

# Critical Endpoint of QCD in a Finite Volume

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Based on:

JB, Fischer, Isserstedt, Schaefer, PRD 104 (2021) 074035

and

JB, Fischer, Isserstedt (in preparation)



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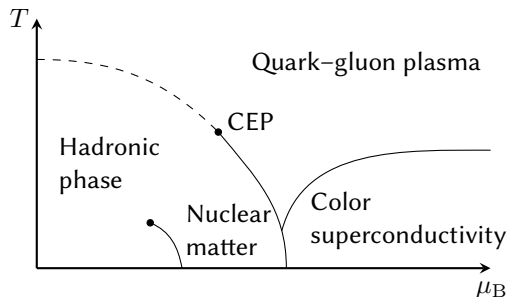
Paralia, Greece

2022-05-26

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- 3 Conclusion and Outlook

# First Objective: QCD Phase Diagram in a Finite Volume

# Motivation: Why Finite Volume?



- Goal of many experiments is to locate critical endpoint in QCD phase diagram
- “Fireball” of heavy-ion collisions has finite spatial extent
- Impact of volume effects on CEP is important for comparison between theory and experiment
- Cross-check between different theoretical approaches: lattice QCD (by construction formulated in a finite volume) vs. functional methods

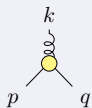
# Functional Framework: Truncated Set of DSEs

## Truncated Dyson–Schwinger Equations for Quark and Gluon Propagators

see Fischer, PPNP 105 (2019) 1  
(and references therein)

$$\begin{aligned}
 \text{---}\bullet\text{---}^{-1} &= \text{---}^{-1} + \text{---}\bullet\text{---}\text{---}\bullet\text{---}^{-1} \\
 \text{---}\bullet\text{---}^{-1} &= \text{---}\bullet\text{---}^{-1} + \sum_{f \in \{u,d,s\}} \text{---}\bullet\text{---}\text{---}\bullet\text{---}^{-1}
 \end{aligned}$$

## Quark–Gluon Vertex Ansatz



$$\Gamma_{\mu}^f(k, p, q) = \Gamma(k, p, q) \Gamma_{\mu}^{f, \text{BC}}(p, q) \quad (\text{Information about quarks})$$

## Quenched Gluon Propagator

$$\text{---}\bullet\text{---} \quad D_{\mu\nu}^{\text{que}}(k) = D_{\mu\nu}^{\text{que}}(k; T) \quad (\text{Temperature-dependent fit to lattice data})$$

reference for lattice data: Fischer, Maas, Müller, EPJC 68 (2010) 165-181

Maas, Pawłowski, von Smekal, Spielmann, PRD 85 (2012) 034037

# Finite Volume Framework: Ansatz

- Feasible shape as ansatz: cube with edge length  $L$ :

$$\int_{\mathbb{R}^3} d^3x \mathcal{L} \rightarrow \int_{[0,L]^3} d^3x \mathcal{L}$$

- For quarks, free to choose between

$$\psi(\mathbf{x} + L\mathbf{e}_i) = +\psi(\mathbf{x}) \quad \text{periodic boundary conditions (PBC)}$$

$$\psi(\mathbf{x} + L\mathbf{e}_i) = -\psi(\mathbf{x}) \quad \text{antiperiodic boundary conditions (ABC)}$$

- For gluons, need PBC for kinematic reasons

→ Only discrete values possible in momentum space

# Finite Volume Framework: Implications

→ Possible discrete momentum values given by:

## Spatial Matsubara Modes

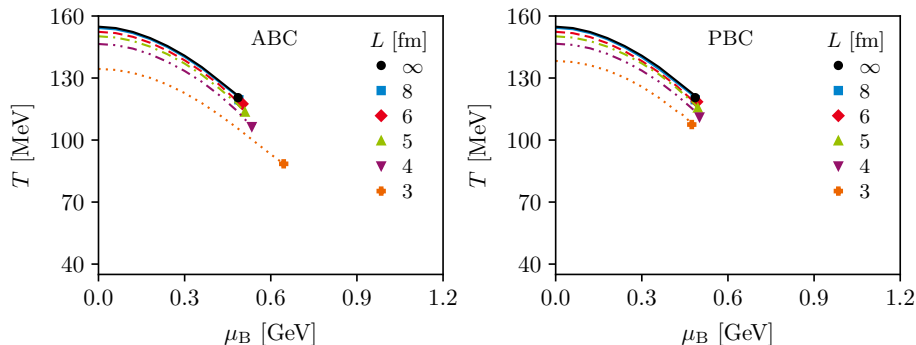
$$\omega_n^L = \begin{cases} 2n\pi/L & \text{for PBC,} \\ (2n+1)\pi/L & \text{for ABC,} \end{cases} \quad n \in \mathbb{Z}$$

- Momentum integrals become sums

$$\int \frac{d^3q}{(2\pi)^3} K(\mathbf{q}) \rightarrow \frac{1}{L^3} \sum_{\mathbf{n} \in \mathbb{Z}^3} K(\mathbf{q}_n),$$

where  $\mathbf{q}_n := \sum_{i=1}^3 \omega_{n_i}^L \mathbf{e}_i$  are allowed momentum vectors

# Results: QCD Phase Diagram in a Finite Volume



- Consistent infinite-volume limit
- For decreasing  $L$ , pseudocritical temperature decreases and CEP (mostly) moves to higher  $\mu$
- Visible volume effects for  $L \leq 4$  fm
- Very similar results for ABC and PBC above  $L \geq 4$  fm



# Baryon Number Fluctuations

## Fluctuations of Conserved Charges from QCD's Grand Potential

$$\chi_{ijk}^{\text{BQS}} = -T^{(i+j+k)-4} \frac{\partial^{i+j+k}}{\partial \mu_{\text{B}}^i \partial \mu_{\text{Q}}^j \partial \mu_{\text{S}}^k} \Omega$$

B: baryon number

Q: electric charge

S: strangeness

- Relation to cumulants of baryon number distribution:

$$C_n^{\text{B}} = VT^3 \chi_n^{\text{B}} \quad \text{where} \quad \chi_n^{\text{B}} = \chi_{n00}^{\text{BQS}}$$

- Directly linked to moments of baryon number distribution:

$$\sigma_{\text{B}}^2 = C_2^{\text{B}}, \quad S_{\text{B}} = C_3^{\text{B}} (C_2^{\text{B}})^{-3/2}, \quad \kappa_{\text{B}} = C_4^{\text{B}} (C_2^{\text{B}})^{-2}, \quad \dots$$

- Ratios relate theoretical and experimental quantities:

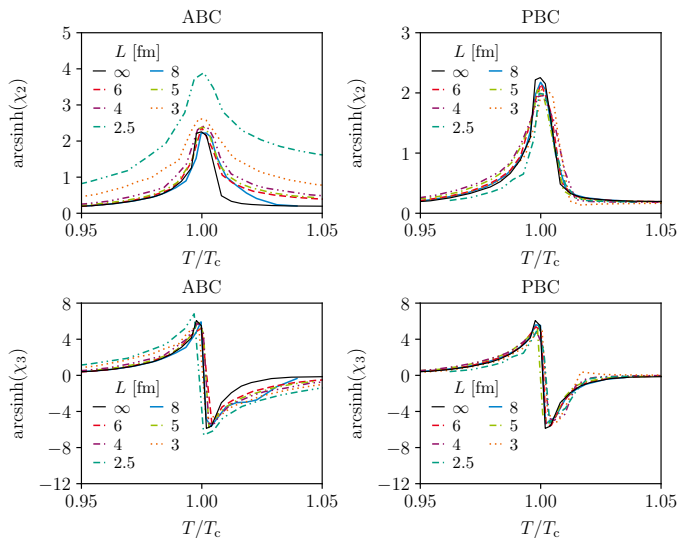
$$\chi_3^{\text{B}} / \chi_2^{\text{B}} = S_{\text{B}} \sigma_{\text{B}}, \quad \chi_4^{\text{B}} / \chi_2^{\text{B}} = \kappa_{\text{B}} \sigma_{\text{B}}^2, \quad \dots$$

→ Explicit volume dependence drops out!

Reviews: Luo, Xu, Nucl. Sci. Tech. 28 (2017) 112

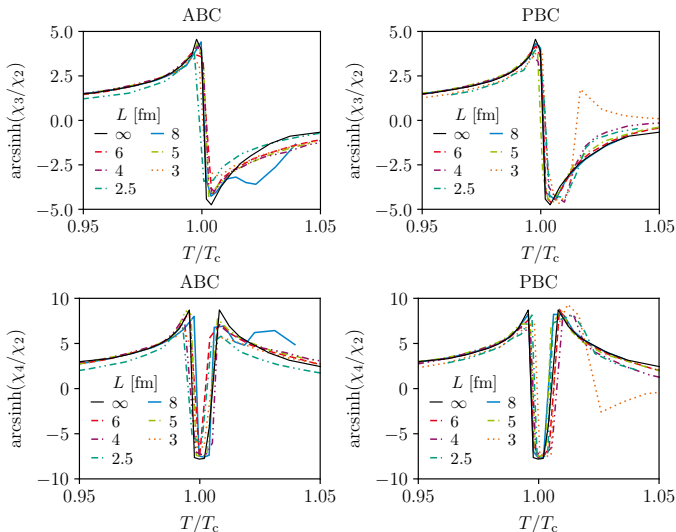
Bzdak, Esumi, Koch, Liao, Stephanov, Xu, Phys. Rep. 853 (2020) 1

# Results: Baryon Number Fluctuations at $\mu = \mu_c$



- Visible volume effects (especially for ABC)

# Results: Baryon Number Fluctuation Ratios at $\mu = \mu_c$

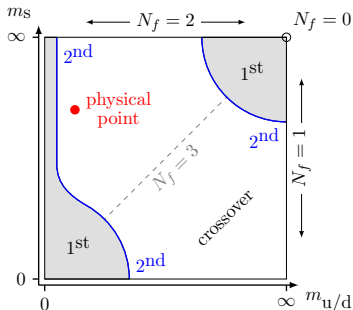
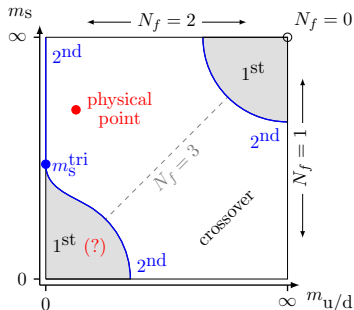


- (Essentially) independent of system size  $\rightarrow$  no implicit volume dependence

## Second Objective: Columbia Plot and (Up Quark) Chiral Limit

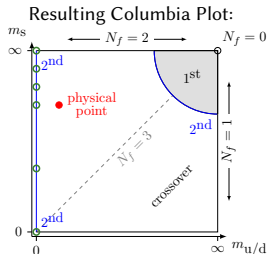
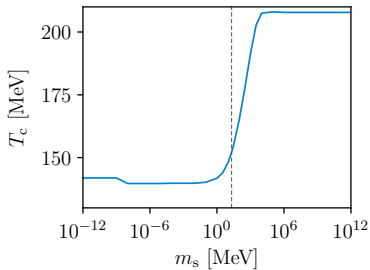
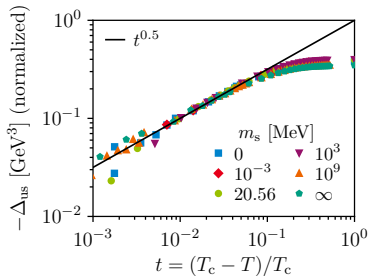
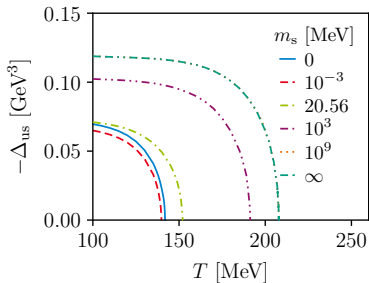
# Motivation: Columbia Plot(s)

for reference on upper right corner in DSE framework,  
see Fischer, Luecker, Pawłowski, PRD 91 (2015) 014024



- Two different scenarios for Columbia Plot: anomalously broken (left) or restored (right)  $U_A(1)$ -symmetry
- Existence of first order region in lower left corner (of left scenario) is not yet clear see Cuteri, Philipsen, Sciarra, JHEP 11 (2021) 141
- Chiral limit is difficult for lattice QCD but no conceptual problem for our framework

# Results: Condensate and Critical Scaling in Chiral Limit



# Meson Backcoupling Ansatz

- Improve truncation in chiral limit: long-range correlations in vertex become important  $\rightarrow$  add meson backcoupling diagram to quark DSE

## Modified Quark DSE

$$\text{quark line with grey blob}^{-1} = \text{quark line}^{-1} + \text{quark line with wavy meson loop} + \text{quark line with double-line meson loop}$$

$\pi, K, \eta_8, \sigma, f_0$

## Bethe-Salpeter Amplitudes



(Obtained from quarks: Goldberger-Treiman-like relations)

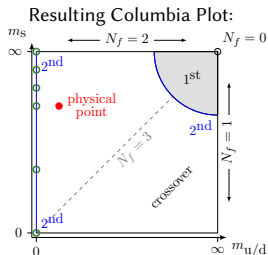
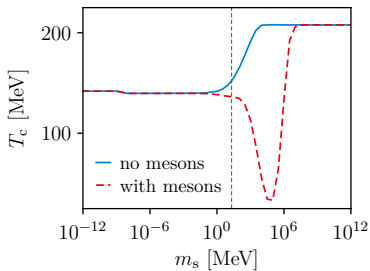
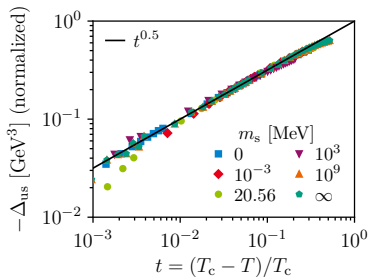
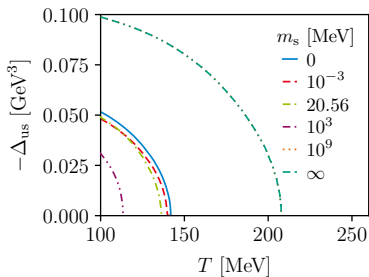
## Free Meson Propagator



(Mass from Gell-Mann-Oakes-Renner fit)

details on meson backcoupling: Fischer, Müller, PRD 84 (2011) 054013

# Results for Meson Backcoupling





# Conclusion and Outlook

## Conclusion:

- Studied finite-volume effects on QCD phase diagram using DSEs beyond rainbow–ladder truncation for ABC and PBC
- Crossover line and CEP exhibit visible volume effects for  $L \leq 4$  fm
- Baryon number fluctuations show volume dependence, ratios do not
- Second order phase transition across whole left edge of Columbia Plot (both with and without meson backcoupling)

## Outlook:

- Implement proper QCD scaling to meson backcoupling
- Study finite-volume effects in chiral limit