Dispersive analysis of the $\gamma\gamma \to D\bar{D}$ data

Oleksandra Deineka

In collaboration with Igor Danilkin and Marc Vanderhaeghen

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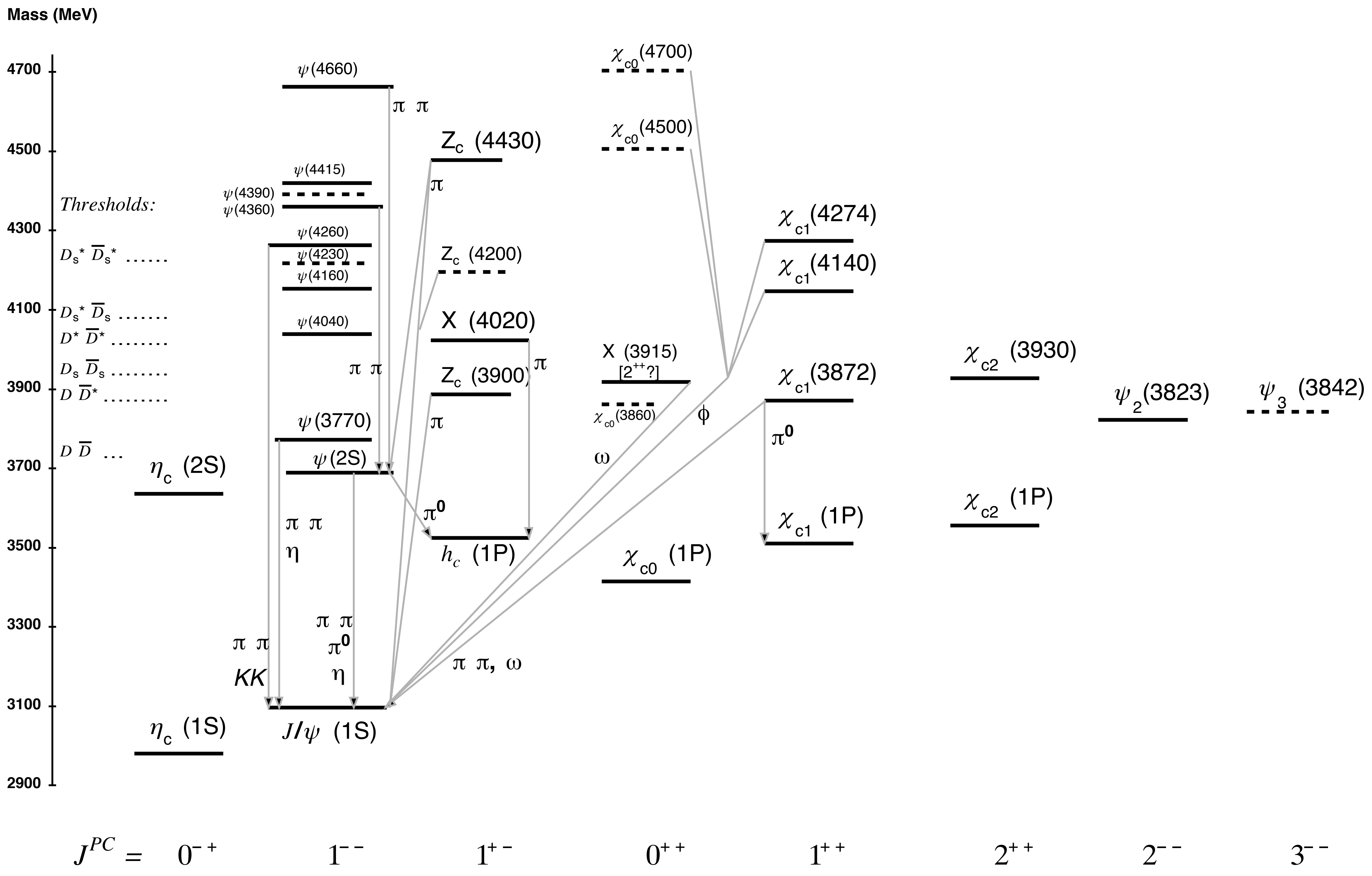
What are we looking for?



[PDG 2019]

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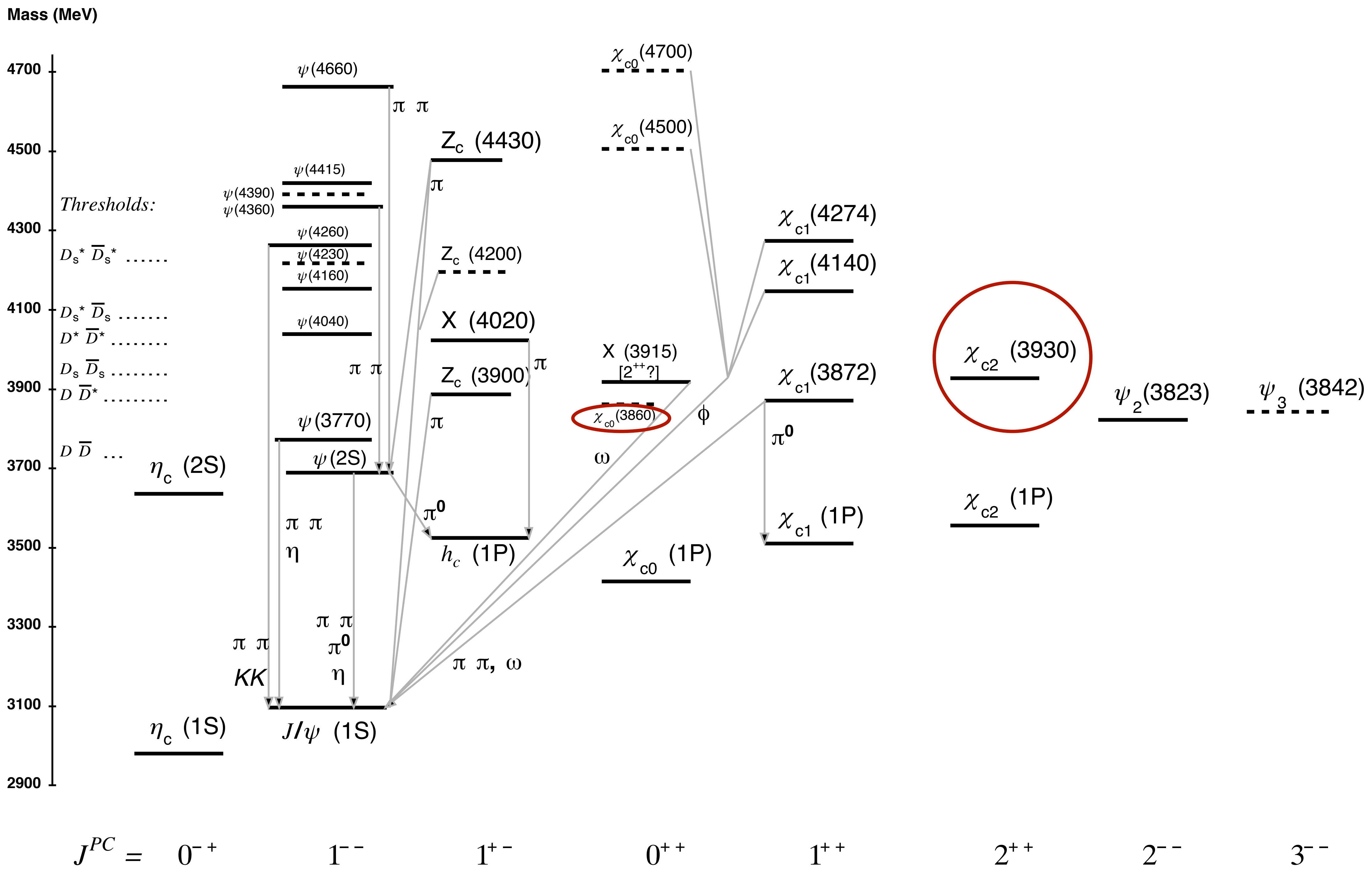

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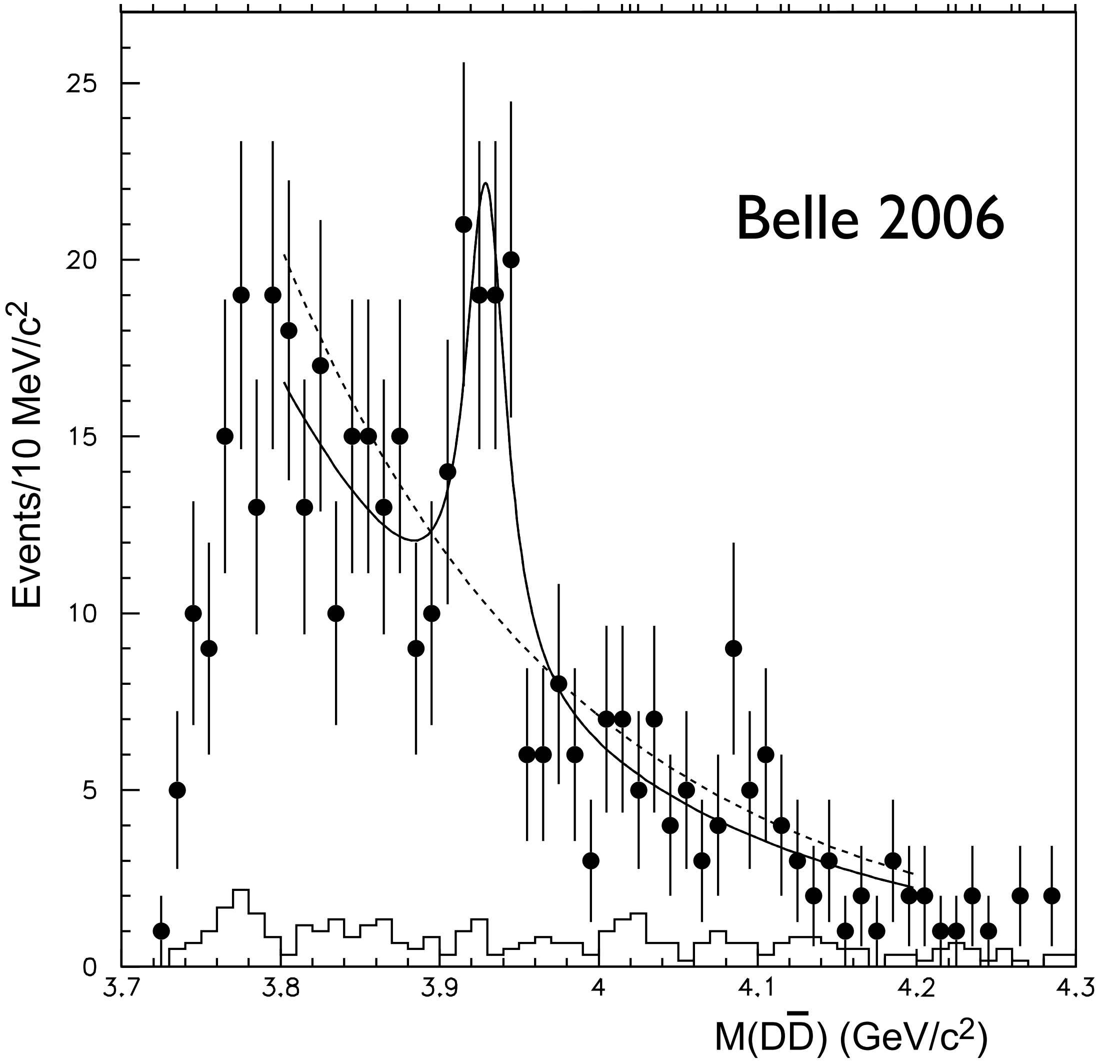
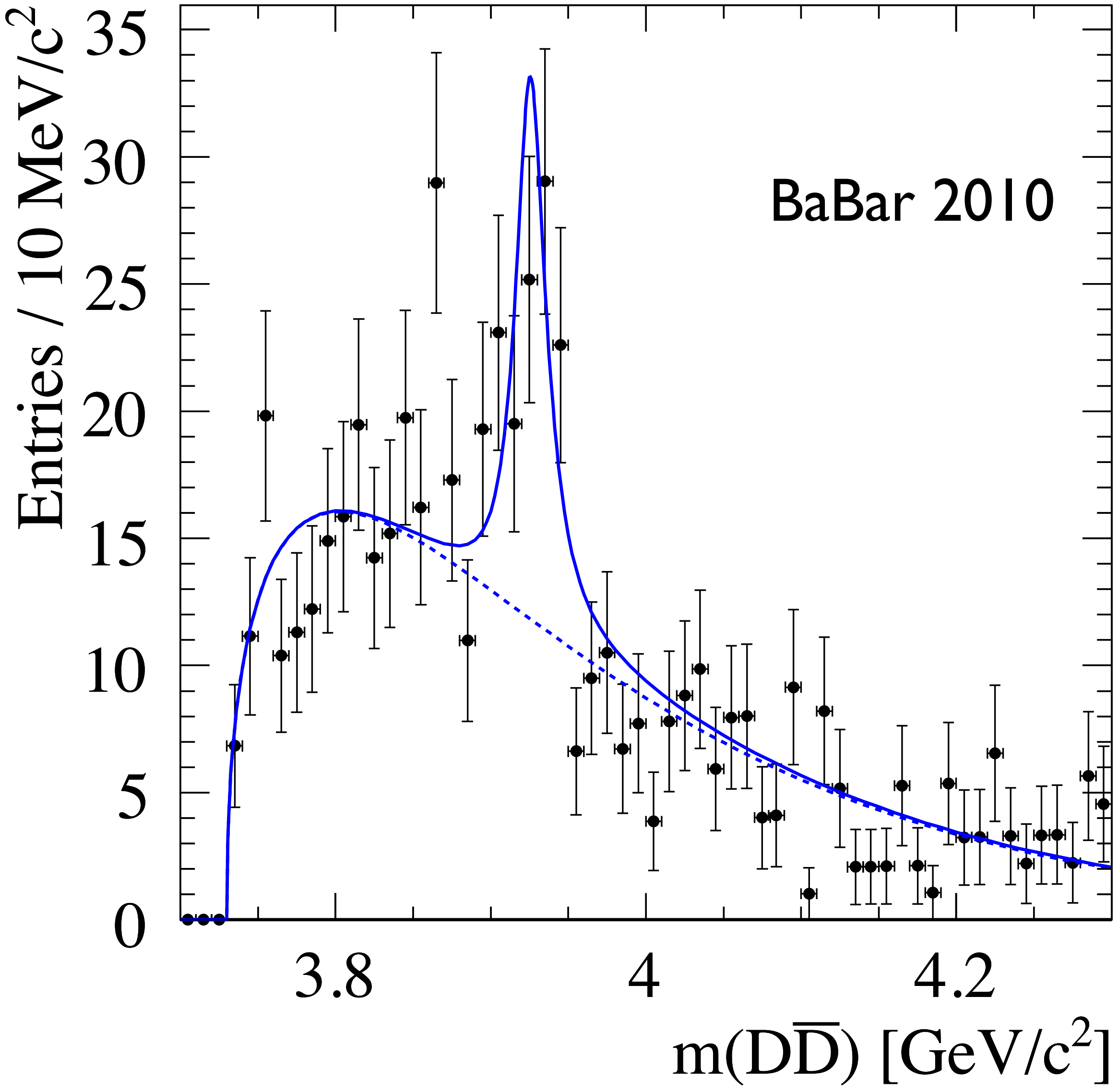
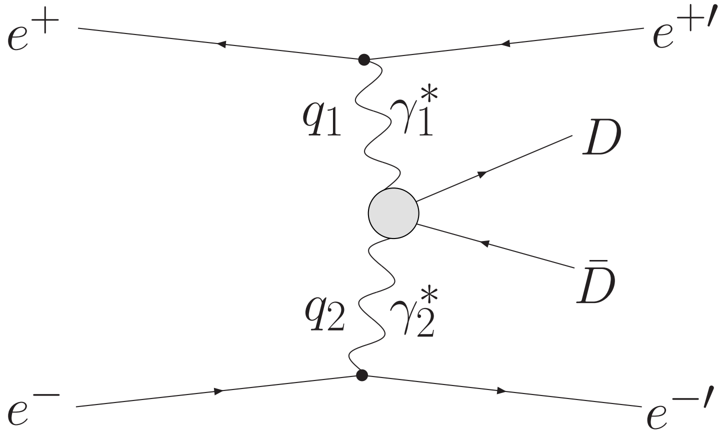
Everything is fine with $\chi_{c2}(2P)$

$\gamma\gamma \to D\bar{D}$







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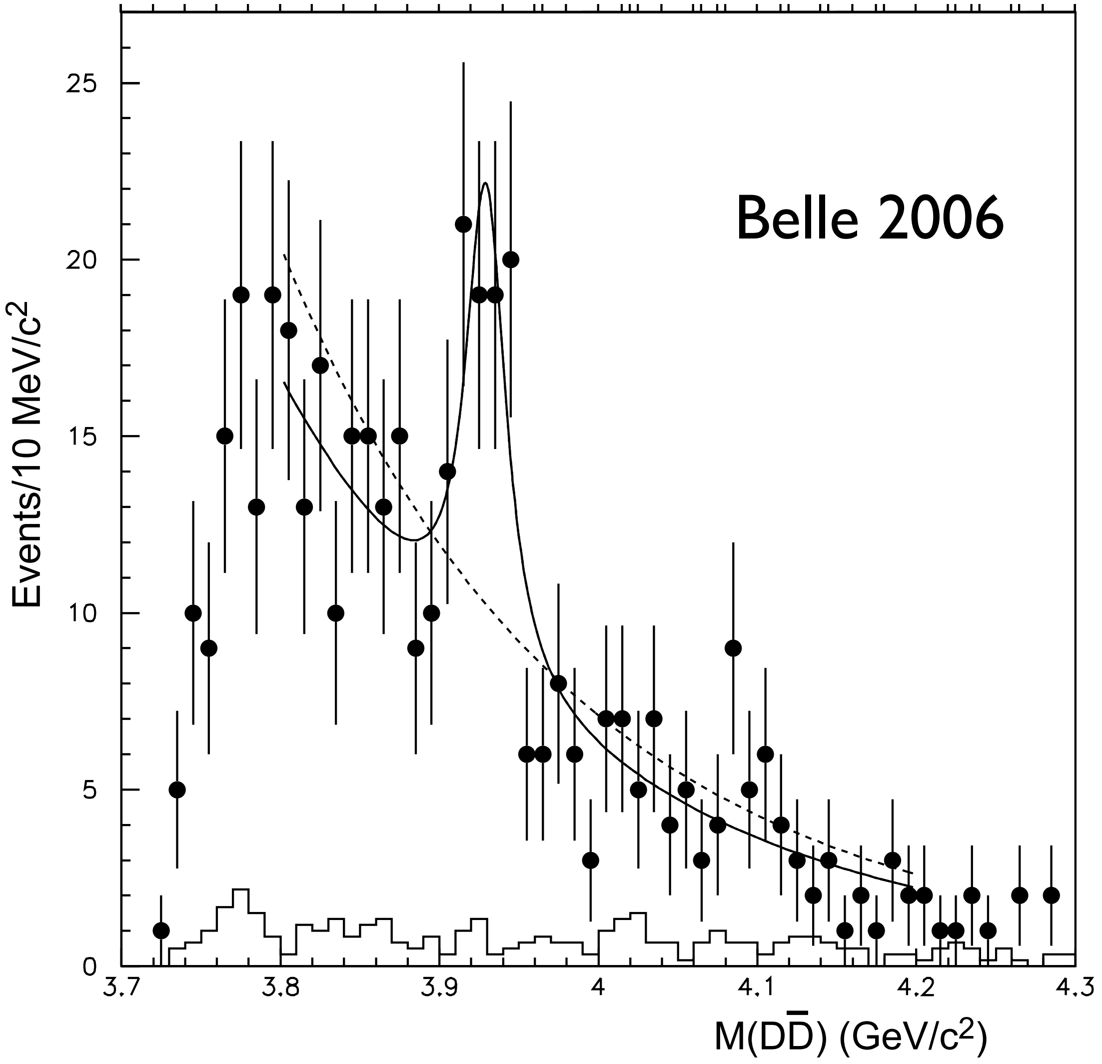
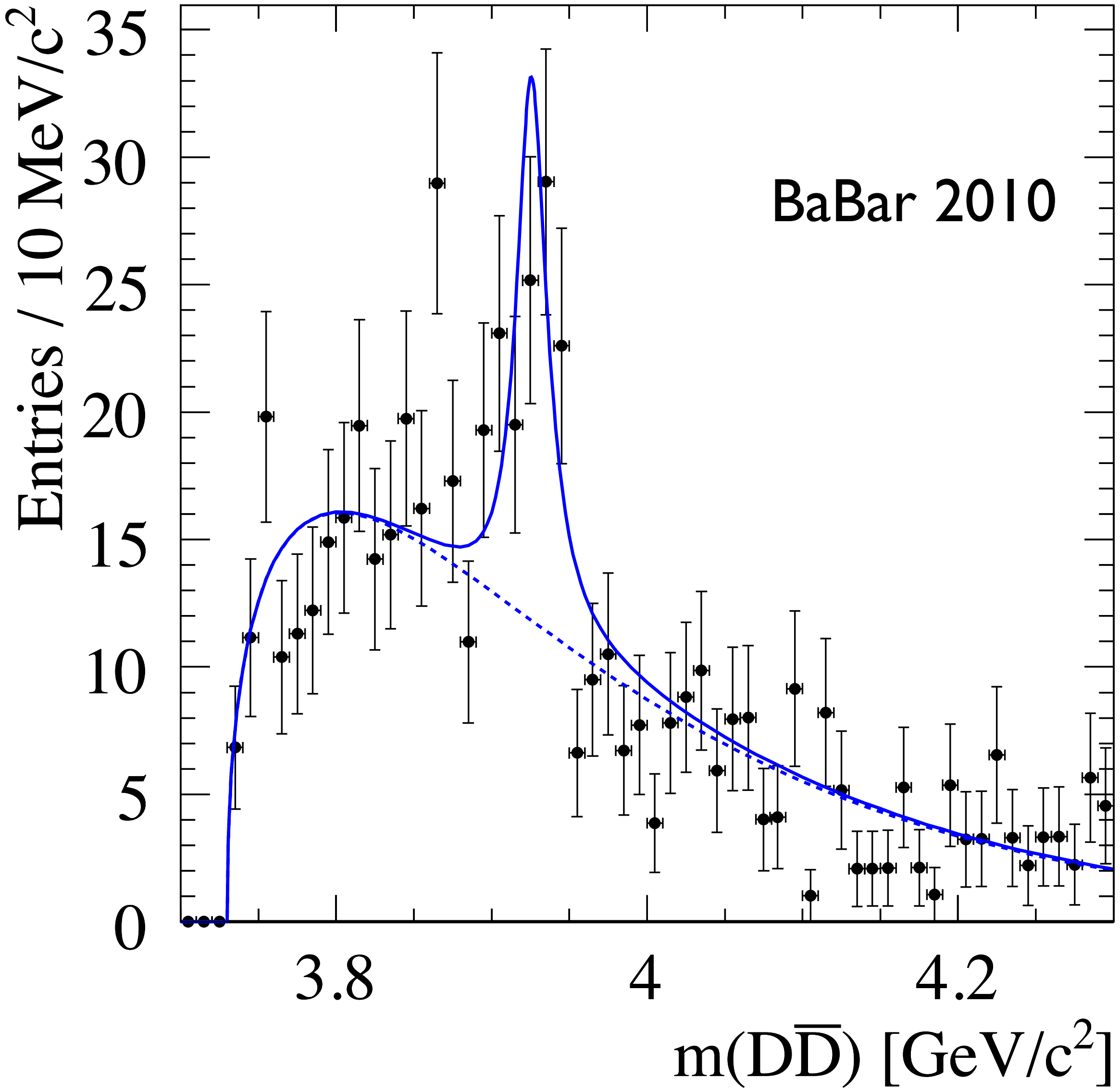
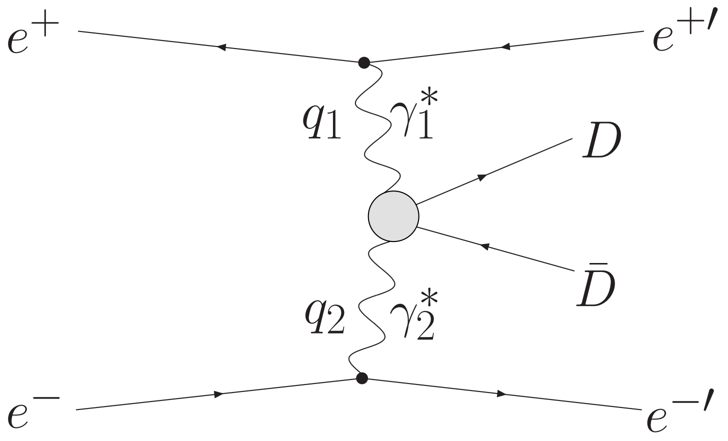
$$\gamma\gamma \to D\bar{D}$$







$X(3930)$

=

$\chi_{c2}(3930)$

$J^{PC} = 2^{++}$

$M_{\chi_{c2}(3930)} = 3922.2 \pm 1.0$ MeV

$\Gamma_{\chi_{c2}(2P)} = 35.3 \pm 2.8$ MeV

+ LHCb

Is $X(3915)$ a $\chi_{c0}(2P)$?

Belle 2005 $B \to J/\psi\omega K : X(3915)$:
later confirmed by BaBar 2008, 2010

Belle 2010 $\gamma\gamma \to X(3915) \to J/\psi\omega$
BaBar 2012 spin-parity analysis: $J^{PC} = 0^{++}$

$M_{X(3915)} = 3921.7 \pm 1.8$ MeV $\Gamma_{X(3915)} = 18.8 \pm 3.5$ MeV + LHCb

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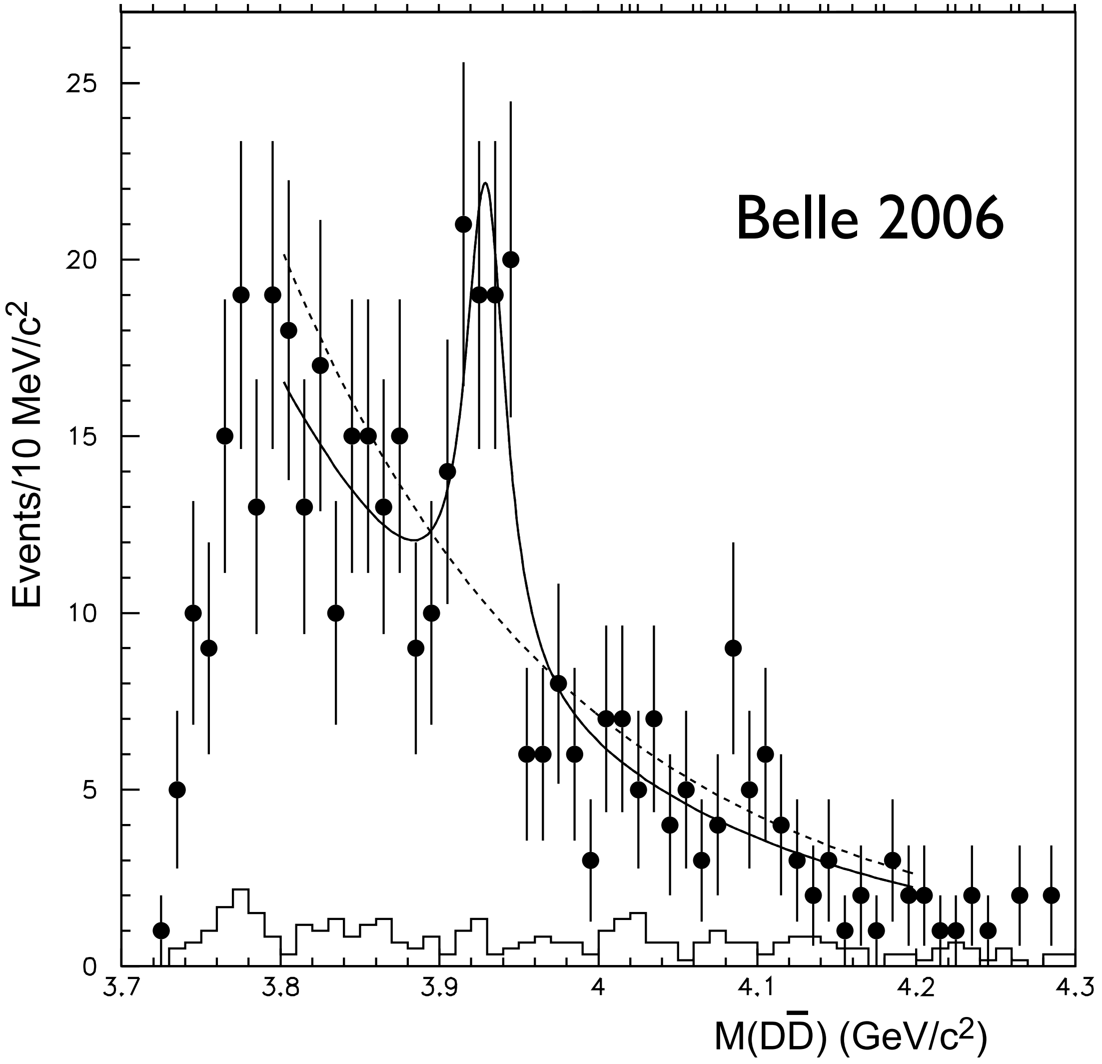
$X(3915)$
?
$\chi_{c0}(3915)$

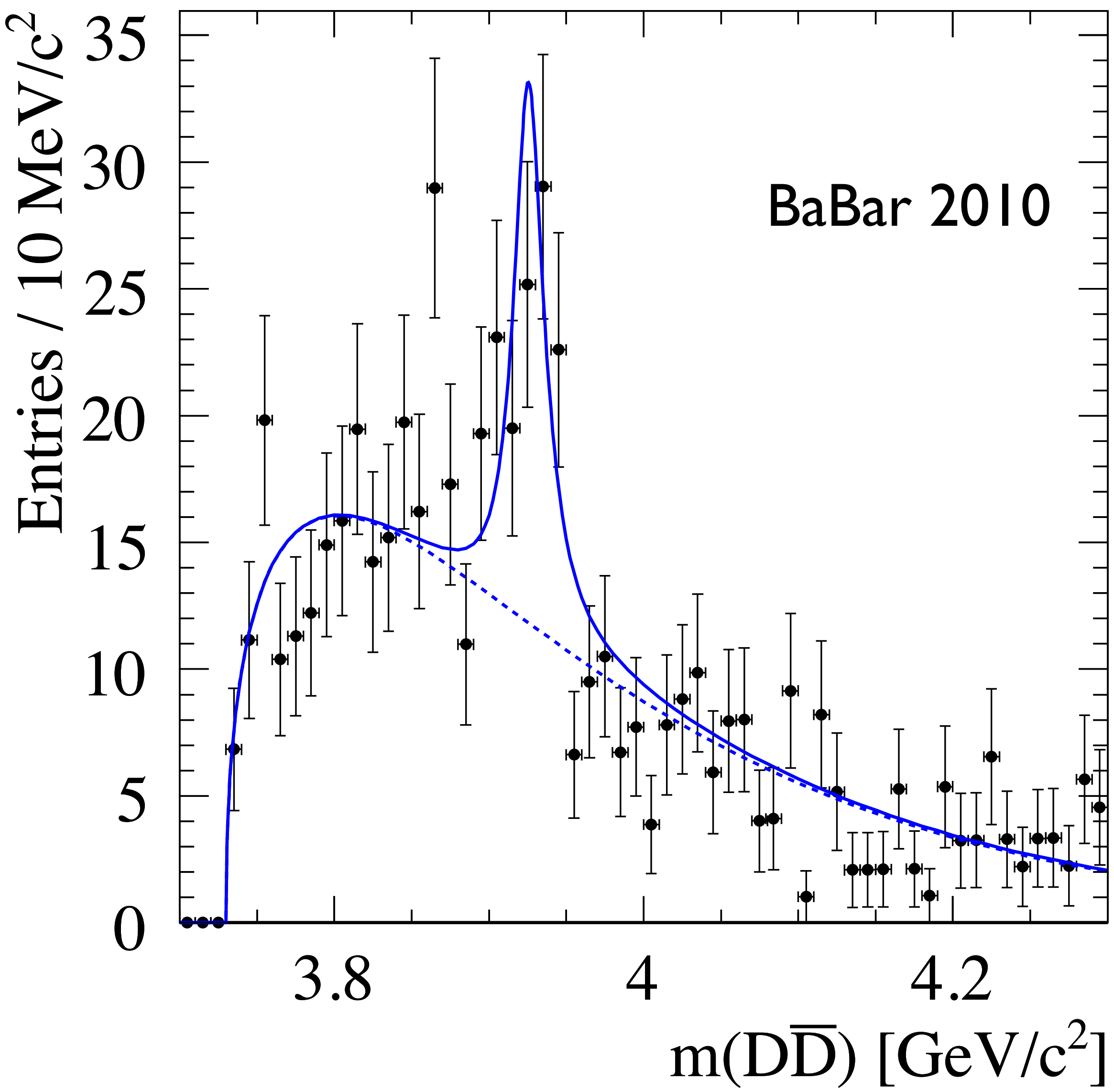
But there are some **problems**:

- No decay mode in S-wave
- The observable decay should be OZI suppressed for $\chi_{c0}(2P)$
- Narrow, width of ~20 MeV
- Small mass splitting with $\chi_{c2}(3930)$
- Might actually be the same state as $\chi_{c2}(3930)$

The dangerous Breit-Wigner

$\gamma\gamma \to D\bar{D}$





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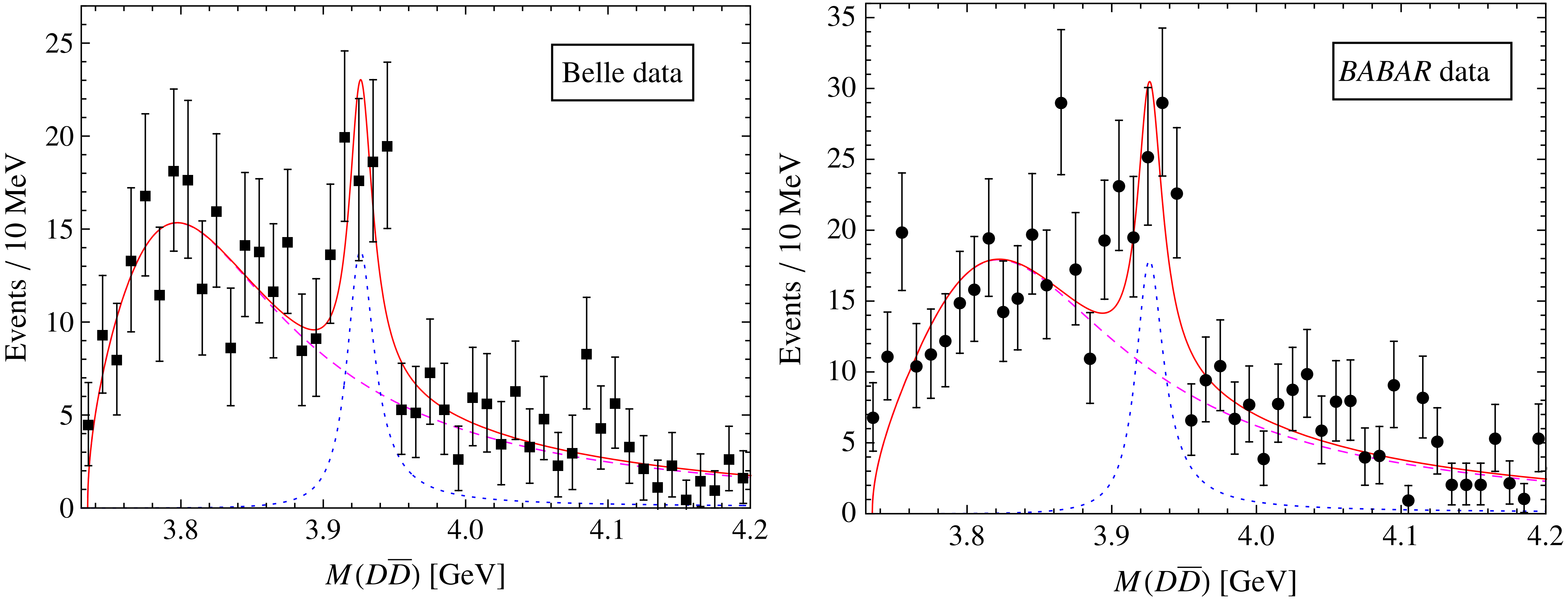

[Guo 2012]

$$\mathsf{BW}(s) = \left(\frac{p(s)}{p(m_R^2)}\right)^{2L+1} \frac{m_R}{\sqrt{s}} \frac{B_L^2(s)}{(s-m_R^2)^2 + m_R^2\Gamma_{\mathsf{BL}}^2(s)}\,, \quad \Gamma_{\mathsf{BL}}(s) = \Gamma_R \left(\frac{p(s)}{p(m_R^2)}\right)^{2L+1} \frac{m_R}{\sqrt{s}} B_L^2(s)\,,$$

$$B_0(s) = 1\,, \quad B_2(s) = \frac{F_2(p(s)r)}{F_2(p(m_R^2)r)}, \quad F_2(x) = \frac{1}{9+3x^2+x^4}\,,$$

The dangerous Breit-Wigner

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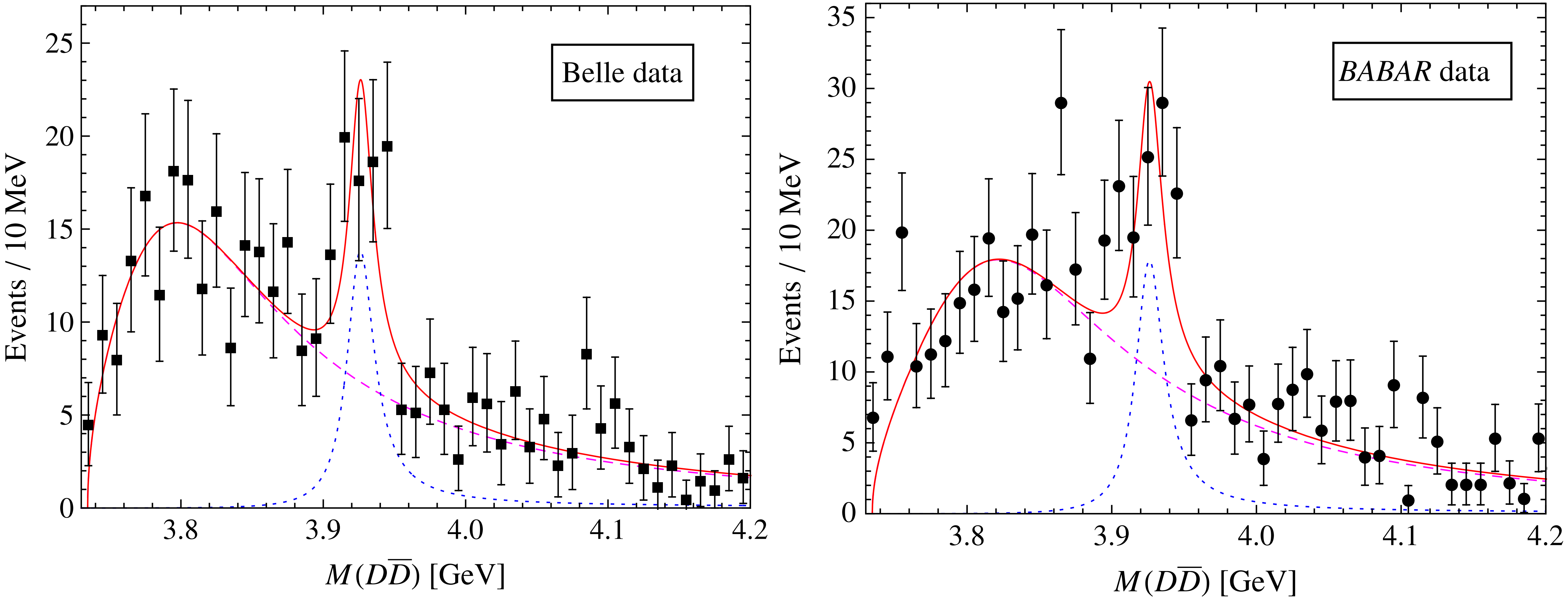
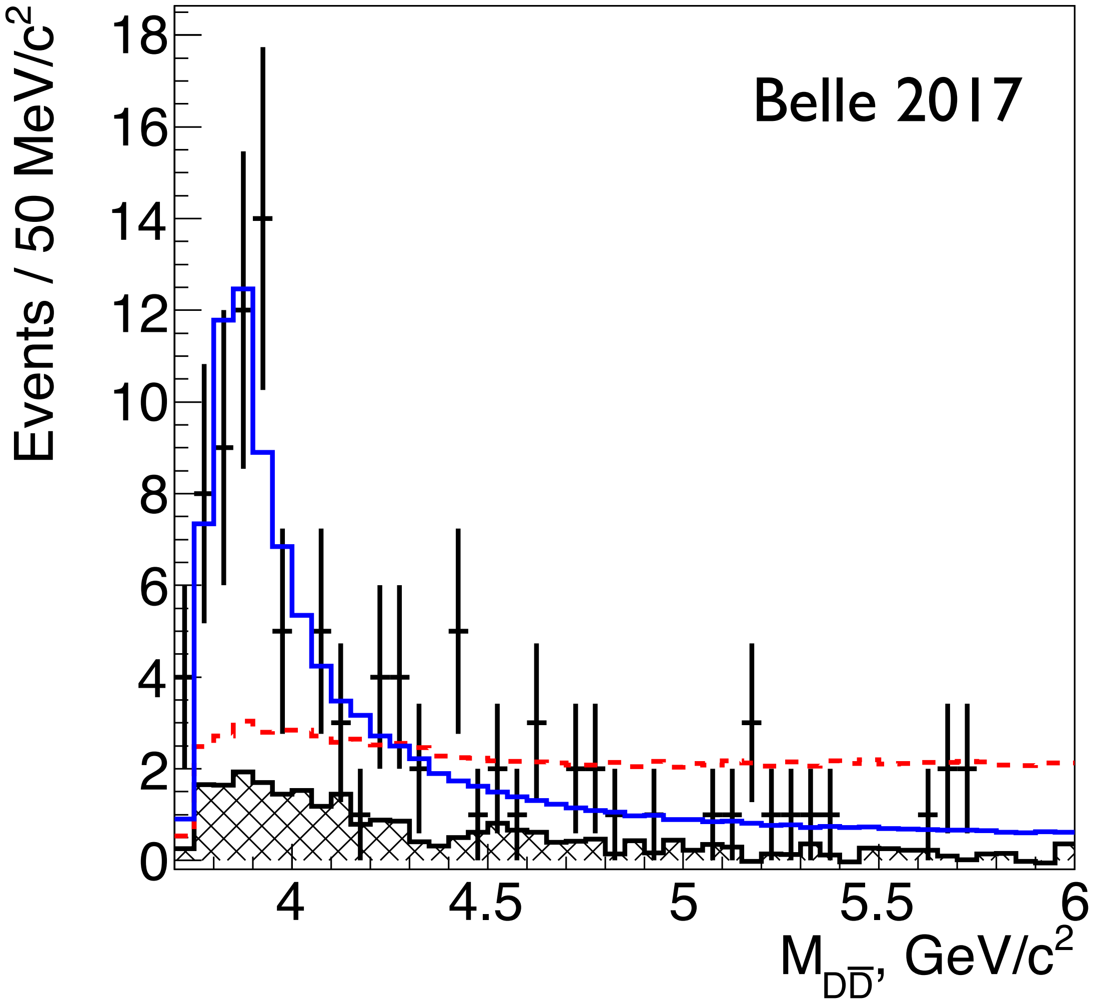


[Guo 2012]

2 Breit-Wigner functions, mass and width of fixed to experimental value

$$M_{\chi_{c0}(2P)} = 3837.6 \pm 11.5\text{MeV}\,, \quad \Gamma_{\chi_{c0}(2P)} = 221 \pm 19\text{MeV}$$

Is $X(3860)$ a $\chi_{c0}(2P)$?



$e^+e^- \to J/\psi D\bar{D}$

$M_{X(3860)} = 3862^{+26+40}_{-32-13}$ MeV

$\Gamma_{X(3860)} = 201^{+154+88}_{-67-82}$ MeV

PDG (2021)

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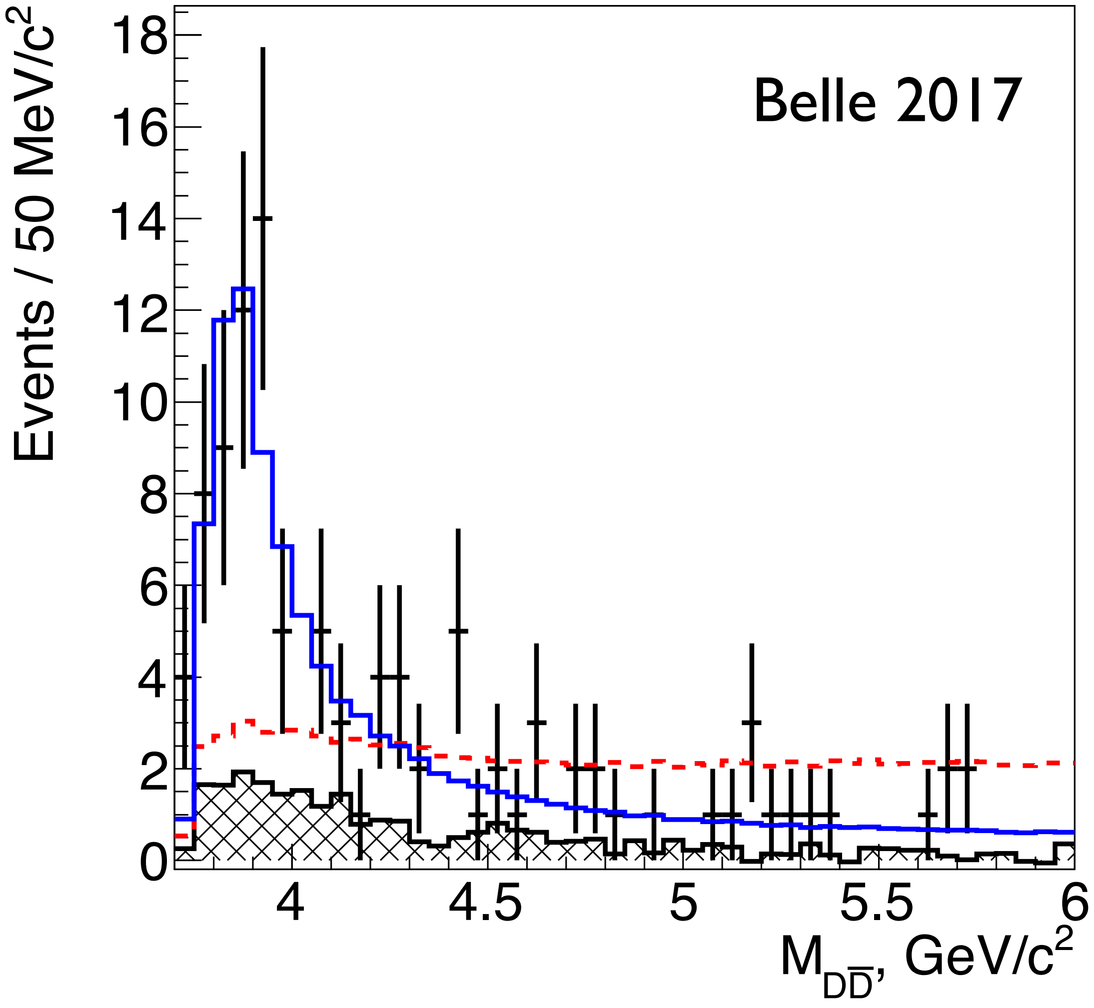

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PDG (2021)

$X(3860)$

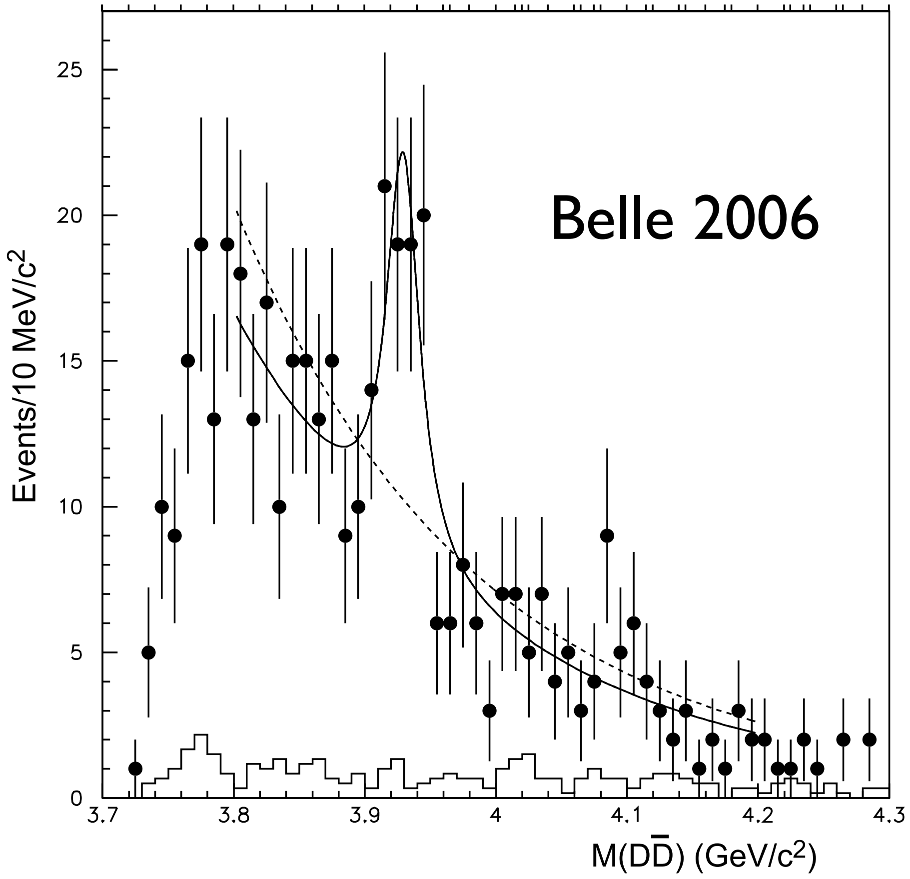
?

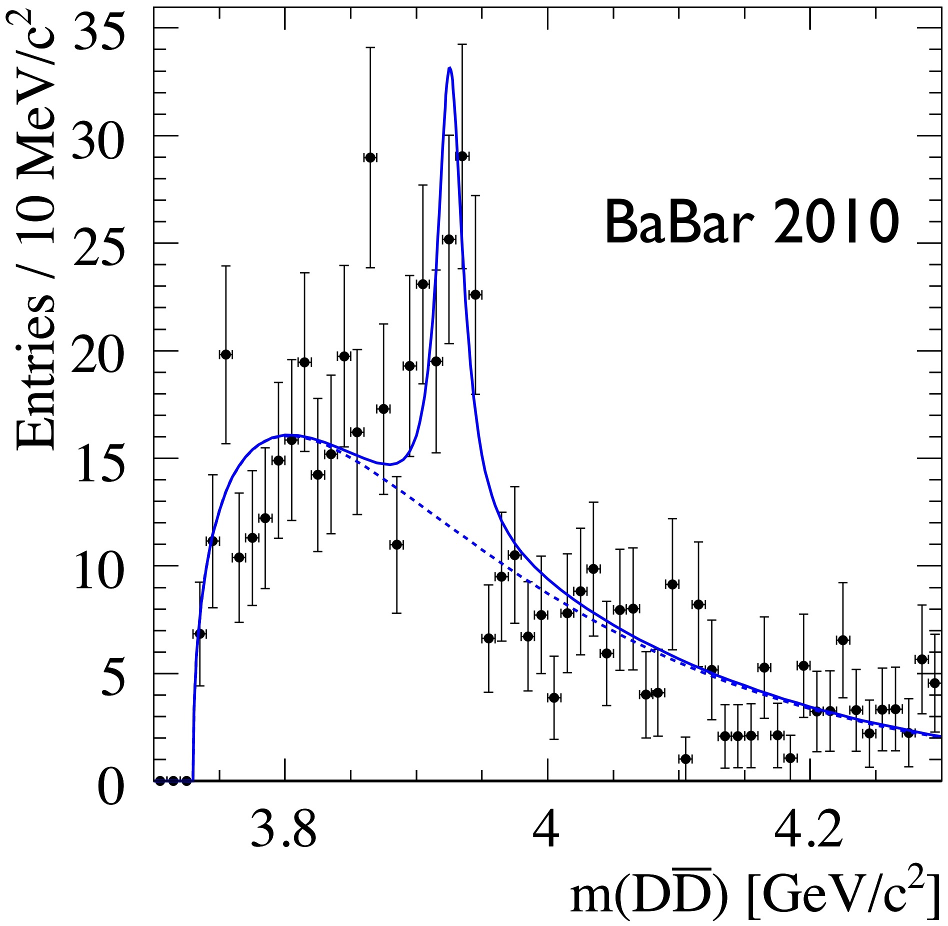
$\chi_{c0}(3860)$

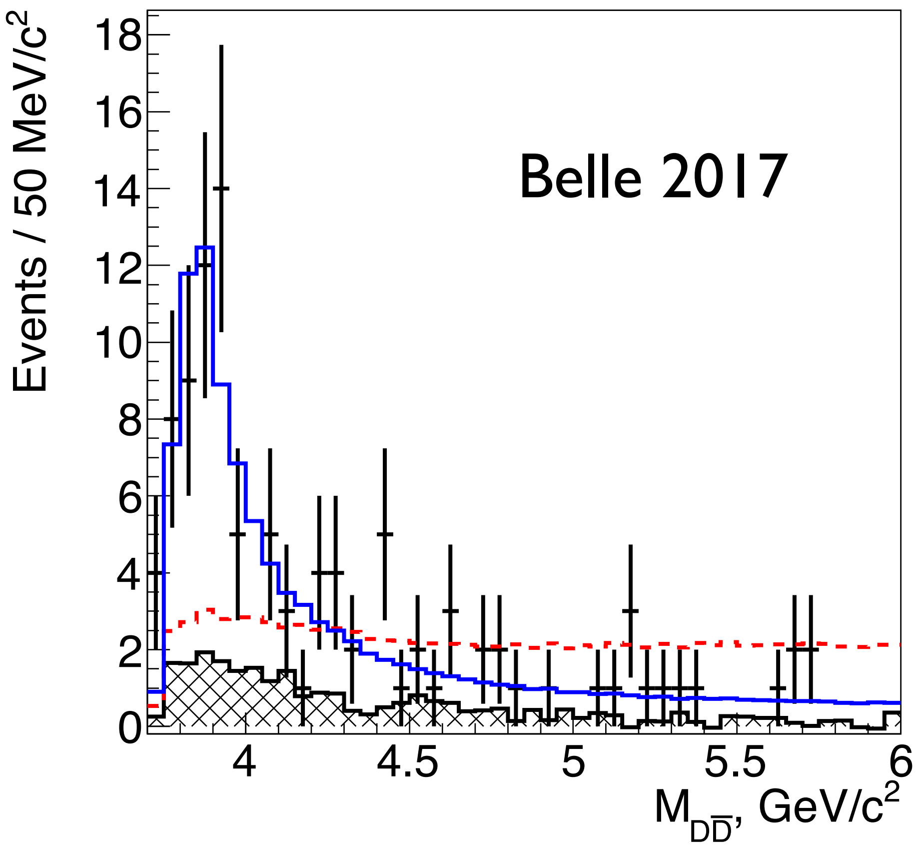
But there are some **problems**:

- The $e^+e^- \to J/\psi D\bar{D}$ statistics is rather limited;
- BW parametrisation does not respect S-matrix constraints;
- Unitary analysis $e^+e^- \to J/\psi D\bar{D}$ & $\gamma\gamma \to D\bar{D}$ [Wang 2020]: **no** $X(3860)$, **bound state**
- LHCb pp collisions: **no** $X(3860)$; $\chi_{c0}(3930)$ and $\chi_{c2}(3930)$ at the same position

What data do we have?





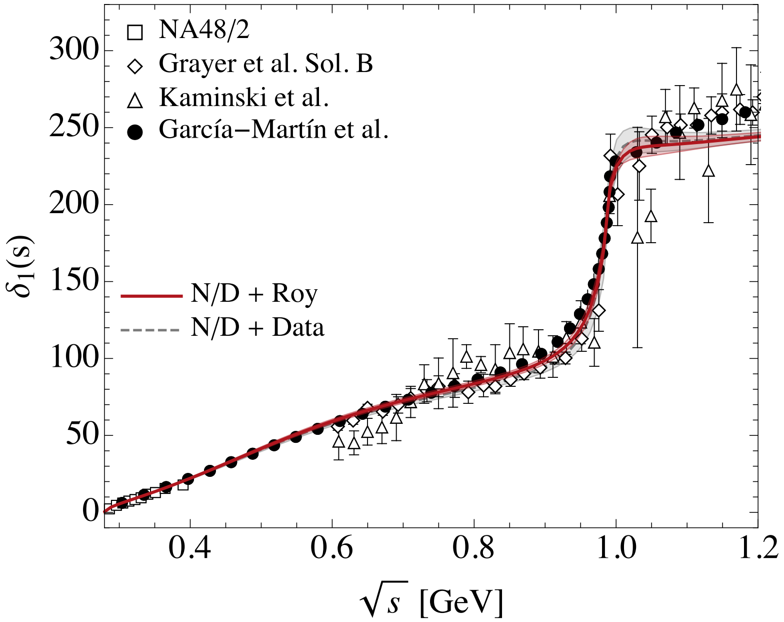


In order to figure out what is going on in the s-wave we need an approach, which respects **unitarity** & **analyticity** properties of the S-matrix

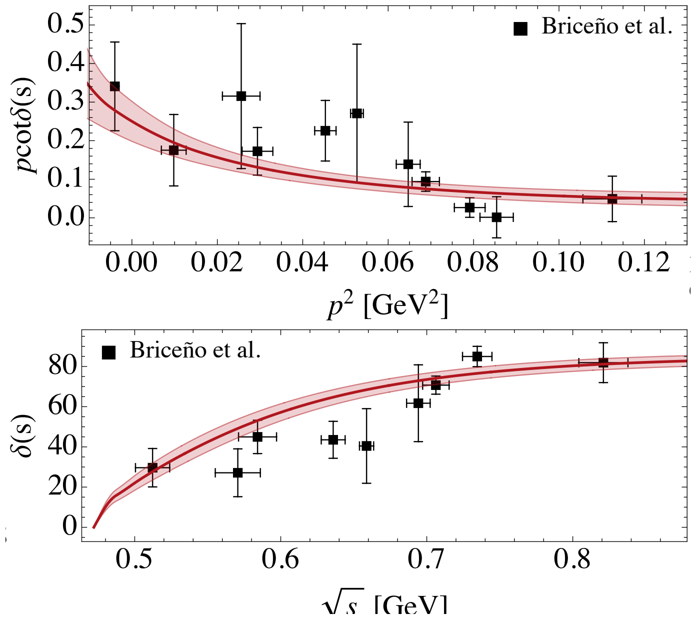
Why are we sure we can analyse it?

Partial wave dispersion relation approach:

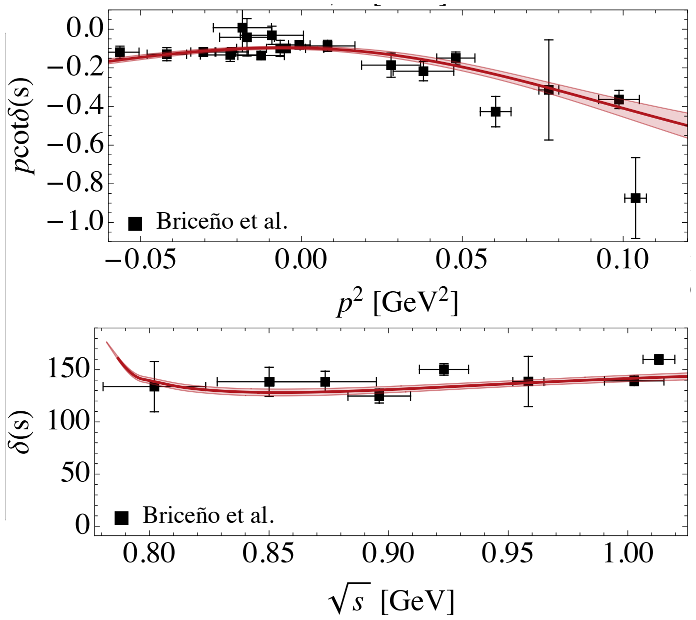
- $\sigma/f_0(500)$ in $\pi\pi$ scattering (real & lattice data); $f_0(980)$ in $\{\pi\pi, K\bar{K}\}$



$$\sqrt{s_\sigma} = 458(10)^{+7}_{-15} - i\,256(9)^{+5}_{-8} \text{ MeV}$$
$$\sqrt{s_{f_0(980)}} = 993(2)^{+2}_{-1} - i\,21(3)^{+2}_{-4} \text{ MeV}$$



$$\sqrt{s_\sigma} = 498(21)^{+12}_{-19} - i\,138(13)^{+5}_{-10} \text{ MeV}$$



$$\sqrt{s_\sigma} = 758(5)(0) \text{ MeV}$$

bound state

[D., Danilkin, Vanderhaeghen 2020]

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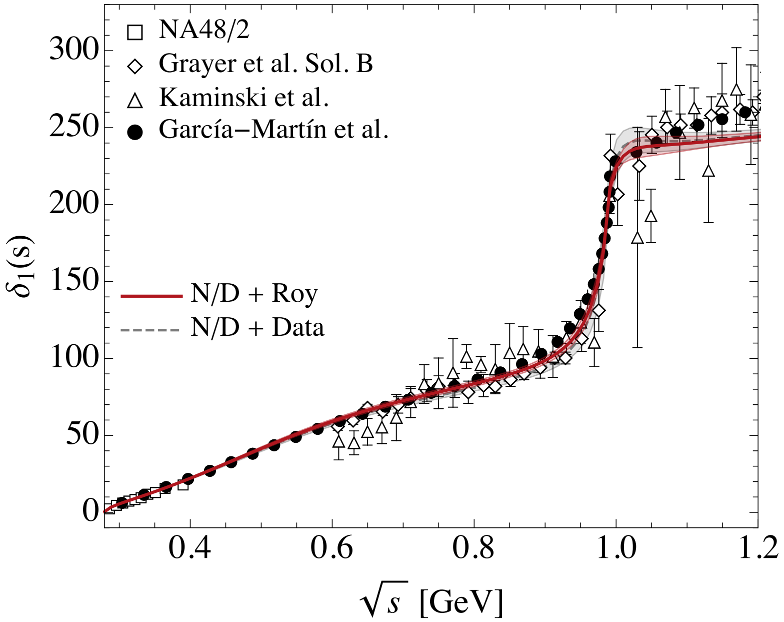

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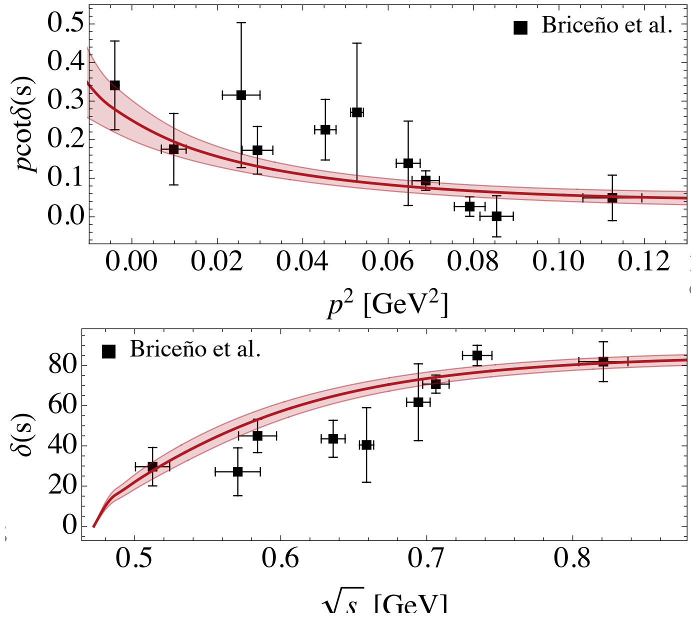

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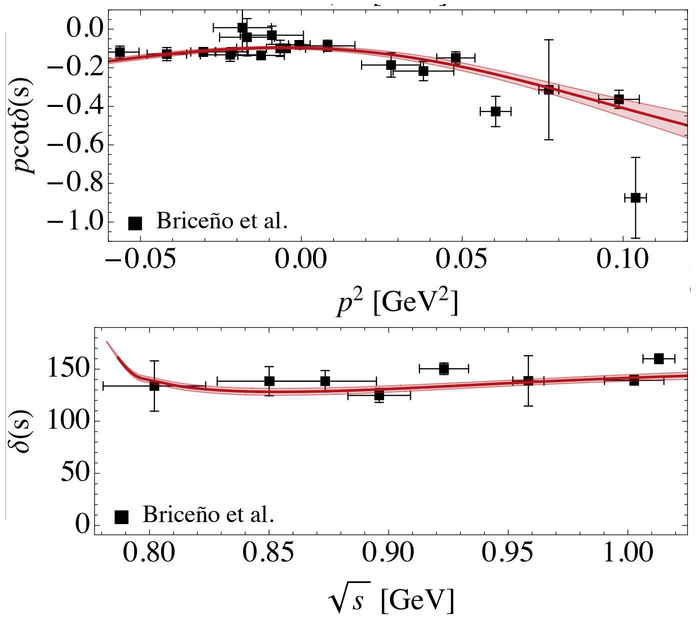

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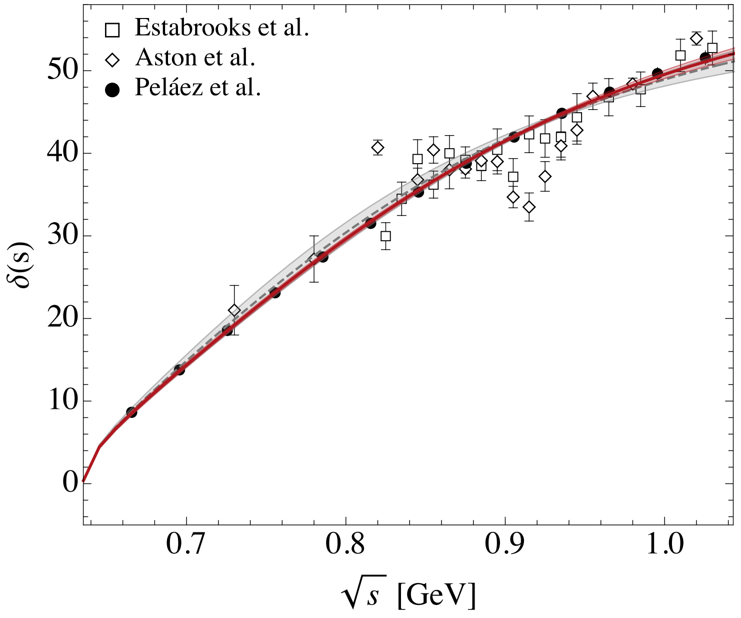
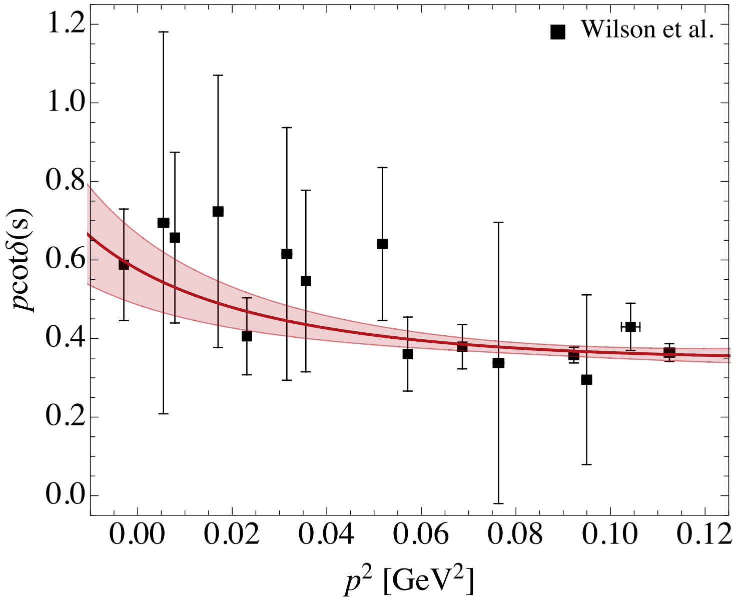
- $\kappa/K^*(700)$ in πK scattering (real & lattice data)



$$\sqrt{s_\kappa} = 702(12)^{+4}_{-5} - i\,285(16)^{+8}_{-13}\ \text{MeV}$$



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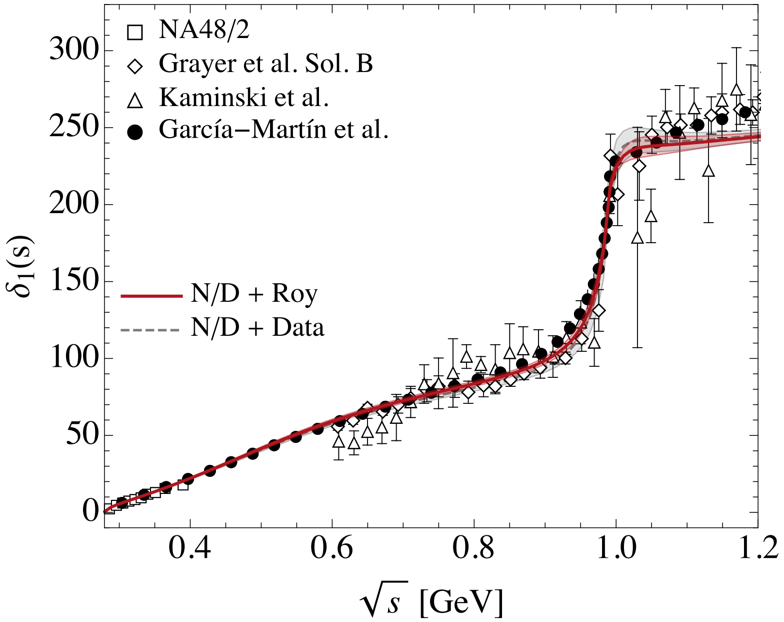

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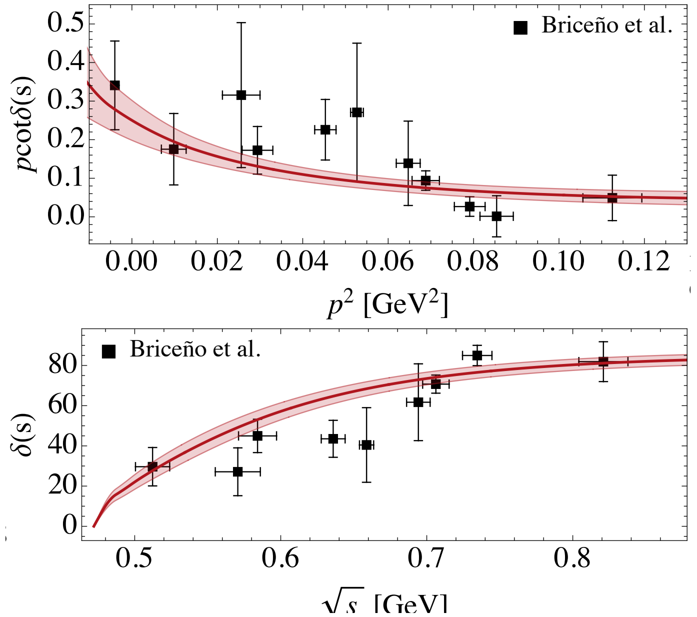

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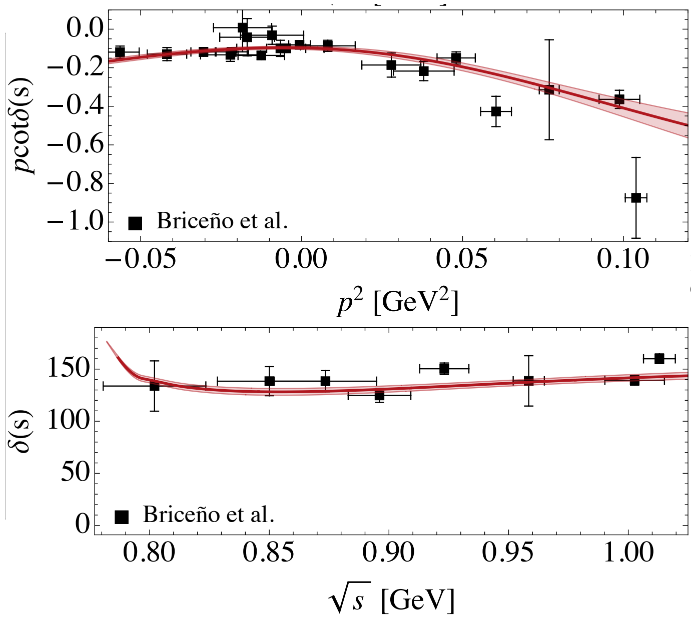

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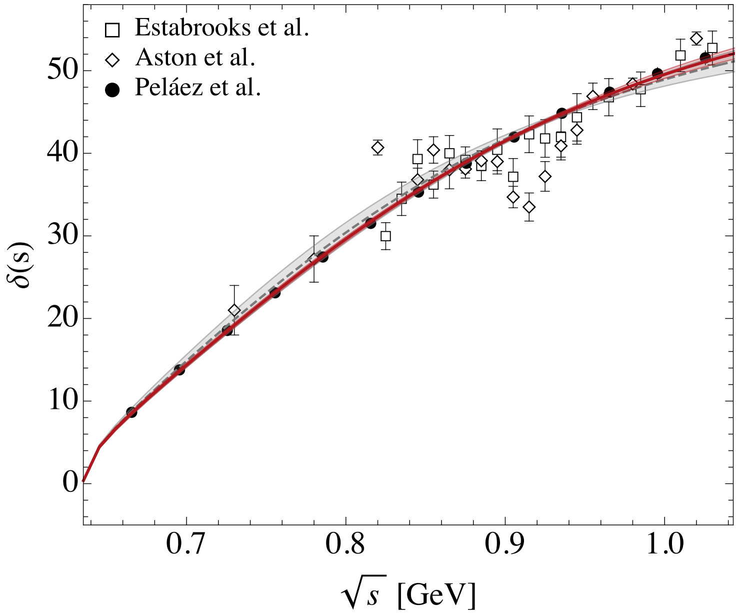

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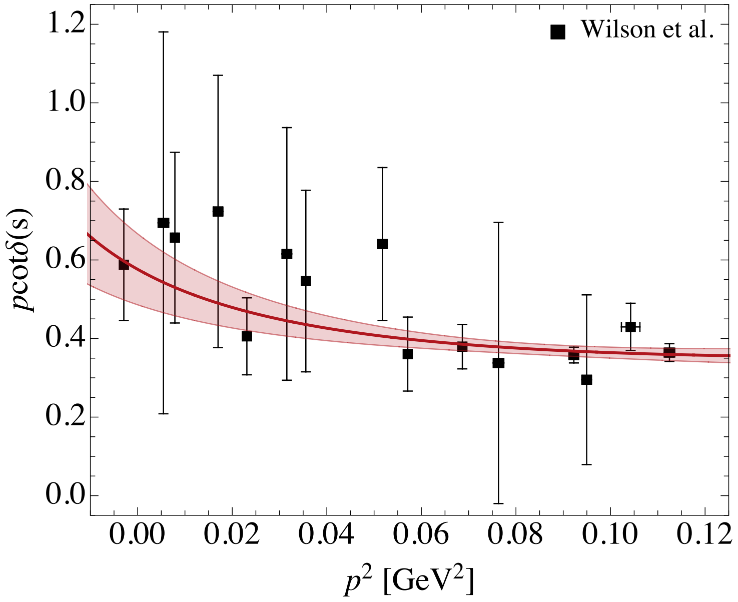

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Can search for resonances & bound states given the data input

[D., Danilkin, Vanderhaeghen 2020]

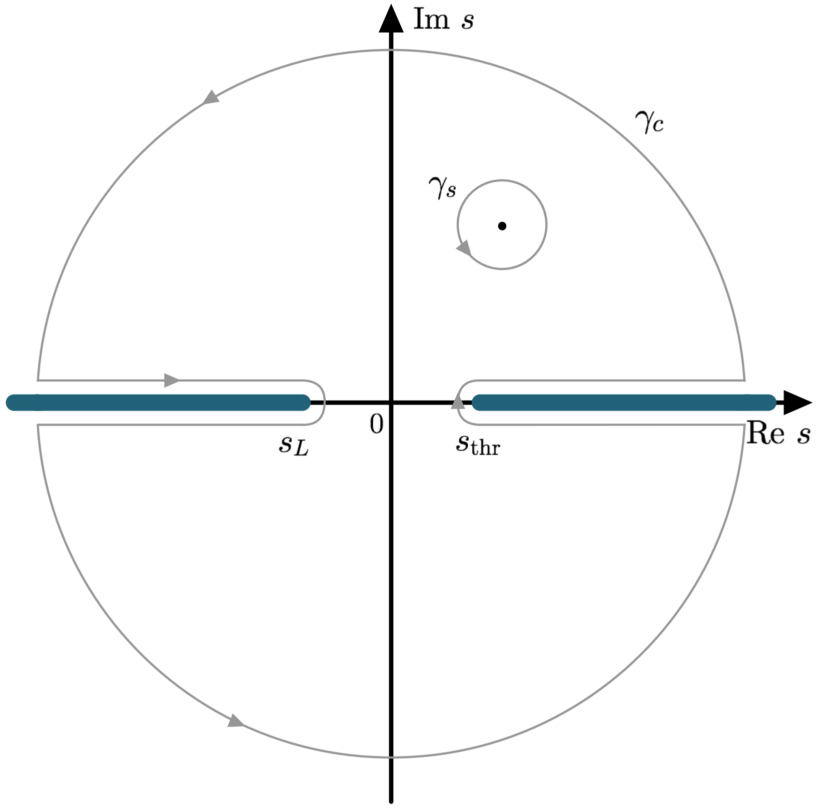
Partial wave dispersion relation

Unitarity relation for the partial wave amplitudes guarantees that p.w. amplitudes behave asymptotically **no worse than a constant**

$$\text{Disc}\, t_{ab}(s) = \sum_c t_{ac}(s)\, \rho_c(s)\, t^*_{cb}(s)$$

From the **maximal analyticity** principle one can write **dispersion relation**

$$t_{ab}(s) = \int_{-\infty}^{s_L} \frac{ds'}{\pi} \frac{\text{Disc}\, t_{ab}(s')}{s'-s} + \int_{s_{th}}^{\infty} \frac{ds'}{\pi} \frac{\text{Disc}\, t_{ab}(s')}{s'-s}$$



Which we subtract once in accordance with **unitarity bound**

$$t_{ab}(s) = U_{ab}(s) + \frac{s - s_M}{\pi} \int_{s_{th}}^{\infty} \frac{ds'}{s'-s_M} \frac{\text{Disc}\, t_{ab}(s')}{s'-s}$$

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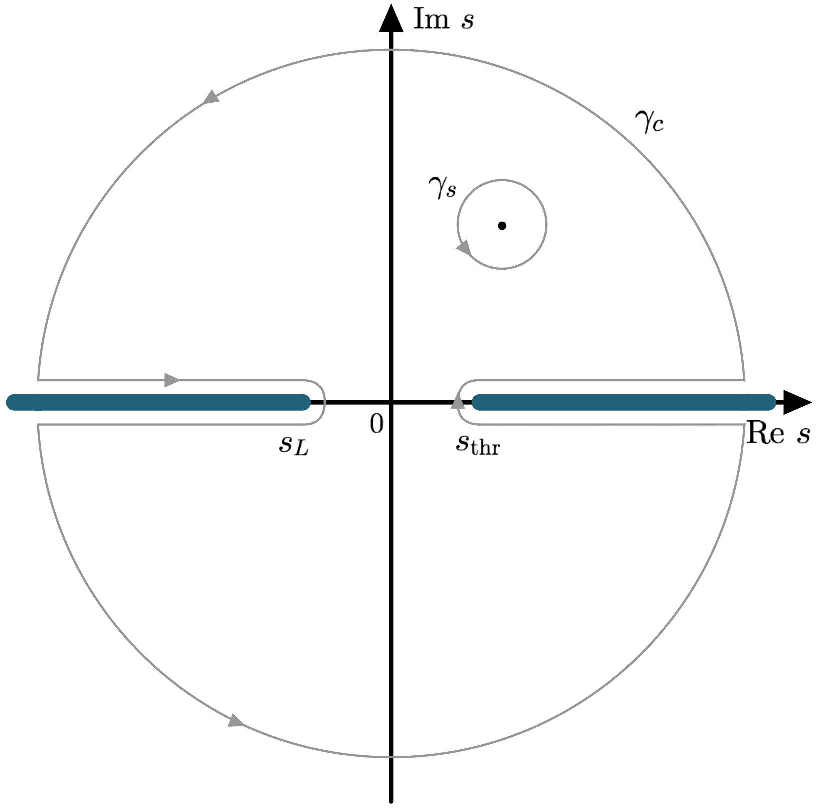

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included subtraction constant and left-hand cuts, asymptotically bounded **unknown function**

N/D method

Once-subtracted p.w. dispersion relation

$$t_{ab}(s) = U_{ab}(s) + \frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{\text{Disc } t_{ab}(s')}{s'-s}$$

can be solved using N/D method with input from $U_{ab}(s)$ **above threshold**

$$t_{ab}(s) = \sum_c D_{ac}^{-1}N_{cb}(s)$$

$$N_{ab}(s) = U_{ab}(s) + \frac{s}{\pi}\sum_c\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{N_{ac}(s')\rho_c(s')(U_{cb}(s')-U_{cb}(s))}{s'-s}$$

$$D_{ab}(s) = \delta_{ab} - \frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{N_{ab}(s')\rho_b(s')}{s'-s}$$

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s-wave $\gamma\gamma \to D\bar{D}$ amplitude is the off-diagonal term of the coupled-channel $\{\gamma\gamma, D\bar{D}\}$ system with $1 = \gamma\gamma, 2 = D\bar{D}$

$$t_{12}(s) = \underbrace{U_{12}(s)}_{\textit{left-hand cuts}} + \underbrace{D^{-1}_{22}(s)}_{\textit{hadronic part}}\left(-\frac{s}{\pi}\int_{4m_D^2}^{\infty}\frac{ds'}{s'}\frac{\text{Disc}(D_{22}(s'))U_{12}(s')}{s'-s}\right)$$

Left-hand cuts

We approximate left-hand cuts as an expansion in a **conformal mapping variable** $\xi(s)$

[Gasparyan, Lutz 2010]

$$U_{22}(s) = t_{22}(0) + \frac{s}{\pi} \int_{-\infty}^{s_L} \frac{ds'}{s'} \frac{\text{Im}\ t_{22}(s')}{s' - s} \simeq \sum_{n=0}^{\infty} C_n \xi^n(s)$$

to be determined from the fits

The exact form of the conformal map

$$\xi(s) = \frac{\sqrt{s - s_L} - \sqrt{s_E - s_L}}{\sqrt{s - s_L} + \sqrt{s_E - s_L}}$$

$$s_L = 4(m_D^2 - m_\pi^2)$$

$$\sqrt{s_E} = \frac{1}{2}\left(\sqrt{s_{th}} + \sqrt{s_{max}}\right)$$

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$$\sqrt{s_E} = \frac{1}{2}\left(\sqrt{s_{th}} + \sqrt{s_{max}}\right)$$

For the s-wave, $I = 0$ photon fusion process $\gamma\gamma \to D\bar{D}$: Born (also for non-resonant $I = 1$)

$$U_{12}(s) = -\frac{2\sqrt{2}\, e^2 m_D^2}{s\beta(s)} \log \frac{1 + \beta(s)}{1 - \beta(s)}, \quad \beta(s) \equiv \frac{2\, p(s)}{\sqrt{s}} = \sqrt{1 - \frac{4\, m_D^2}{s}}$$

Naïve analysis of the combined data

s-wave: $I = 0$ with rescattering, $I = 1$ only Born
d-wave: Breit-Wigner with fixed $\chi_{c2}(3930)$ mass and width

$\Longrightarrow$

2 parameters from N/D
+ normalisations N_0, N_2

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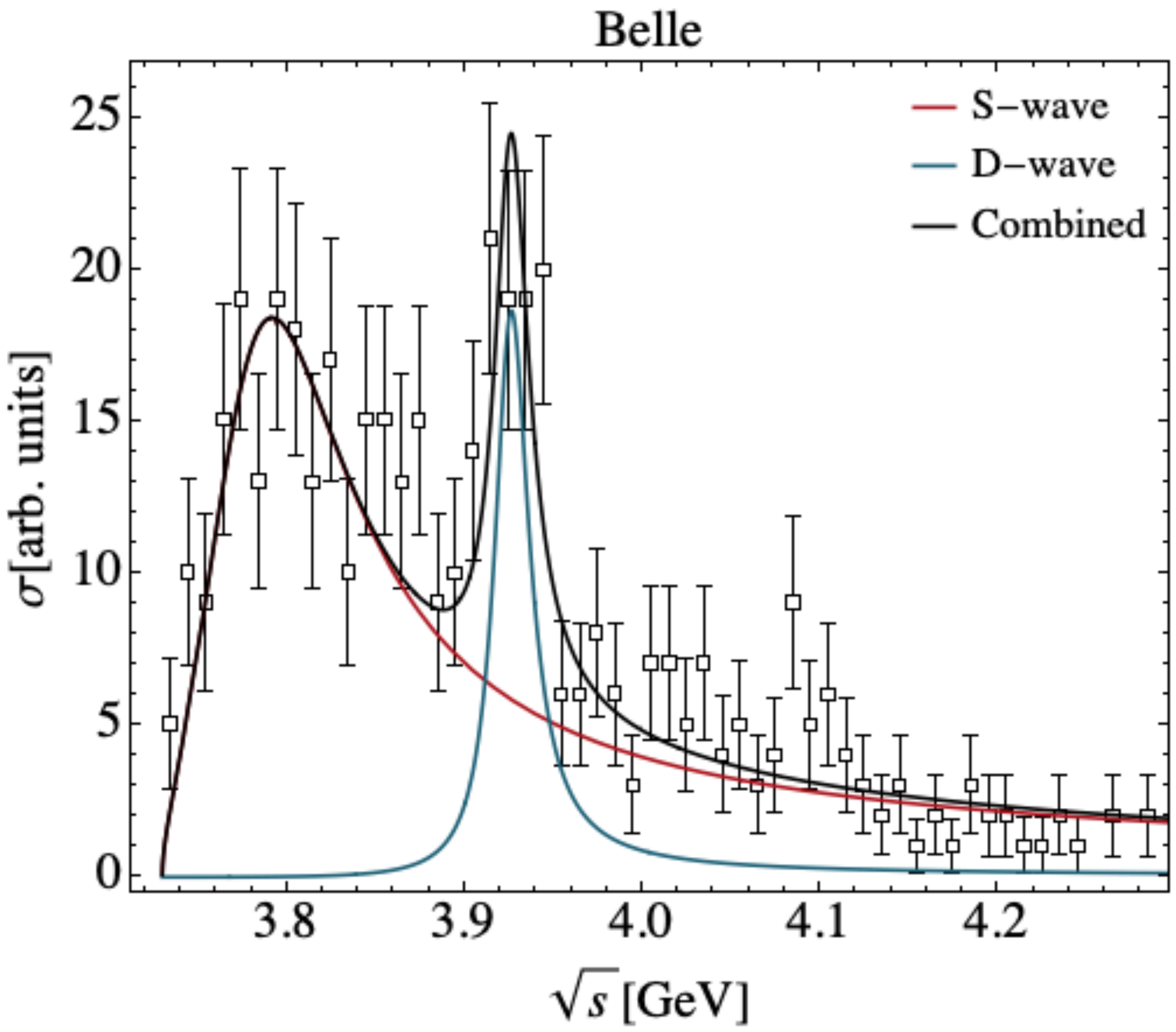
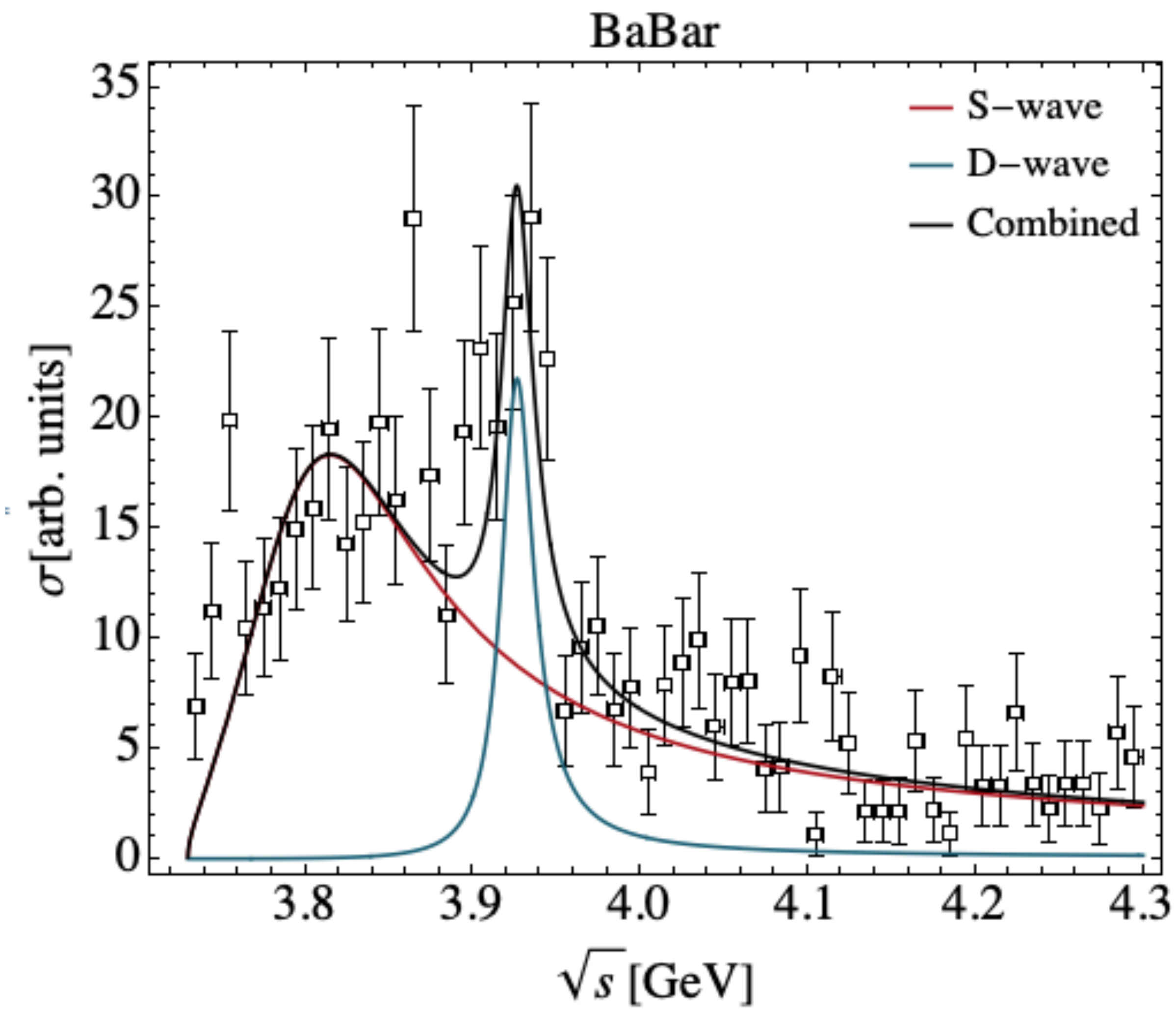


$3772.2 - 50i$ MeV



$3788.6 - 68i$ MeV

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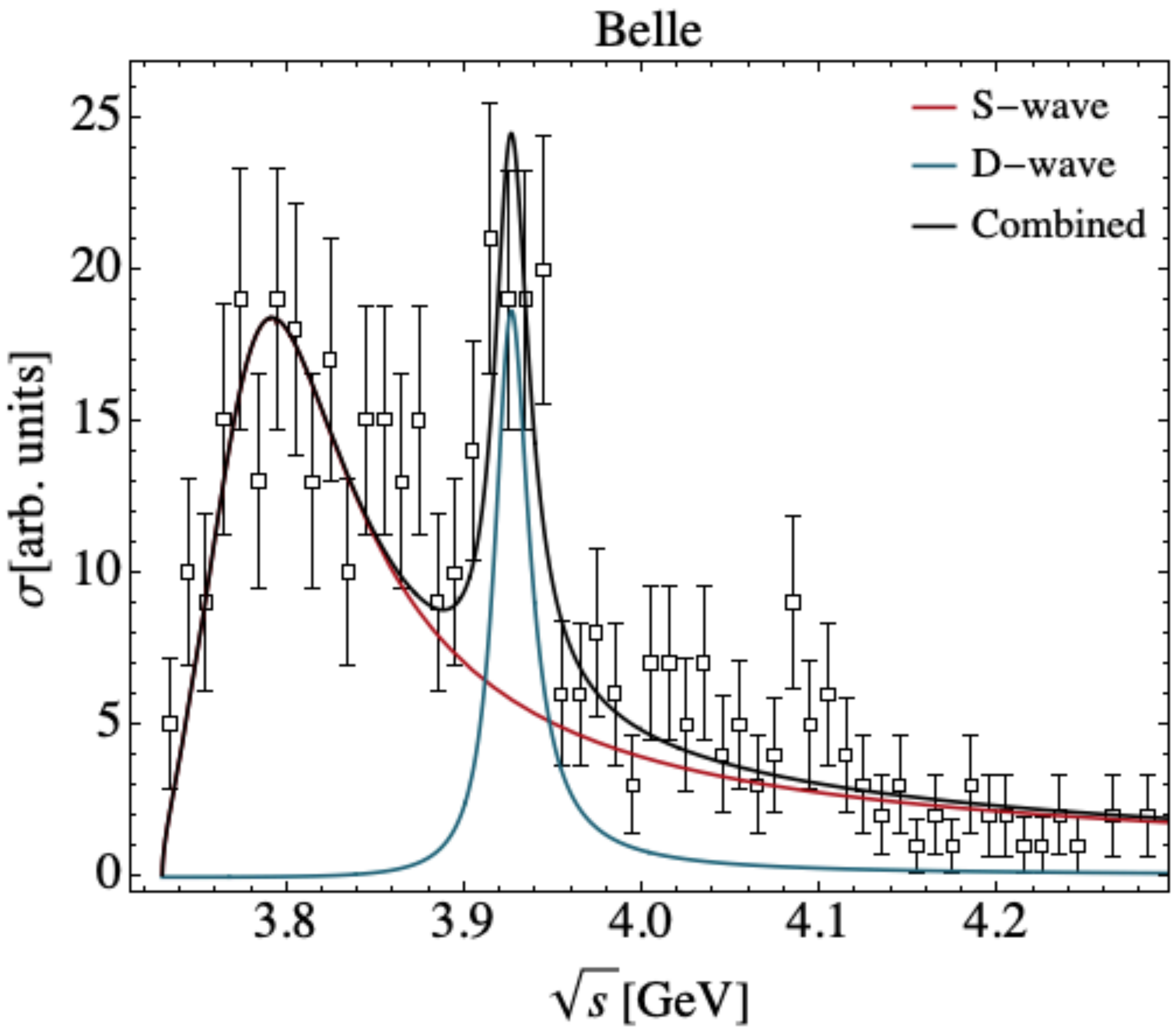
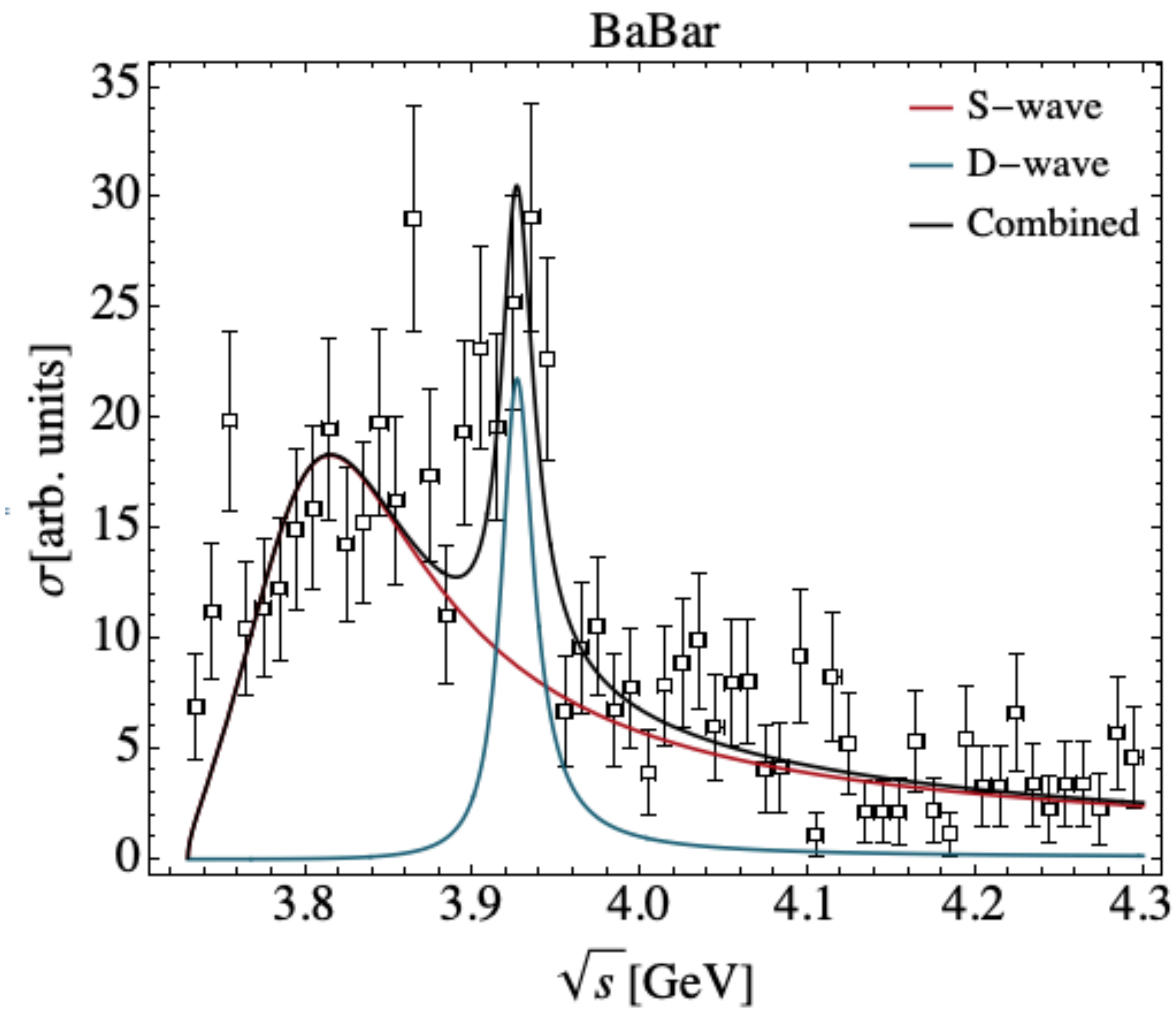


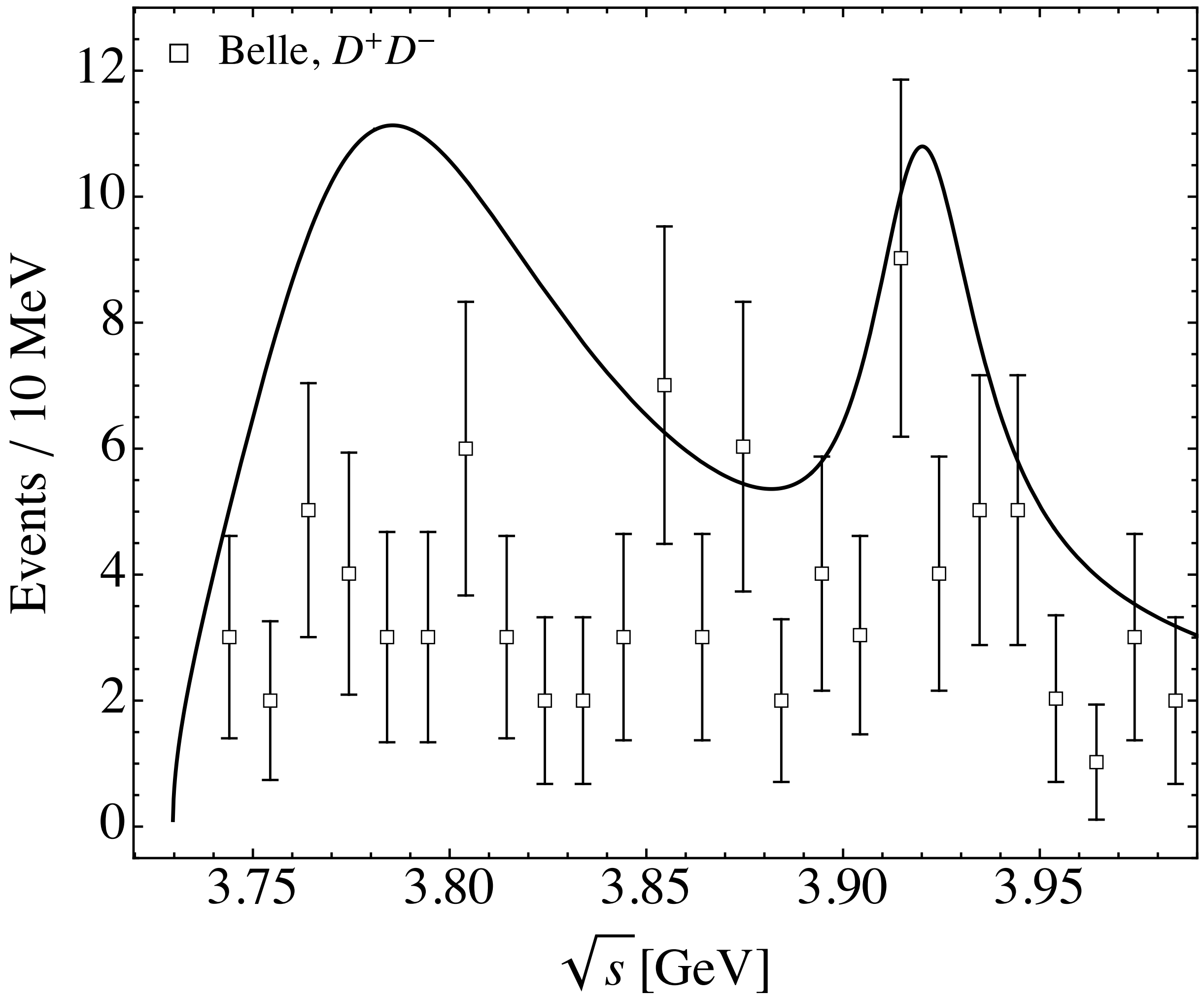
$3772.2 - 50i$ MeV

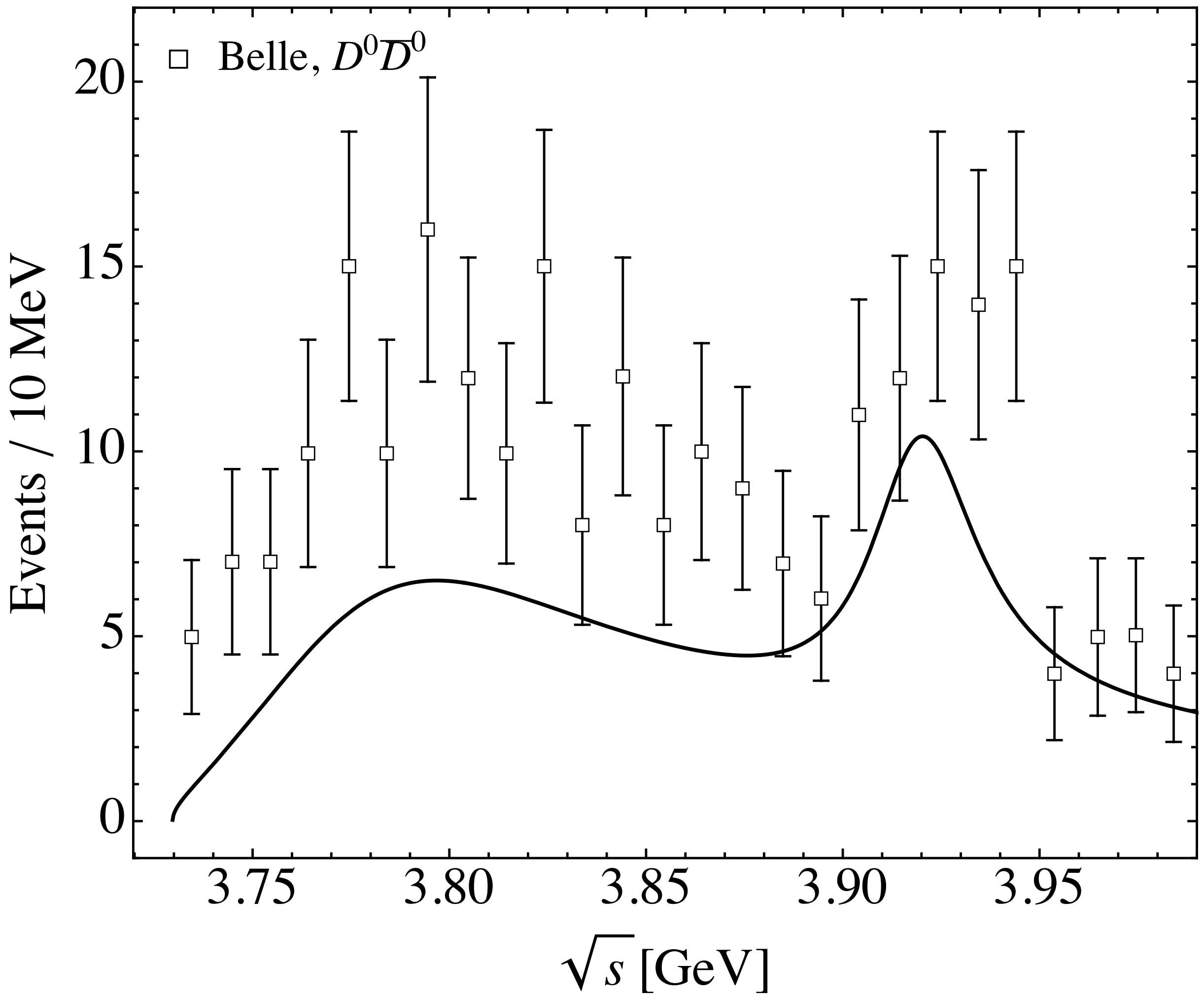


$3788.6 - 68i$ MeV

Everything is perfect, right? **Wrong**.

Surprise for the naïve analysis





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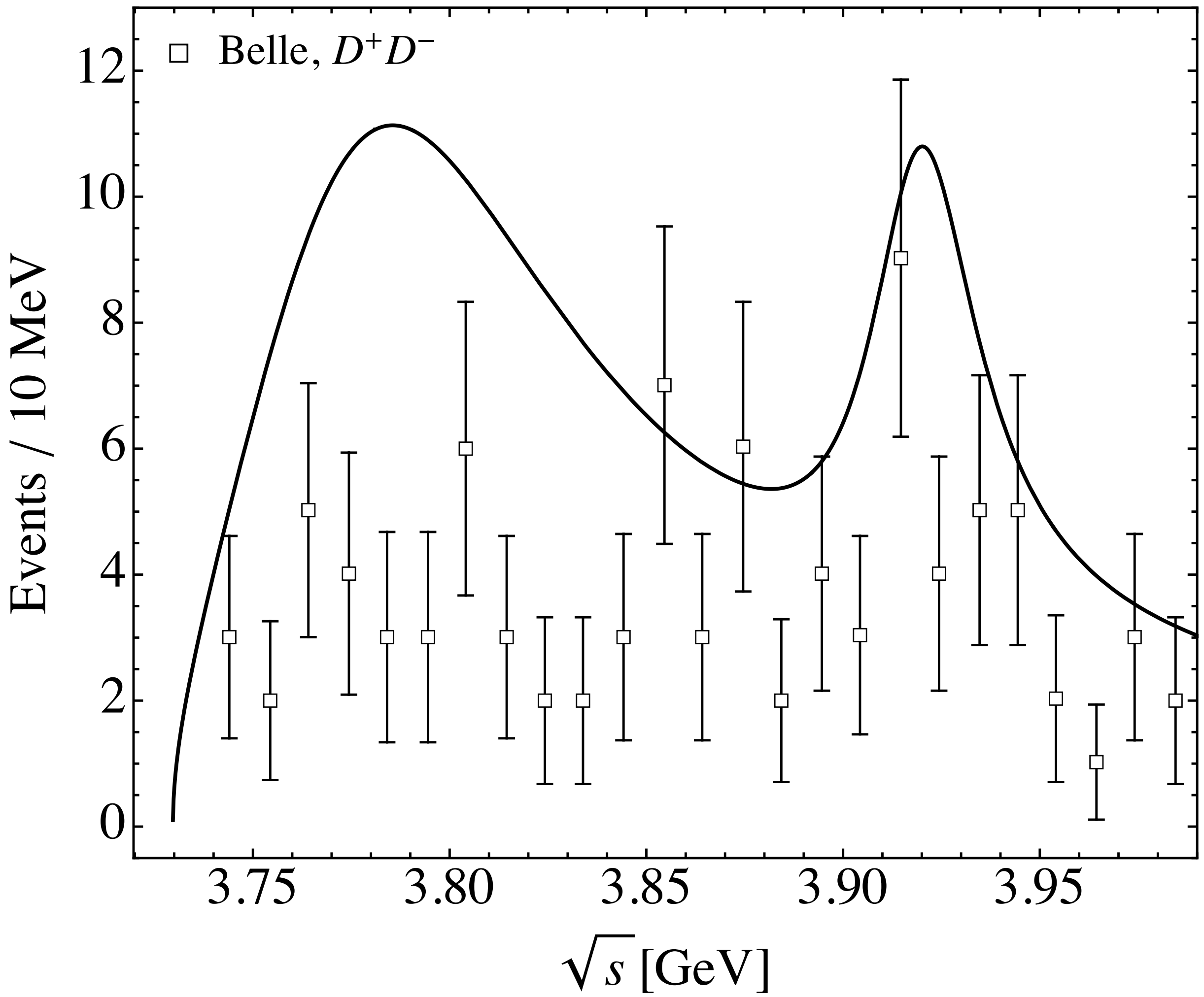
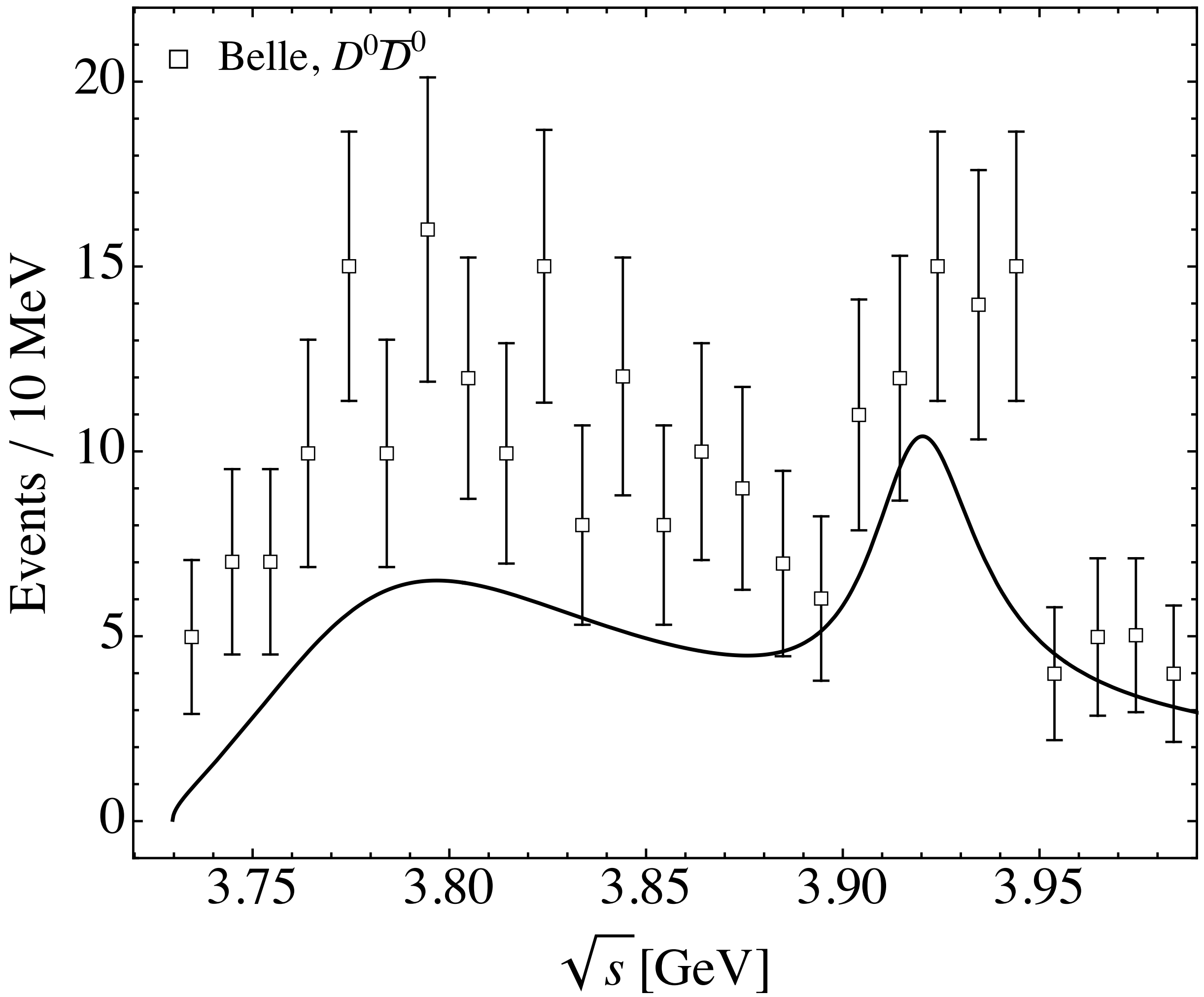
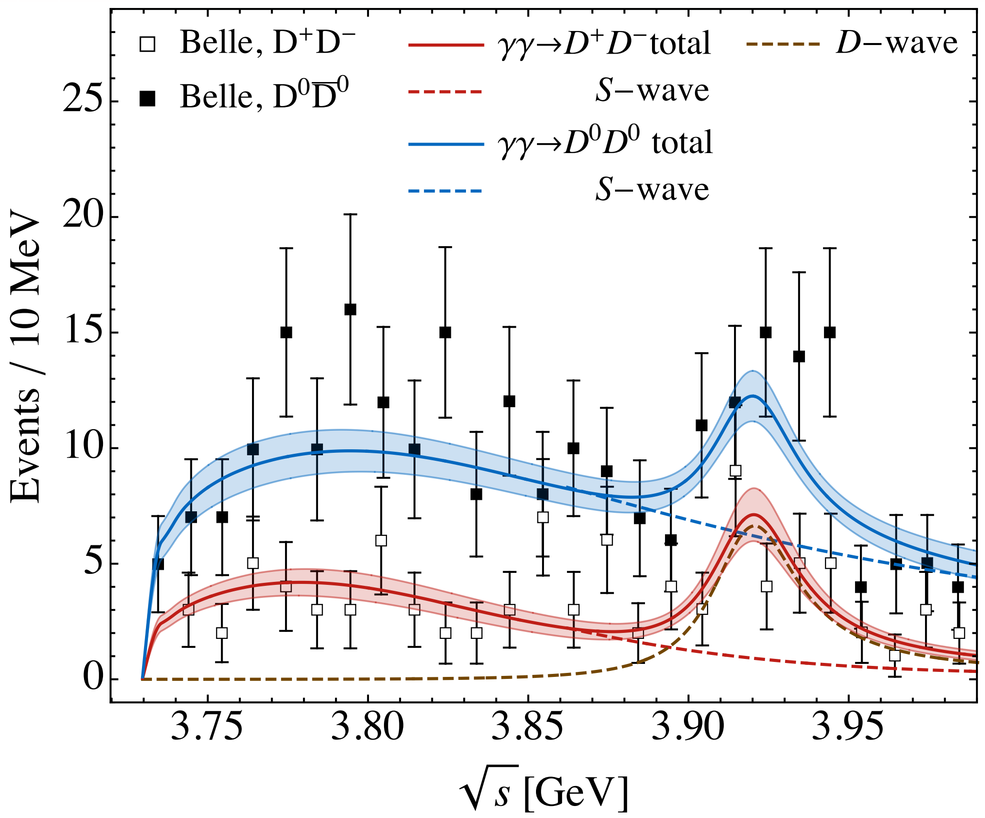




The fit to combined data **do not** describe charged and neutral channels
(the same holds for BaBar data)

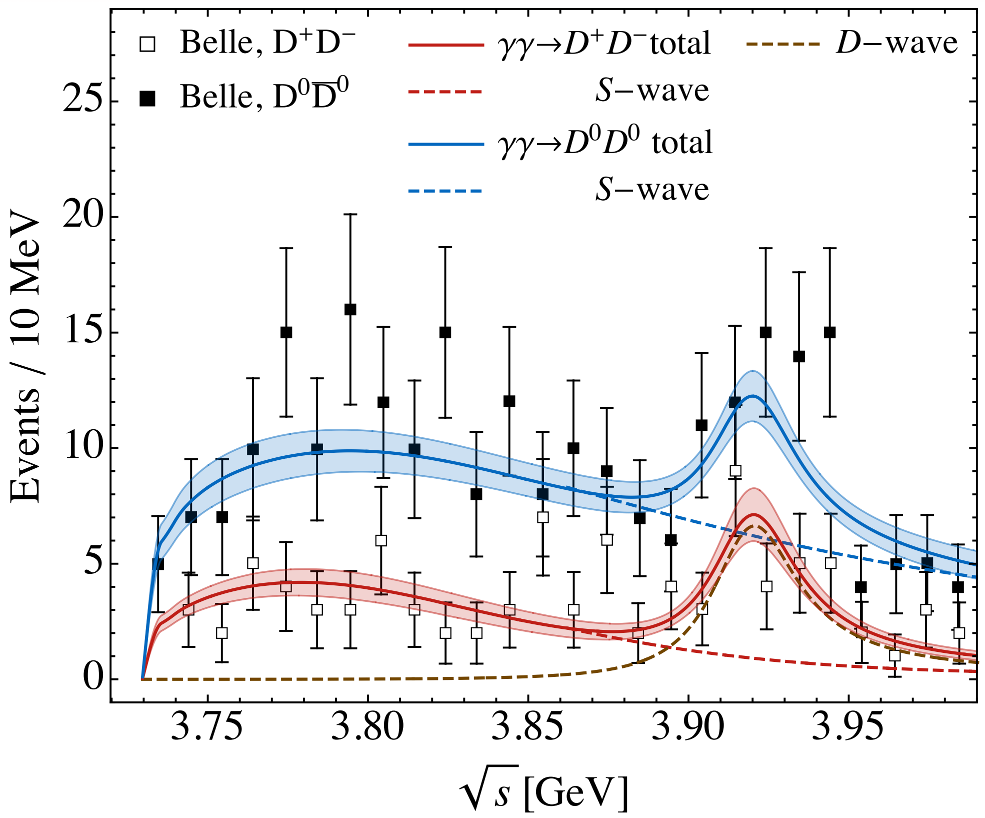
Maybe there is something wrong with the data itself? Or maybe we've
already seen something similar?…

Analysis of charged and neutral data



$s_B = 3695(4)$ MeV

Analysis of charged and neutral data



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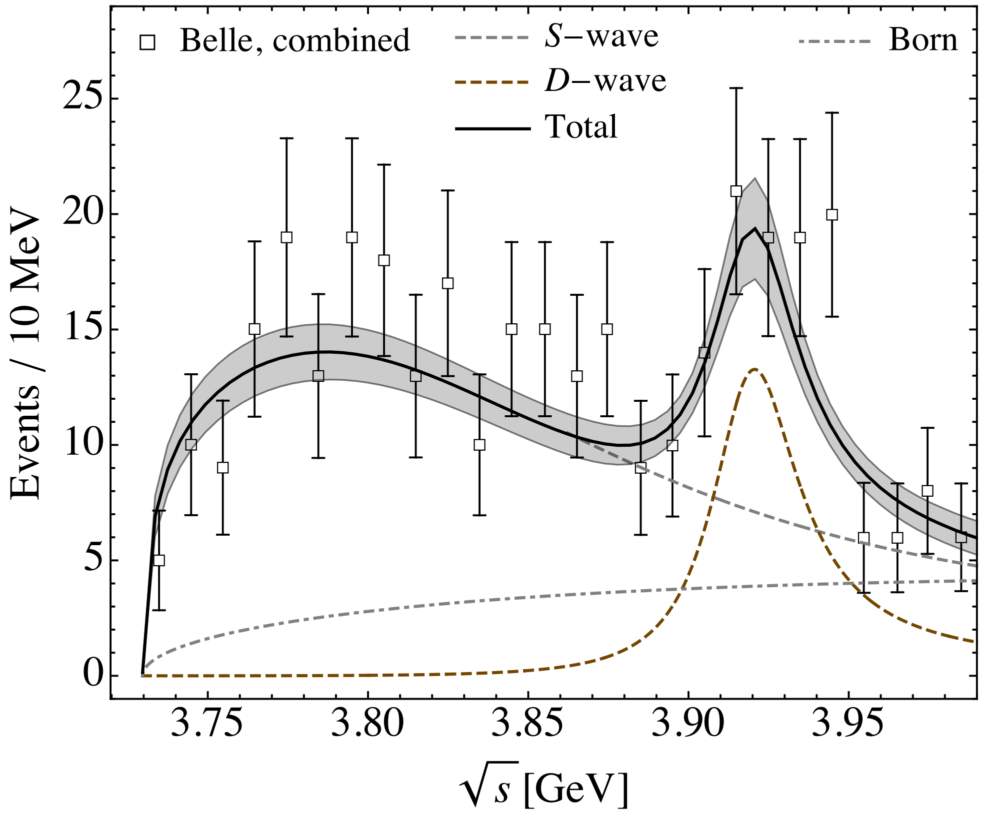

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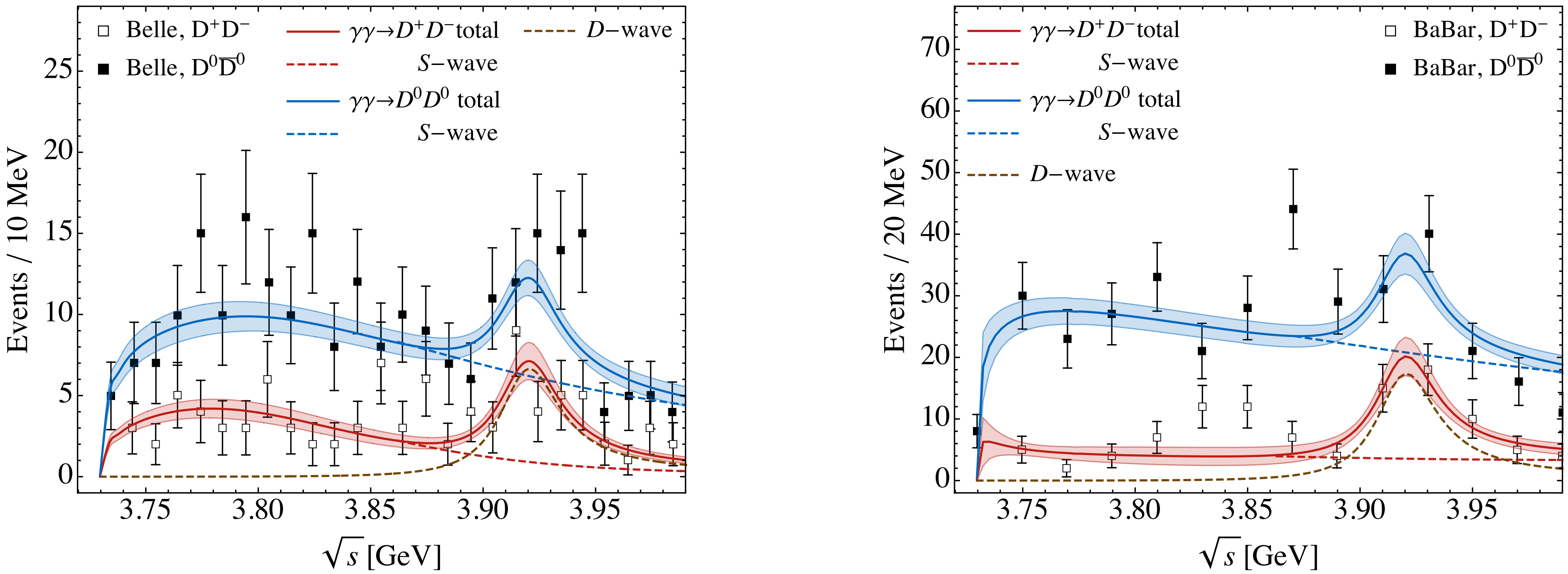
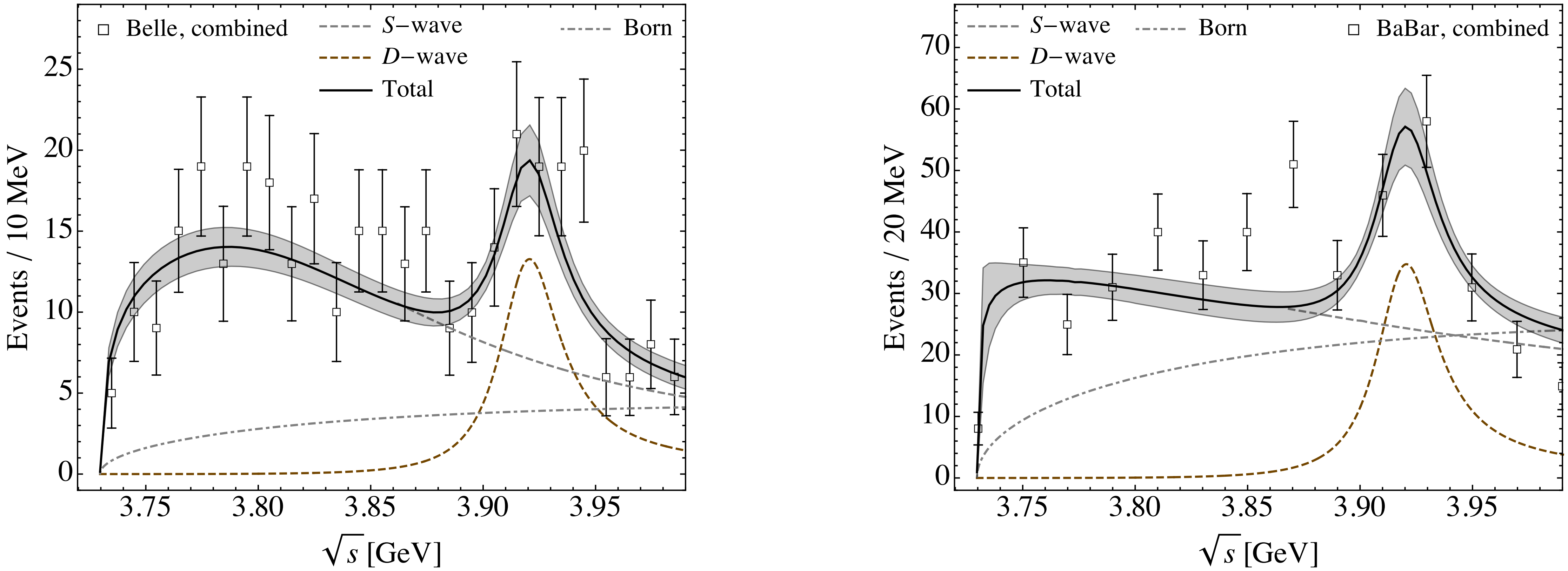
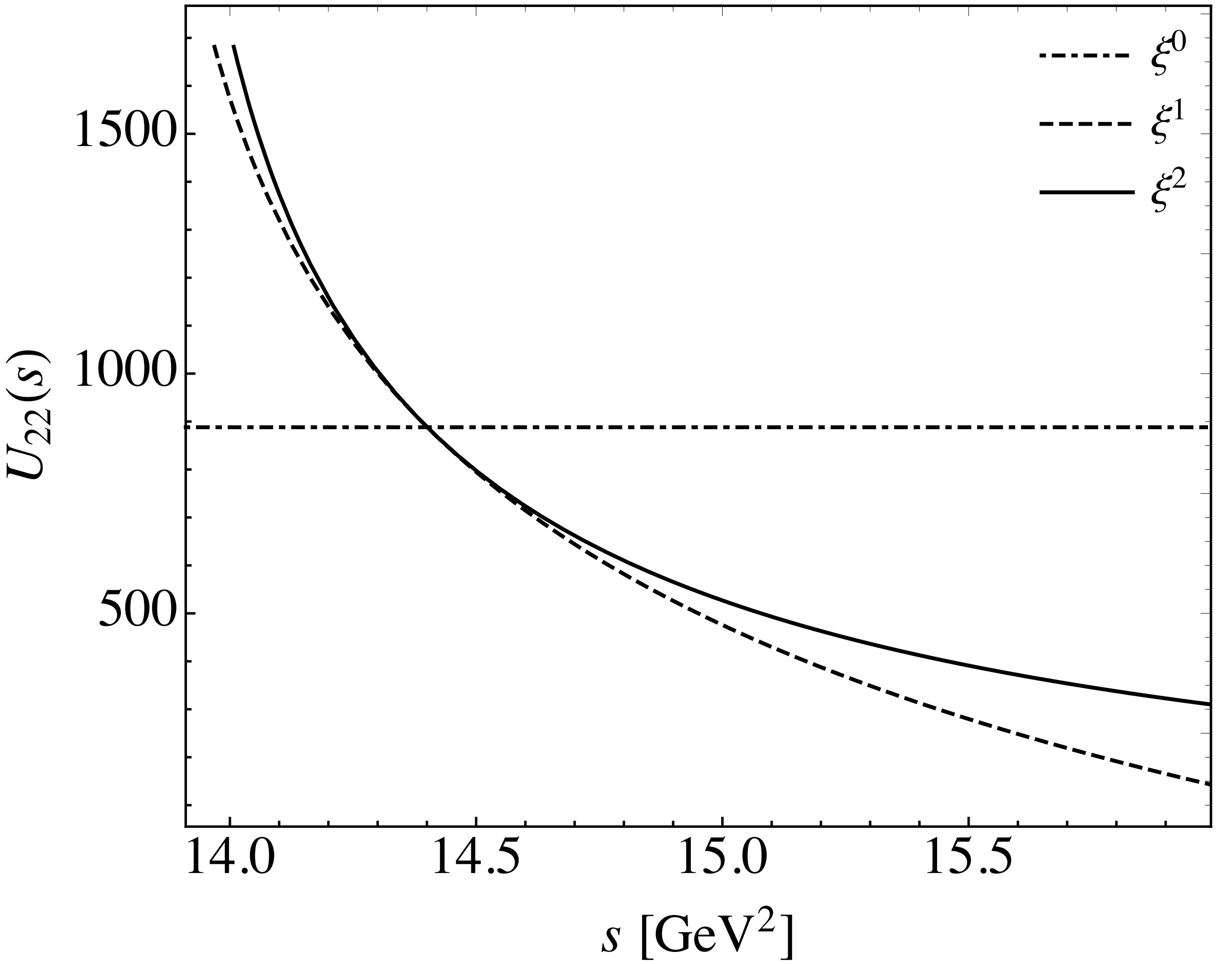


$s_B = 3695(4)$ MeV

$\sqrt{s_B} = 3669.4(18.0)$ MeV



Convergence & angular distribution



Good convergence with 3 parameters
in conformal mapping expansion $\Longrightarrow$
there is no need for more parameters

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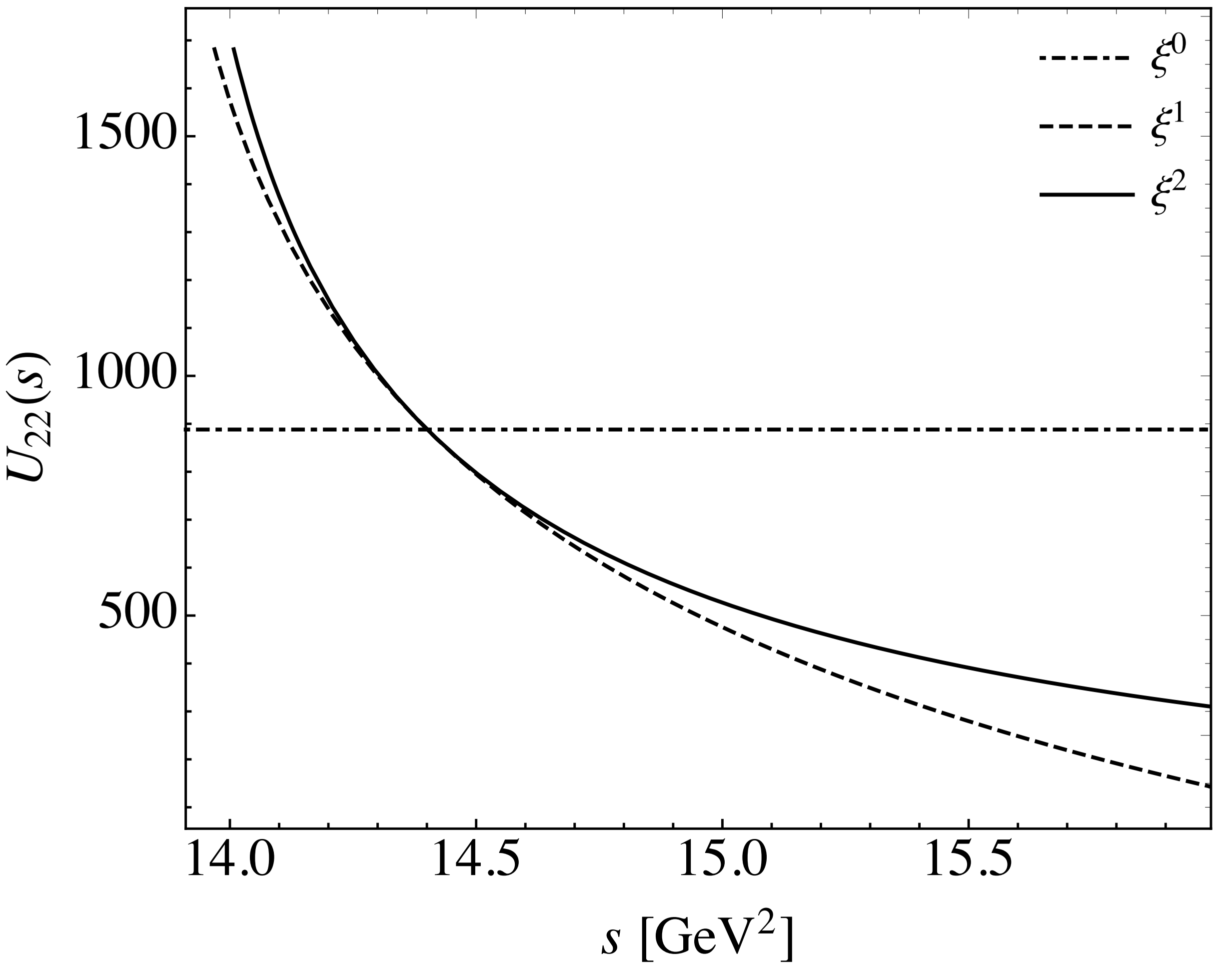
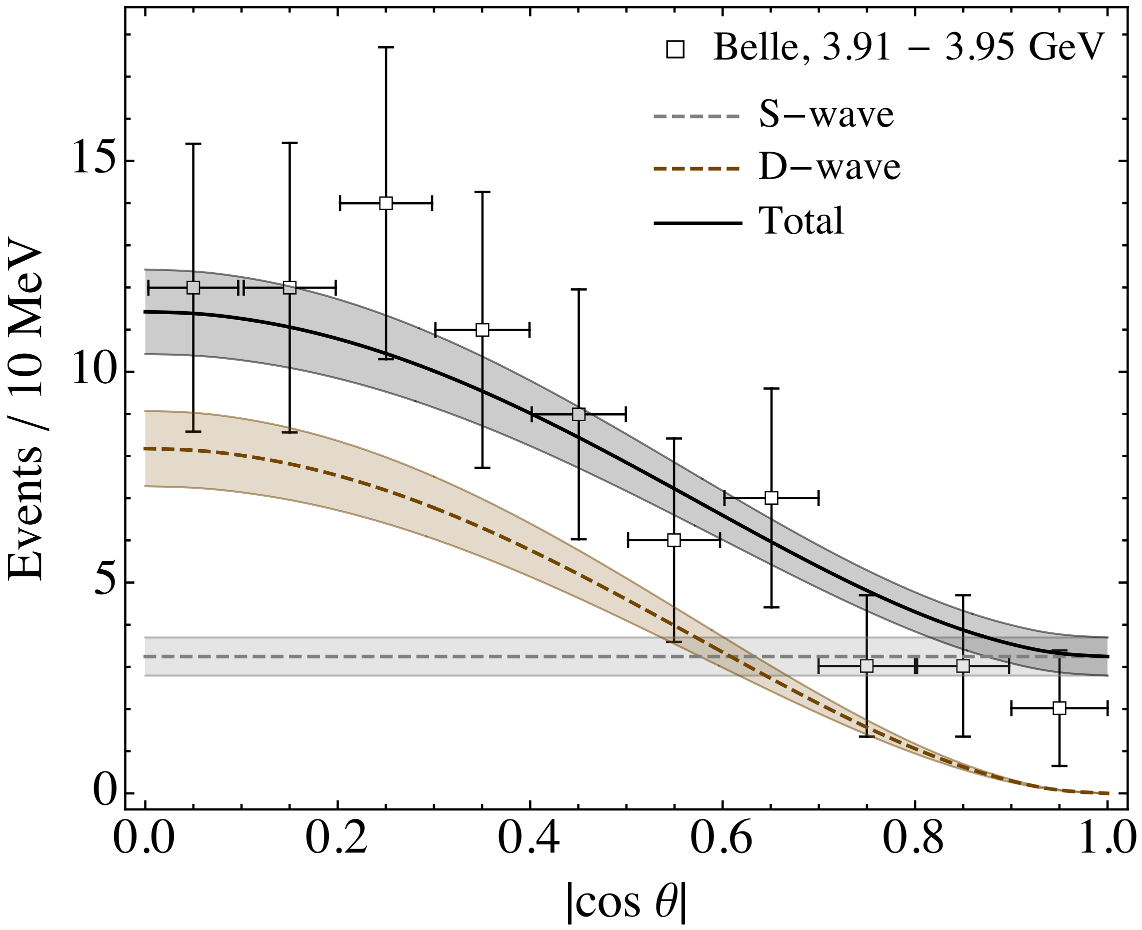
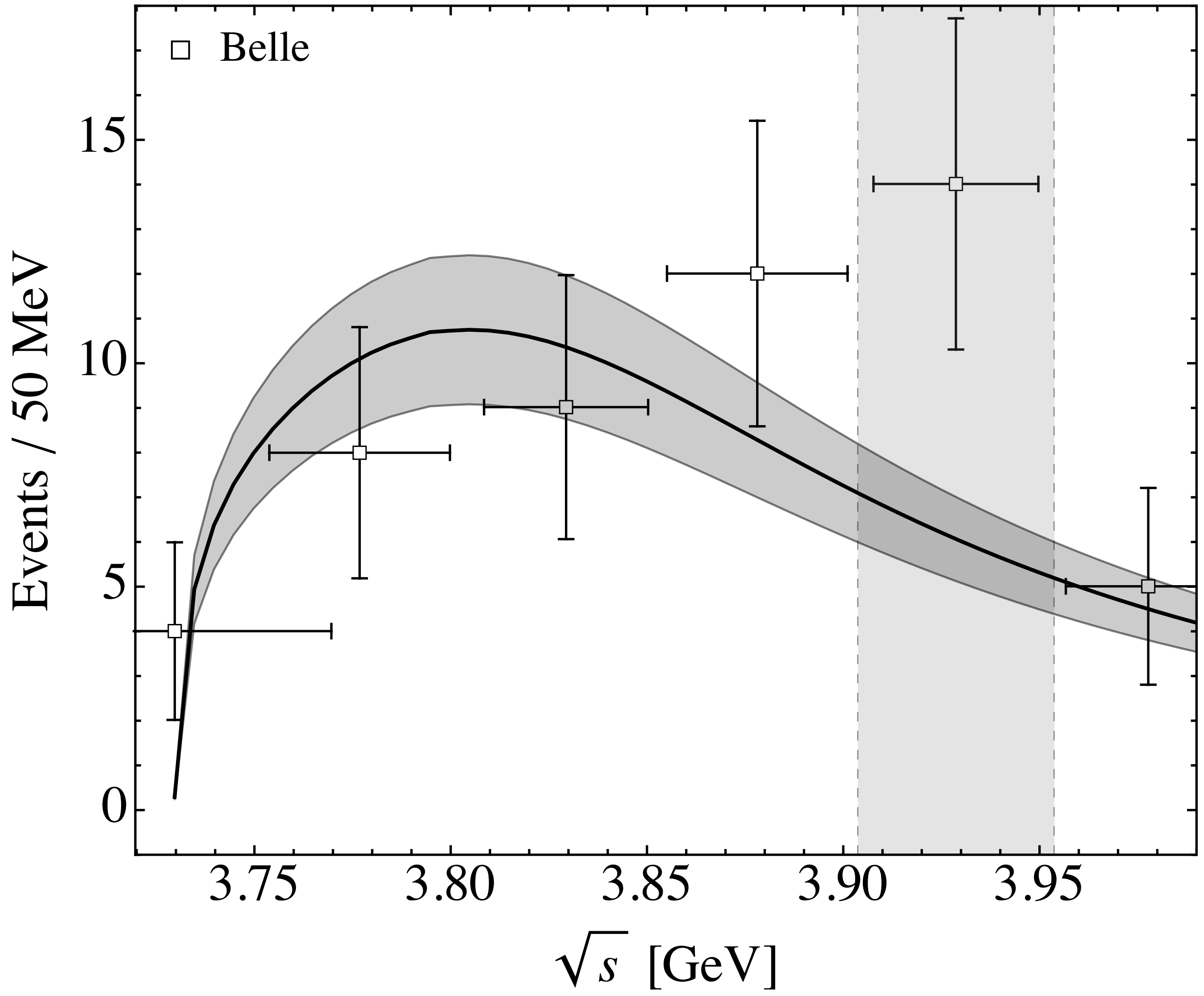




Good convergence with 3 parameters in conformal mapping expansion $\Longrightarrow$ there is no need for more parameters

Claims regarding $X(3915)$ presence also in $\gamma\gamma \to D\bar{D}$ data [Chen 2012] : no evidence in angular distribution

Analysis of data



Only 6 data points below 4 GeV Minus one point at ~ 3.93 GeV where $\chi_{c2}(3930)$ resides = 5 points

No realistic estimates can be done from this data alone; full experimental dataset is needed

Now $\chi_{c2}(3860)$ is in PDG ¯_(ツ)_/¯

Also lattice [Prelovsek 2020] but ~100 MeV bigger (maybe $X(3915)$)

$$\frac{d\sigma}{d\sqrt{s}} = \boxed{N}\,\frac{\lambda^{1/2}(s,q^2,m_{J/\psi}^2)\,\lambda^{1/2}(s,m_D^2,m_D^2)}{q^6\sqrt{s}}\left|D_{22}^{-1}(s)\right|^2$$

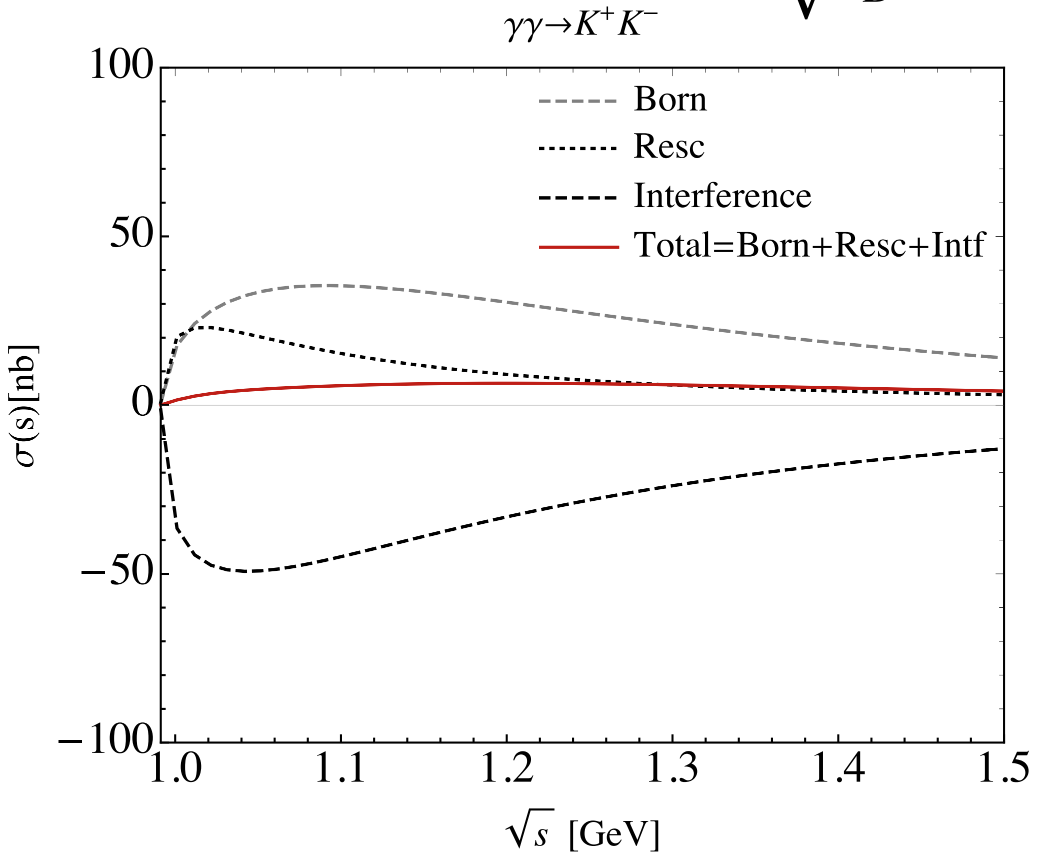
the only fitting parameter

Our analysis of $\gamma\gamma \to D\bar{D}$ is consistent

$$\$\lambda(x,y,z) \equiv x^2+y^2+z^2-2(xy+xz+yz)\$$$

Comparison with the case

Consider again $\{\pi\pi, K\bar{K}\}$ coupled channel system and switch off $\pi\pi$ channel $\Longrightarrow f_0(980)$ becomes a $K\bar{K}$ bound state with $\sqrt{s_B} = 961$ MeV



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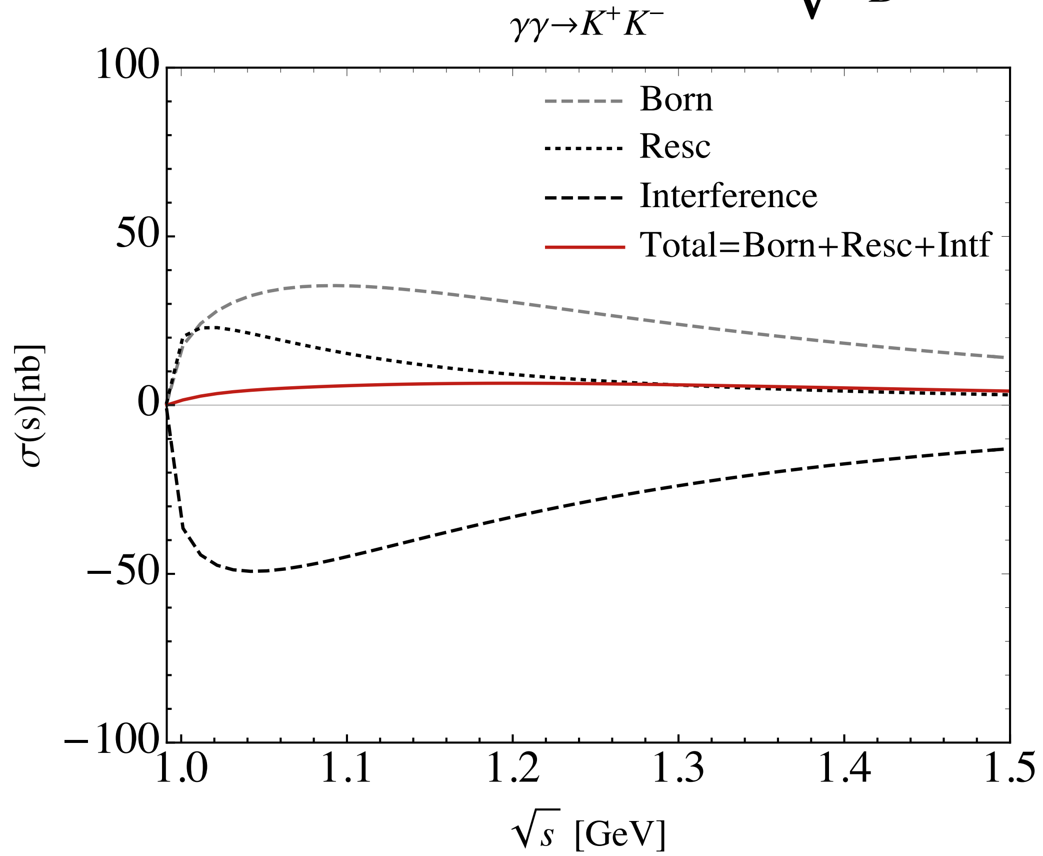

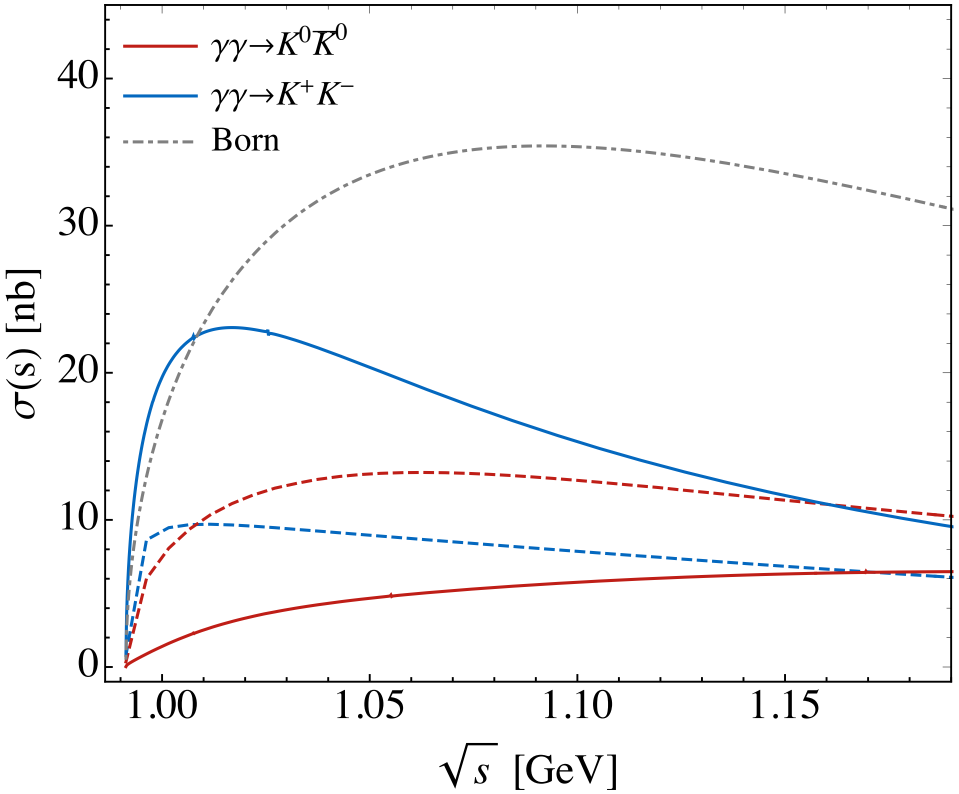

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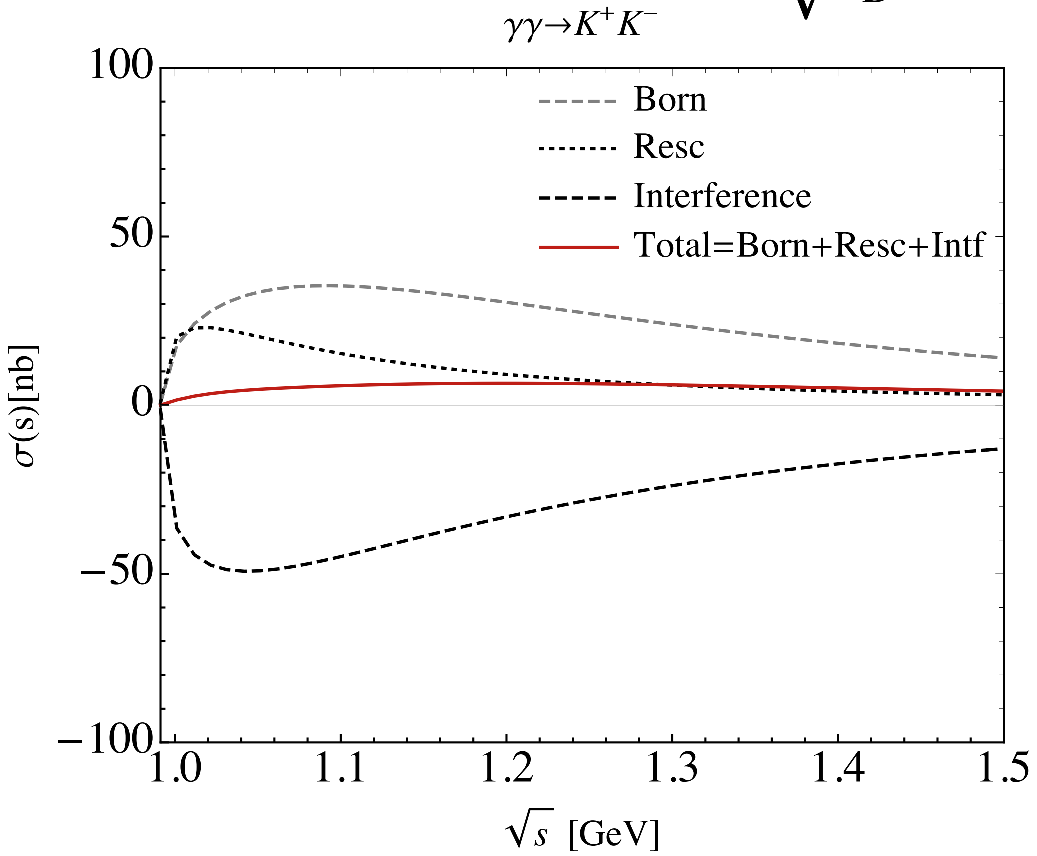

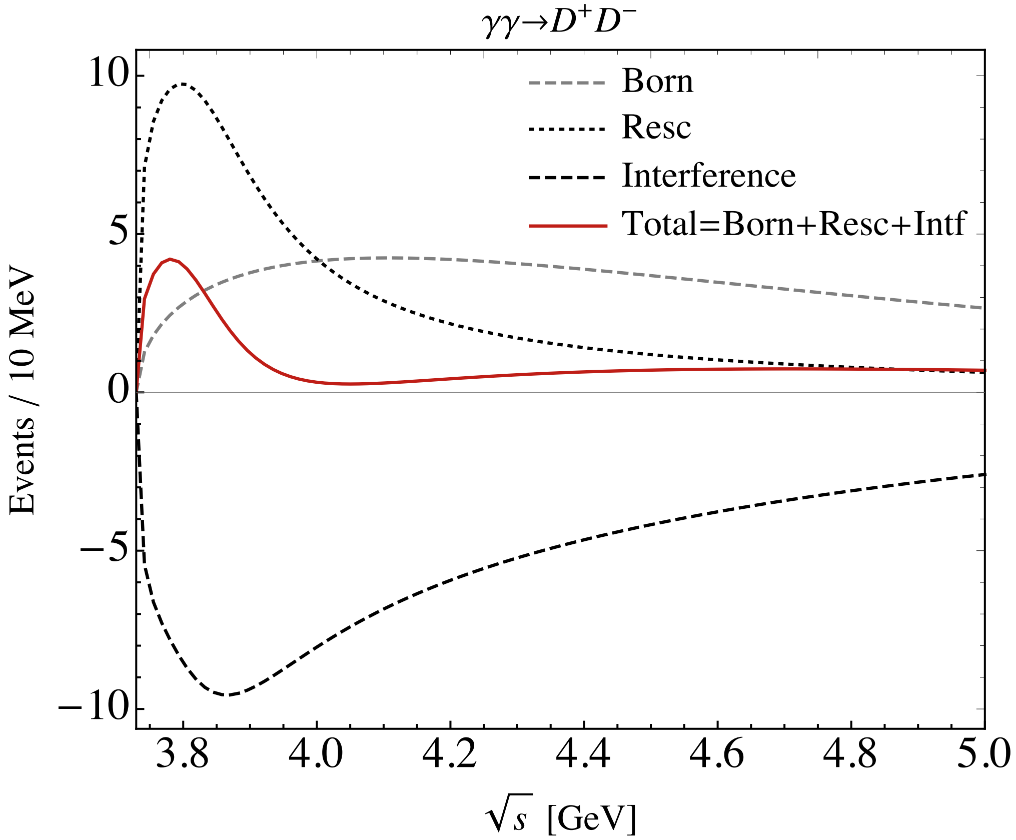

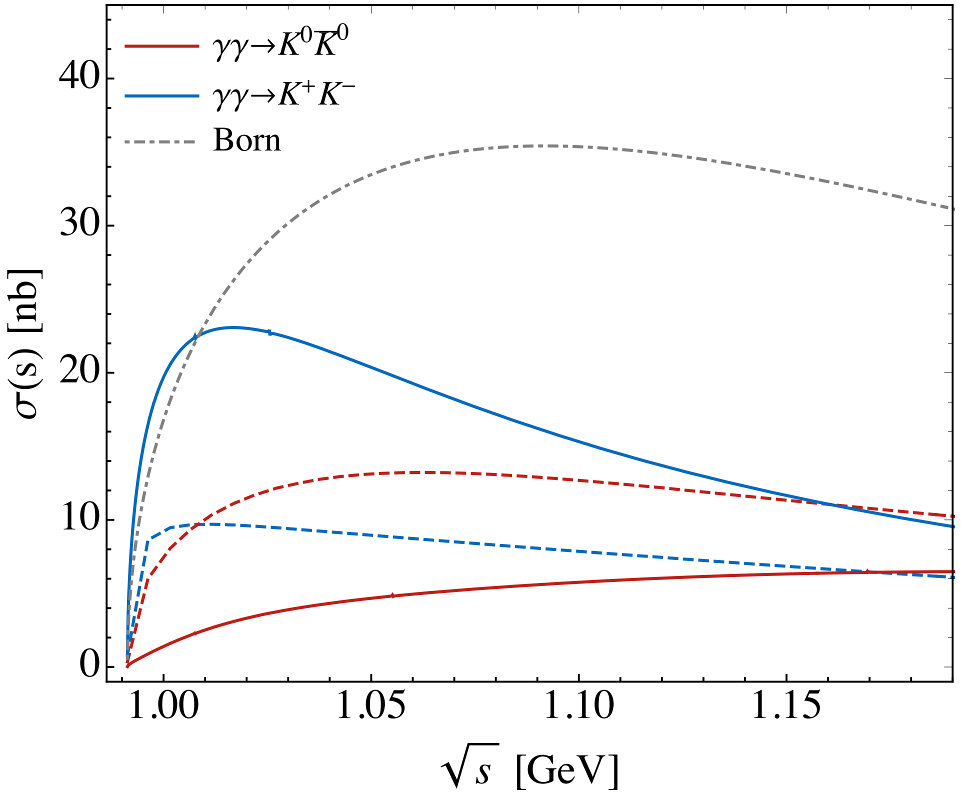

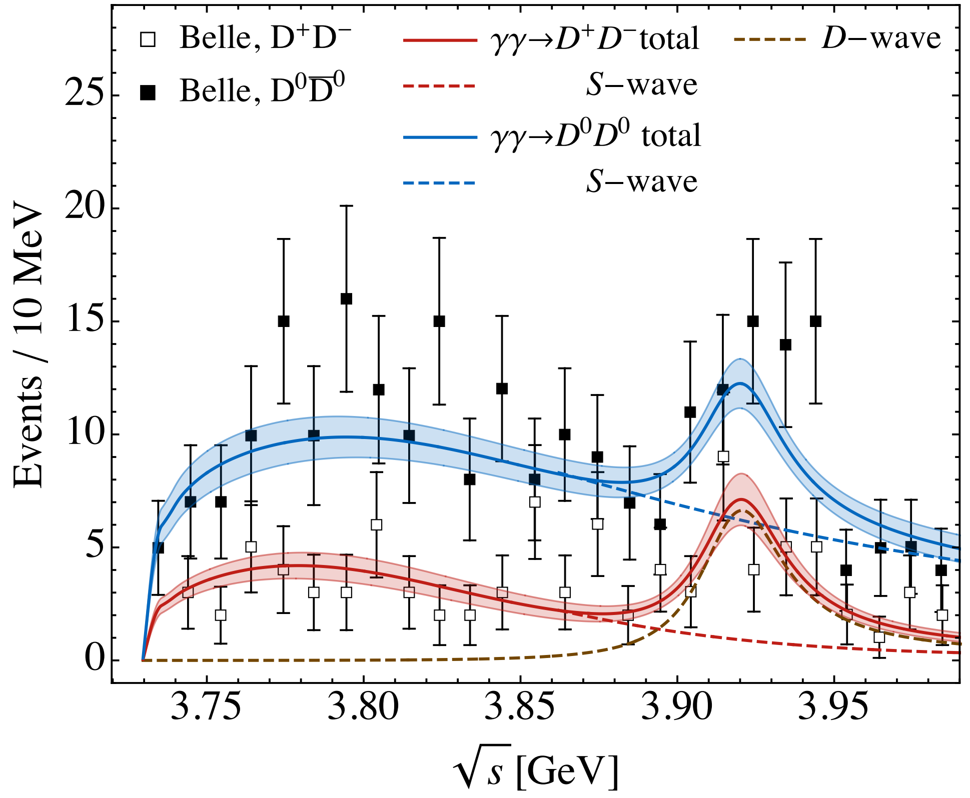

What's next?

☑ Dispersive analysis of the $\gamma\gamma \rightarrow D^+D^-, D^0\bar{D}^0$ data

☑ Consistency check with the $e^+e^- \rightarrow J/\psi D\bar{D}$ data

☑ No broad resonance corresponding to $X(3860)$ found

☑ Bound state below the $D\bar{D}$ threshold, $\sim D\bar{D}$ molecule

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☐ Light: $\gamma\gamma \to \pi^0\eta, K\bar{K}; (g-2)_\mu$ HLBL contributions

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Thank you!