

# Statistical Thermodynamics within Hadron resonance gas (HRG) model including magnetic field at CBM Experiment

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## Abstract

In this paper, we have calculated statistical thermodynamic quantities such as pressure density ( $P/T^4$ ), energy density ( $\epsilon/T^4$ ), and trace anomaly ( $\epsilon-3p)/T^4$  using HRG model in the presence of magnetic field ( $eB$ ) effect. The mentioned thermodynamics have been calculated at both vanishing (**zero**) and finite (**170, 340, and 425**) baryon chemical potential ( $\mu_B$ ) where  $eB$  are taken to be **1, 1.2, and 1.5**. The obtained results are compared with the recent lattice QCD data over temperatures ranging from **130 MeV** to **200 MeV** at the mentioned values of  $eB$  and  $\mu_B$ . For equilibrium temperatures up to the vicinity of the chiral phase transition temperature  $\approx 160$  MeV, a decent fitting between the model and the lattice data is observed for different values of  $eB$  and  $\mu_B$ , especially at  $(eB, \mu_B) = (1.2, 170)$ ,  $(1.2, 340)$  and  $(1.2, 425)$ , where  $eB$  is in  $\text{GeV}^2$  and  $\mu_B$  is in MeV. For  $\mu_B = 0$  and **170 MeV**, the ideal HRG model seems to offer the best fit. The effect of magnetic field seems to be more probable at non-vanishing chemical potentials, especially within the range  $\mu_B \in [170, 340]$  MeV.

## Introduction

One of the primary objectives of ultra relativistic nuclear collisions is to learn more about the hadron-parton phase diagram, which is characterized by many phases and types of phase transitions [1]. The gauge field theory of quantum chromodynamics (QCD) describes the strong interactions of coloured quarks and gluons and their colourless bound states. The study of strongly interacting matter under severe temperature  $T$  and baryon chemical potential  $\mu_B$  has sparked a spectacular global theoretical and experimental effort.

The bulk thermodynamic quantities such as temperature and density (chemical potential) dependency, typically described as the equation of state (EoS) is the most fundamental characterization of the equilibrium properties of highly interacting matter. Its analysis within the framework of lattice QCD has been refined ever since the early calculations performed in pure SU(N) gauge theories [2]. The EoS at vanishing chemical potentials does already provide important input into the modeling of the hydrodynamic evolution of hot and dense matter created in heavy-ion collisions [3, 4]. While this is appropriate for the thermal conditions met in these collisions at the LHC and the highest RHIC beam energies, knowledge of the EoS at non-vanishing baryon, strangeness, and electric charge chemical potentials is indispensable for the hydrodynamic models of the conditions met in the beam energy scan (BES) at RHIC [5] and in future experiments at future facilities like FAIR at GSI and NICA at JINR [6, 7].

In this paper, we have used HRG model with magnetic field effect to obtain better results consistent with that obtained from the recent lattice (QCD) theory at both vanishing and finite baryon chemical potential.

## The Used Approach

In a grand canonical ensemble, the partition function reads [8]

$$Z(T, V, \mu) = \text{Tr} \left[ \exp \left( \frac{\mu N - H}{T} \right) \right], \quad (1)$$

where  $H$  is the Hamiltonian combining all relevant degrees of freedom and  $N$  is the number of constituents of the statistical.

Eq. (1) Would be expressed as a sum overall hadron resonances which are taken from the particle data group (PDG) [9] with masses up to **12 GeV**,

$$\ln Z(T, V, \mu) = \sum_i \ln Z(T, V, \mu) = V \sum_i \frac{g_i}{2\pi^2} \int_0^\infty p^2 dp \ln \left[ 1 \pm \exp \left( \frac{\mu_i - \epsilon_i}{T} \right) \right], \quad (2)$$

Where  $\pm$  stands for fermions and bosons, respectively.  $\mu_i$  represents the chemical potential.  $\epsilon_i$  is the dispersion relation and it is defined as,

$$\epsilon_i = \sqrt{p^2 + m_i^2}. \quad (3)$$

All thermodynamic quantities can be derived from Eq. (2) as,

$$n_i(T, \mu) = \sum_i \frac{g_i}{2\pi^2} \int_0^\infty p^2 dp \frac{1}{\exp[(\mu_i - \epsilon_i)/T] \pm 1}, \quad (4)$$

$$p_i(T, \mu) = \sum_i \frac{g_i}{2\pi^2} \int_0^\infty p^2 dp \frac{-\epsilon_i \pm \mu_i}{\exp[(\mu_i - \epsilon_i)/T] \pm 1}, \quad (5)$$

$$\epsilon_i(T, \mu) = \sum_i \frac{g_i}{2\pi^2} \int_0^\infty p^2 dp \frac{1}{\exp[(\mu_i - \epsilon_i)/T] \pm 1}. \quad (6)$$

Eqs. (4), (5), and (6) gives the number density, pressure density, and energy density in the ideal HRG model, respectively.

In case of applying magnetic field effect, Eq. (3) will be modified depending on Landau quantization suggestions [10] as the following ,

$$\epsilon_i = \sqrt{p^2 + \{2l - 1 - 2s\}e_i B + m_i^2} \quad (5)$$

with  $s = -s_i, -s_i + 1, \dots, s_i$  and  $l=0,1,2,\dots$  are the spin and orbital quantum numbers, respectively. where  $e_i$  is the electrical charge of the  $i$ -th hadron. The statistical thermodynamics such as pressure density ( $P/T^4$ ), energy density ( $\epsilon/T^4$ ), and trace anomaly ( $\epsilon-3p)/T^4$  are calculated using HRG model in the presence of magnetic field ( $eB$ ) effect.

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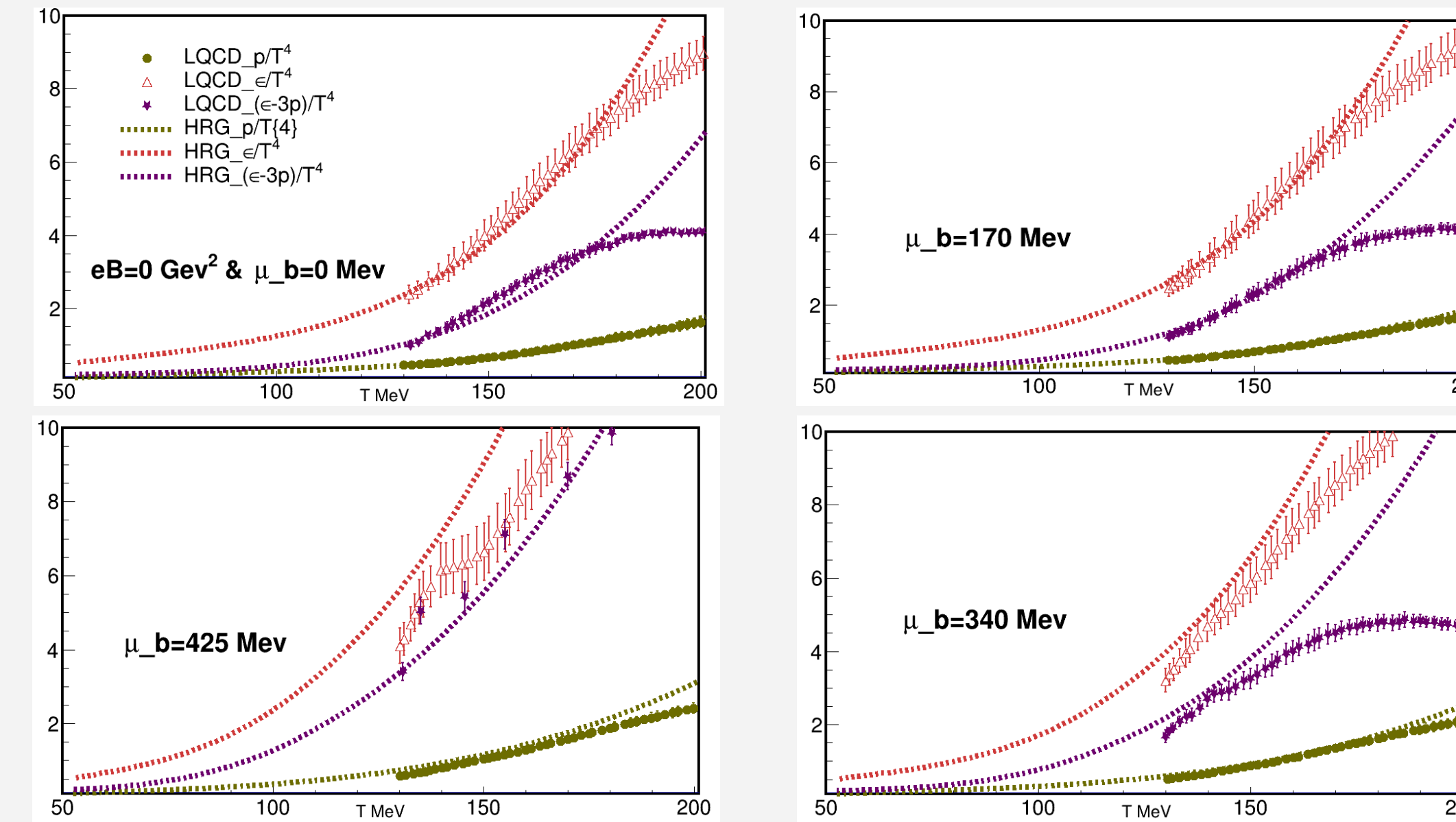
## Results and Discussions

We confront the data of the calculated thermodynamics using the HRG model based on magnetic field effect with the corresponding lattice thermodynamics data in the temperature range  $T \in [130, 200]$  MeV.

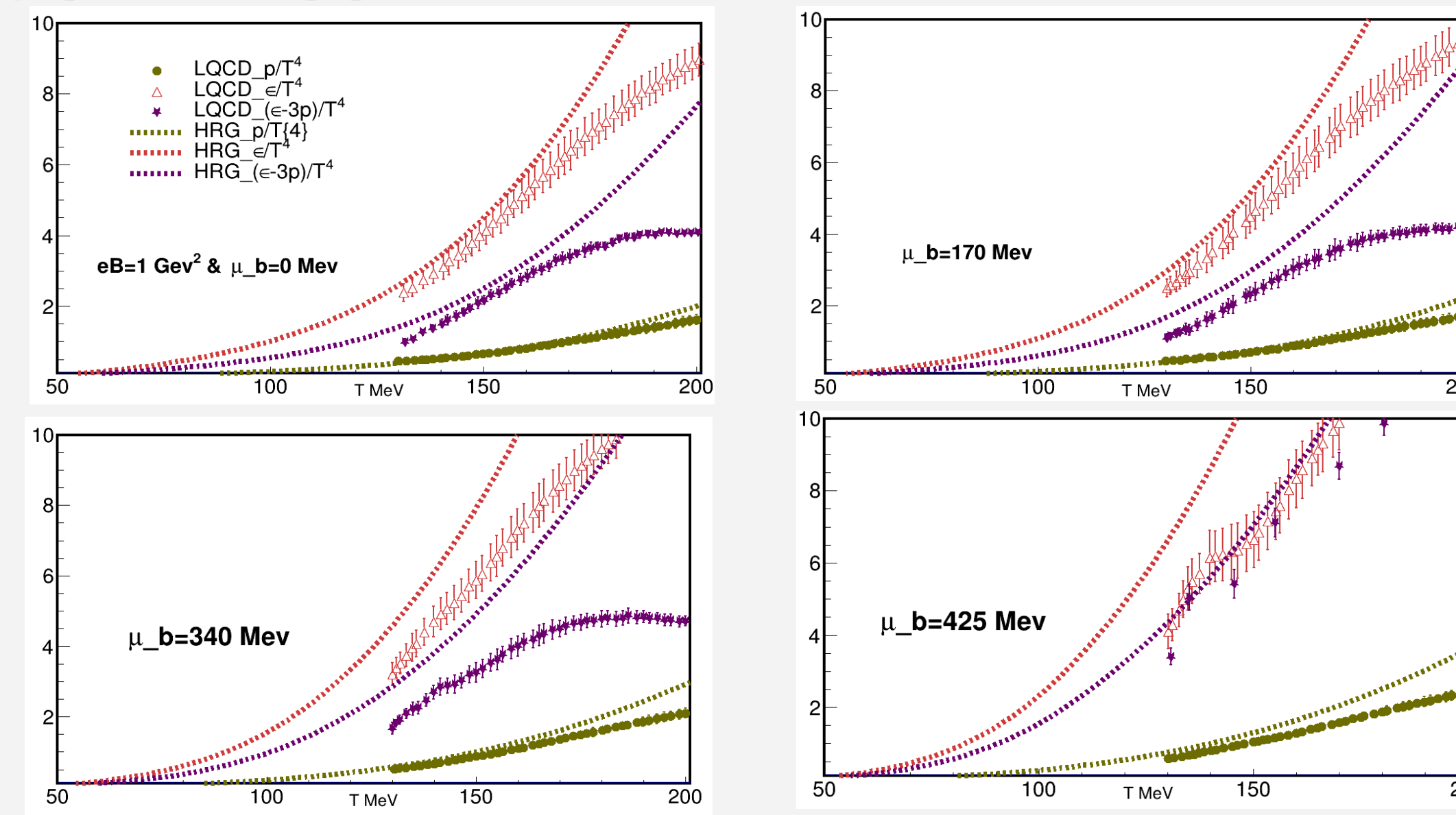
Fig. (1) depicts the temperature dependence of the pressure density ( $P/T^4$ ), energy density ( $\epsilon/T^4$ ), and trace anomaly ( $\epsilon-3p)/T^4$  (dashed curves) calculated using the ideal HRG model.

For  $\mu_B = 170$  MeV as it appears in Fig. (2), the best fit occurs for zero and **1.2  $\text{GeV}^2$**  magnetic field. This good-fit temperature extends from temperatures below  $T_c$  till  $T \gtrsim T_c \approx 160$  MeV. It is quite obvious here that the model fits the lattice data better compared to the corresponding vanishing chemical potential case(s) [11, 12]. However, in the temperature range  $T \in [170, 200]$  MeV, the mismatch of the model with the lattice data becomes more pronounced compared to the corresponding range of the vanishing chemical potential case(s). For the case of  $\mu_B = 340$  MeV, see Fig. (3), the only interesting observation is for  $eB = 1.2 \text{ GeV}^2$ , the model data well below and in the vicinity of  $T_c$  and up to  $T \approx 170$  MeV significantly approaches the corresponding lattice data. The data mismatch then diverges for higher temperatures. For the case of  $\mu_B = 425$  MeV, Fig. (4), the data mismatch is generally too expect at **1.2  $\text{GeV}^2$** .

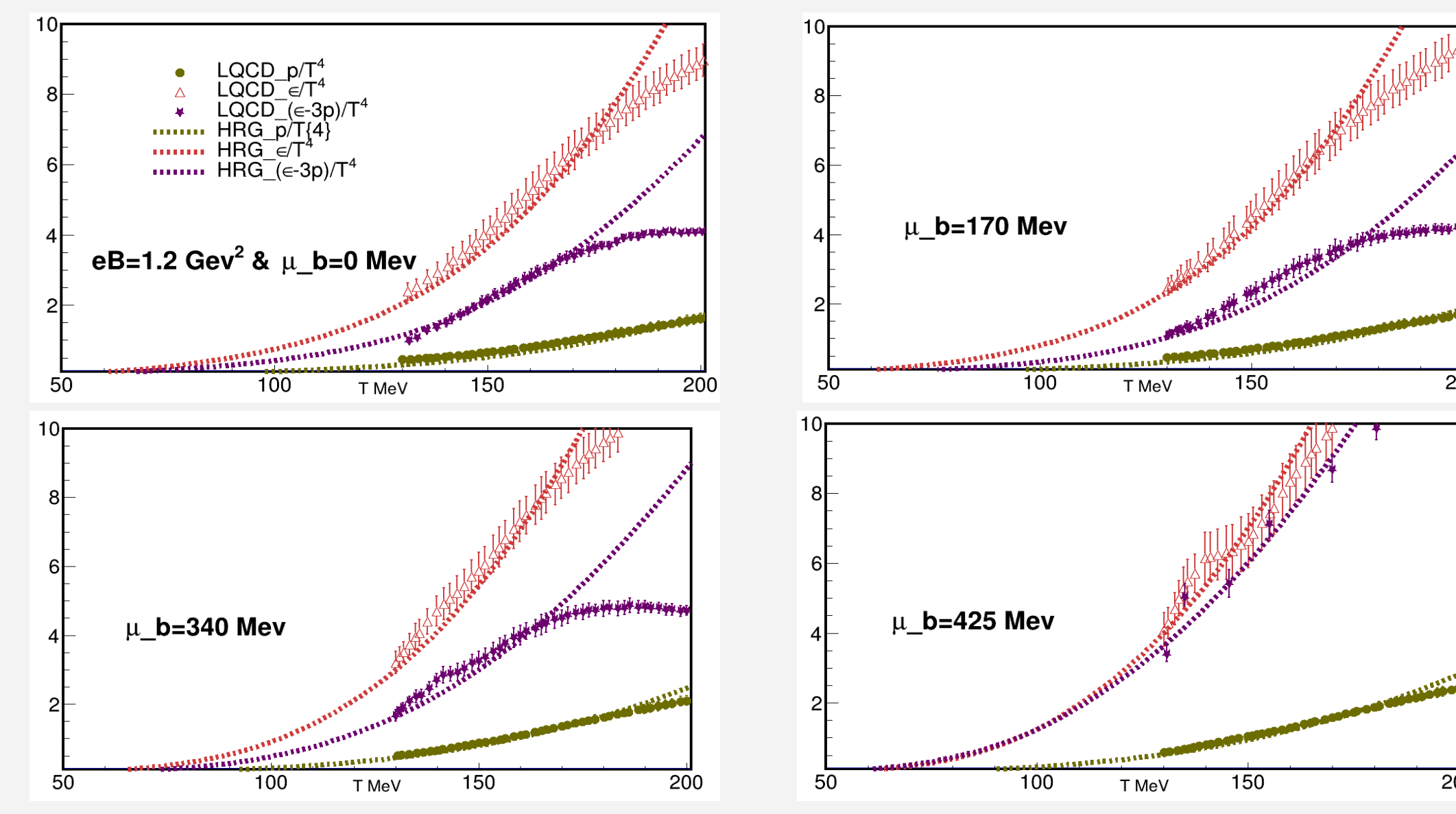
### At $eB = 0 \text{ GeV}^2$



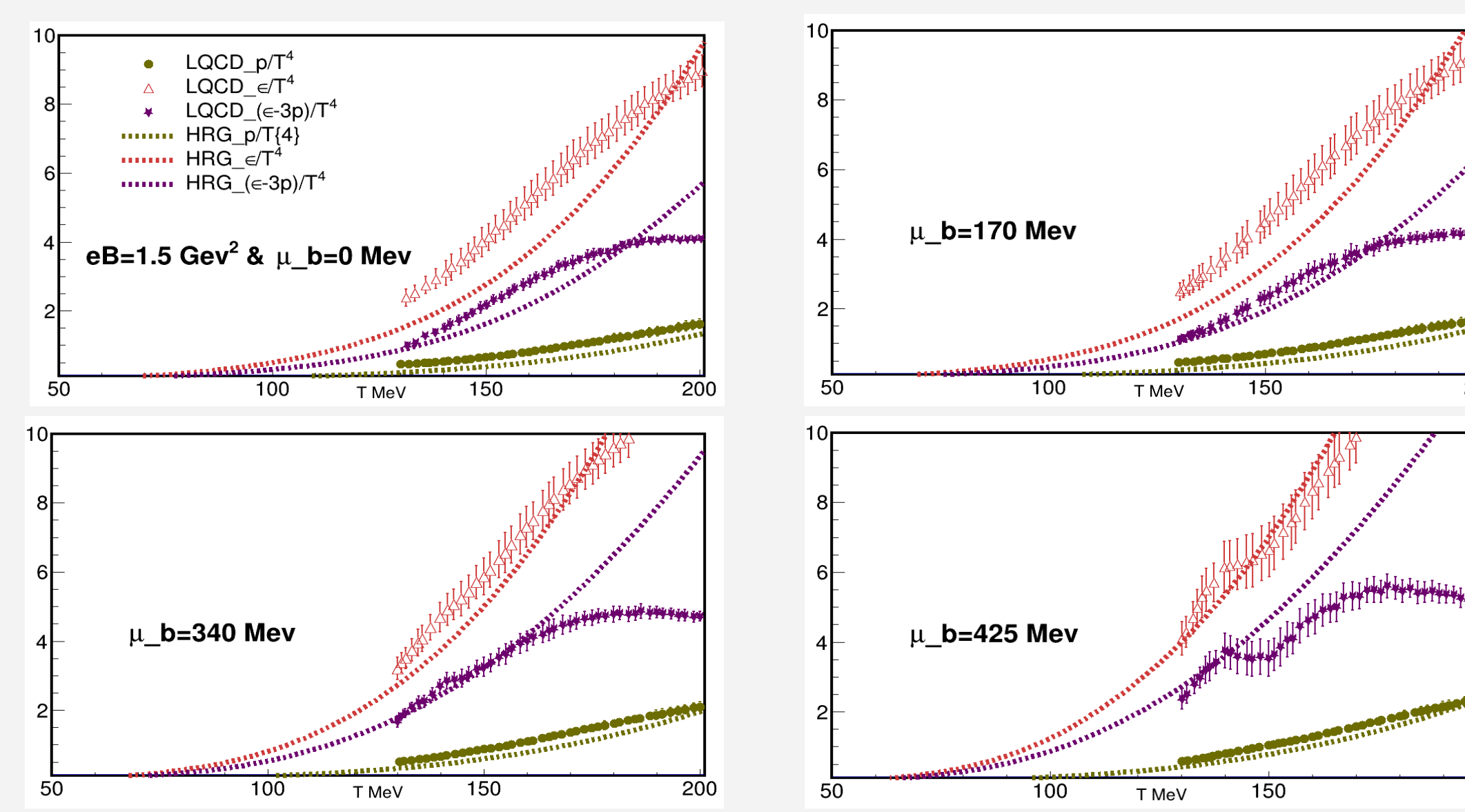
### At $eB = 1 \text{ GeV}^2$



### At $eB = 1.2 \text{ GeV}^2$



### At $eB = 1.5 \text{ GeV}^2$



## Conclusions

We confronted the HRG model in the presence of magnetic field effect with recent lattice data. All our model calculations considered in this study do not seem to satisfactorily mimic the corresponding lattice data in the full temperature range under investigation,  $T \in [130, 200]$  MeV. However, the best matching occurs locally in the vicinity of  $T_c$  in the range  $T \in [140, 170]$  MeV for the case of  $\mu_B = 170$  MeV at zero and **1.2  $\text{GeV}^2$**  magnetic field, respectively. Another remarkable matching between our model data with the corresponding lattice data occurs for the case of  $\mu_B = 340$  MeV, at **1.2  $\text{GeV}^2$**  magnetic field for temperatures  $T \lesssim T_c$  and up to  $T \approx 170$  MeV. In the lower temperature range,  $T \in [130, 160]$  MeV, most of the cases investigated in this research show reasonable match with the corresponding lattice data for different magnetic field values except for the case in which  $\mu_B = 425$  MeV where no good fitting is observed for any magnetic field value expect at **1.2  $\text{GeV}^2$** .

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