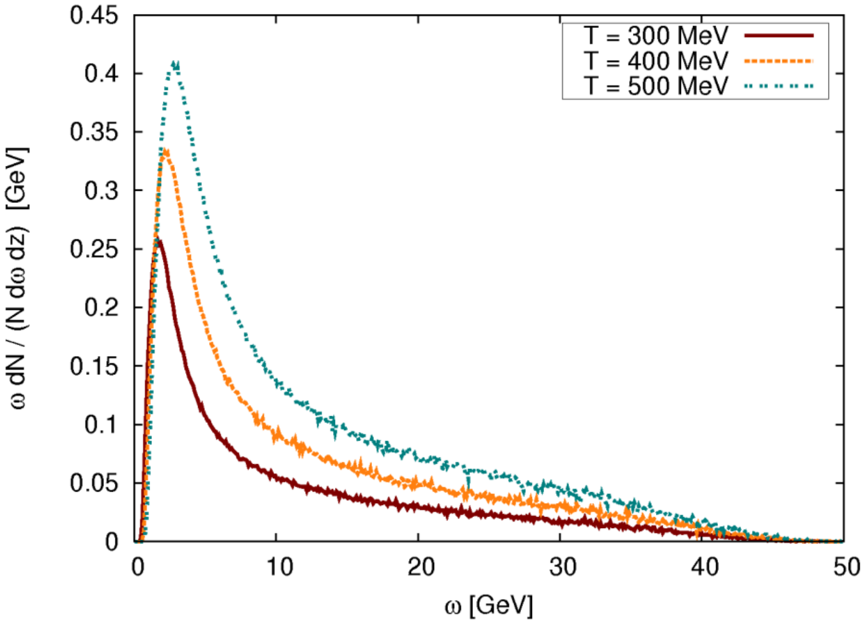
total boost $\gamma = \gamma'\gamma''(1+\vec{\beta}'\vec{\beta}'') = \frac{\cosh y}{\sqrt{1-\beta'^2}}\left(1 + \beta' \tanh y \cos\Theta\right)$

$$\boxed{\Theta(\lambda - \tau) \to \Theta\left(k_\perp - \frac{\gamma}{\lambda}\right)}$$

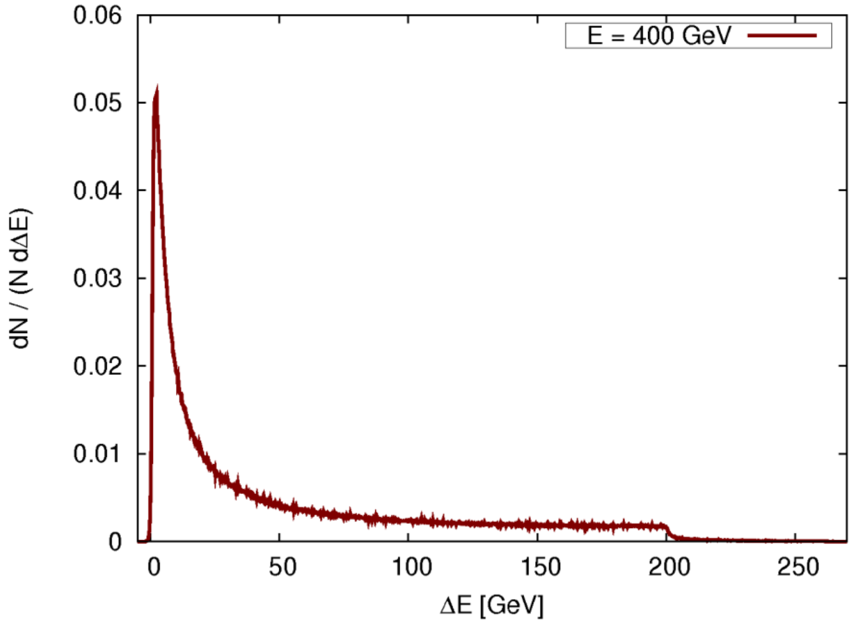


Gluon Radiation and Energy Loss

Radiaton spectrum (E = 50 GeV)

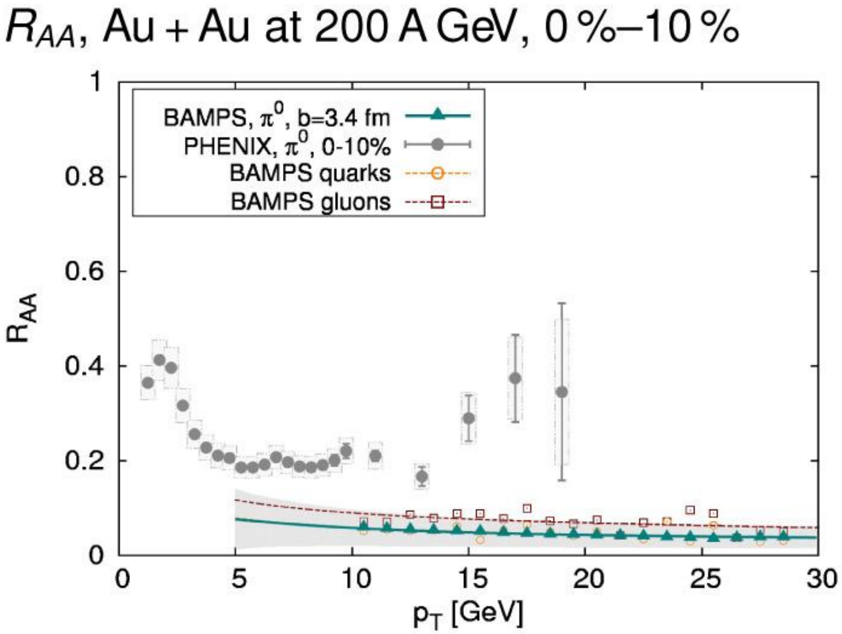


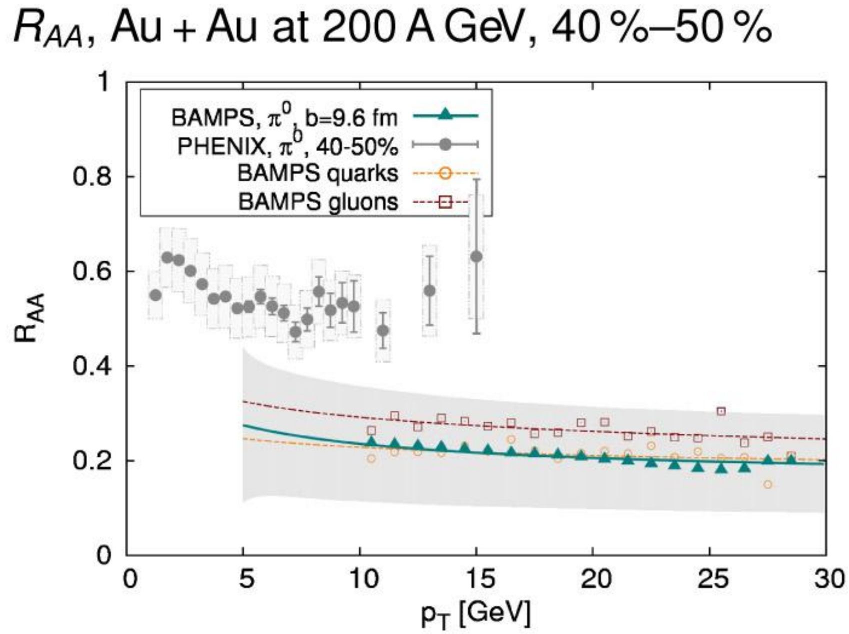
ΔE distribution (E = 400 GeV)



- Heavy tail in ΔE distribution leads to large mean <ΔE>

Jet Suppression in BAMPS Simulations at RHIC

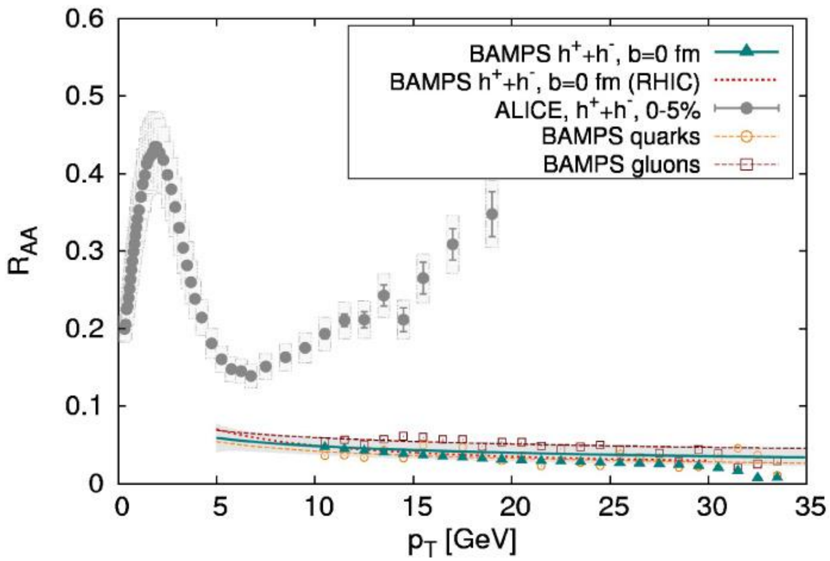




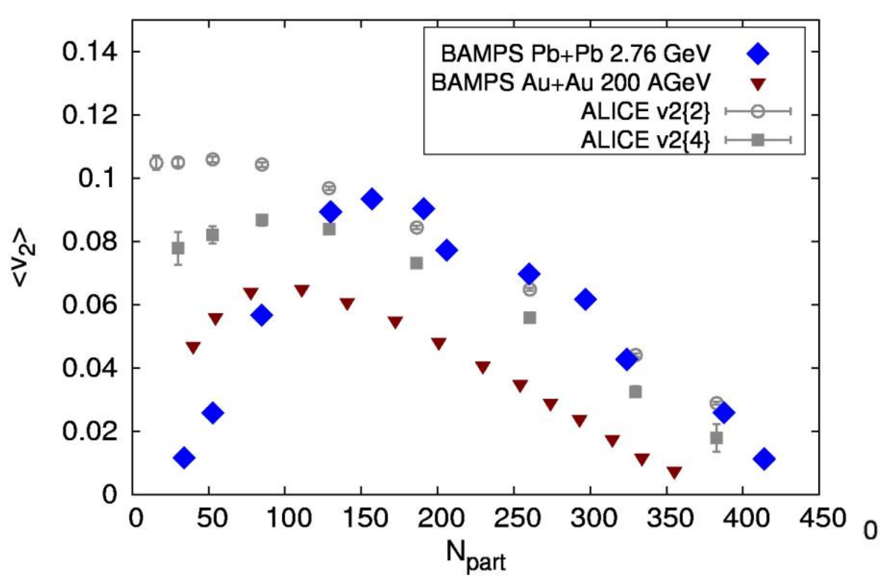
- Hadronization via AKK fragmentation functions
- Suppression in BAMPS is too strong
  - Strong mean energy loss in $2 \rightarrow 3$ processes
  - Sizeable conversion of quark jets into gluon jets
  - Small difference in the energy loss of quarks and gluons

Jet Suppression and Elliptic Flow at LHC

R_{AA}, Pb + Pb at 2.76 A TeV, 0 %–5 %



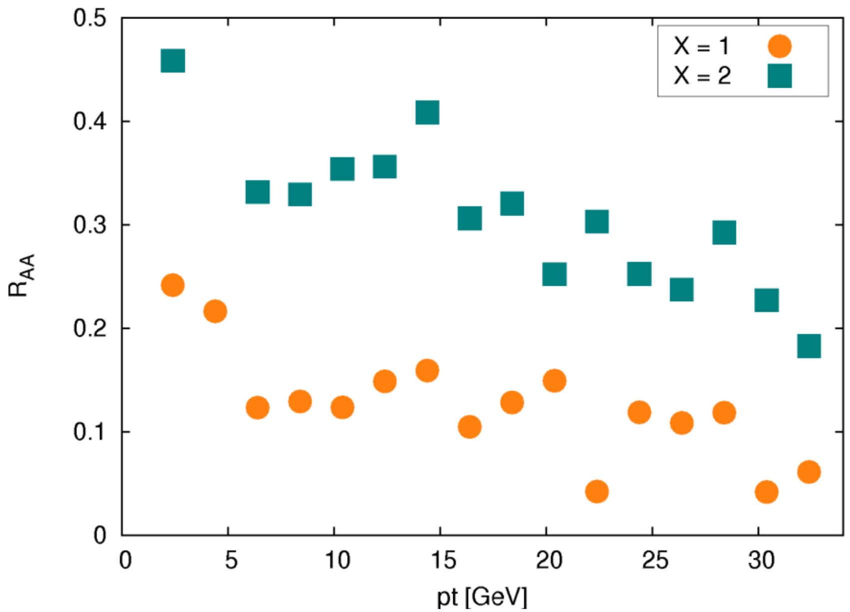
v_2, Pb + Pb at 2.76 A TeV



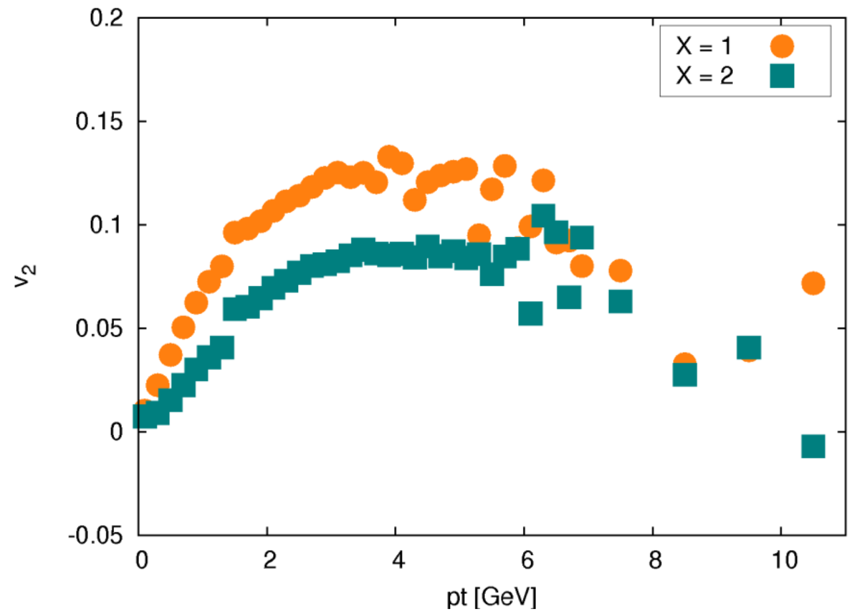
- PYTHIA initial conditions (Uphoff, OF et al. PRC 82 (2010)), $\alpha_s = 0.3$
- R_{AA} almost identical to RHIC, does not reproduce rise towards large p_T
- Integrated v_2 shows increase, drops below data at about 50 % centrality

Sensitivity on the LPM Cut-Off

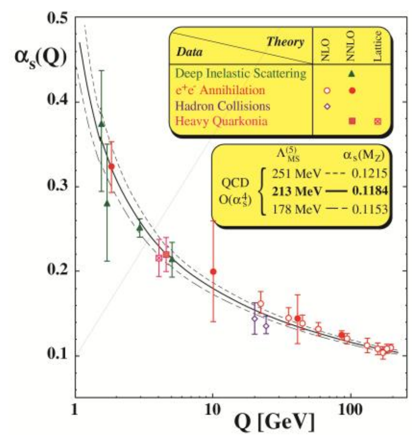
R_{AA} for different X (*b = 7 fm*)

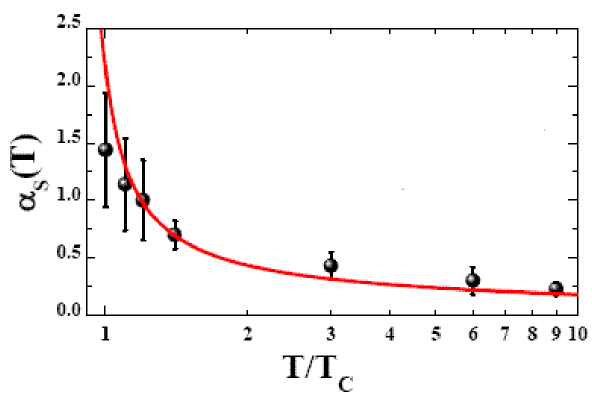


v_2 for different X (*b = 7 fm*)

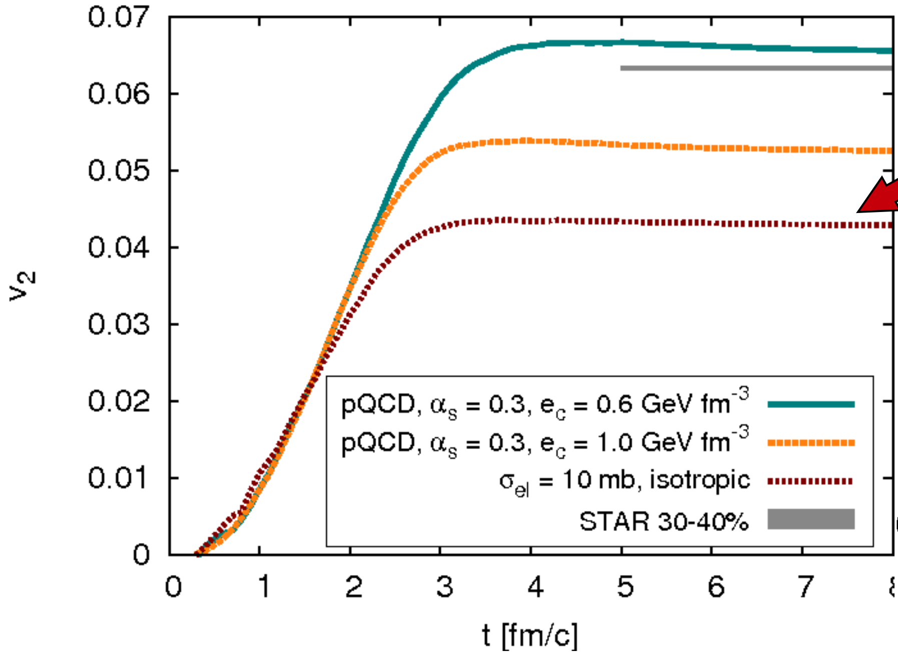


further idea: running coupling

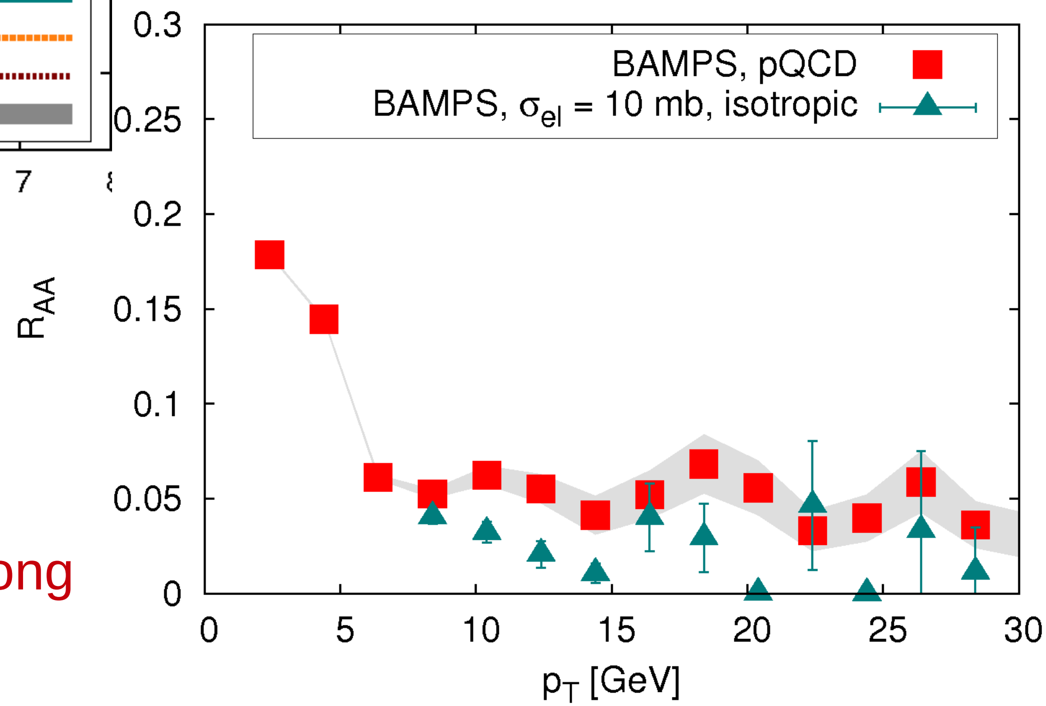




Binary processes – R_{AA} and v_2?

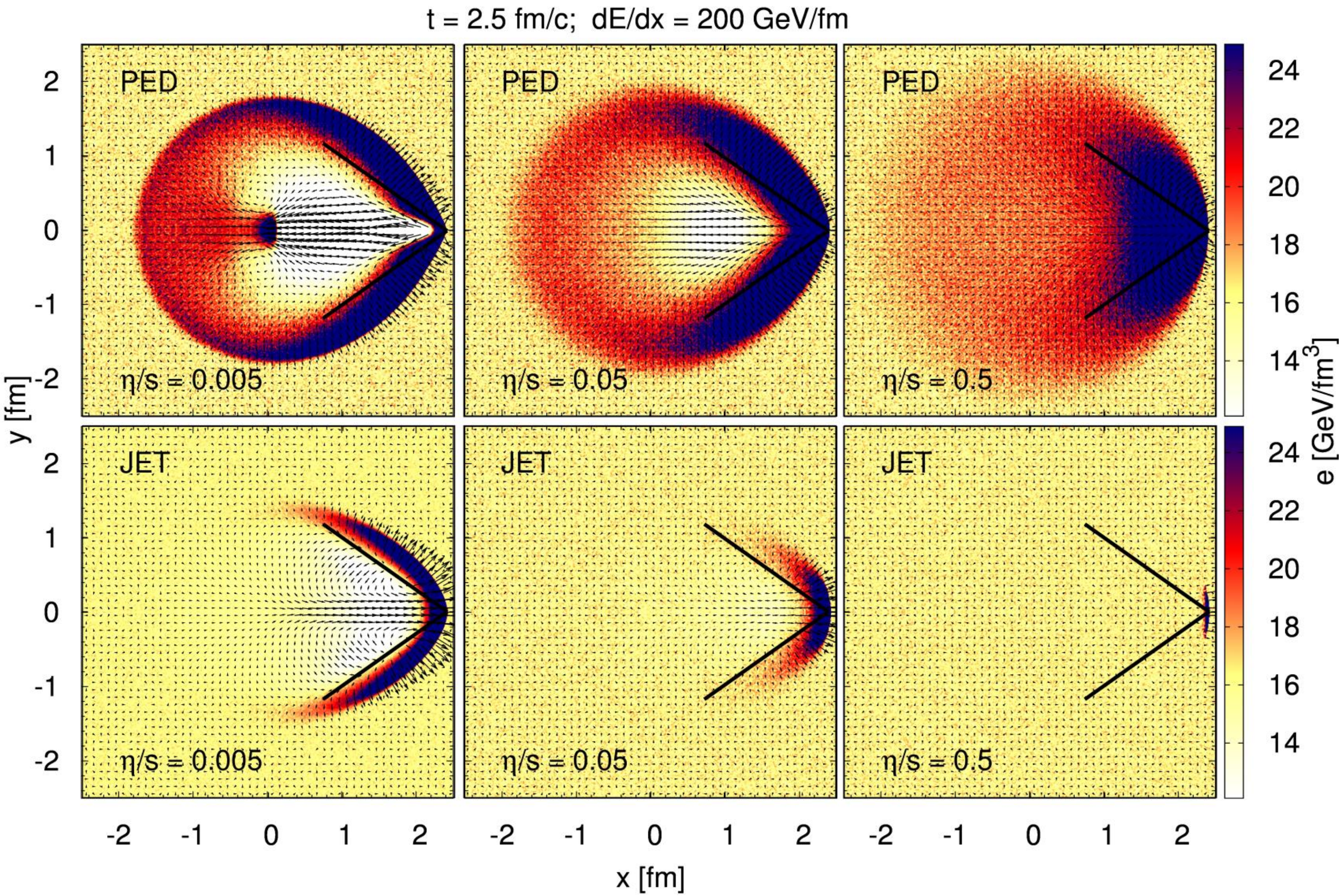


10 mb, isotropic 2->2:
v_2 is still to low



10 mb, isotropic 2->2:
jet quenching is already to strong

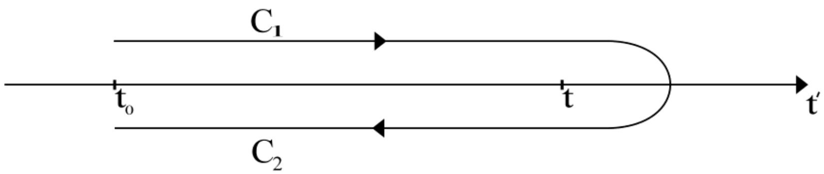
VISCOUS Solutions



… the death of Mach Cones !

Realtime formalism – Kadanoff-Baym equations

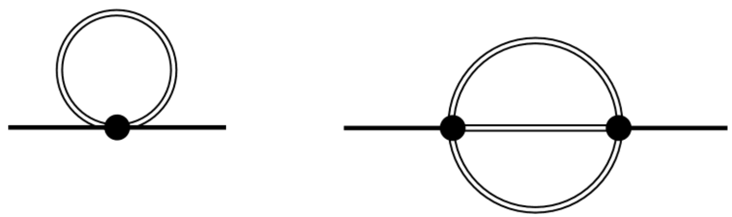
- Evaluation along Schwinger-Keldysh time contour

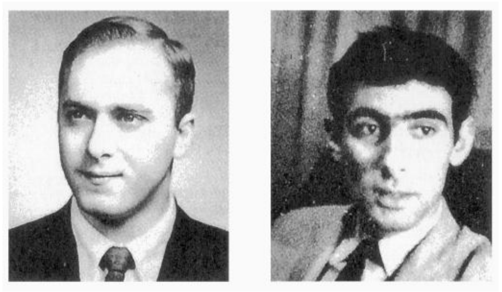


$$\langle \hat{O}_H(t) \rangle = \left\langle T^C \left\{ \exp \left[-i\hbar^{-1} \int_{t_0}^{t_0} dt' \hat{H}_1(t') \right] \hat{O}_I(t) \right\} \right\rangle$$

- nonequilibrium Dyson-Schwinger equation

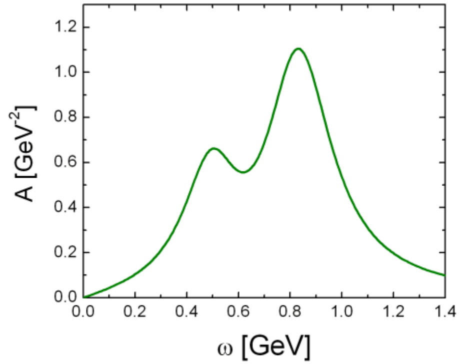
$$D \otimes G =$$





$$\begin{aligned}
\Rightarrow & \hat{D} G^{\gtrless}(1,1') \\
& = \int d^3x_2 \Sigma_{\mathrm{HF}}(\mathbf{x}_1, t_1, \mathbf{x}_2, t_1) G^{\gtrless}(\mathbf{x}_2, t_1, 1') \\
& \quad + \int_{t_0}^{t_1} d2 (\Sigma^{>}(1,2) - \Sigma^{<}(1,2)) G^{\gtrless}(2,1') \\
& \quad - \int_{t_0}^{t_{1'}} d2 \Sigma^{\gtrless}(1,2)(G^{>}(2,1') - G^{<}(2,1')),
\end{aligned}$$

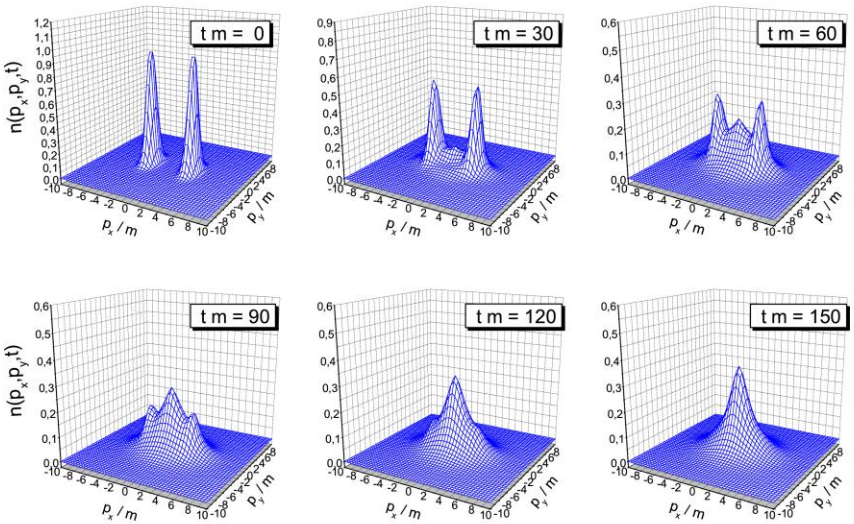
- spectral information:



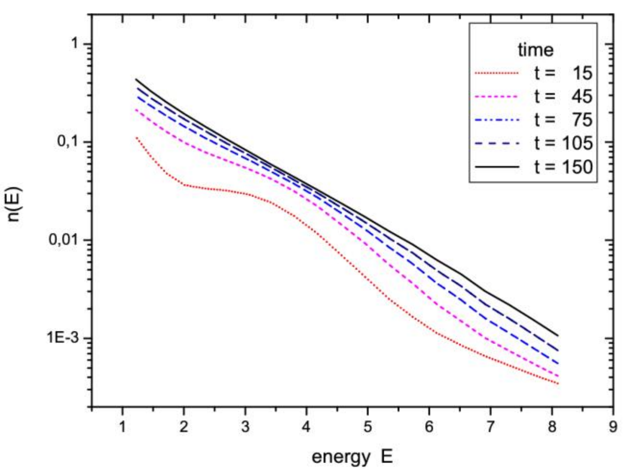
- Kadanoff-Baym equations are non-local in time → memory - effects

Equilibration of quantum fields from first principles

- Evolution of occupation number

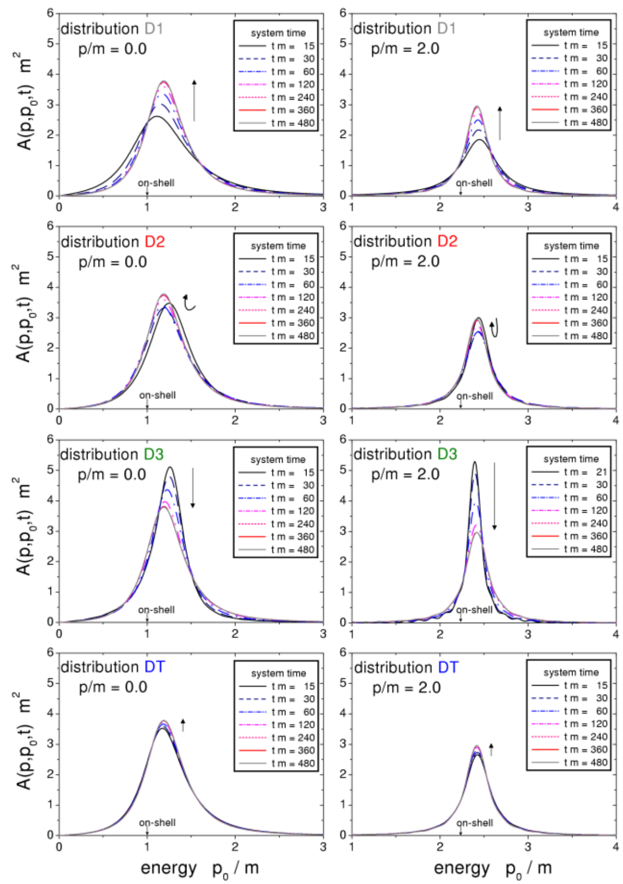


- Transverse energy spectrum



Exponential mass spectra occur before full kinetic equilibrium

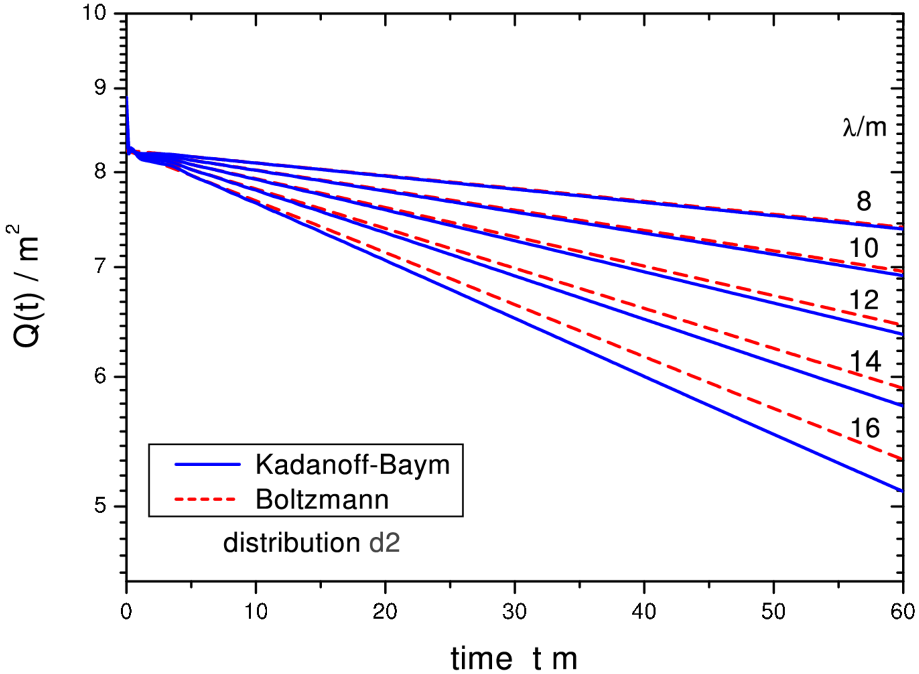
- spectral functions $\Gamma \sim \dfrac{\hbar}{\tau_{\text{coll.}}}$



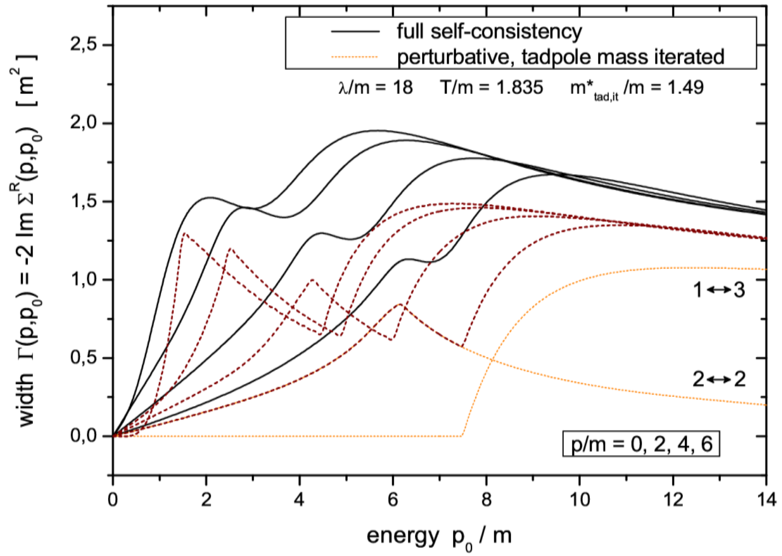
S. Juchem, W. Cassing, C.G., PRD**69**:025006, 2004

Kinetic Boltzmann versus full strongly coupled quantum evolution

quadrupole moment $Q(t)$



width Γ

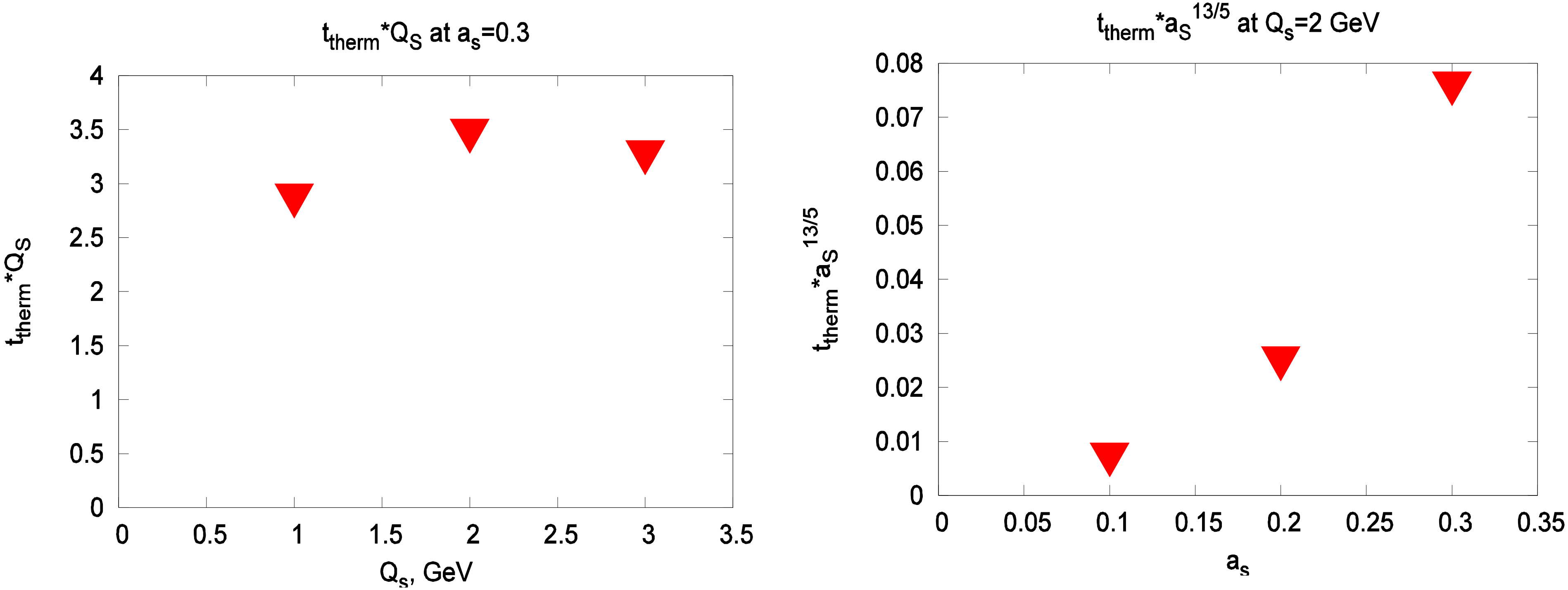


$2 \leftrightarrow 2 \qquad 1 \leftrightarrow 3$

- Boltzmann a little weaker than KB approach for $\Gamma / E \sim 0.5$

- Offshell $1 \leftrightarrow 3$- processes drive system into chemical equilibrium; this corresponds appr. to $2 \leftrightarrow 4$ kinetic onshell processes

Thermalization times: comparison with bottom-up prediction



- $1/Q_s$ behavior seems to be correct.
- instead $\alpha^{-13/5}$ behavior but α^{-x} with $x < 13/5$