

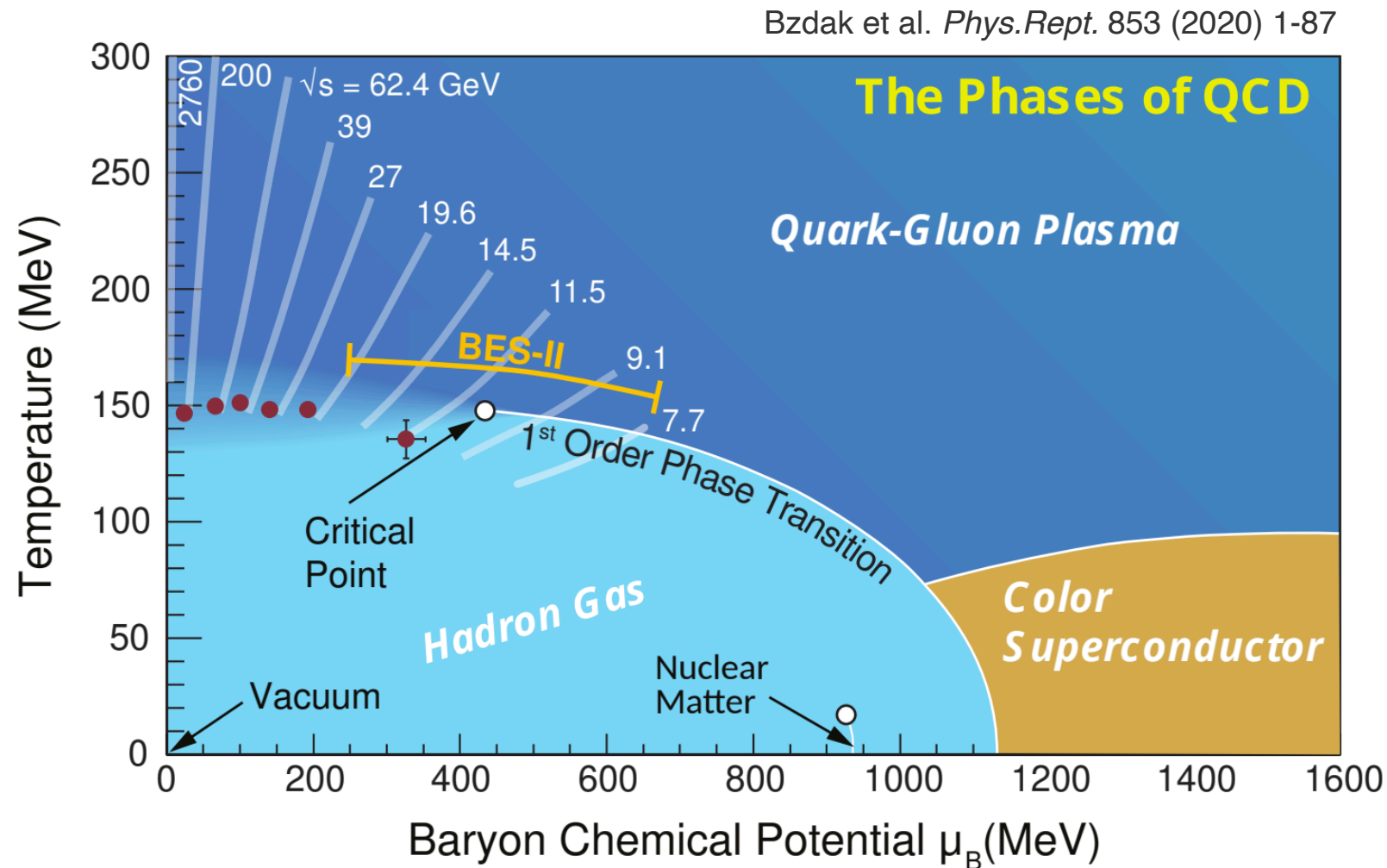
Impact of hadronic interactions and conservation laws on cumulants of conserved charges in a dynamical model

Jan Hammelmann and Hannah Elfner

arXiv: 2202.11417



Introduction



Explore the phase diagram of QCD by using heavy-ion collisions

Measure cumulants of conserved charges

Introduction

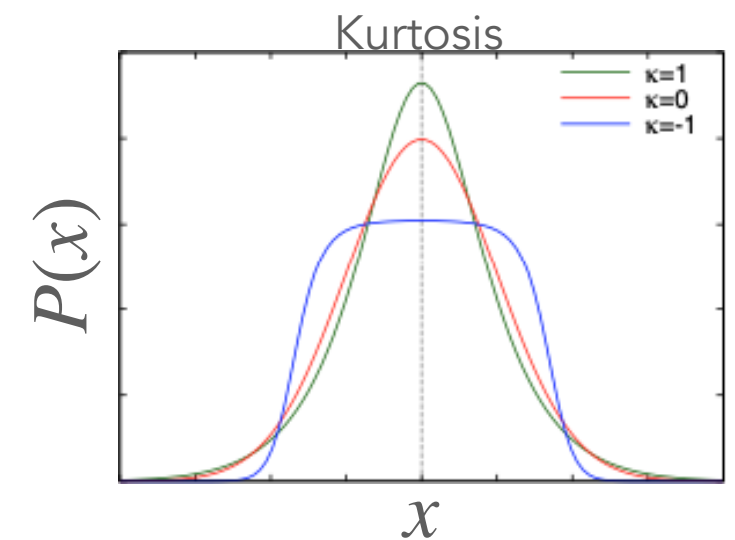
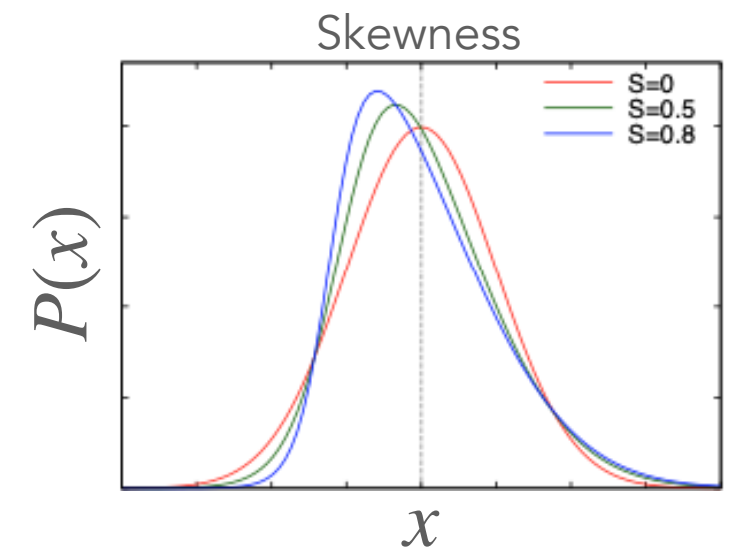
Information of the physical system is encoded in the grand-canonical partition function Z^{gce}
The cumulants can be calculated via

$$C_k \approx \frac{\partial^n (\ln Z^{gce})}{\partial \mu^n}$$

Present ratios to get rid of additional factors

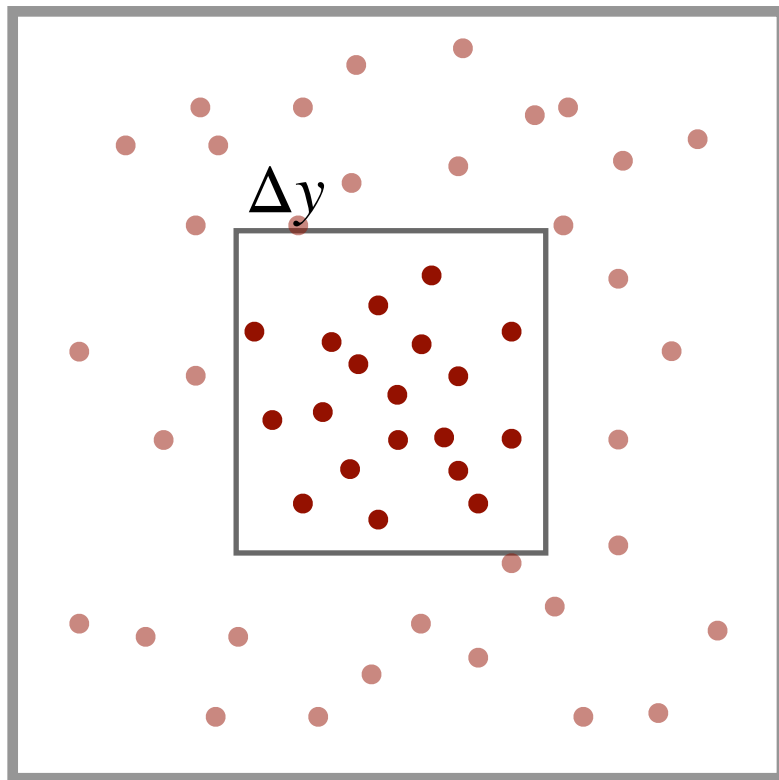
$$S\sigma = \frac{C_3}{C_2}$$

$$\kappa\sigma^2 = \frac{C_4}{C_2}$$



arXiv: 1512.05038

Introduction



Clear signs of conservation effects in measurements of heavy-ion collisions

How to compare theory calculations performed in the grand-canonical ensemble to experiments?

Net protons are used as a proxy of the net baryon number fluctuations

Perform dynamical simulations to model the background signal

Model

Simulating **M**any **A**ccelerated **S**trongly-interacting **H**adrons (SMASH)

<https://smash-transport.github.io/>

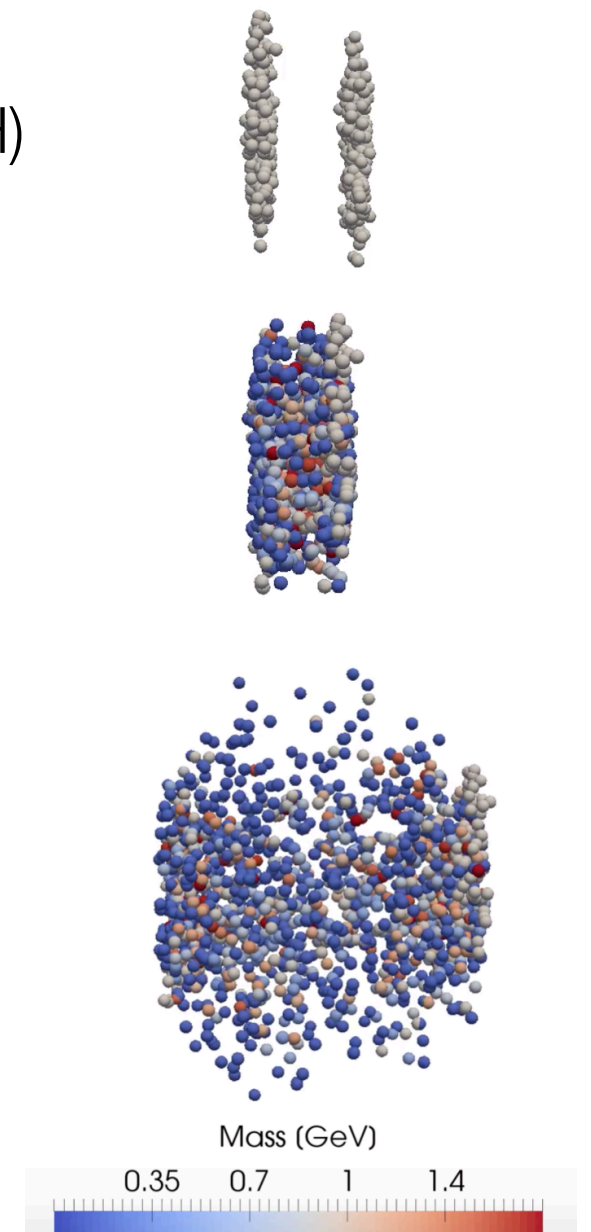
Uses a geometric collision criterion

$$\pi d_{\perp}^2 < \sigma_{\text{tot}}$$

Incorporates particles with masses up to $\sim 2 \text{ GeV}$

Types of processes are:

- $2 \leftrightarrow 1$ resonance formation / decay
- $2 \leftrightarrow 2$ elastic / inelastic interactions
- String excitation (not used in this work)



Pb-Pb collision with 17.3 GeV
center-of-mass energy
(by J. Mohs)

Model

Degrees of freedom

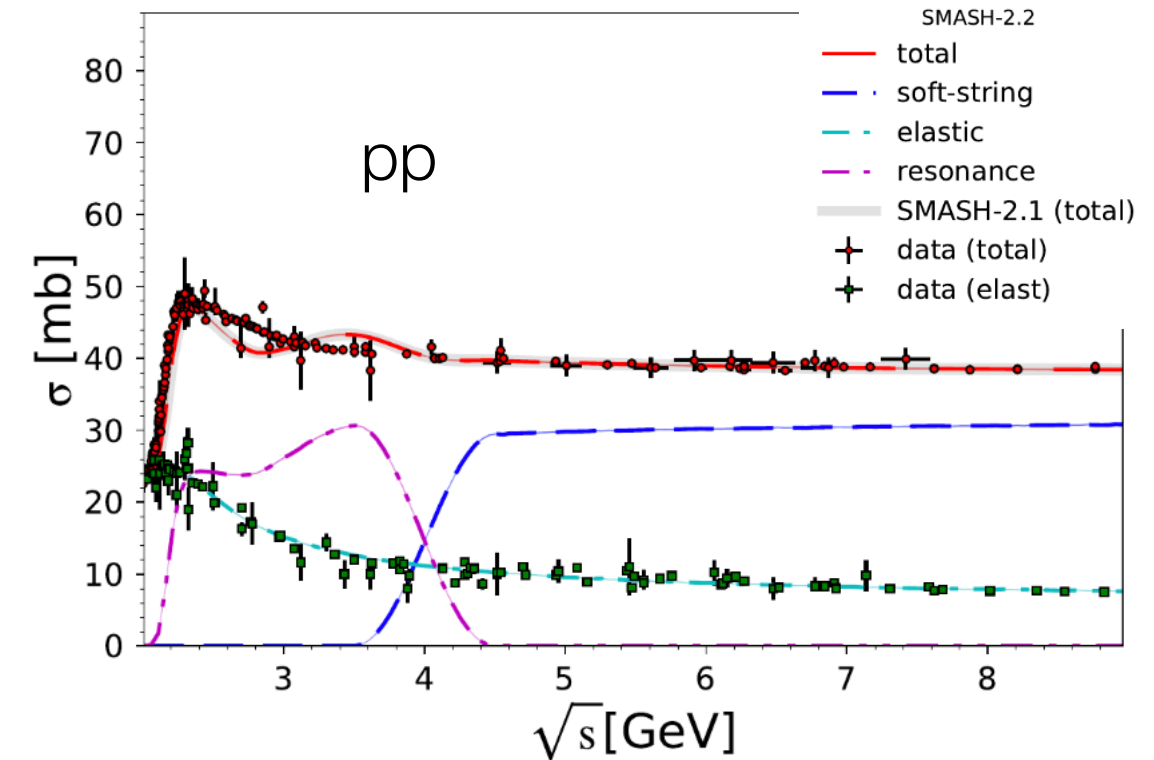
N	Δ	Λ	Σ	Ξ	Ω	Unflavored				Strange
N ₉₃₈	Δ_{1232}	Λ_{1116}	Σ_{1189}	Ξ_{1321}	Ω_{1672}^-	π_{138}	$f_{0\ 980}$	$f_{2\ 1275}$	$\pi_{2\ 1670}$	K_{494}
N ₁₄₄₀	Δ_{1620}	Λ_{1405}	Σ_{1385}	Ξ_{1530}	Ω_{2250}^-	π_{1300}	$f_{0\ 1370}$	$f_{2'\ 1525}$		K^{*}_{892}
N ₁₅₂₀	Δ_{1700}	Λ_{1520}	Σ_{1660}	Ξ_{1690}		π_{1800}	$f_{0\ 1500}$	$f_{2\ 1950}$	$\rho_{3\ 1690}$	$K_{1\ 1270}$
N ₁₅₃₅	Δ_{1905}	Λ_{1600}	Σ_{1670}	Ξ_{1820}			$f_{0\ 1710}$	$f_{2\ 2010}$		$K_{1\ 1400}$
N ₁₆₅₀	Δ_{1910}	Λ_{1670}	Σ_{1750}	Ξ_{1950}		η_{548}		$f_{2\ 2300}$	$\phi_{3\ 1850}$	K^{*}_{1410}
N ₁₆₇₅	Δ_{1920}	Λ_{1690}	Σ_{1775}	Ξ_{2030}		η'_{958}	$a_{0\ 980}$	$f_{2\ 2340}$		$K^{*}_{0\ 1430}$
N ₁₆₈₀	Δ_{1930}	Λ_{1800}	Σ_{1915}			η_{1295}	$a_{0\ 1450}$		$a_{4\ 2040}$	$K^{*}_{2\ 1430}$
N ₁₇₀₀	Δ_{1950}	Λ_{1810}	Σ_{1940}			η_{1405}		$f_{1\ 1285}$		K^{*}_{1680}
N ₁₇₁₀		Λ_{1820}	Σ_{2030}			η_{1475}	ϕ_{1019}	$f_{1\ 1420}$	$f_{4\ 2050}$	$K_{2\ 1770}$
N ₁₇₂₀		Λ_{1830}	Σ_{2250}				ϕ_{1680}			$K^{*}_{3\ 1780}$
N ₁₈₇₅		Λ_{1890}				σ_{800}		$a_{2\ 1320}$		$K_{2\ 1820}$
N ₁₉₀₀		Λ_{2100}					$h_{1\ 1170}$			$K^{*}_{4\ 2045}$
N ₁₉₉₀		Λ_{2110}				ρ_{776}		$\pi_{1\ 1400}$		
N ₂₀₈₀		Λ_{2350}				ρ_{1450}	$b_{1\ 1235}$	$\pi_{1\ 1600}$		
N ₂₁₉₀						ρ_{1700}				
N ₂₂₂₀							$a_{1\ 1260}$	$\eta_{2\ 1645}$		
N ₂₂₅₀						ω_{783}				
						ω_{1420}		$\omega_{3\ 1670}$		
						ω_{1650}				

► + corresponding antiparticles

► Perturbative treatment of photons and dileptons

► Isospin symmetry

Cross section



From A. Schaefer, see <https://smash-transport.github.io/>

https://theory.gsi.de/~smash/analysis_suite/SMASH-2.2/

Methodology

Initialization

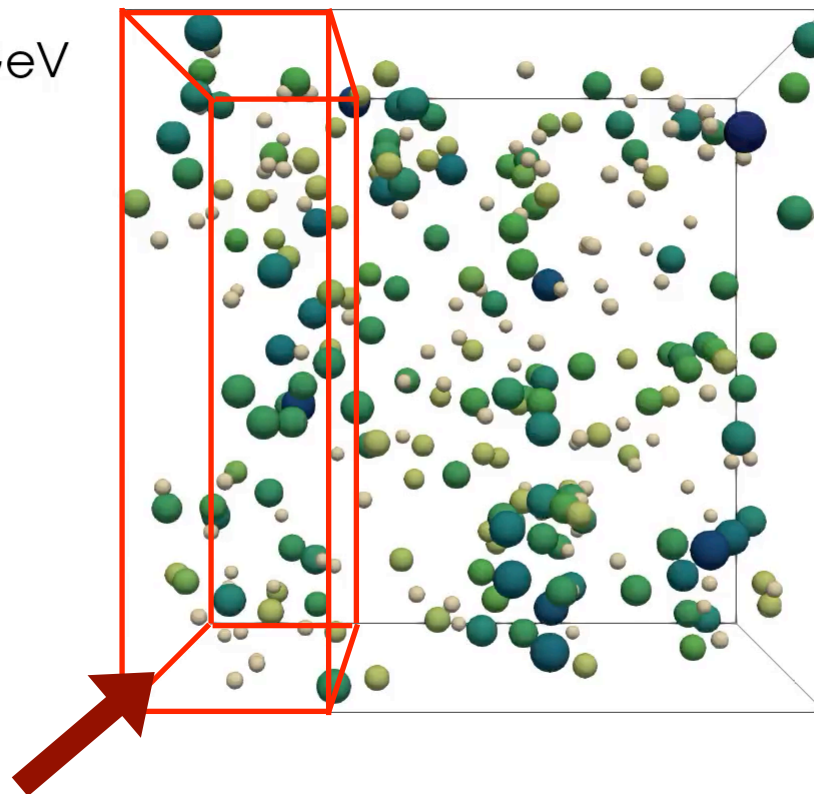
Initialize a box event-by-event with the same number of particles with momenta according to the Boltzmann distribution

Box

Time: 0 fm

Box Width: 10 fm

Temperature: 0.15 GeV

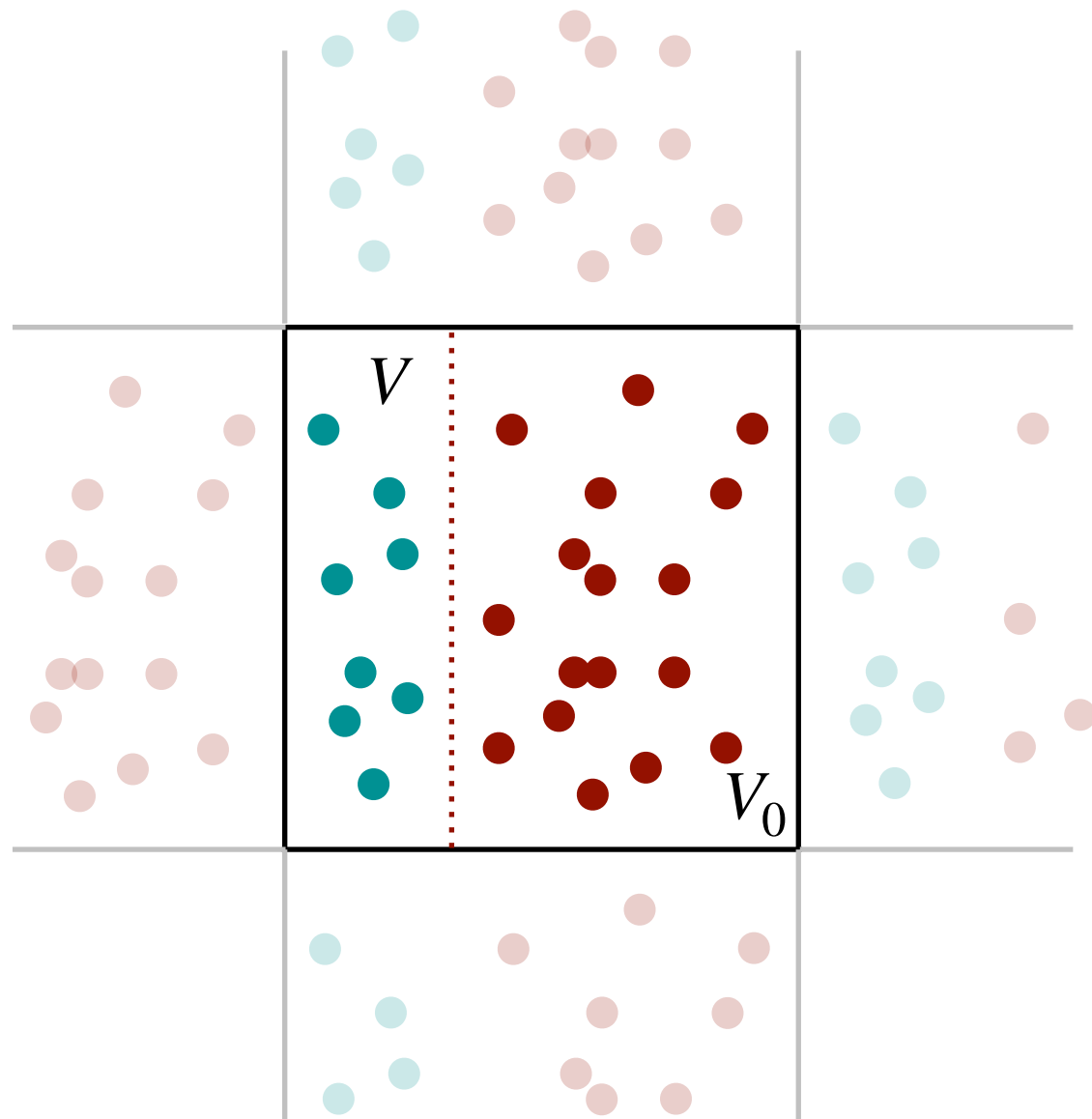


Calculate cumulants in this sub volume



<https://smash-transport.github.io/>

Methodology



Infinite matter simulation

Calculate cumulants of particle distributions as a function of the size of the subvolume

$$x = V/V_0$$

$$\omega = \frac{C_2}{C_1} = \frac{\langle(\delta N)^2\rangle}{\langle N\rangle}$$

$$S\sigma = \frac{C_3}{C_2} = \frac{\langle(\delta N)^3\rangle}{\langle(\delta N)^2\rangle}$$

$$\kappa\sigma^2 = \frac{C_4}{C_2} = \frac{\langle(\delta N)^4\rangle - 3\langle(\delta N)^2\rangle^2}{\langle(\delta N)^2\rangle}$$

Additionally study the effects of cuts on transverse momentum $0.4 < p_T < 2 \text{ GeV}$

Methodology

Analytic Comparison

Assume only fixed number of stable particles

Probability of finding a particle within a sub volume is given by the binomial distribution

$$B(x, n, k) = \binom{n}{k} x^k (1 - x)^{n-k}$$

The cumulants are as follows:

$$\kappa_1 = Nx$$

$$\kappa_2 = Nx(1 - x)$$

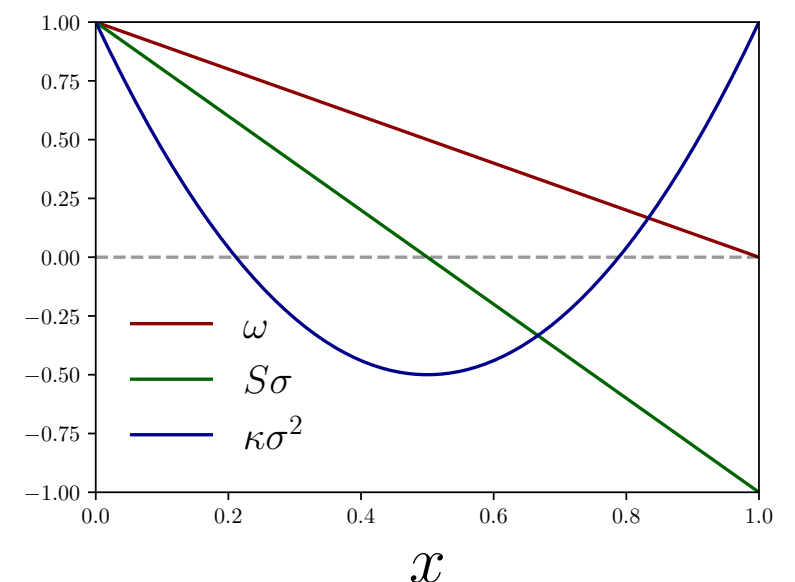
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Ratios:

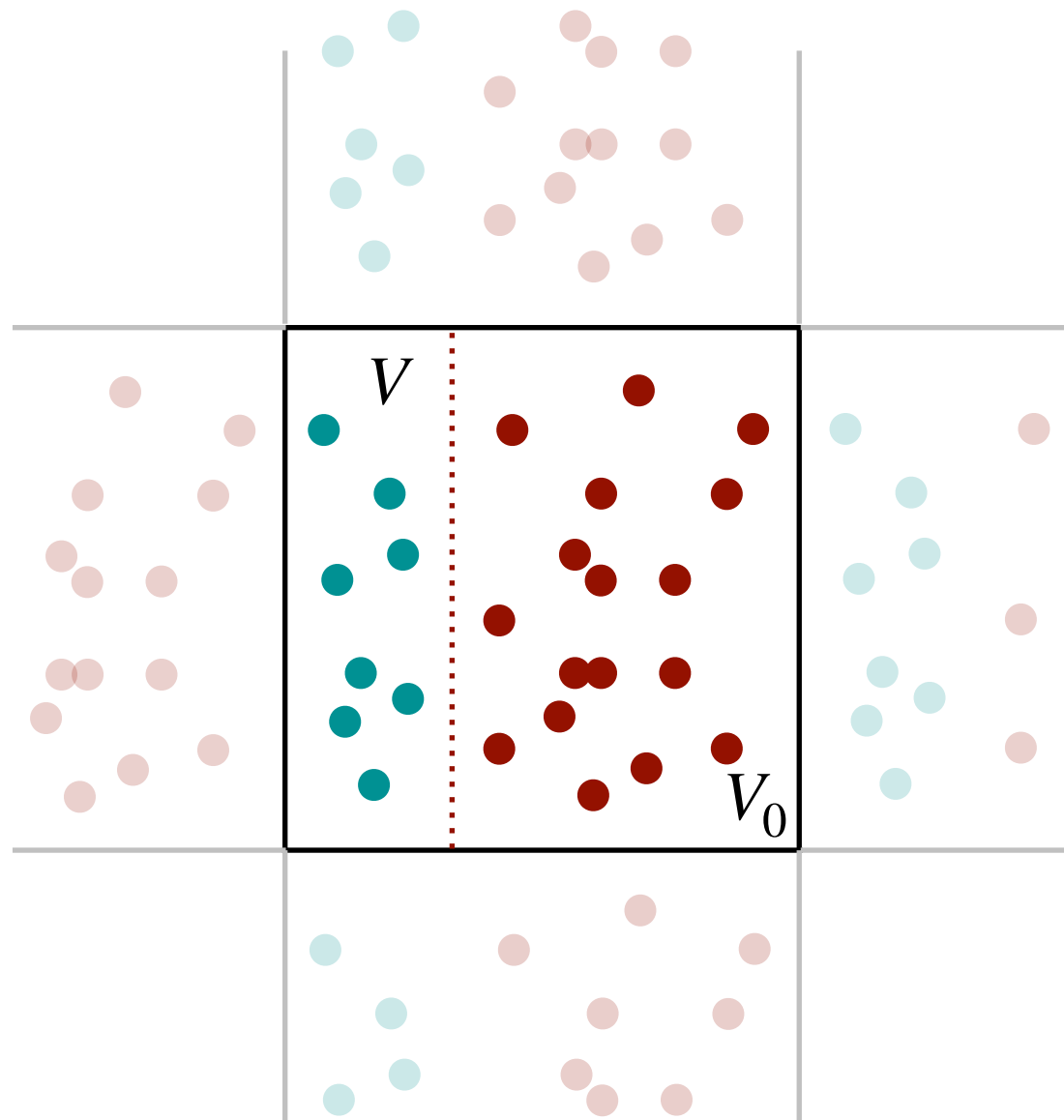
$$\omega = 1 - x$$

$$S\sigma = 1 - 2x$$

$$\kappa\sigma^2 = 1 + 6x(1 - x)$$



Methodology



Analytic Comparison

When the net charge number is conserved and the total charge number can fluctuate

$$P(n_+, n_-; x) = \sum_{N_+, N_-} P(N_+, N_-) \underbrace{B(N_+, n_+; x)}_{\text{Binomial distribution}} \underbrace{B(N_-, n_-; x)}$$

$$P(N_+, N_-) = \underbrace{\delta(N_+ - N_- - Q)}_{\text{Net charge conservation}} \underbrace{P(N_{ch})}_{\text{Fluctuation of the total charge number}}$$

Net charge conservation Fluctuation of the total charge number

➔ Analytic expressions for cumulants as function of x

$$\omega = 1 - x$$

$$S\sigma = \frac{Q}{\langle N_{ch} \rangle} (1 - 2x)$$

$$\kappa\sigma^2 = 1 + 3x(1 - x)(\omega[N_{ch}] - 2)$$

For more details see *Phys.Rev.C* 101 (2020) 2, 024917

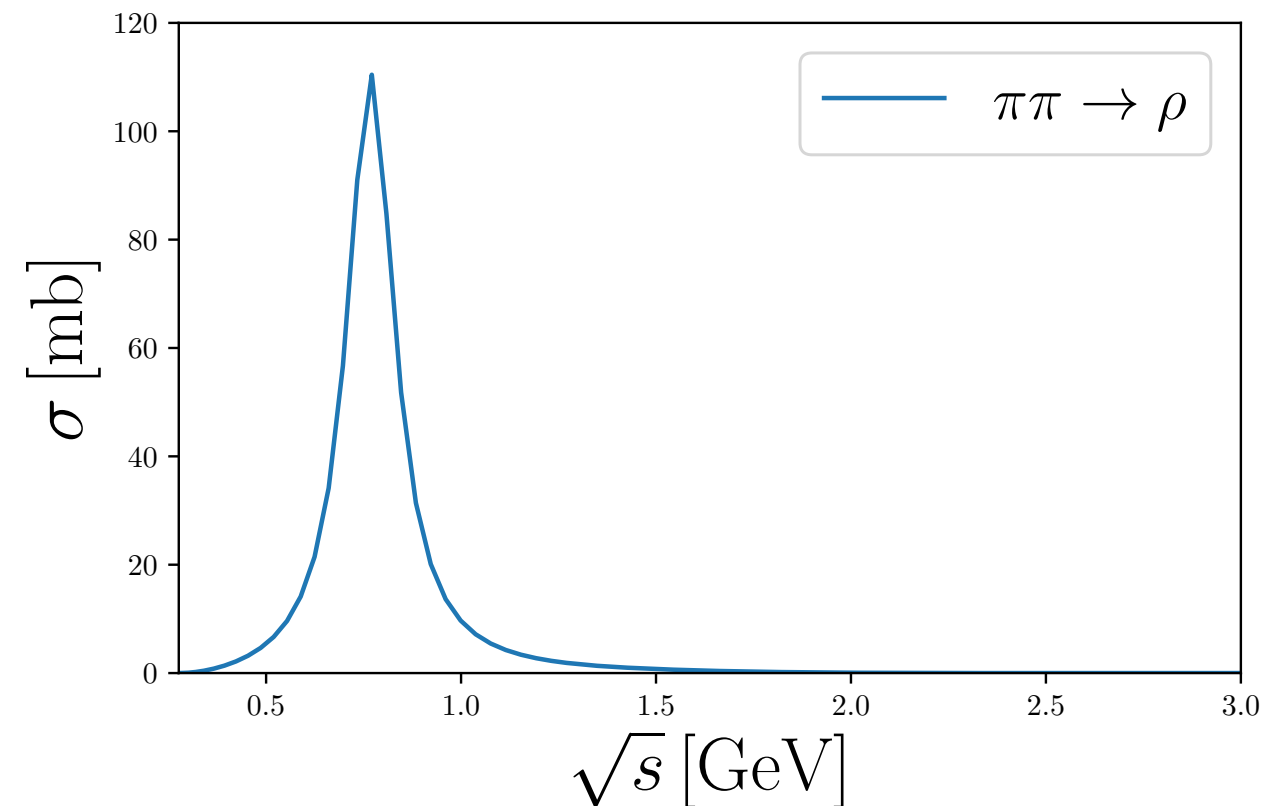
Test System

Setup

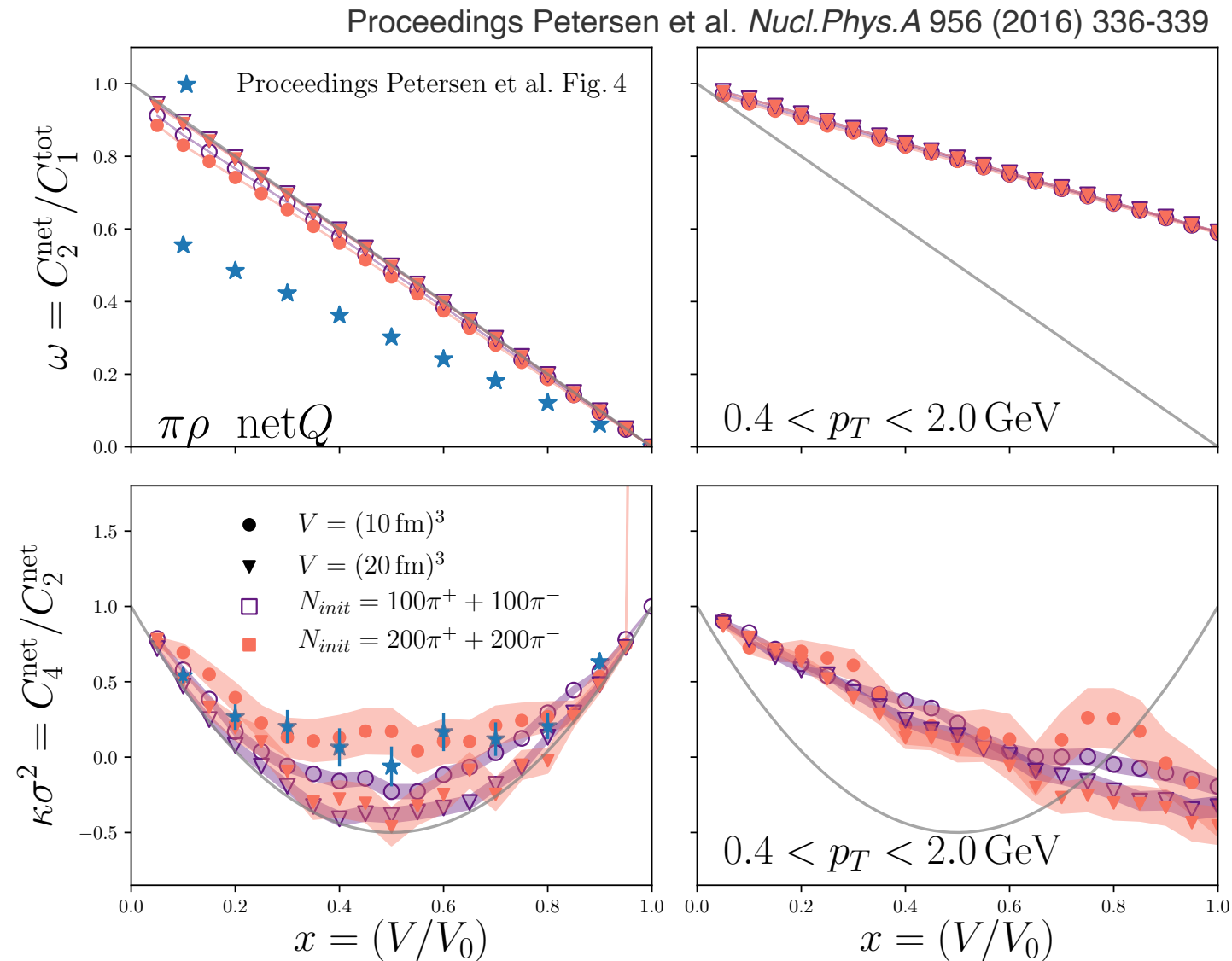
Interacting $\pi\rho$ gas with an energy dependent cross section at $T = 160 \text{ MeV}$ charge annihilation process $\pi^\pm \pi^\mp \leftrightarrow \rho^0$

Fixed initial number of pions (e.g. $N_{\pi^+} + N_{\pi^-}$)

Run the evolution in different volumes and number of particles



Test System



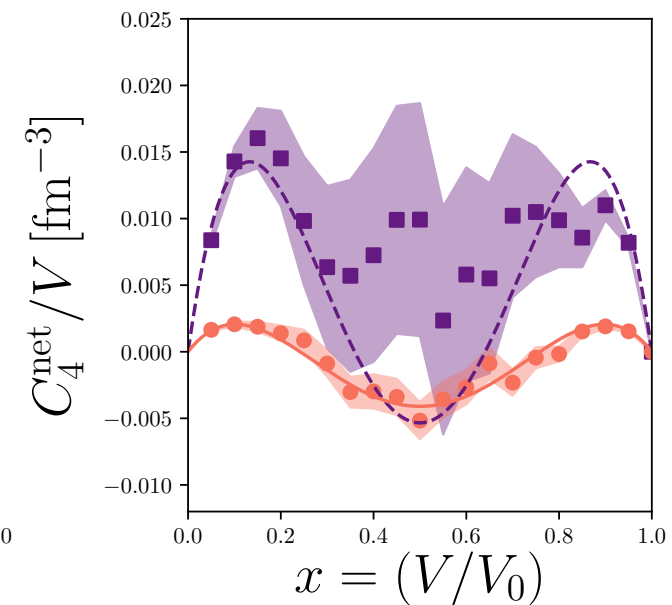
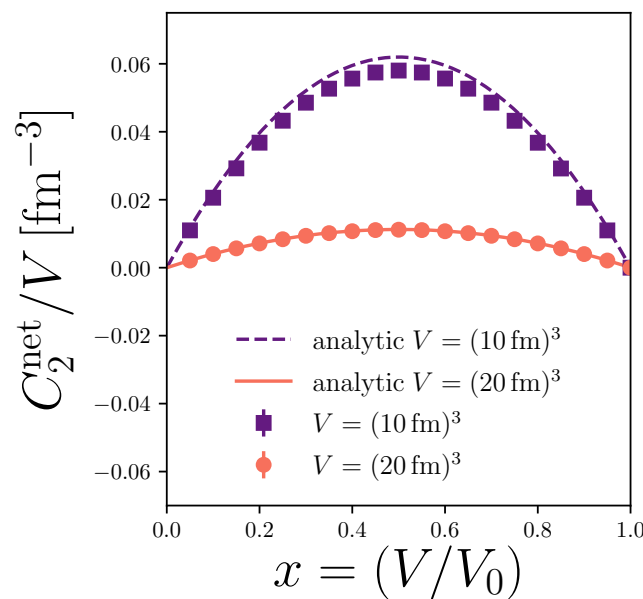
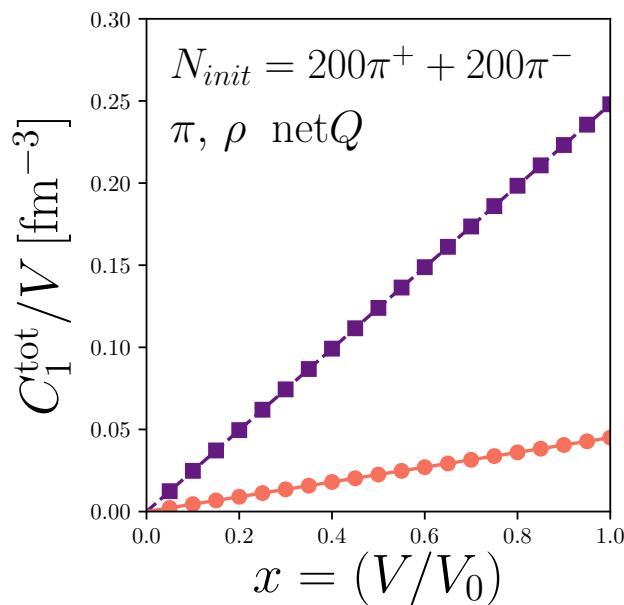
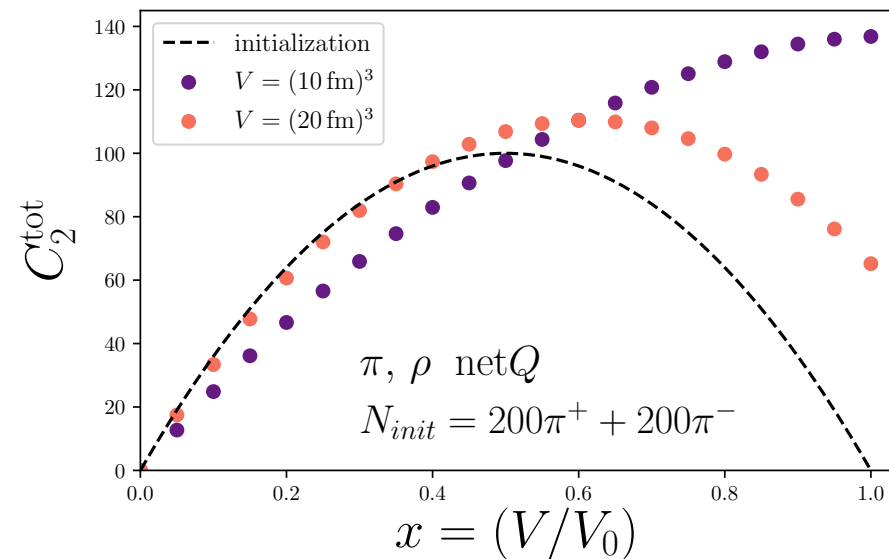
Scaled variance is well described by the analytic expectation
Differences in $\kappa\sigma^2$ around $x = 0.5$ depending on the charge density

Test System

Fluctuations of the total charge number $C_2[N_{ch}]$ becomes important at large densities



More charge annihilation processes (C_4 depends on $C_2[N_{ch}]$)



Fluctuations can be described by analytic formulas with the input of $\langle N_{\pm} \rangle$ and $C_2[N_{ch}]$

Full Hadron Gas

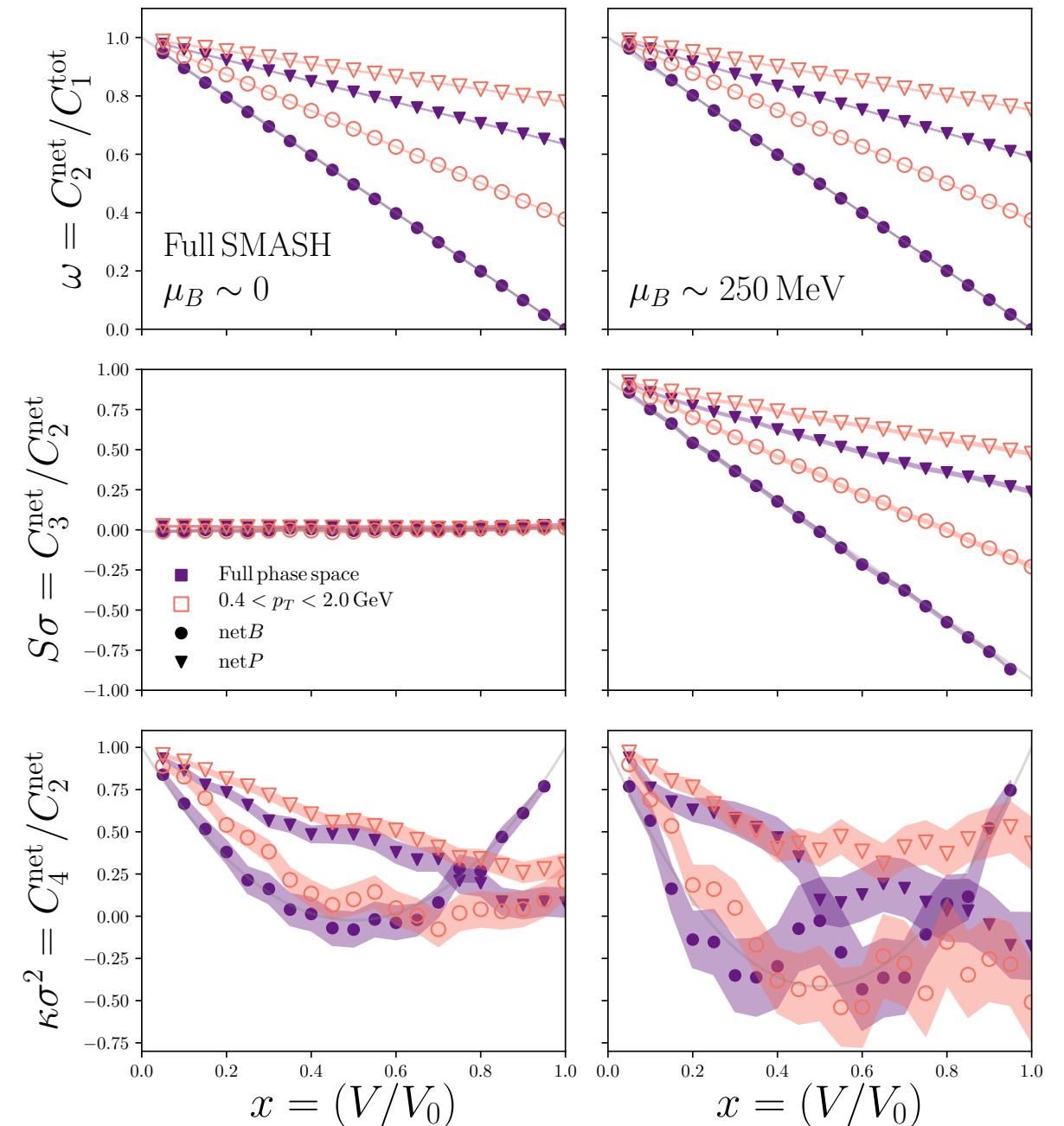
Full set of hadrons from SMASH with all possible interactions

Perform final decays after the dynamical evolution

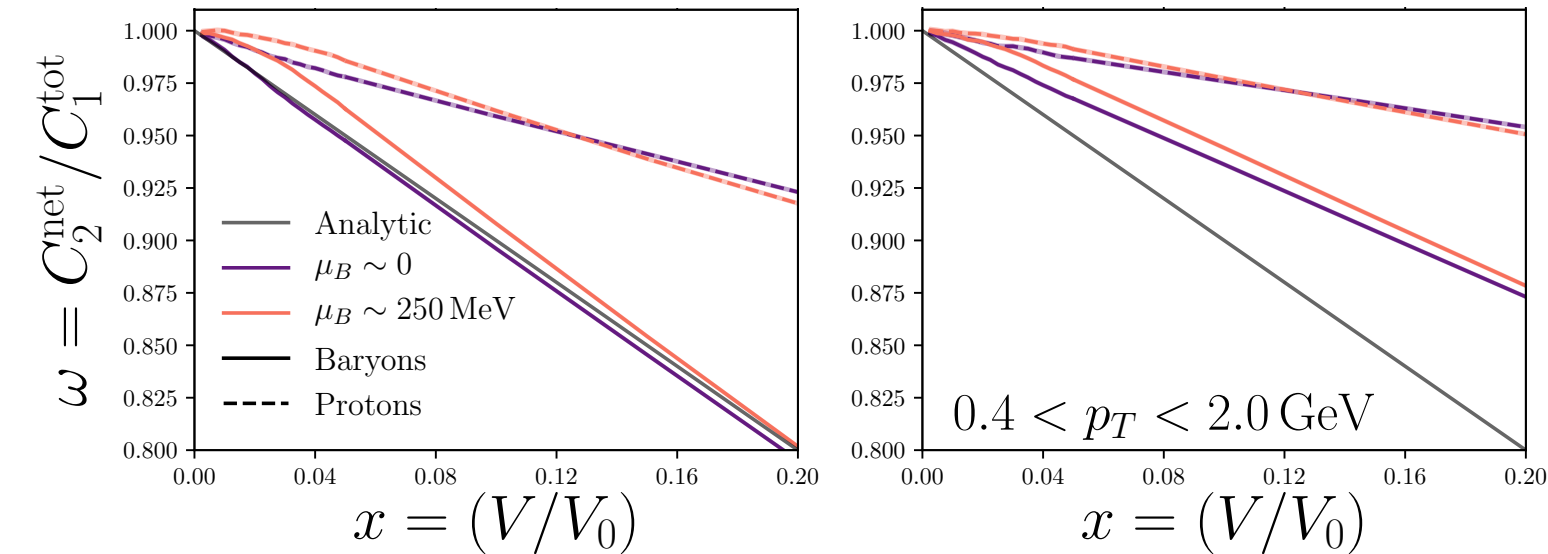
Possibility to extract $C_2[N_{ch}]$ for a realistic interacting hadron gas

Extract cumulants of protons which are not directly accessible in theory

net baryons and net protons

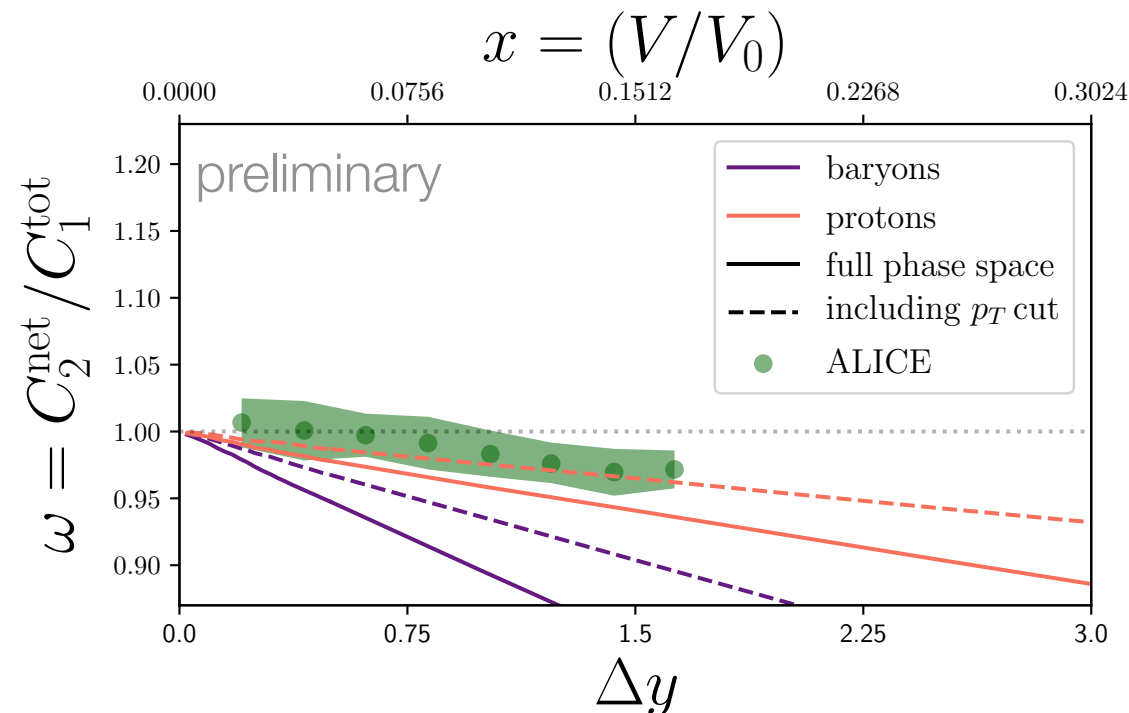


Full Hadron Gas



In the limit $x \rightarrow 0$, $\omega / \kappa \sigma^2 \rightarrow 1$
(Poisson Limit)

For small volumes and large baryon chemical potentials, the cumulants don't strictly follow the analytic expressions



ALICE: *Phys.Lett.B* 807 (2020) 135564

Comparison to experimental data using

$$x \approx N_{ch}(\Delta y) / N_{ch}(\infty) \approx \text{erf} \left(\Delta y / (2\sqrt{2}\sigma_y) \right)$$

$$\sigma_y = 3.86$$

Coordinate space and momentum space is not directly equivalent!

arXiv: 2110.02446

Phys.Lett.B 726 (2013) 610-622

Mapping between protons and baryons

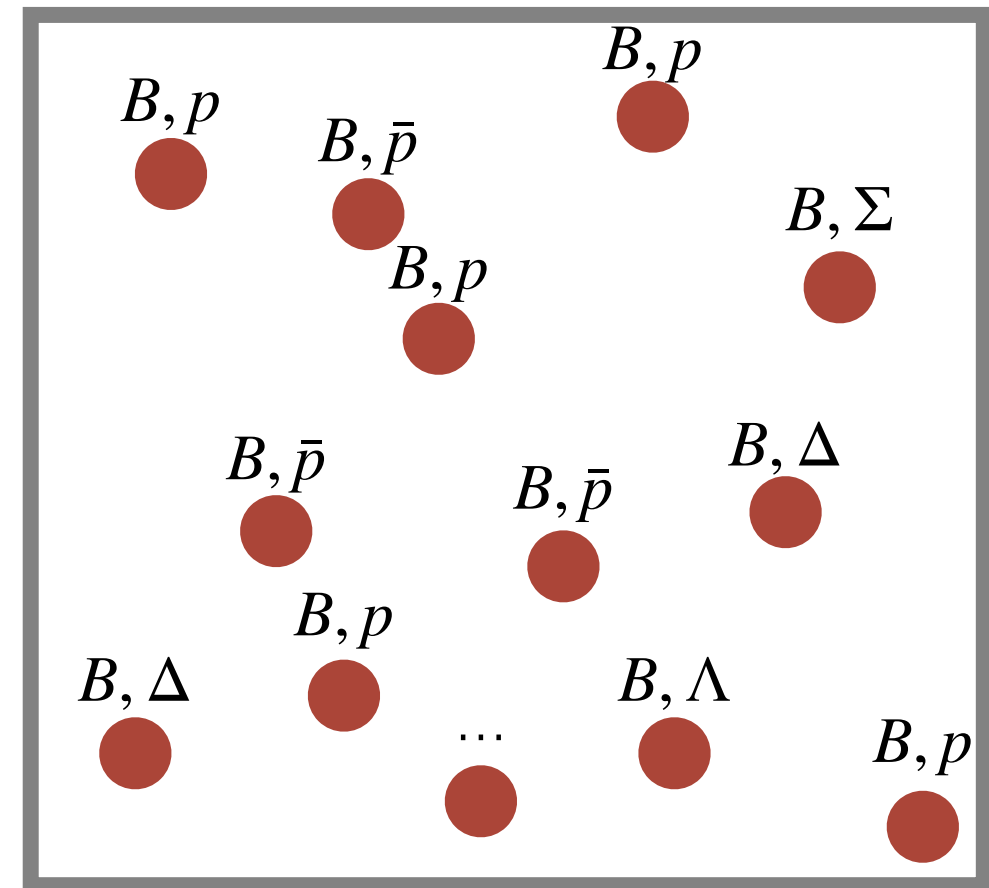
δB fluctuations are of interest, however only δP fluctuations are accessible

Is it possible to map fluctuations $\delta N_p \rightarrow \delta N_B$?

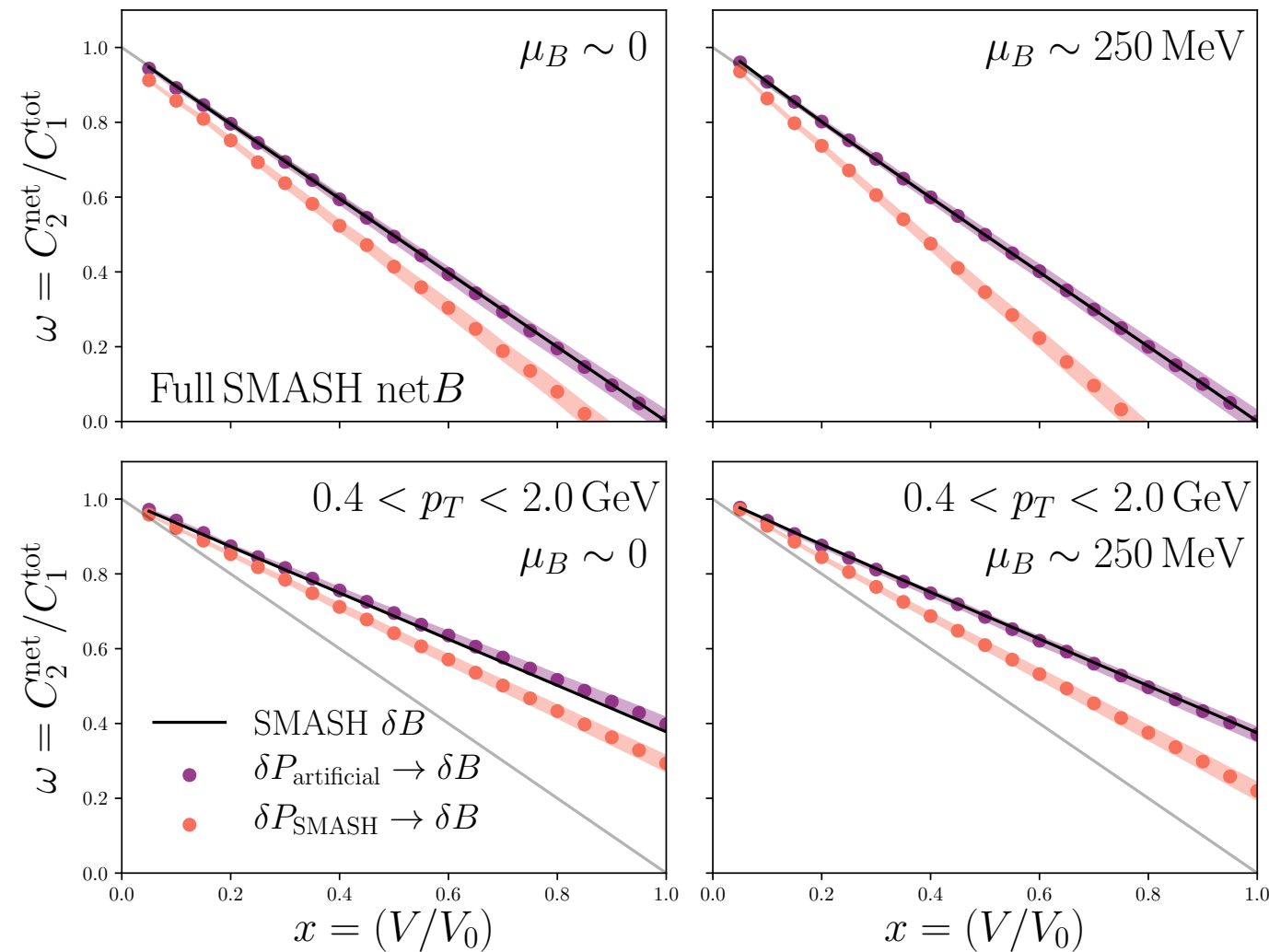
Mapping between proton and baryon number fluctuations $\delta P \leftrightarrow \delta B$

Kitazawa and Asakawa *Phys.Rev.C* 85 (2012) 021901

Based on binomial unfolding procedure
input: Probability $p = \langle N_p \rangle / \langle N_B \rangle$



Mapping between protons and baryons



Dynamical correlations within the model limits the applicability in large acceptance regions

Dependence on the complexity of the system

Conclusion

Summary

Charge annihilation processes affects the kurtosis at large densities

At large baryon chemical potentials the scaled variance is modified in small volumes

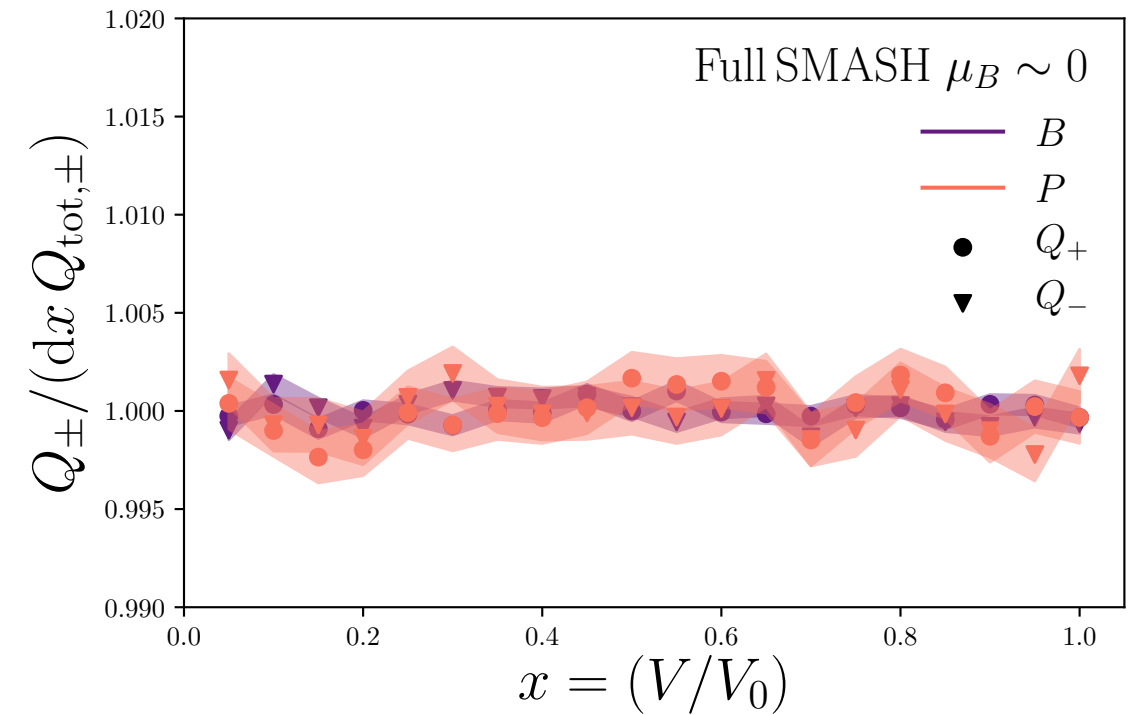
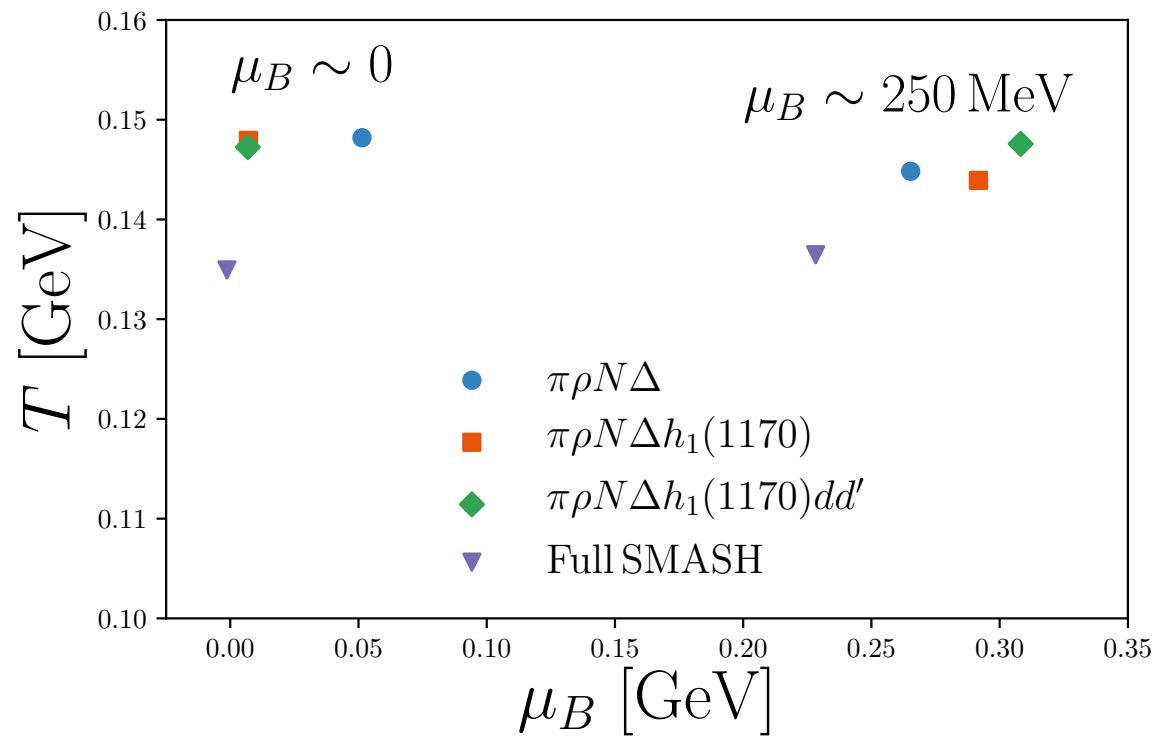
Proton number fluctuations cannot be recovered from baryon number fluctuations due to dynamical correlations

Outlook

Critical initial conditions in an expanding sphere

Impact of baryon annihilation and rescattering in more realistic scenarios

Methodology



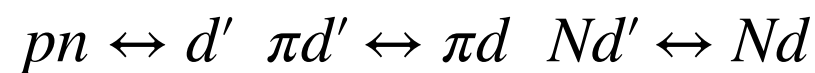
- Thermal and chemical equilibrium as well as isotropic densities are a pre-requisite to perform these kind of calculations
- Temperature and baryon chemical potential are calculated assuming

$$dN/dp \sim \exp(-(\sqrt{p^2 + m^2} - \mu_B)/T)$$
- The density is distributed isotropically in the system

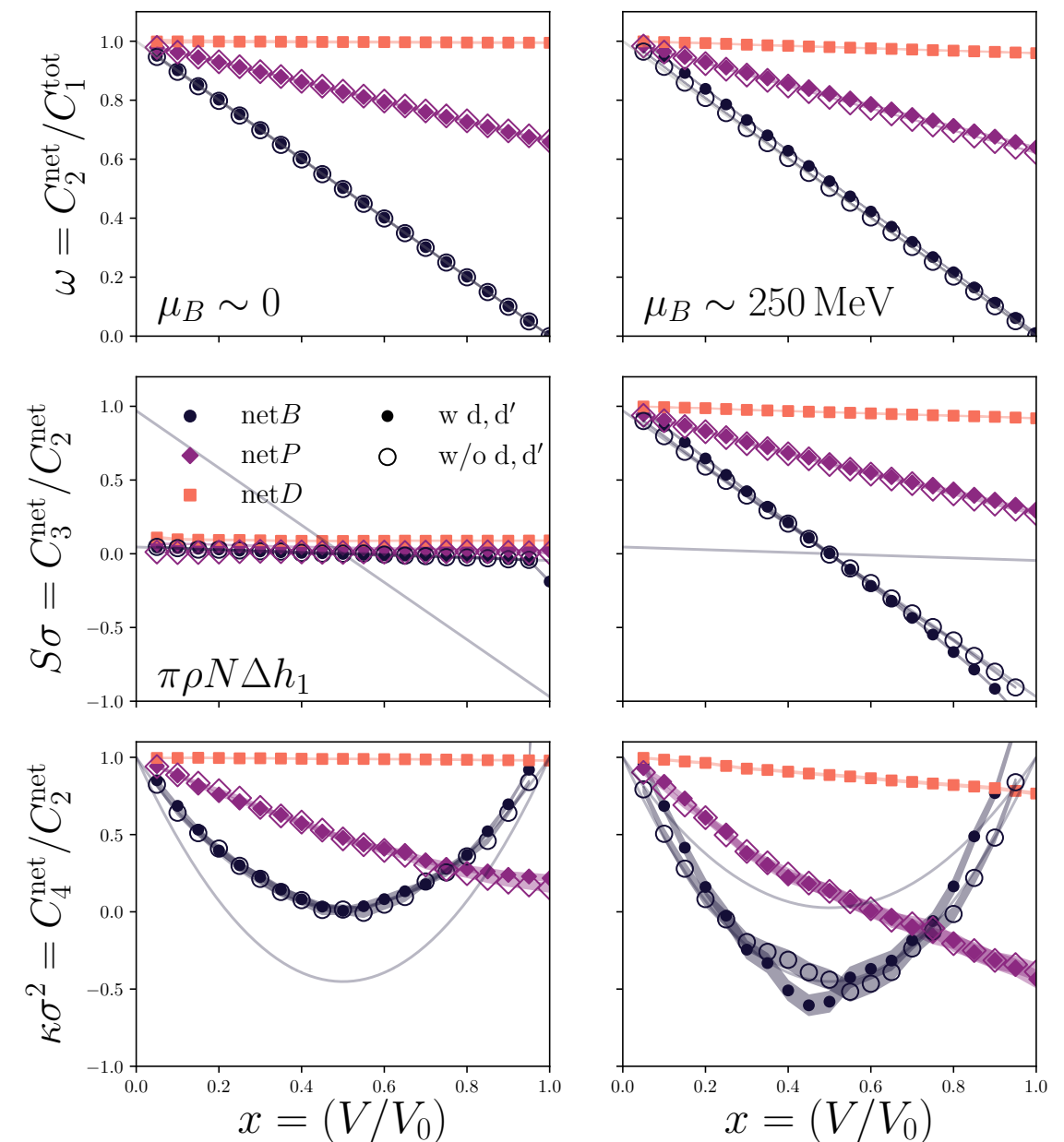
Impact of deuteron formation

Build a relatively easy hadron gas such that the impact of deuterons can be extracted

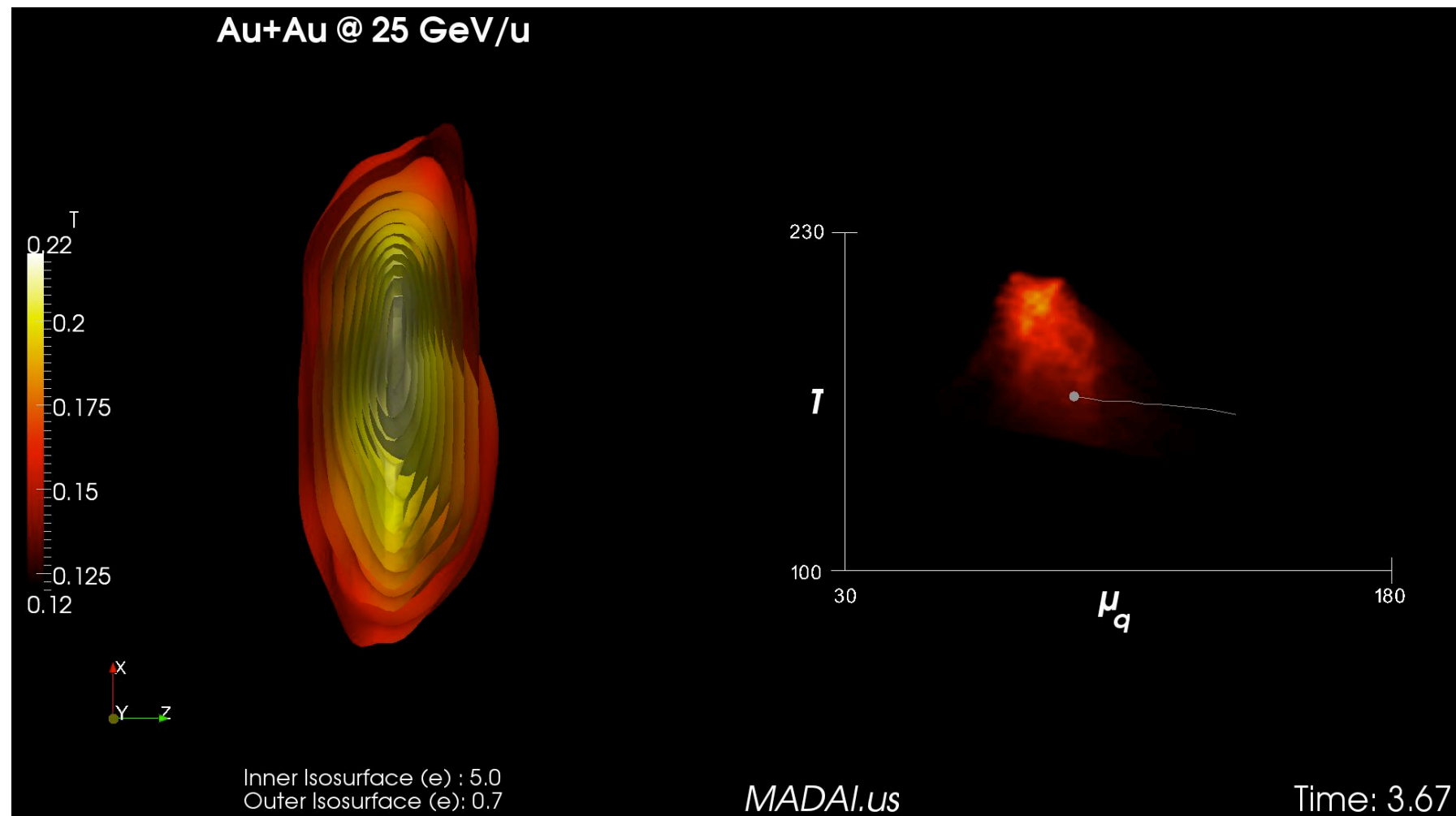
Deuteron formation on the basis of a fictional resonance d'



Proton number fluctuations are not affected by deuteron formation as they are produced very rarely

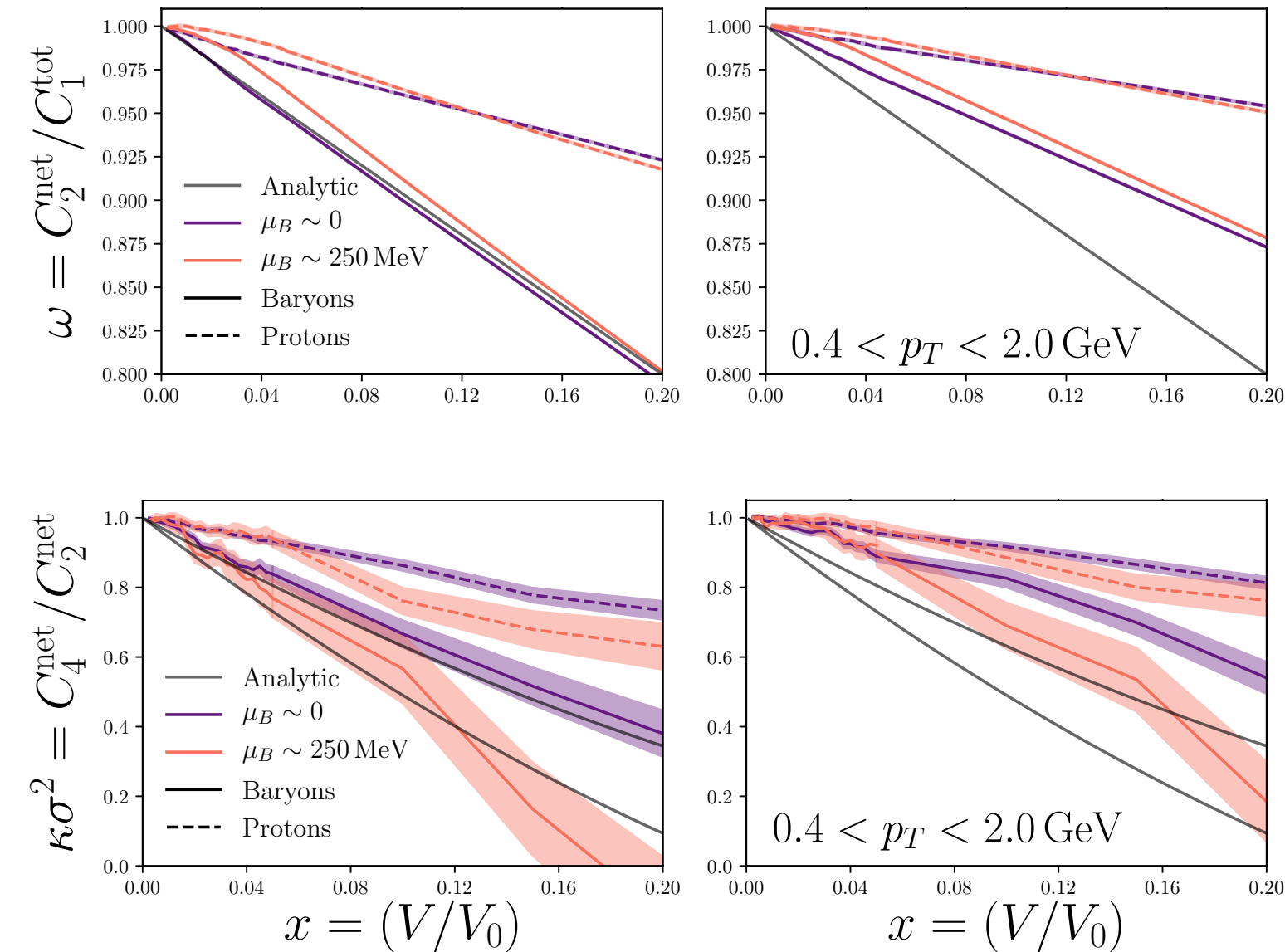


Introduction



System has a finite volume and spans a region in the phase diagram

Full Hadron Gas



In the limit $x \rightarrow 0$, $\omega/\kappa\sigma^2 \rightarrow 1$
(Poisson Limit)

For small volumes and large baryon
chemical potentials, the cumulants
don't strictly follow the analytic
expressions