THE UPPER RIGHT CORNER OF THE COLUMBIA PLOT WITH STAGGERED FERMIONS

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and
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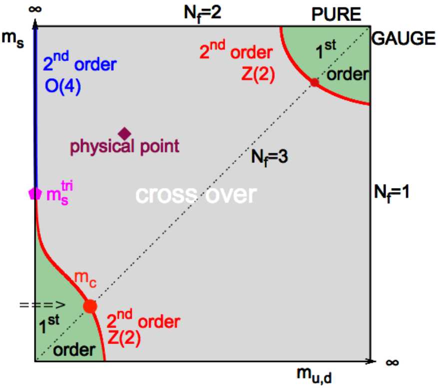
Fairness 2022



Forcrand et. al. 1702.00330

Columbia Plot for $\mu = 0$

- $m_q \to \infty$: Pure gauge theory (quenched case)
- Latent heat in cont. lim. Shirogane PRD 16, Borsanyi PRD 22 $\Rightarrow$ 1st order
- Phys. quark masses and $\mu = 0$: Crossover
- Which value takes the (heavy) critical mass m_c?

What are the reasons for the structures?

QCD Lagrangian

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a$$

- Exact global and local SU(3) gauge theory

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Center Symmetry $\mathcal{Z}_3$ for $m \to \infty$

- Invariance under $U_4(\vec{x}, t_0) \to zU_4(\vec{x}, t_0), z \in \{1, e^{\pm i2\pi/3}\}, U \in \mathrm{SU}(3)$
- SSB at high temperatures (deconfinement)
- Order parameter Polyakov loop: $|\langle P \rangle| \sim e^{-F/T}$

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Chiral symmetry for $m \to 0$

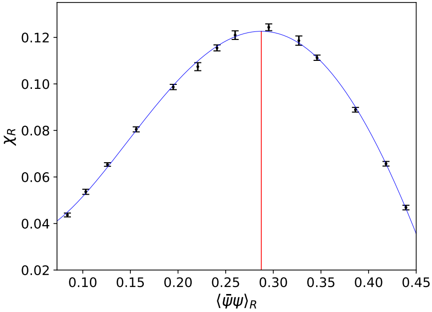
- $\mathcal{L}_F = \bar{\psi}_L i\not{D}\psi_L + \bar{\psi}_R i\not{D}\psi_R$
- SSB at moderate temperatures
- Order parameter chiral condensate $\langle\bar{\psi}\psi\rangle = \langle\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\rangle$

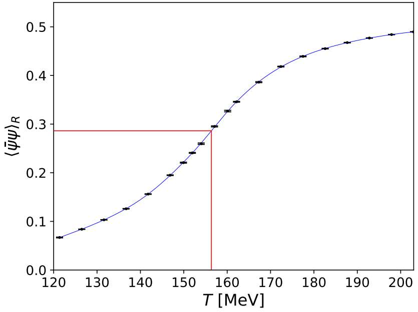
Chiral observables

$$\langle\bar\psi\psi\rangle = \frac{T}{V}\frac{\partial \log Z}{\partial m} \qquad \langle\bar\psi\psi\rangle_R = -\left[\langle\bar\psi\psi\rangle_T - \langle\bar\psi\psi\rangle_{T=0}\right]\frac{m}{f_\pi^4}$$

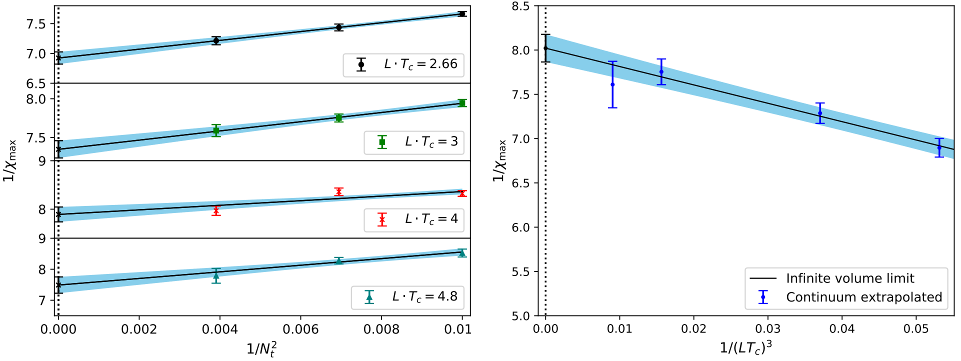
$$\chi = \frac{T}{V}\frac{\partial^2 \log Z}{\partial m^2} \qquad \chi_R = \left[\chi_T - \chi_{T=0}\right]\frac{m^2}{f_\pi^4}$$

How to determine precisely the inflection point of $\langle\bar\psi\psi\rangle$ or $\chi_{\max}$?





What is the type of transition?



Analysis

- No real phase transition
- Weak crossover[1] for $\mu = 0$ (early universe)

[1] First time: Fodor et. al. Nature 2006

Dependence of the strength and width on μ

Definition of the transition width (Gaussian)

$$(\Delta T)^2 = -\chi(T_c)\left[\frac{\mathrm{d}^2\chi}{\mathrm{d}T^2}(T_c)\right]^{-1}$$

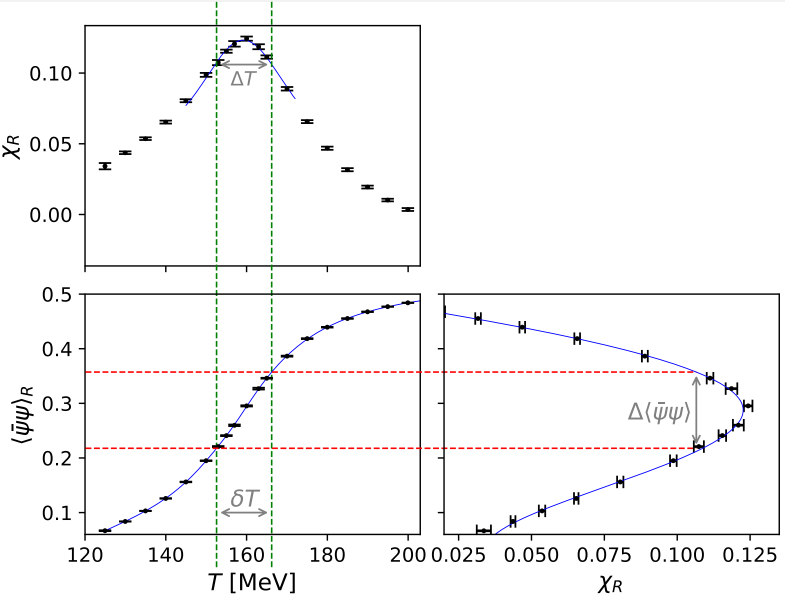
- Second derivative without appropriate fitfunction?

$$\left(\Delta\left\langle\bar\psi\psi\right\rangle\right)^2 = -\chi(T_c)\left(\left.\frac{\mathrm{d}^2\chi}{\mathrm{d}\left\langle\bar\psi\psi\right\rangle^2}\right|_{\left\langle\bar\psi\psi\right\rangle_c}\right)^{-1}$$

$$\delta T = \left\langle\bar\psi\psi\right\rangle^{-1}\left(\left\langle\bar\psi\psi\right\rangle_c + \frac{\Delta\left\langle\bar\psi\psi\right\rangle}{2}\right) - \left\langle\bar\psi\psi\right\rangle^{-1}\left(\left\langle\bar\psi\psi\right\rangle_c - \frac{\Delta\left\langle\bar\psi\psi\right\rangle}{2}\right)$$

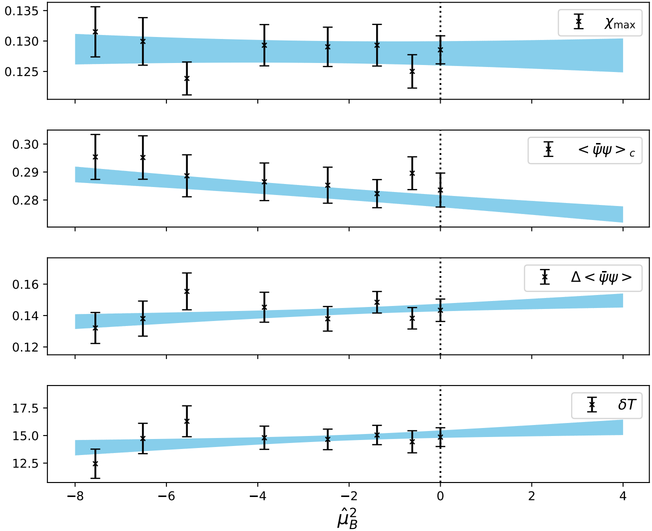
Let's visualize the approach

What is ΔT and δT?

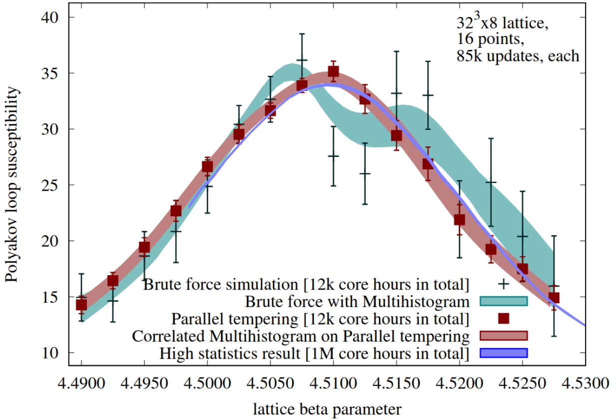


Continum extrapolated observables as functions of $\hat{\mu}_B^2$

Borsanyi, R.K. et. al. PRL (2020)

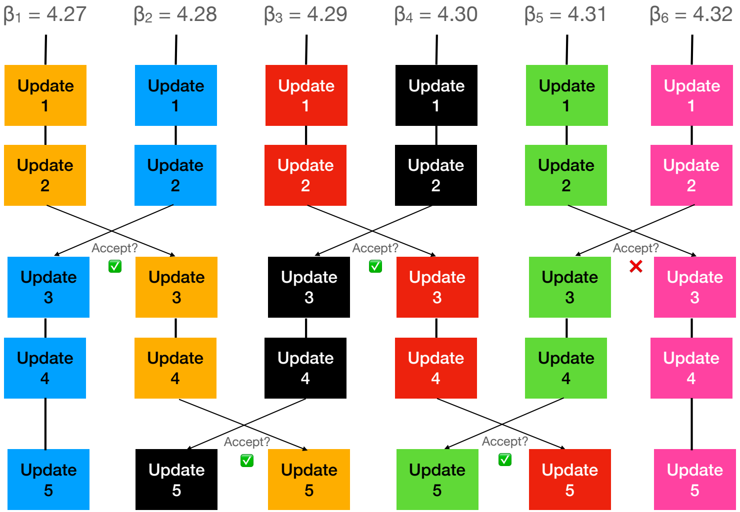


Quenched case: (Super-)critical slowing down



Tempering algorithms

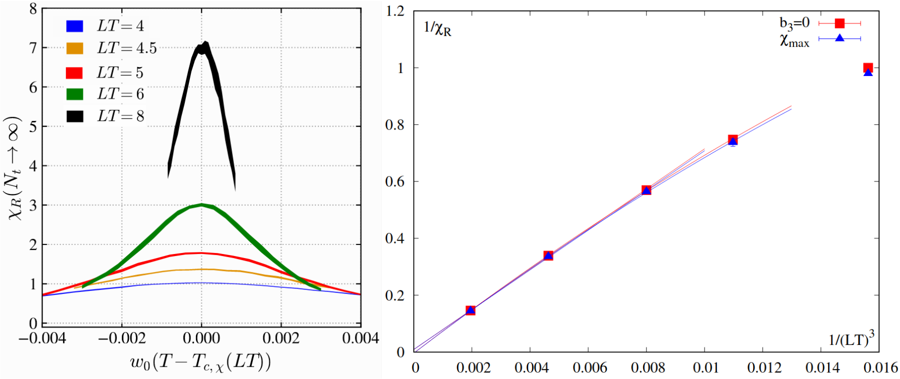
- Multiple simulations at different β [Marinari 9205018] [Joó 9810032]
- Swap configurations between subensembles $\Rightarrow$ reduced auto. corr.



$\beta_1 = 4.27$
$\beta_2 = 4.28$
$\beta_3 = 4.29$
$\beta_4 = 4.30$
$\beta_5 = 4.31$
$\beta_6 = 4.32$
Update 1
Update 2
Accept?
Update 3
Update 4
Update 5

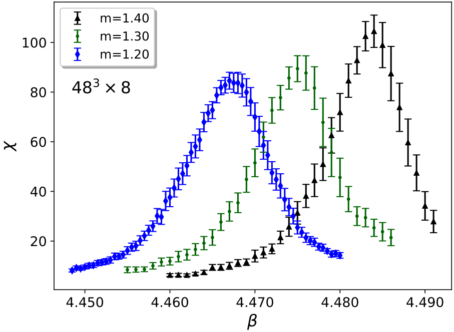
Polyakov loop and its susceptibility

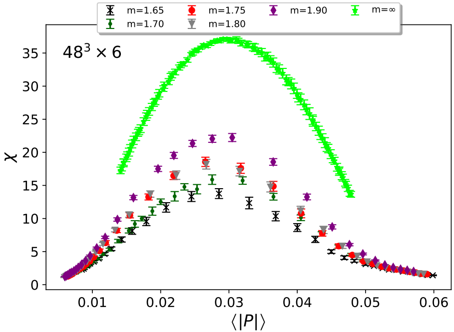
$$P = \frac{1}{N_s^3}\sum_{\vec{x}} P_{\vec{x}} = \frac{1}{N_s^3}\sum_{\vec{x}} \mathrm{tr}\left[\prod_{\tau} U_4(\vec{x},\tau)\right] \qquad \chi = N_s^3\left(\langle|P|^2\rangle - \langle|P|\rangle^2\right)$$



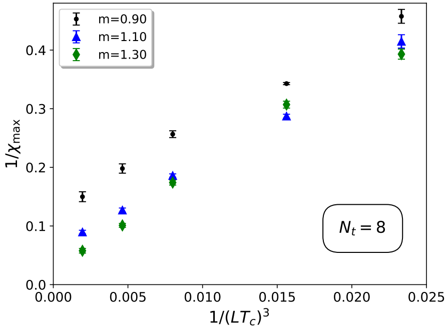
Borsanyi, R.K. et. al. PRD (2022)

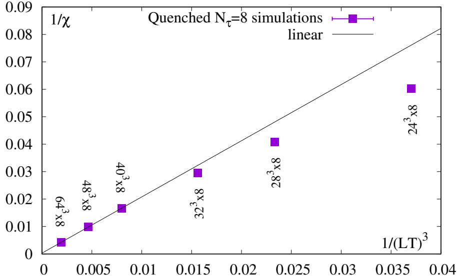
Heavy but finite masses



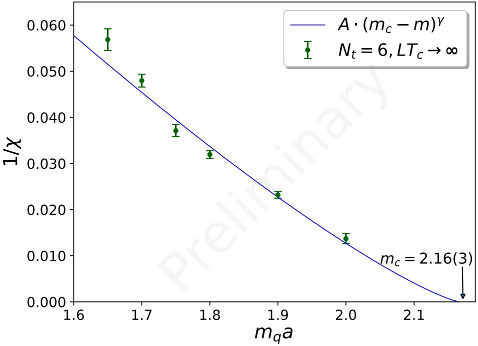


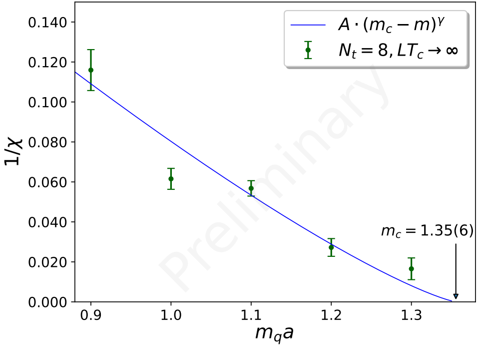
Volume scaling for $N_t = 8$





Determination of the critical mass m_c





Critical region

- $\chi_{\max}^{-1}$ follows a power law near the transition
- m_q represents symmetry breaking field

$$\chi_{\max}^{-1}(LT_c \to \infty) = A \cdot (m_c - m)^\gamma$$

Summary

Results of dynamical quark simulations

- β tempering algorithms to reduce auto-correlation
- $\chi(\langle|P|\rangle)$ allows precise peak determination
- Estimate of the critical mass: $N_t = 6$ $m_c = 2.16$; $N_t = 8$ $m_c = 1.35$

Outlook

- Systematic error analyzis
- Continuum limit of the critical mass

Thank you for your attention!

Tempering algorithm

Background

- Tempering simulation is collection of sub-simulations
- Overall phase space as direct product of sub-ensemble phase spaces
- Equilibrium distribution of configs. in individual sub-ensembles overlap
- Independent Markov-processes in each sub-ensemble
- Swap configs. between sub-ensemble pairs

$\Longrightarrow$ Auto-corr. is reduced

Tempering algorithm

Transition between sub-ensembles

- a is config. in sub-ensemble i, b config. in sub-ensemble j
  $(a, b) \to (b, a) \Leftrightarrow$ swap accepted
  $(a, b) \to (a, b) \Leftrightarrow$ swap rejected

Detailed balance condition

$$P_s(i,j) e^{-\mathcal{H}_i(a)} e^{-\mathcal{H}_j(b)} = P_s(j,i) e^{-\mathcal{H}_j(a)} e^{-\mathcal{H}_i(b)}$$

$$P_s(i,j) = \min\left(1, e^{\Delta\mathcal{H}}\right)$$

$$\Delta\mathcal{H} = \{\mathcal{H}_j(a) + \mathcal{H}_i(b)\} - \{\mathcal{H}_i(a) + \mathcal{H}_j(b)\}$$

Cubic spline Interpolation

$$s_i(x) = a_i + b_i x + c_i x^2 + d_i x^3 \Longrightarrow 4(N-1) \text{ conditions}$$

Smoothness conditions

$$\begin{aligned} s_i(x_{i+1}) &= s_{i+1}(x_{i+1}) \\ s_i'(x_{i+1}) &= s_{i+1}'(x_{i+1}) \\ s_i''(x_{i+1}) &= s_{i+1}''(x_{i+1}) \end{aligned}$$

$\Longrightarrow 3(N-2)$ constraints

Interpolation condition

$$S(x_j) = y_j$$

$\Longrightarrow N$ conditions

Endpoints

$s_0''(x_0) = 0 = s_{N-2}''(x_{N-1}) \Longrightarrow 2$ conditions