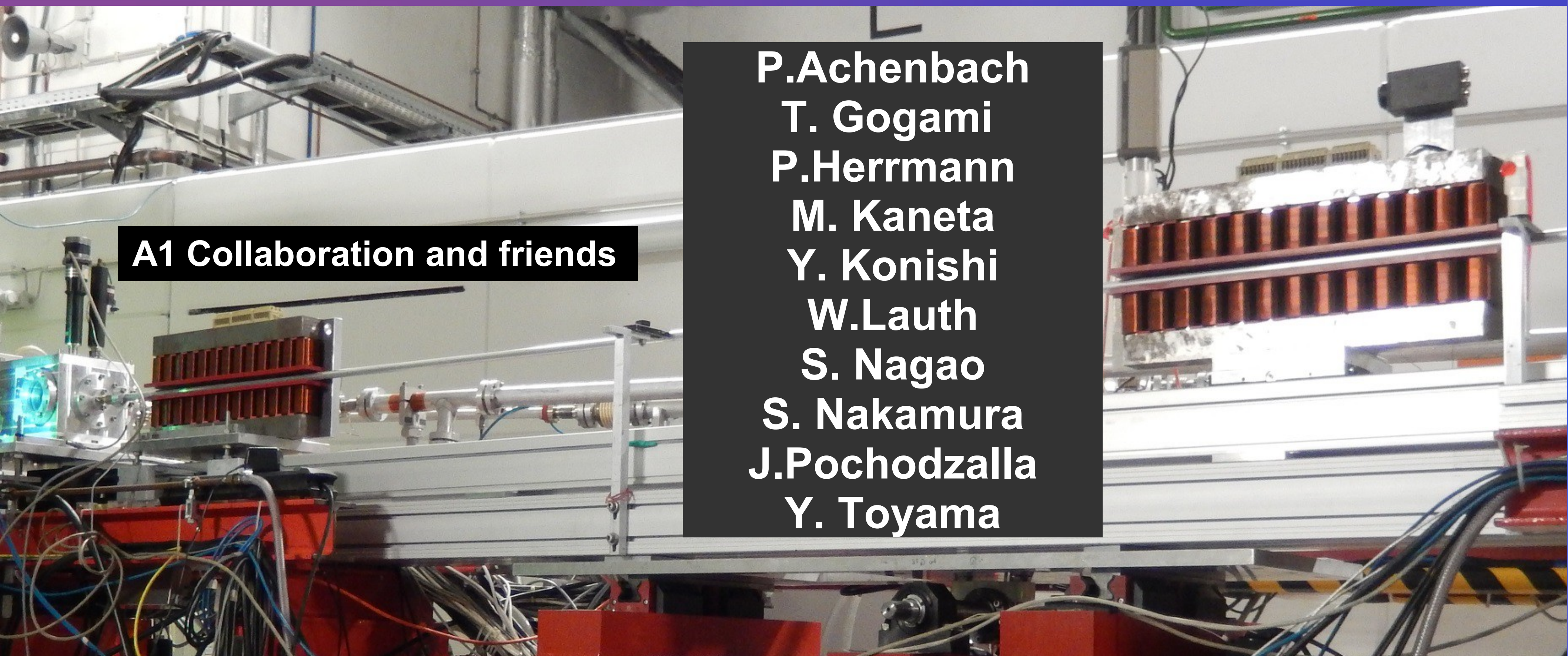
Shared parameters Fresnel diffraction fitting

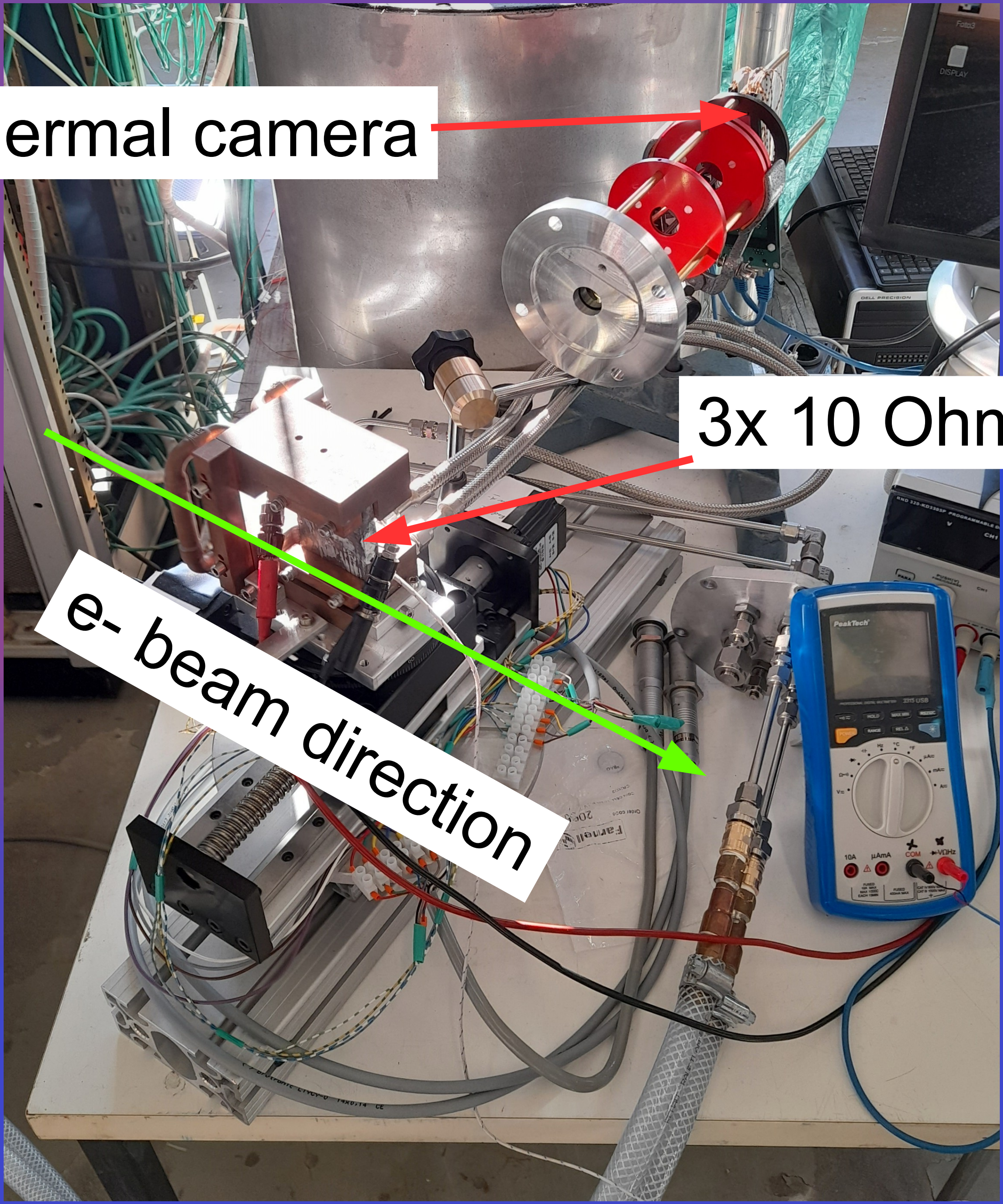


A1 Collaboration and friends

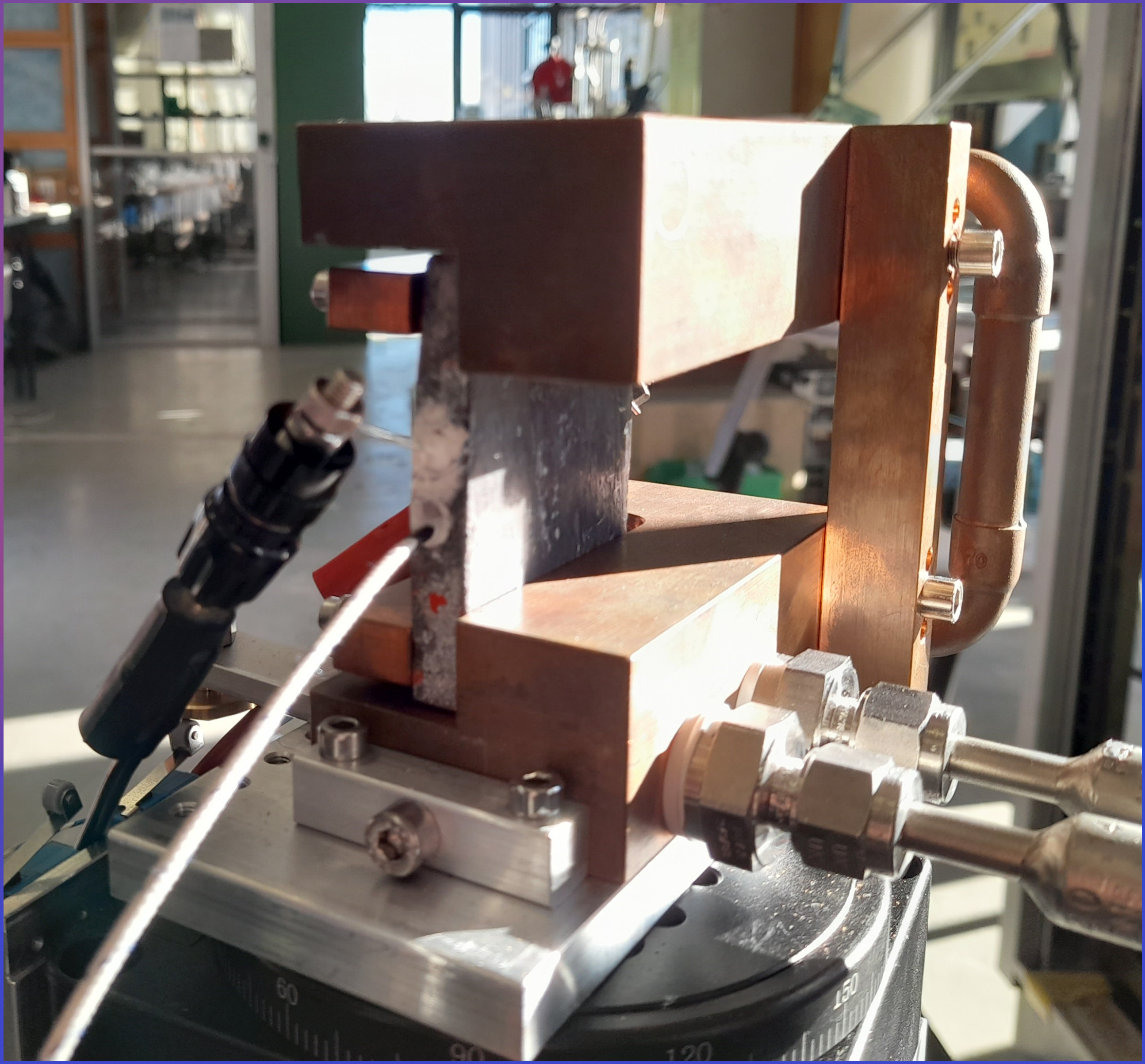
P.Achenbach
T. Gogami
P.Herrmann
M. Kaneta
Y. Konishi
W.Lauth
S. Nagao
S. Nakamura
J.Pochodzalla
Y. Toyama

Pascal Klag
10.03.2022

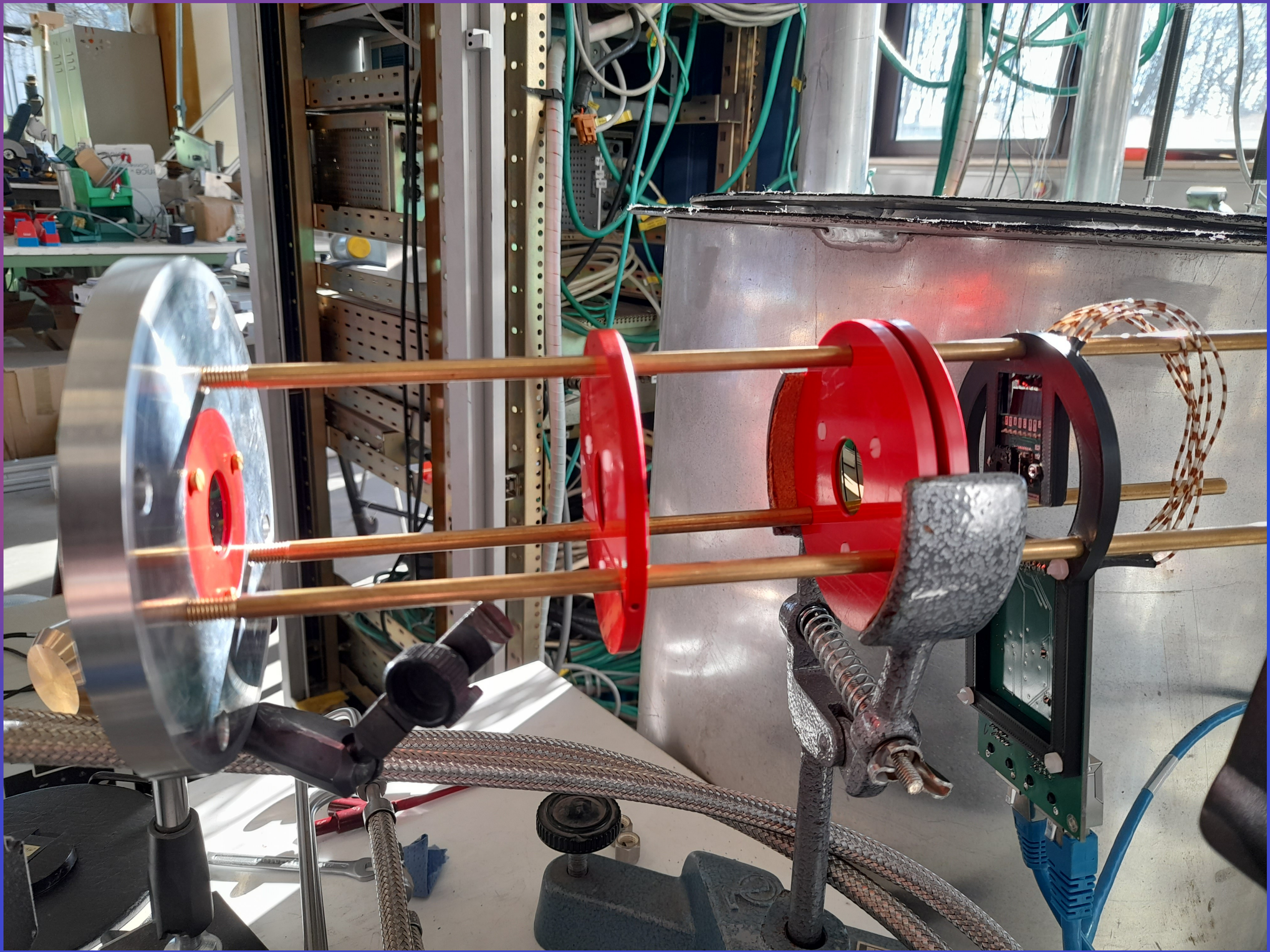
Target and lens system



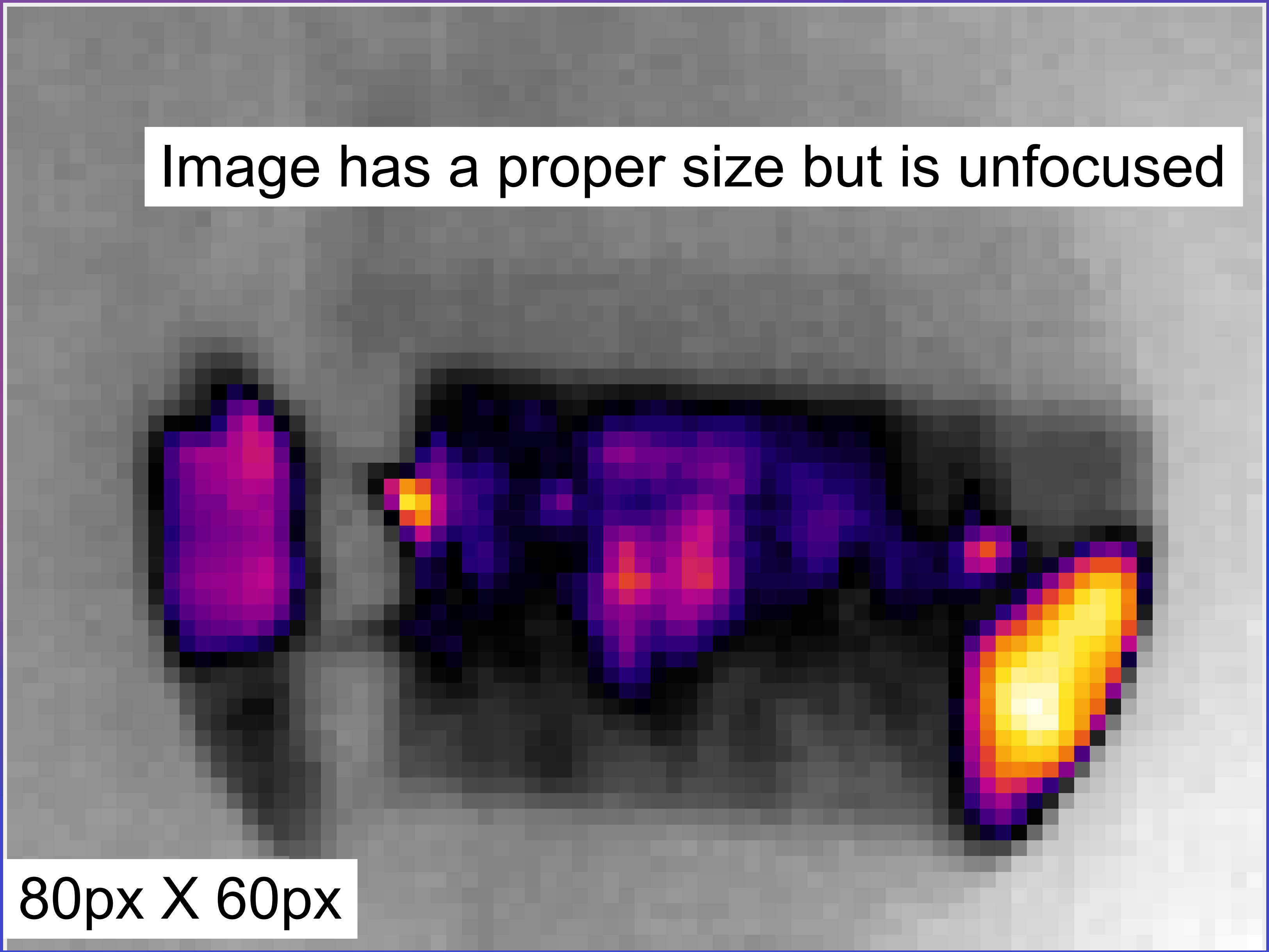
Water cooled target holder



Lens system

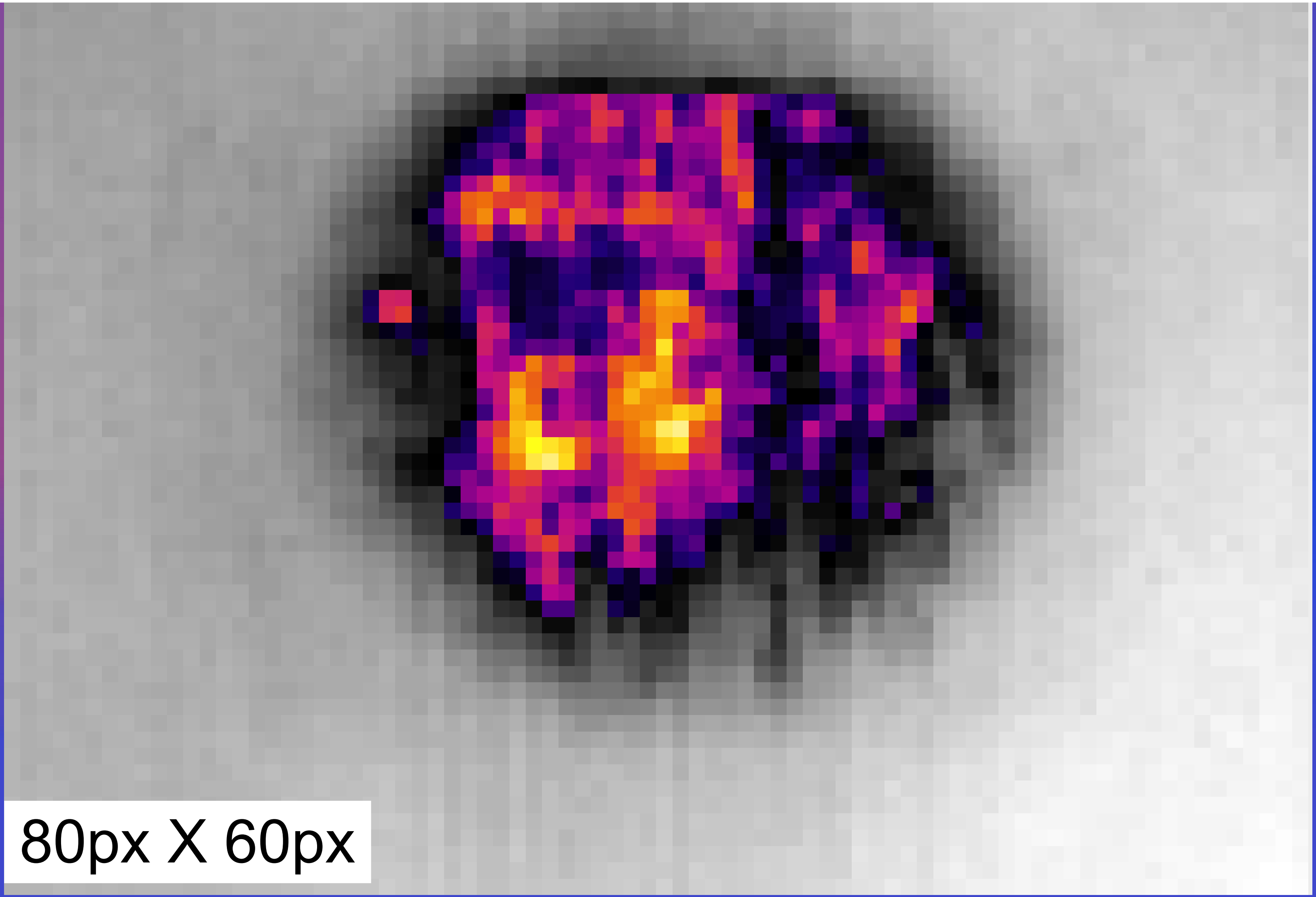


Camera image arbitrary lens config

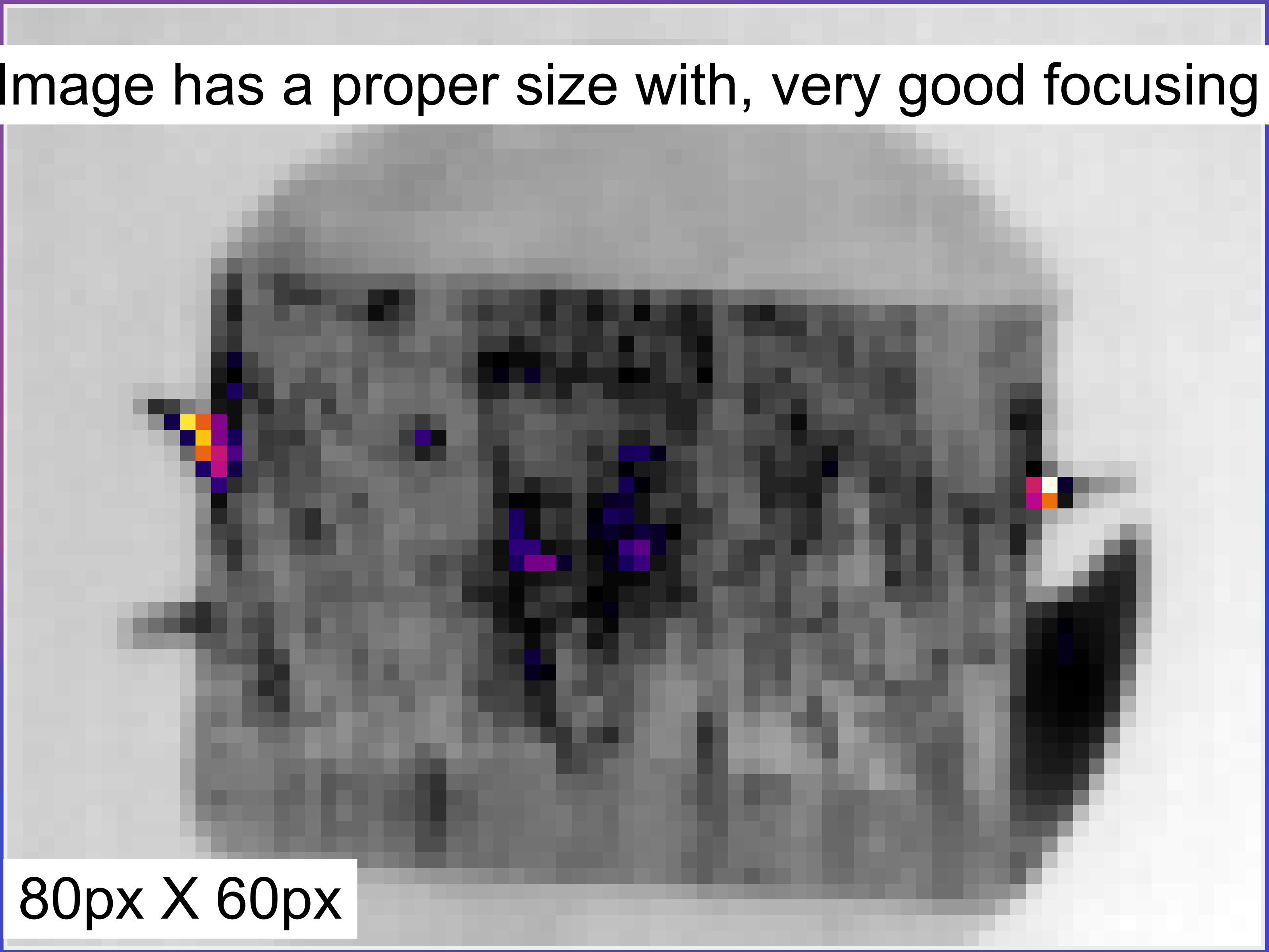


Camera image arbitrary lens config

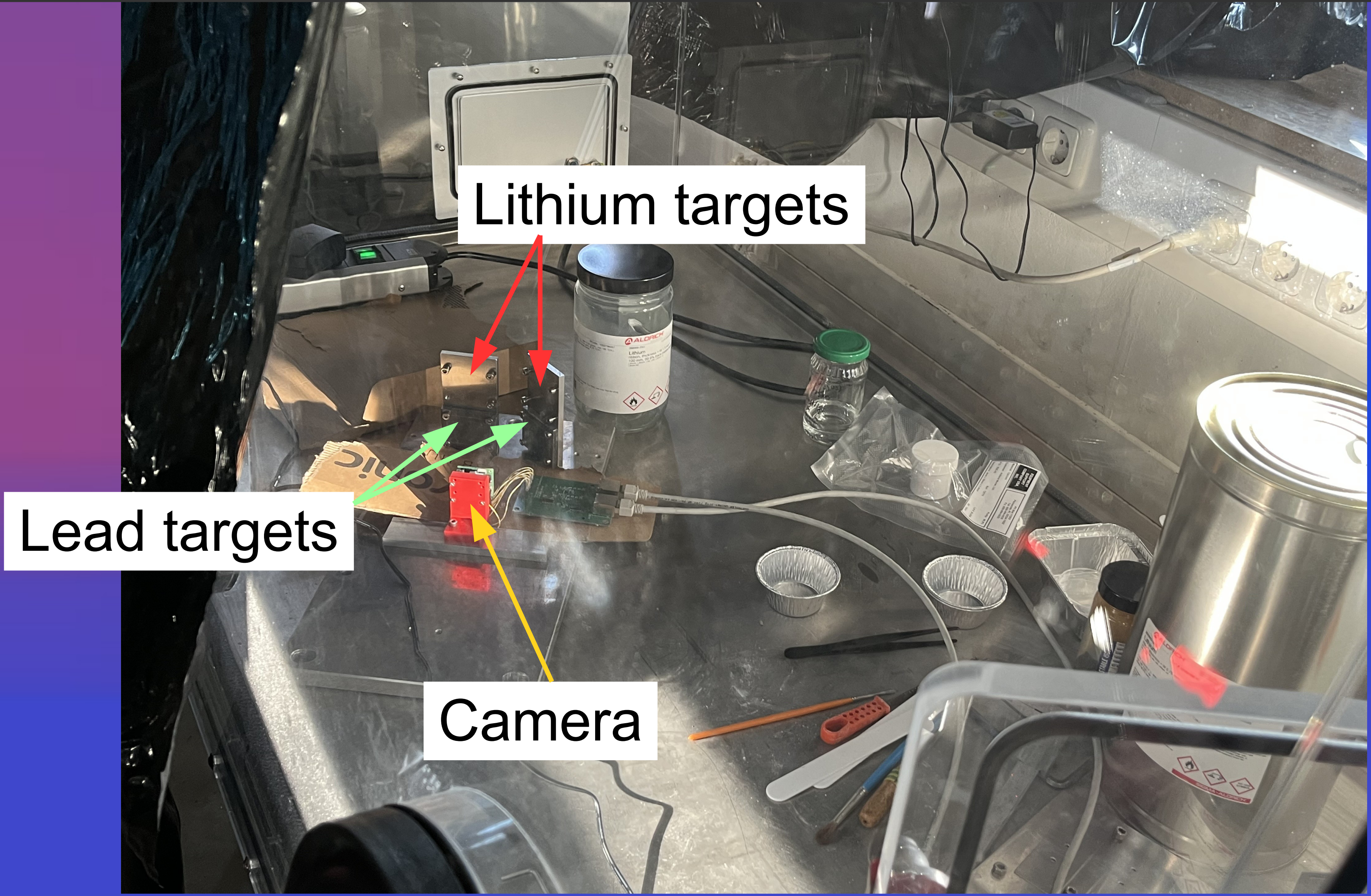
Image is limited by one lens aperture, this time focused



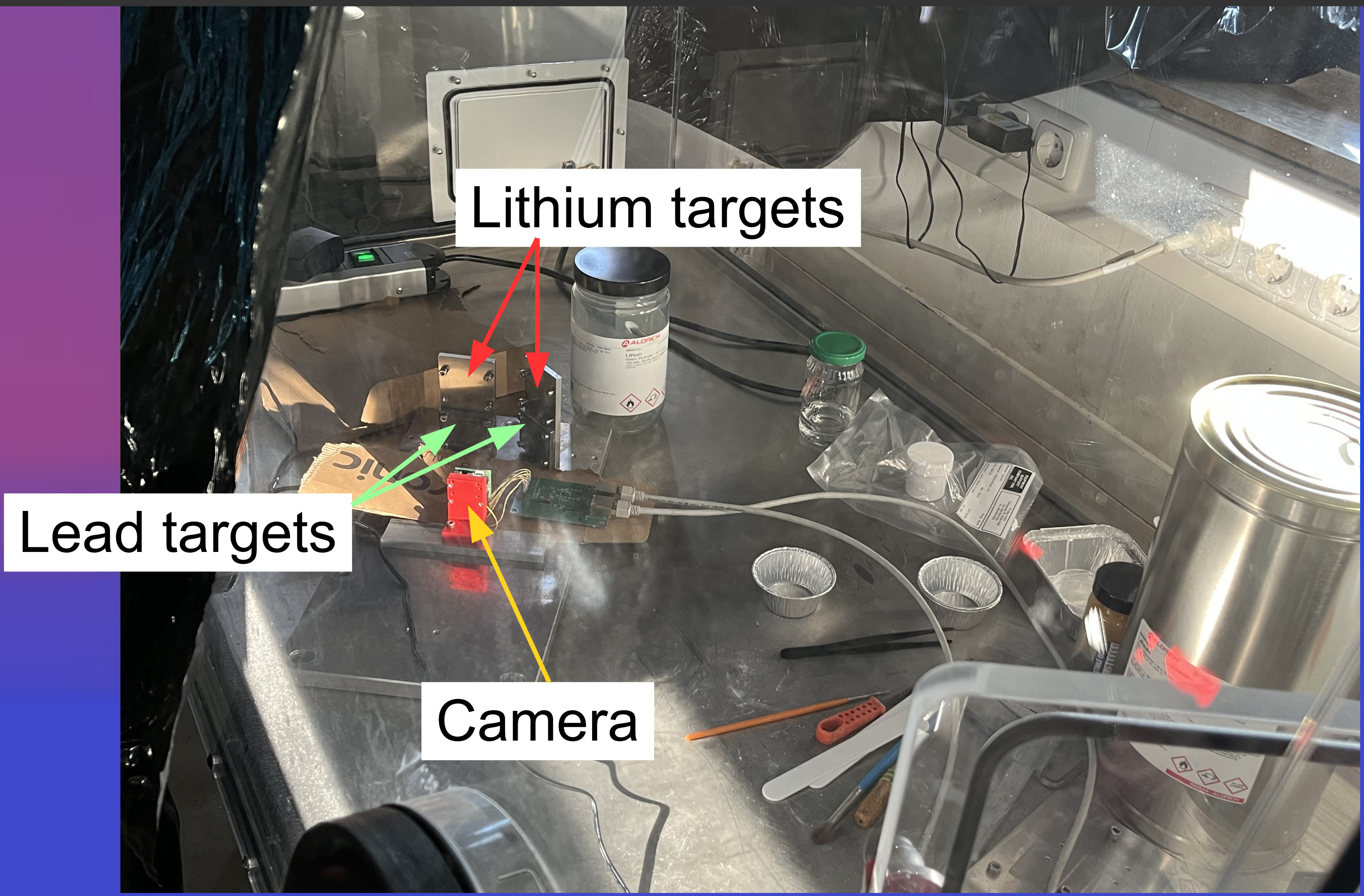
Camera image proper lens config



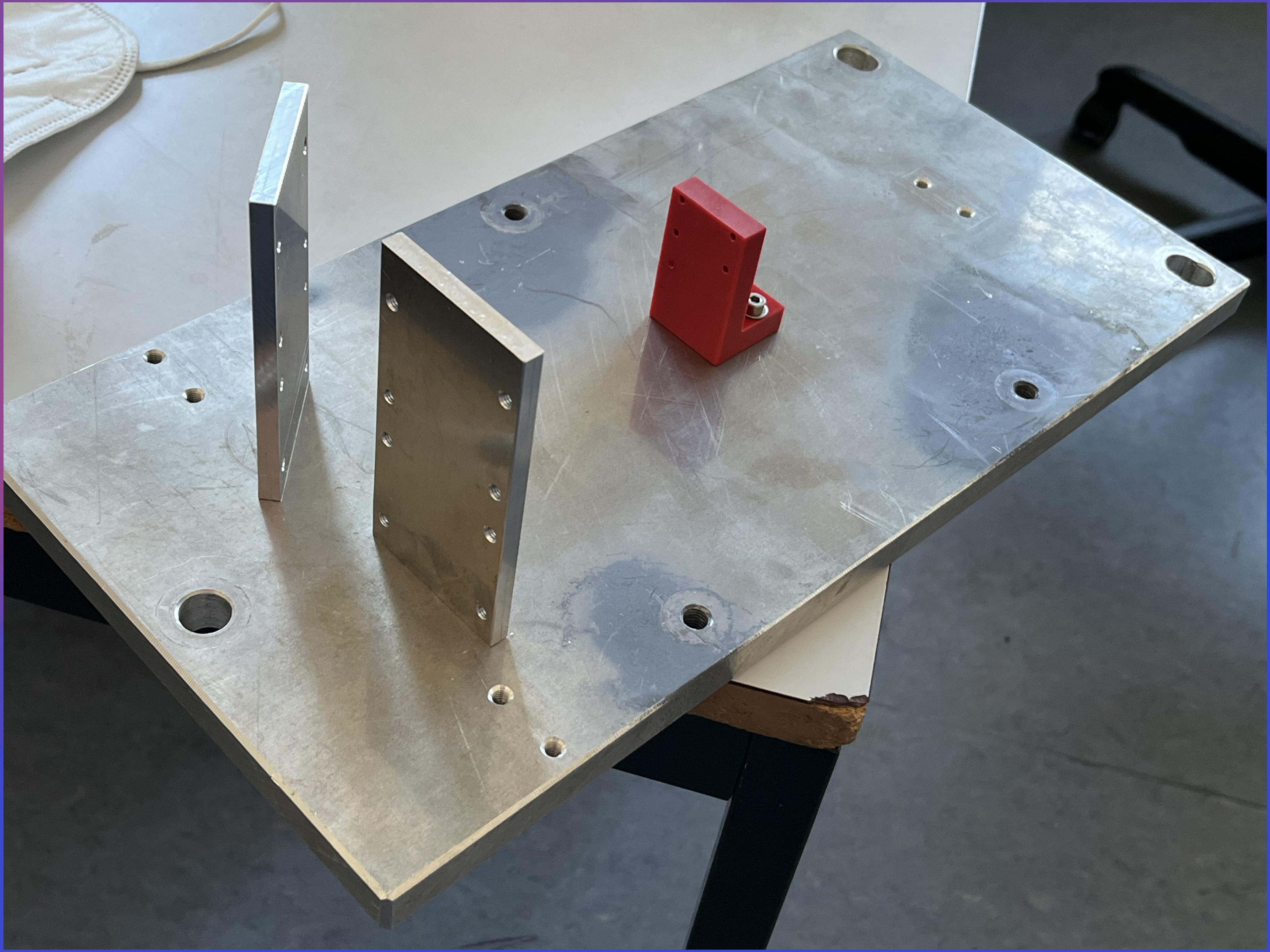
Camera inside the glovebox monitoring the deterioation of Lithium



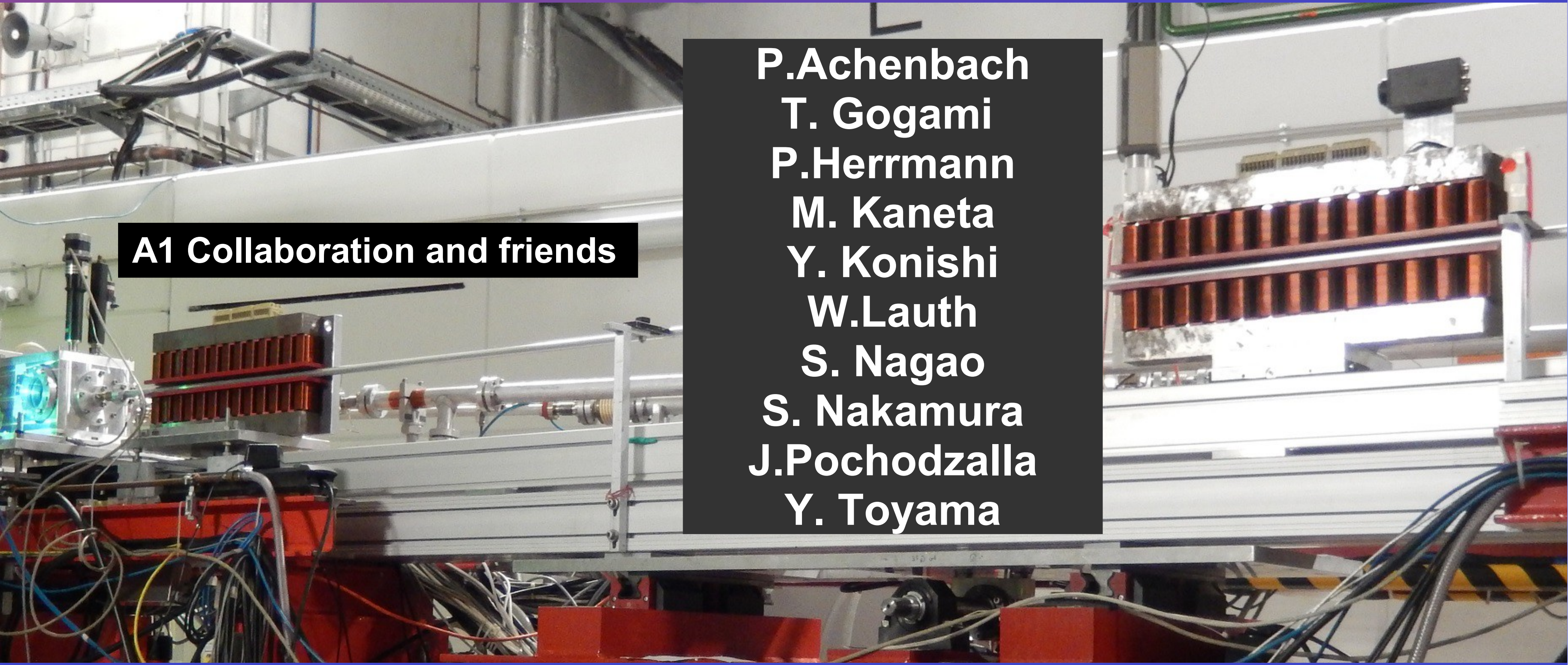
Camera inside the glovebox monitoring the deterioation of Lithium



Test target holders



Shared parameters Fresnel diffraction fitting



A1 Collaboration and friends

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10.03.2022

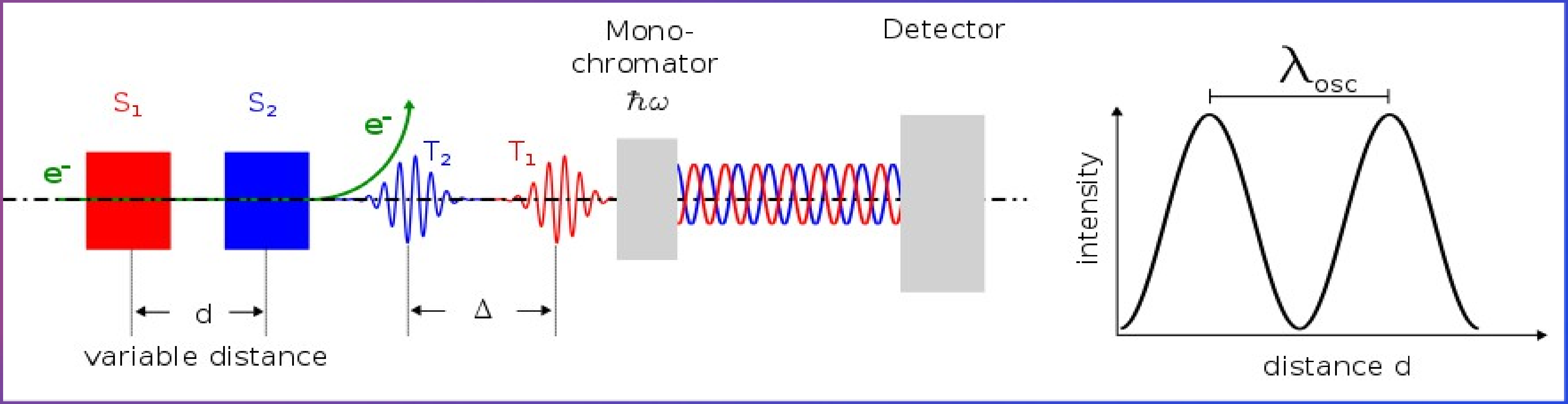
Method

Coherent sources

Wave packets

Monochro-matic light

Light intensity of selected wavelength

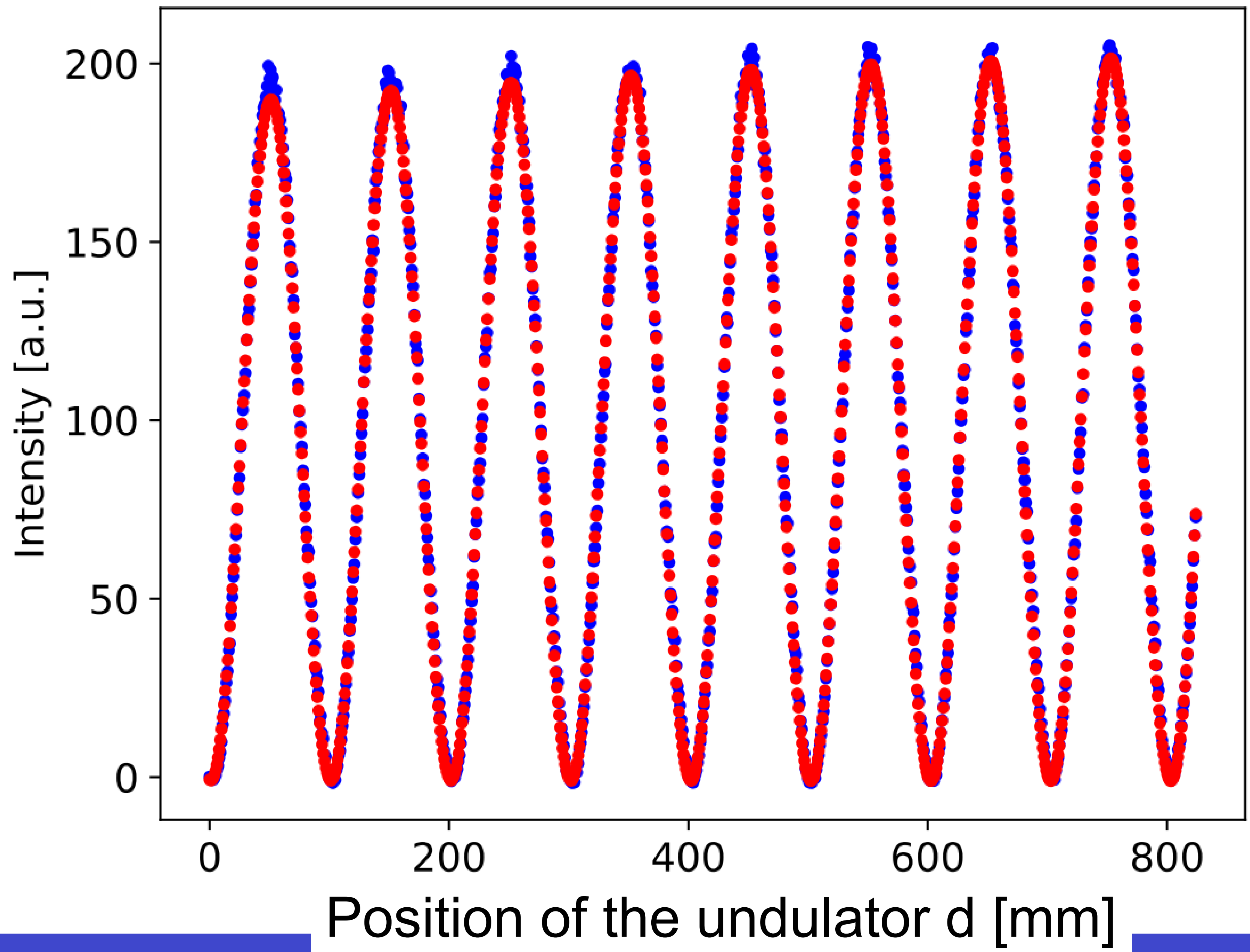


$$\lambda_{osc} = 2\gamma^2 \lambda_L$$

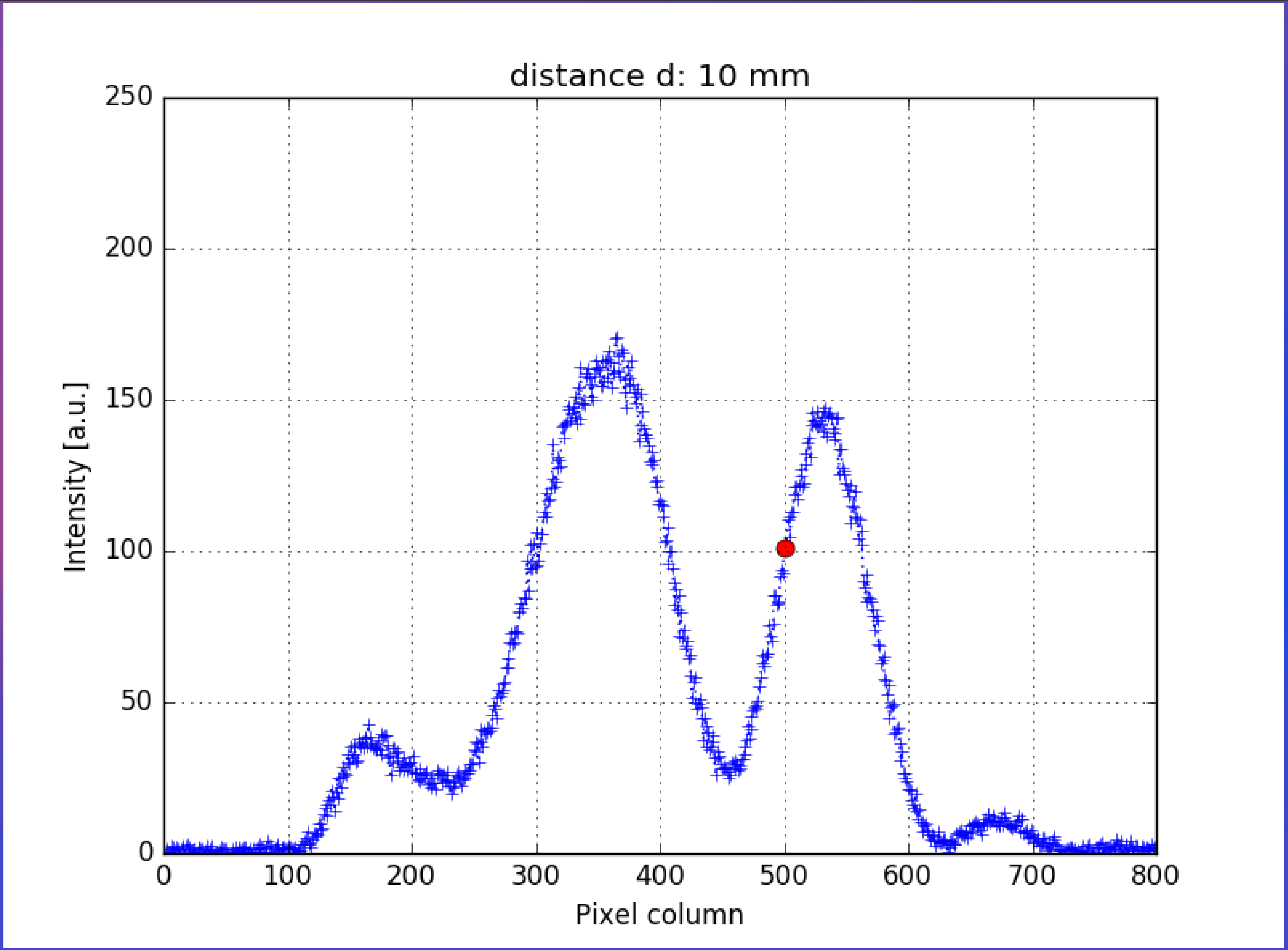
Example for wavelength and period

$$\left.\begin{array}{l}\lambda_L \approx 400\text{nm} \\ \gamma \approx 381\text{ ,E=195MeV}\end{array}\right\} \lambda_{osc} \approx 116\text{mm}$$

Intensity oscillation



For each wavelength a different period and phase is present for the oscillation

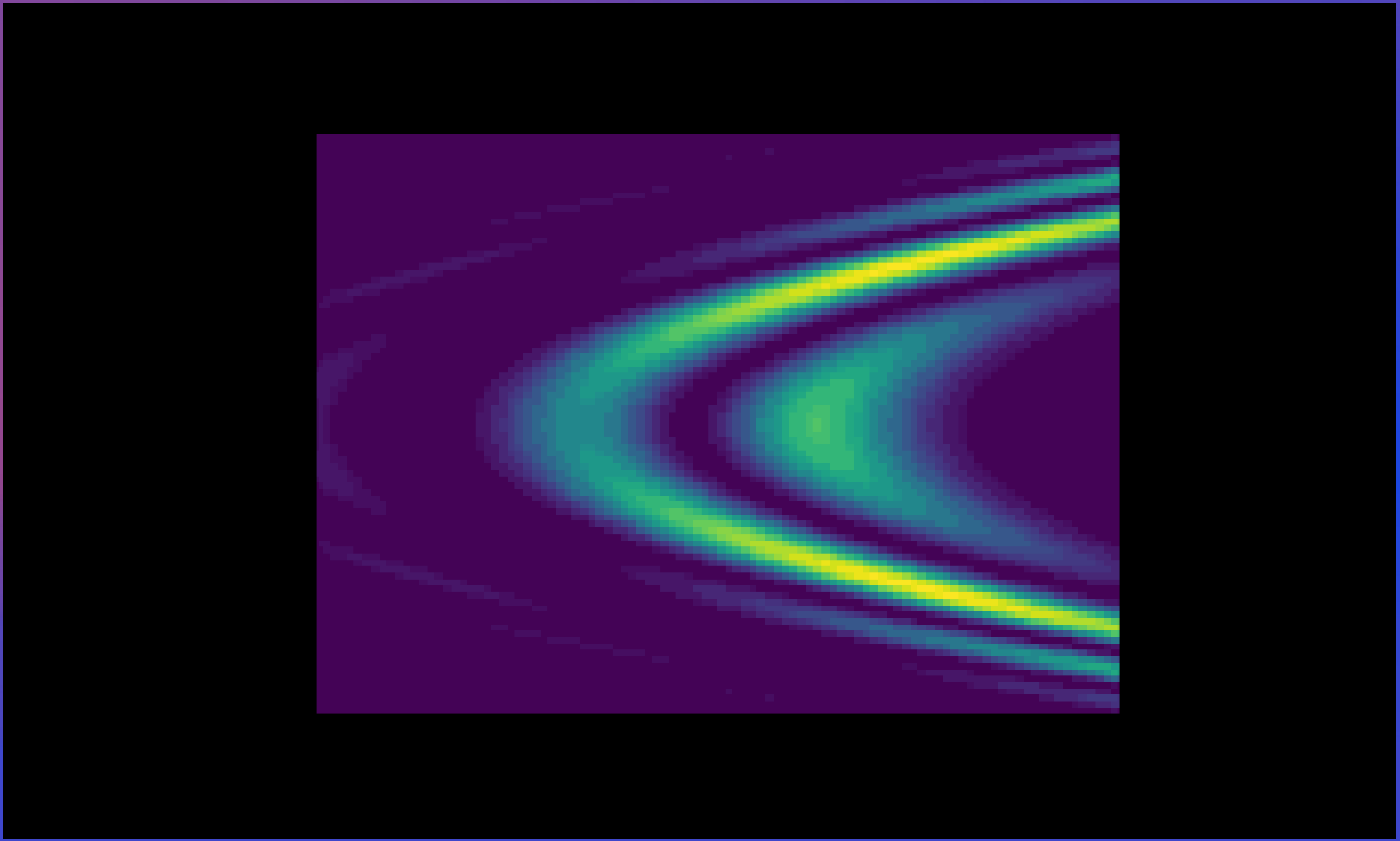


Intensity [a.u.]

Wavelengths λ [px]

(Complex) pattern
High intensity Yellow, low dark Blue

Observation angle θ



Wavelengths λ [px]

Fit model and parameters

Fit model

$$I(d) = a\sin\left(\frac{2\pi}{\lambda_L}\left(\frac{d}{2\gamma^2}\right) + \Phi + \frac{\tau}{\lambda_L}\right) + b$$

Parameters are:

a,b,γ,Φ,τ

But there exists a relationship between λ and γ

$$\gamma^2 = \frac{\lambda_{osc}}{2\lambda_L}$$

τ represents the distance shift to the undulator starting position

Fit model and Unshared parameters

$$I_1(d)=a_1\sin\left(\frac{2\pi}{\lambda_{L1}}\left(\frac{d}{2\gamma_1^2}\right)+\Phi_1+\frac{\tau_1}{\lambda_L}\right)+b_1$$

$$I_2(d)=a_2\sin\left(\frac{2\pi}{\lambda_{L2}}\left(\frac{d}{2\gamma_2^2}\right)+\Phi_2+\frac{\tau_2}{\lambda_L}\right)+b_2$$

$$I_3(d)=a_3\sin\left(\frac{2\pi}{\lambda_{L3}}\left(\frac{d}{2\gamma_3^2}\right)+\Phi_3+\frac{\tau_3}{\lambda_L}\right)+b_3$$

● ● ● ●

Fit model and shared parameters

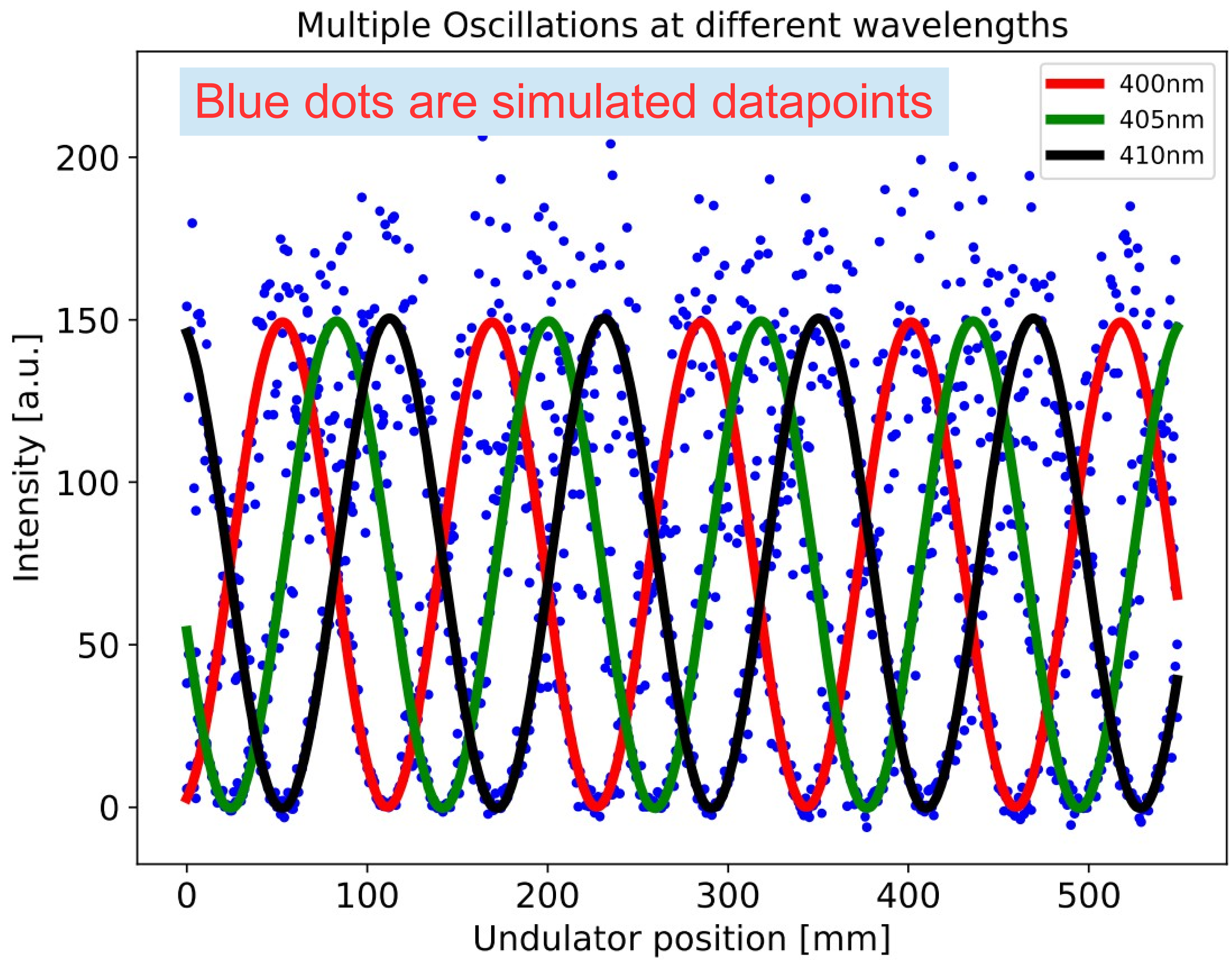
$$I_1(d)=a_1\sin\left(\frac{2\pi}{\lambda_{L1}}\left(\frac{d}{2\gamma^2}\right)+\Phi_1+\frac{\tau_1}{\lambda_L}\right)+b_1$$

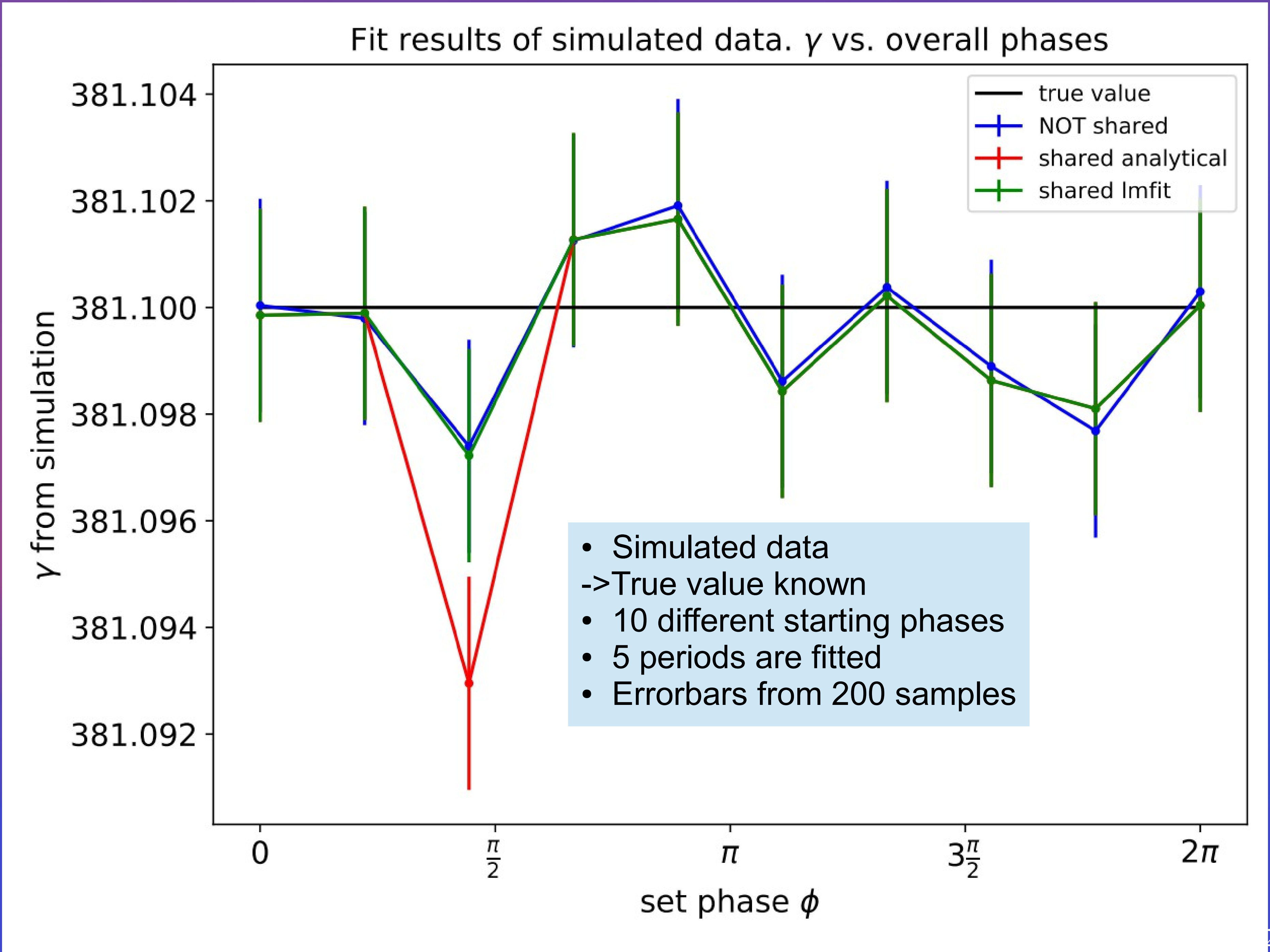
$$I_2(d)=a_2\sin\left(\frac{2\pi}{\lambda_{L2}}\left(\frac{d}{2\gamma^2}\right)+\Phi_2+\frac{\tau_2}{\lambda_L}\right)+b_2$$

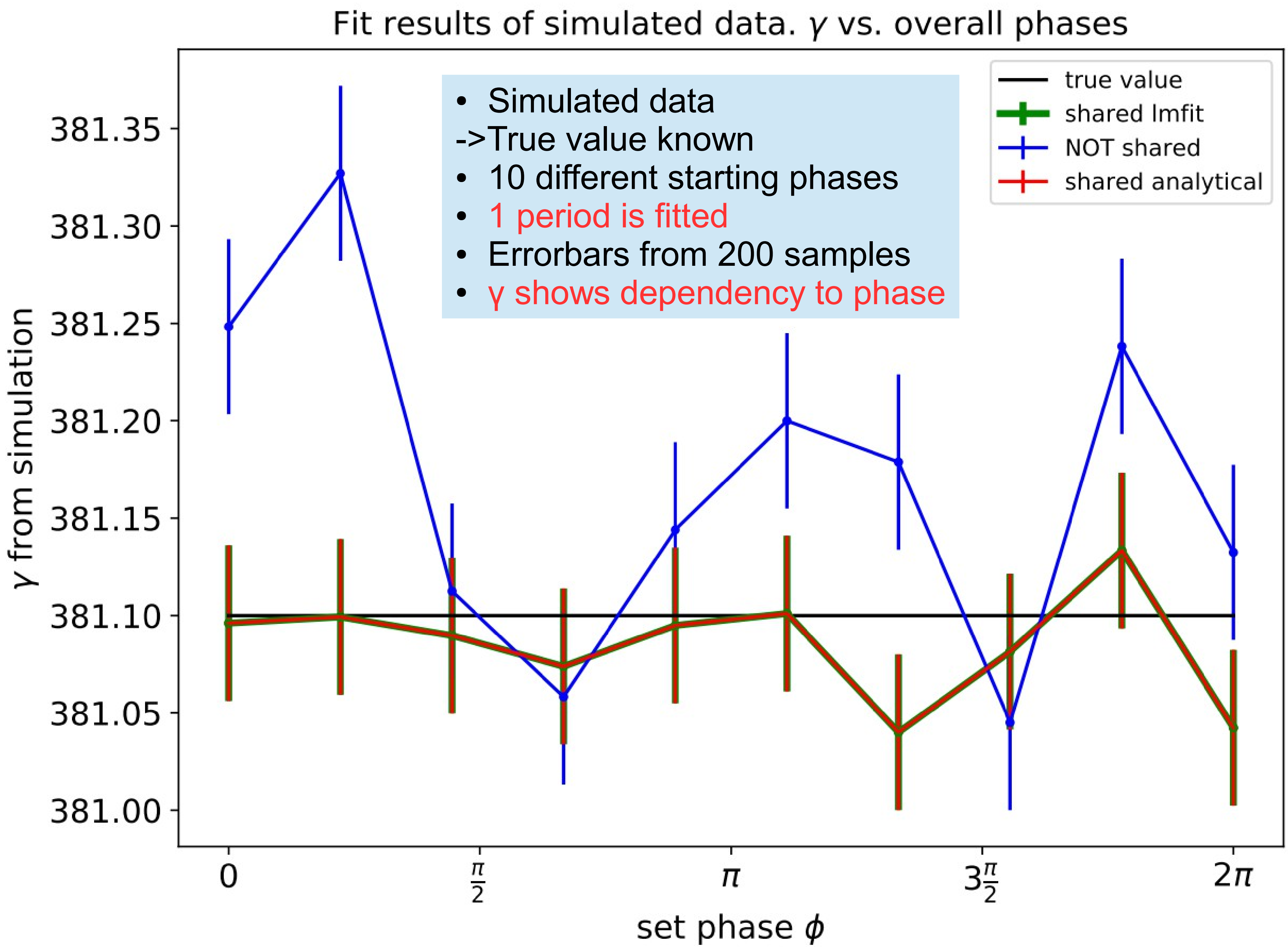
$$I_3(d)=a_3\sin\left(\frac{2\pi}{\lambda_{L3}}\left(\frac{d}{2\gamma^2}\right)+\Phi_3+\frac{\tau_3}{\lambda_L}\right)+b_3$$

● ● ● ●

γ is shared


Multiple Oscillations at different wavelengths
Blue dots are simulated datapoints
400nm
405nm
410nm
Intensity [a.u.]
0
50
100
150
200
0
100
200
300
400
500
Undulator position [mm]


Fit results of simulated data. γ vs. overall phases
381.104
381.102
381.100
381.098
381.096
381.094
381.092
γ from simulation
true value
NOT shared
shared analytical
shared lmfit
0
π/2
π
3π/2
2π
set phase ϕ
• Simulated data
->True value known
• 10 different starting phases
• 5 periods are fitted
• Errorbars from 200 samples

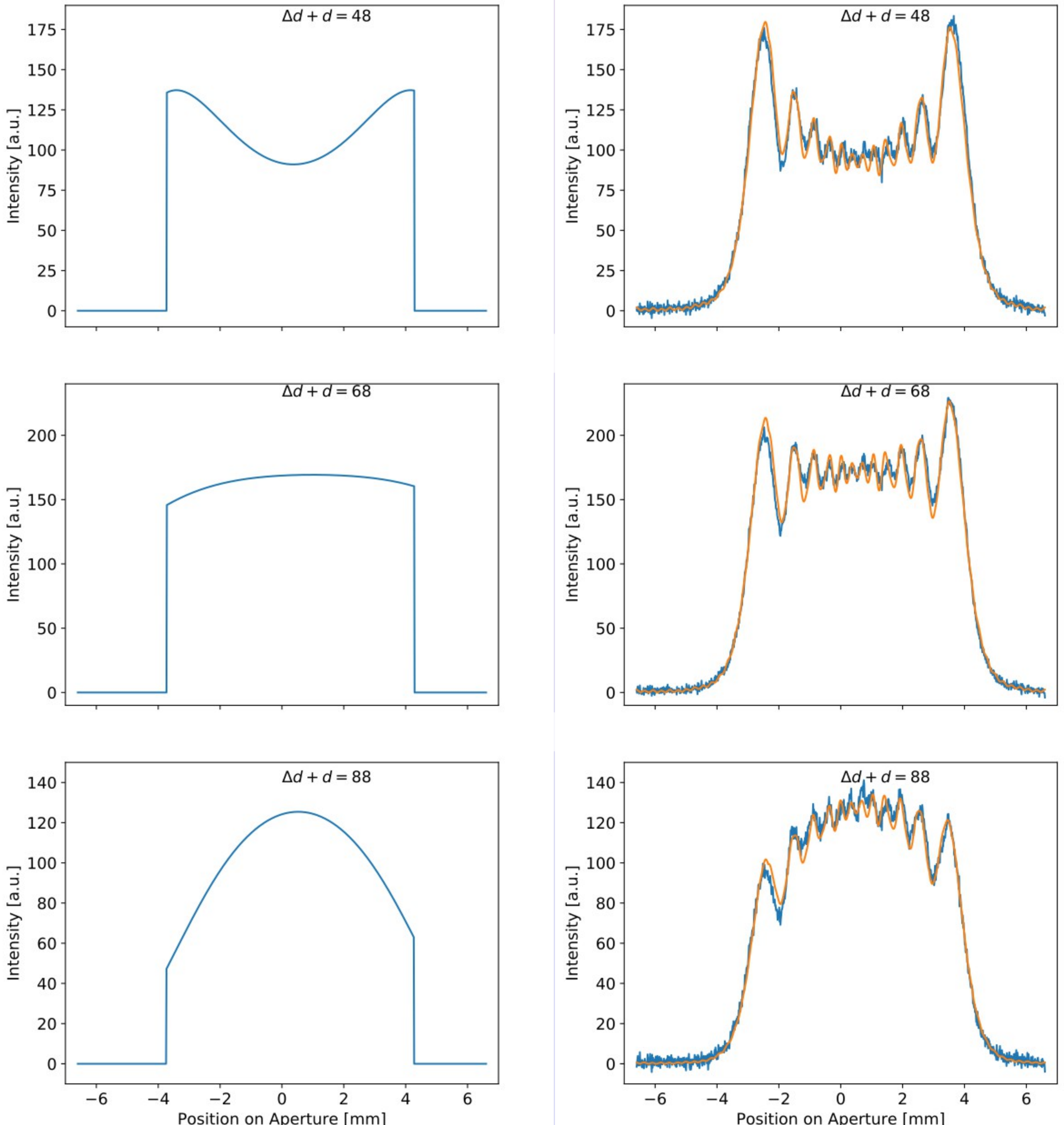

Fit results of simulated data. γ vs. overall phases
• Simulated data
->True value known
• 10 different starting phases
• 1 period is fitted
• Errorbars from 200 samples
• γ shows dependency to phase
true value
shared lmfit
NOT shared
shared analytical
381.35
381.30
381.25
381.20
381.15
381.10
381.05
381.00
γ from simulation
0
π/2
π
3π/2
2π
set phase ϕ

First conclusion

- Performing nonlinear LMA-fits with shared parameters reduces systematic phase dependency.

  (try it) :)

  LMA = Levenberg Marquardt Algorithm



$\Delta d + d = 48$
$\Delta d + d = 68$
$\Delta d + d = 88$
Intensity [a.u.]
Position on Aperture [mm]

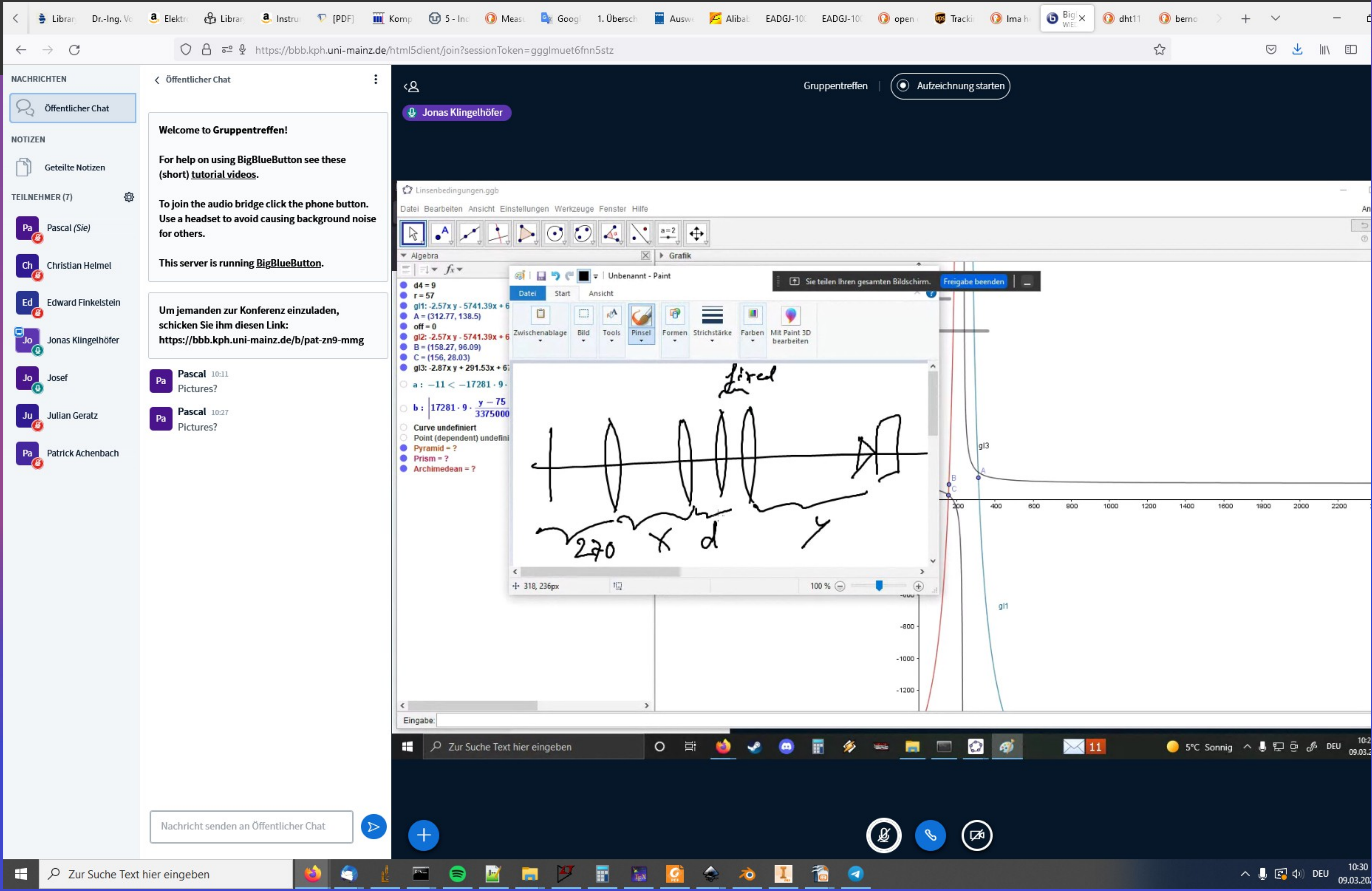
Fitting theory

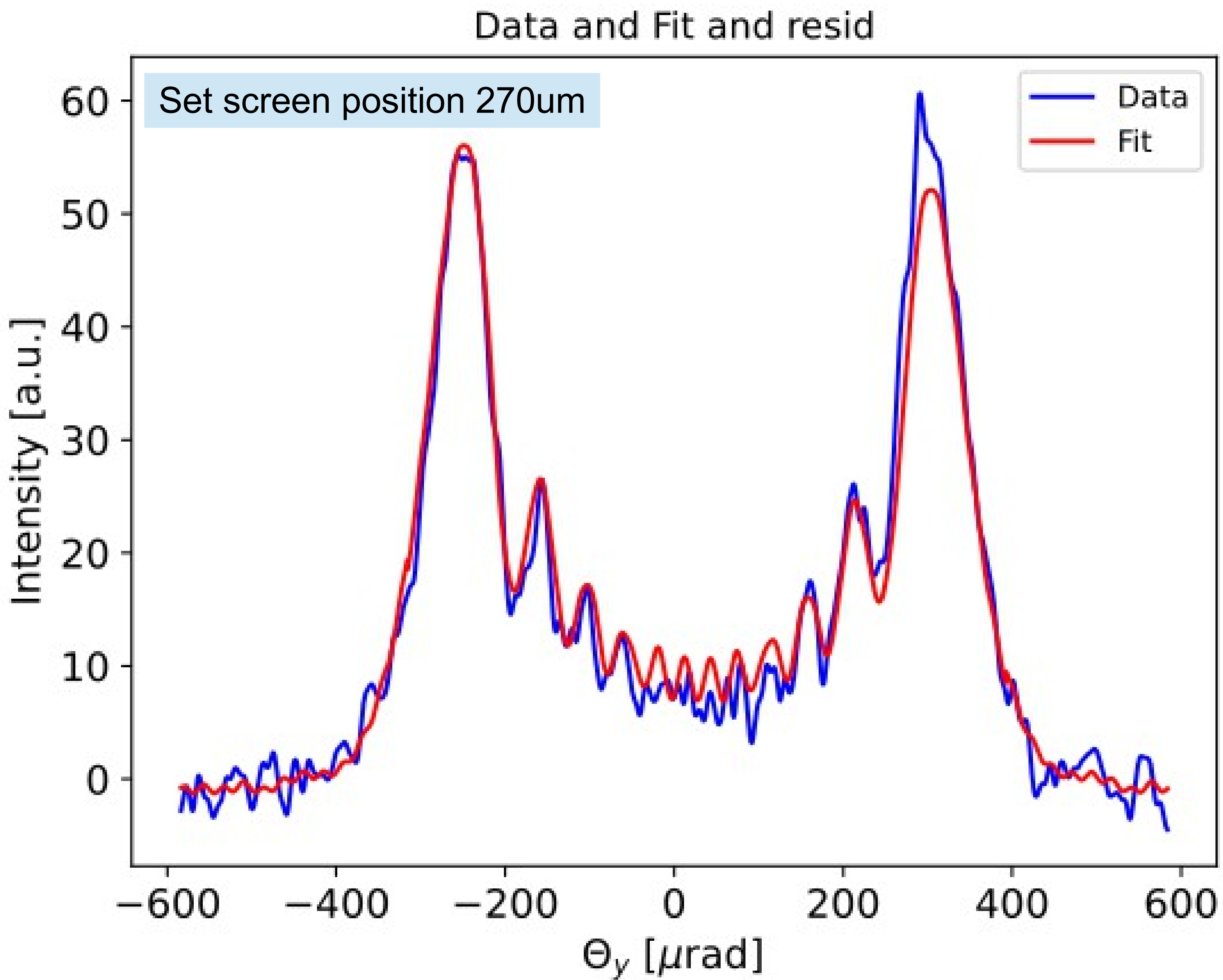
- N: Number of Curves.
- A(n): Number of nodes belonging to the nth measured curve

$$\chi^2 = \sum_{n=1}^{N} \chi_n^2$$

$$\chi_n^2 = \sum_{i=1}^{A(n)} \left(\frac{1}{\sigma_{n,i}^2}\right)\left(y_{n,i} - y_n\left(x_{n,i,a}\right)\right)^2$$

First conclusion




Data and Fit and resid
Set screen position 270um
Data
Fit
Intensity [a.u.]
60
50
40
30
20
10
0
−600
−400
−200
0
200
400
600
Θy [μrad]

Fit model and parameters

Fit model

$$I(d) = a \sin\left(\frac{2\pi}{\lambda}\left(\frac{d}{2\gamma^2}\right) + \Phi\right) + b$$

Parameters are:
a,b,γ,Φ
But there exists a relationship between λ and γ

$$\gamma^2 = \frac{\lambda_{osc}}{2\lambda_L}$$

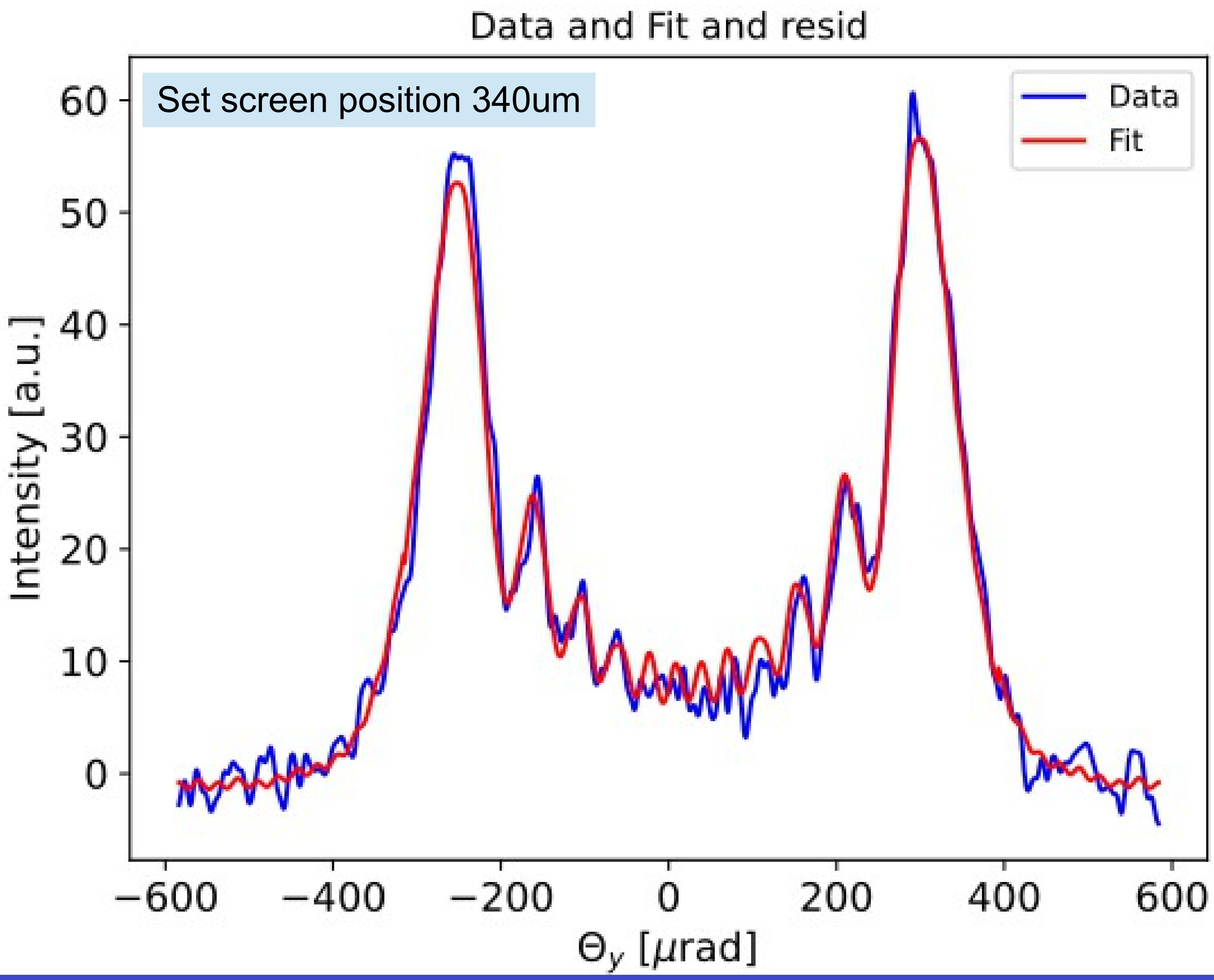
Error to the energy due to uncertainty in the observation

Interference phase Φ depends also on the angle θ:

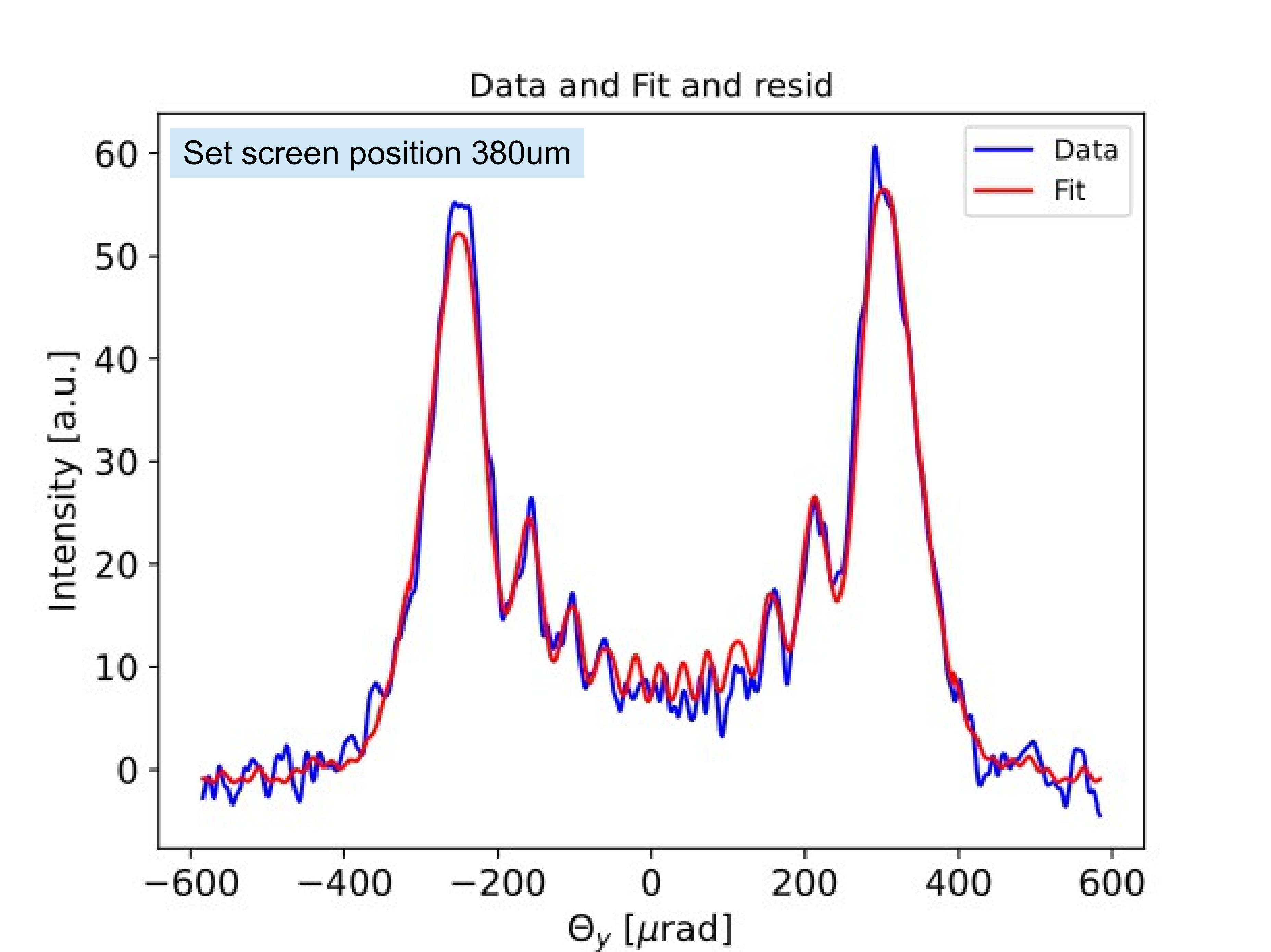
$$\Phi(d) = k_L\left((L_U + d)\frac{\Theta^2}{2} + \frac{d}{2\gamma^2}\right)$$

The uncertainty in the central angle θ, (δθ) results in an error of the γ

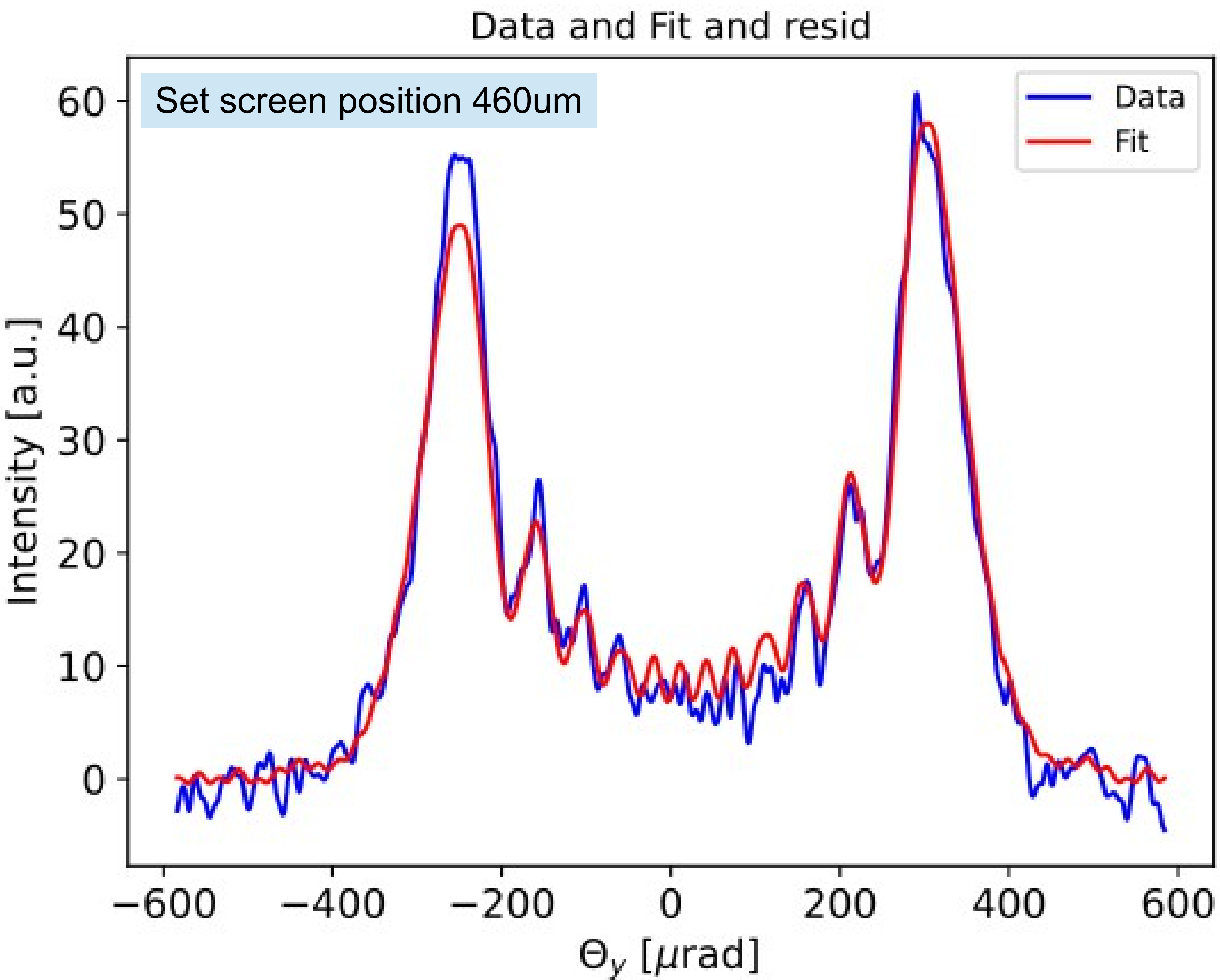
$$\frac{\delta\gamma}{\gamma} = \frac{1}{2}\sqrt{\left(\frac{2\gamma^2\delta\Theta^2}{1+\gamma^2\delta\Theta^2}\right)^2}$$

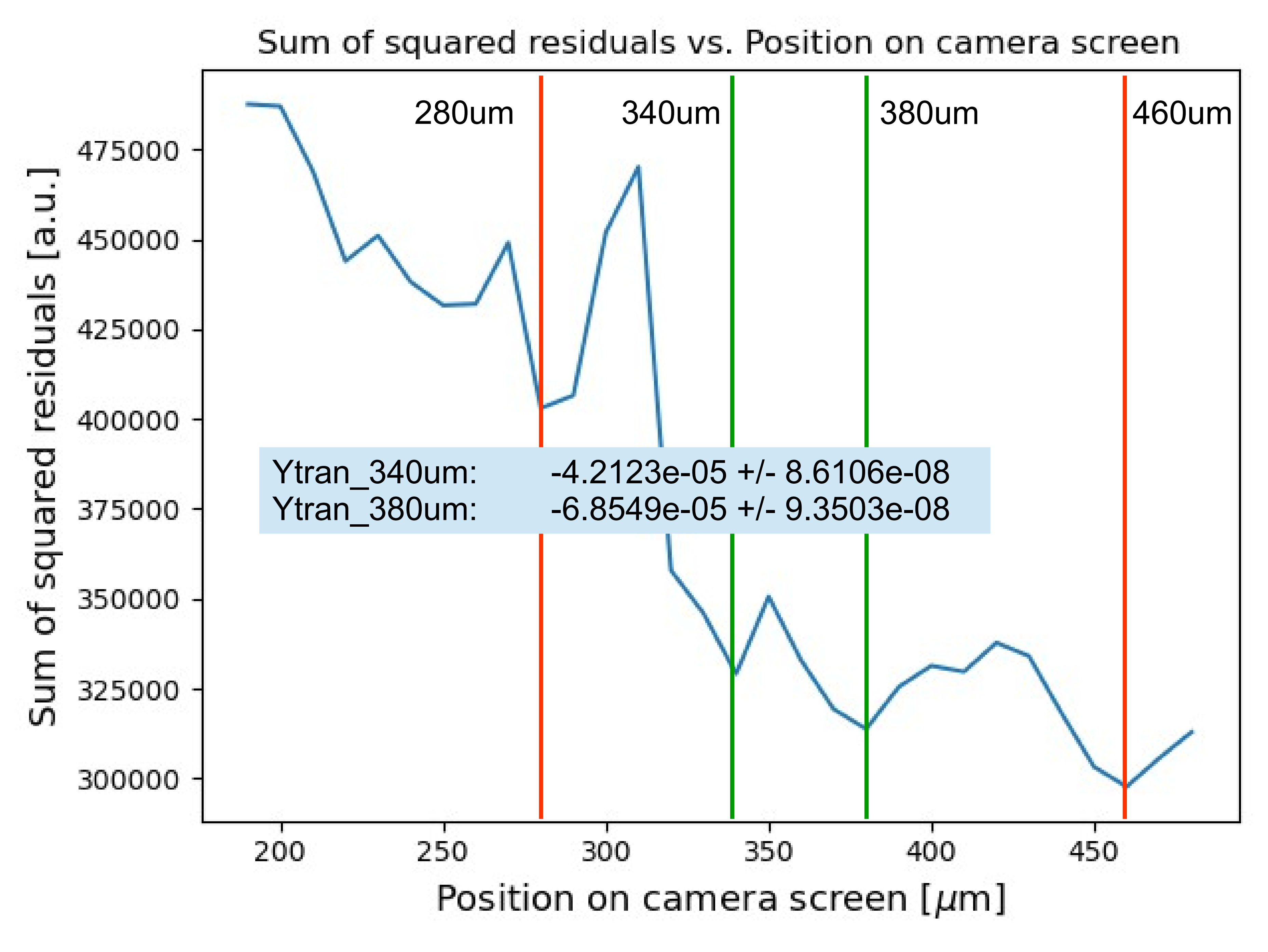


Data and Fit and resid
Set screen position 340um
Data
Fit
Intensity [a.u.]
60
50
40
30
20
10
0
−600
−400
−200
0
200
400
600
Θ_y [μrad]



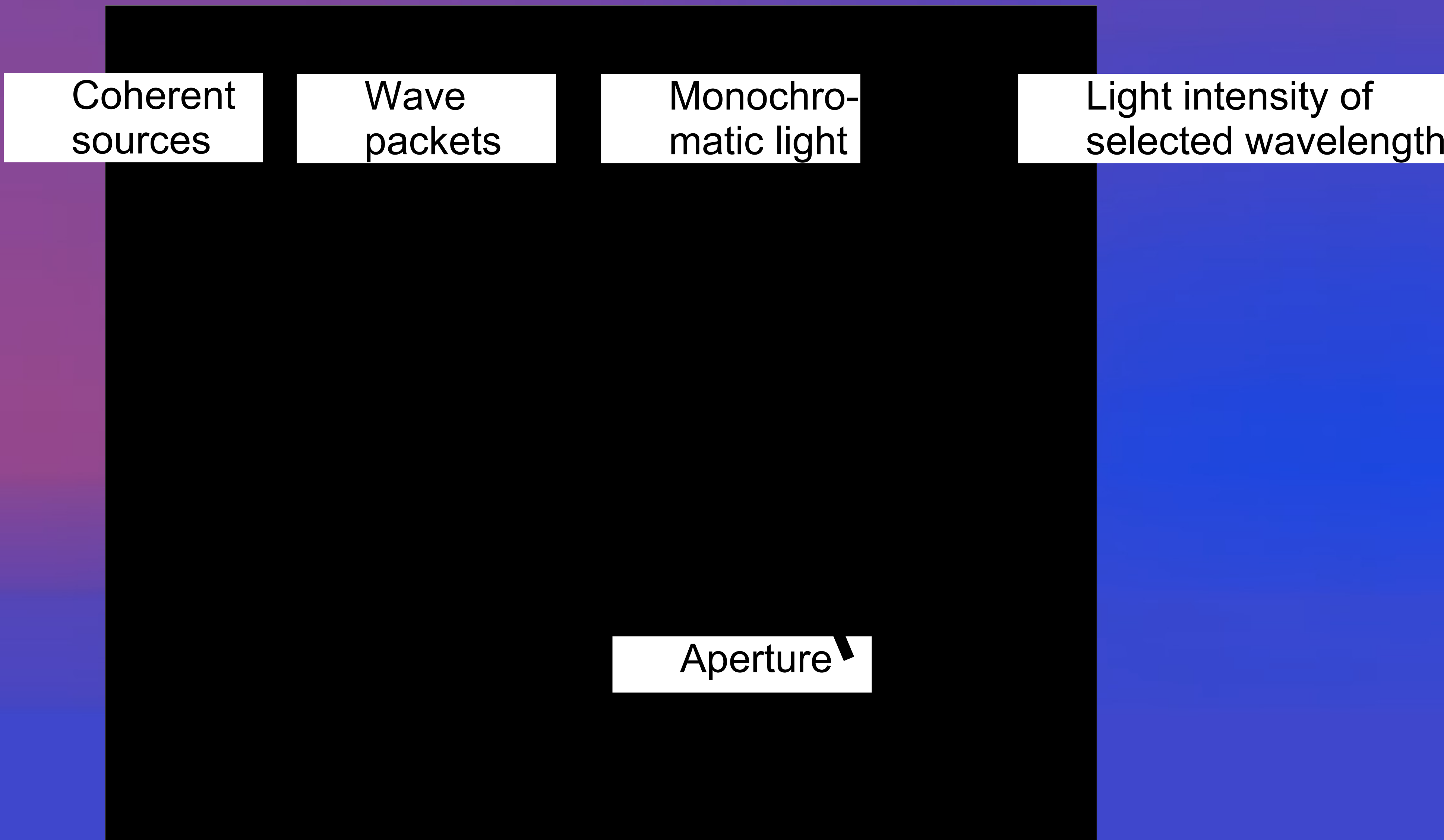
Data and Fit and resid
Set screen position 380um
Data
Fit
Intensity [a.u.]
60
50
40
30
20
10
0
−600
−400
−200
0
200
400
600
Θy [μrad]

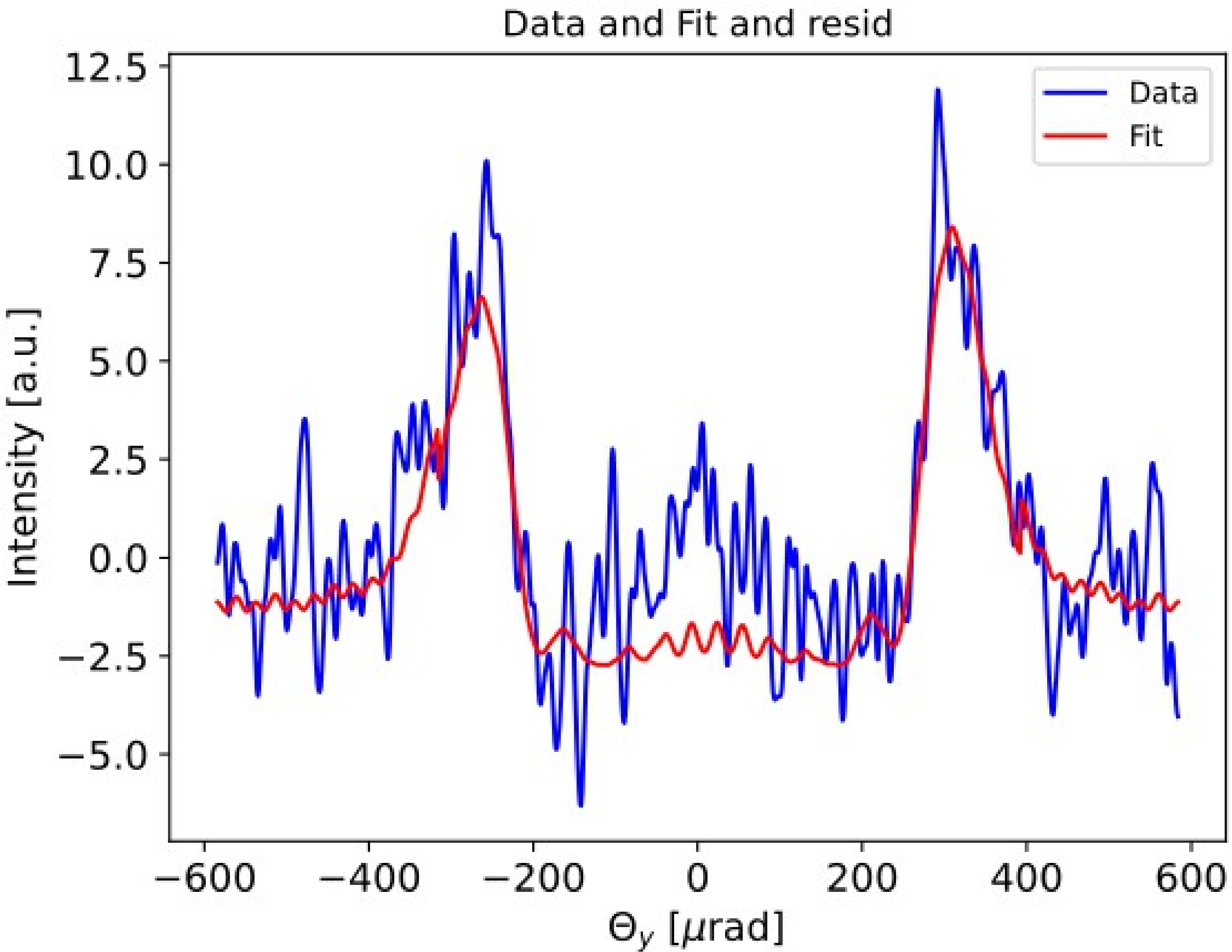

Data and Fit and resid
Set screen position 460um
Data
Fit
Intensity [a.u.]
60
50
40
30
20
10
0
−600
−400
−200
0
200
400
600
Θy [μrad]

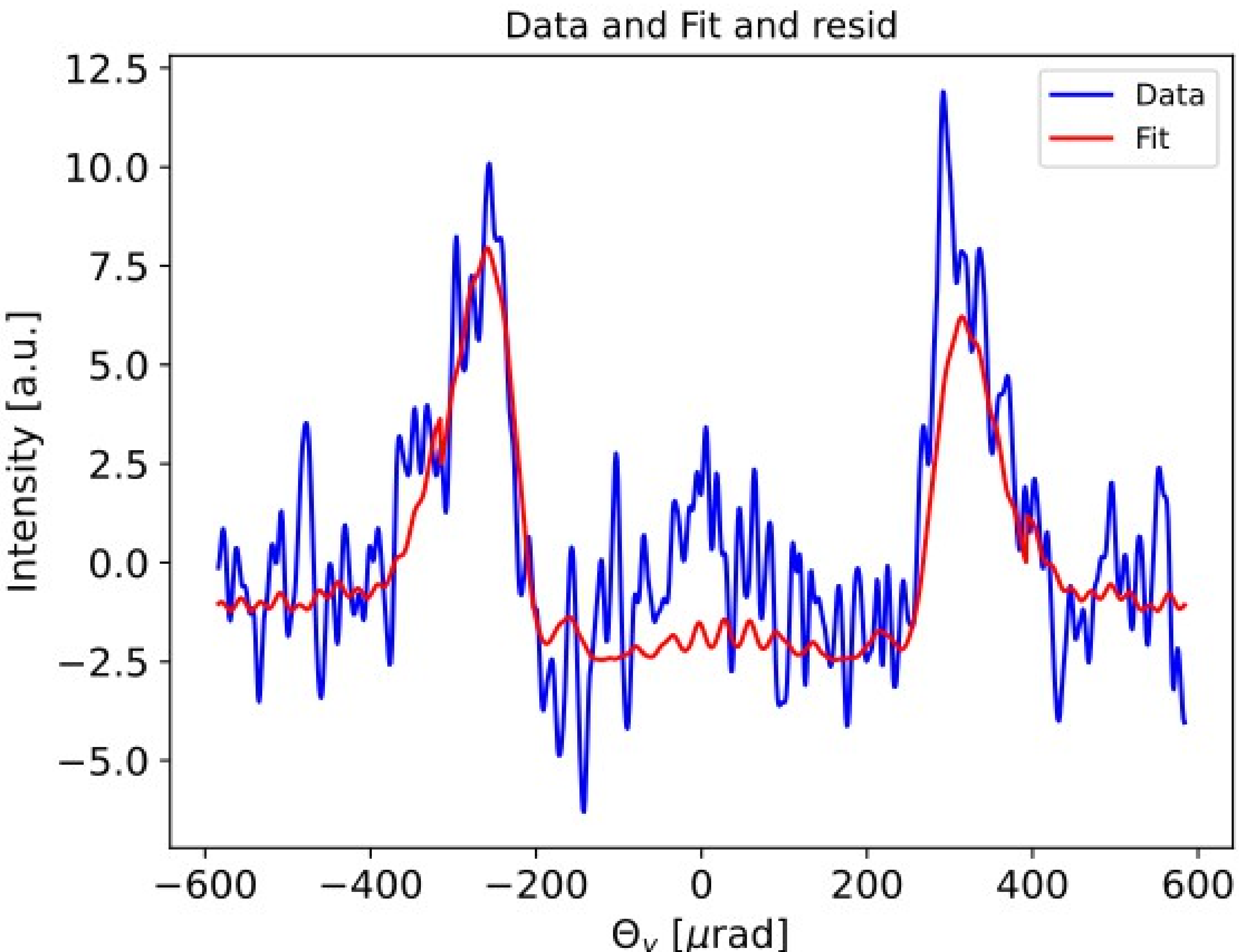


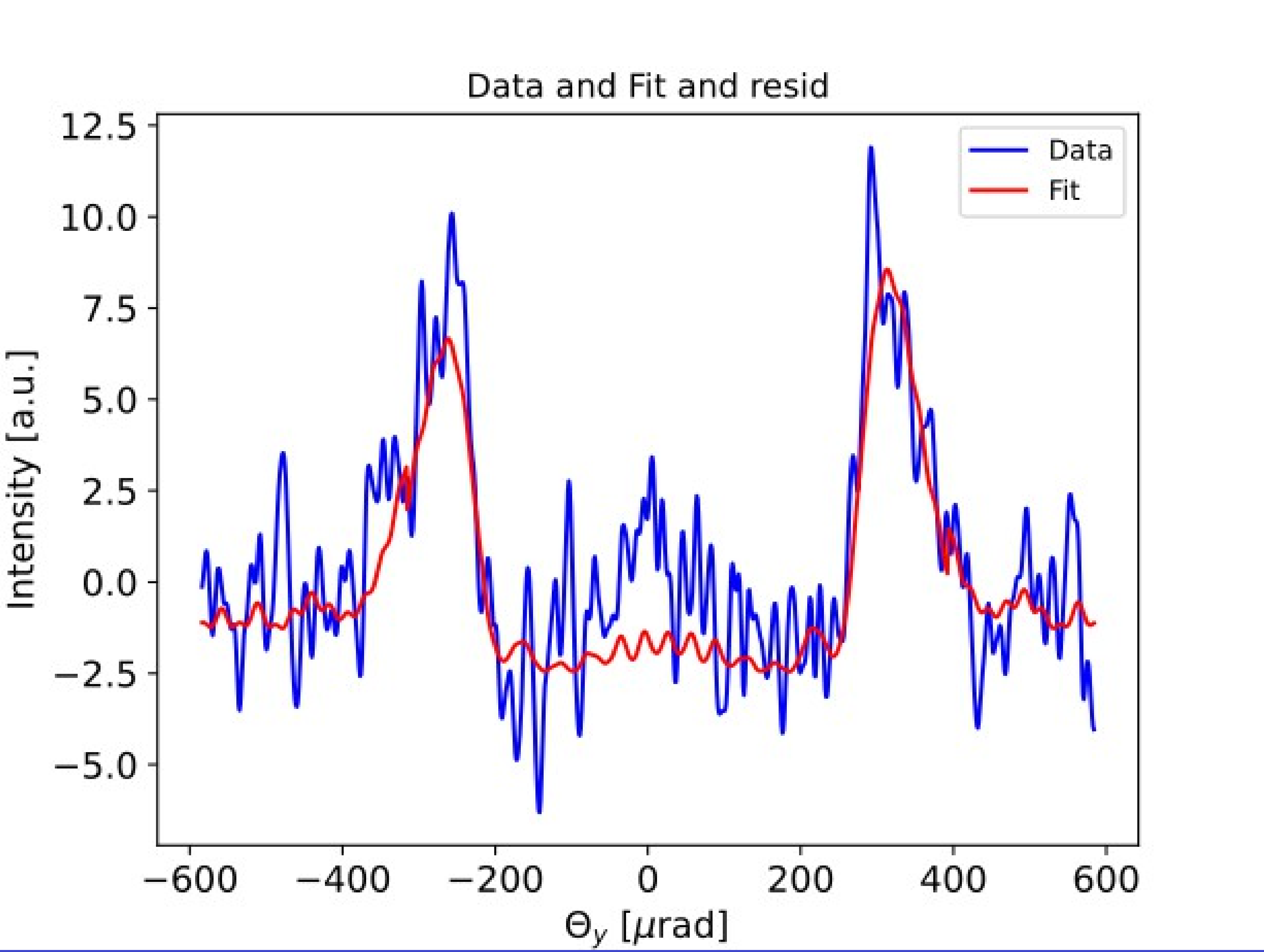
Sum of squared residuals vs. Position on camera screen
280um
340um
380um
460um
475000
450000
425000
400000
375000
350000
325000
300000
Sum of squared residuals [a.u.]
Ytran_340um: -4.2123e-05 +/- 8.6106e-08
Ytran_380um: -6.8549e-05 +/- 9.3503e-08
200
250
300
350
400
450
Position on camera screen [μm]

Method

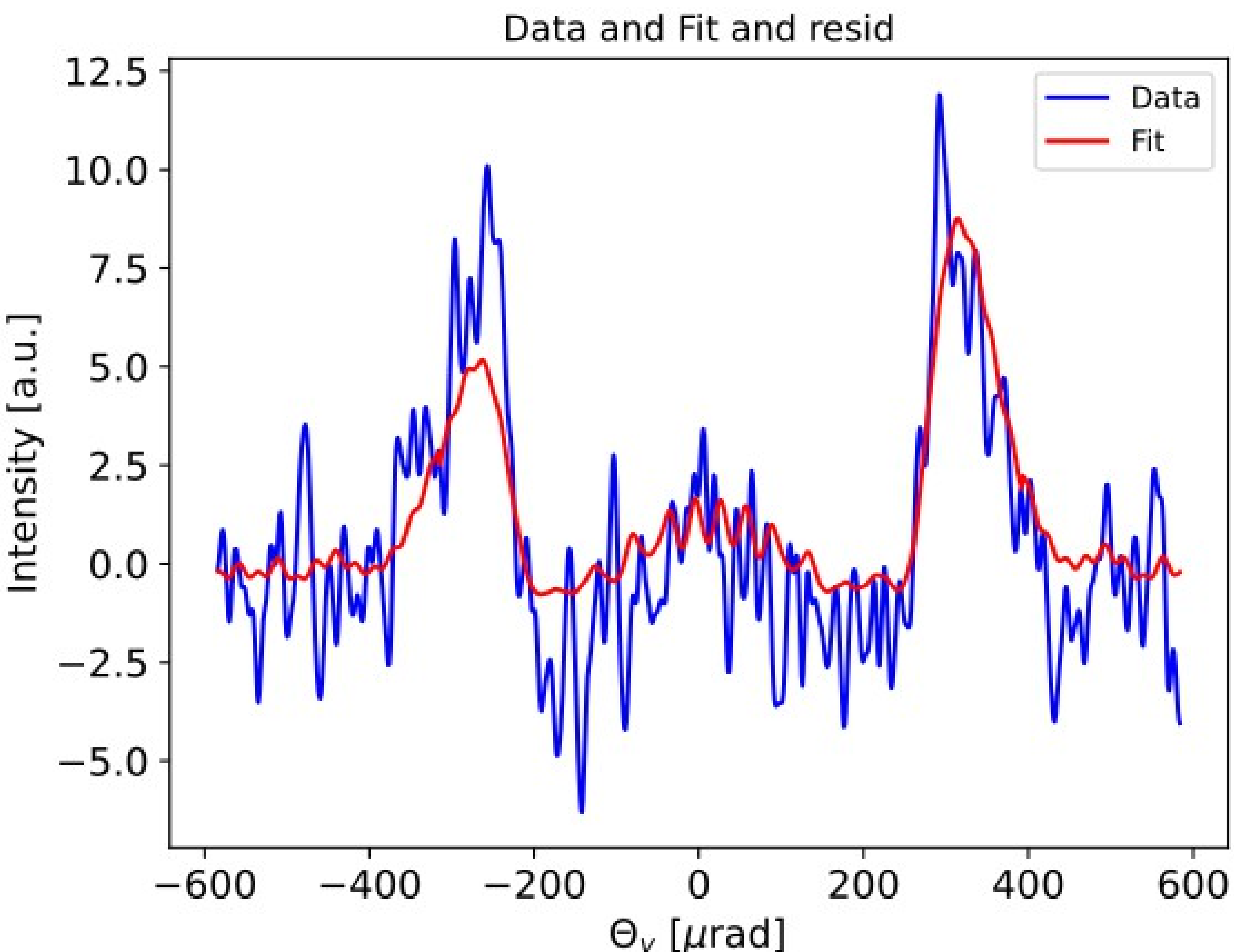

Coherent sources
Wave packets
Monochro-matic light
Light intensity of selected wavelength
Aperture


Data and Fit and resid
Data
Fit
12.5
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
Intensity [a.u.]
−600
−400
−200
0
200
400
600
Θy [μrad]

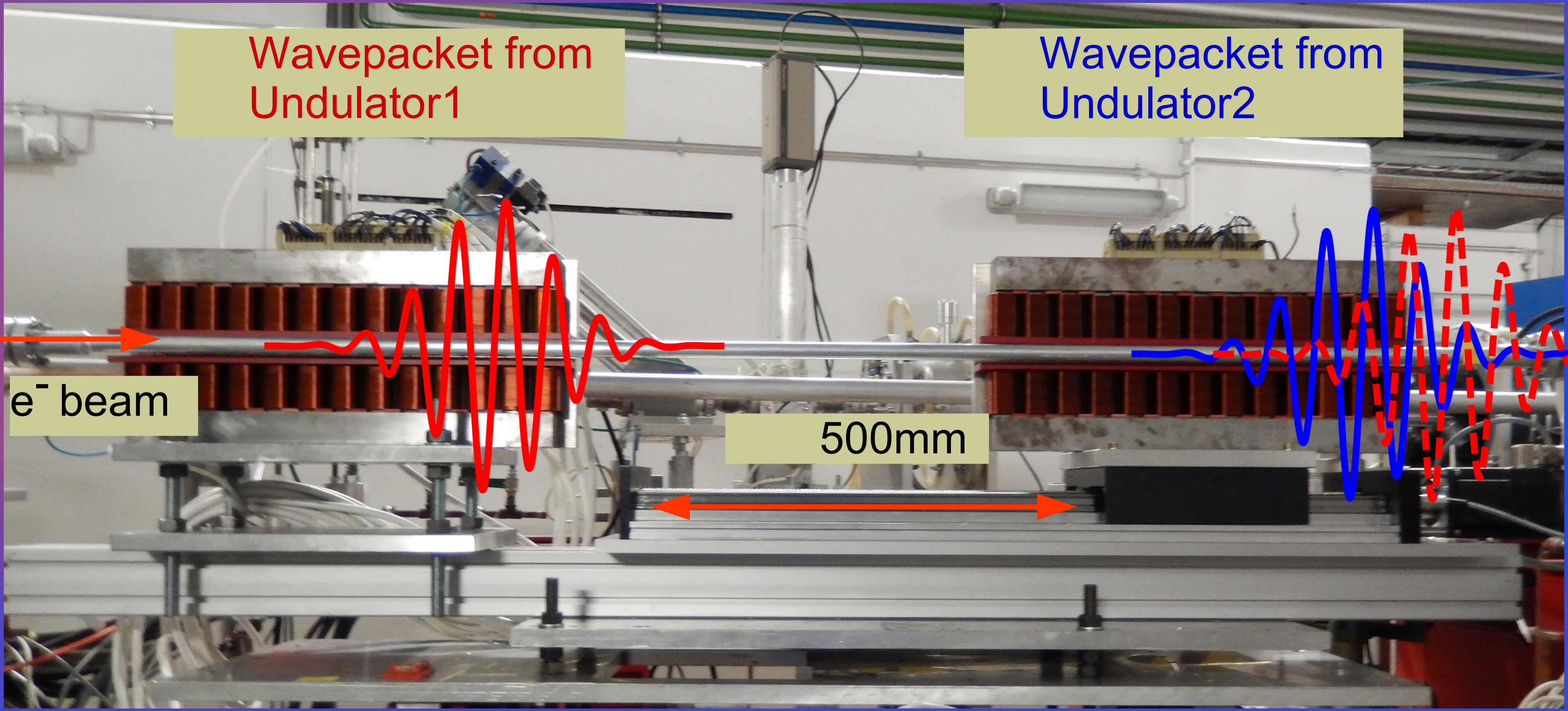

Data and Fit and resid
Data
Fit
Intensity [a.u.]
12.5
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−600
−400
−200
0
200
400
600
Θ_y [μrad]



Data and Fit and resid
Data
Fit
Intensity [a.u.]
12.5
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−600
−400
−200
0
200
400
600
Θ_y [μrad]


Data and Fit and resid
Data
Fit
Intensity [a.u.]
12.5
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−600
−400
−200
0
200
400
600
Θ_y [μrad]

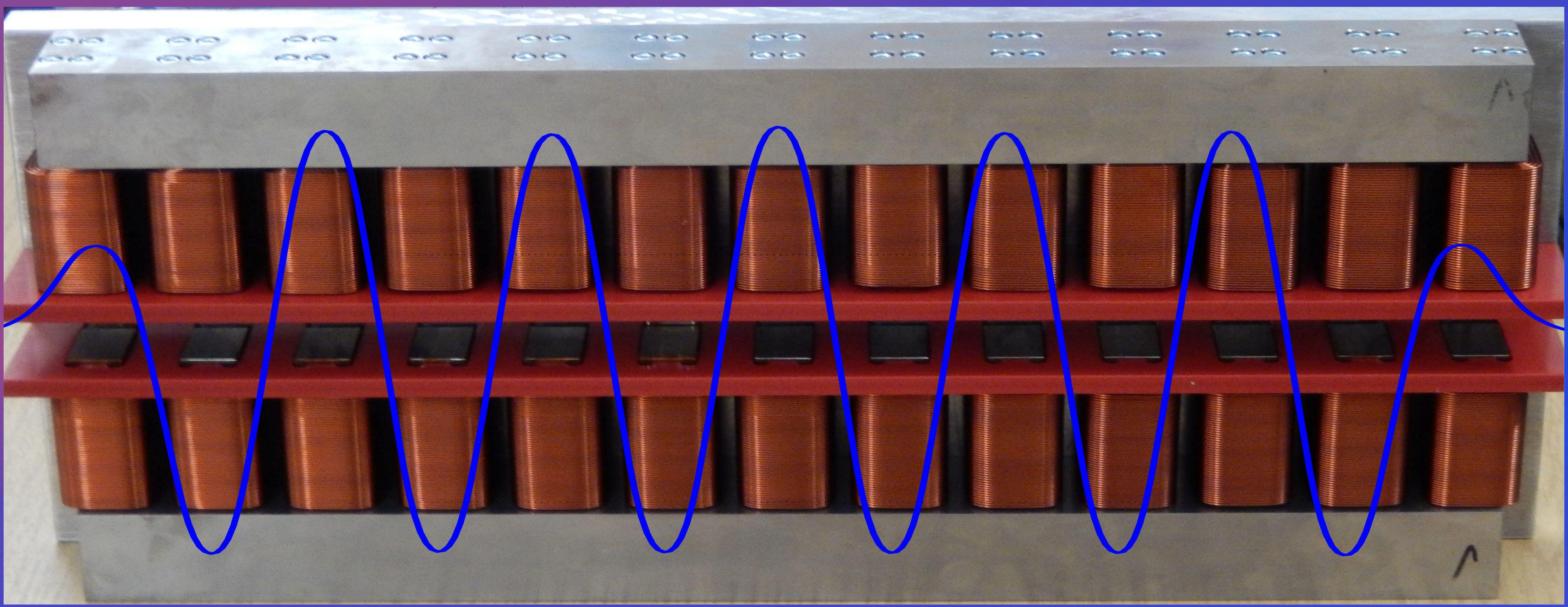
Undulators as sources for coherent radiation (former setup)



To Measure the oszillation period the second undulator ist moved by a motorized stage

Undulatorfield

500mm

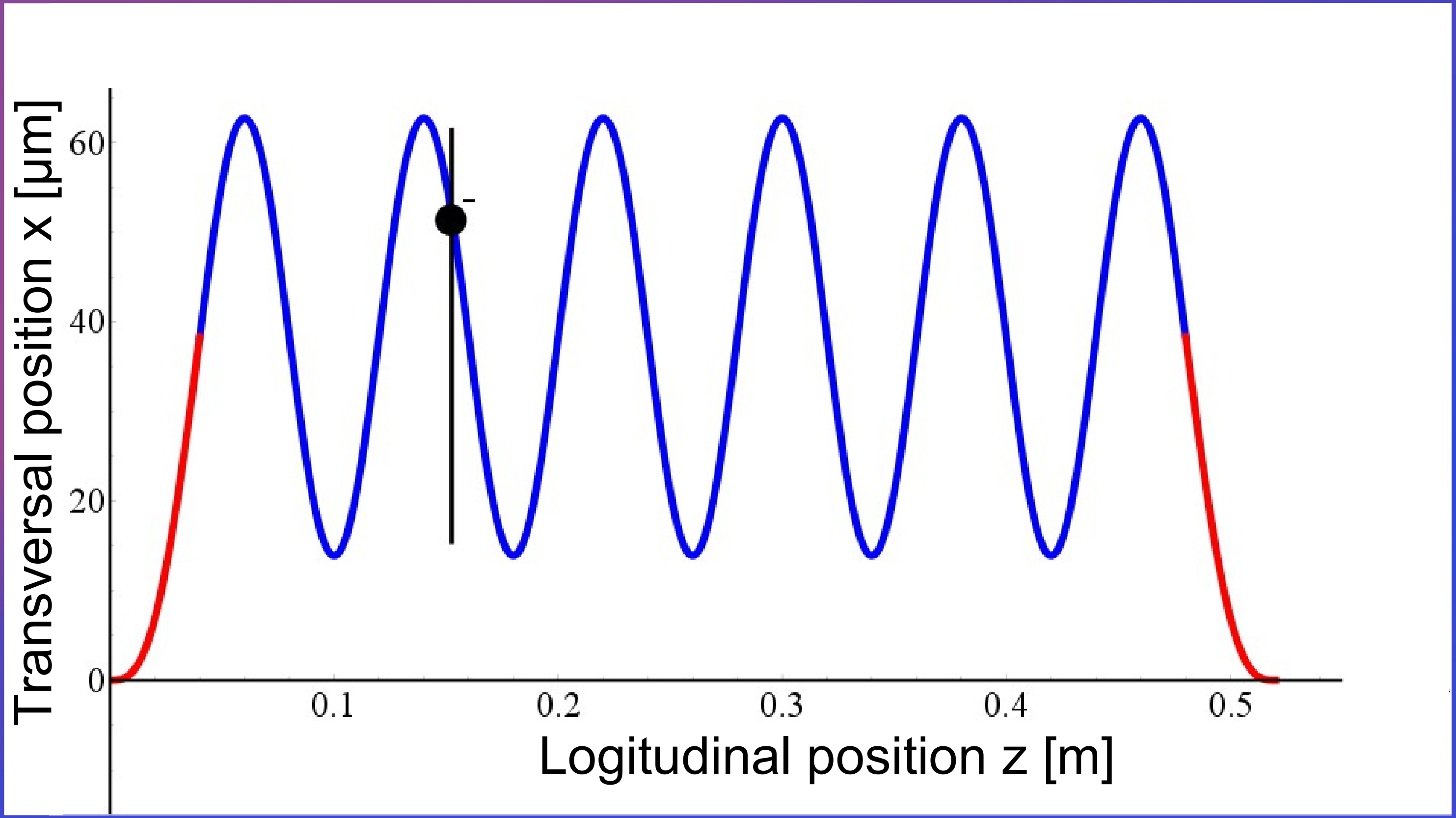


$B = \pm 100\,mT$

80mm

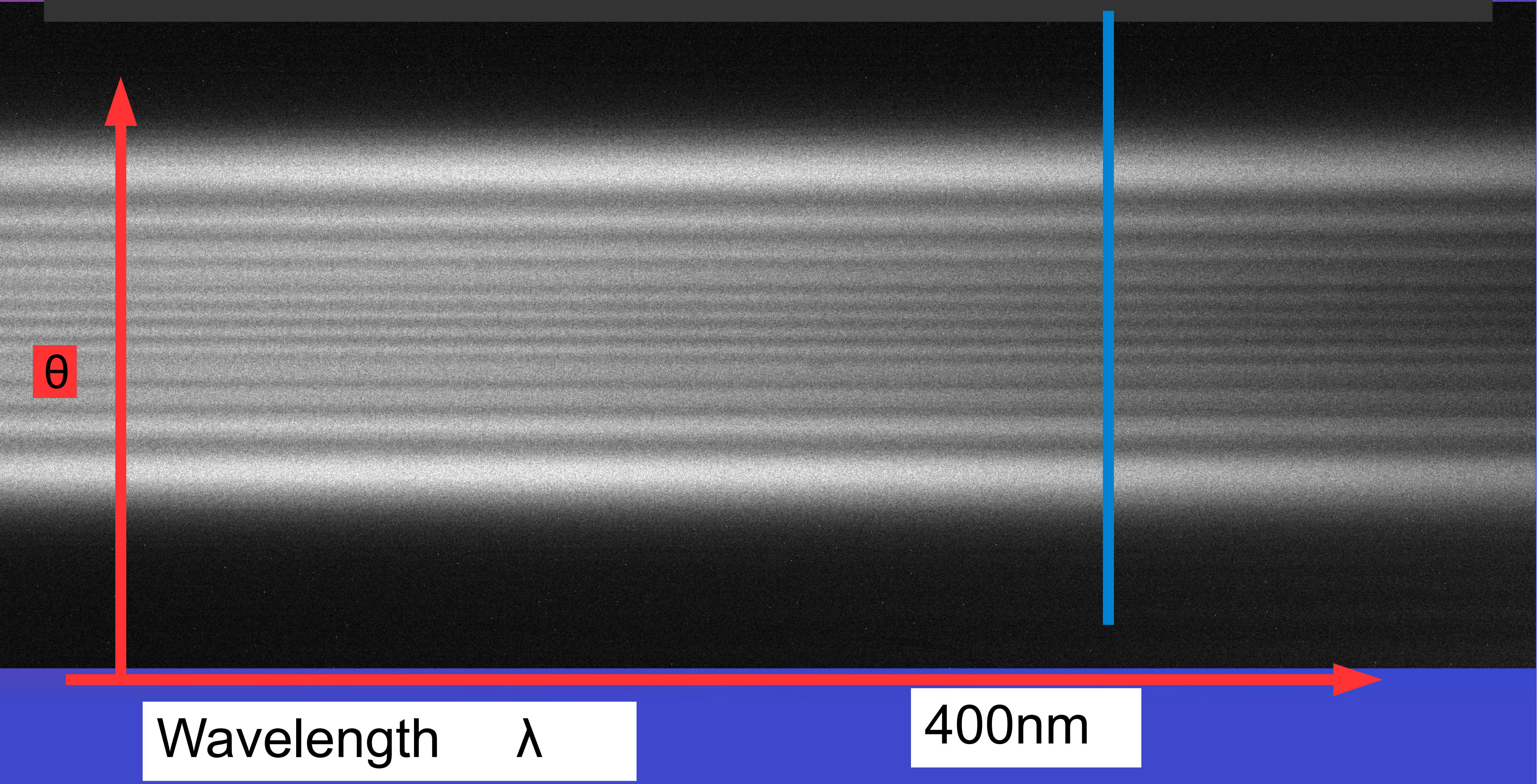
Undulator period λ_U

Emission of synchrotron radiation

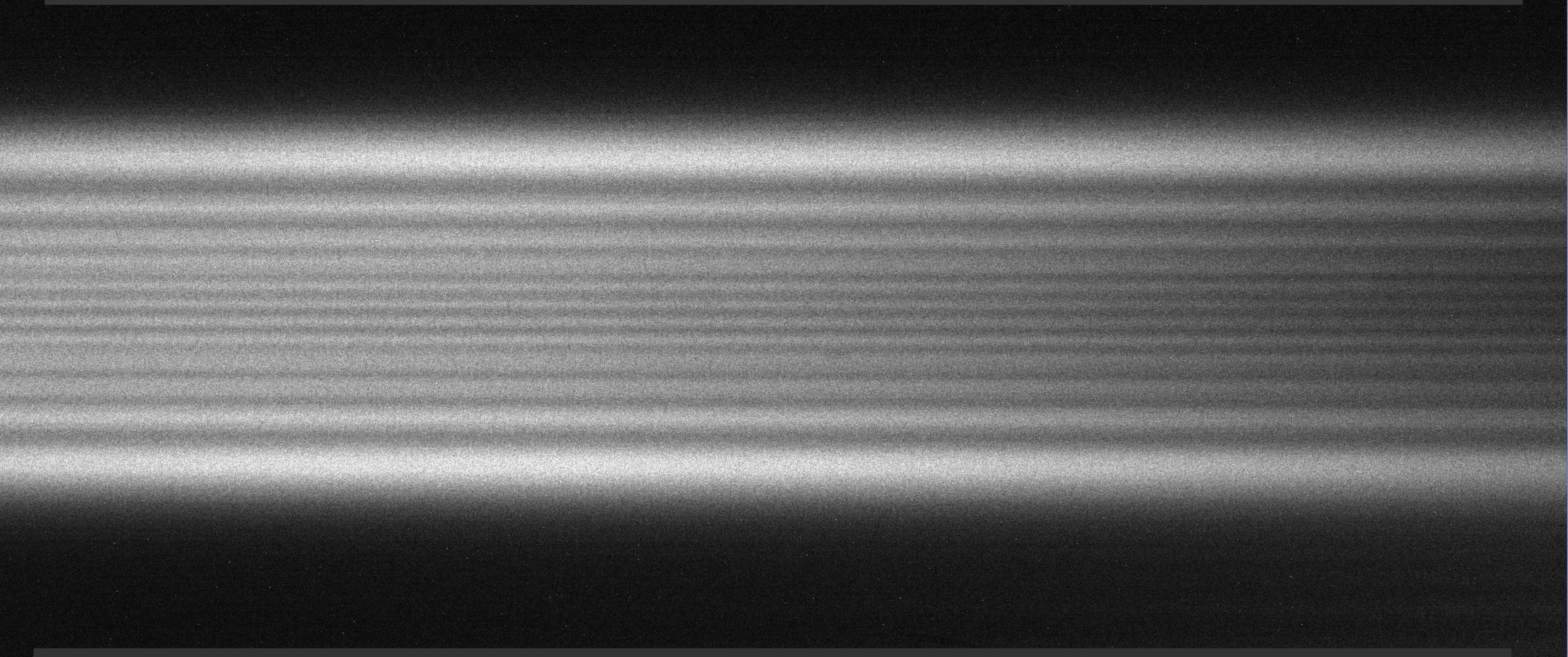


- Electrons oscillate perpendicular to the z-Axis
- The black bar suggests the idea of a high relativistic antenna moving towards the observer
- Emission takes place only in a finite length

Typical Undulator spectrum

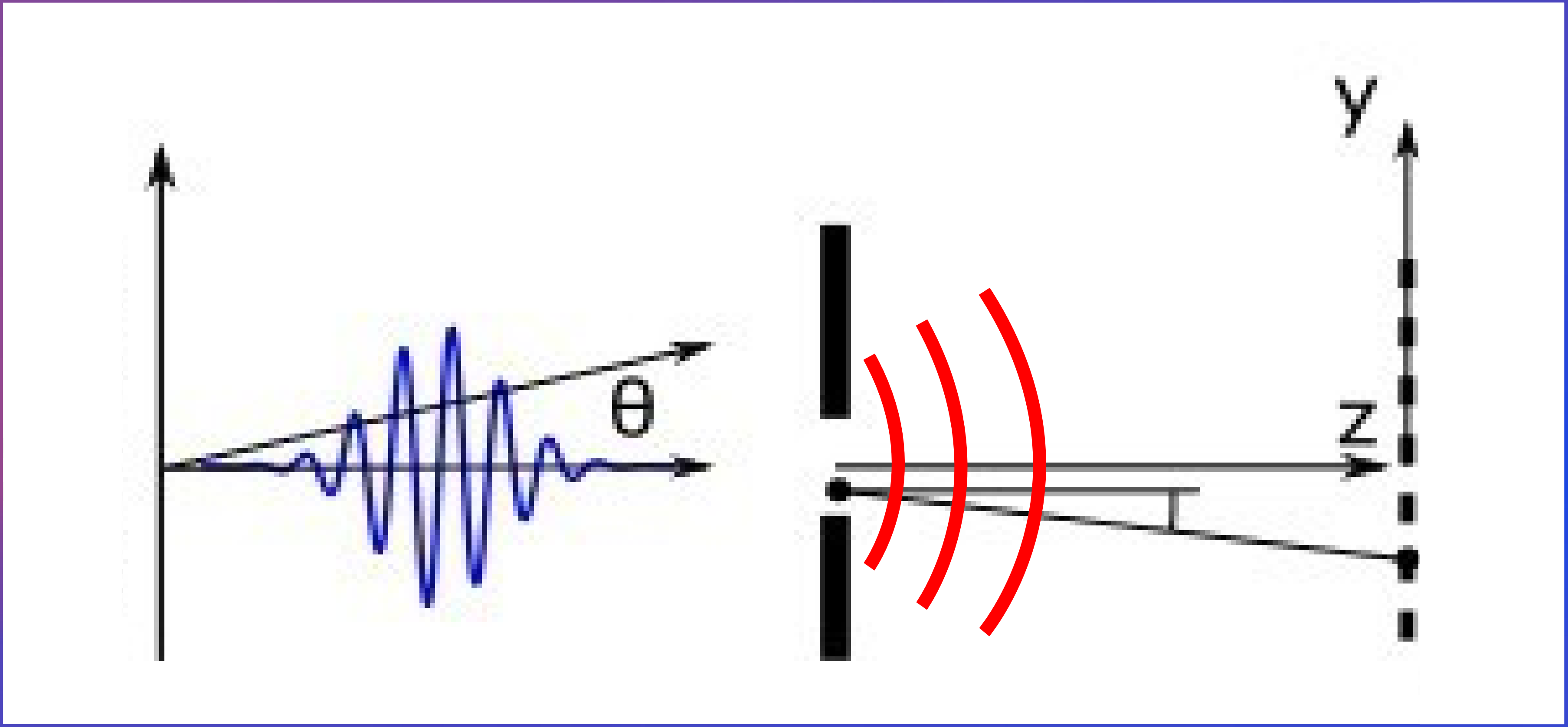


Typical Undulator spectrum

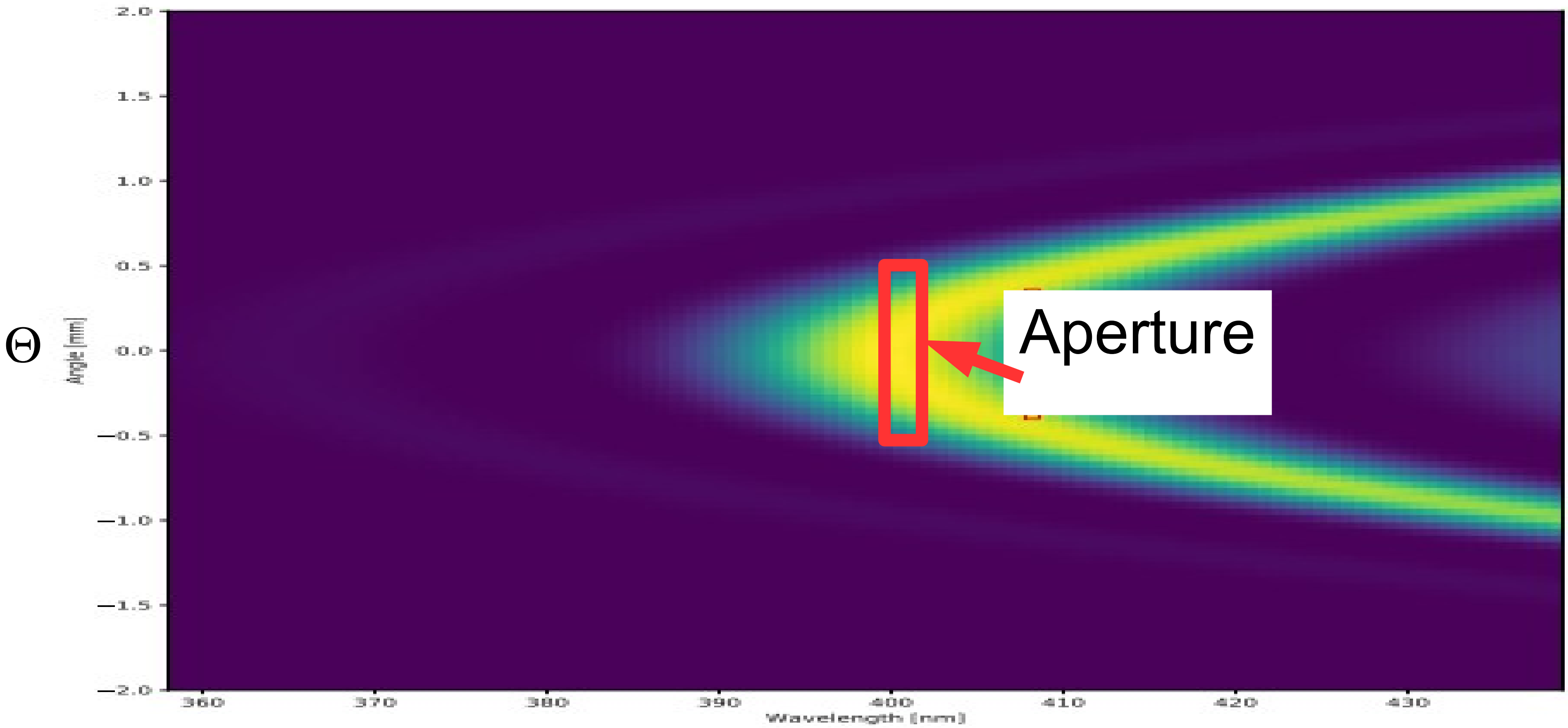


The stripes arise from Fresnel diffraction

Where diffraction occures



Undulator spectrum no Diffraction



Apparent gamma

Phase Φ depends also on the angle θ:

$$\Phi(d)=k_L\left((L_U+d)\frac{\Theta^2}{2}+\frac{d}{2\gamma^2}\right)$$

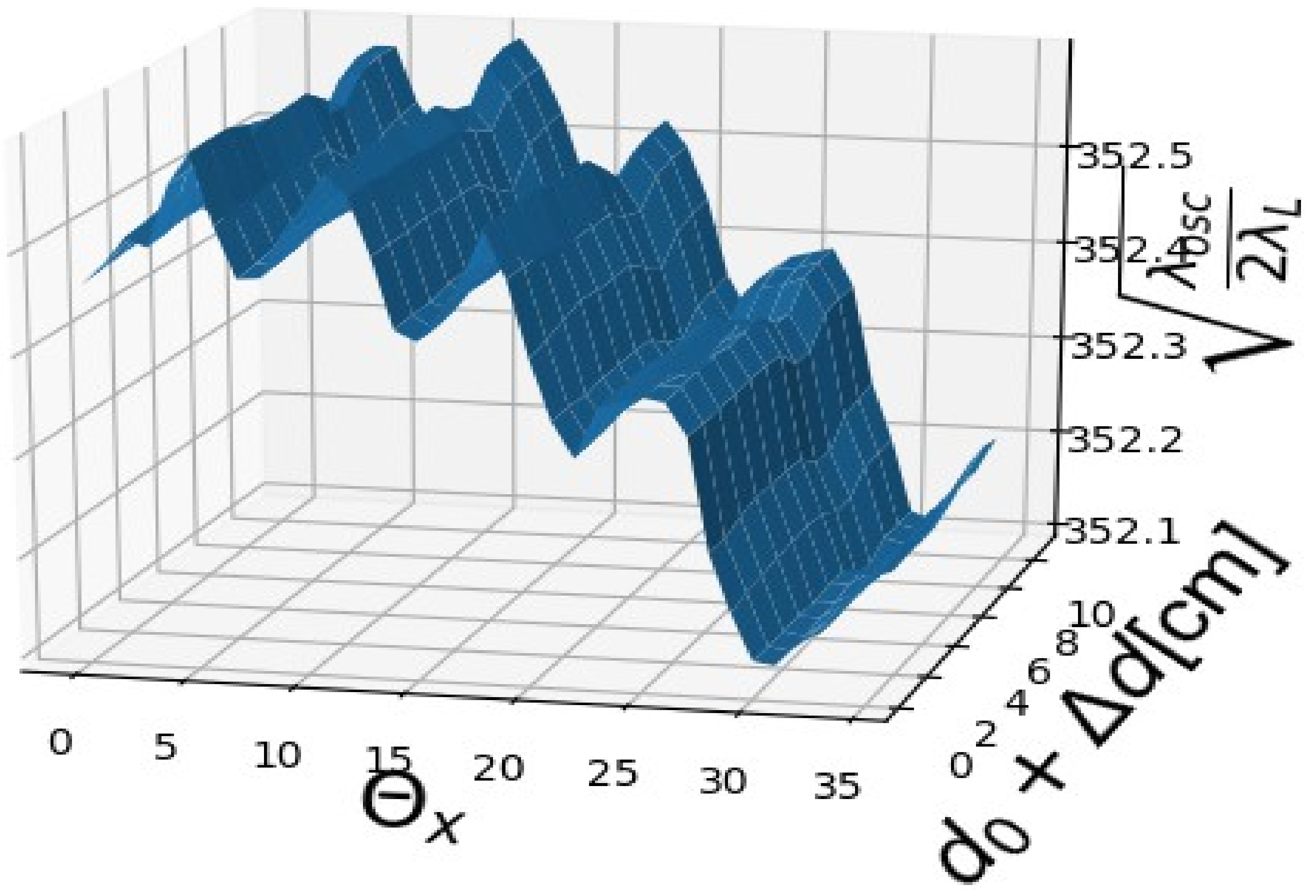
By setting Φ=2π an apparent gamma, „depends“ on the angle

$$\gamma(\Theta)=\sqrt{\frac{\lambda_{osc}}{2\lambda_L}}(\Theta)\propto\sqrt{\frac{1}{1+\Theta^2}}$$

(The real physics is, λosc depends on θ!!)

Gamma is affected by the Pattern

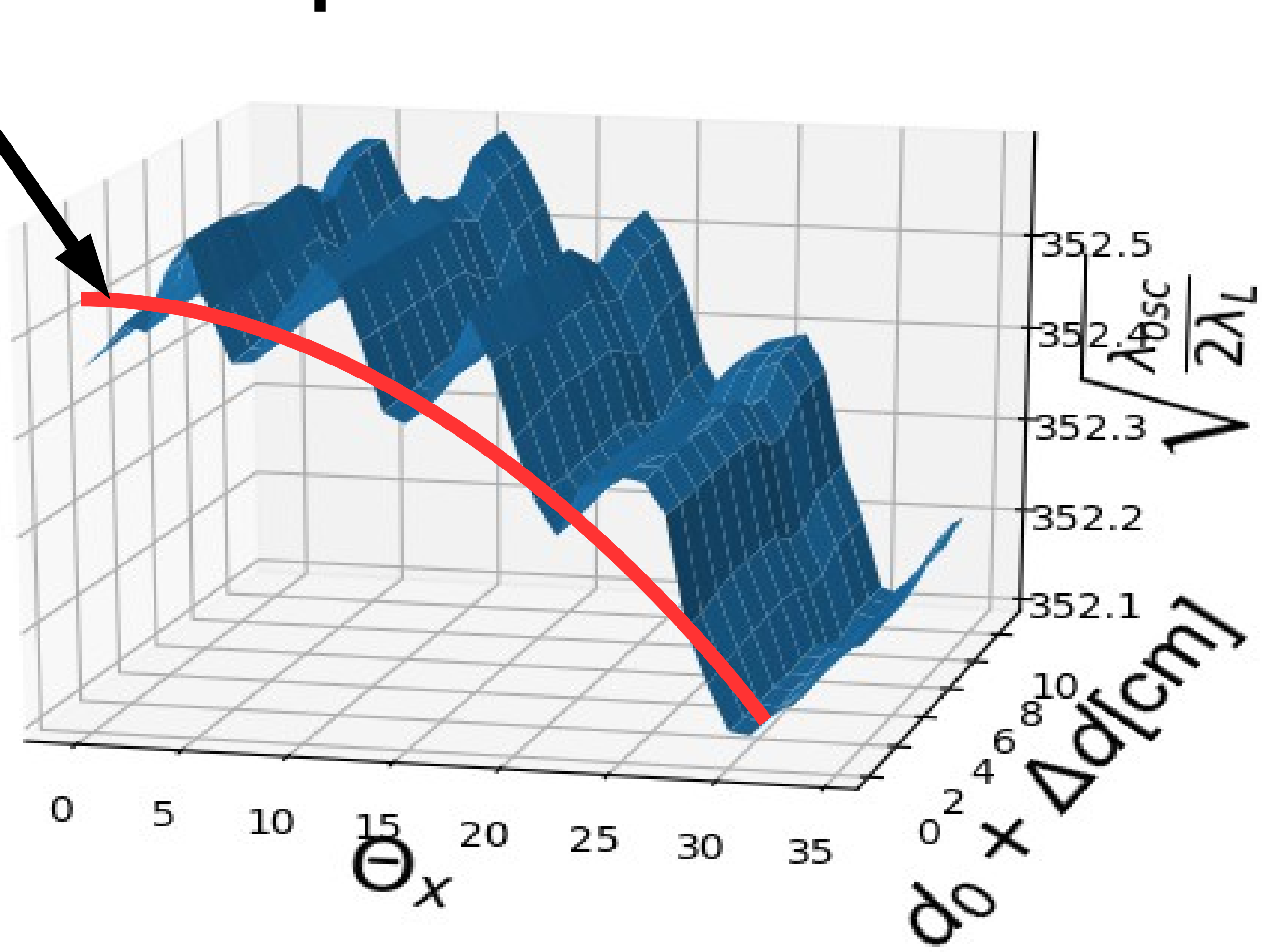
Analysis at one specific wavelength λ_L

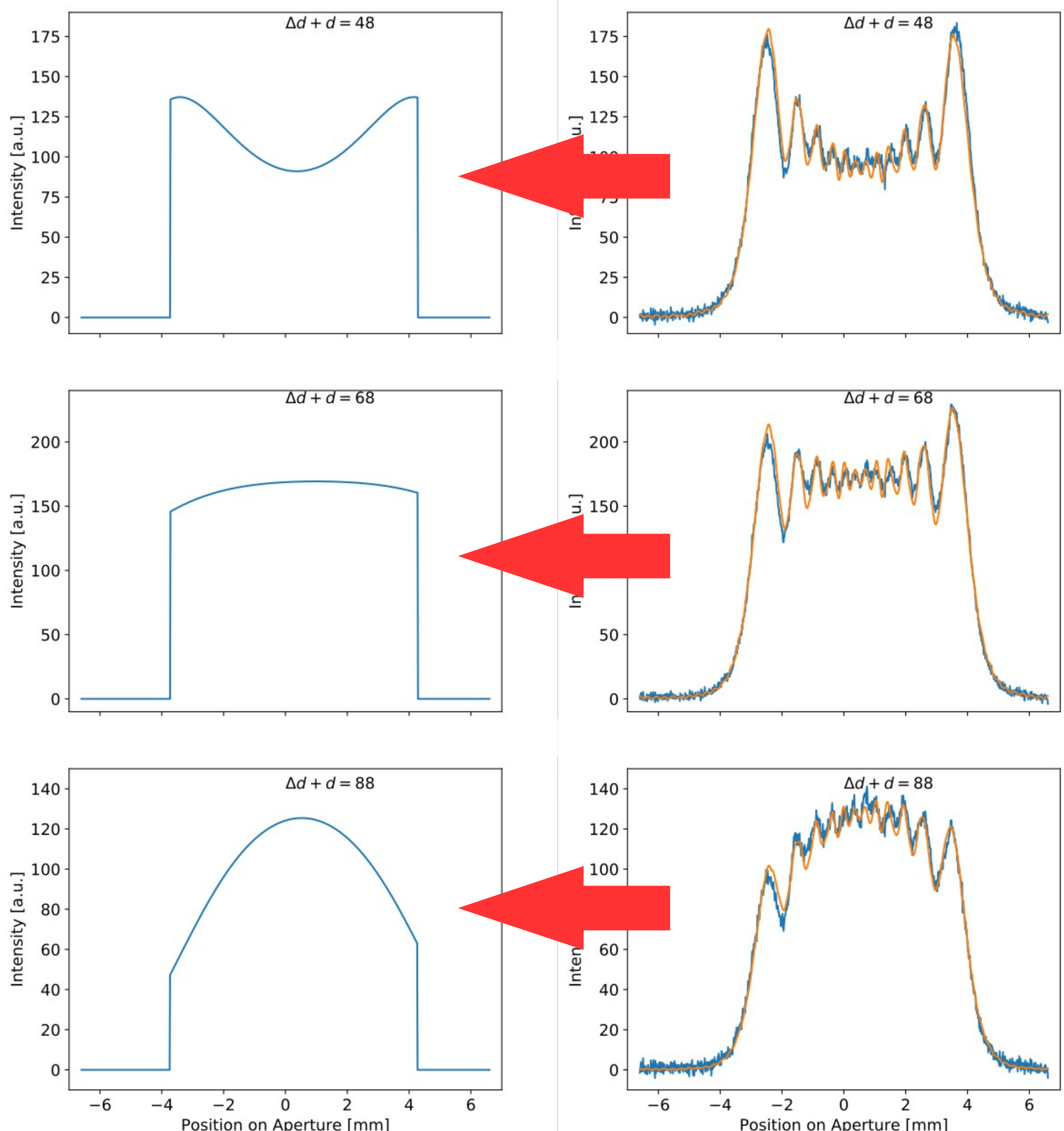


Gamma is affected by the Pattern

According to the equation it must be monotonic

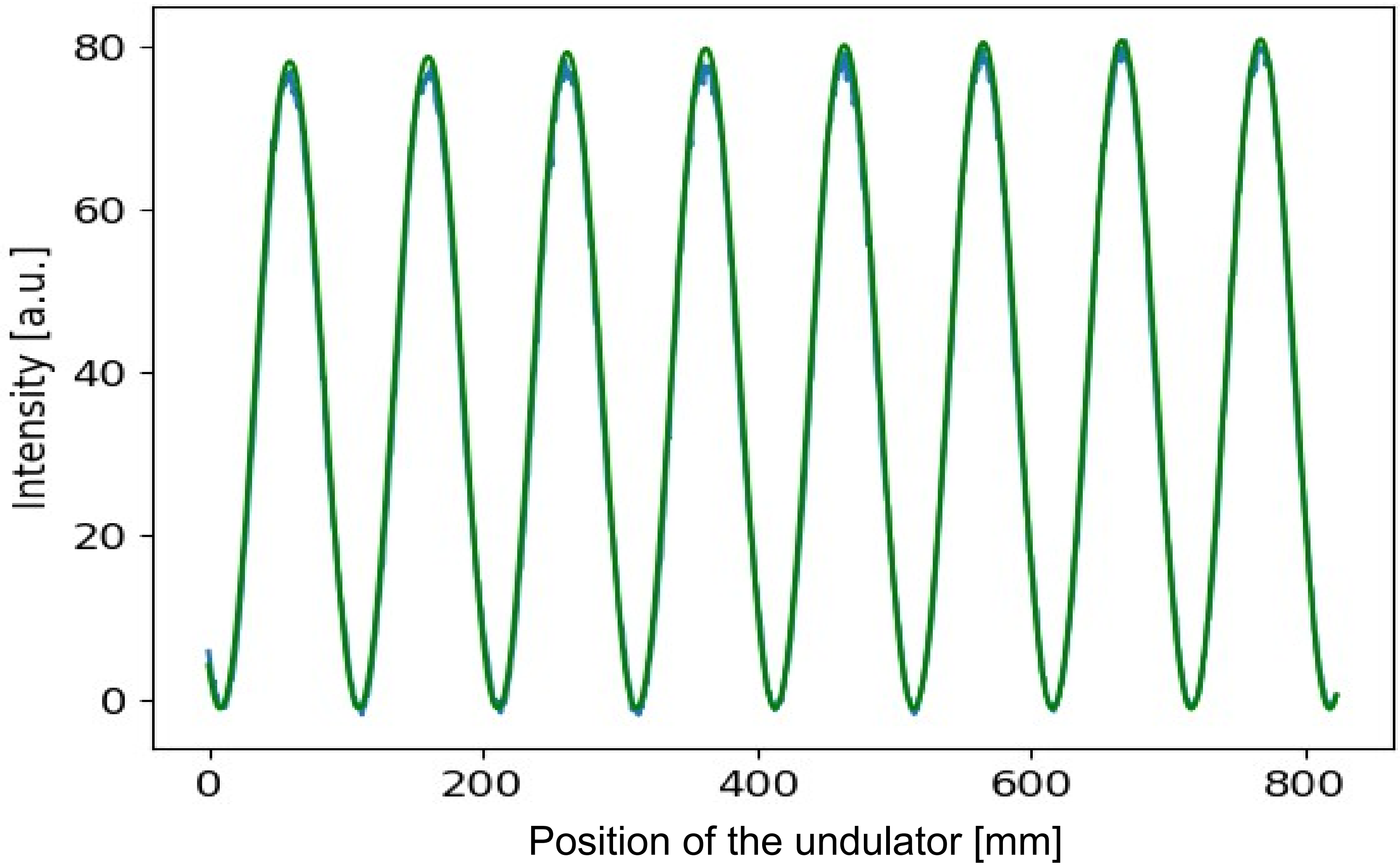
$$\sqrt{\frac{1}{1+\Theta_2^2}}$$





Δd + d = 48
Δd + d = 48
Δd + d = 68
Δd + d = 68
Δd + d = 88
Δd + d = 88
Intensity [a.u.]
Position on Aperture [mm]
Position on Aperture [mm]

Fitting works but how to determine the error for the fit?



Thank you for you attention