Low-energy charmonium, bottomonium and tetraquark production cross-sections from a statistical model

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Outline

- Motivation
- Statistical method for low energy cross section calculations

Eur. Phys. J. A (2018) 54:25

Eur. Phys. J. A (2020) 56:174

Eur. Phys. J. A (2020) 56:237

- X(3872) production cross sections

Eur. Phys. J. A (2021) 57: 246

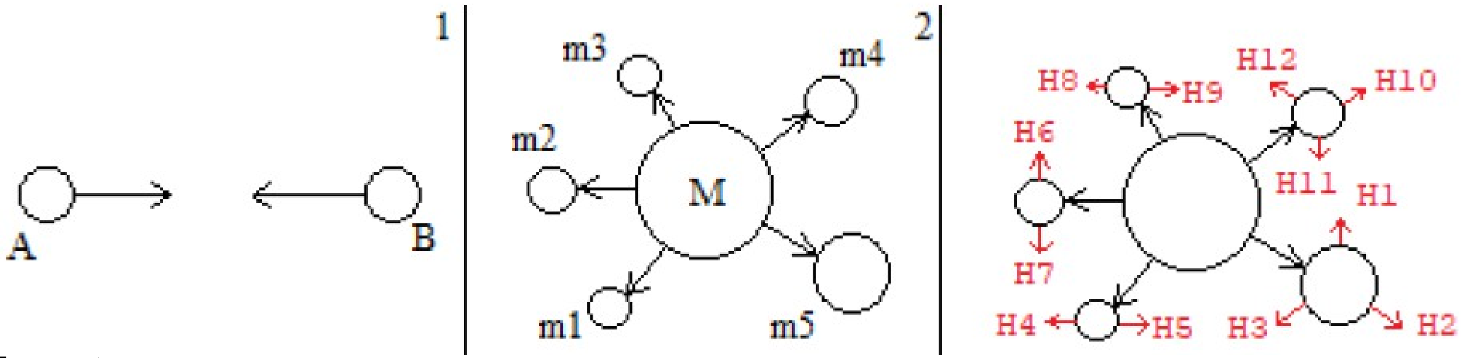
- Summary, future plans ...

Motivation

- Charmonium mass shifts in antiproton induced reactions with off-shell BUU transport
- Low-energy charmonium production cross-sections are needed
  - *proton+proton, pion+proton* → usually measurements + extrapolation
  - *antiproton+proton* → only a few meas. points, Breit-Wigner cross-sections
  - Different models (NRQCD, CSM, SPS,...)
- *2 → 2,* ***2*** *→* ***N*** *, (N → M)* reactions

Statistical model

- Model to calculate low energy (~GeV) cross sections based on the Fermi model and the Statistical bootstrap approach.
- Fermi model: $\sigma \propto \left(\frac{\Omega}{V}\right)^{n-1} \left(\frac{V}{8\pi^3}\right)^{n-1} \rho(\sqrt{s}, m_1, ..., m_n)$
- Main idea: During the collision a fireball is formed, which will decay into smaller fireballs and eventually to hadrons.



- **Ingredients**:
  - Fireball formation probability
  - Phase space factors, DOS from Bootstrap, Breit-Wigner factors for resonances, spin factors
  - Quark combinatorial factors

- $P_k^{fb}(\sqrt{s})$: probability of the formation of n-fireballs.
- $T_i(x) = C_{Q_i}(x) P_{n_i}^{H,i}(x)$: Hadronization probability of a specific fireball.
  - $P_{n_i}^{H,i}(x) = P_n^d \frac{\Phi_n(x, m_1, .., m_n)}{(2\pi)^{3n-3}\rho(x) N_I!} \prod_{l=1}^{n}(2s_l + 1)$
  - Statistical bootstrap → P_n^d : n-hadron formation probability

    → $\rho(x)$: Density of states
  - k-body phase space integral for resonances and stable particles:

$$\Phi_k(x, m_1, ..., m_k) = V^{k-1}\Big(\int \prod_{i=1}^{k} d^3\vec{q}_i\Big)\Big(\int \prod_{r\in R} dE_r F_r^{BR}(x, m_r)\Big) \times$$

$$\times\, \delta\Big(\sum_{j=1}^{k} E_j - x\Big)\delta\Big(\sum_{j=1}^{k} \vec{q}_j\Big),$$

The quark combinatorial factors

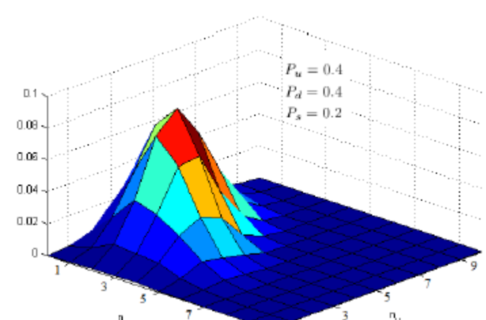
- Number of colorless quark (antiquark) combinations, which can form a specific hadronic final state (~parton model, parton distribution functions...)
- Number of quarks at a specific invariant mass:

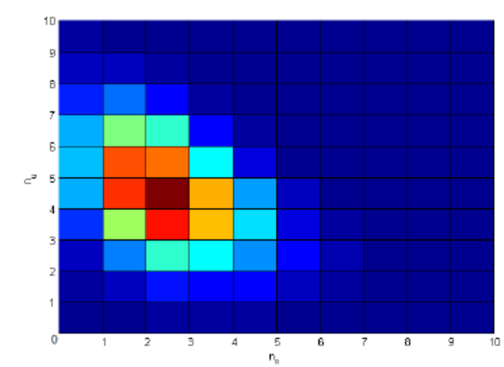
$$\Phi_N^{kvark}(x) = \int \prod_{i=1}^{N} \frac{d^3 p_i}{2E_i(2\pi)^3}(2\pi)^4 \delta^{(4)}\left(P^\mu - \sum_{i=1}^{N} p_{\mu,i}\right) = \\ = \frac{1}{2(4\pi)^{2N-3}} \frac{x^{2N-4}}{\Gamma(N)\Gamma(N-1)}$$

$$\langle x^2 \rangle = \frac{\int dx x^2 \Phi_N^{kvark}(x) e^{-x/T_0}}{\int dx \Phi_N^{kvark}(x) e^{-x/T_0}} = 4N(N-1)T_0^2$$

- Quark number distribution for the different flavours → multinomial distribution, with *Pu,Pd,Ps,Pc,Pb* **<u>quark creational probabilities</u>**.

$$F(N(x), n_i) = \frac{N(x)!}{\prod_i n_i!} \prod_i P_i^{n_i}$$

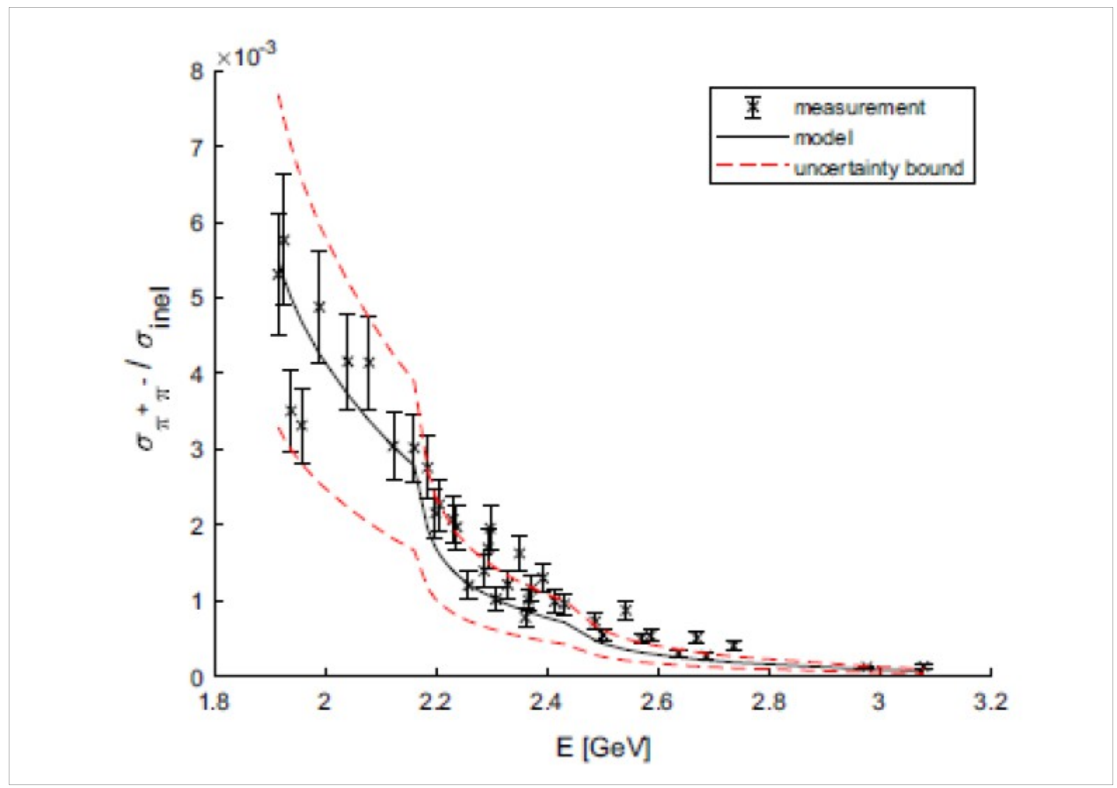


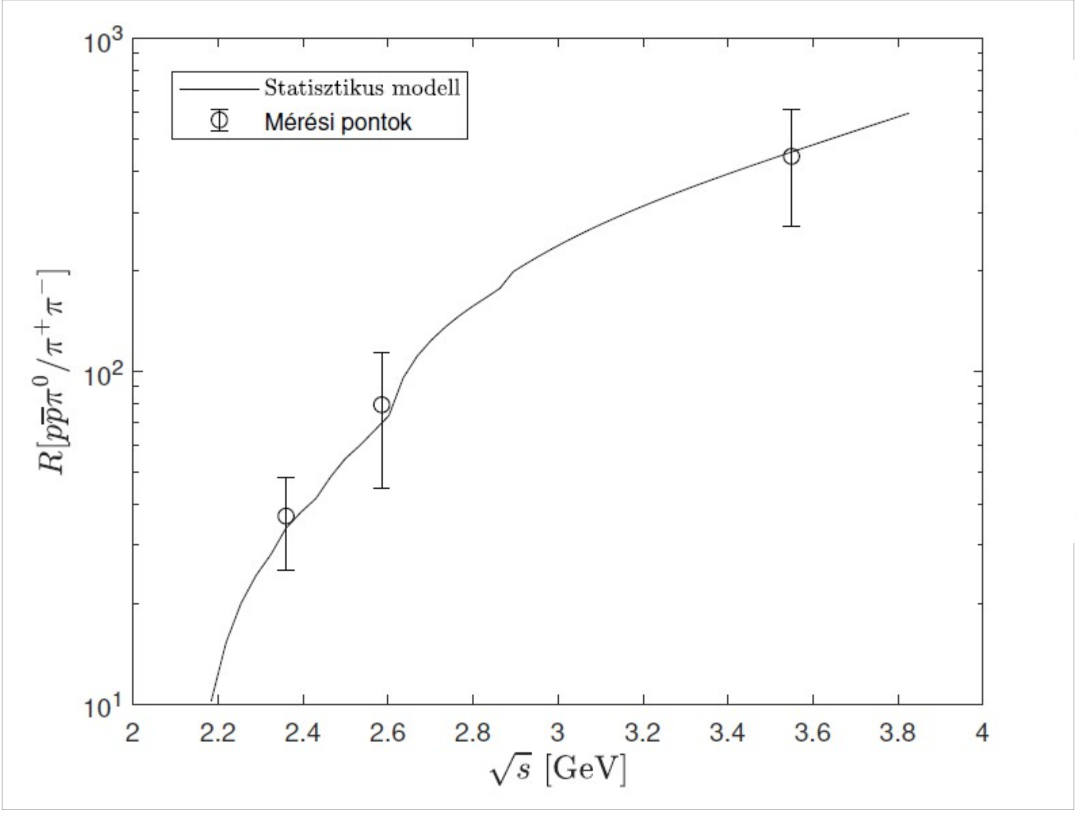


- The normalized quark combinatorial factors are the probabilities that N quarks(antiquarks) could build up a specific 2-, or 3-body hadronic final state.
- Has to be calculated for each created fireball.

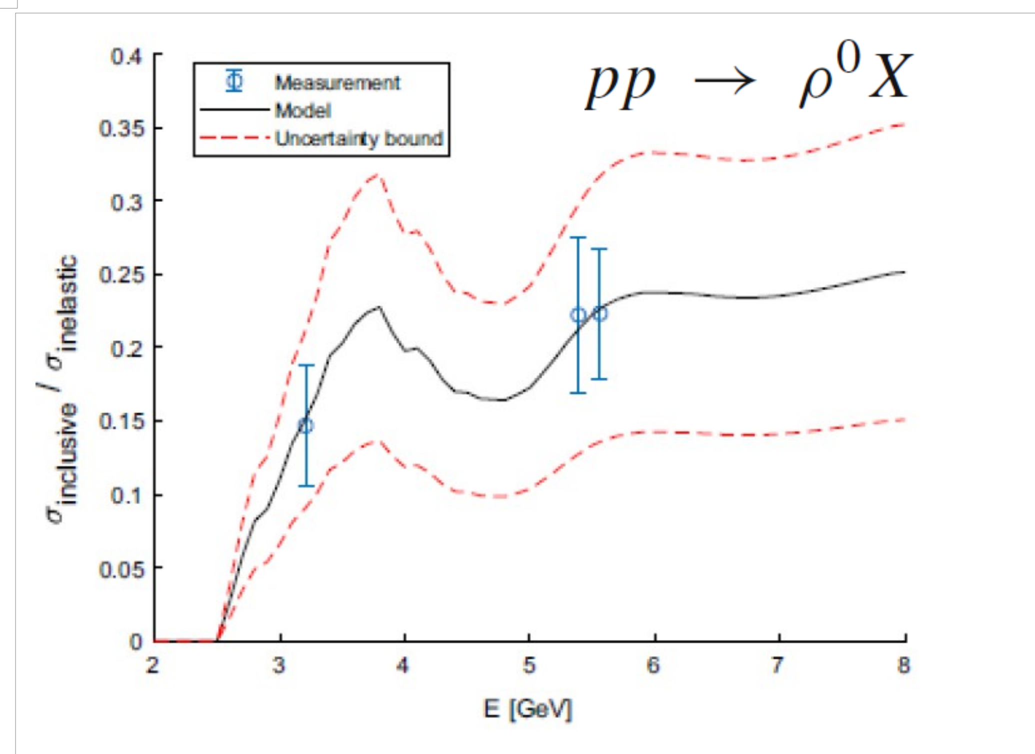
$$C_{Q_k,(AB,ABC)} = \frac{1}{\mathcal{N}_k^{(2,3)}} F(N; \langle n_i \rangle) \left[\prod_{i=1}^{2,3} \mathcal{C}_i \right] \left[\prod_{i=1}^{M_{2,3}} \frac{\Gamma(\langle n_i \rangle + 1)}{\Gamma(\langle n_i \rangle - n_i^0 + 1)} \right]$$

- Free parameters:
  - T^0 (~*130-170* MeV)
  - *V* – interaction volume
  - P_i (i=u,d,s,c,b)

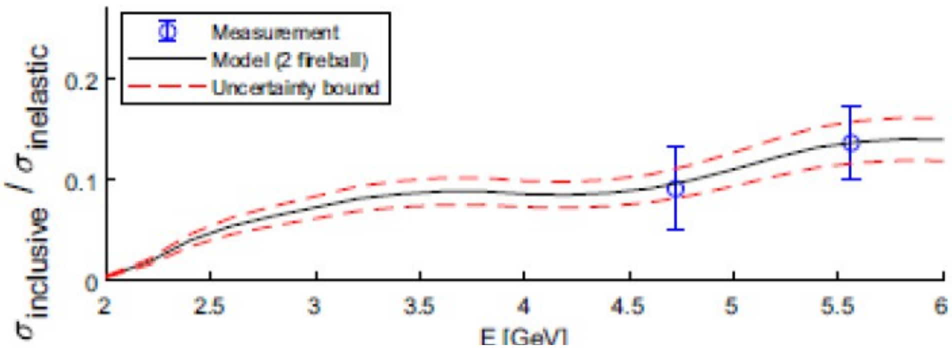




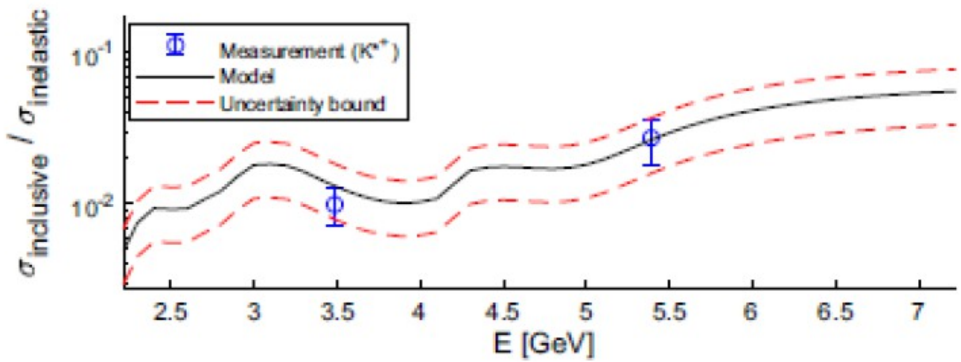
	R_i	m_{R_i} [GeV]	s_{R_i}	$B_i^{p\pi^0}$
1	N_{1440}	1.43	1/2	0.22
2	N_{1520}	1.515	3/2	0.2
3	N_{1535}	1.535	1/2	0.15
4	N_{1650}	1.655	1/2	0.23
5	N_{1680}	1.685	5/2	0.23
6	Δ_{1232}	1.232	3/2	0.66
7	Δ_{1620}	1.63	1/2	0.17
8	Δ_{1910}	1.89	1/2	0.15
9	Δ_{1950}	1.93	7/2	027



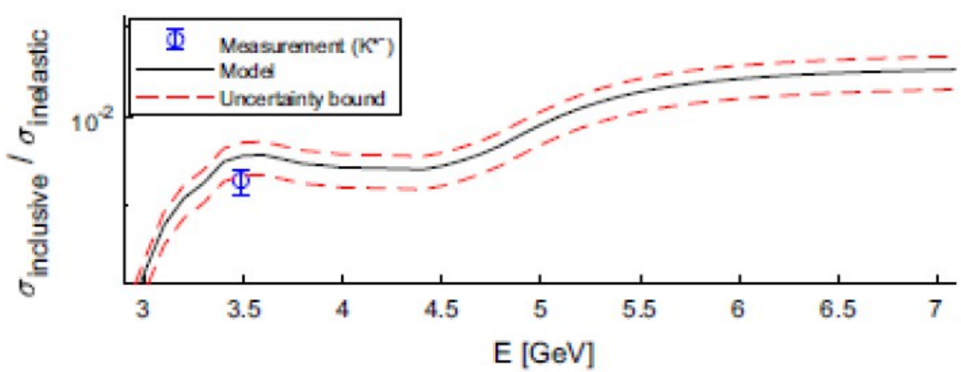
Inclusive Kaon production



$$p\pi^- \rightarrow K^0 X$$

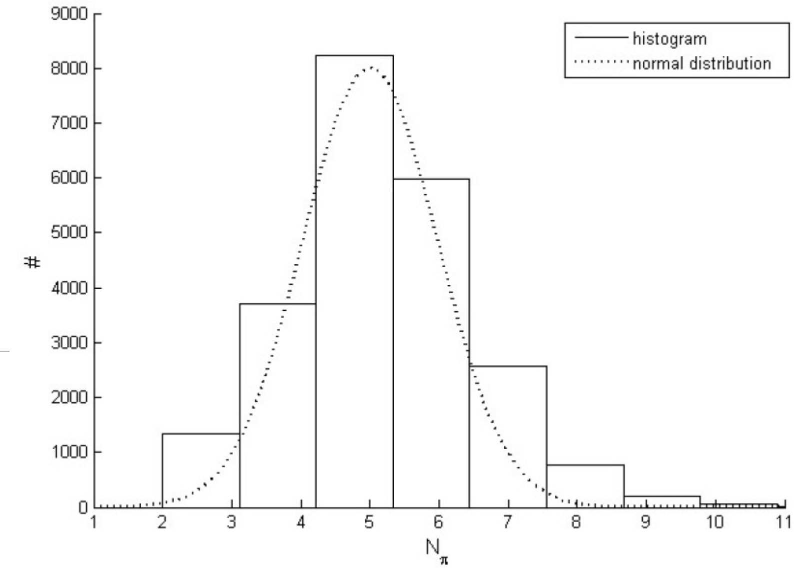


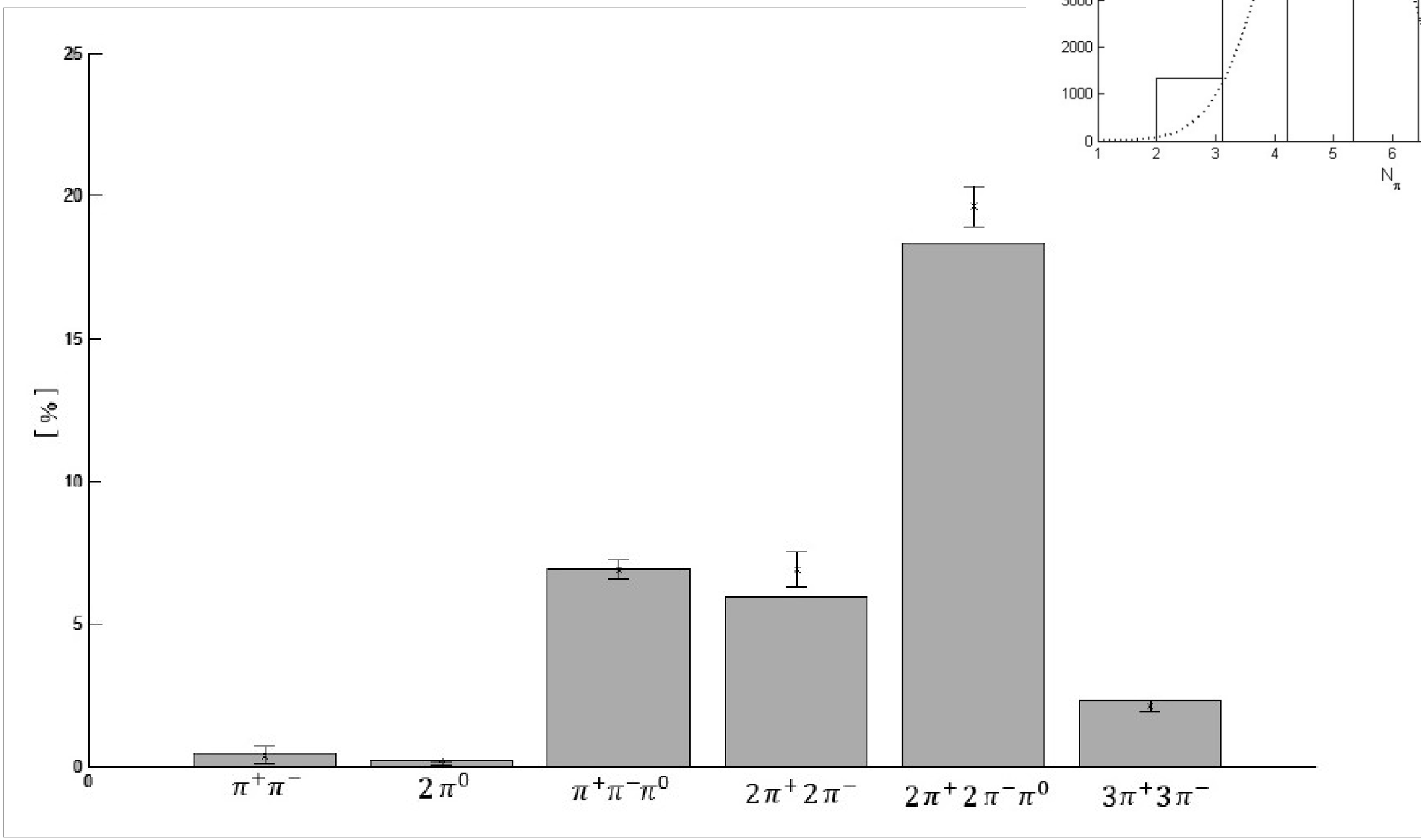
$$p\pi^- \rightarrow K^*(892)^+ X$$



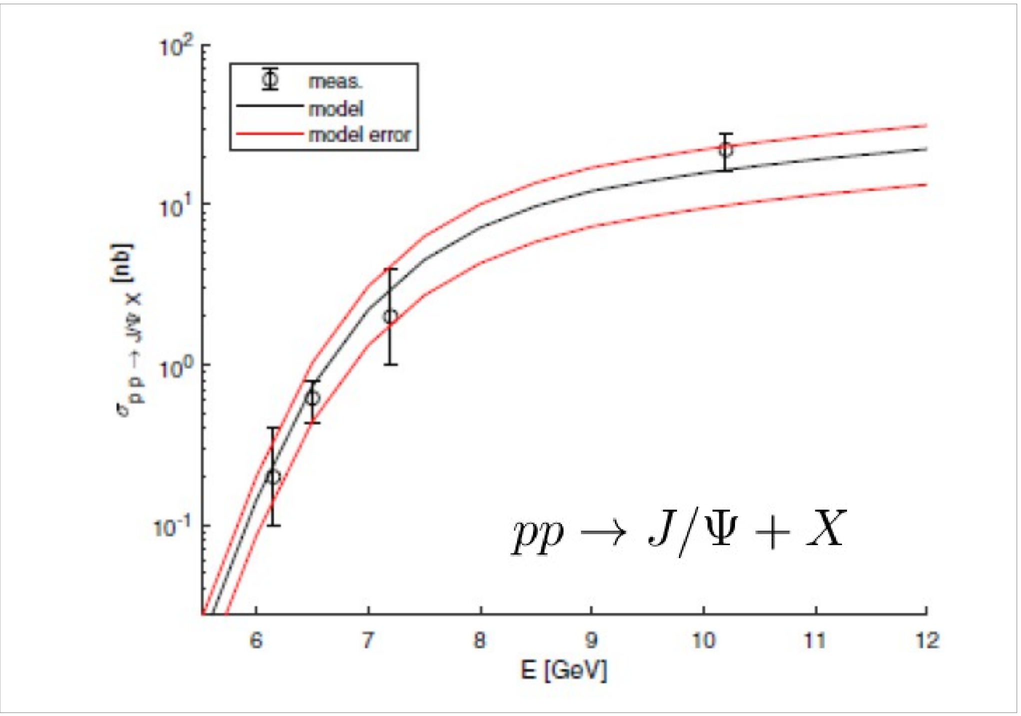
$$p\pi^- \rightarrow K^*(892)^- X$$

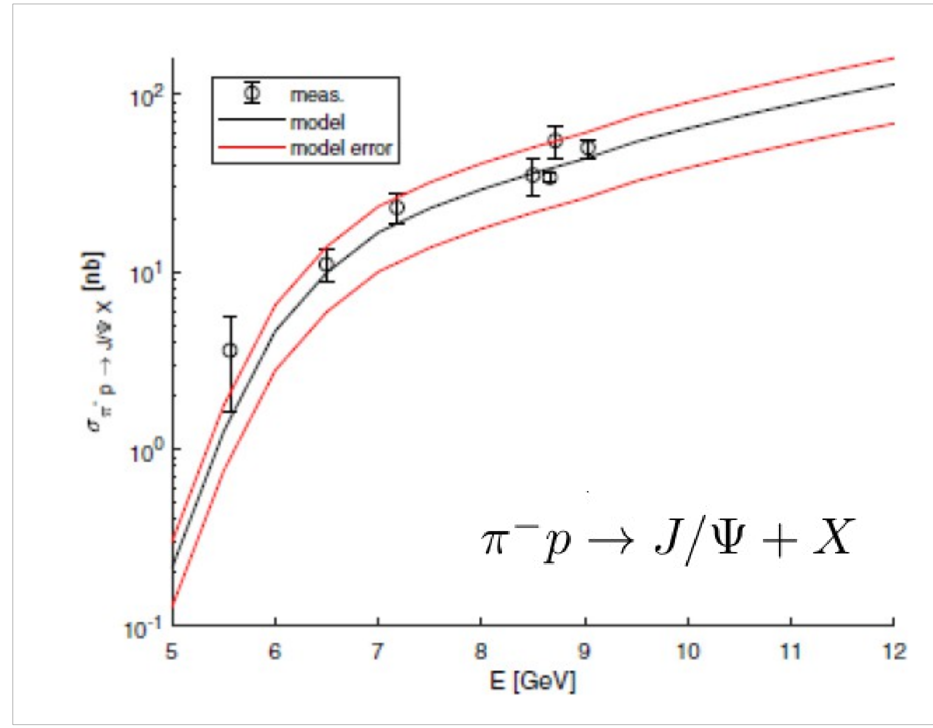
Proton-antiproton annihilation at rest

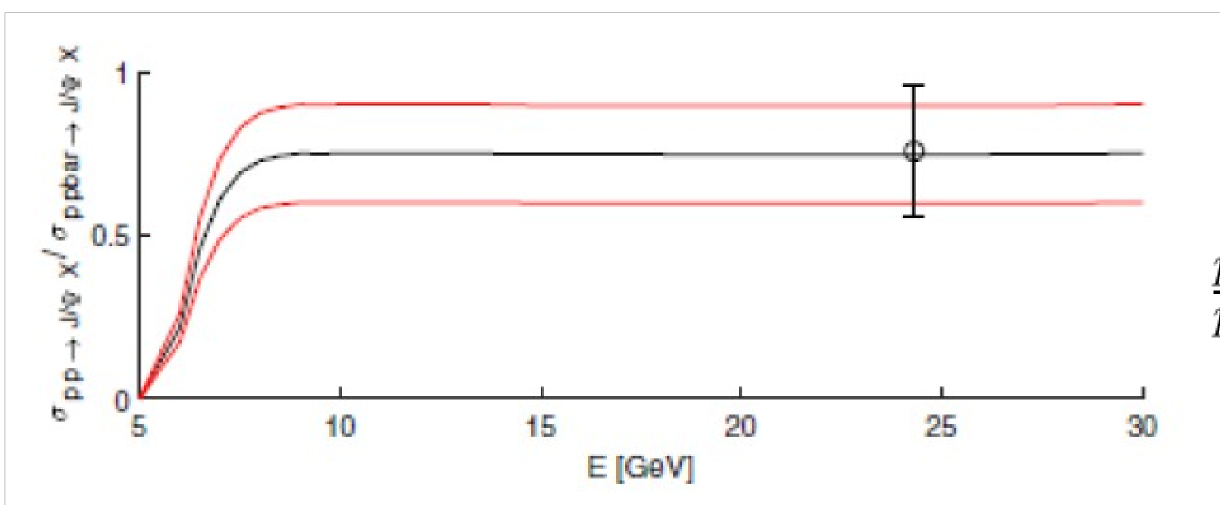




Inclusive charmonium production cross-sections in *proton-proton, pion-proton,* and *proton-antiproton* collisions

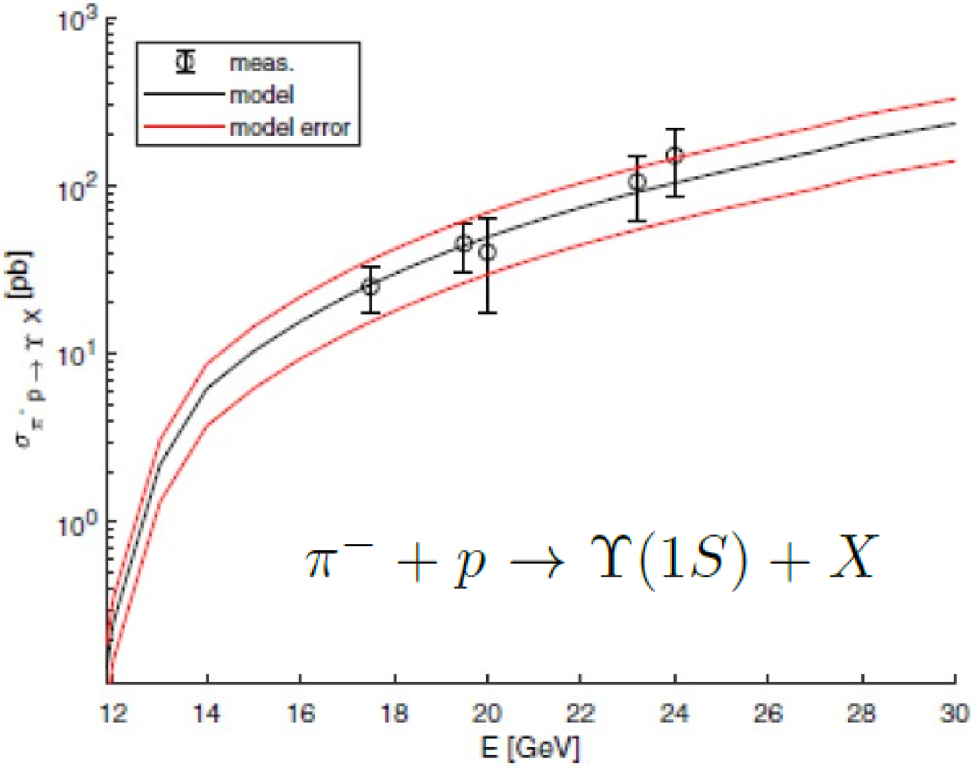


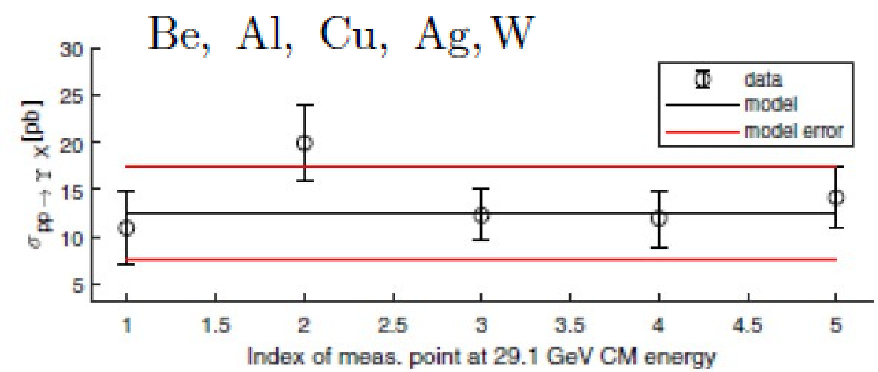


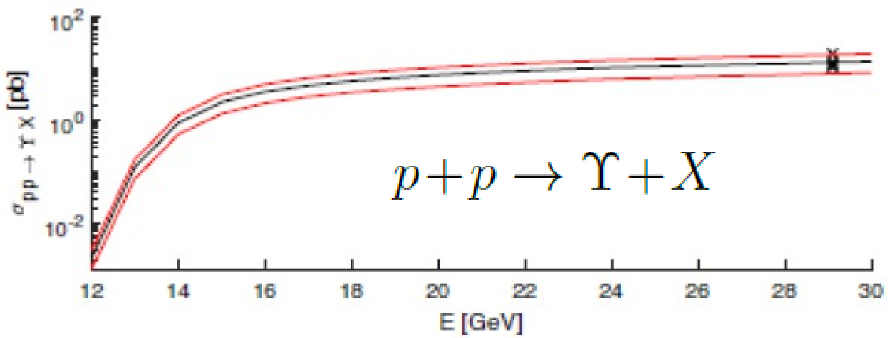


$$\frac{pp \to J/\Psi + X}{p\overline{p} \to J/\Psi + X}$$

Bottomonium production







$$\Upsilon = \Upsilon(1S) + \Upsilon(2S) + \Upsilon(3S)$$

Introduction to tetraquarks

- Exotic states: glueballs, pentaquarks, tetraquarks etc...
- Tetraquarks are bound states of 2 quarks and 2 antiquarks
  - Example: *diquark*+*antidiquark* color algebra

$$3_c \otimes 3_c = \overline{3}_c \oplus 6_c \qquad 3_c \otimes \overline{3}_c = 1_c \oplus 8_c$$

$$\overline{3}_c \otimes \overline{3}_c = 3_c \oplus \overline{6}_c \qquad 6_c \otimes \overline{6}_c = 1_c \oplus 8_c \oplus 27_c$$

- Long time predicted by QCD
  - Properties e.g. masses, decay widths described with:
    - *Bag model* calculations
    - *NRQCD* and *DPS* for heavy light tetraquarks
    - Potential models, Schrödinger equation
    - Functional methods (Dyson-Scwinger, Bethe-Salpeter,...)

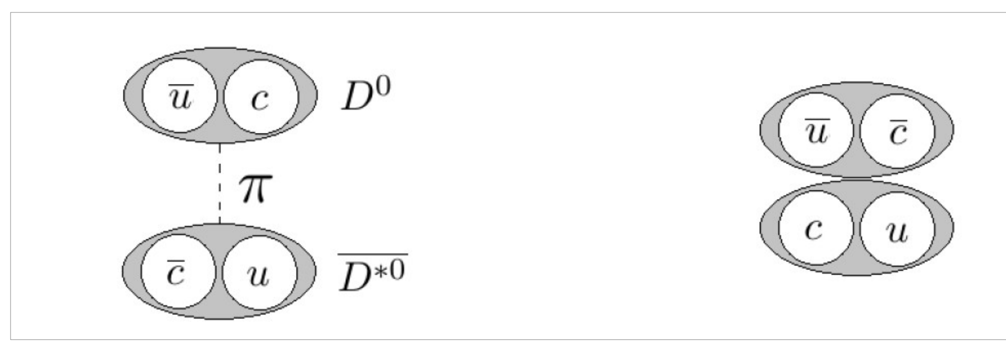
The X(3872) „possible" tetraquark state

- Measured in 2003 by the Belle collaboration from the process:

$$B^+ \to X(3872)K \to (J/\Psi\pi^+\pi^-)K^+$$

Γ ~ 1 MeV, $J^{PC}=1^{++}$, quark configuration : $[c\bar{c}u\bar{u}]$

- Its quantum numbers and the obtained mass difference from potential models suggest it is an exotic state → but which one?
  - Compact 4 quark
  - Diquark-antidiquark
  - molecule state $D^0\overline{D^{*0}}$
  - charmonium hibrid $c\bar{c}g$
  - ...



X(3872) production in the statistical approach

- Diquark-antidiquark approximation.
- It could be a triplet-antitriplet or a sextet-antisextet state → (p_3, p_6=1-p_3) is a free parameter at the validation step, but p_3=1 in the calculations.
- To get the correct quantum numbers the spin configuration should be (1,0) or (0,1).
- Assumption to the diquark formation probability: $P_{ij} = P_i P_j$
- The quark number distribution will be:

$$F(x, n_i, n_{ij}) = \frac{N(x)!}{\prod_i n_i! \prod_{ij} n_{ij}!} \prod_i P_i^{n_i} \prod_{ij} P_{ij}^{n_{ij}}$$

- Measurement in pp collisions at $\sqrt{s} = 7$ TeV using the decays $X(3872) \to J/\Psi\pi^+\pi^-$ and $\Psi(2S) \to J/\Psi\pi^+\pi^-$ in the kinematical region $p_T \in [10, 50]$ GeV, $|y| < 1.2$.

$$\frac{\sigma_{X(3872)} \cdot \mathrm{Br}(J/\Psi\pi^+\pi^-)}{\sigma_{\Psi(2S)} \cdot \mathrm{Br}(J/\Psi\pi^+\pi^-)} = 0.0656 \pm 0.0094$$

- The branching fraction for the X(3872) is not well measured...only an upper and lower bound is available:

$$\mathrm{Br}(X(3872) \to J/\Psi\pi^+\pi^-) = [0.042, 0.093]$$

- The measured cross section ratio is: $\frac{\sigma_{\Psi(2S)}}{\sigma_{X(3872)}} \approx [1.88, 4.16]$

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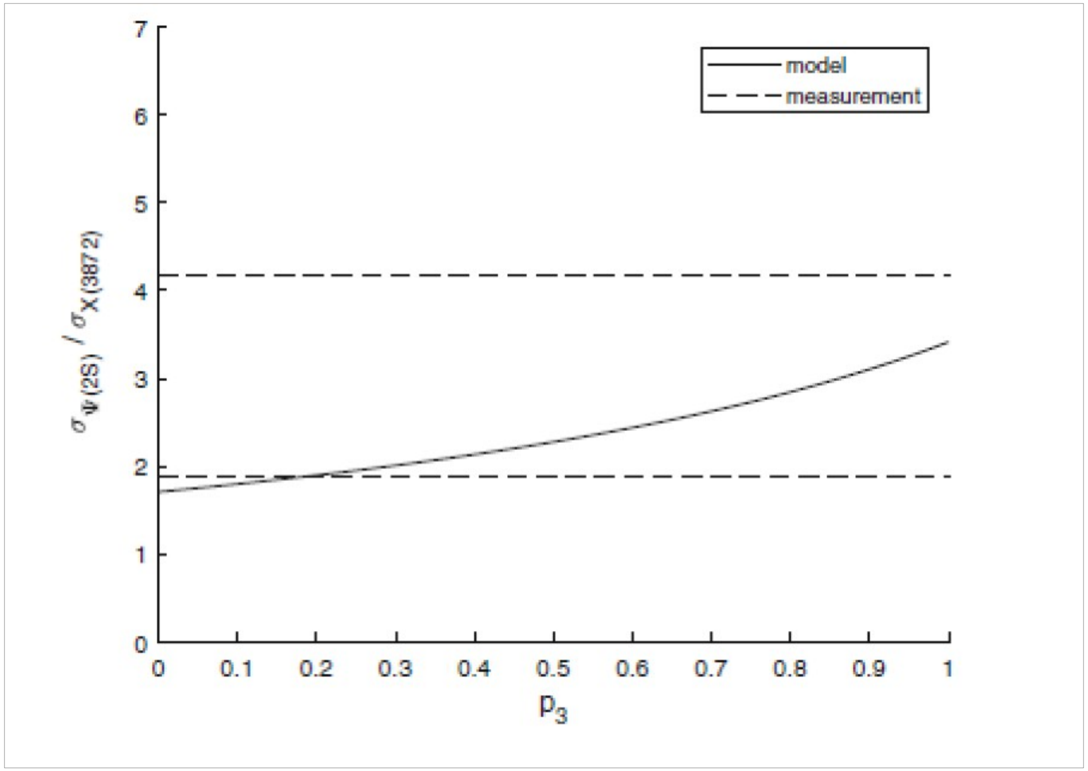
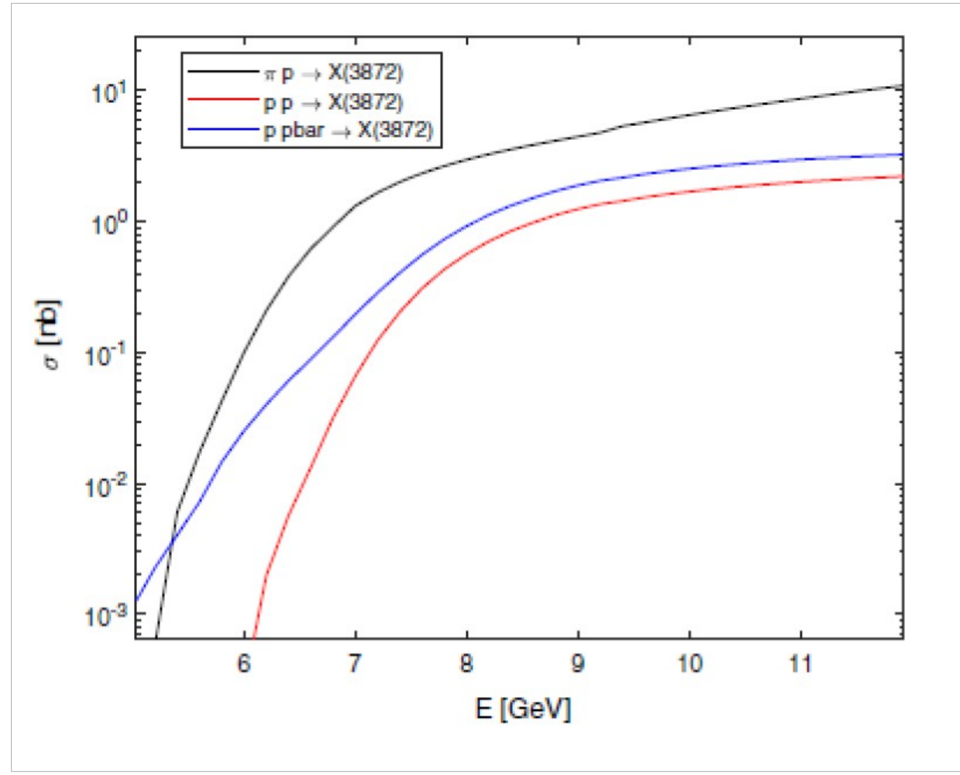
Validation at $\sqrt{s} = 7\ \mathrm{TeV}$ Estimation at low energies





- Results:The results are satisfactory for almost every p_3 value, but it seems that the best results are achieved if the tetraquark is mostly in the triplet-antitriplet configuration

Summary

- Elementary hadronic cross-sections are important input parameters in heavy-ion transport.
- Need for production cross-sections → a new statistical model is used.
- The statistical model is able to describe many exclusive and inclusive hadronic reactions. It is also possible to reproduce the high energy X(3872) cross section data.
- The Monte-Carlo version can be used as a low-energy event generator.
- Future plans:
  - Tetraquark production in heavy-ion collisions with BUU transport → structure studies
  - N → M collisions in heavy-ion transport