GENCO Award Ceremony
Annual NUSTAR Meeting 2012
GSI
March 1, 2012

60 years of nuclear shell model

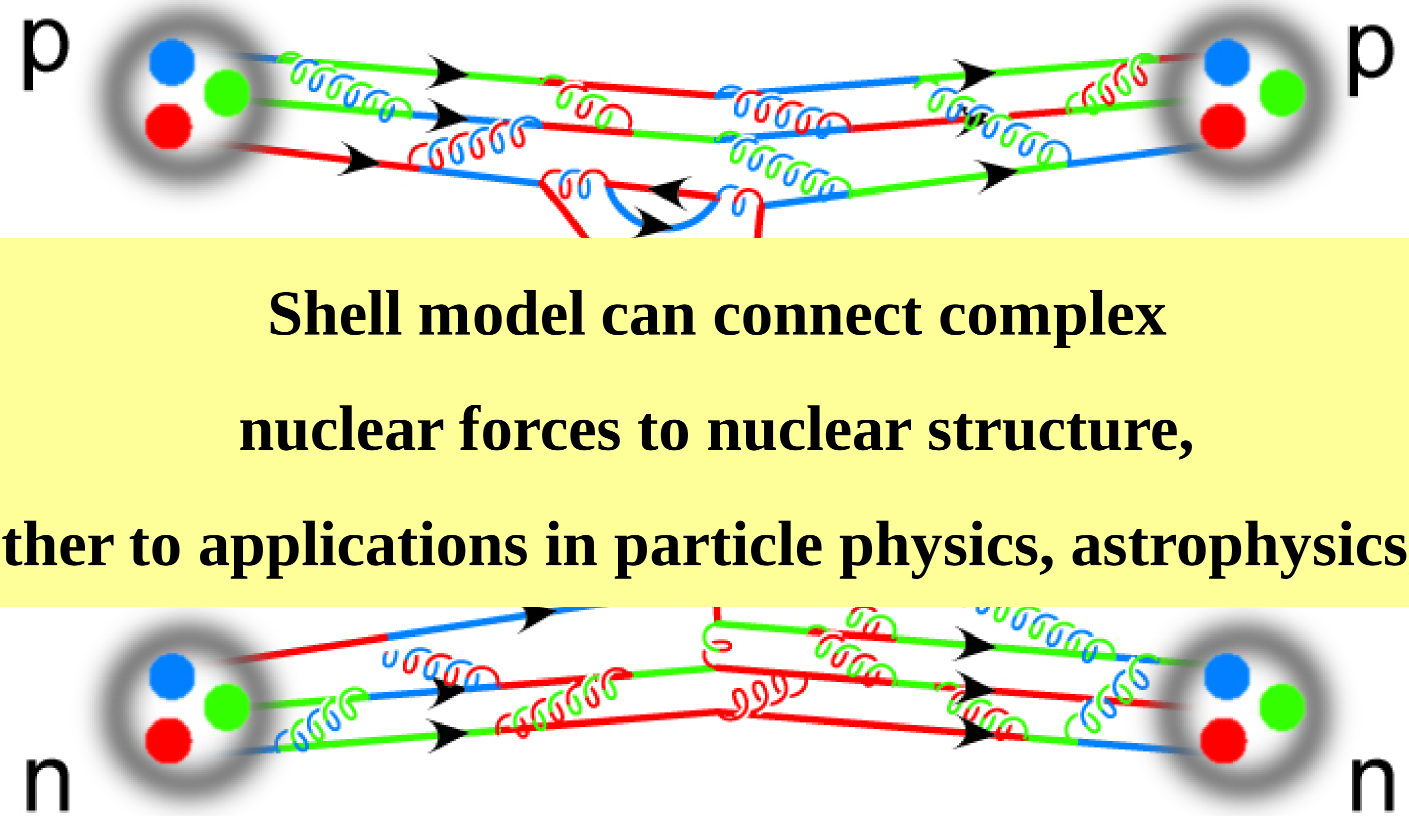
- paradigm, achievement and future -

Takaharu Otsuka

東京大学
THE UNIVERSITY OF TOKYO

NSCL

Image of NN force by Hadronic Physicist

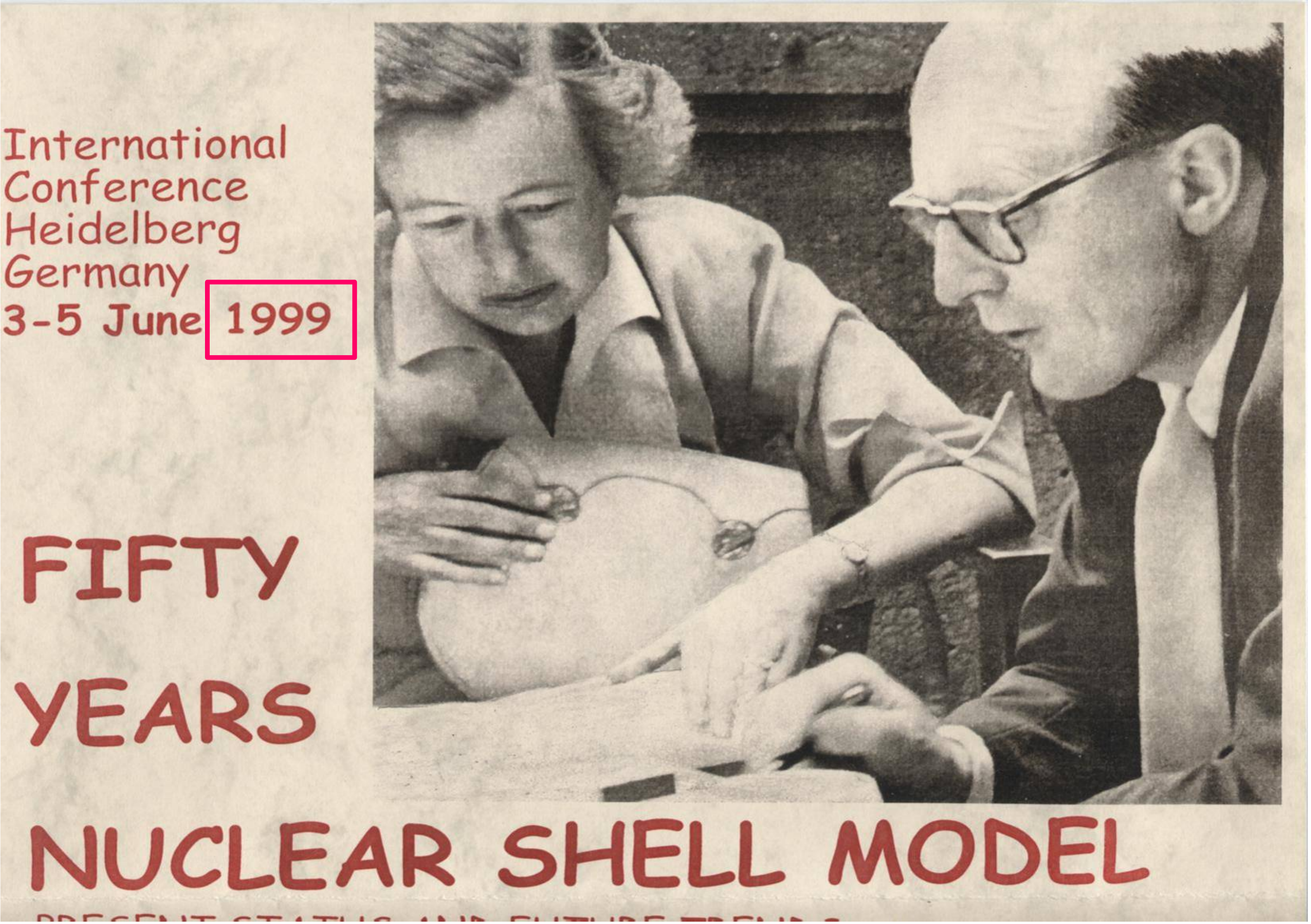


Shell model can connect complex

nuclear forces to nuclear structure,

further to applications in particle physics, astrophysics, etc.

Image Bank (ph004), School of Science, University of Tokyo

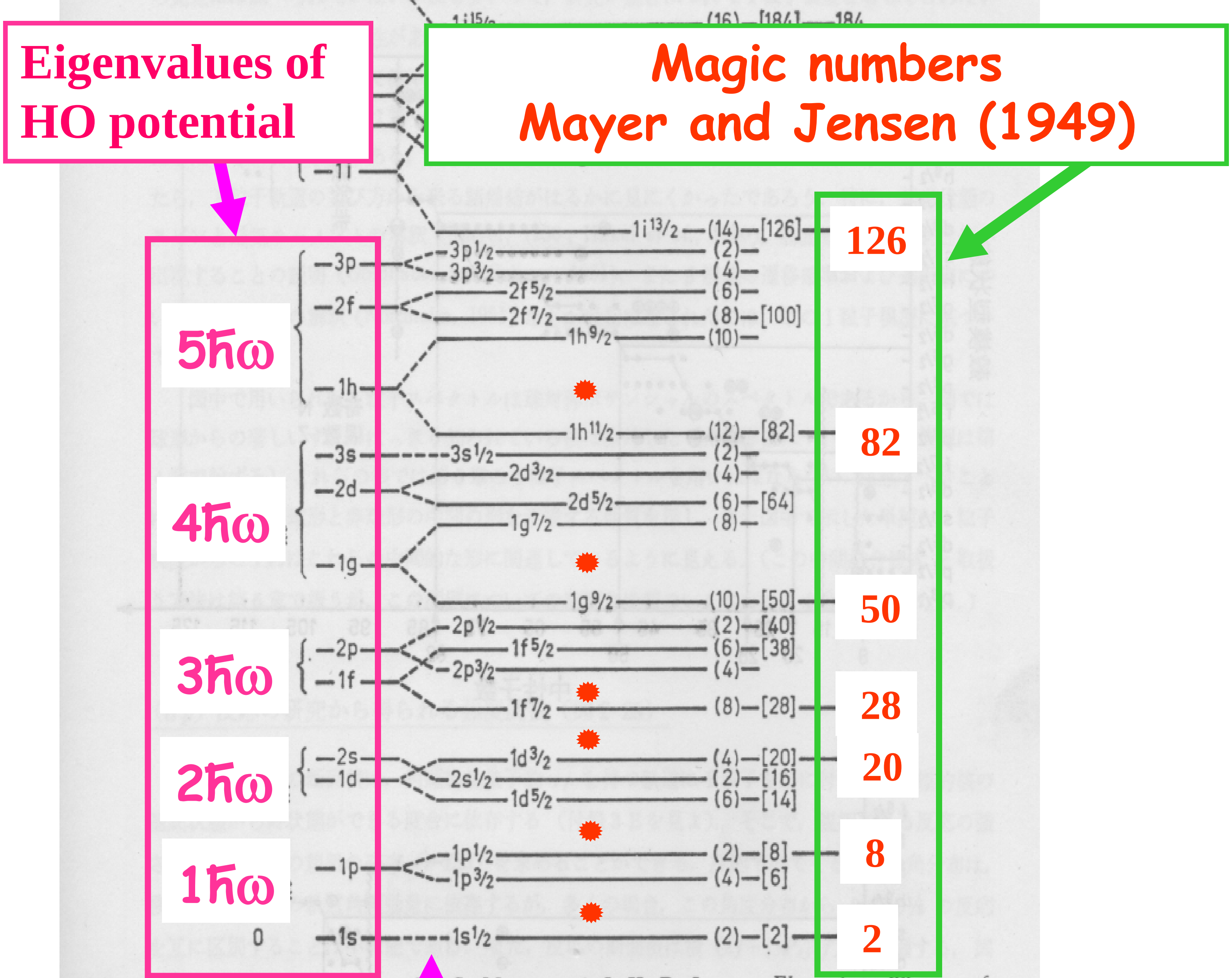


International
Conference
Heidelberg
Germany
3-5 June 1999

FIFTY
YEARS

NUCLEAR SHELL MODEL

The 60th anniversary has just passed.



Eigenvalues of HO potential

Magic numbers
Mayer and Jensen (1949)

$5\hbar\omega$

$4\hbar\omega$

$3\hbar\omega$

$2\hbar\omega$

$1\hbar\omega$

126

82

50

28

20

8

2

図2-23　1粒子軌道の順序．図は G. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structur* った.

Spin-orbit splitting

60 year anniversary is special in Japan (or Asia)

Ancient (~3000 year ago in China) way to count the year

cycle of 10 years

5 elements x 2

木 tree < 陰 dark / 陽 bright

火 fire

土 soil

金 metal

水 water < 陰 dark

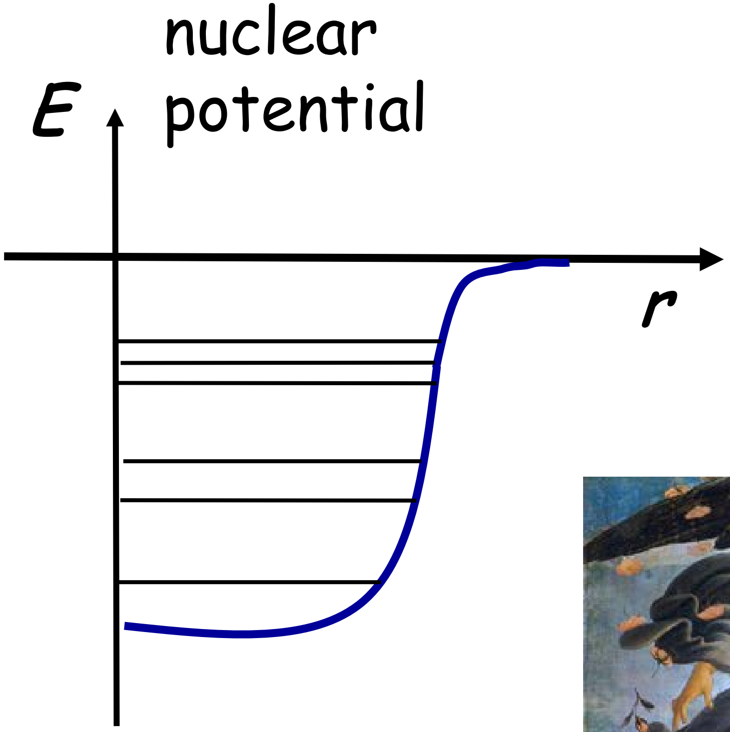
×

cycle of 12 years

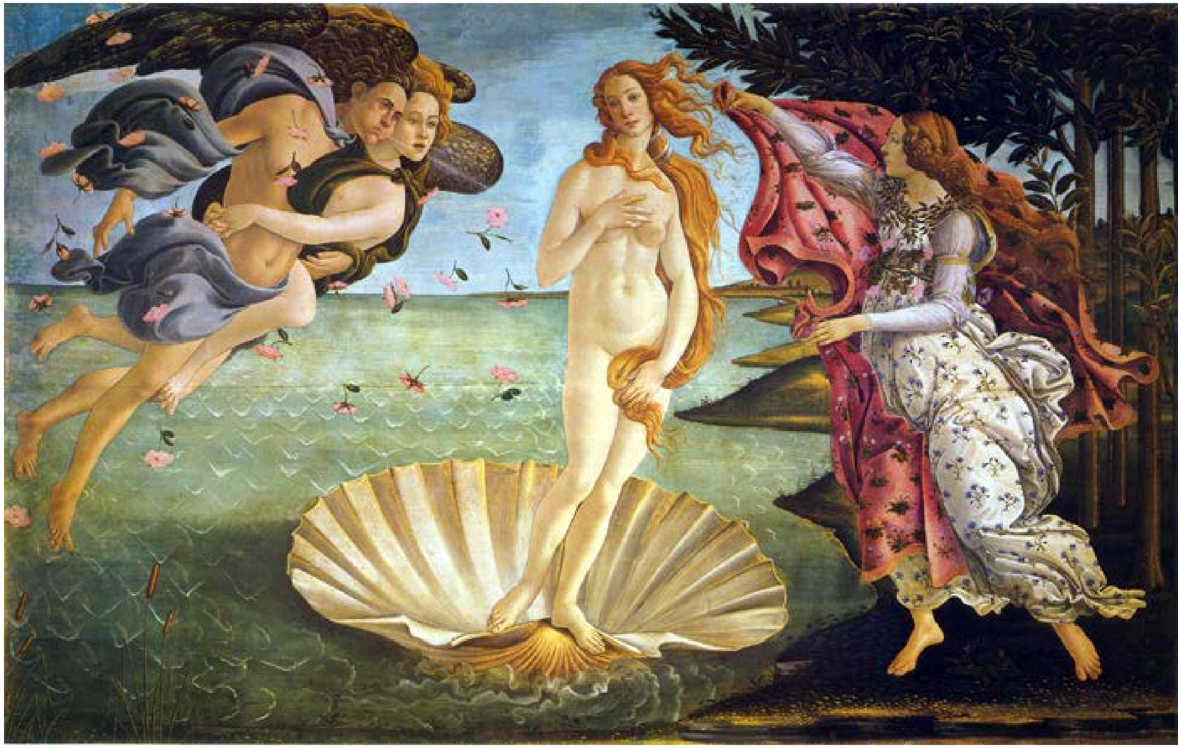
12 animals (spirits)

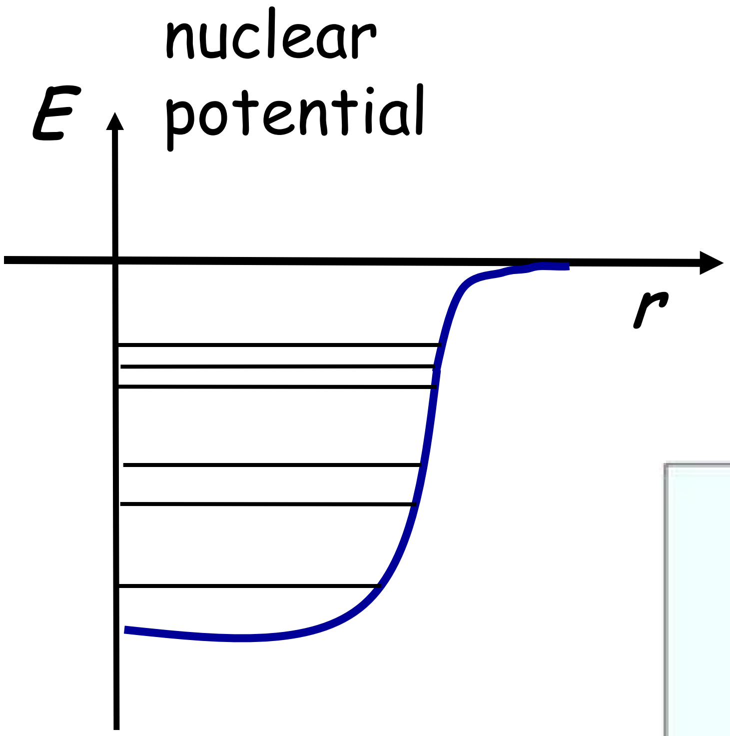
子	mouse
丑	cow
寅	tiger
卯	rabbit
辰	dragon
巳	snake
午	horse
未	sheep
申	monkey
酉	hen
戌	dog
亥	wild boar

Things are reborn every 60 years, as the age is reset.



What can we create from this vessel with beautiful shell pattern ?





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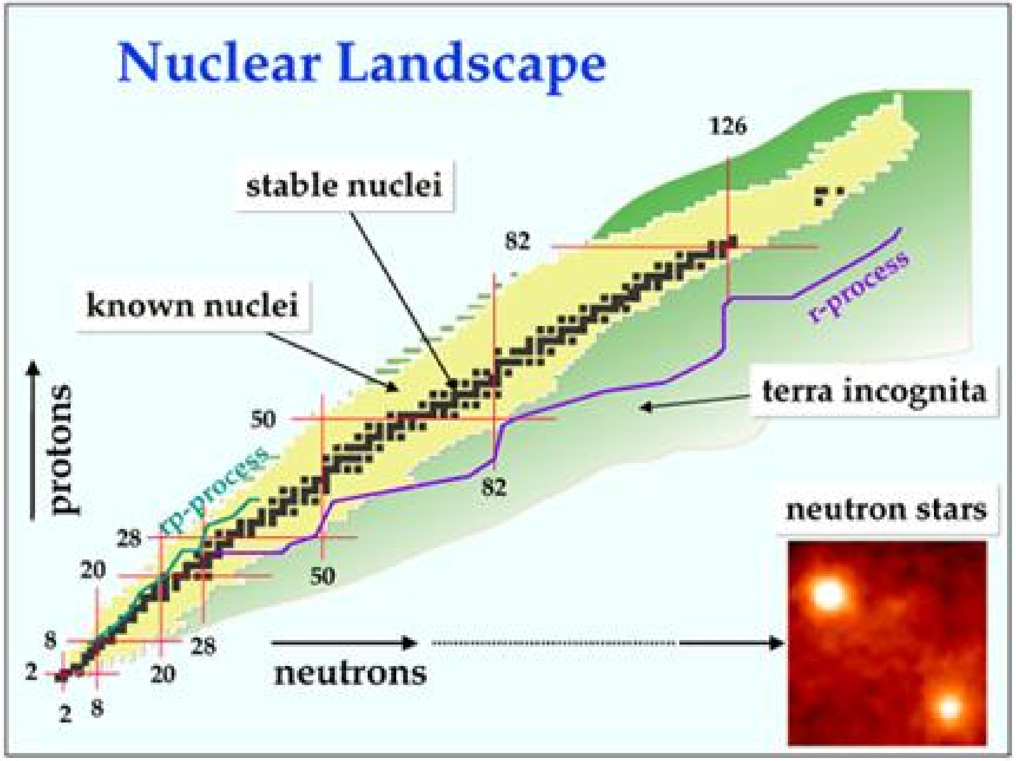


Building blocks of shell model

Model space (set of orbits for active particles)

→ Combination of the model space and the number of nucleons determines the dimension

Effective Interaction

$$H = \Sigma_i \boxed{\epsilon_i}\, n_i + \Sigma_{i,j,k,l} \boxed{v_{ij,kl}}\, a_i^\dagger\, a_j^\dagger\, a_l\, a_k$$

Two-Body Matrix Element (TBME)

Single Particle Energy (SPE)

History of the shell model

larger dimension ……… many-body structure

more precise TBME ……… nuclear forces

interplay between structure and force→ paradigm

Dimension

Matrix of
Hamiltonian **H**
→ diagonalized

Slater determinants

$\phi_1 = a_\alpha^{\;+}\; a_\beta^{\;+}\; a_\gamma^{\;+} \;\ldots.. \,|\, 0 >$

$\phi_2 = a_{\alpha'}^{\;+}\; a_{\beta'}^{\;+}\; a_{\gamma'}^{\;+} \;\ldots.. \,|\, 0 >$

$\phi_3 = \ldots.$

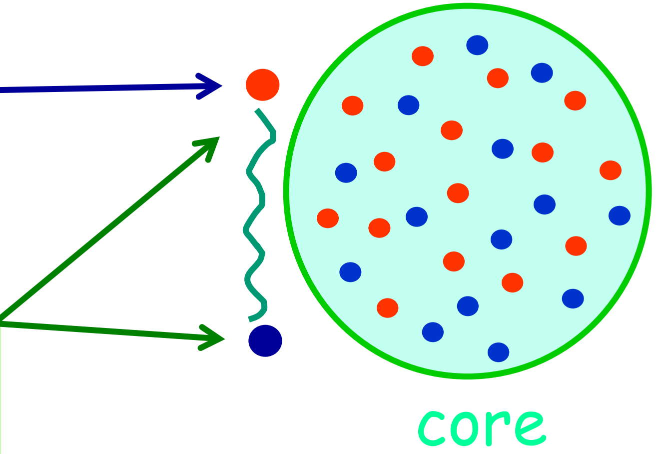
shell-model dimension

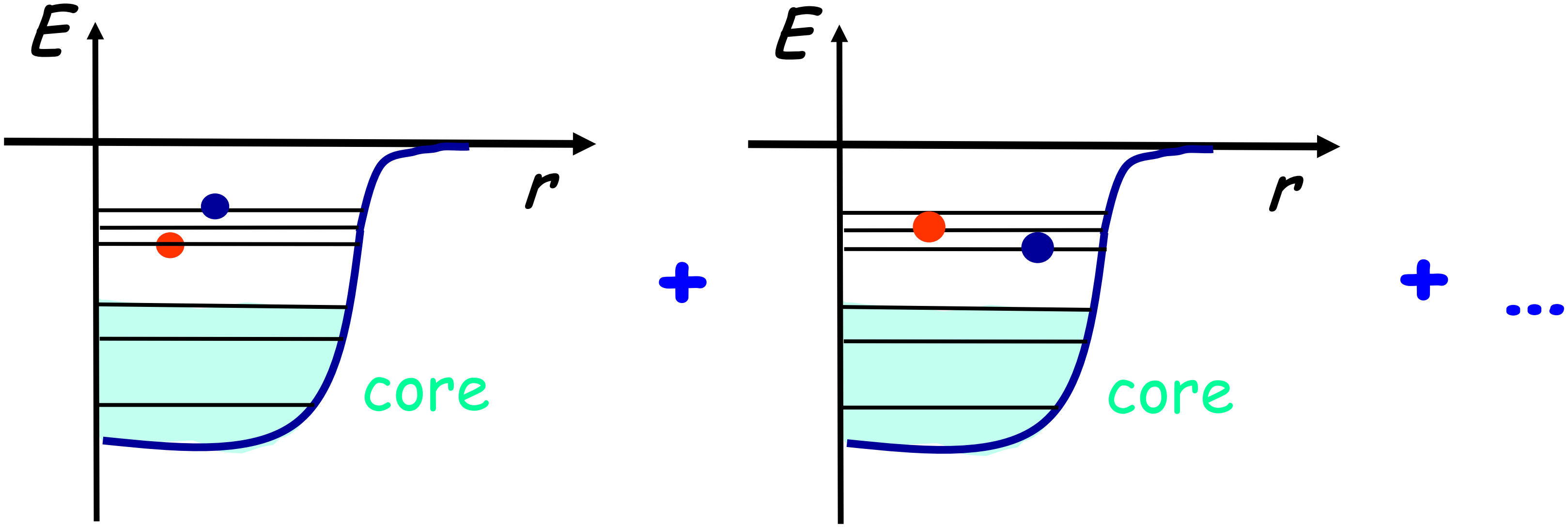
$$H = \begin{bmatrix} <\phi_1 |H| \phi_1> & <\phi_1 |H| \phi_2> & <\phi_1 |H| \phi_3> & \ldots. \\ <\phi_2 |H| \phi_1> & <\phi_2 |H| \phi_2> & <\phi_2 |H| \phi_3> & \ldots. \\ <\phi_3 |H| \phi_1> & <\phi_3 |H| \phi_2> & <\phi_3 |H| \phi_3> & \ldots. \\ <\phi_4 |H| \phi_1> & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix}$$

Shell-model dimension

1 for one valence particle
on top of the core

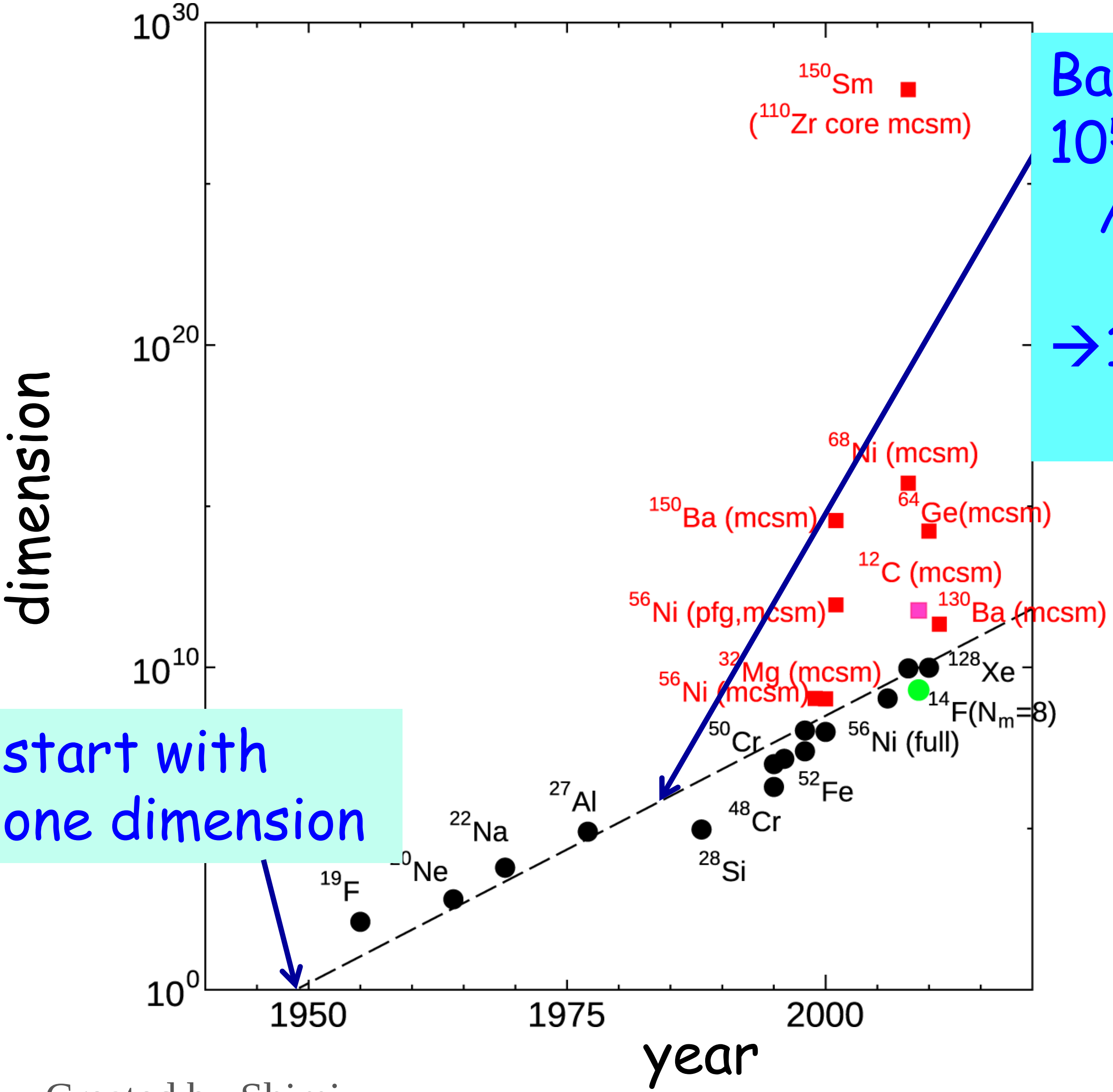
several for two valence particles
on top of the core





many for many valence particles
on top of the core

Increase of shell-model dimension



Basic trend :
10^5 times
/ 30 years

→10 billion dimension
after 60 years

start with
one dimension

black, green
circles :
conventional
shell model

red circles :
Monte Carlo
shell model

Created by Shimizu

About TBME (two-body matrix element)

At the beginning, χ^2 fit is made as usual.

Example : 0^+, 2^+, 4^+ in ^{18}O (oxygen) : d5/2 & s1/2

< d5/2, d5/2, J, T=1 | V | d5/2, d5/2, J, T >,
< d5/2, s1/2, J, T=1 | V | d5/2, d5/2, J, T >, etc.

Arima, Cohen, Lawson and McFarlane (Argonne group) 1968

Later and till now, combination between fit and microscopic calculations is the major way.

Example : USD interaction by Wildenthal & Brown

sd shell d5/2, d3/2 and s1/2

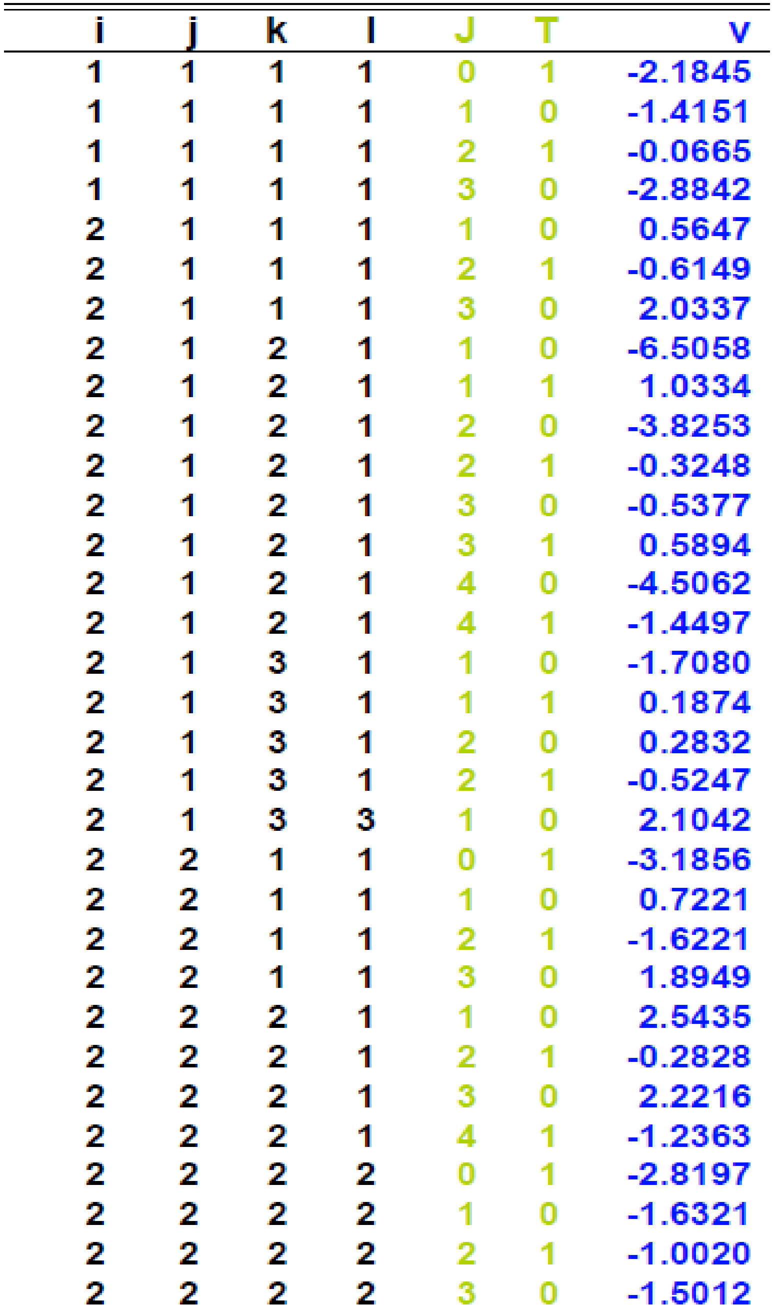
63 matrix elements
3 single particle energies

USD interaction

1 = d3/2

2= d5/2

3= s1/2



i	j	k	l	J	T	V
1	1	1	1	0	1	-2.1845
1	1	1	1	1	0	-1.4151
1	1	1	1	2	1	-0.0665
1	1	1	1	3	0	-2.8842
2	1	1	1	1	0	0.5647
2	1	1	1	2	1	-0.6149
2	1	1	1	3	0	2.0337
2	1	2	1	1	0	-6.5058
2	1	2	1	1	1	1.0334
2	1	2	1	2	0	-3.8253
2	1	2	1	2	1	-0.3248
2	1	2	1	3	0	-0.5377
2	1	2	1	3	1	0.5894
2	1	2	1	4	0	-4.5062
2	1	2	1	4	1	-1.4497
2	1	3	1	1	0	-1.7080
2	1	3	1	1	1	0.1874
2	1	3	1	2	0	0.2832
2	1	3	1	2	1	-0.5247
2	1	3	3	1	0	2.1042
2	2	1	1	0	1	-3.1856
2	2	1	1	1	0	0.7221
2	2	1	1	2	1	-1.6221
2	2	1	1	3	0	1.8949
2	2	2	1	1	0	2.5435
2	2	2	1	2	1	-0.2828
2	2	2	1	3	0	2.2216
2	2	2	1	4	1	-1.2363
2	2	2	2	0	1	-2.8197
2	2	2	2	1	0	-1.6321
2	2	2	2	2	1	-1.0020
2	2	2	2	3	0	-1.5012

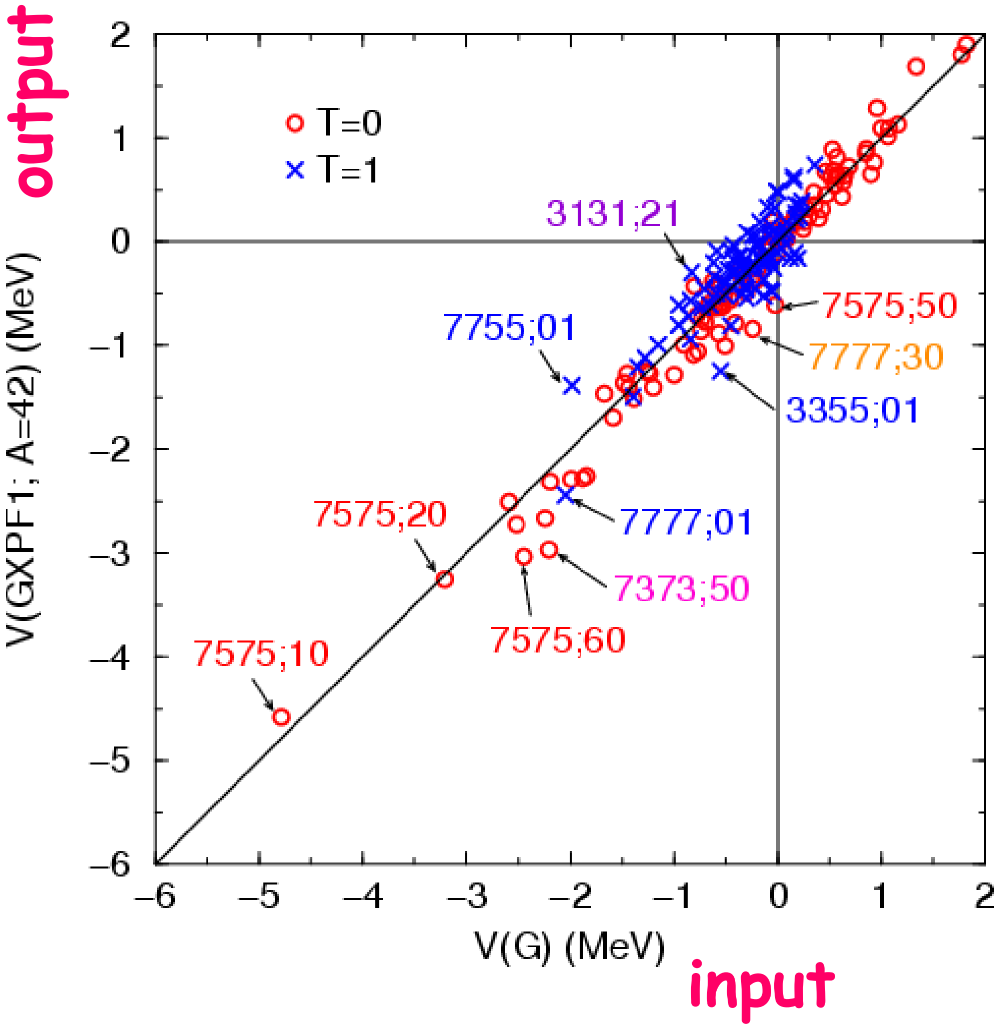
Changes by the fit : big or small ?

TBME

two-body matrix element

$\langle ab;\, JT \,|\, V \,|\, cd\, ;\, JT \rangle$

$7 = f_{7/2},\ \ 3 = p_{3/2},\ \ 5 = f_{5/2},\ \ 1 = p_{1/2}$



By the fit,

- T=0 … more attractive
- T=1 … more repulsive

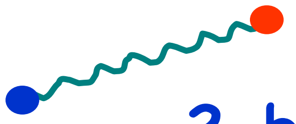
For two-body interaction, our understanding from microscopic basis (i.e. nucleon level) has been advanced enormously

NN interaction	potentials from scattering (Hamada-Johnston to CD-Bonn), EFT, Lattice QCD
Renormalization	G-matrix, SRG, MBPT Renormalization Persistency

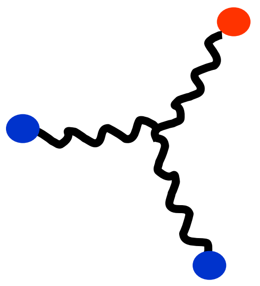
USD family	sd shell
KB3 family	pf shell
GXPF1 family	pf shell
SDPF-M	sd-f7p3
SDPF-U	sd-pf

……

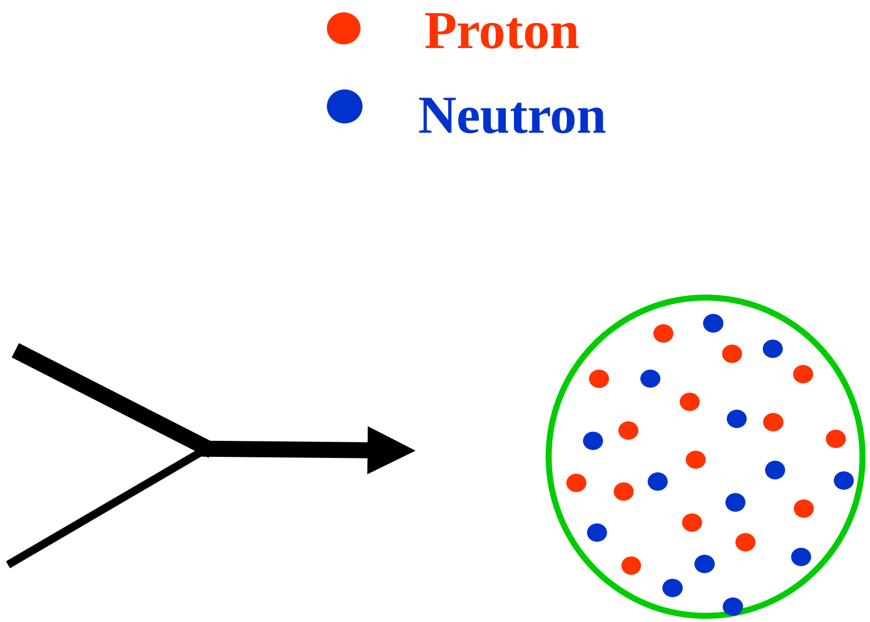
Recent interactions are more independent of fit



2-body interaction



3-body interaction



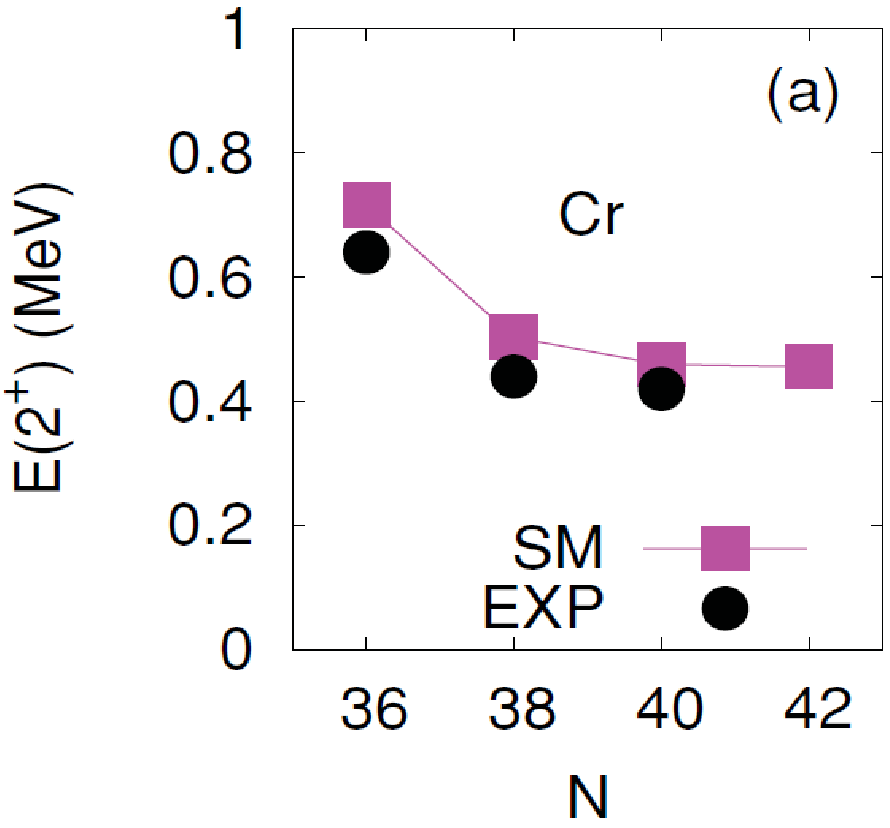
Effects of 3-body interaction are unknown to a larger extent than those of 2-body interaction

→ We still need partial fit

Achievement

selected from recent examples
of conventional shell-model calculations

A frontier of shell-model calculation :



2^+ level of Cr isotopes calculated by the Strasbourg+Madrid group.
Model space: full pf for proton, f5, p3, p1, g9, d5 for neutron with 14p-14h truncation (from Z=28 N=40 config.). Up to 10^{10} m-scheme dimension in Fe.

S. M. Lenzi, F. Nowacki, A. Poves, and K. Sieja, Phys. Rev. C 82, 054301 (2010).

Courtesy of Utsuno

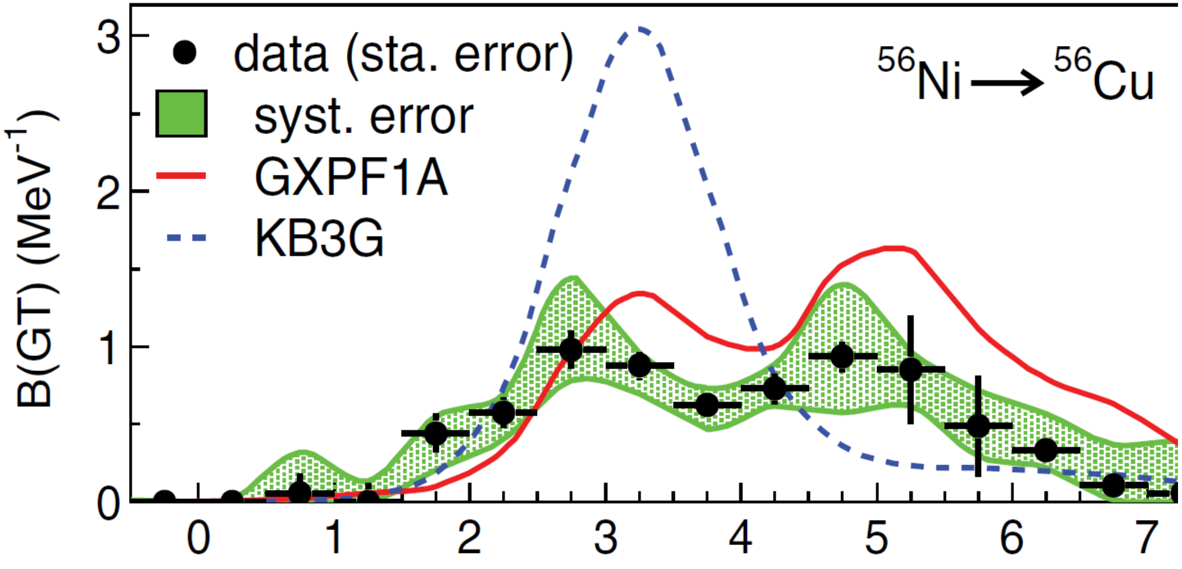
^{56}Ni GT- → ^{56}Cu = ^{56}Ni GT+ → ^{56}Co

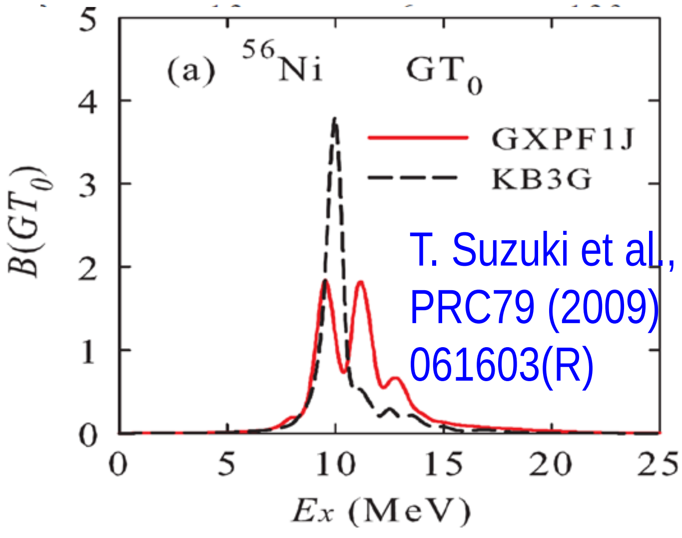
e-capture rate at supernovae

PRL **107,** 202501 (2011) PHYSICAL REVIEW LETTERS 2011

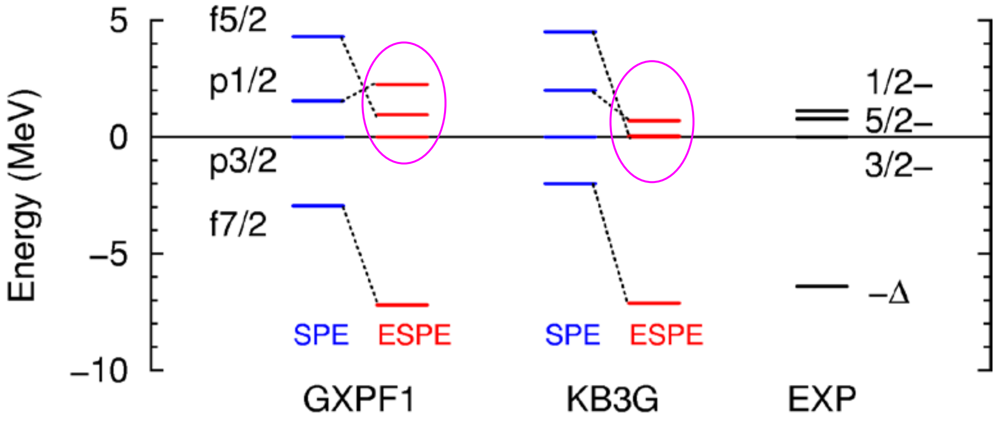
Gamow-Teller Transition Strengths from ^{56}Ni

M. Sasano,[1,2] G. Perdikakis,[1,2] R. G. T. Zegers,[1,2,3] Sam M. Austin,[1,2] D. Bazin,[1] B. A. Brown,[1,2,3] C. Caesar,[4] A. L. Cole,[5]





T. Suzuki et al., PRC79 (2009) 061603(R)



Courtesy of Honma

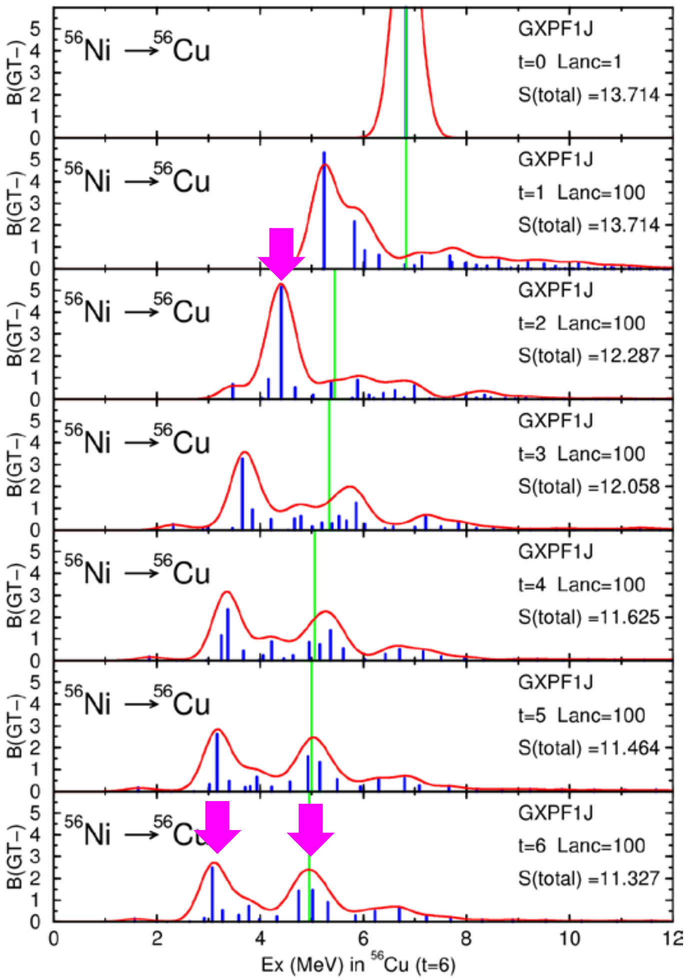
^{56}Ni GT- → ^{56}Cu

Correlations generate double peaks

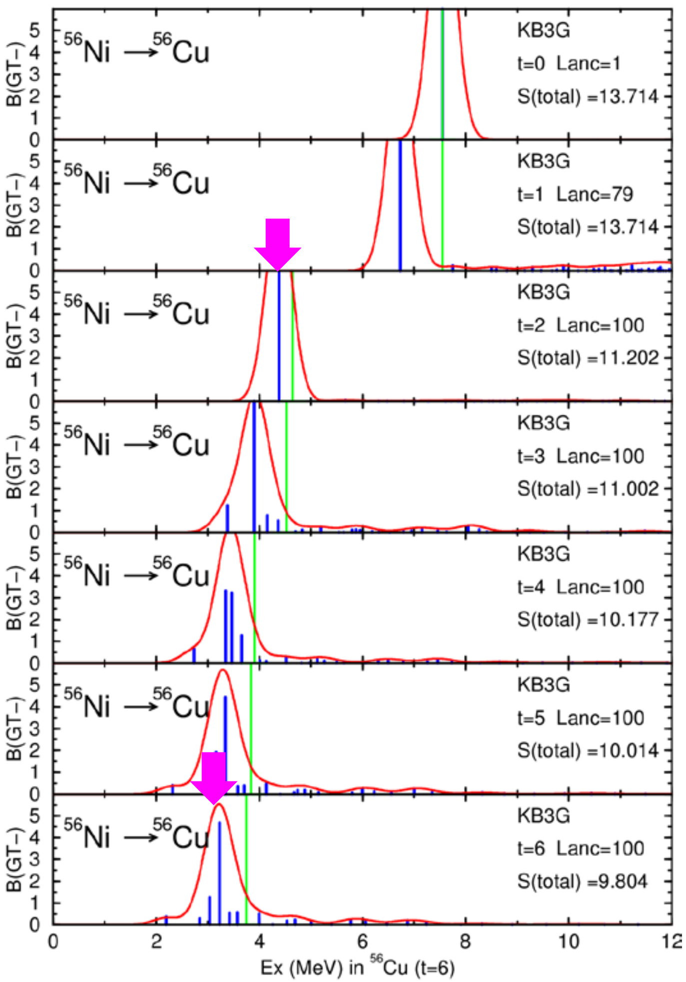
- Truncation by $f_{7/2}$ core excitation : $(f_{7/2})^{16-t}\,(p_{3/2},f_{5/2},p_{1/2})^{t}$
- Double-peak structure appears for $t \geq 3$ 2p-2h crucial ?

more excitations from $f_{7/2}$

GXPF1J

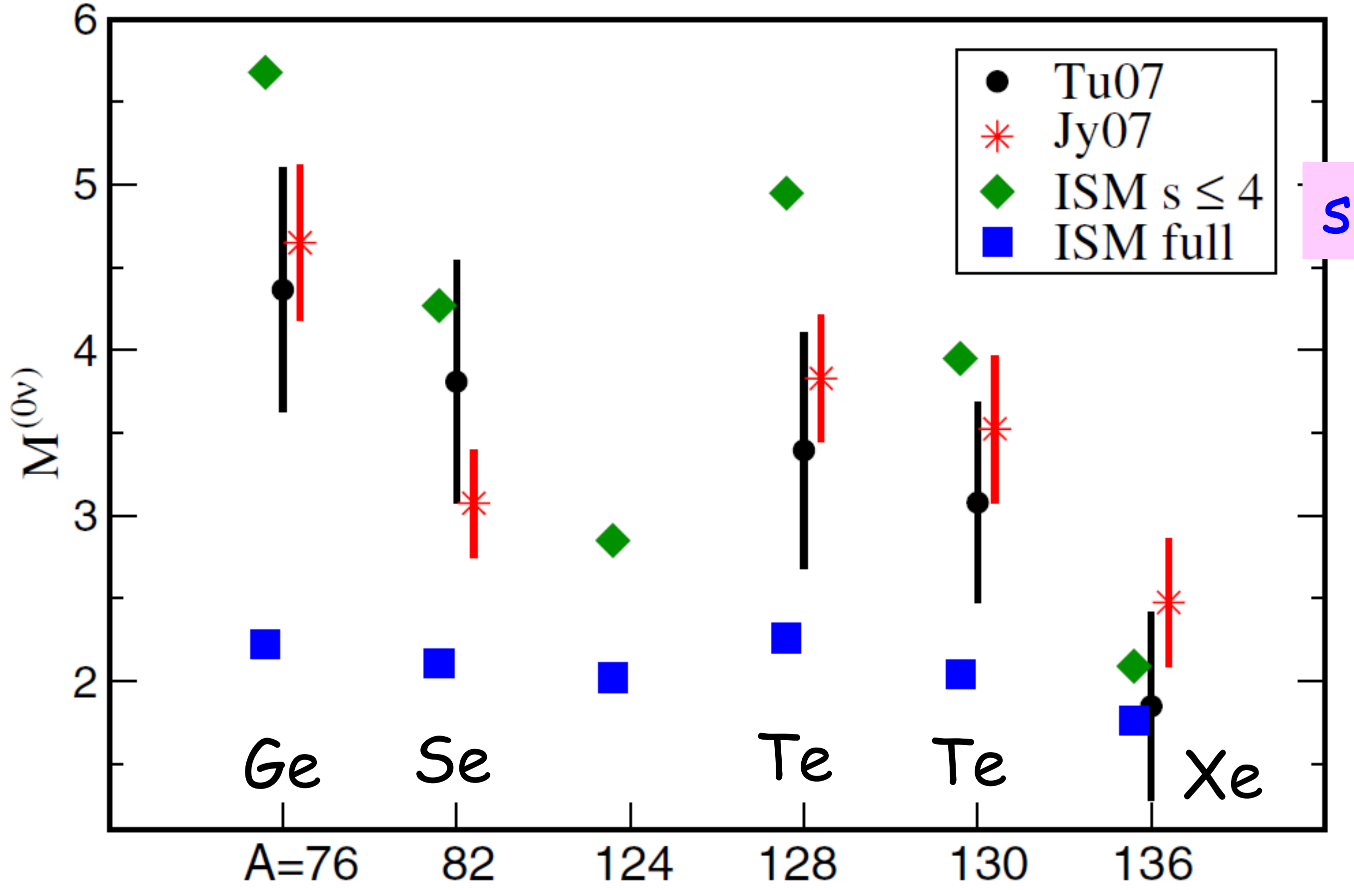


KB3G



Courtesy of Homma

Applications : Double beta decay



shell model

Comparison of neutrinoless double beta decay nuclear matrix elements Between QRPA calculations and the shell model.
Tu07 and Jy07: QRPA by different groups; ISM: shell model
(green is truncated calculation up to seniority=4)

E. Caurier, J. Menendez, F. Nowacki, and A. Poves, PRL 100, 052503 (2008).

Courtesy of Utsuno

Paradigm

Paradigms

Paradigm 1 - Foundation of Shell Model -
Shell model works even if full microscopic basis is not given (for ever or for the moment).

It is still missing to derive shell model from the first principle. *Needed?* *Possible?*
ab initio calculations may give us answer or hint (*skipped*).

Paradigm 2 - Robustness of shell structure -
Shell structure conceived by Mayer and Jensen is robust, and should be valid to basically all nuclei.

This has been one of the focuses of RI-beam physics in recent years. It seems that this paradigm should be changed
all nuclei → all stable nuclei
→ *Next slides*

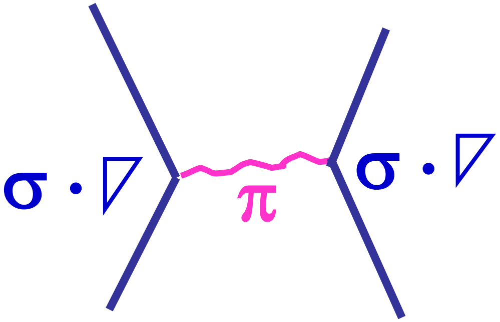
Tensor force

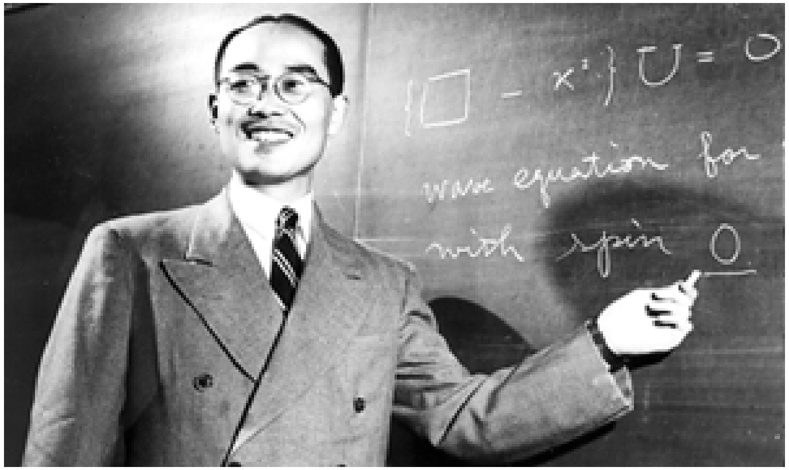
$$V_T = (\tau_1\tau_2)\,(\,[\sigma_1\sigma_2]^{(2)}\,Y^{(2)}(\Omega)\,)\,Z(r)$$

contributes
only to **S=1** states

relative motion

π meson : primary source





Yukawa

ρ meson (~ π+π) : minor (~1/4) cancellation

Ref: Osterfeld, Rev. Mod. Phys. 64, 491 (92)

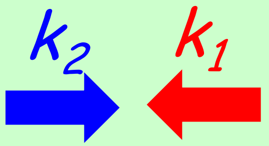
Monopole component of tensor force

- An intuitive picture -

TO, Suzuki, et al.
PRL 95, 232502

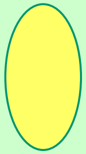
At collision point:

$$\Psi \propto e^{ik_1x_1}e^{ik_2x_2} + e^{ik_2x_1}e^{ik_1x_2} = 2\,e^{iKX}\cos(kx)$$



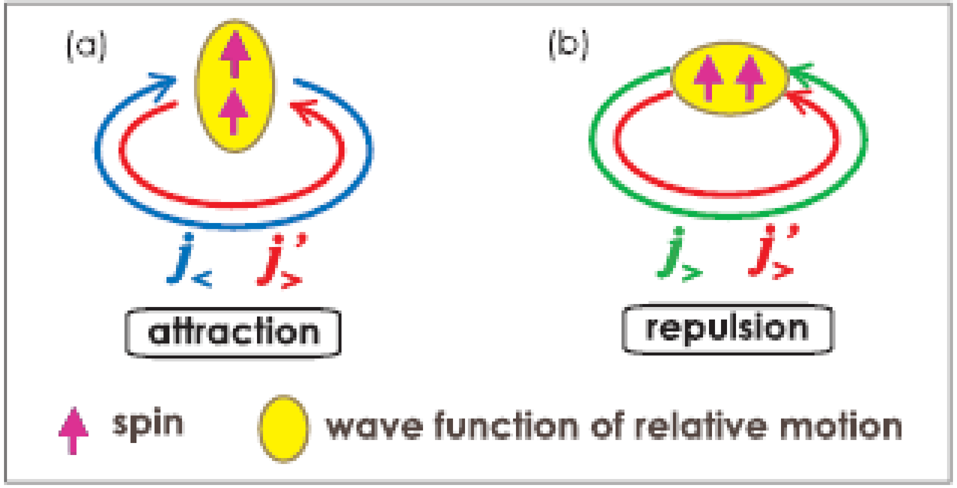
large relative momentum k

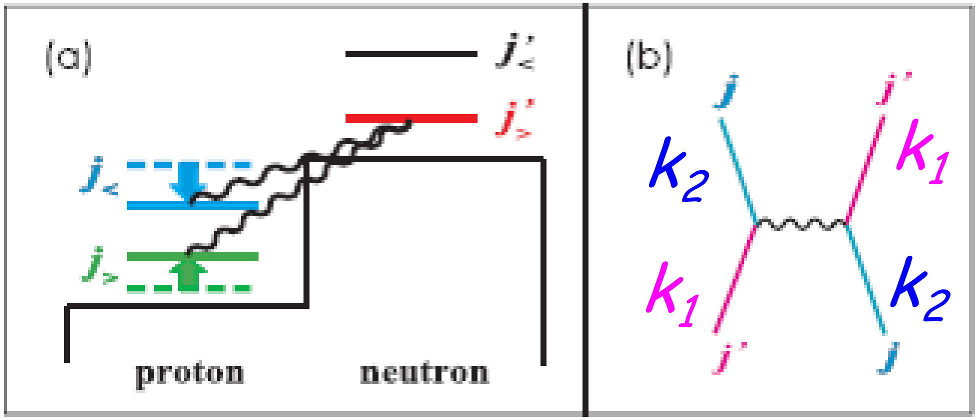
strong damping

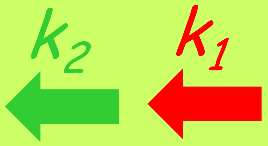


wave function of relative coordinate

$$k = k_1 - k_2\,,\quad K = k_1 + k_2$$

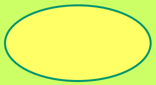






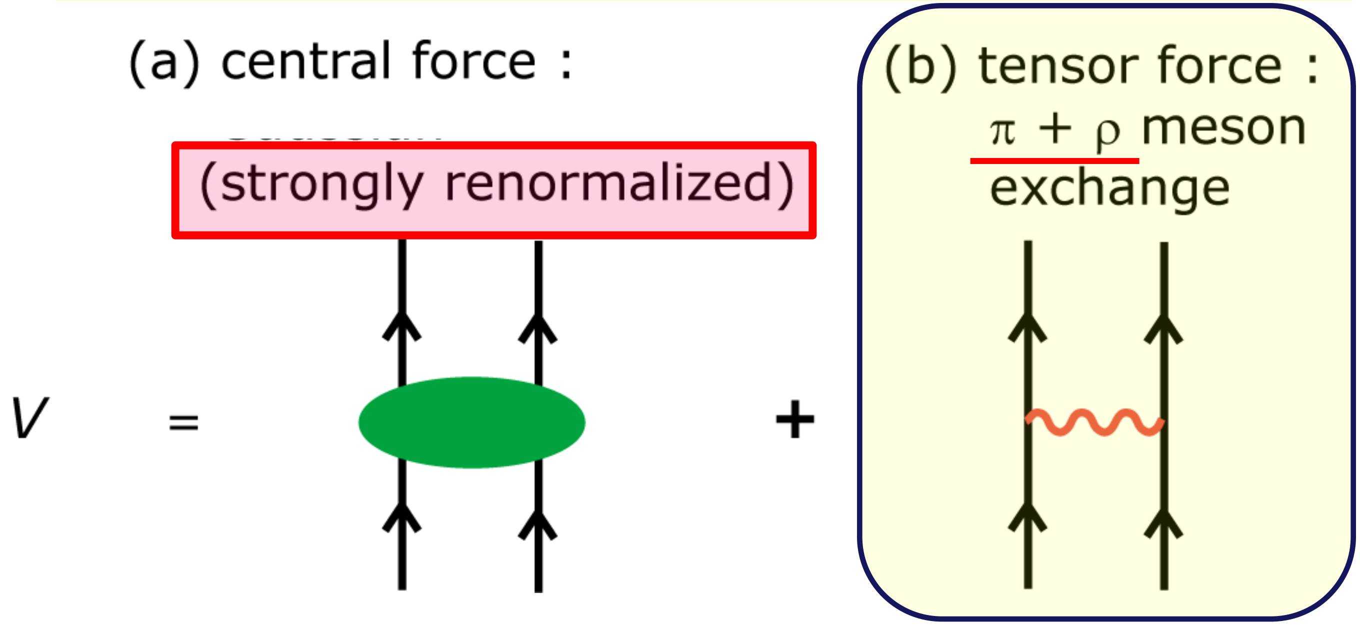
small relative momentum k

loose damping



wave function of relative coordinate

Two major components in nuclear force



monopole component of

tensor force in nuclear medium

Renormalization Persistency

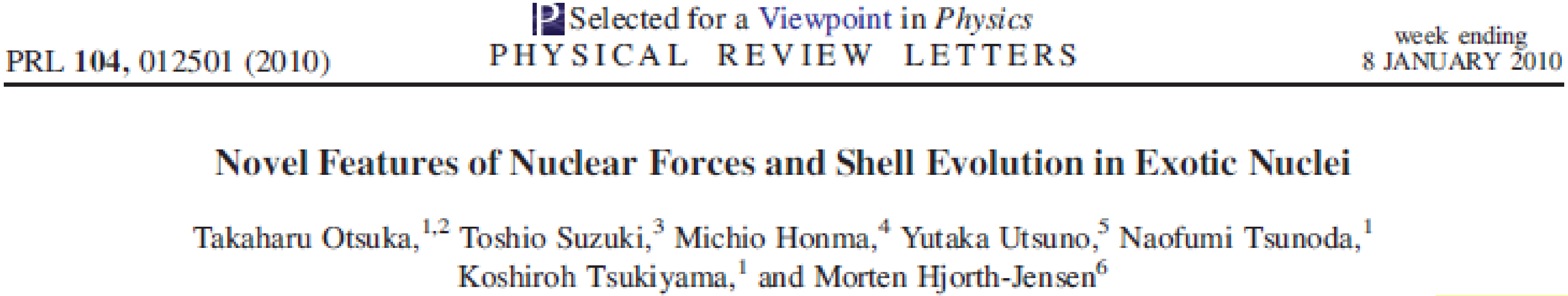
almost equal (no renormalization)

monopole component of

tensor force in free space

Tsunoda, O,
Tsukiyama,
H.-Jensen, PRC (2011)

Shell evolution in exotic nuclei due to tensor + central forces
tensor → sharp local variation


Selected for a Viewpoint in *Physics*

PRL 104, 012501 (2010) PHYSICAL REVIEW LETTERS week ending 8 JANUARY 2010

Novel Features of Nuclear Forces and Shell Evolution in Exotic Nuclei

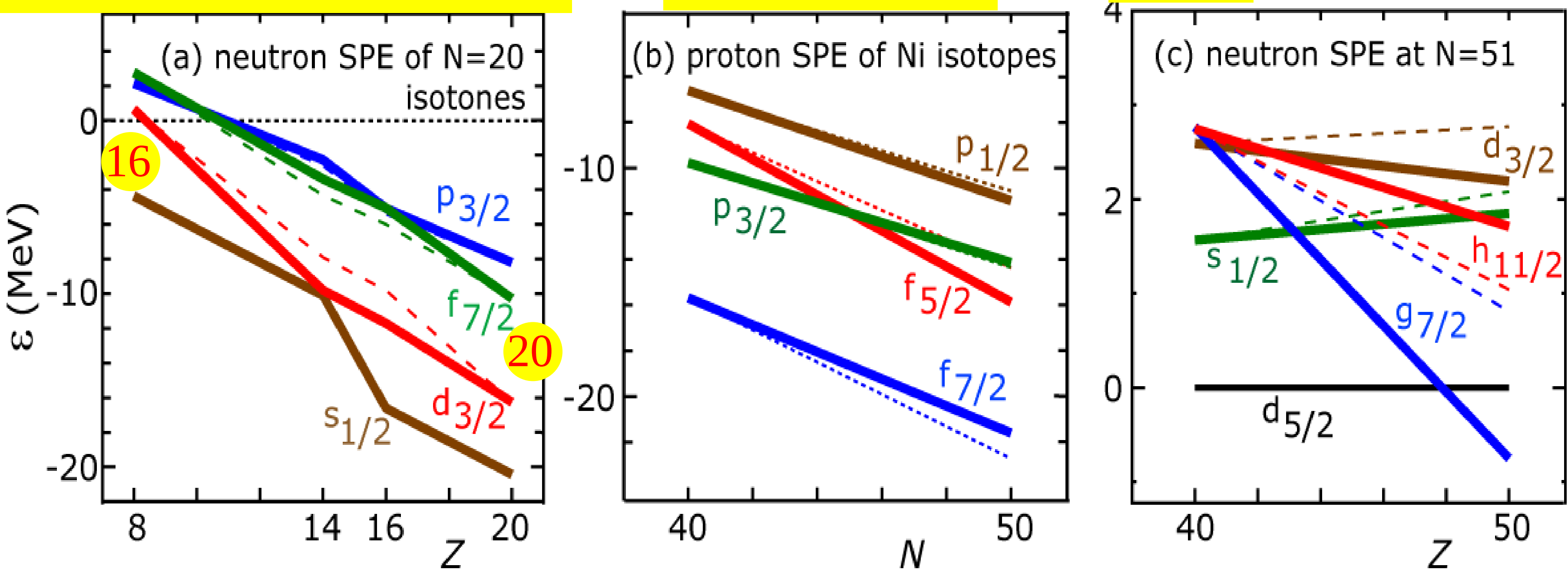
Takaharu Otsuka,[1,2] Toshio Suzuki,[3] Michio Honma,[4] Yutaka Utsuno,[5] Naofumi Tsunoda,[1]
Koshiroh Tsukiyama,[1] and Morten Hjorth-Jensen[6]

N~20 island of inversion

^{68}Ni → ^{78}Ni

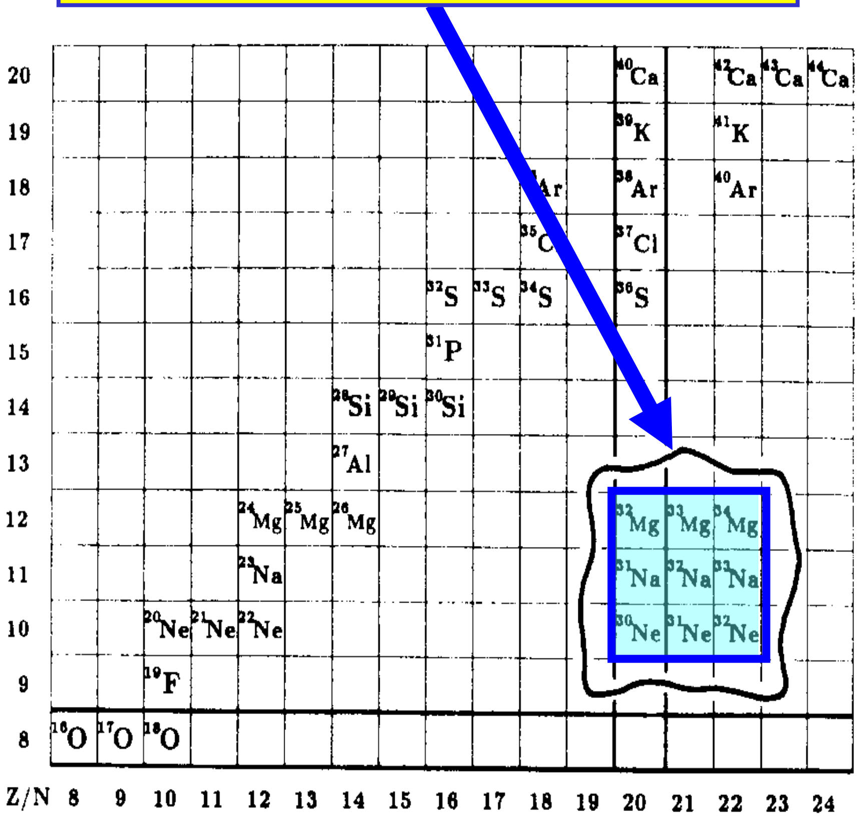
^{90}Zr

^{100}Sn



+ proton $h_{11/2}$-$g_{7/2}$ Sb isotopes + …

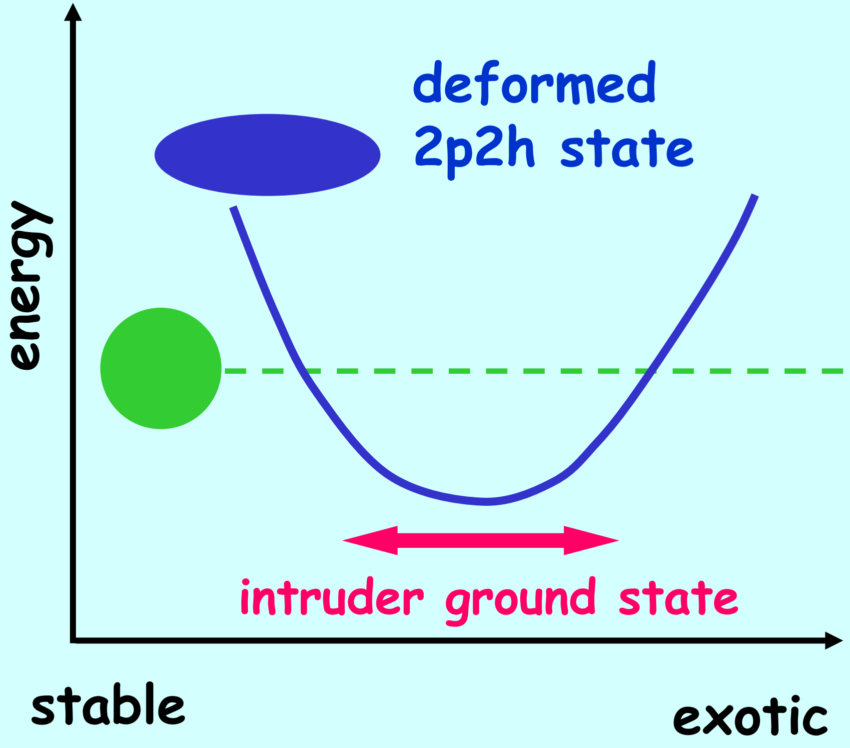
Island of Inversion

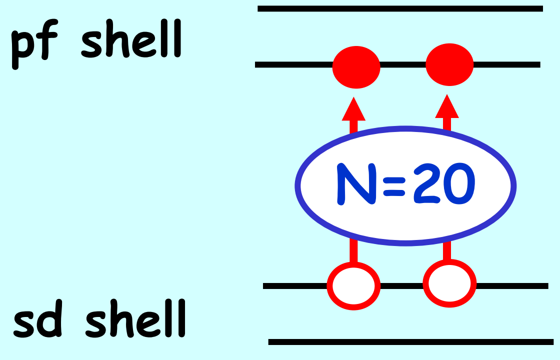


9 nuclei:
Ne, Na, Mg with N=20-22

Phys. Rev. C 41, 1147 (1990),
Warburton, Becker and
Brown

Basic picture was





gap ~
constant

What is the boundary (shape) of the Island of Inversion ?

- Are there clear boundaries in all directions ?

- Is the Island really like the square ?

Which type of boundaries ?


Shallow
(diffuse & extended)


Steep (sharp)

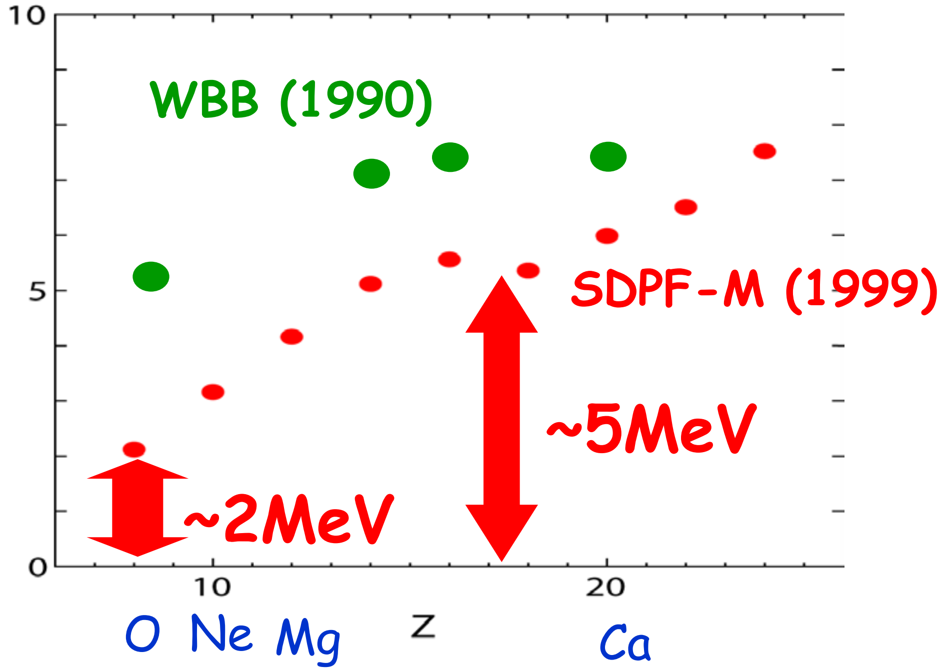

Straight lines

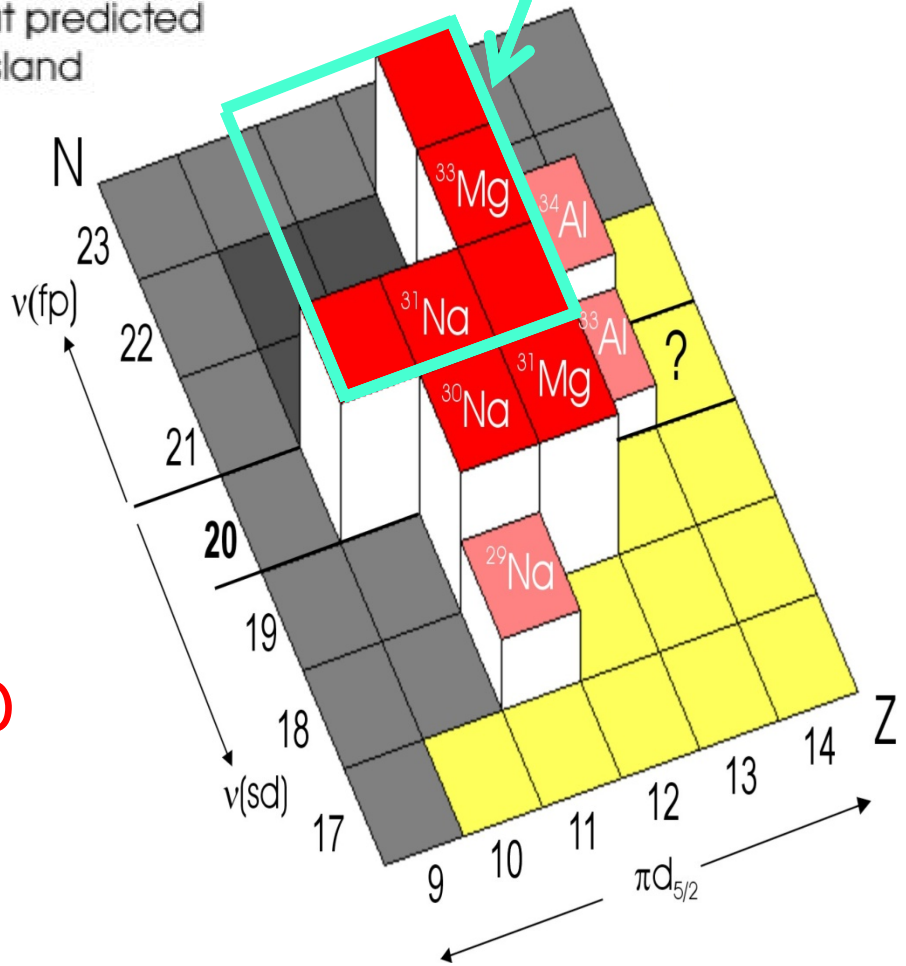
Re-definition of Island of Inversion

- 🟥 Intruder dominates
- 🟧 Mixed
- 🟨 Normal dominates
- ▨ Unknown
- ■ Unknown but predicted inside the island

Original Island of Inversion

Physics behind :
Changing N=20 gap
between sd and pf shells





From Himpe *et al.*,
Phys. Lett. B658, 203 (2008)

~~Large f = Large Gap~~

~~Sharp boundary, small territory~~

The gap changes due to shell evolution


Iou jima

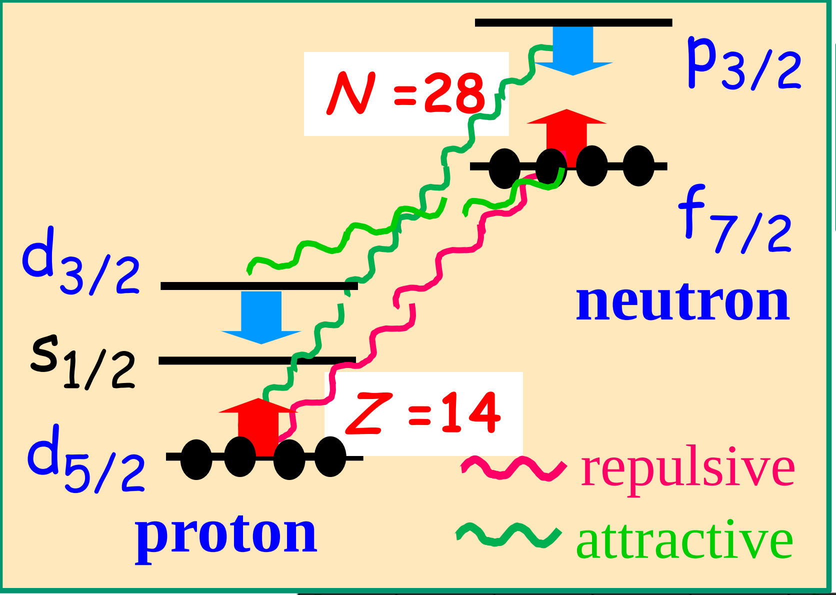

Chubu airport

Smaller f = Smaller Gap

Diffuse boundary, wide territory


Borabora

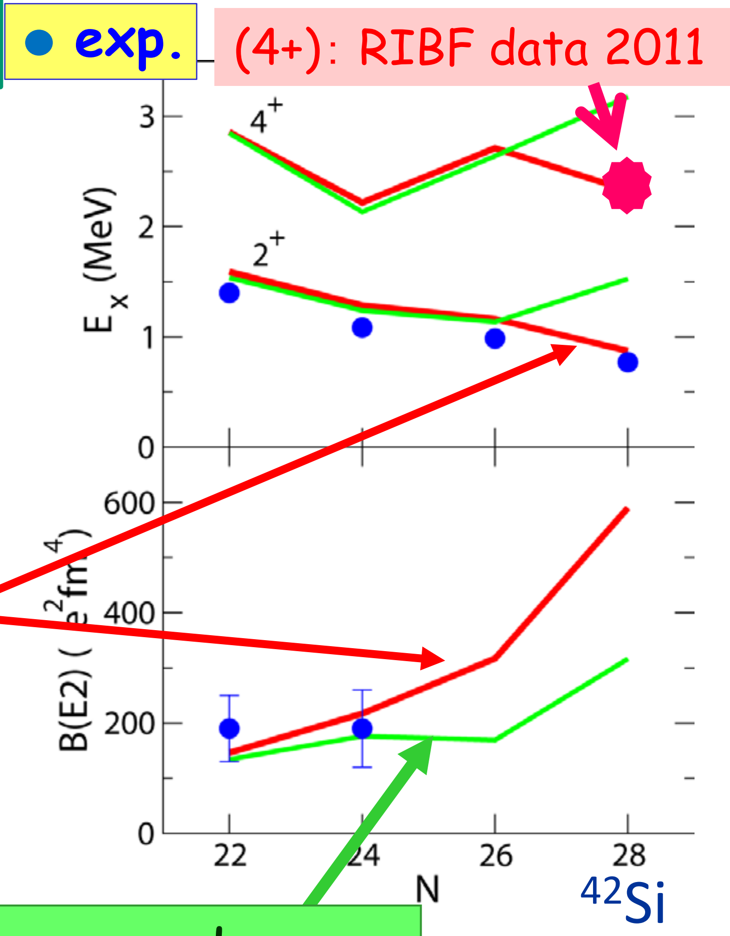
Island of inversion is like a coral reef paradise !



$^{42}_{14}\mathrm{Si}_{28}$

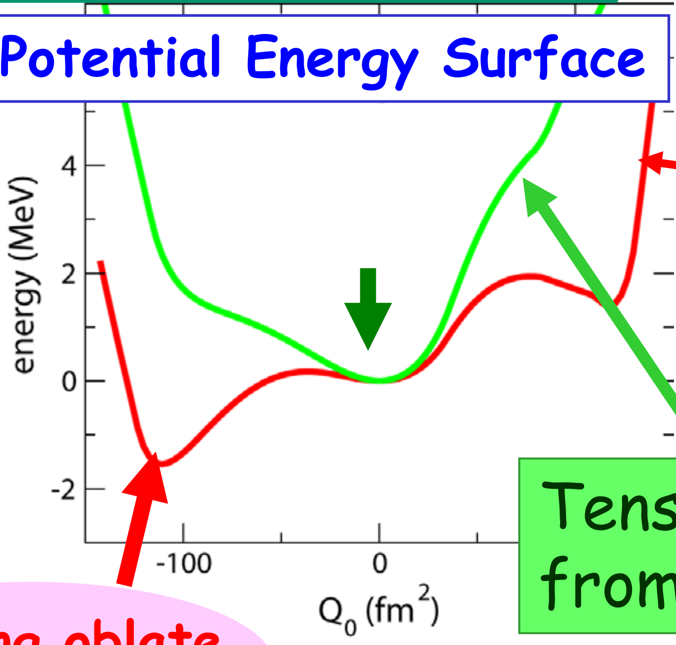
doubly magic ?

Otsuka, Suzuki and Utsuno, Nucl. Phys. A805, 127c (2008)



(4+): RIBF data 2011

Potential Energy Surface



full

Tensor force removed from cross-shell interaction

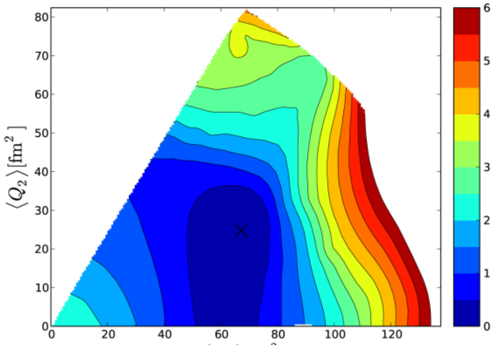
Strong oblate deformation

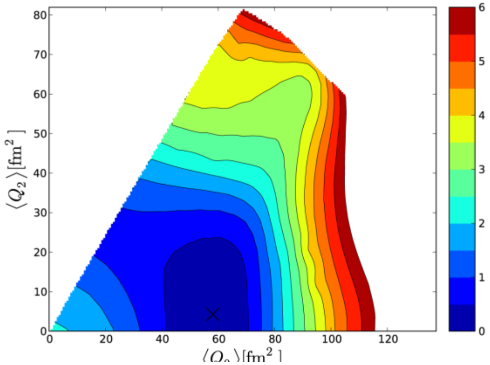
Other calculations (RMF, Gogny) show oblate shape.

^{42}Si 2+: *Bastin, Grévy et al., PRL 99 (2007) 022503* with tensor in sd-pf

without tensor in sd-pf

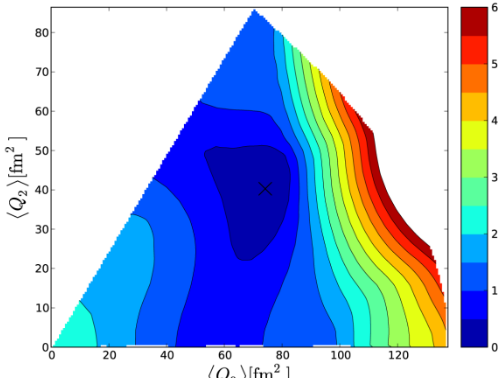
38 Si

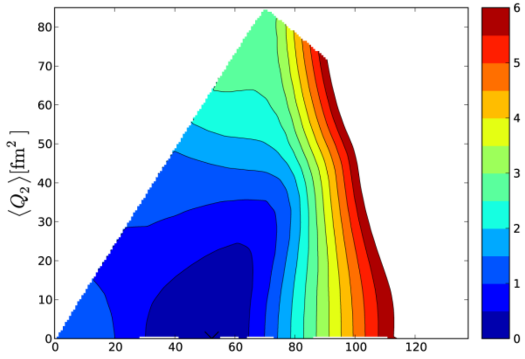




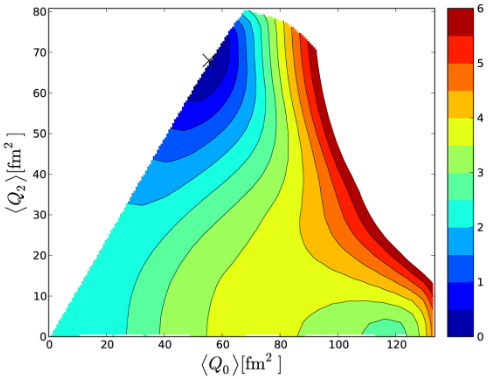
Clean textbook example of Jahn-Teller effect on shape change

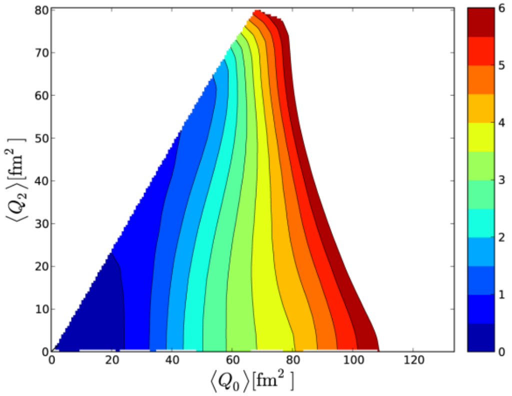
40 Si





42 Si





Underlying robust mechanism ?

Primary mean effect of proton – neutron correlation is modeled by

- $f\ Q_0$ (proton) * Q_0 (neutron)

Q_0 : quadrupole moment

Max {Q_0 (proton) * Q_0 (neutron) }

→ shape of ground state

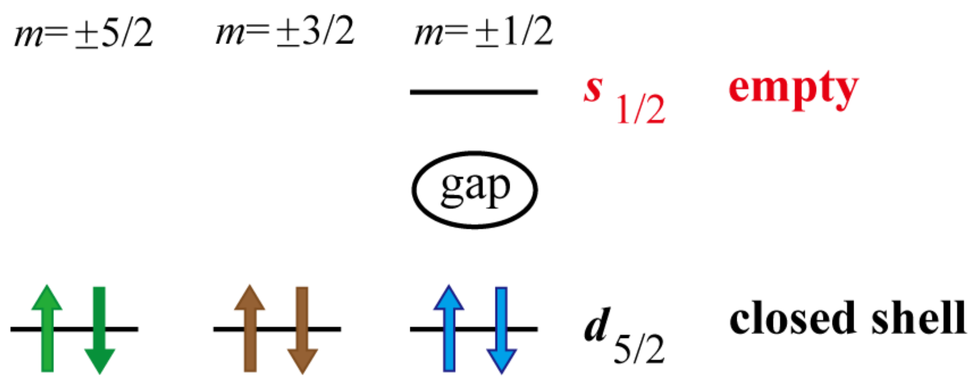
In many stable nuclei, $Q_0 > 0$ **prolate dominance**

A question : Also true for exotic nuclei ?

Why oblate deformation in ^{42}Si ground state ?

Proton wave function of intrinsic state with axial symmetry

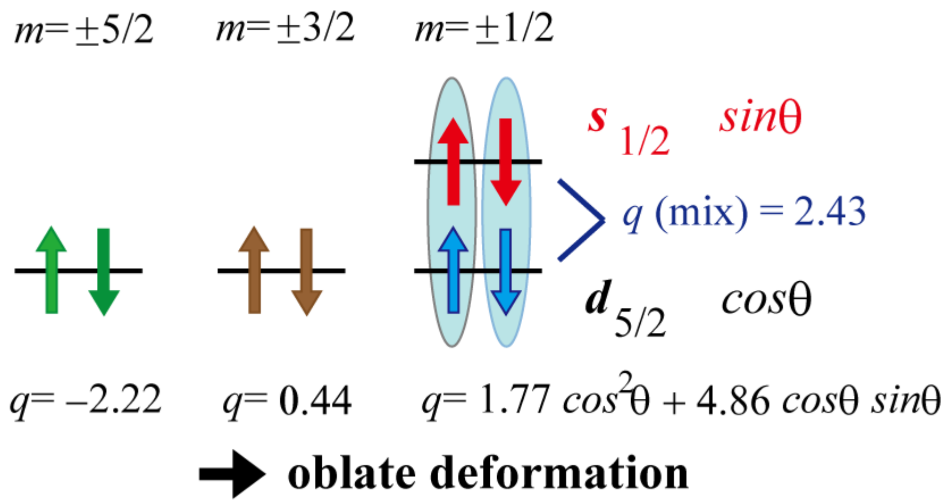
(a) large gap (no tensor effect)



Spherical magic

$Q_0 = 0$

(b) small or no gap (strong tensor effect)



Oblate shape

intrinsic quadrupole moment

$$Q_0 = 2 \{ q(m=5/2) + q(m=3/2) + \cos^2\theta \; q(m=1/2) \} + 4 \cos\theta \sin\theta \; q(\mathrm{mix})$$

$\{ \ldots \} < 0$ for $\cos^2\theta < 1$

$|Q_0|$ larger, if $Q_0 < 0$ (oblate)

future

new aspect of forces

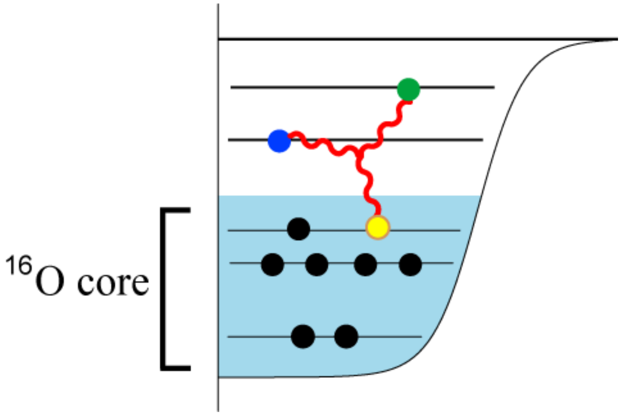
faster computers with advanced methodologies

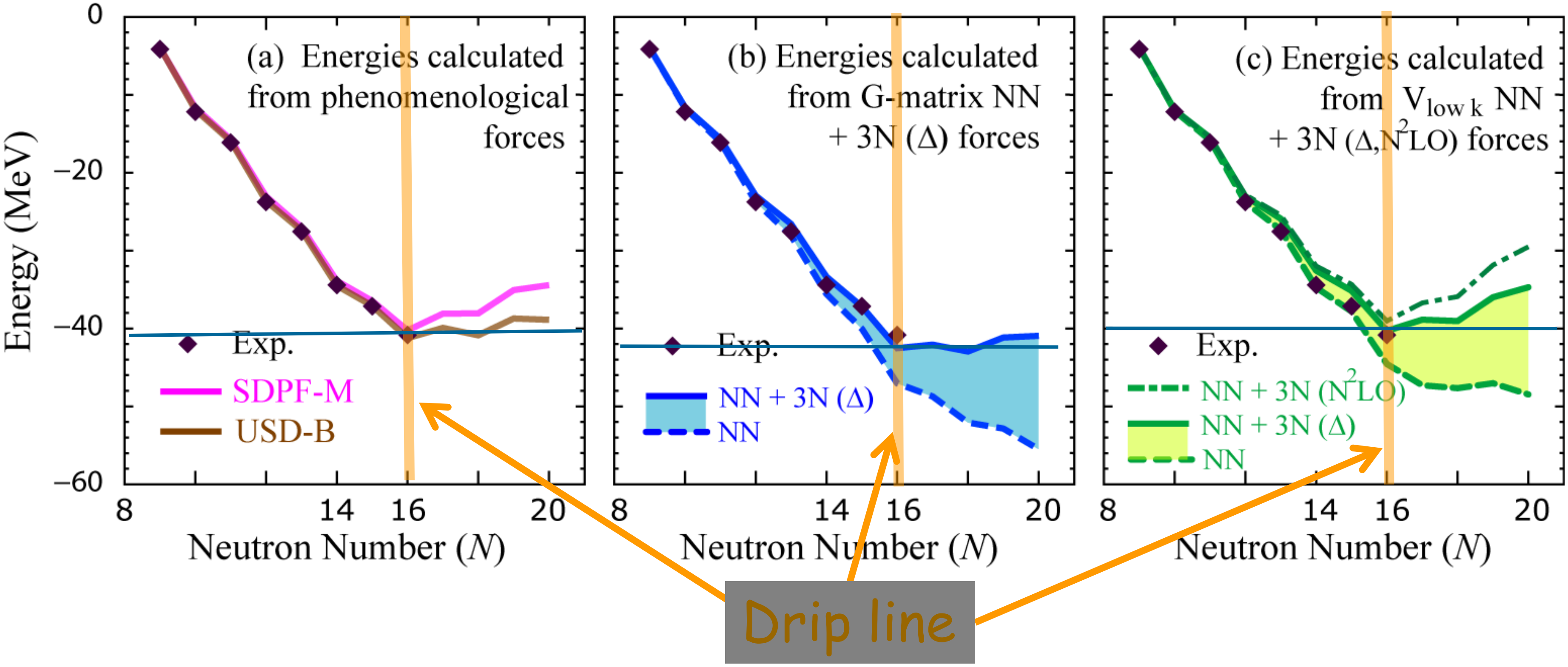
3-body forces

Ground-state energies of oxygen isotopes

NN force + *3N*-induced *NN* force

(Fujita-Miyazawa force)





Drip line

Monte Carlo Shell Model calculations

$$H = \begin{pmatrix} \ast & \ast & \ast & \ast & \ast & \cdot & \cdot \\ \ast & \ast & \ast & \ast & \cdot & \cdot & \cdot \\ \ast & \ast & \ast & \cdot & \cdot & \cdot & \\ \ast & \ast & \cdot & \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdot & & & \\ \cdot & & & & & & \end{pmatrix} \xrightarrow{\text{diagonalization}} \begin{pmatrix} \varepsilon_1 & & & & & 0 \\ & \varepsilon_2 & & & & \\ & & \varepsilon_3 & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ 0 & & & & & \cdot \end{pmatrix}$$

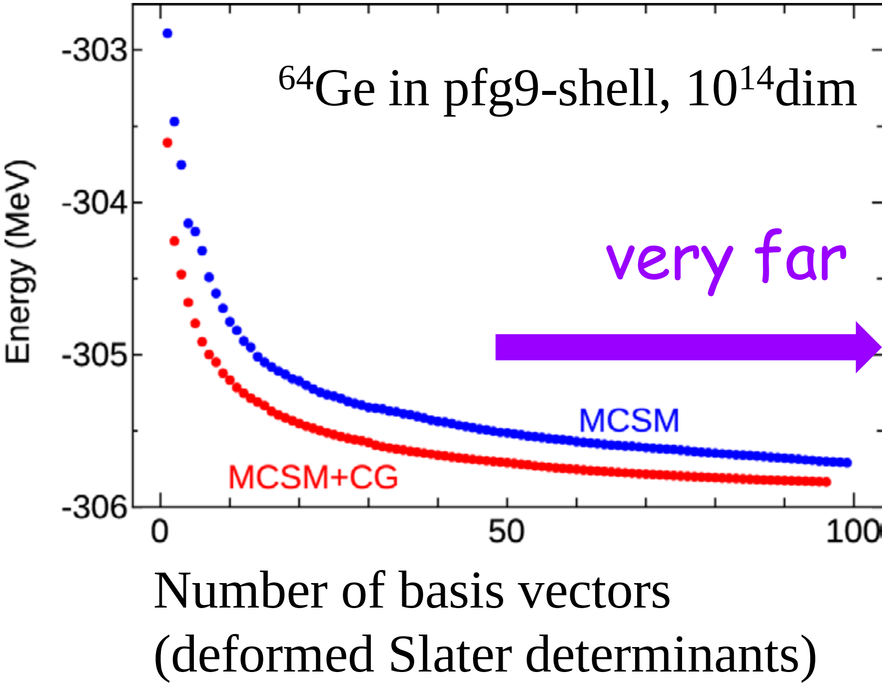
Conventional Shell Model
all Slater determinants

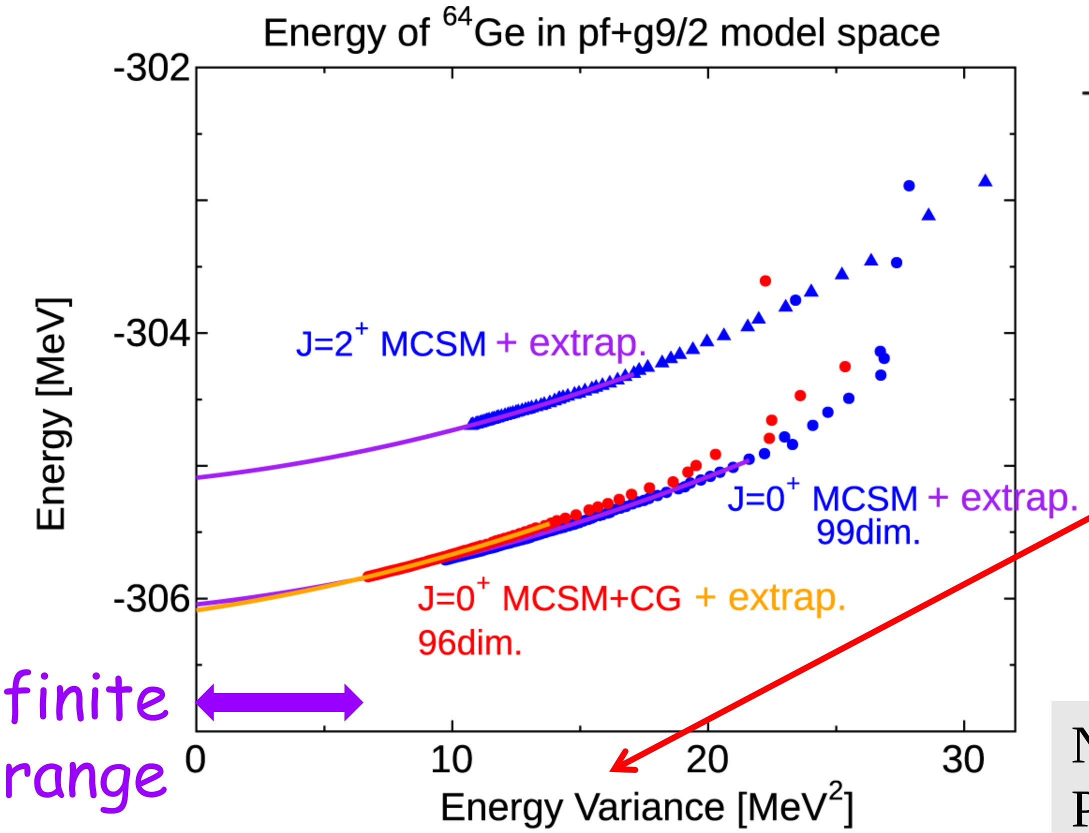
$$H \approx \begin{pmatrix} \ast & \ast & \ast & \cdot \\ \ast & \ast & \ast & \cdot \\ \ast & \ast & \cdot & \\ \cdot & \cdot & & \end{pmatrix} \xrightarrow{\text{diagonalization}} \begin{pmatrix} \varepsilon'_1 & & 0 \\ & \varepsilon'_2 & \\ & & \cdot \\ 0 & & \cdot \end{pmatrix}$$

Monte Carlo Shell Model
bases important for a specific eigenstate

optimized basis vectors selected by quantum Monte Carlo
and by variational method

Energy minimization by Conjugate Gradient method + Energy Variance Extrapolation



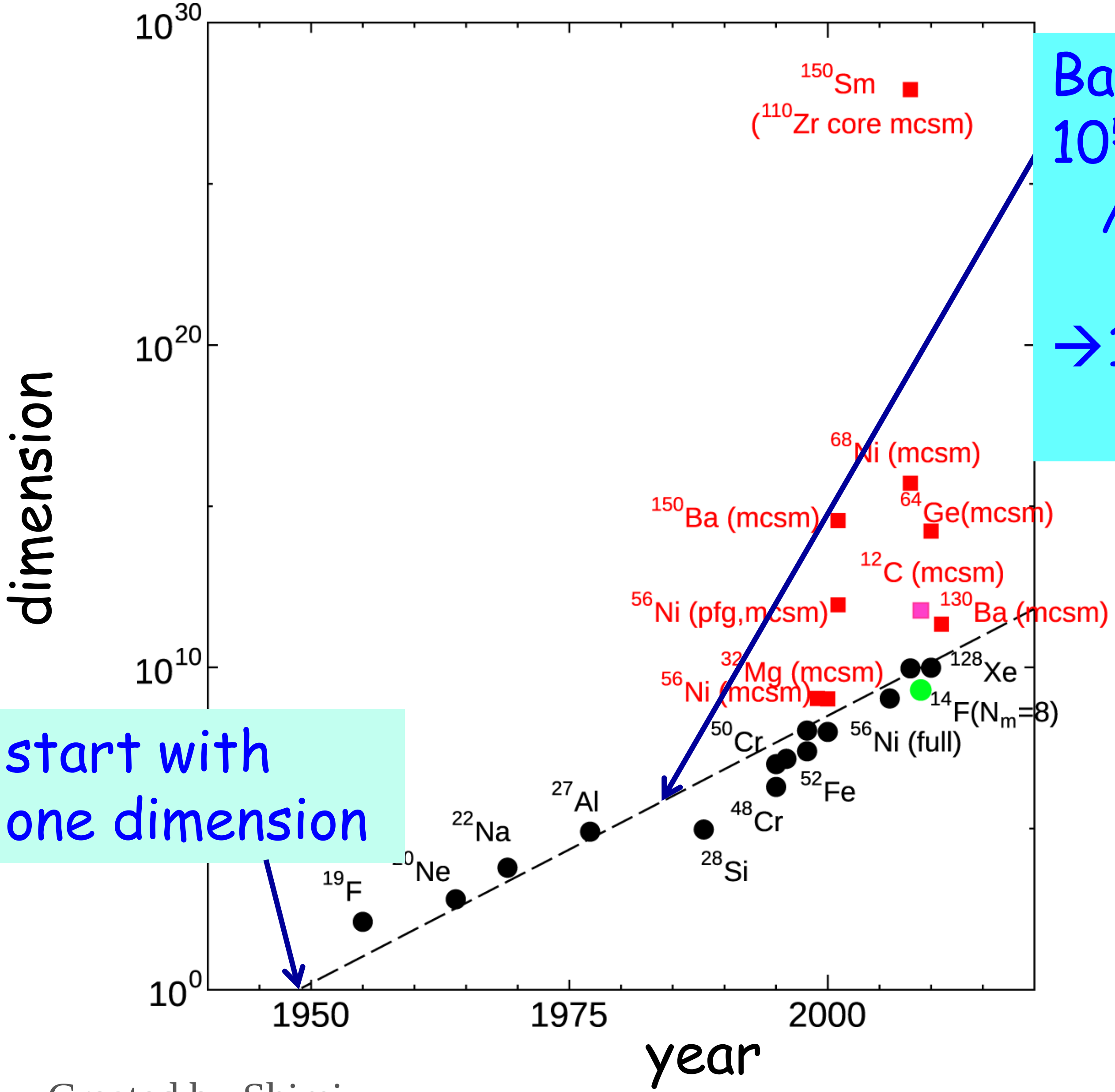


Variance : $\langle \Delta H^2 \rangle = \langle H^2 \rangle - \langle H \rangle^2$

$$\langle H \rangle = E_0 + a\langle \Delta H^2 \rangle + b\langle \Delta H^2 \rangle^2 + \ldots$$

N. Shimizu, et al.,
Phys. Rev. C **82,** 061305(R) (2010).

Increase of shell-model dimension



Basic trend :
10^5 times
/ 30 years

→10 billion dimension
after 60 years

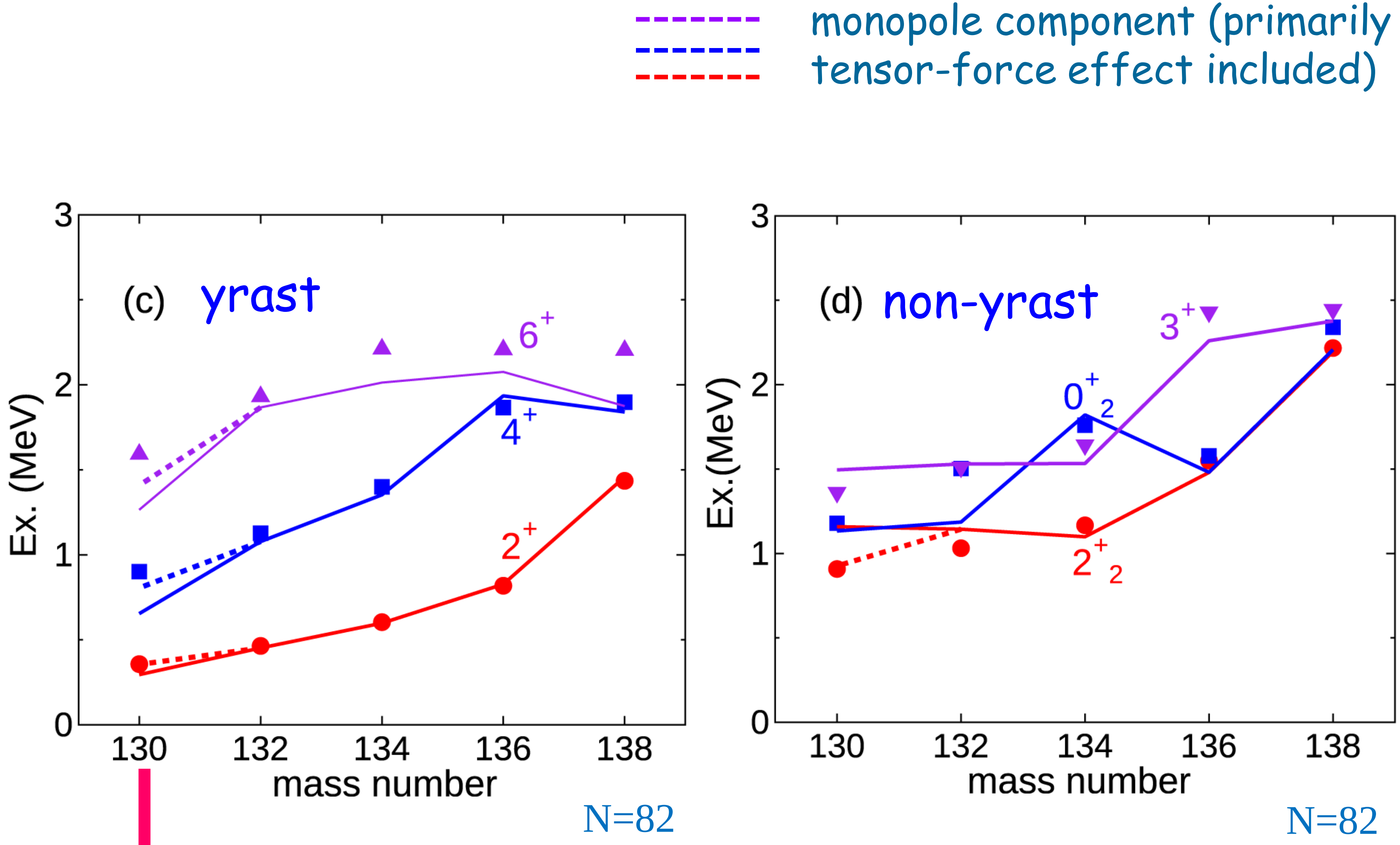
start with
one dimension

black, green
circles :
conventional
shell model

red circles :
Monte Carlo
shell model

Created by Shimizu

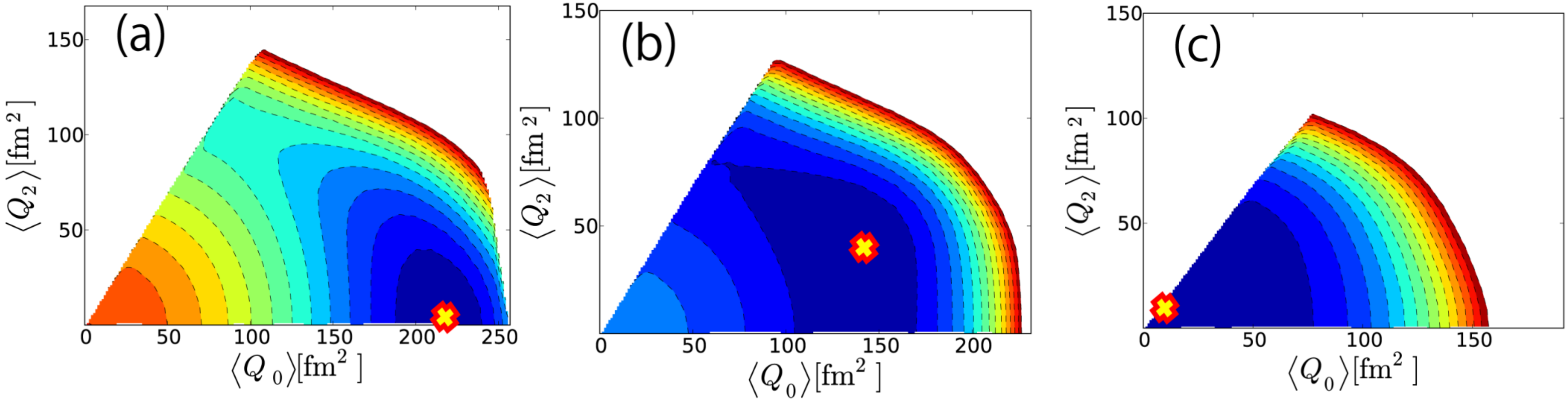
Energy levels of Ba isotopes



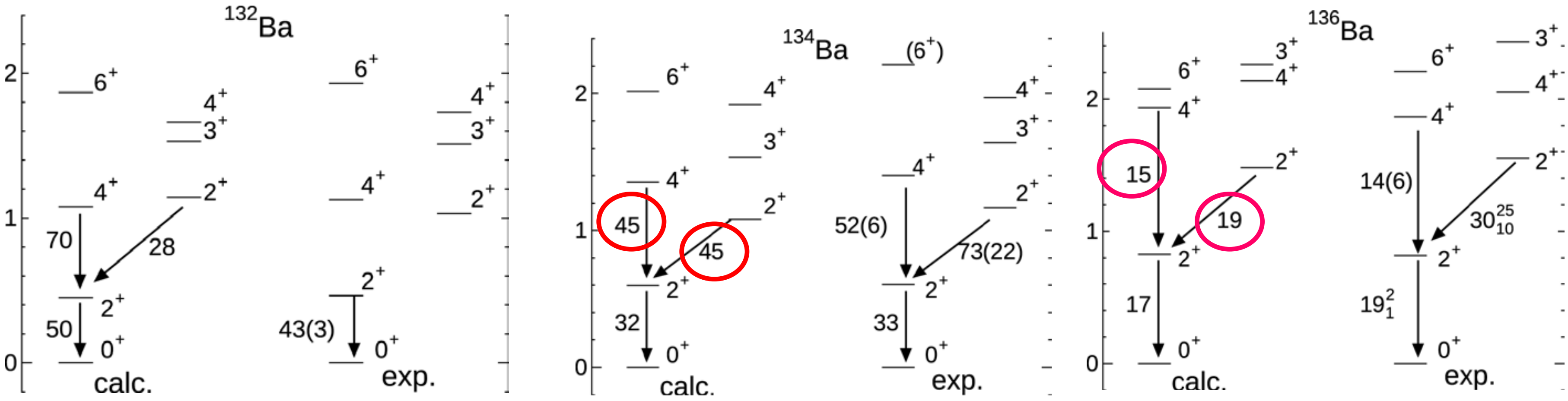
2×10^{11} dimension

Shell model result exhibits rapid shape transition

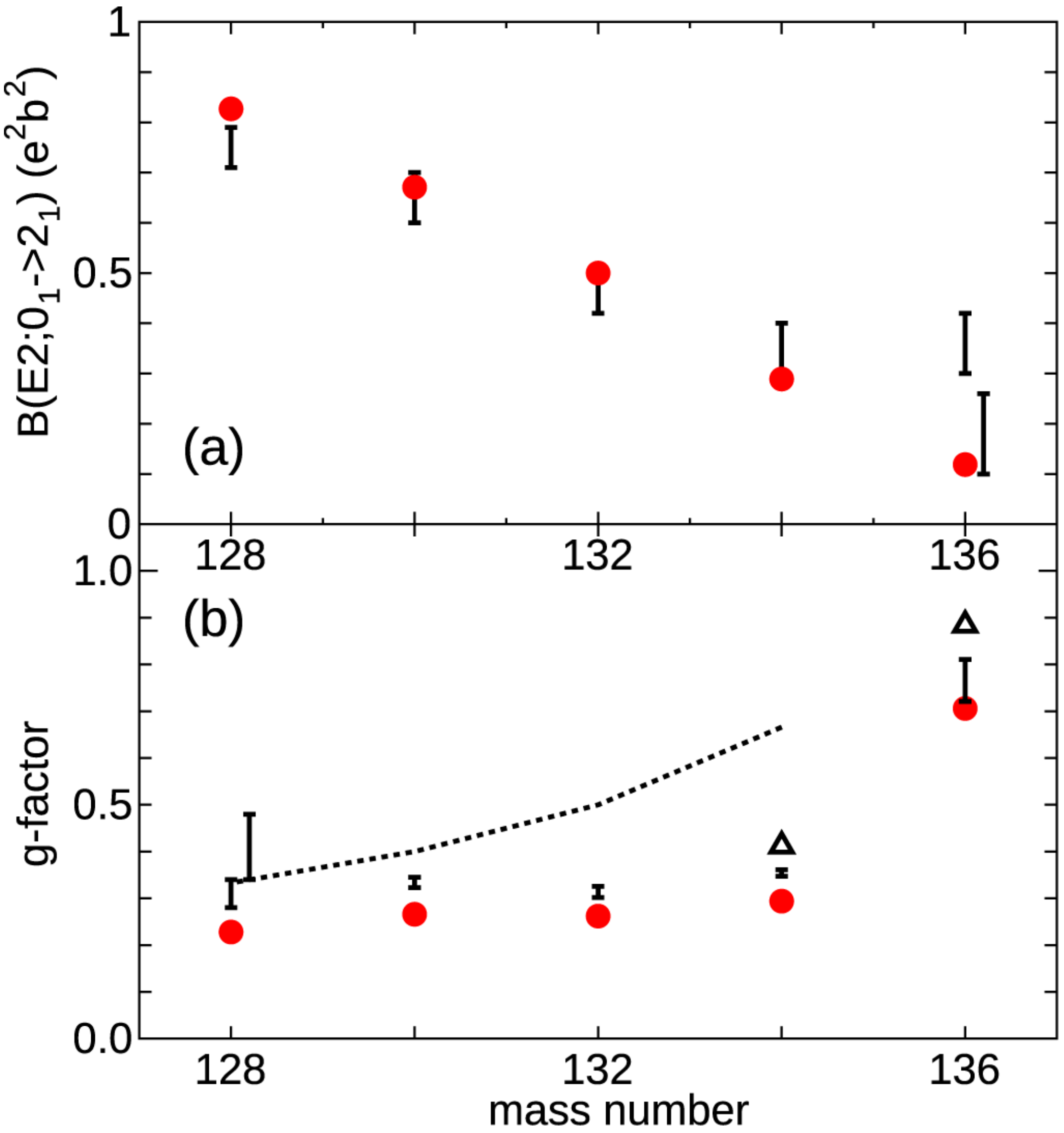
^{132}Ba N=76 ^{134}Ba N=78 ^{136}Ba N=80



moderately prolate γ-unstable/triaxial spherical



B(E2) and g-factor of Xe isotopes



● present calc.

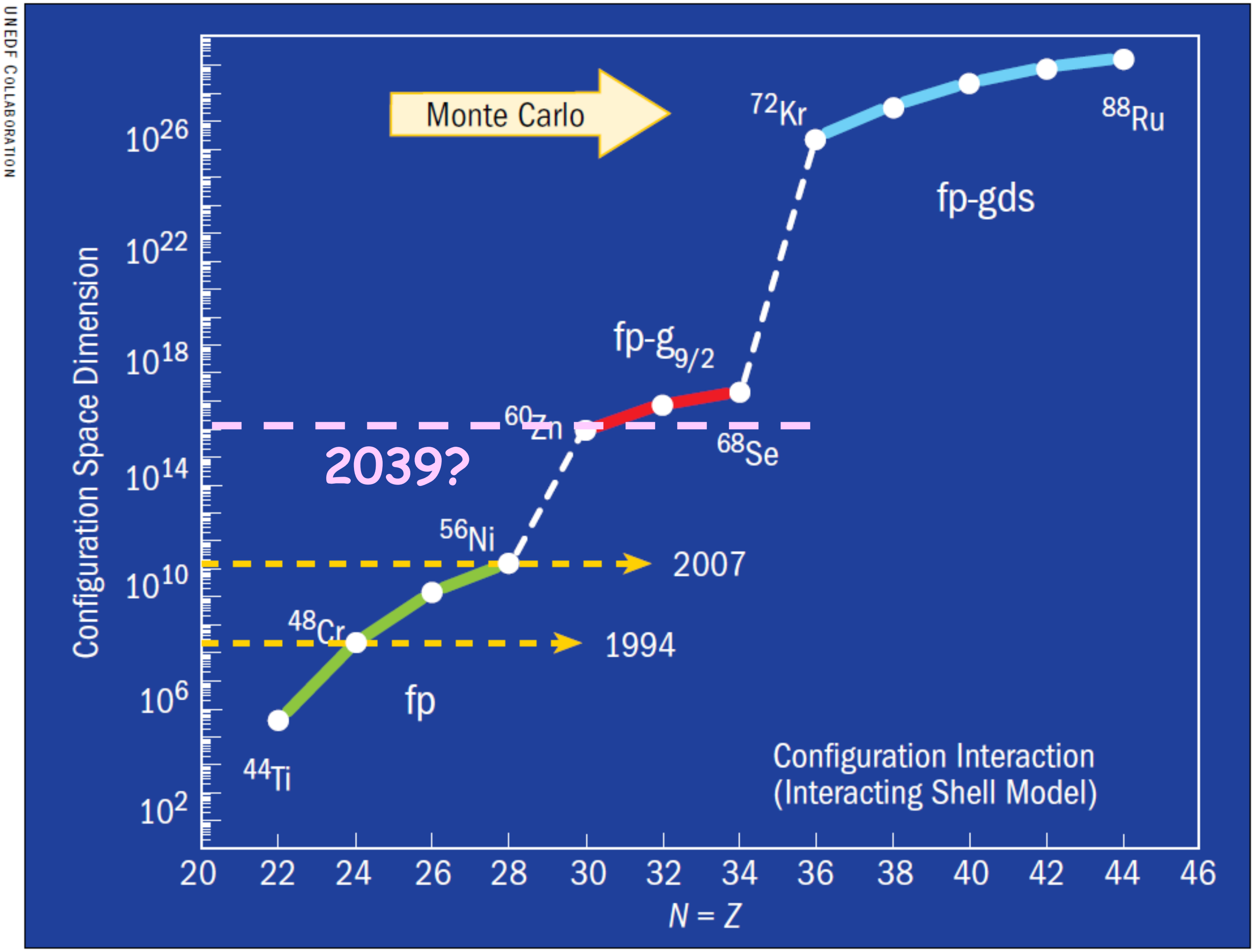
spin quenching 0.65

I exp.

Jacob et al. PRC65, 2002
Raman et al. NDT 78, 2001
Gordon et al. PRC12, 1975
Arnesen et al., Hyp. Int. 5, 1977
Mukhopadhay et al. PRC78, 2008

.... IBM (totally symmetric state)

Δ Conventional Shell Model calc.
Brown et al.
PRC71 (2005)

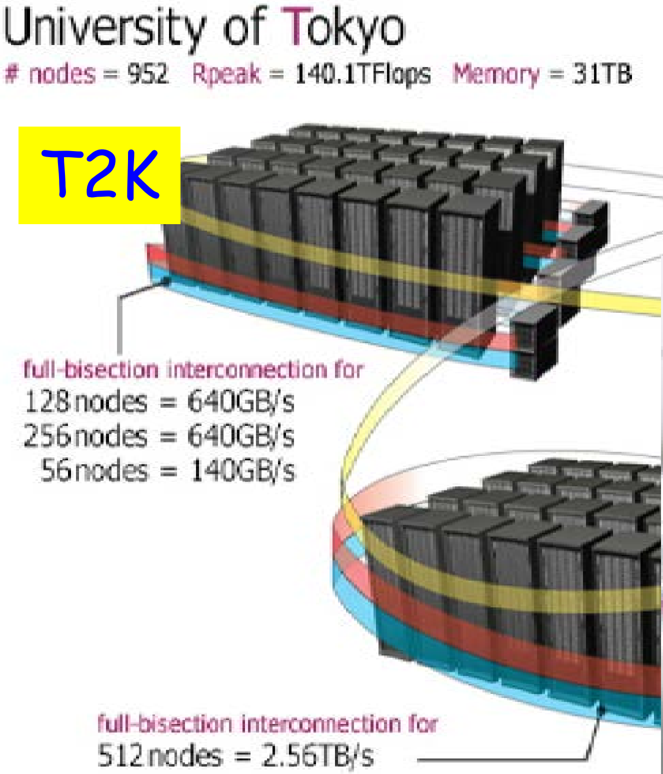


Shell-model dimension (without symmetry consideration) for the pf- to pf-g- shell nuclei.

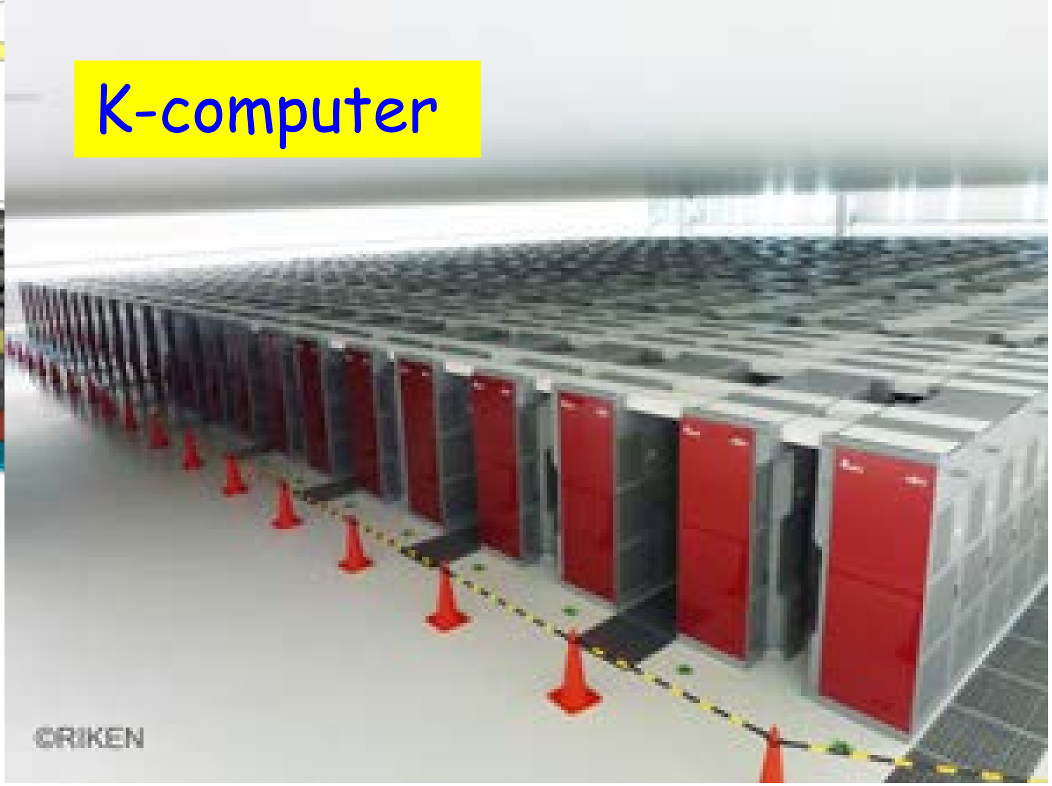
SciDAC Review, winter issue 2007 + personal hunch.

Courtesy of Utsuno

One of the future directions is to use supercomputers



T2K



K-computer

©RIKEN

Concluding remarks

Nuclear shell model has achieved recently
- shell-model dimension = a few billions
  systematic studies up to around A=90
- applications to astrophysics, particle physics
- interactions for higher and wider model spaces
- advanced Monte Carlo Shell Model → bigger dimension

Paradigm of foundation of nuclear shell model
- being studied in the framework of *ab initio* calculations and modern theories of nuclear forces
  → skipped
  looks much more feasible (compared to the past)
  still needs a lot of time and effort

Paradigm on robustness of shell structure

- large-scale calculation is not the whole story
- shell model can link nuclear forces to structure in visible/intuitive ways, making simple predictions
- RI-beam can clarify various structural evolutions as functions of N and Z
- shell evolution due to nuclear forces (tensor, 3NF, …) occurs along many trails on the nuclear chart
- shell evolution can lead to unexpected shapes
- shell evolution can change driplines and halo formation, perhaps affects continuum properties

- shell/magic structure of exotic nuclei may differ from what Mayer and Jensen conceived in 1949

Future : exciting, unexpected, but demanding