

Thermal Equilibration in Scalar Field Theory

Advisor: Francois GELIS

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environnement + énergie + santé

Introduction

Fixed volume

Expanding volume

Conclusion

Thomas EPELBAUM
IPhT

Resummation formula [GELIS, LAPPI, VENUGOPALAN (2008)]

NLO

$$T_{\text{NLO}}^{\mu\nu} = \hat{O} T_{\text{LO}}^{\mu\nu}[\varphi_0]$$

$$\hat{O} \approx \frac{1}{2} \int G(x, y) \frac{\delta}{\delta \varphi_0(x)} \frac{\delta}{\delta \varphi_0(y)}$$

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anomalous dimensions + examples of renormalization

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Resummation formula [GELIS, LAPPI, VENUGOPALAN (2008)]

Resummation

$$T_{\text{resum}}^{\mu\nu} = e^{\hat{O}} T_{\text{LO}}^{\mu\nu}[\varphi_0]$$

$$\hat{O} \approx \frac{1}{2} \int G(x, y) \frac{\delta}{\delta \varphi_0(x)} \frac{\delta}{\delta \varphi_0(y)}$$

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enseignement - exemples - formations

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Resummation formula [GELIS, LAPPI, VENUGOPALAN (2008)]

Resummation

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anisotropie + examples + normes

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Equivalent formulation

$$T_{\text{resum}}^{\mu\nu} = \int [D\mathbf{a}(\mathbf{u})] e^{-\frac{1}{2} \iint d^3\mathbf{u} d^3\mathbf{v} \mathbf{a}(\mathbf{u}) \mathbf{G}^{-1}(\mathbf{u}, \mathbf{v}) \mathbf{a}(\mathbf{v})} T_{\text{LO}}^{\mu\nu}[\varphi_0 + \mathbf{a}]$$

- Solve the EOM for the classical background field φ_0 with random gaussian fluctuations \mathbf{a} on top of it.
- Semi-classical calculation that takes into account some quantum corrections.
- Monte Carlo Simulation.

Scalar field theory

Lagrangian of the theory

$$\mathcal{L}(\phi) = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \underbrace{\frac{g^2}{4!}\phi^4}_{V(\phi)} + J\phi$$

where

$$J \propto \theta(-x^0)$$

Why do we use this model?

- Scale invariance in $3 + 1$ dimensions
- Parametric resonance
- A lot simpler!

anisotropie, instabilités, + examples, applications

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Form of the solution

Initial condition of the EOM

$$\phi_{\text{init}}(t, \mathbf{x}) = \varphi_0(\mathbf{x}) + \sum_{\mathbf{k}} \Re [c_{\mathbf{k}} e^{i\omega_{\mathbf{k}} t} f_{\mathbf{k}}(\mathbf{x})]$$

with

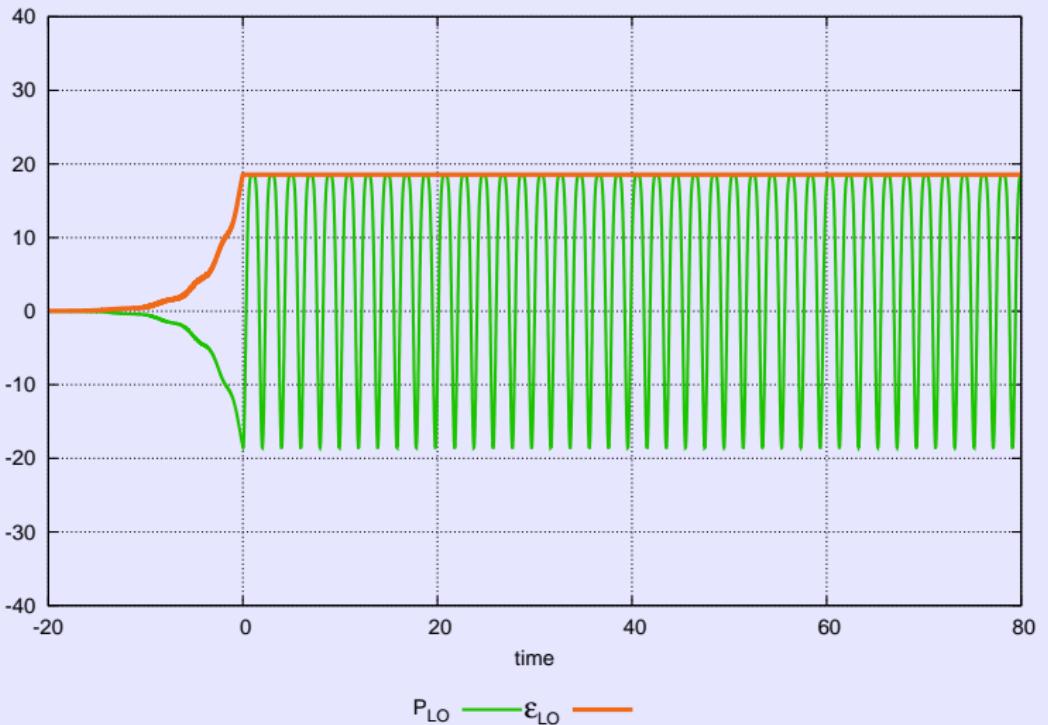
$$[-\Delta + V''(\varphi_0)] f_{\mathbf{k}}(\mathbf{x}) = \omega_{\mathbf{k}}^2 f_{\mathbf{k}}(\mathbf{x})$$

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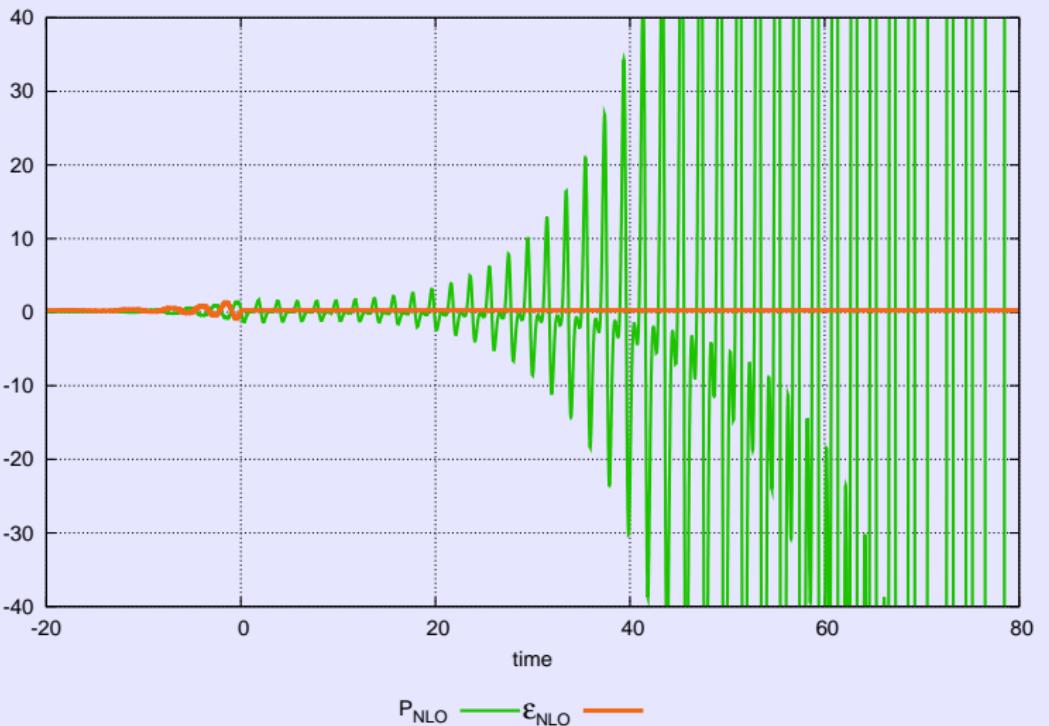
enseignement - exemples algorithmes

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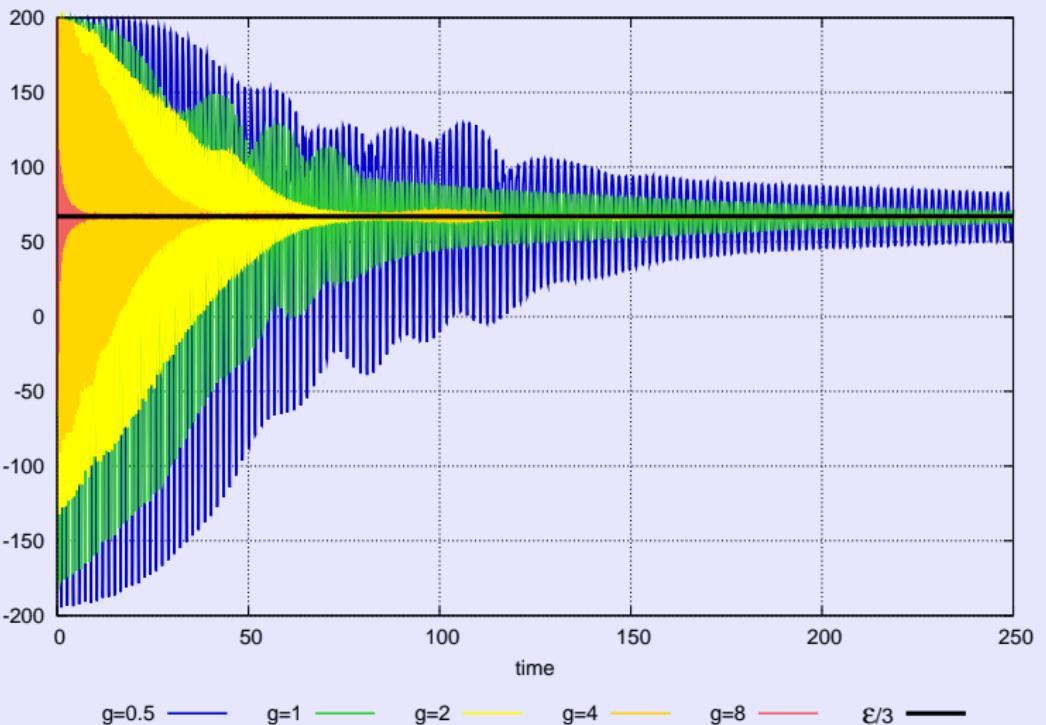
environnement + énergie + sécurité

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$T^{\mu\nu}_{\text{resum}}$ [DUSLING, TE, GELIS, VENUGOPALAN (2010)]

ensembles thermiques + exemples algorithmiques

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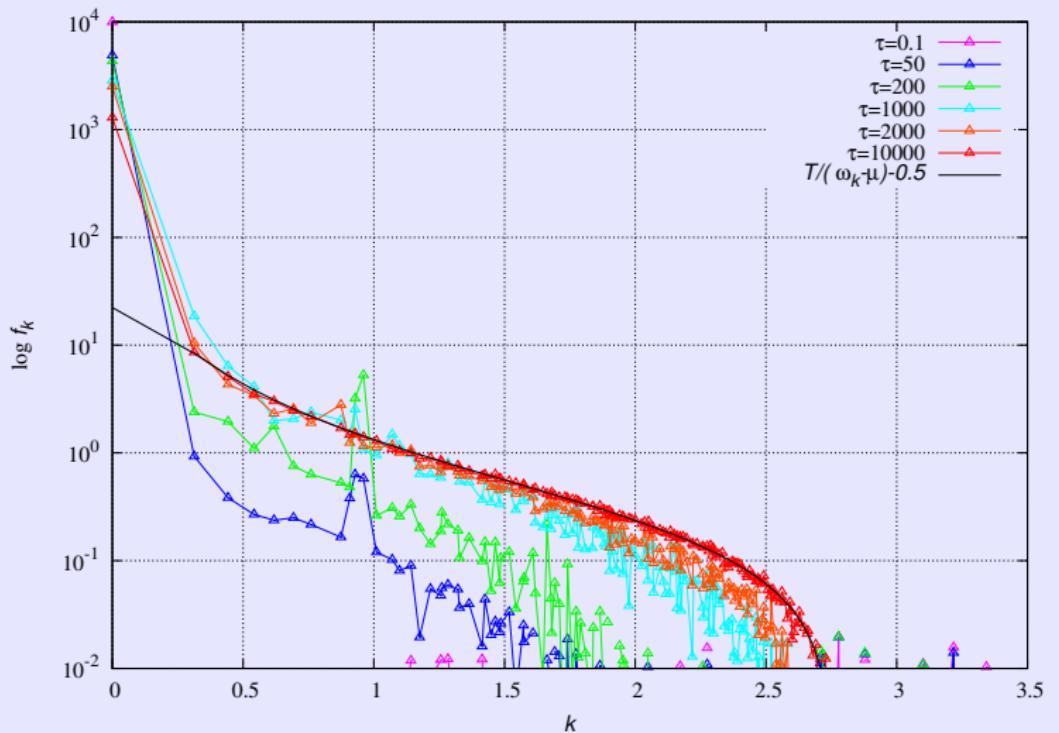
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Occupation number, constant φ_0 [TE, GELIS (2011)]



ensembles thermiques + exemples algorithmiques

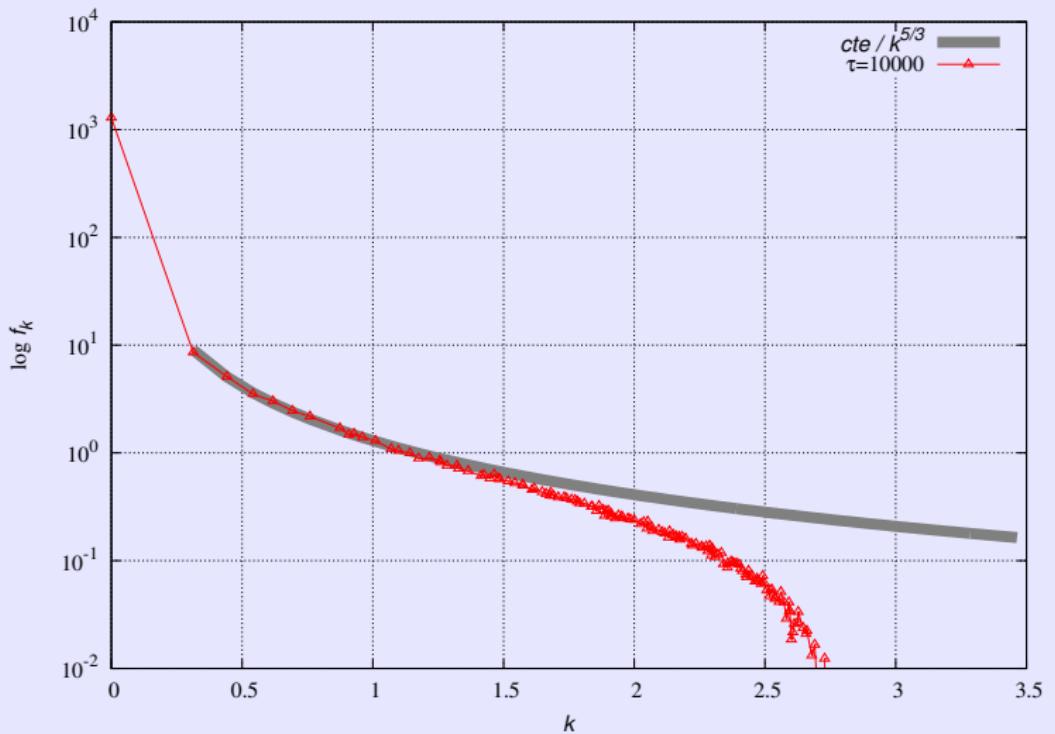
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Kolmogorov scaling? (constant φ_0) [TE, GELIS (2011)]



ceci n'est pas un exemple d'optimisation

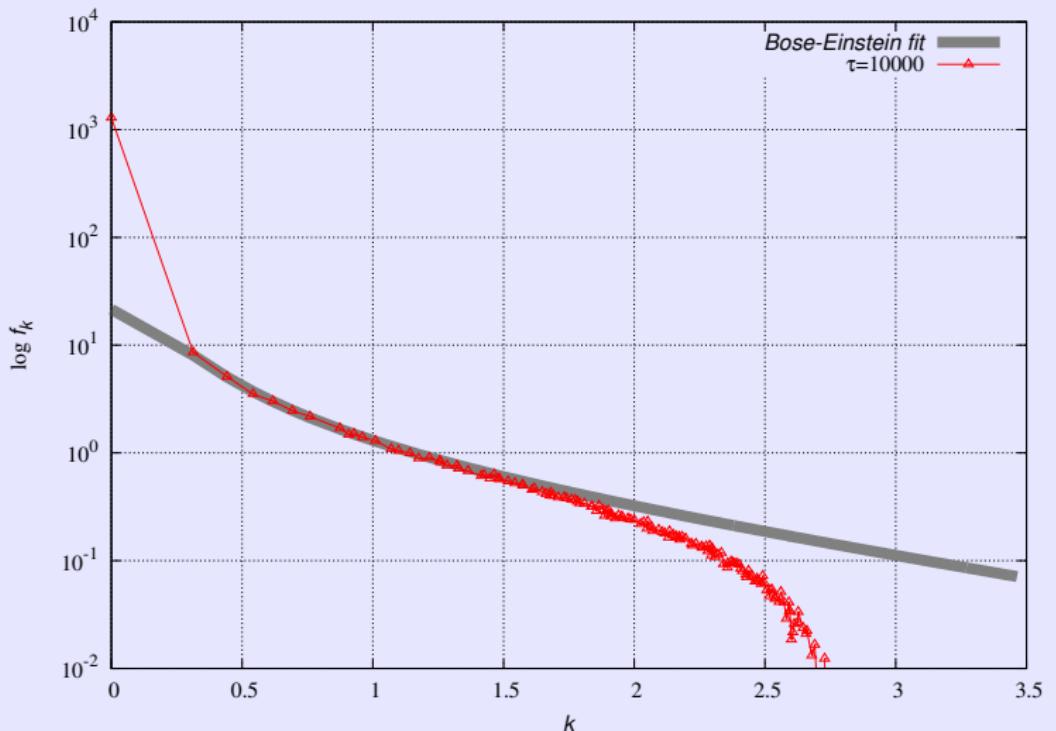
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BOSE-EINSTEIN fit? (constant φ_0) [TE, GELIS (2011)]



ceci n'est pas un exemple d'optimisation

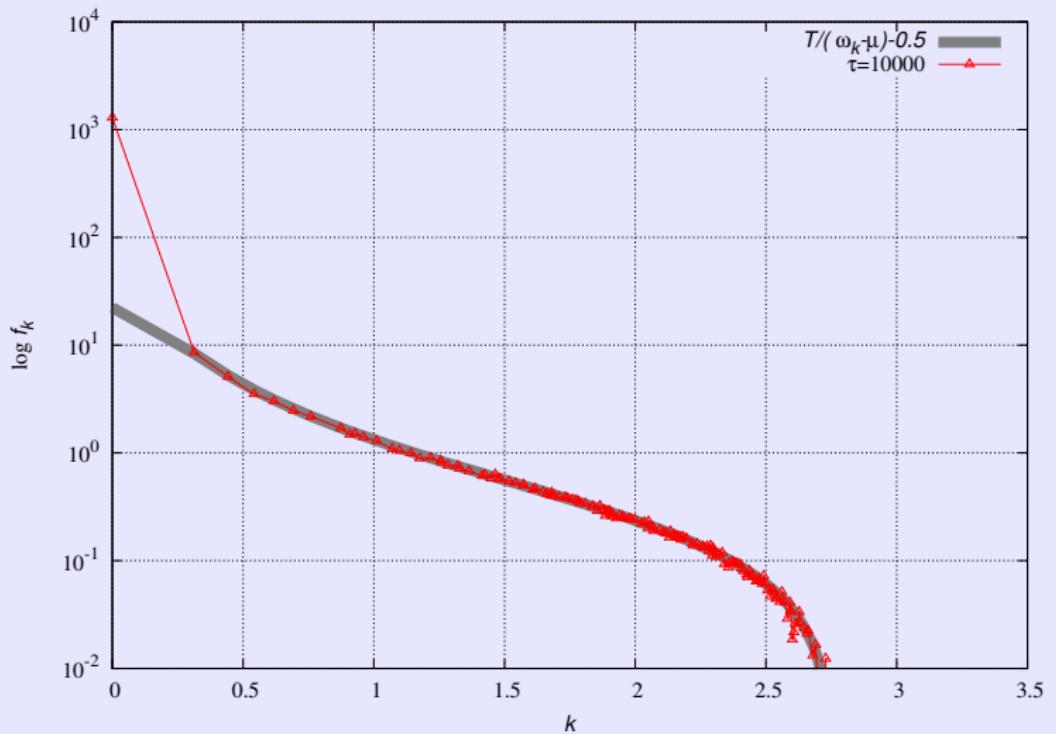
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Classical fit, constant φ_0 [TE, GELIS (2011)]



anisotropie et turbulences < examples optimisation

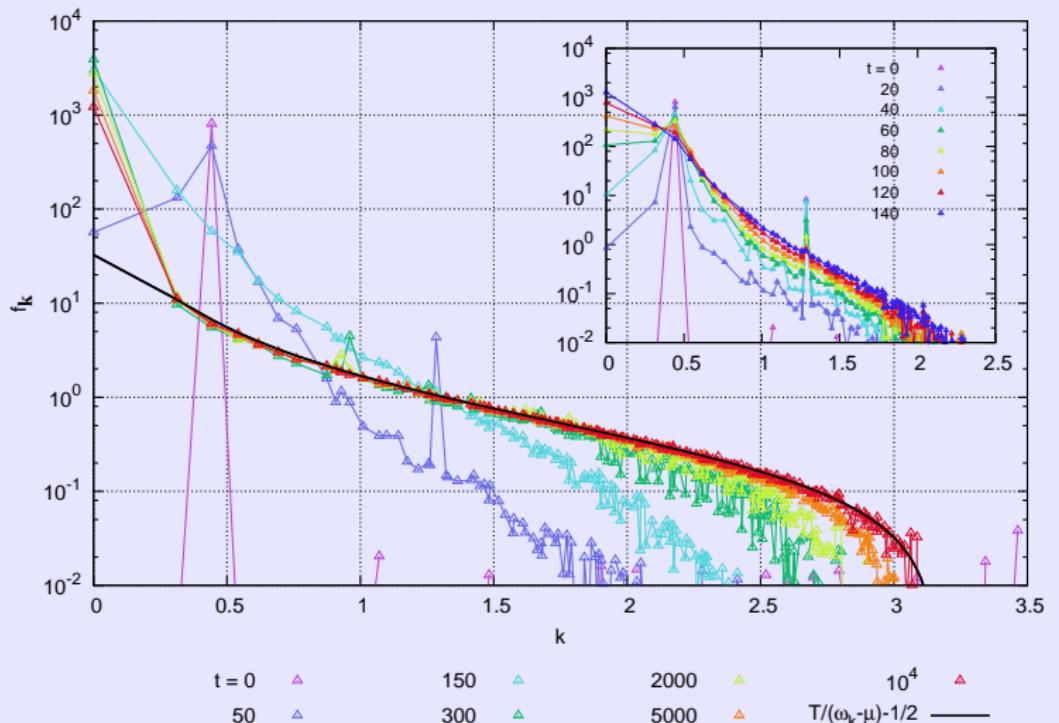
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Non-zero initial mode [TE, GELIS (2011)]



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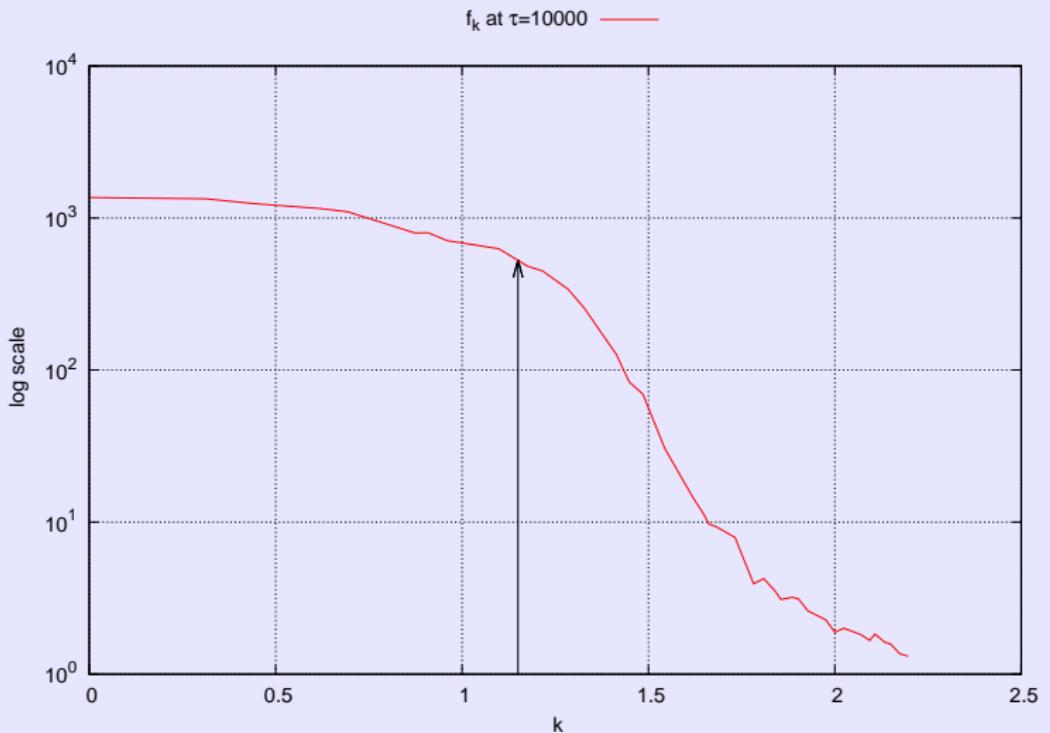
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$$\varphi_0(x, y) = M_0 \cos(k_x x + k_y y)$$
 [Work in progress]



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enseignement - exemples d'algorithmes

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Initial condition of the EOM

$$\phi_{\text{init}}(\tau, \mathbf{x}_\perp, \eta) = \varphi_0(\mathbf{x}_\perp) + \sum_{\mathbf{k}_\perp, \nu} \Re \left[c_{\mathbf{k}, \nu} H_{i\nu}^{(2)}(\omega_{\mathbf{k}_\perp} t) e^{i\nu\eta} f_{\mathbf{k}_\perp}(\mathbf{x}_\perp) \right]$$

with

$$[-\Delta_\perp + V''(\varphi_0)] f_{\mathbf{k}_\perp}(\mathbf{x}_\perp) = \omega_{\mathbf{k}_\perp}^2 f_{\mathbf{k}_\perp}(\mathbf{x}_\perp)$$

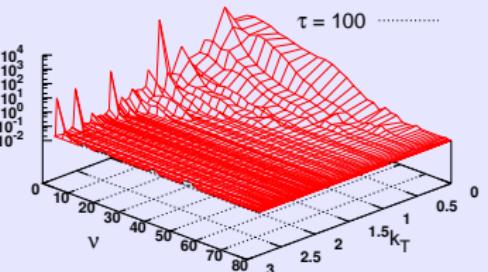
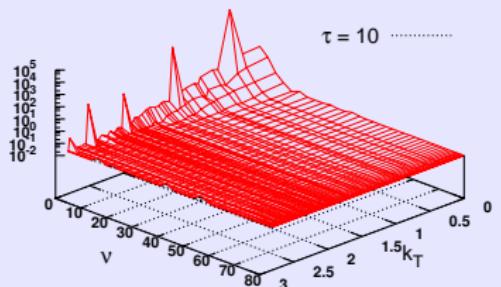
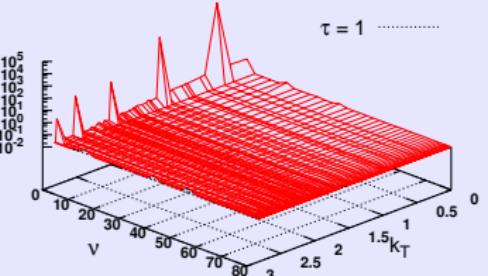
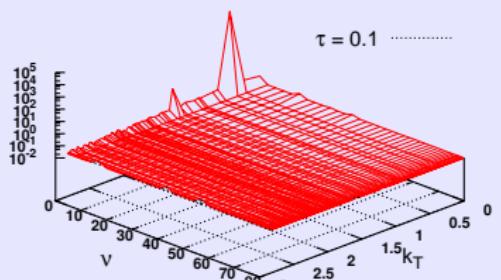
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BOSE-EINSTEIN Condensate [Preliminary results]



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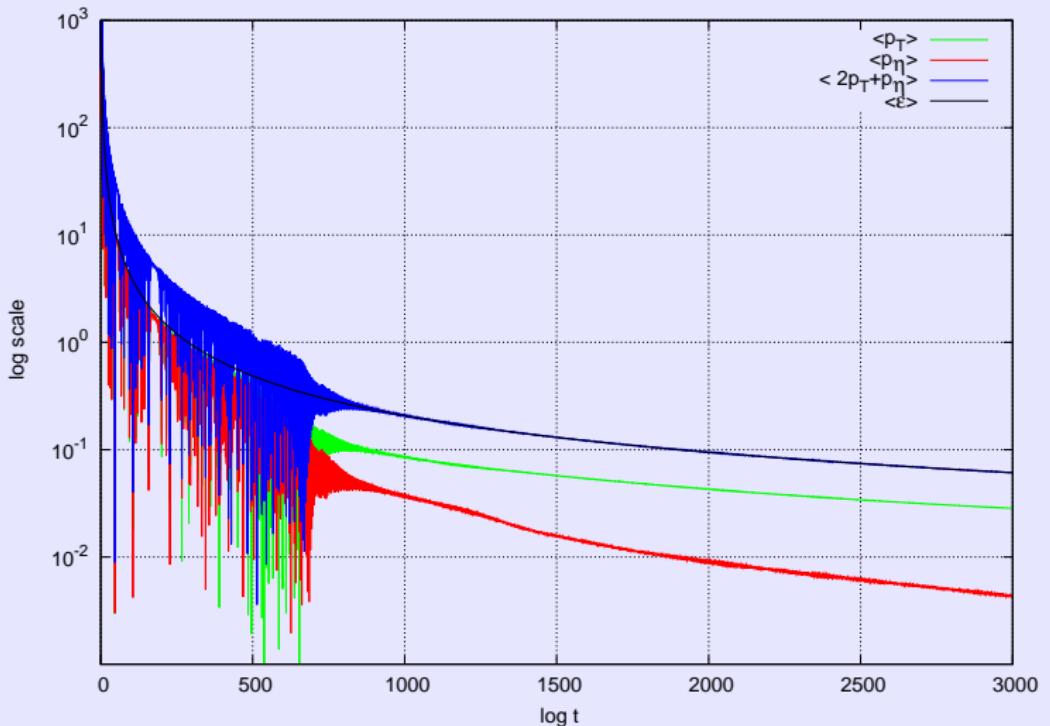
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environnement + energies alternatives

$T^{\mu\nu}_{\text{resum}}$ [Preliminary results]



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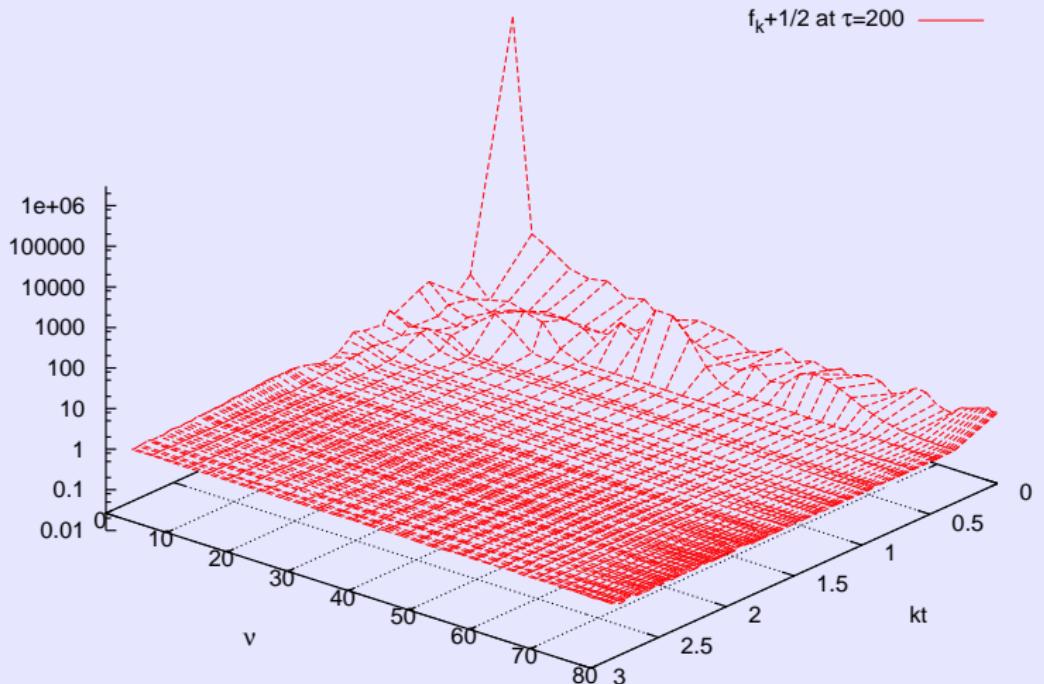
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anisotropie <examples options>

Lattice artifacts [Preliminary results]



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[examples_elasticus](#) < [examples_elasticus](#)

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enseignement + exercices + méthodes

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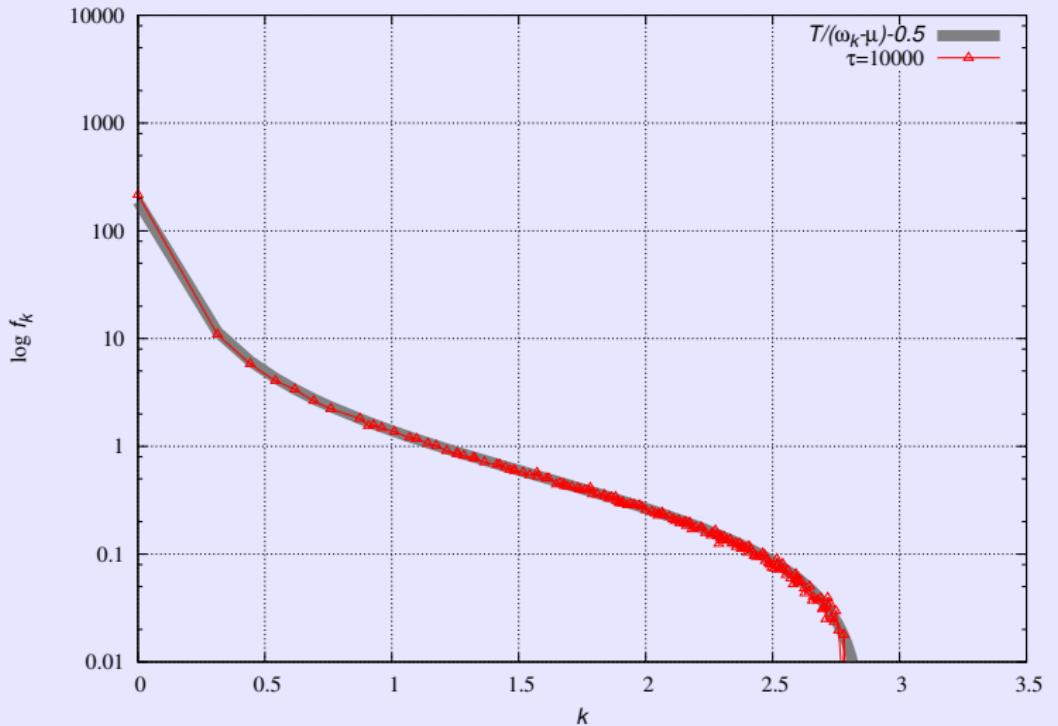
Principal results for the fixed volume

- EOS $\epsilon = 3p$
- $p_x = p_y = p_z$
- $f_{\mathbf{k}} \propto \frac{T}{\omega_{\mathbf{k}} - \mu} - \frac{1}{2}$ at late times
- BOSE-EINSTEIN condensate

Principal results for the expanding volume

- BOSE-EINSTEIN condensate?
- EOS?
- $p_\eta \neq p_T$?

Backup slide



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