Finite volume effects in the extended linear sigma model

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Introdution

Contrary to the field theoretical calculations in the thermodynamic limit where the volume is assumed to be infinitely large, the heavy-ion collisions always carry the effects of the finite size. Sufficiently small system size is expected to affect the thermodynamical quantities and the phase diagram of the strongly interacting matter. To study these effects one can take into account the finite spatial extent of the system within the framework of an effective model, too, via the restriction of the momentum integrals using discretization or in a simplified case using a low momentum cutoff. We investigated the effects of the finite volume in a vector meson extended Polyakov quark-meson model both at the physical case and in the chiral limit and found a remarkable change in the thermodynamics and the phase transition especially in the location of the critical endpoint and the tricritical point.

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ELSM

The vector and axial vector meson **E**xtended Polyakov **L**inear **S**igma **M**odel is an advanced quark-meson model, including four full meson nonets and $2+1$ flavor constituent quarks in the fermion sector. The mesonic part of the Lagrangian contains the dynamical and the meson-meson interaction terms up to fourth order, taking care of the symmetry properties and also symmetry breaking, namely the $U(1)_A$ anomaly and the explicit breaking of the chiral symmetry to $SU(2)_I \times U(1)_V$. The constituent quarks are included in a Yukawa-type Lagrangian

$$\mathcal{L}_Y = \bar{\psi}\left(i\gamma^\mu\partial_\mu - g_F(S - i\gamma_5 P)\right)\psi.$$

Functional integration over the fermionic fields and proper renormalization of the vacuum contribution are carried out.

Thermodynamics determined from the mean field level grand potential built up from

- the classical potential,
- the fermionic one-loop correction,
- and the Polyakov loop potential.

$$\Omega(T,\mu_q) = U_{Cl} + \Omega^{(0)}_{\bar{q}q}(T,\mu_q) + U(\Phi,\bar{\Phi})$$

The field equations are given by minimazing the grand potential in the order parameters, ϕ_N, ϕ_S, Φ and $\bar{\Phi}$, that are the scalar-isoscalar meson condensates and the Polyakov loop variables, respectively.

The parametrization of the model was caried out at $T=0$, $\mu=0$ with χ^2 method using $\sim$30 physical quantities like meson masses and decay widths for the 14 model parameters.The thermodynamics and each physical quantities are calculated using the grand potential. Particularly for the mesons the curvature masses are defined as

$$m^2_{ab} = \frac{\partial^2\Omega(T,\mu_q)}{\partial\varphi_a\partial\varphi_b}.$$

Finite volume effects on physical quantities

The finite volume effects are usually studied within effective field-theoretical models in the thermodynamic limit by taking into account the discretization in momentum space given by the finite spatial extent in the direct space. Although this method does not include each aspects of the finite size physical system like the surface effects, it can be a good approximation to study the difference between infinite or large volume systems – like several of our theoretical models or the compact stars – and the small systems like the fireball in a HIC. A further simplification can be made by using a low momentum cutoff instead of discretization, by making use of the observation that the zero momentum modes are the most relevant ones for the criticality of the system. Similar approximation was also studied in HRG model and showed a good agreement with the direct finite volume calculations [2]. Therefore, it seems to be a good possibility to study the finite volume effects within the ELSM by using a low momentum cutoff, that can be implemented in the fermionic part of the grand potential, which includes a properly renormalized vacuum, and a matter part.

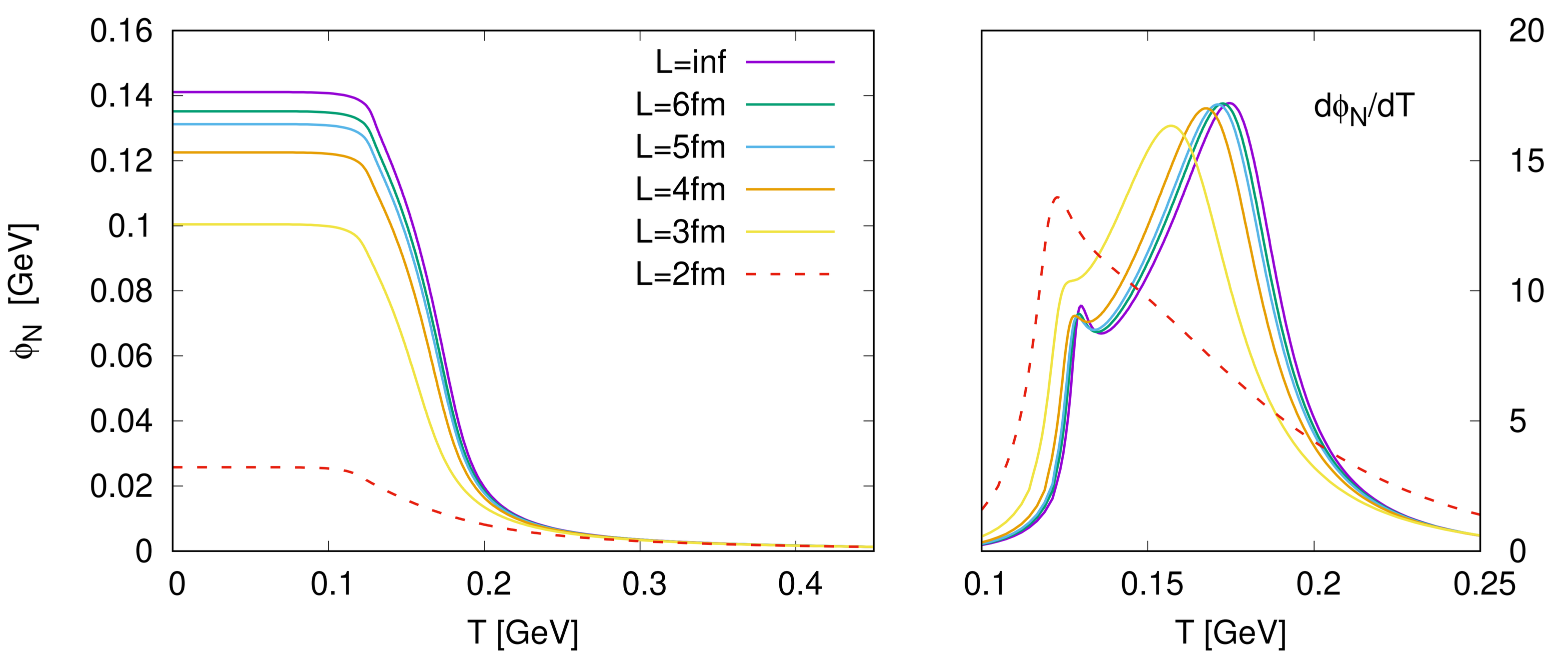

Fig. 1. The temperature dependence of the non-strange meson condensate and its derivative for different system sizes.

The solution of the finite size modified field equations changes and already the vacuum value of the meson condensates – that are our order parameters – decreases with the decreasing volume. This give rise to the modification of the quantities depending on them, thus, keeping the original set of parameters fixed one can see that the physical quantities change significantly with the system size below 10 fm.

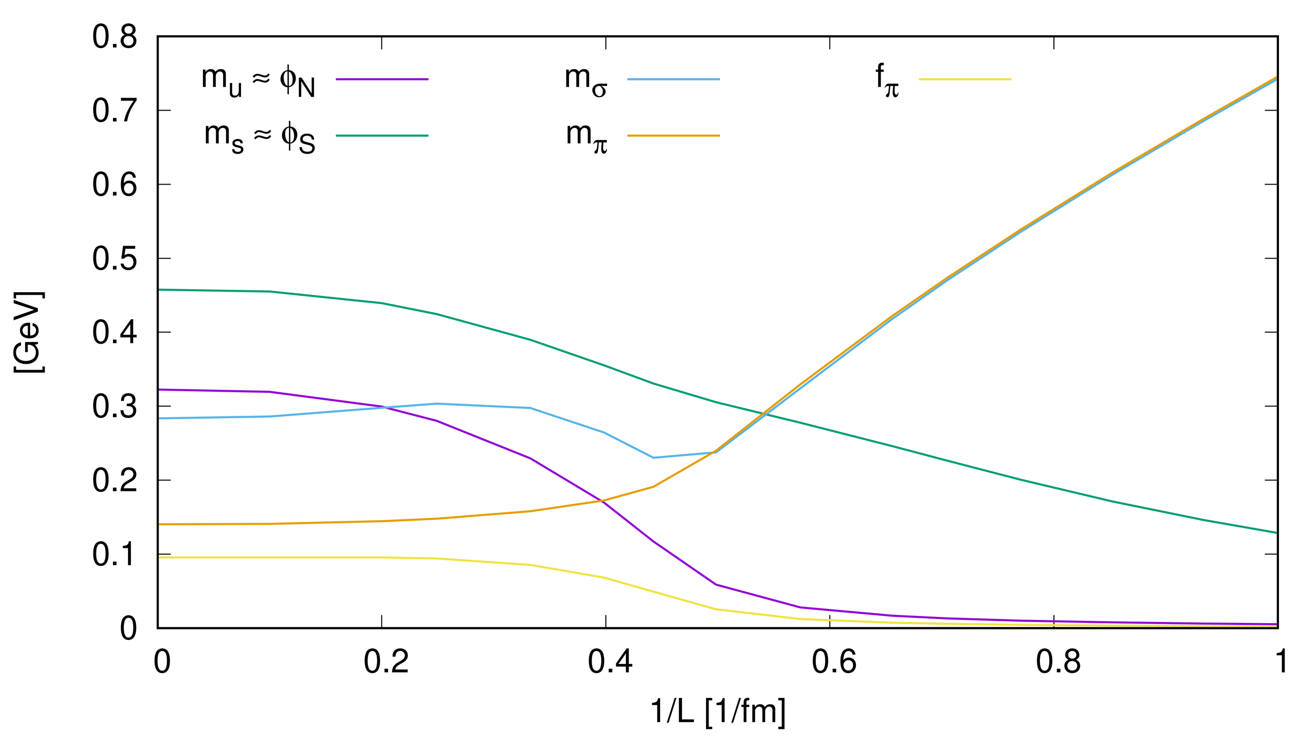

Fig. 2. The modification of physical quantities at $T=0$, $\mu=0$ with $1/L$.

Although, the changing initial value of the condensates as an order parameters has no physical meaning, the physical quantities – depending on them directly – shows a crossover-like behavior in $1/L$ similar to that in T. Treating $1/L$ as an external parameter similarly to T and μ_q one may need to be careful by identifying the temperature dependence at very small volumes as a phase transition. We should also note, that below a certain size the validity of the models with thermodynamical approach is questionable as well.

Modification of the phase diagram

In the phase diagram the transition temperature at $\mu_q = 0$ decreases with the decreasing volume, while the transition becomes smoother. At the same time the CEP moves to lower temperatures and slightly higher chemical potentials until it disappears around $L = 2.5$ fm after which the criticality of the system is washed out.

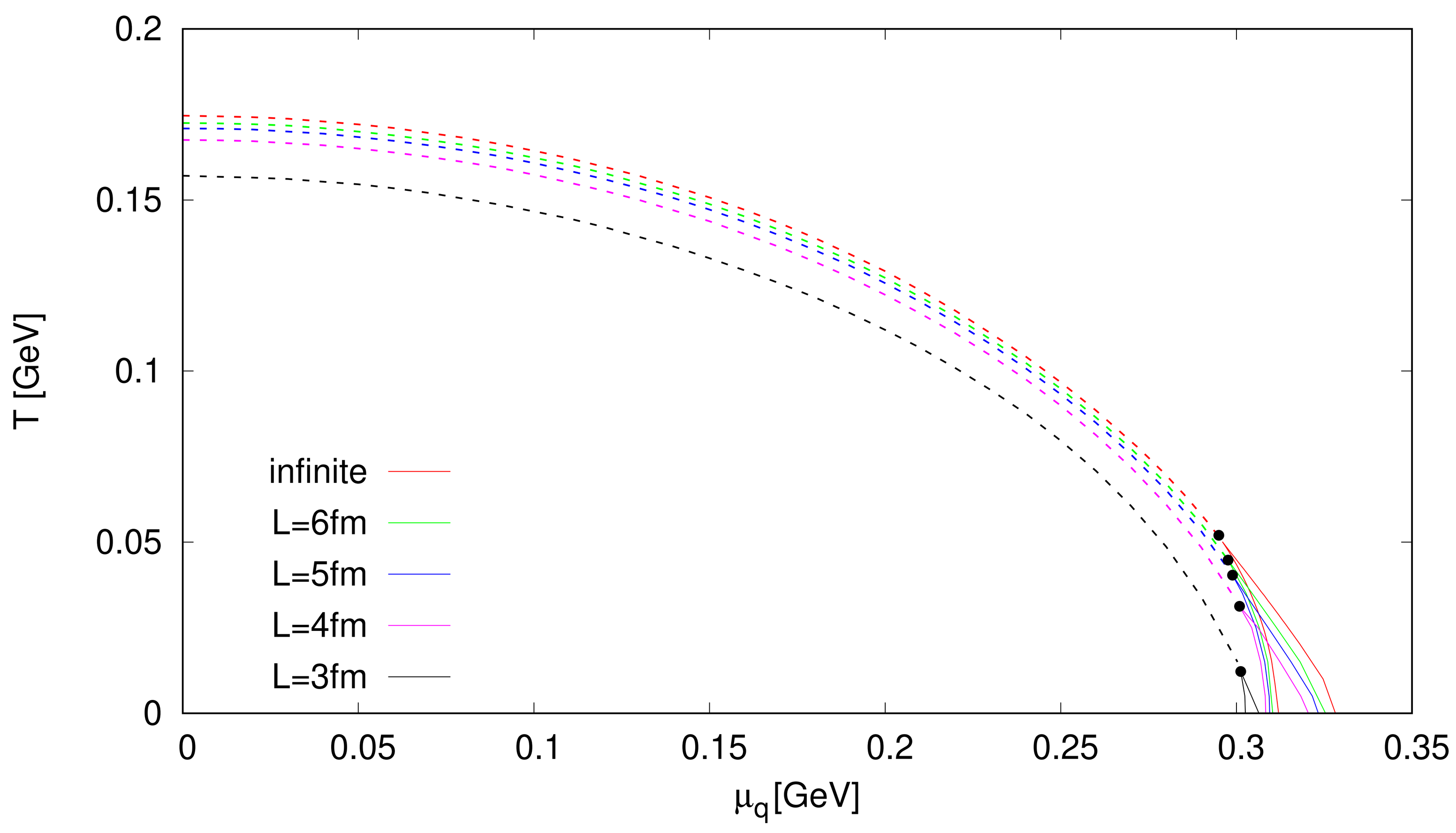

Fig. 4. The finite size dependence of the phase diagram and the CEP.

In the chiral limit, where the explicit symmetry breaking vanishes the location of the tricritical point changes similarly to the CEP in the explicitly broken case, while the critical temperature at $\mu_q = 0$ decreases with the decreasing size.

Summary and outlook

- Finite volume effects on thermodynamics and the phase diagram of strong interaction was studied via a low momentum cutoff.
- The meson masses and other physical quantities start to significantly change with the system size under $\sim 5-10$ fm.
- The CEP and the first order region disappear at a small finite size ~ 2.5 fm.
- We want to understand connection between different constraints on momentum space (low cutoff, discretisation with different boundary conditions) and the role of the soft modes.
- We shall clarify the limit of the validity of our model at very small volumes.

References

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[2] F. Karsch, K. Morita and K. Redlich, Phys. Rev. C **93**, no.3, 034907 (2016) [arXiv:1508.02614 [hep-ph]].