Approaching the Continuum Limit of the Deconfinement Critical Point for $N_\mathsf{f}=2$ Staggered Fermions

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in collaboration with

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26/05/2022 - 16:40

FAIRness 2022 (Paralia Katerinis, Greece)

Outline

1 The QCD Phase Diagram and the Columbia Plot

2 Parameters of Thermal LQCD

3 A Strategy to Analyze Phase Transitions on the Lattice

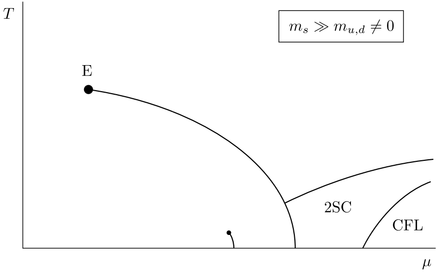
4 Results for the Z_2 Critical Point

5 Conclusion and Outlook

The QCD Phase Diagram and the Columbia Plot

Conjectured QCD Phase Diagram

(Rajagopal and Wilczek 2001)



- transition line separates hadronic phase from quark gluon plasma
- sign problem prevents lattice QCD from simulating at real $\mu \neq 0$
- thermal transition at $\mu = 0$ is analytic crossover[1] → Columbia plot

[1] Aoki et al. 2006

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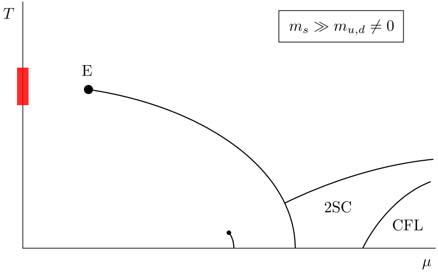

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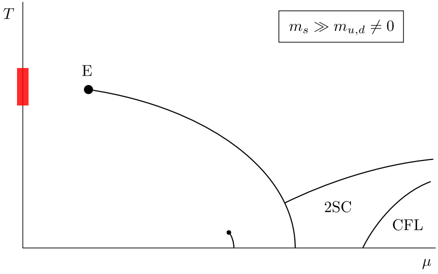
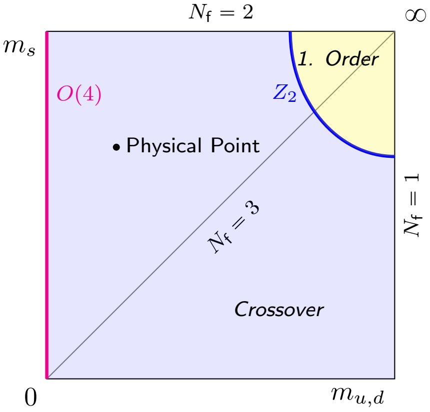


Columbia Plot

(Cuteri, Philipsen, and Sciarra 2021)



The QCD Phase Diagram and the Columbia Plot

- type of thermal transition as function of $m_{u,d}, m_s$
- pure gauge theory: deconfinement due to the spontaneous breaking of the Z_3 center symmetry
- dynamical quarks break center symmetry explicitly

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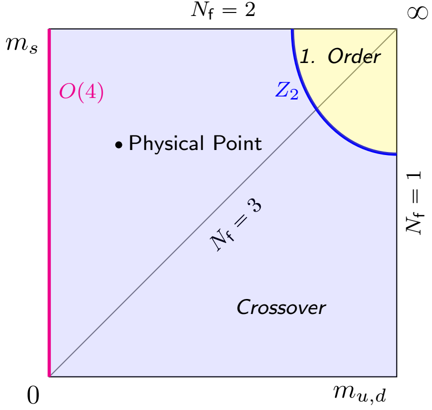

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Goal

Localize the Z_2 critical point for $N_f = 2$ with lattice QCD approaching the continuum limit.

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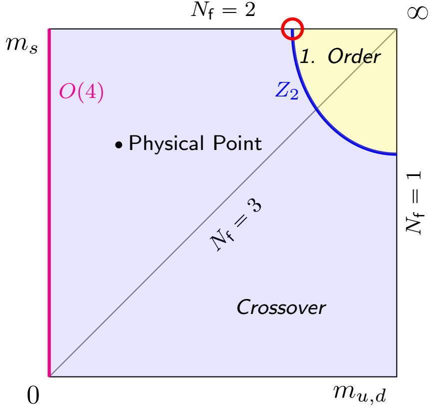

Why Study Thermodynamics of Unphysical Heavy Quarks?

- (it is computationally cheaper than physical quarks)
- knowledge of phase boundaries in the whole parameter space is valuable (scaling regions)
- first principles benchmarks for effective theories without sign problem (lattice or continuum theories)
- study the interplay between dynamical screening and Debye screening
  - dynamical screening: driven by quark mass value (string breaking)
  - Debye screening: occurs in medium with deconfined color charges

Parameters of Thermal LQCD

- Euclidean 4D space-time lattice

$$\Lambda = \{n = (n_1, n_2, n_3, n_4) \mid n_{1,2,3} \in [0, N_\sigma - 1]; n_4 \in [0, N_\tau - 1]\}$$

- fermion fields $\psi(n)$ on the the lattice sites
- gauge fields $U_\mu(n)$ on the links connecting the sites
- discretized action (Wilson gauge action & staggered fermion action)
- apply Monte Carlo importance sampling: gauge configurations $\{U\}$ sampled $\propto \det(D^{\mathsf{st}}[U])^{1/2} \exp\left(-S_g^W[U]\right)$

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Observable: Polyakov Loop

$$L = \frac{1}{N_\sigma^3} \sum_{\boldsymbol{n}} \frac{1}{3} \operatorname{Tr}\left[\prod_{n_4=0}^{N_\tau-1} U_4(\boldsymbol{n}, n_4)\right]$$

Parameters to Tune

- inverse gauge coupling $\beta(a)$
- bare quark mass am
- N_σ: $\rightarrow$ spatial system size
- N_τ: $(T = 1/(aN_\tau))$

A Strategy to Analyze Phase Transitions on the Lattice

- distribution of order parameter $|L|$ is continuous for finite systems
- analyze skewness B_3 and kurtosis B_4 of $|L|$
- $B_3(\beta_c) = 0$ determines β_c
- $B_4(\beta_c) \rightarrow$ information on type of transition

infinite volume kurtosis values

Type	1. Order	Z_2 (Ising 3D)	Crossover
$\boldsymbol{B_4(T_c)}$	1	$1.604(1)$[1]	3

[1] Blote, Luijten, and Heringa 1995
[2] Ferrenberg and Swendsen 1989

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- analyze quantities using jackknife resampling $\rightarrow$ determine errors
- reweight with respect to β using multiple histogram method[2]

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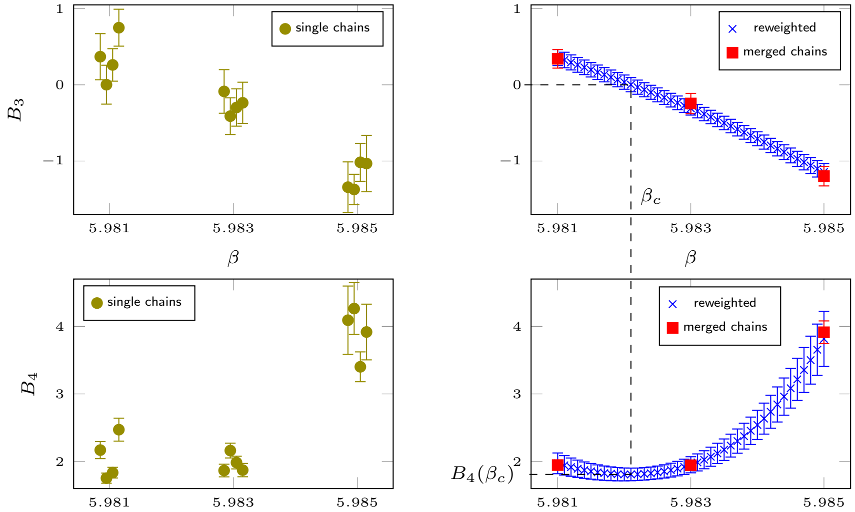
[2] Ferrenberg and Swendsen 1989

Exemplary Analysis of $B_{3,4}$ for am=0.55, N_τ=8, N_σ=56



Kurtosis Finite Size Scaling Formula

- finite size scaling (FSS) formula for kurtosis of observable[3] $O = c_M \cdot M$

$$B_4(N_\sigma, \beta_c, m) = \quad A + B \cdot x + \mathcal{O}\left(x^2\right)$$

scaling variable

$$x = \left(\frac{1}{m} - \frac{1}{m_c}\right) N_\sigma^{1/\nu}$$

critical exponents from Ising 3D universality class[4]

$y_t = 1/\nu$	y_t
$1.5870(10)$	$2.4818(3)$

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Kurtosis Finite Size Scaling Formula

- finite size scaling (FSS) formula for kurtosis of observable[3] $O = c_M \cdot M + c_E \cdot E$

$$B_4(N_\sigma, \beta_c, m) = \left(A + B \cdot x + \mathcal{O}(x^2)\right) \times \left(1 + C N_\sigma^{y_t - y_h} + \mathcal{O}\left(N_\sigma^{2(y_t - y_h)}\right)\right)$$

- correction term becomes irrelevant for sufficiently large volumes
- fit kurtosis data to FSS formula to determine m_c

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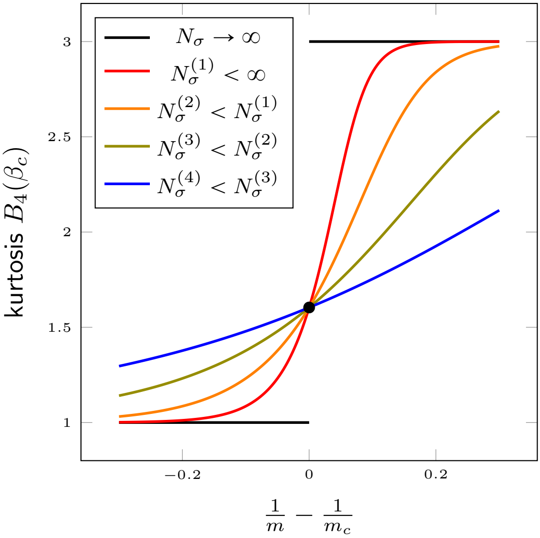
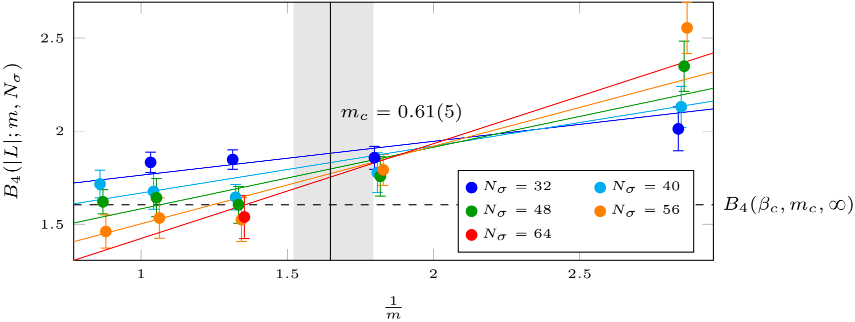
Model of the Course of the Kurtosis



- course of $B_4(\beta_c)$ without correction
- step function in thermodynamic limit
- smoothed in finite size systems

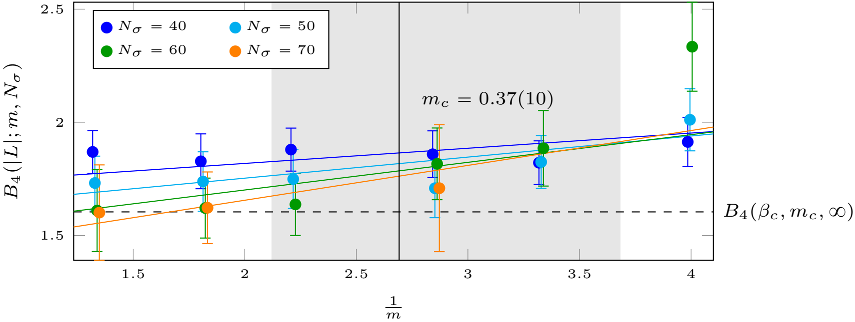
- correction term shifts the lines up, depeding on N_σ

Results for the Critical Mass for $N_\tau = 8$



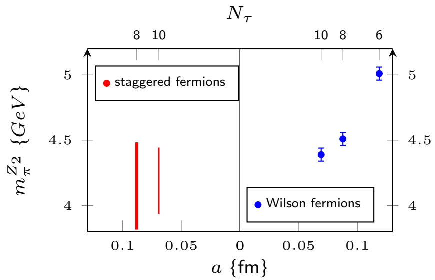
m_c	a_1	c	ndf	χ^2_{ndf}	Q
0.61(5)	$6.4(7) \cdot 10^{-4}$	3.8(6)	17	1.03	42.0%

Preliminary Results for the Critical Mass for $N_\tau\!=\!10$



m_c	a_1	c	ndf	χ^2_{ndf}	Q
0.37(10)	$1.7(6) \cdot 10^{-4}$	4.4(1.3)	18	0.47	97.1%

Results for the Critical Point



- set the scale using w_0 scale[5] based on Wilson flow[6]
- pion mass is not resolved by the lattice
- comparison with Wilson fermions[7]

$N_\tau=8$:

am	β_c	am_π	a {fm}	m_π {GeV}	T_c {MeV}	m_π/T_c
0.55	5.9821	1.72039(7)	0.0888(10)	3.82(4)	278(3)	13.7
0.75	6.0129	1.98121(7)	0.0872(9)	4.48(5)	283(3)	15.8

$N_\tau=10$:

am	β_c	am_π	a {fm}	m_π {GeV}	T_c {MeV}	m_π/T_c
0.35	6.0828	1.38124(11)	0.0691(7)	3.95(4)	285(3)	13.8
0.45	6.1139	1.55837(15)	0.0693(8)	4.44(5)	284(3)	15.6

[5] Borsányi et al. 2010
[6] Lüscher 2010
[7] Cuteri, Philipsen, Schön, et al. 2021

Conclusion and Outlook

- strategy to analyze LQCD transitions and to localize m_c is presented
- critical quark mass in lattice units has been obtained for $N_\tau = 8$
- preliminary critical mass region for $N_\tau = 10$ is found
- increasing computational effort due to larger finite size effects for larger N_τ
- too early to perform continuum limit

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- too early to perform continuum limit

- fit results for $N_\tau = 10$ will improve (currently running simulations)
- at least one larger N_τ must be added for continuum limit

Thank you for your attention!

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