

Exploring the nucleon structure: Fragmentation Functions from $e+e-$ annihilation experiment

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di Ferrara



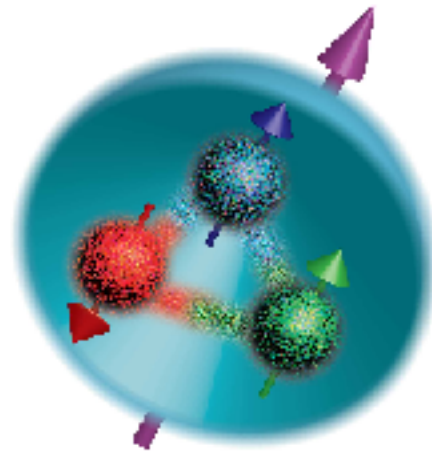
HOTEL & SPA MEDITERRANEAN VILLAGE, PARALIA (PIERIA, GREECE)

THURSDAY, MAY 26, 2022

Introduction

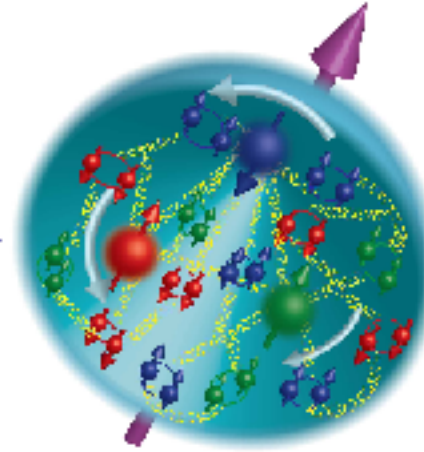
QCD picture of the nucleon

Naive picture



three non-relativistic quarks

Realistic picture



*infinite number of relativistic quarks
and gluons*

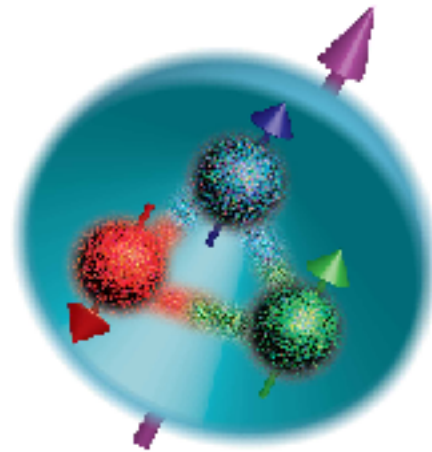
*Parton Distribution
Functions
(PDFs)*

<https://www.annualreviews.org/doi/pdf/10.1146/annurev-nucl-011720-042725>

Introduction

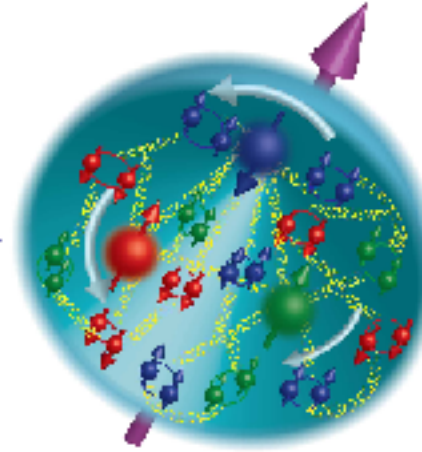
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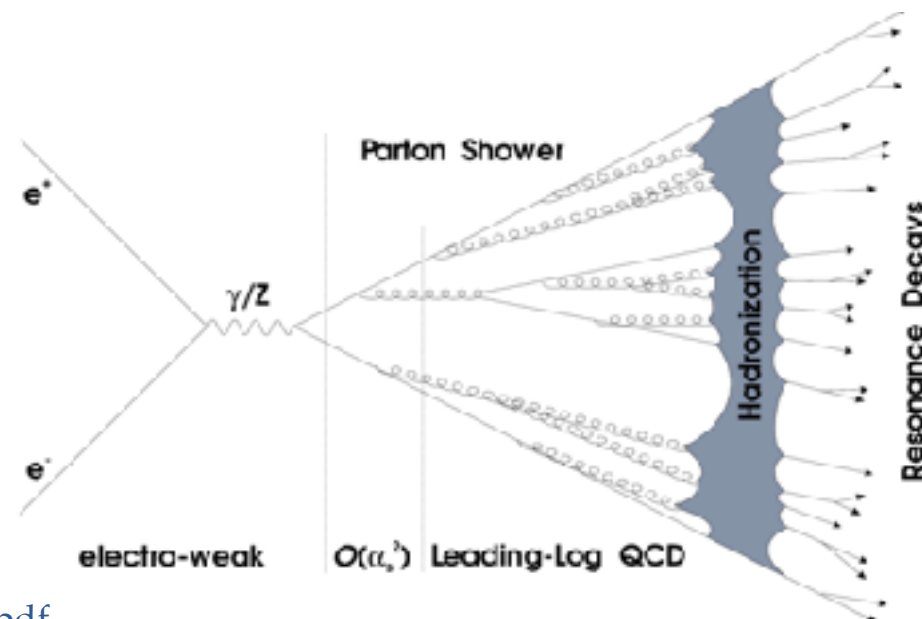
infinite number of relativistic quarks and gluons

Parton Distribution Functions (PDFs)

<https://www.annualreviews.org/doi/pdf/10.1146/annurev-nucl-011720-042725>

Hadron formation

- How many particles and how many jets created?
- What fraction of the initial parton momenta do they carry?



Fragmentation Functions (FFs)

<https://pdg.lbl.gov/2019/reviews/rpp2019-rev-frag-functions.pdf>

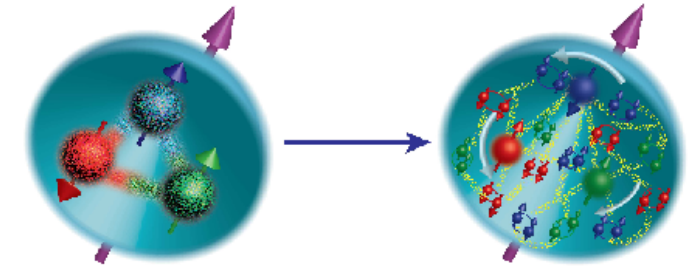
Universal and non-perturbative objects

Parton Distribution Functions

		Quark polarization		
		Unpolarized (U)	Longitudinally polarized (L)	Transversely polarized (T)
Nucleon polarization	U	$f_1 = \text{[Diagram: circle with dot]}$		$h_1^\perp = \text{[Diagram: circle with dot]} - \text{[Diagram: circle with dot]}$ Boer-Mulder
	L		$g_1 = \text{[Diagram: circle with arrow]} - \text{[Diagram: circle with arrow]}$ Helicity	$h_{1L}^\perp = \text{[Diagram: circle with arrow]} - \text{[Diagram: circle with arrow]}$
	T	$f_{1T}^\perp = \text{[Diagram: circle with dot]} - \text{[Diagram: circle with dot]}$ Sivers	$g_{1T}^\perp = \text{[Diagram: circle with arrow]} - \text{[Diagram: circle with arrow]}$	$h_{1T}^\perp = \text{[Diagram: circle with dot]} - \text{[Diagram: circle with dot]}$ Transversity $h_{1T}^\perp = \text{[Diagram: circle with arrow]} - \text{[Diagram: circle with arrow]}$



Transverse Momentum Dependent (TMD)
TMD independent

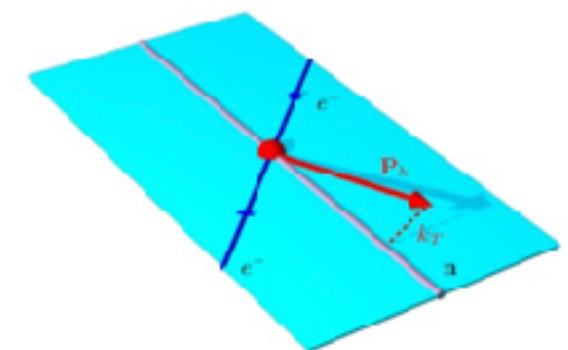


f_1 , g_1 , and h_1 : three leading-twist PDFs

- probability interpretation;
- complete description of the nucleon structure at leading-twist level;
- provide 2-dim (collinear) picture of the nucleon structure
- TMD: 3-D picture

h_1 : Transversity PDF

- Describe the distribution of quark's transverse spin in a transversely polarized nucleon
- Chiral-odd



Fragmentation Functions

FFs: formation of colourless hadrons starting from a coloured partonic initial state;

- probability that a parton i fragments into an hadron h carrying away a fraction z of the parton's momentum

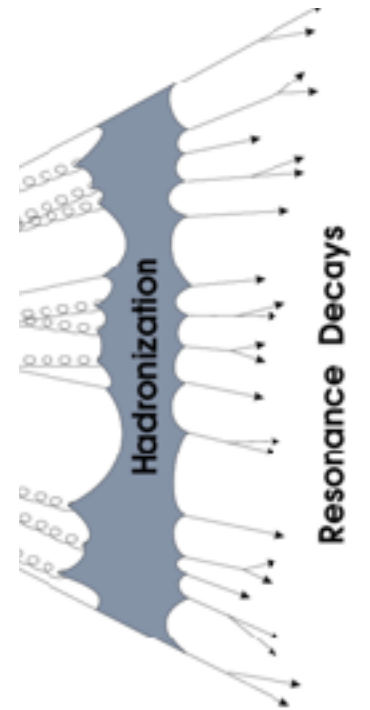
		quark pol.		
		U	L	T
hadron pol.	U	D_1		H_1^\perp
	L		G_{1L}	H_{1L}^\perp
	T	D_{1T}^\perp	G_{1T}	H_1, H_{1T}^\perp

H_1^\perp : Collins FF

- Chiral-odd function

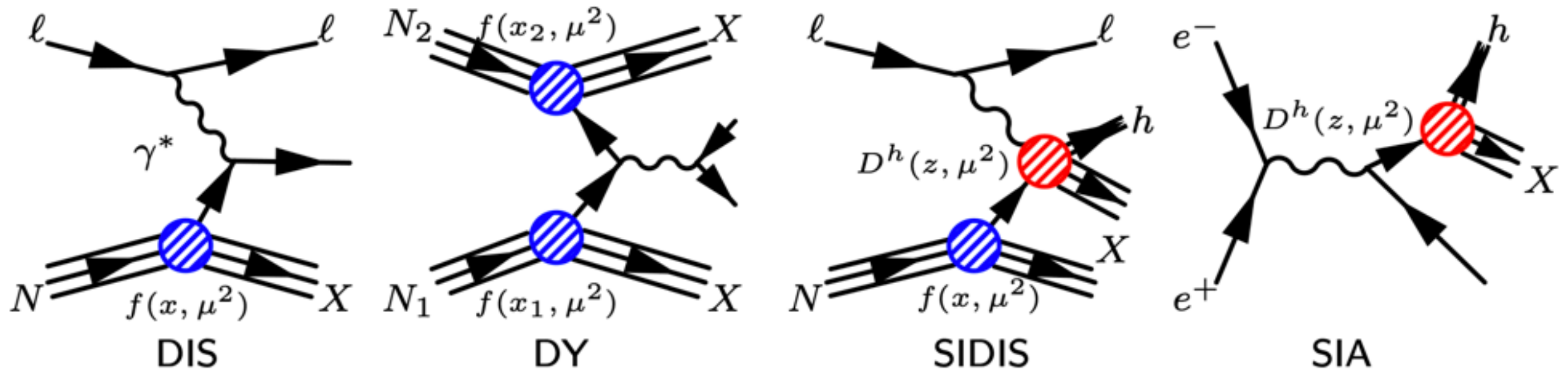
Transverse Momentum Dependent (TMD) FFs \Rightarrow to study the spin-dependent observables

- tools to investigate the 3D-structure of nucleons
- when only spinless hadrons (π , K) are considered, we have only D_1 and H_1^\perp
- TMDs evolution



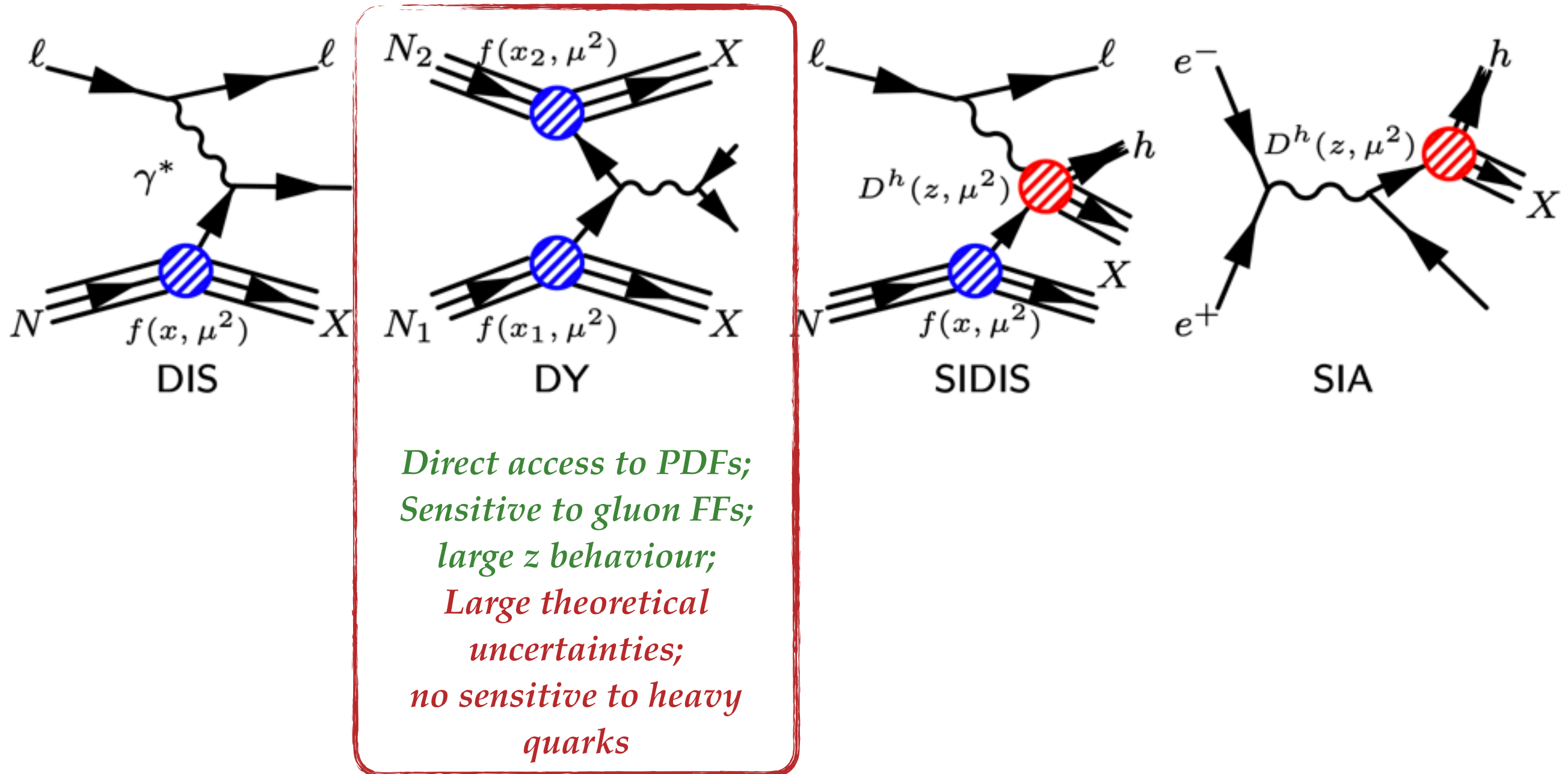
How to access PDFs and FFs

- Physical observables are written as a convolutions of coefficient functions and PDFs/FFs
- QCD factorization theorem*: separate the cross sections into a perturbatively calculable scattering contributions and non-perturbative one encoded in PDFs/FFs



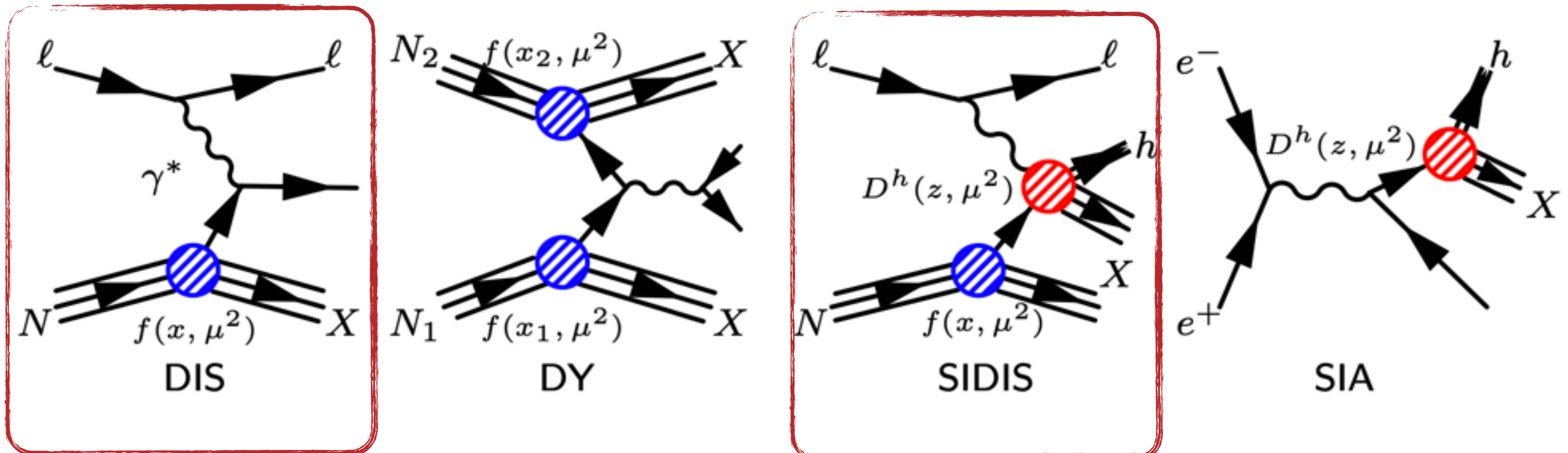
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How to access PDFs and FFs

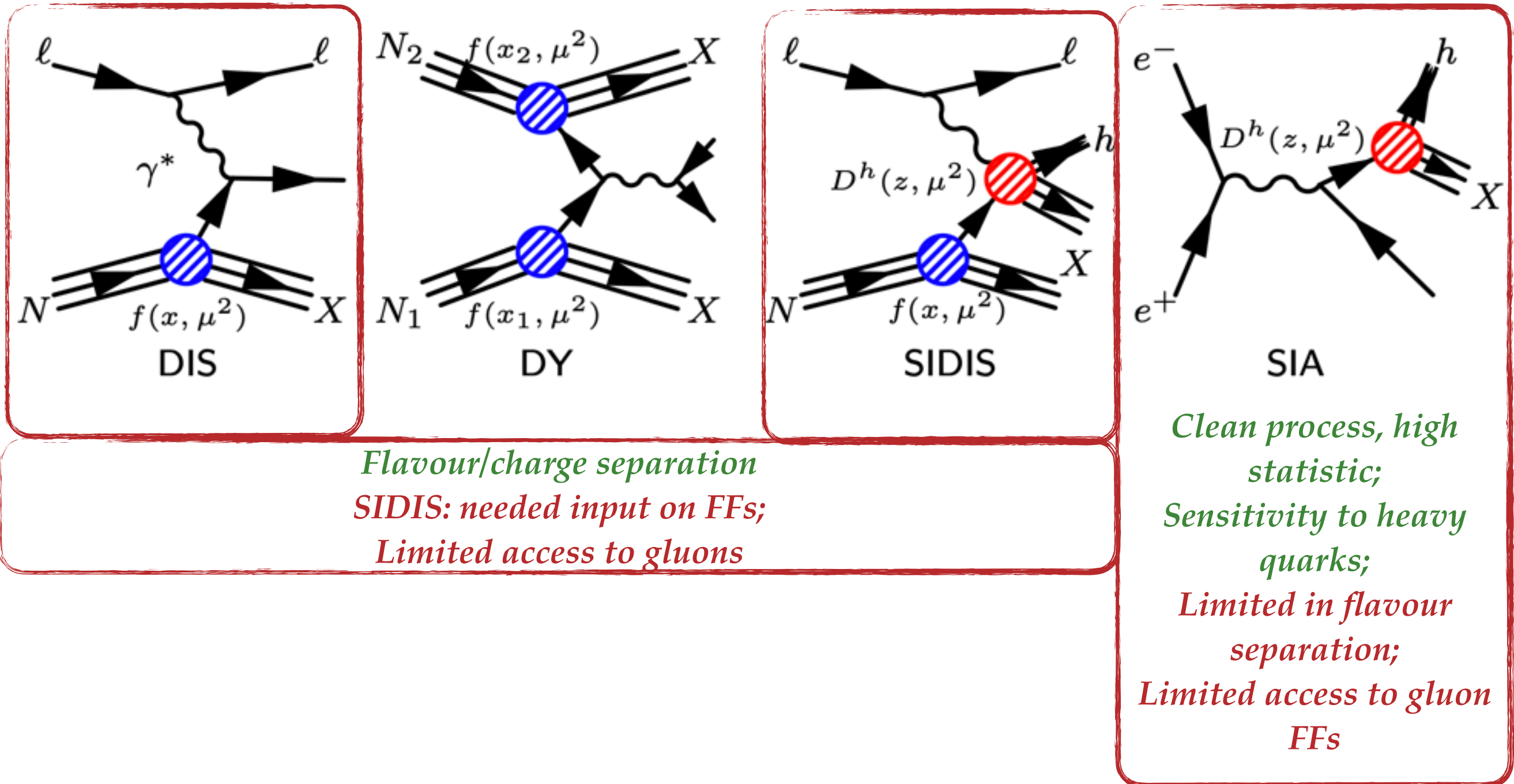
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Flavour/charge separation
SIDIS: needed input on FFs;
Limited access to gluons

How to access PDFs and FFs

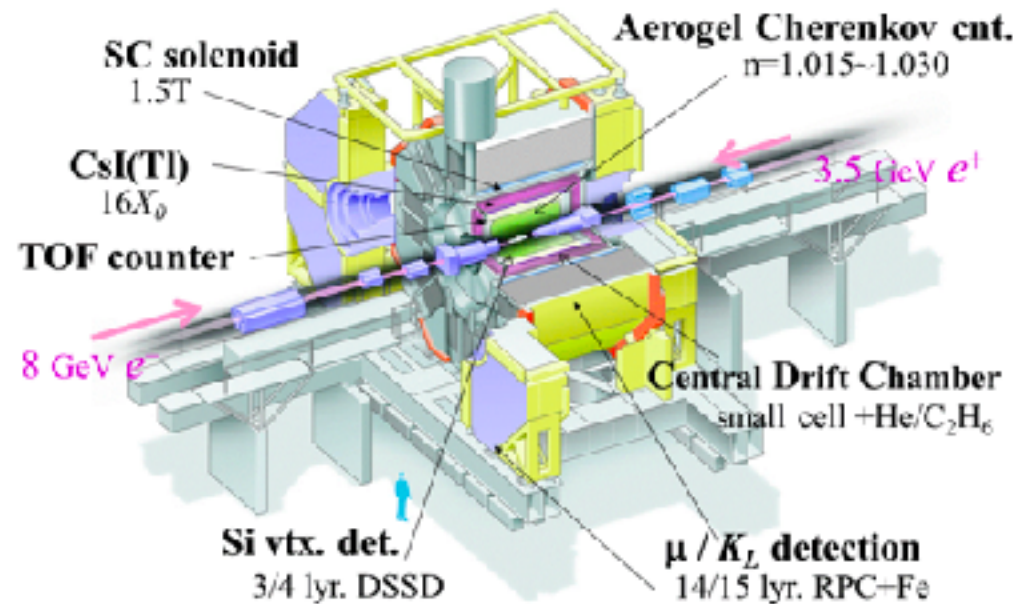
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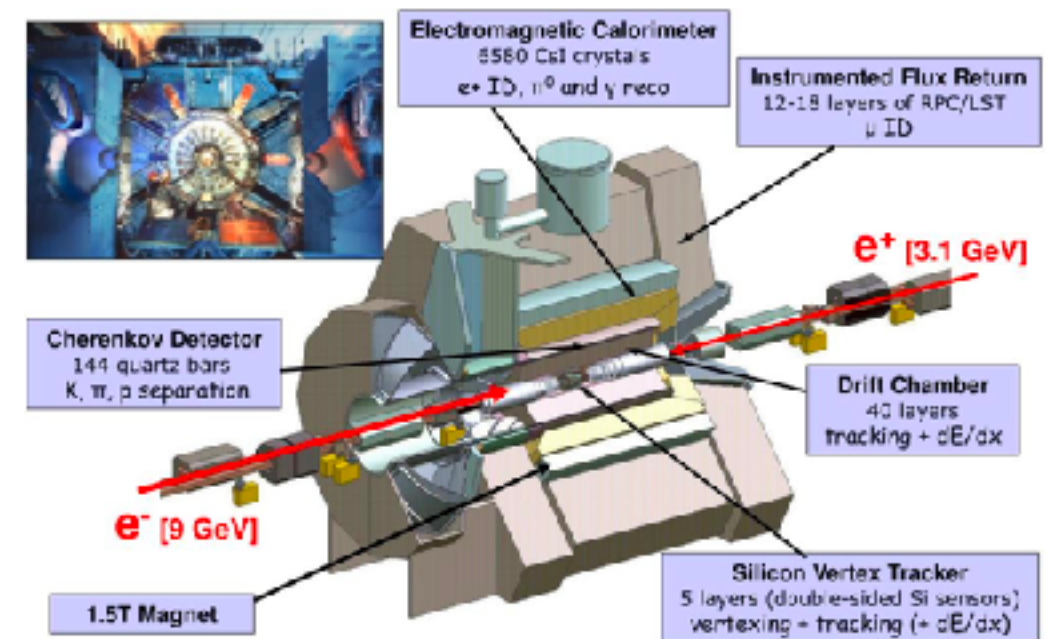
Belle, BaBar, BESIII detectors



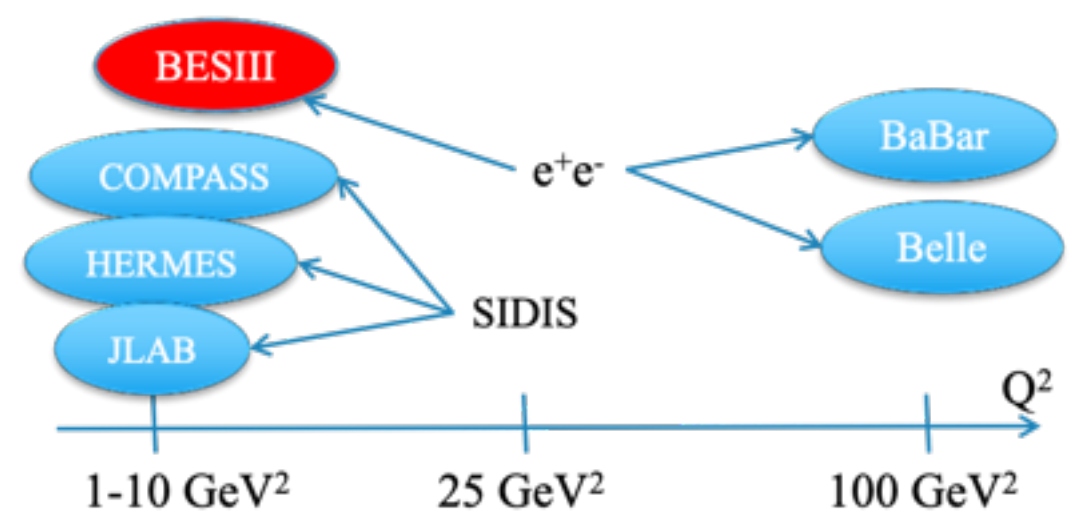
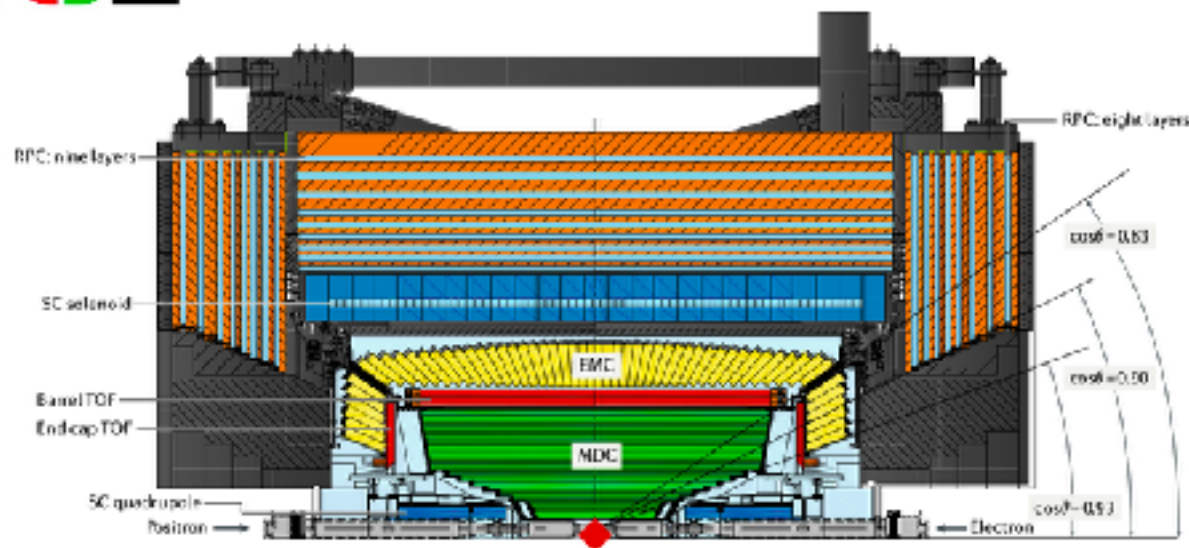
Belle Detector



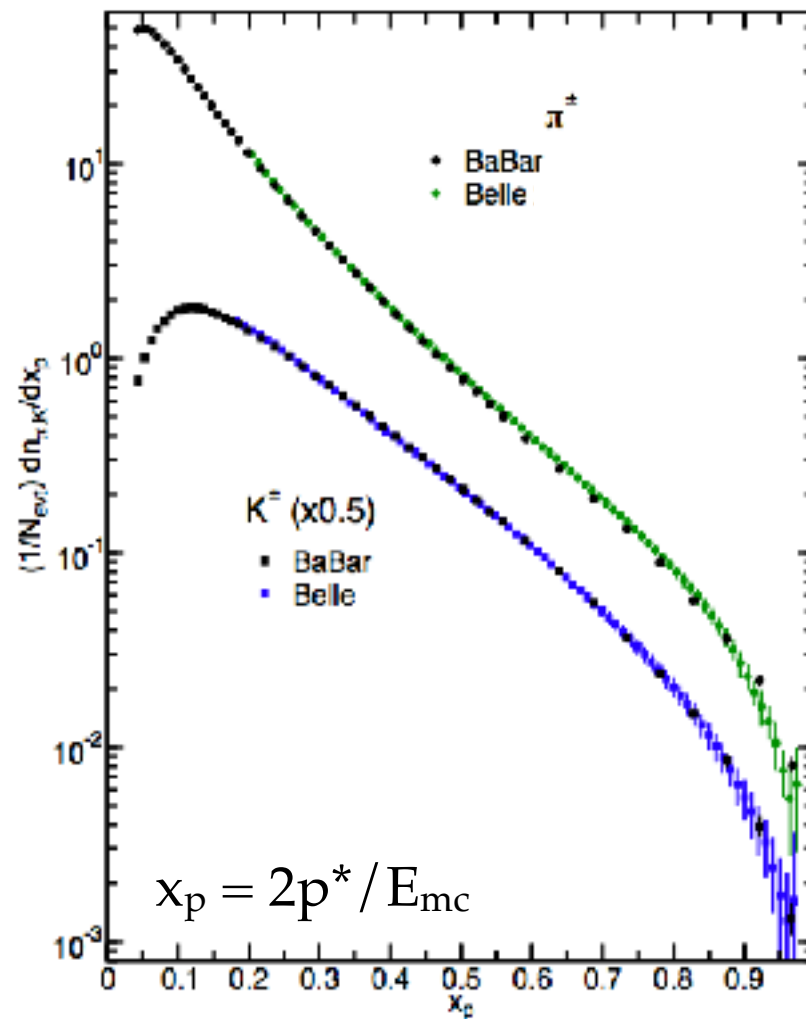
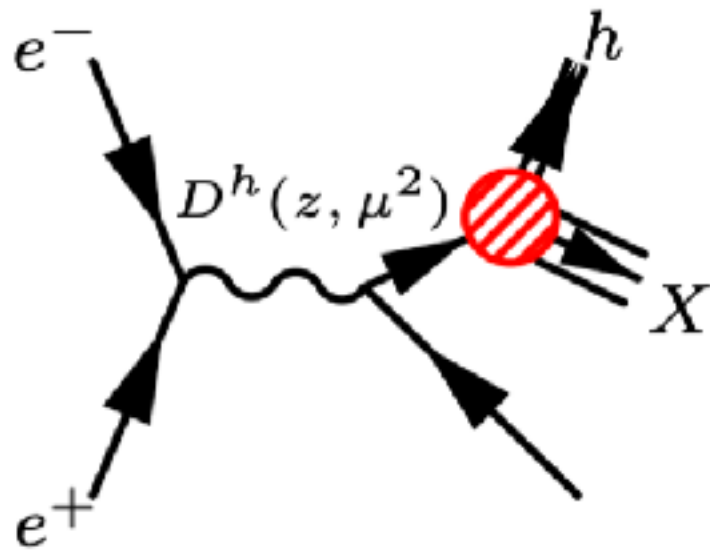
BaBar Detector



BESIII Detector

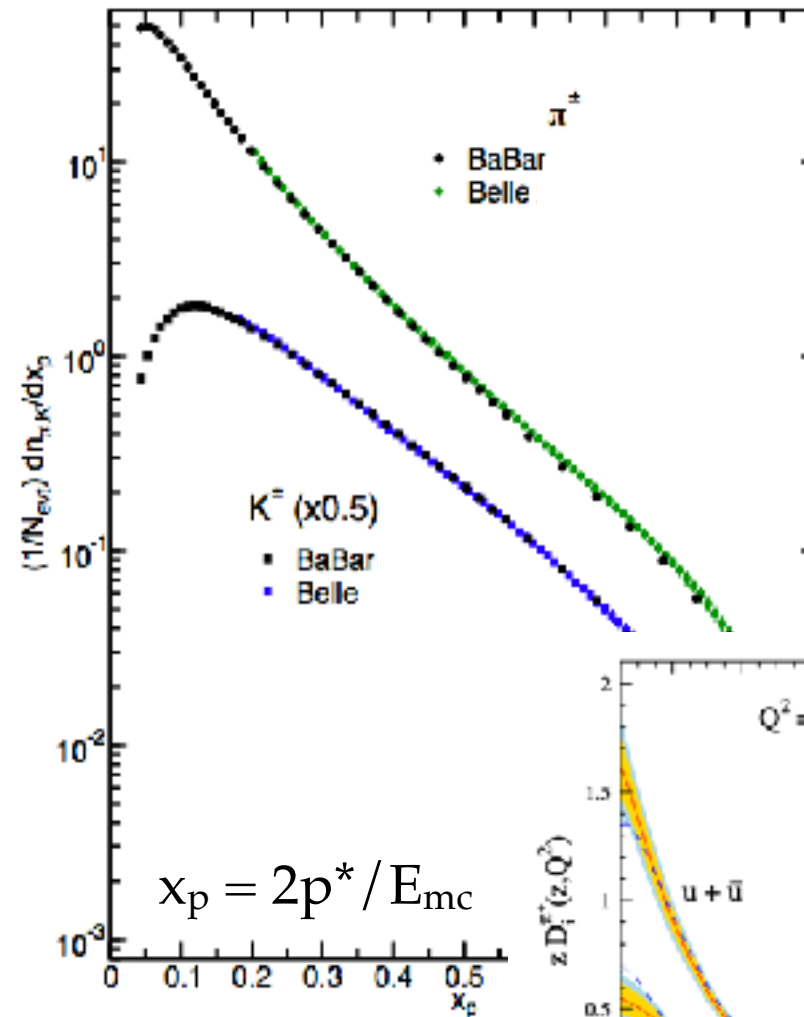
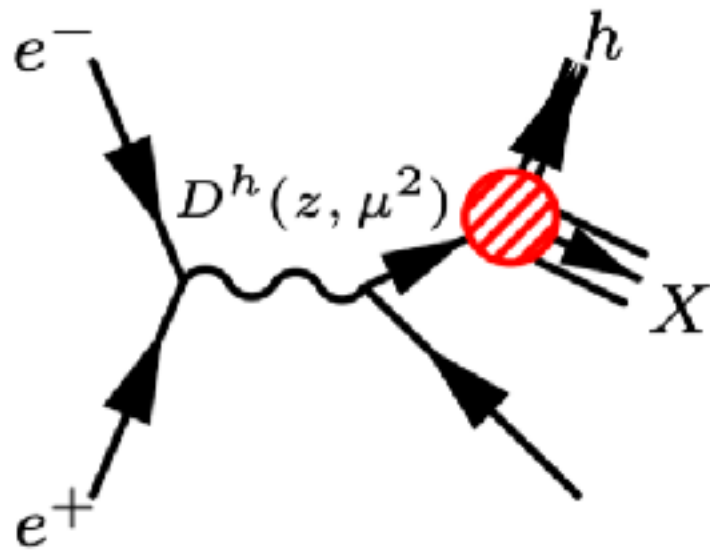


Unpolarised FFs



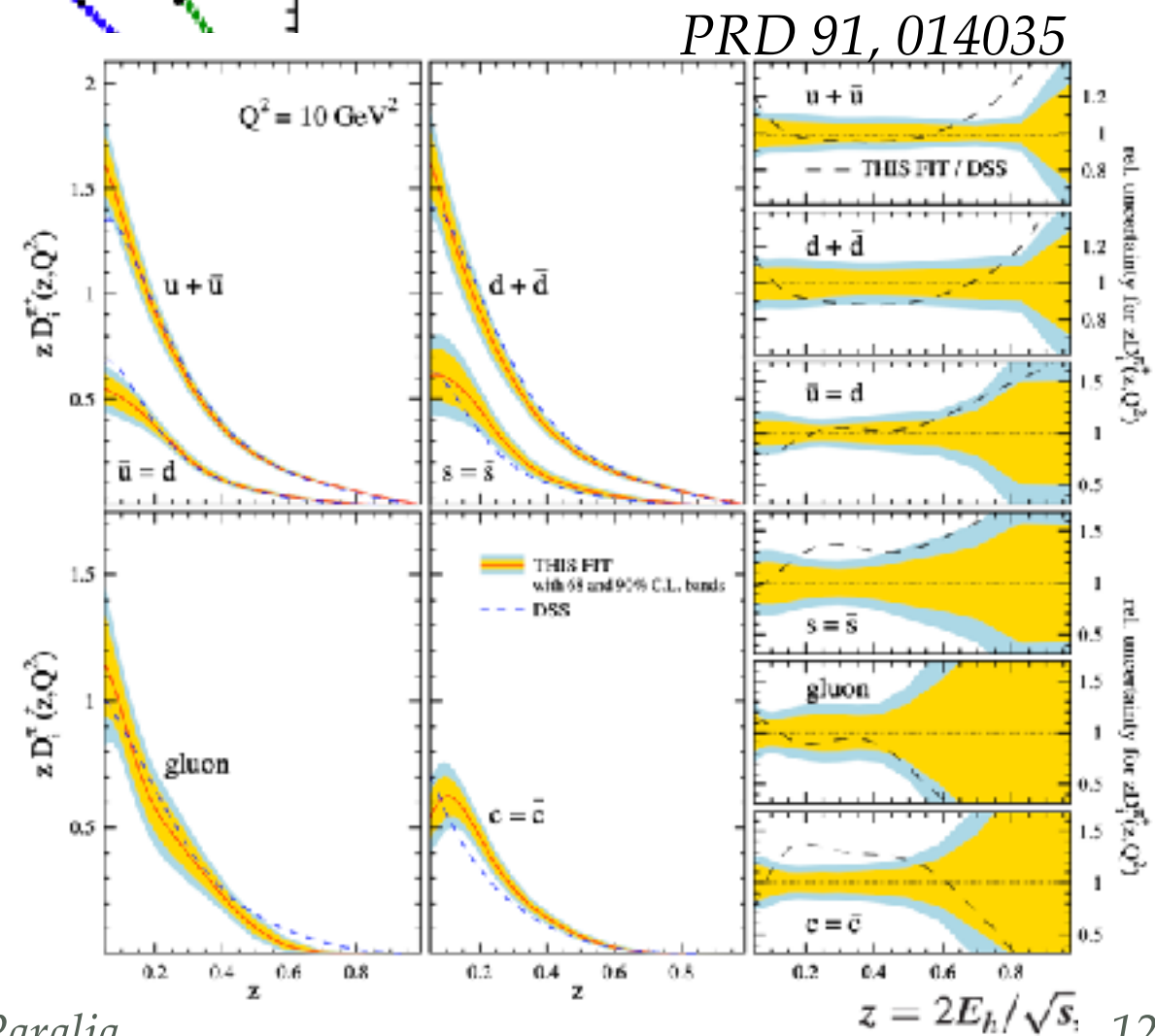
BaBar: [*PhysRevD.88.032011*](#)
 Belle: [*PhysRevLett.111.062002*](#)
 *arbitrary normalisation in order to
 compare the shape

Unpolarised FFs



BaBar: PhysRevD.88.032011
Belle: PhysRevLett.111.062002
 *arbitrary normalisation in order to
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- Global fit analysis (e+e-, SIDIS and pp data) performed in order to extract the FFs for gluons and different quark flavours
 - DSS fit: PRD 91, 014035 (it includes Belle and BaBar data)



$$z = 2E_h/\sqrt{s}, \quad 12$$

Inclusive light hadrons productions

- Data set used: 0.9 fb^{-1} @ $\Upsilon(4S)$ and 3.6 fb^{-1} at 10.54 GeV
- measured both *prompt* and *conventional* hadron cross section
 - *prompt*: primary hadrons or products of a decay chain where all particles have a lifetime shorter than 10^{-11} s
 - *conventional*: includes weak decay products of KS and strange baryons
- Scaled momentum distribution: $x_p = 2p^*/E_{\text{mc}}$ ($0.2 < p < 5.27 \text{ GeV}/c$)

JETSET model:

- represent the color field between the parton by a “string”, and according to an iterative algorithm breaks the string into several pieces, each corresponding to a primary hadron
- **large number of free parameters** (models many hadron species)

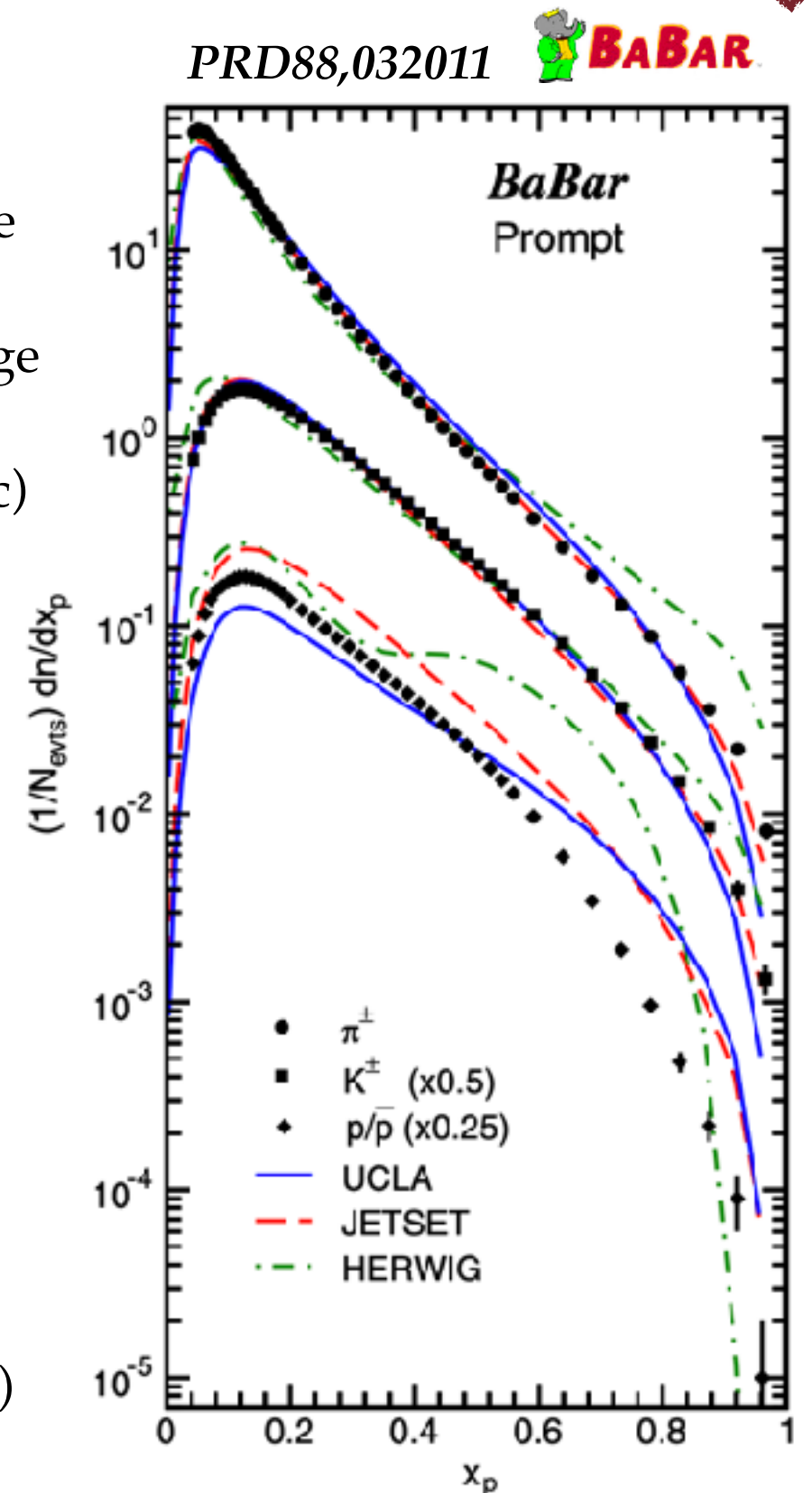
HERWIG model:

- splits the gluons produced into $q\bar{q}$ pairs, combines these quark and antiquark locally to form colorless “clusters”, and decay these “clusters” into primary hadrons
- **few free parameters**

UCLA model:

- generates whole events according to weights derived from phase space and Clebsch-Gordan coefficients
- **few free parameters**

- Good qualitative description of the bulk spectra (no the details)

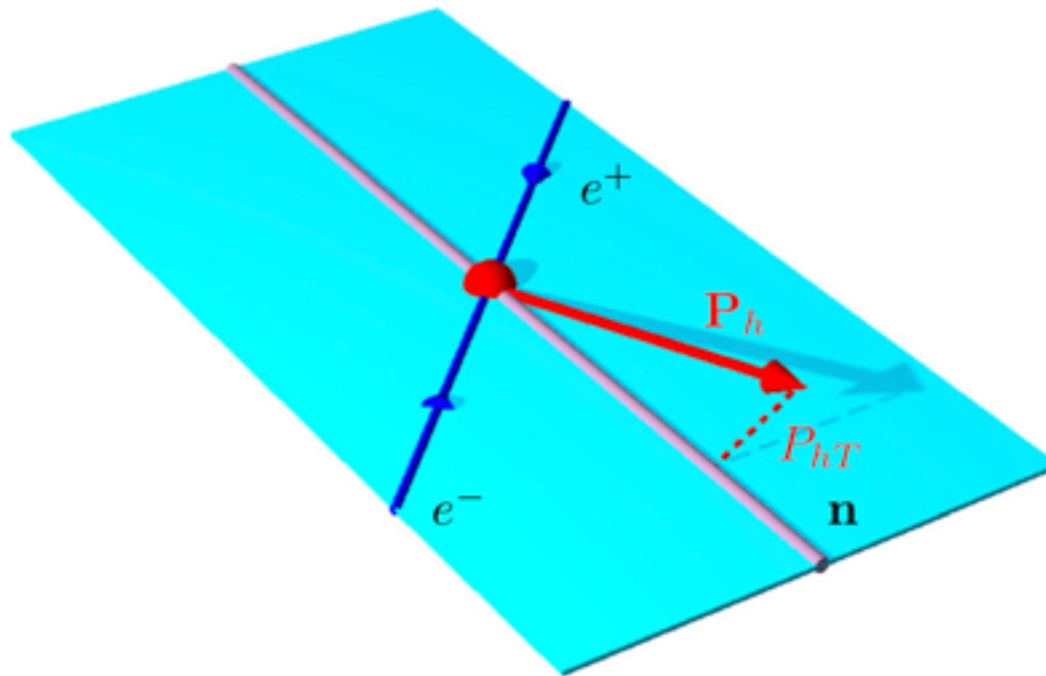


TMD single-hadron production cross sections @ Belle



$$D_1^h(z, \mu^2, k_T)$$

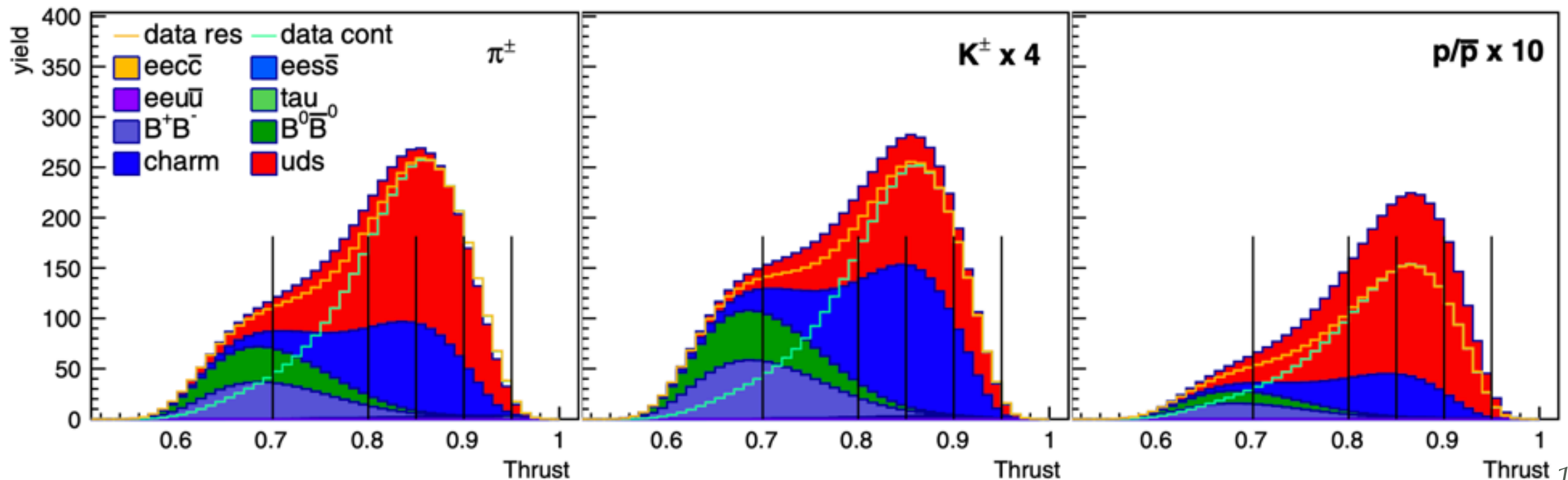
PRD 99, 112006



Definition of thrust event shape variable:

$$T_{\max} \equiv \frac{\sum_h |\mathbf{P}_h^{\text{CMS}} \cdot \hat{\mathbf{n}}|}{\sum_h |\mathbf{P}_h^{\text{CMS}}|}$$

$$z = 2E_h/\sqrt{s}, \quad \mathbf{p}_T = -z\mathbf{k}_T$$

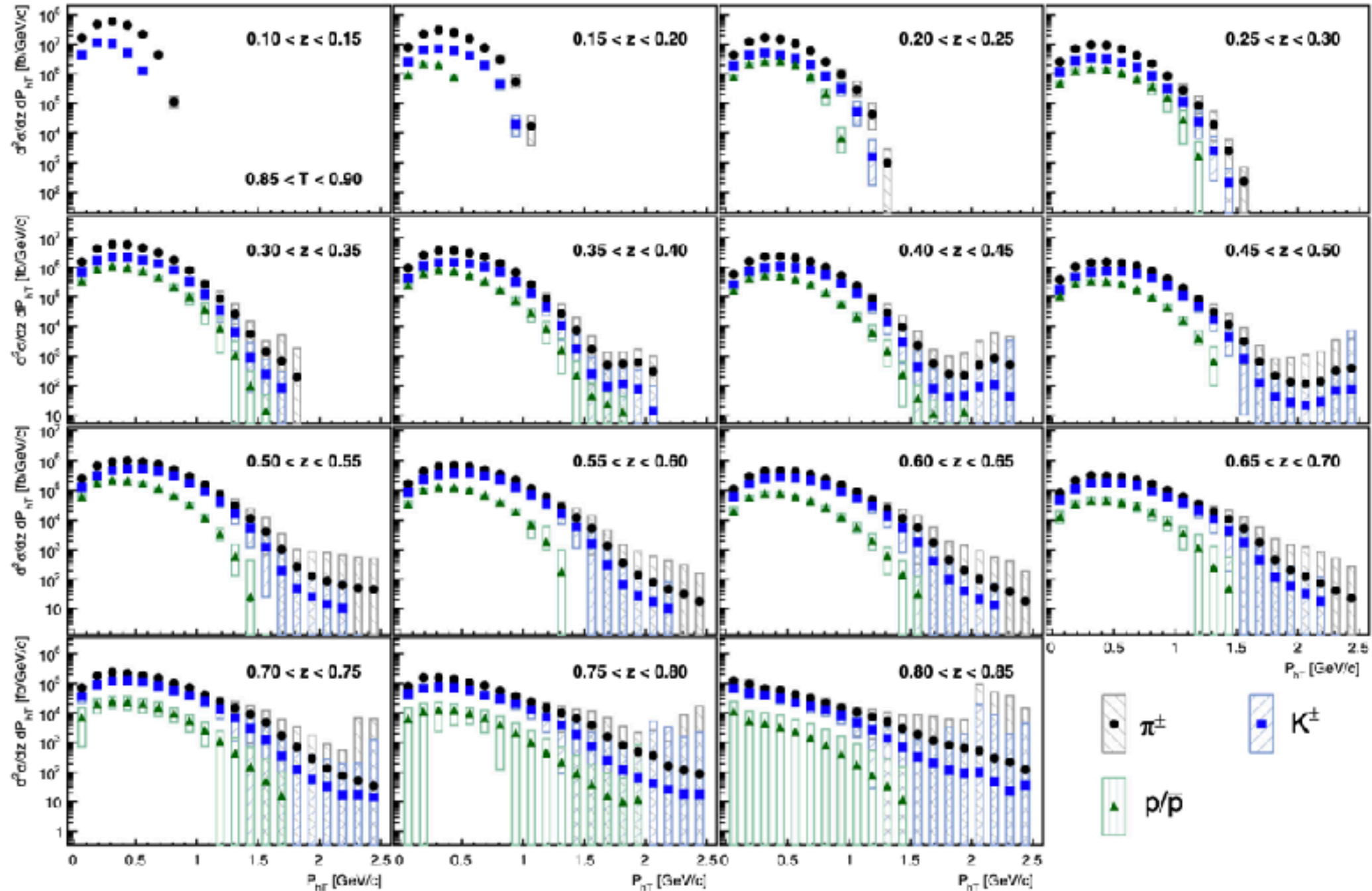


TMD single-hadron production cross sections @ Belle



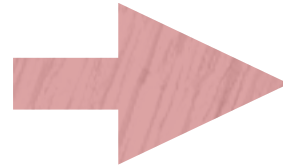
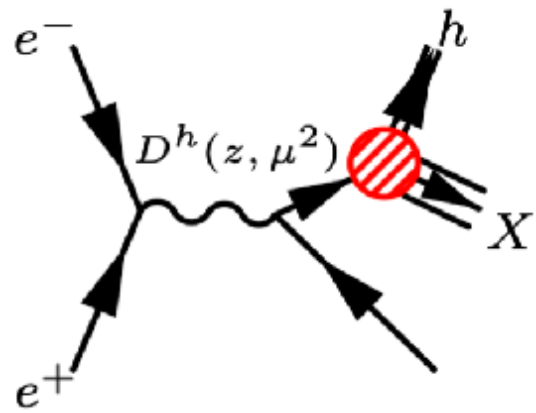
$0.85 < T < 0.9$

PRD 99, 112006



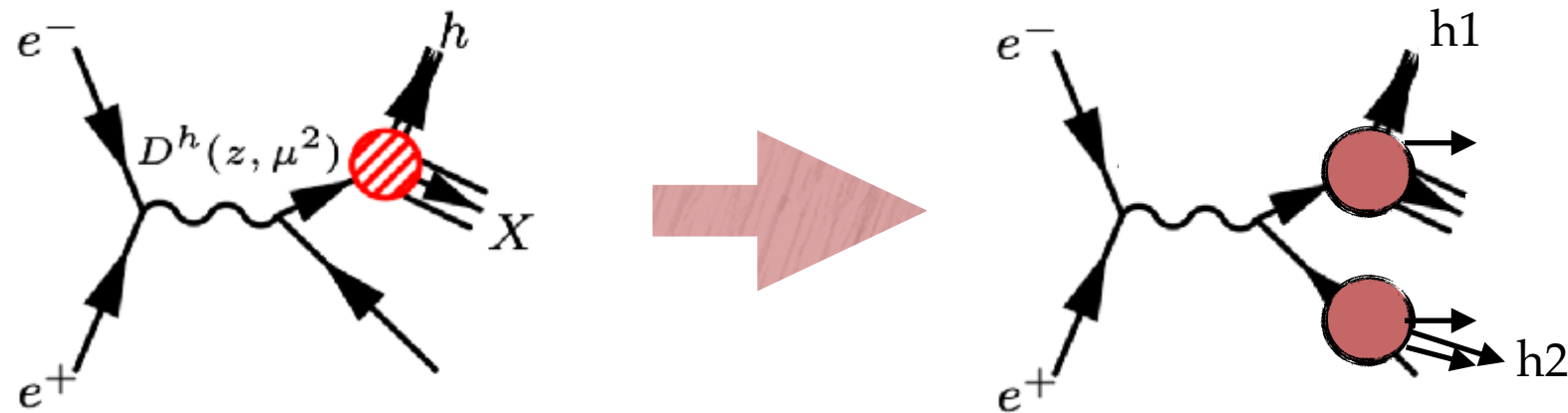
First direct measurements for pions, kaons and protons: better understanding of transverse spin phenomena and important role in the future electron-ion collider to pin down the transverse-momentum structure of nucleons and its transverse spin

Polarised FF: the Collins effect



H_1^\perp ?
chiral-odd function

Polarised FF: the Collins effect



$$q^\uparrow \rightarrow hX: \quad D_1^{q^\uparrow}(z, \mathbf{P}_\perp; s_q) = D_1^q(z, P_\perp) + \frac{P_\perp}{zM_h} H_1^{\perp q}(z, P_\perp) \mathbf{s}_q \cdot (\mathbf{k}_q \times \mathbf{P}_\perp)$$

Unpolarized FF

Collins FF [NPB 396, 161 (1993)]:
related to the probability that a transversely polarized quark (q^\uparrow) fragments into a spinless hadron

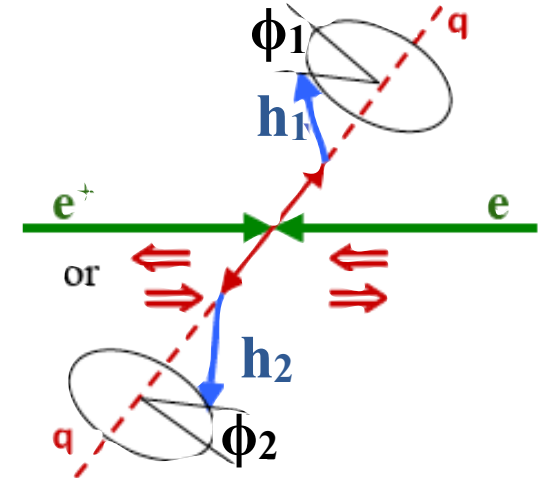
- Evolution of TMD objects
- Global analysis (PRD 78,032011 (2007); PRD 87,094019 (2013), PRD 91,014034 (2015)):
 - combines Semi Inclusive Deep Inelastic Scattering (SIDIS) and e^+e^- data
 - extraction of H_1^\perp and transversity parton distributions h_1 for the “u” and “d” quarks

Collins effect in e^+e^- annihilation

In $e^+e^- \rightarrow q\bar{q}$, spins unknown, but $s_q \parallel s_{\bar{q}}$

- exploit this correlation by using hadrons in opposite jets
- define **favored** ($u \rightarrow \pi^+$, $d \rightarrow \pi^-$) and **disfavored** ($d \rightarrow \pi^+$, $u \rightarrow \pi^-$, $s(\bar{s}) \rightarrow \pi^\pm$) FFs

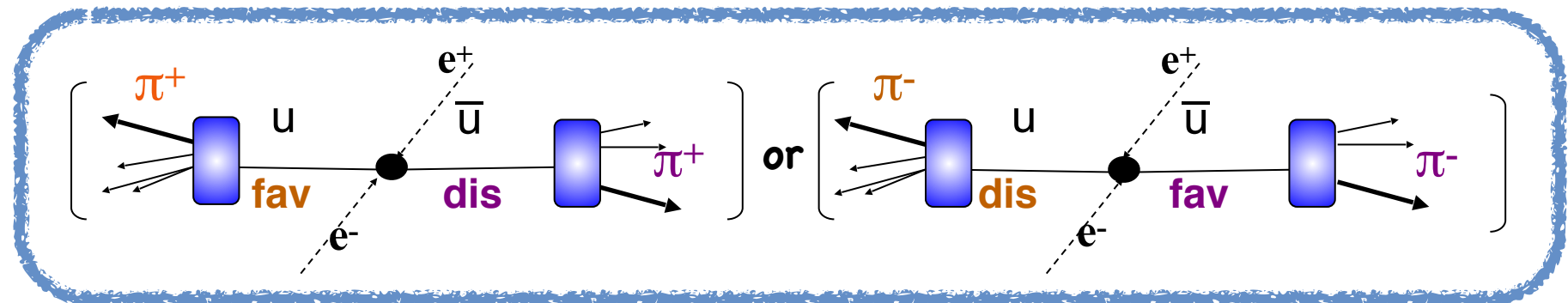
$$e^+e^- \rightarrow q\bar{q} \rightarrow h_1 h_2 X \quad (q=u,d,s) \Rightarrow \sigma \propto \underbrace{\cos(\phi_1 + \phi_2)}_{\text{Collins effect}} H_{1\perp}^{(h_1)} \times H_{1\perp}^{(h_2)}$$



**Azimuthal modulation wrt the quark spin direction:
Collins effect (or Collins asymmetry)**

Example: Like $\pi\pi$ pairs (L)

Collins asymmetry for $\pi\pi$

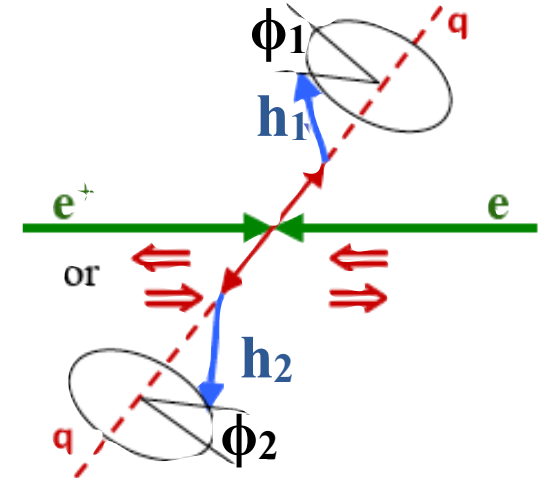


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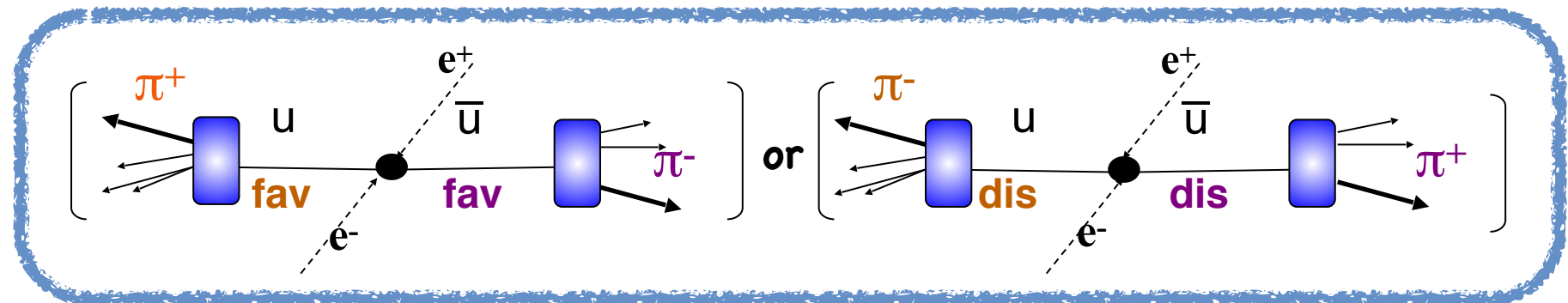
$$e^+e^- \rightarrow q\bar{q} \rightarrow h_1 h_2 X \quad (q=u,d,s) \Rightarrow \sigma \propto \cos(\phi_1 + \phi_2) H_{1\perp}^{(h_1)} \times H_{1\perp}^{(h_2)}$$



**Azimuthal modulation wrt the quark spin direction:
Collins effect (or Collins asymmetry)**

Example: Unlike $\pi\pi$ pairs (U)

Collins asymmetry for $\pi\pi$

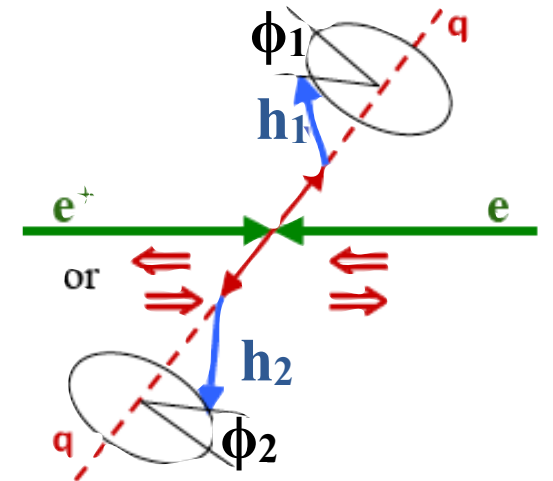


Collins effect in e^+e^- annihilation

In $e^+e^- \rightarrow q\bar{q}$, spins unknown, but $s_q \parallel s_{\bar{q}}$

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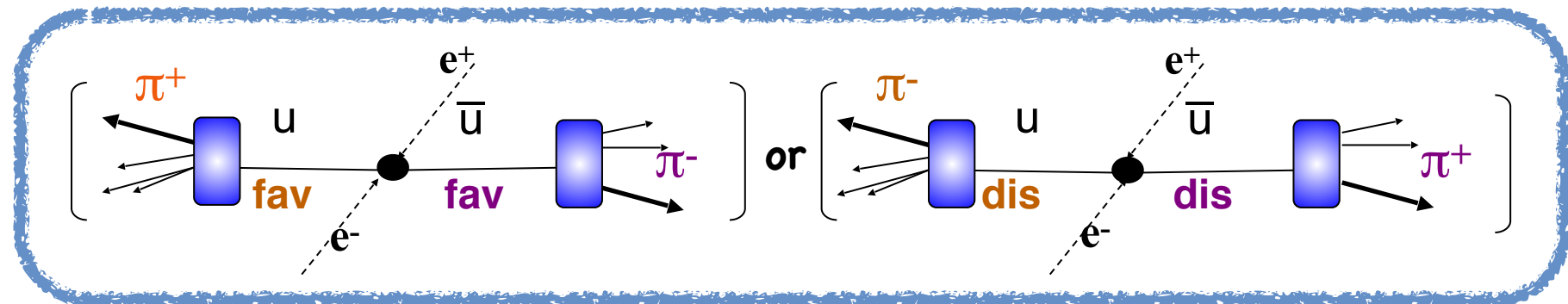
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**Azimuthal modulation wrt the quark spin direction:
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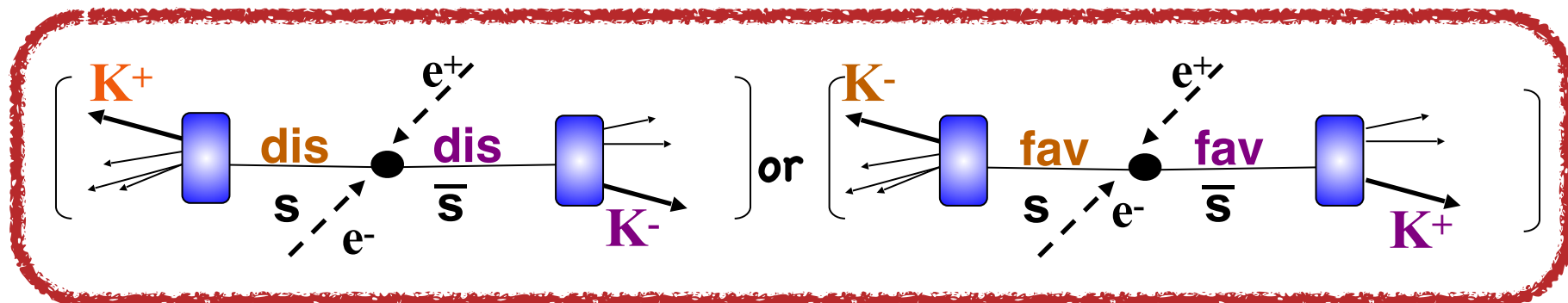
Example: Unlike $\pi\pi$ pairs (U)

Collins asymmetry for $\pi\pi$



Example: Unlike KK pairs (U)

Collins asymmetry for KK:



Reference Frames

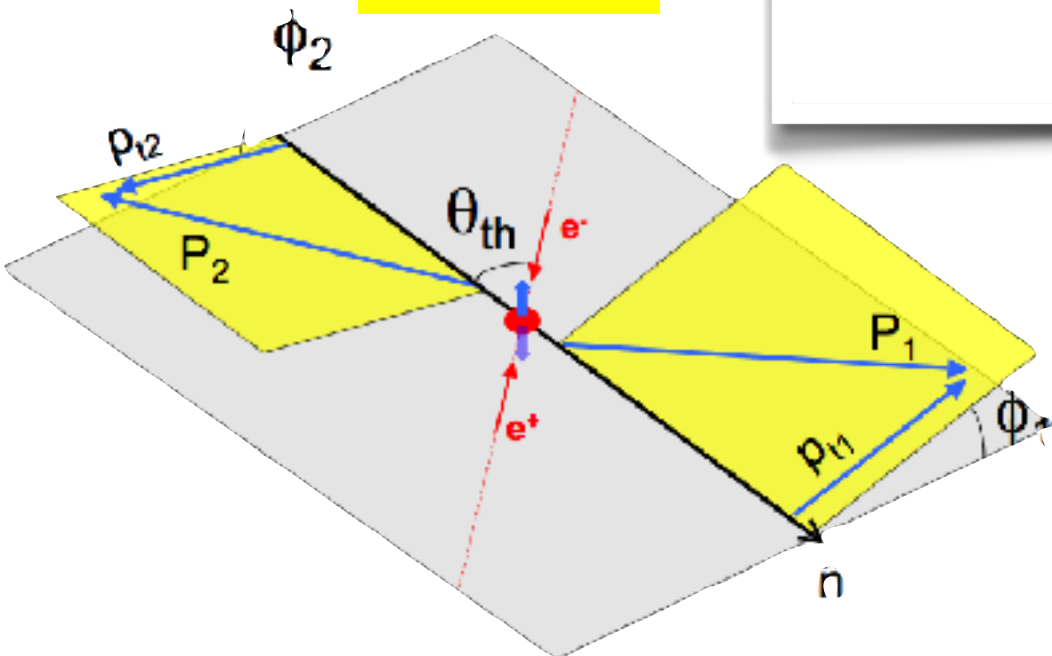
See D. Boer, NPB 806, 23 (2009)

RF12

$$\frac{d\sigma(e^+e^- \rightarrow h_1 h_2 X)}{d\Omega dz_1 dz_2 d\phi_1 d\phi_2} = \sum_{q,\bar{q}} \frac{3\alpha^2}{Q^2} \frac{e_q^2}{4} z_1^2 z_2^2 \left[(1 + \cos^2\theta) D_1^{q,(0)}(z_1) \bar{D}_1^{q,(0)}(z_2) + \sin^2(\theta) \cos(\phi_1 + \phi_2) H_1^{\perp,(1),q}(z_1) \bar{H}_1^{\perp,(1),q}(z_2) \right]$$

All quantities in e+e- center of mass

θ : angle between the e⁺e⁻ axis and the thrust axis;
 $\phi_{1,2}$: azimuthal angles between P_{h1(h2)} and the scattering plane

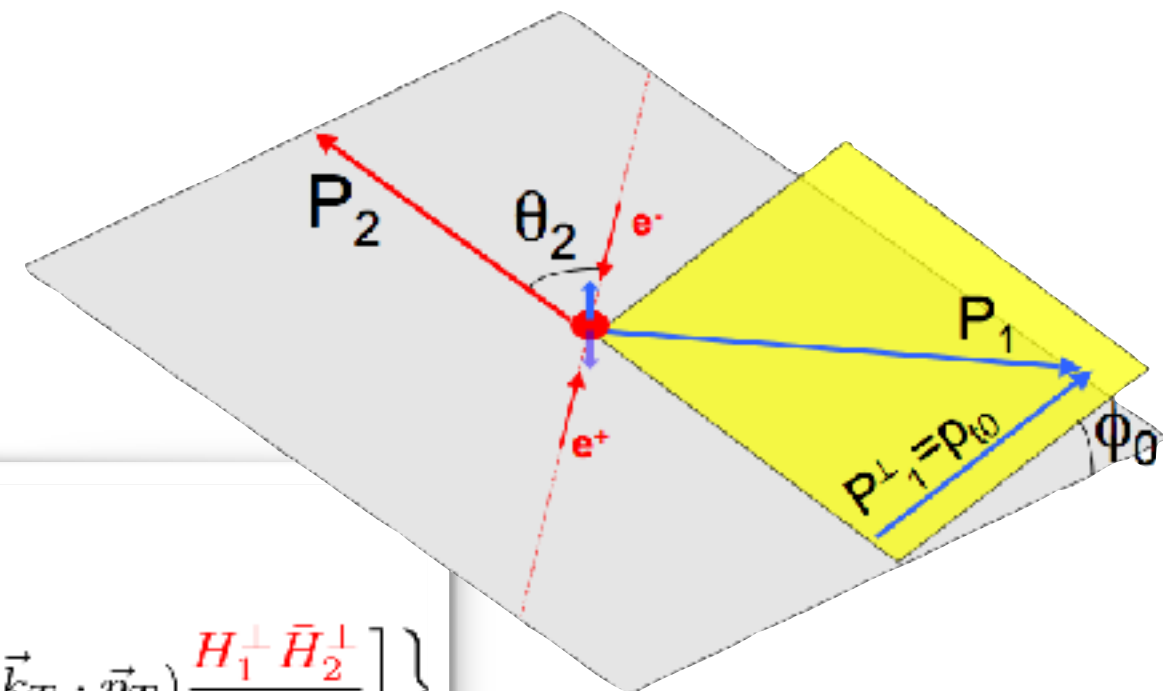


θ_2 : angle between the e⁺e⁻ axis and P_{h2};
 ϕ_0 : angle between the plane spanned by P_{h2} and the e⁺e⁻ axis, and the direction of P_{h1} perpendicular to P_{h2}.

All quantities in e+e- center of mass

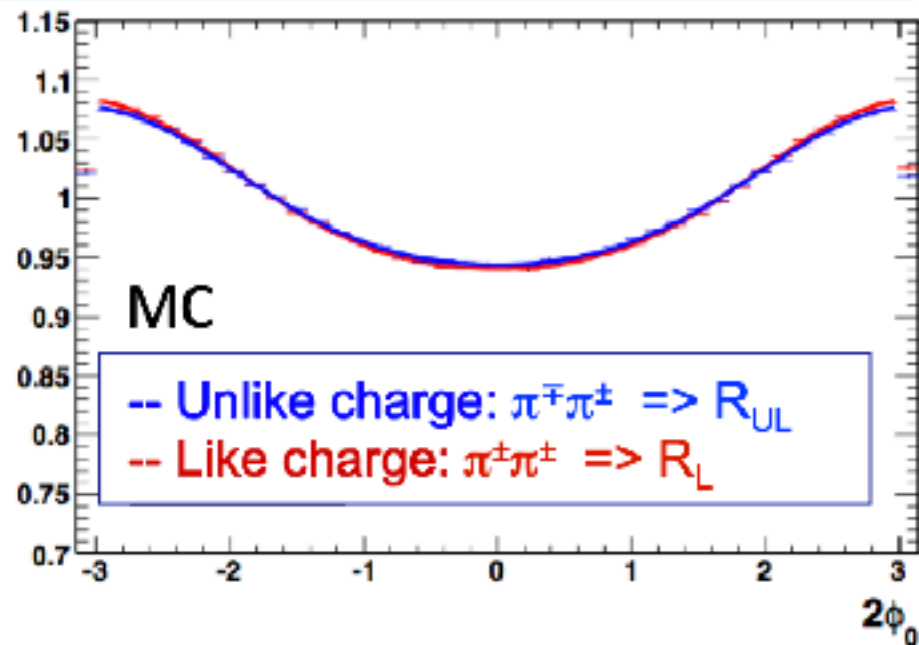
$$\frac{d\sigma(e^+e^- \rightarrow h_1 h_2 X)}{d\Omega dz_1 dz_2 d^2\vec{q}_T} = \frac{3\alpha^2}{Q^2} z_1^2 z_2^2 \left\{ A(y) \mathcal{F}[D_1 \bar{D}_2] + B(y) \cos(2\phi_0) \mathcal{F} \left[(2\hat{h} \cdot \vec{k}_T \hat{h} \cdot \vec{p}_T - \vec{k}_T \cdot \vec{p}_T) \frac{H_1^\perp \bar{H}_2^\perp}{M_1 M_2} \right] \right\}$$

RF0

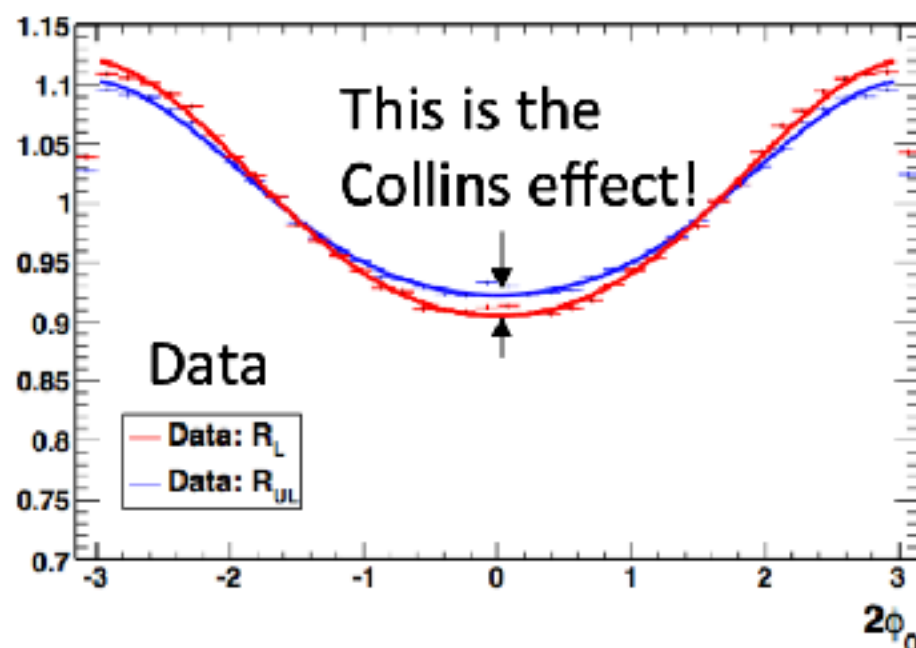


Asymmetry Measurements from Data

$N^{(U,L)}(\phi)/\langle N^{(U,L)}(\phi) \rangle$ in MC sample



$N^{(U,L)}(\phi)/\langle N^{(U,L)}(\phi) \rangle$ in data sample



- Collins asymmetries extracted from fit to the normalised azimuthal distribution

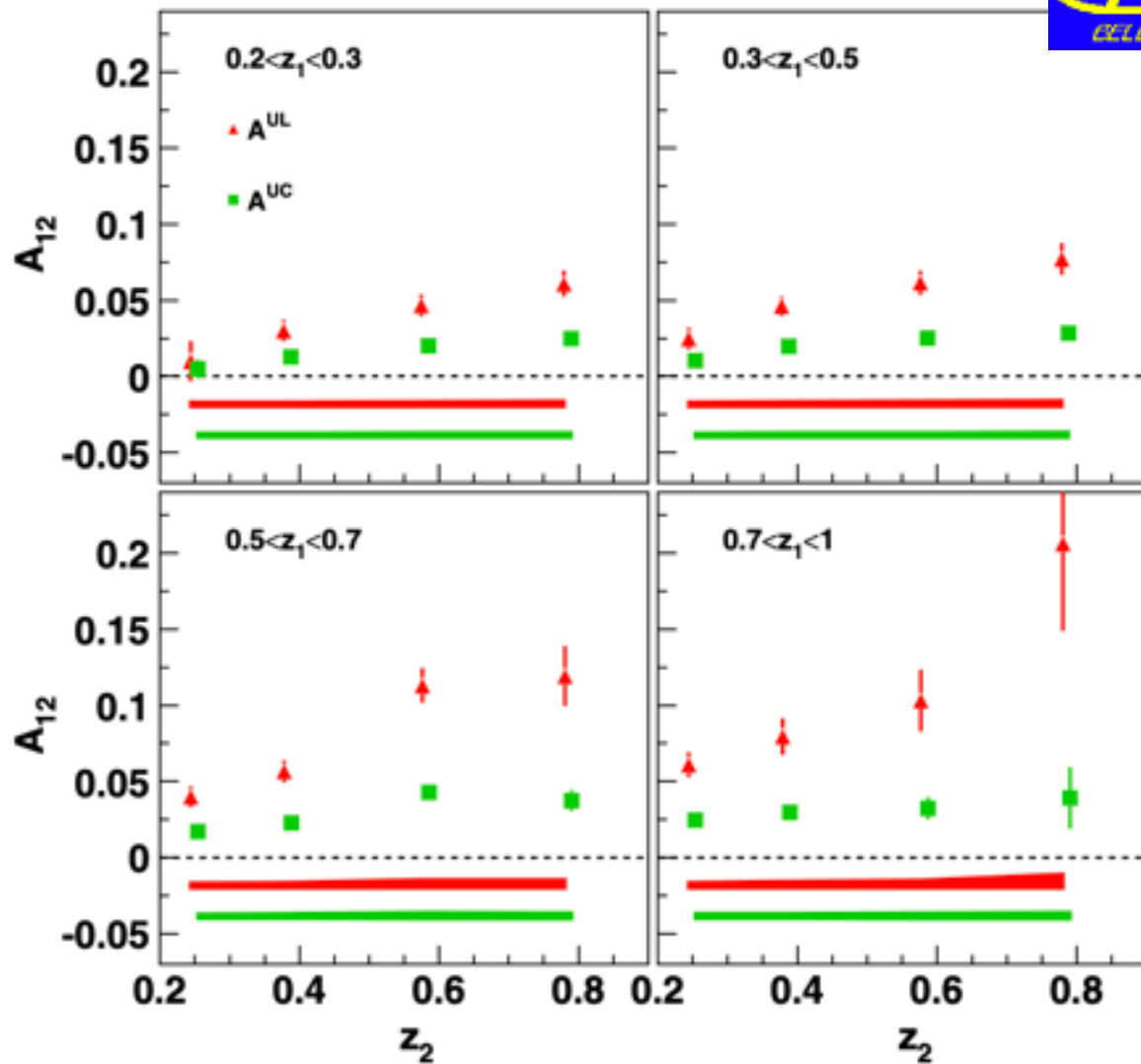
$$R_{\alpha} = \frac{N(\phi_{\alpha})}{\langle N_{\alpha} \rangle} = a + b \cdot \cos(\phi_{\alpha})$$

- MC generator does not include polarised FFs as the Collins FF
 - modulation observed in MC sample produced by detector acceptance
 - correction of these effects would bring too large systematic uncertainties
- Ratio of U and L distributions
 - Collins effect is not sensitive to the electric charge
 - U and L sigli different in data due to the different contribution of favoured and disfavoured FFs

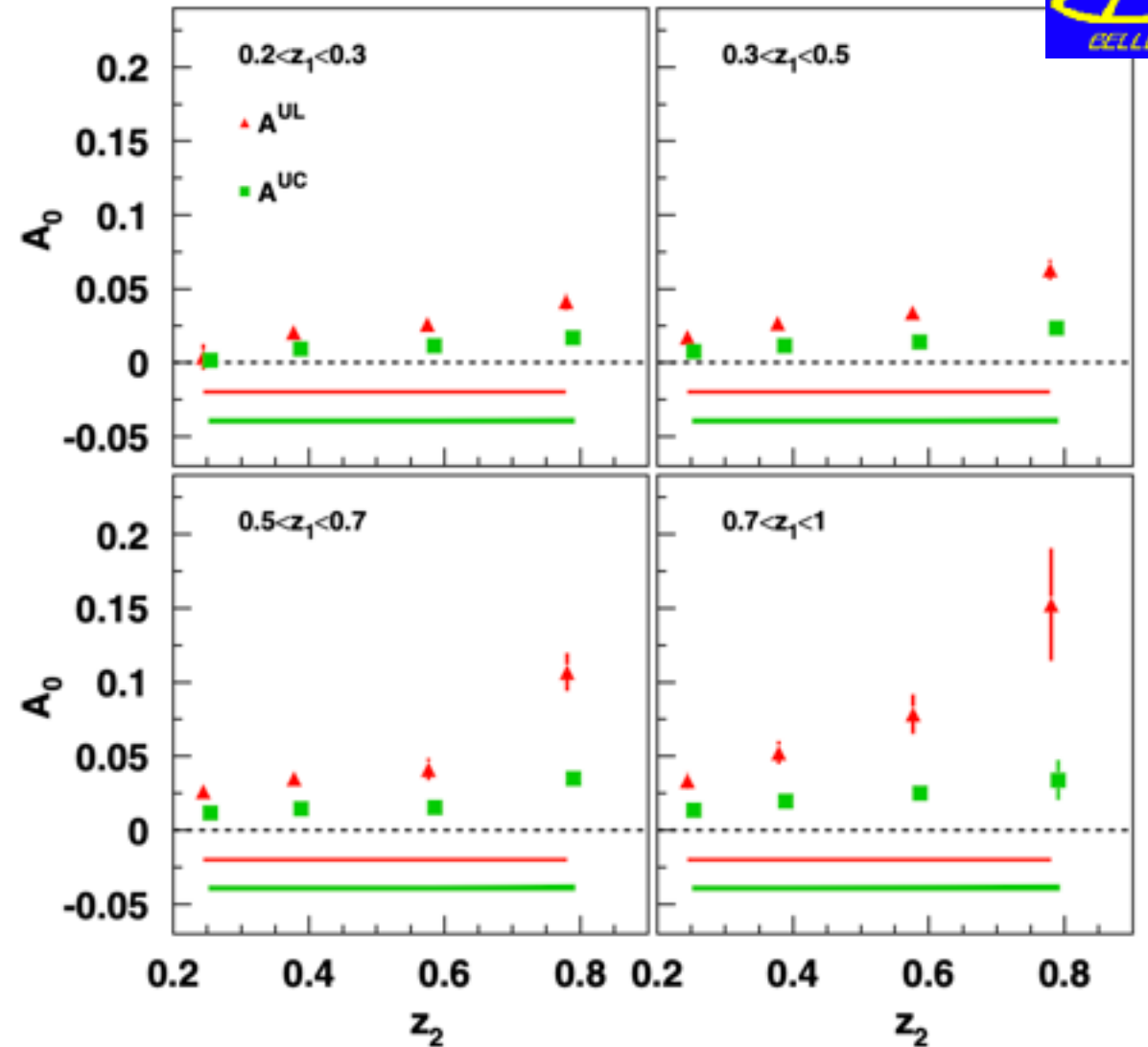
Collins FFs Results

First measurement of Collins effects in e^+e^- annihilation (PRL 96,232002) !!!

PRD86,039905



PRD86,039905

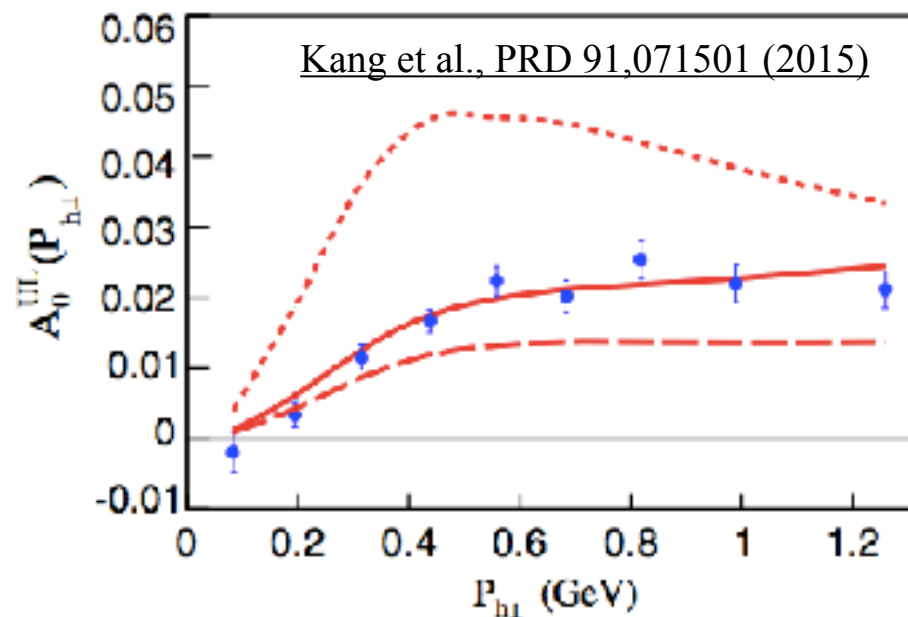
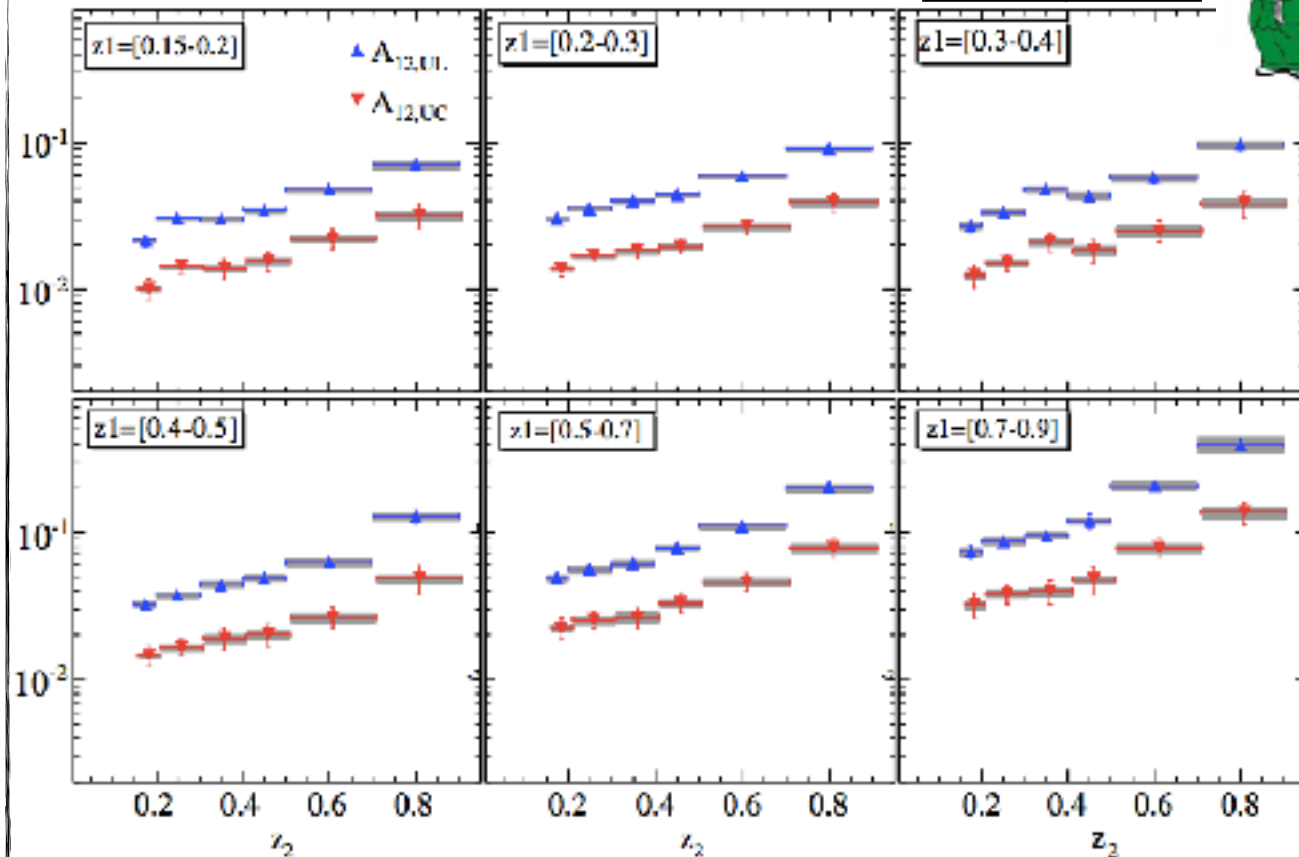


- $\mathcal{L} \sim 547 \text{ fb}^{-1}$ at $\sim 10.58 \text{ GeV}$
- Non-zero asymmetries increasing with z

Collins FFs Results

$\mathcal{L} \sim 468 \text{ fb}^{-1}$ at $\sim 10.58 \text{ GeV}$

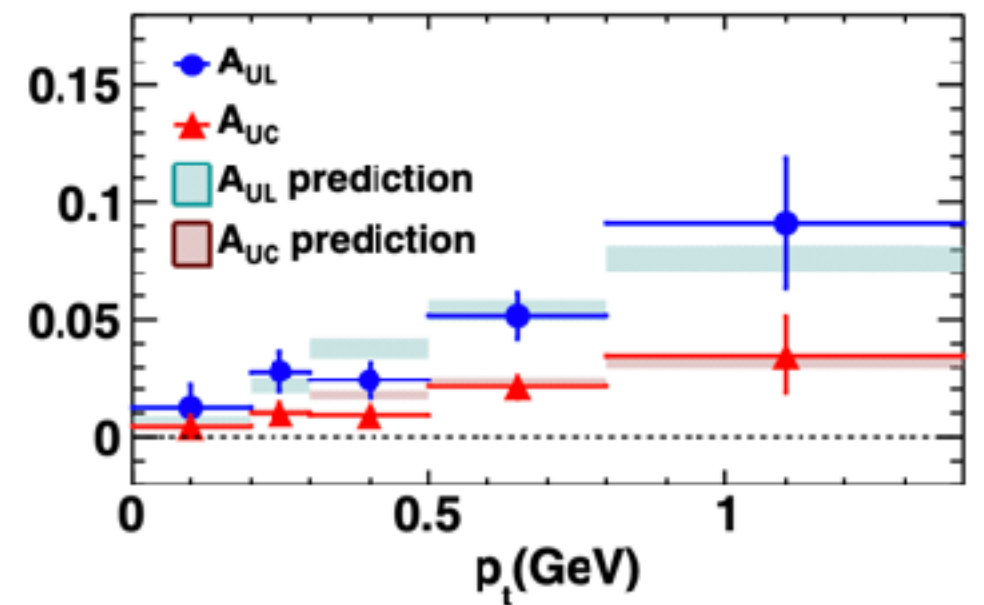
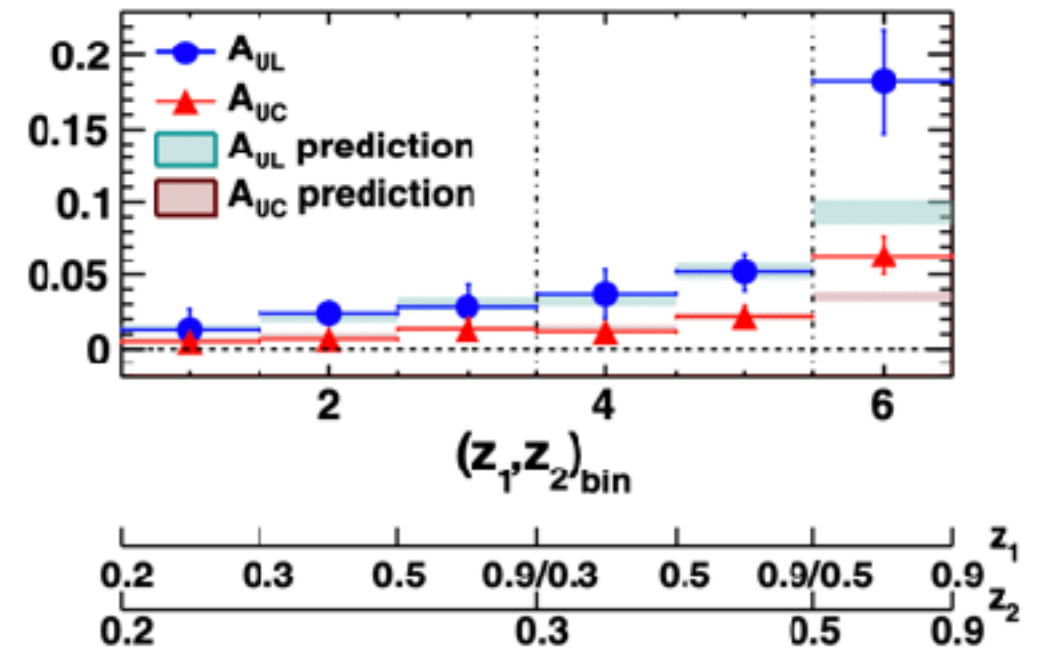
PRD 90,052003



$\mathcal{L} \sim 62 \text{ pb}^{-1}$ at $\sim 3.65 \text{ GeV}$

PRL116, 042001

BES III



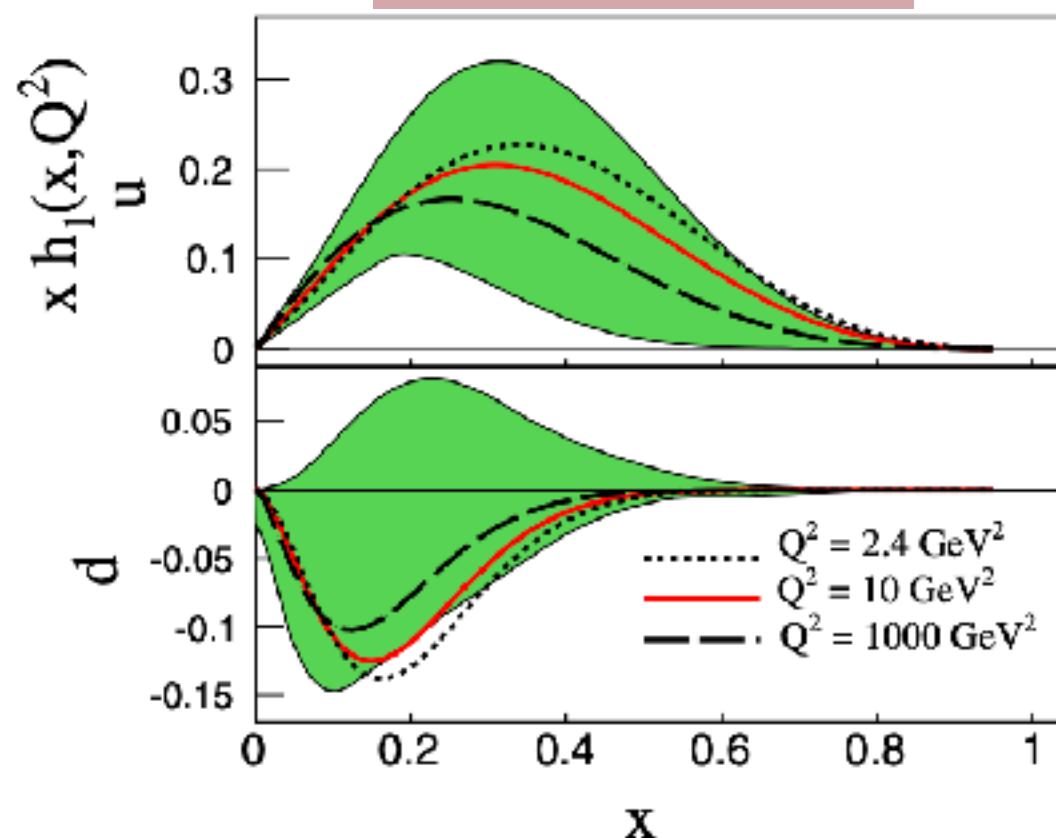
Extraction of h_1 and Collins FF

PRD93,014009

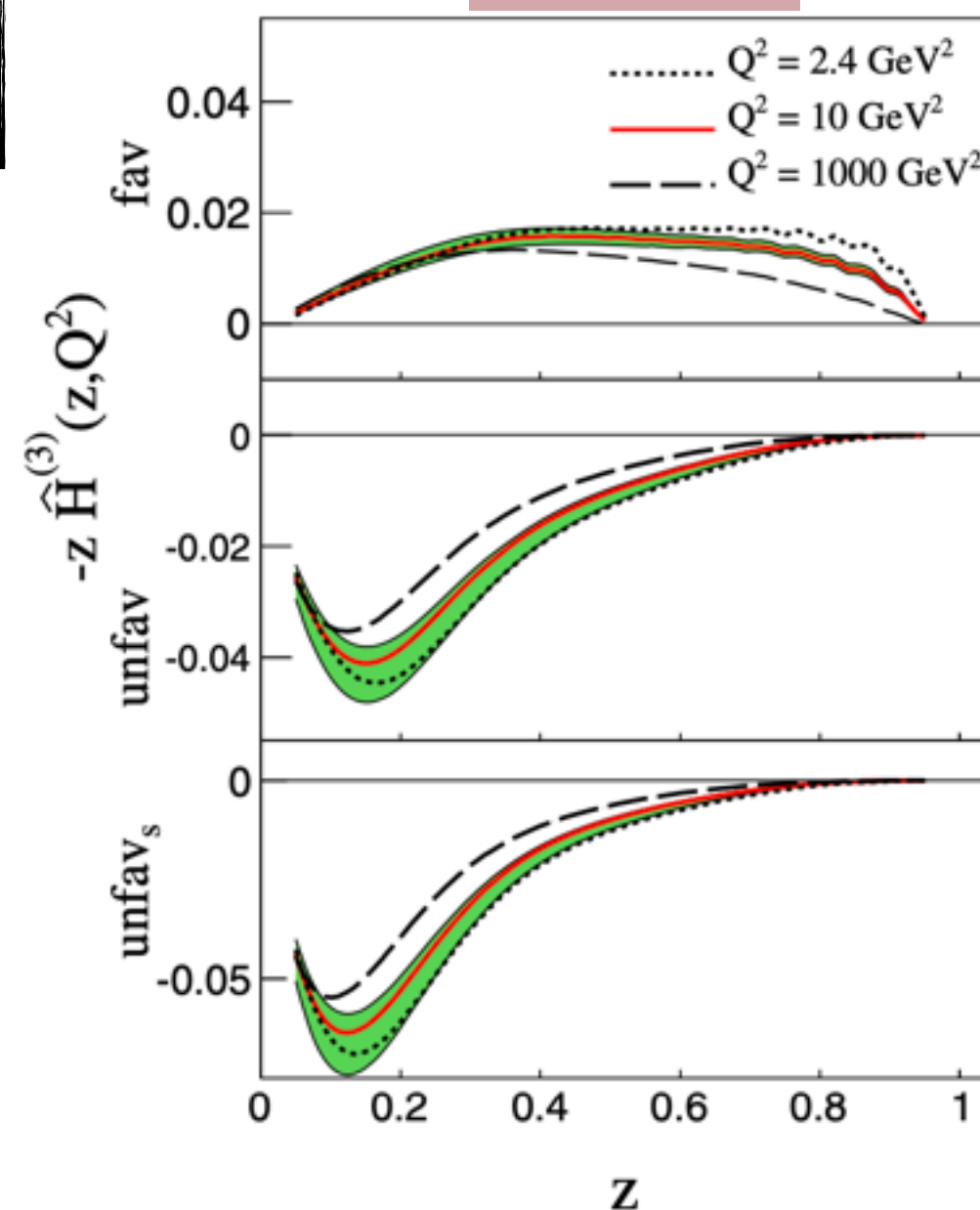
- First extraction: Anselmino et al. PRD75,054032
- Global analysis with SIDIS (HERMES (PRL103,152002), COMPASS (PLB744,250; PLB673,127)) + BaBar RF0 data (PRD90,052003) and Belle RF0 (PRD78,032011) data

Full QCD dynamics taken into account, including TMD evolution effects at the NLL' order and perturbative QCD corrections at the NLO

Transversity PDFs

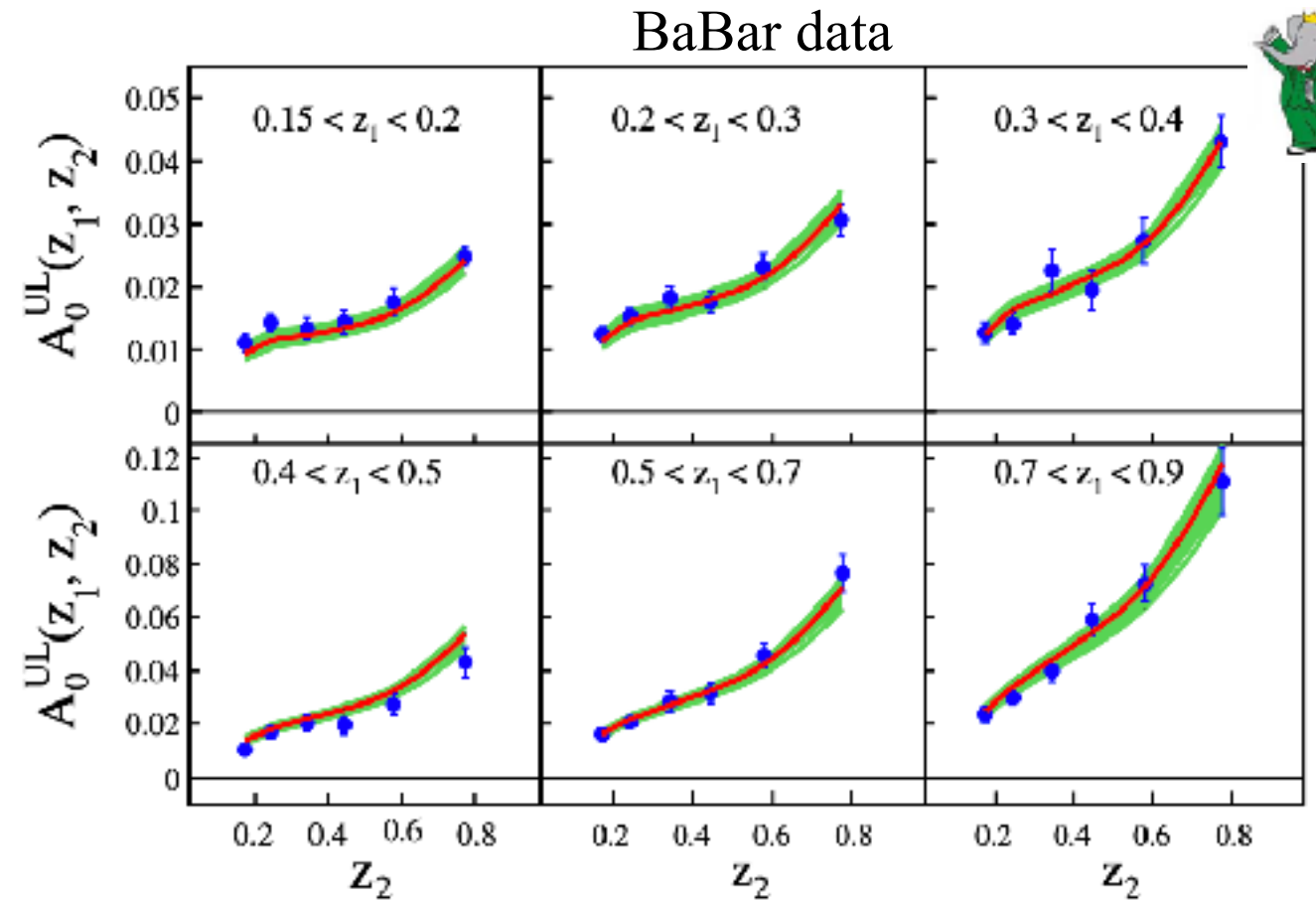
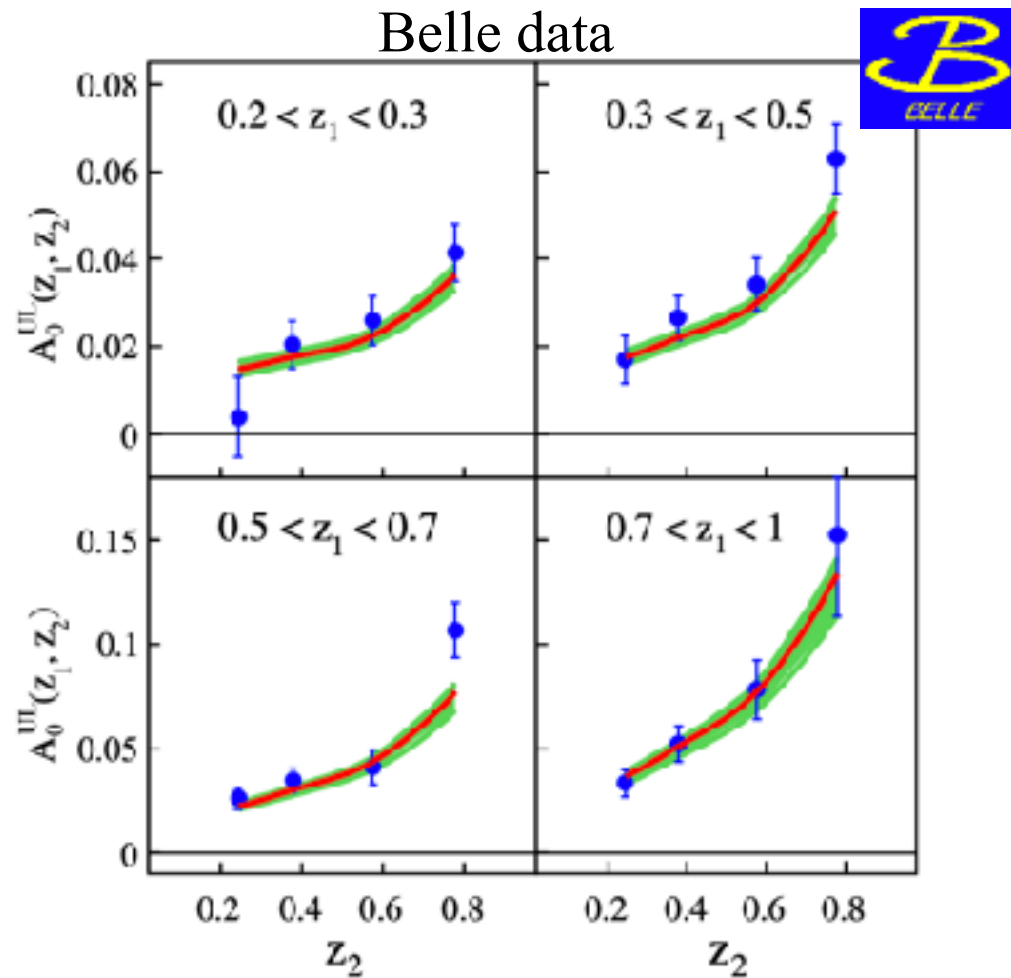


Collins FFs



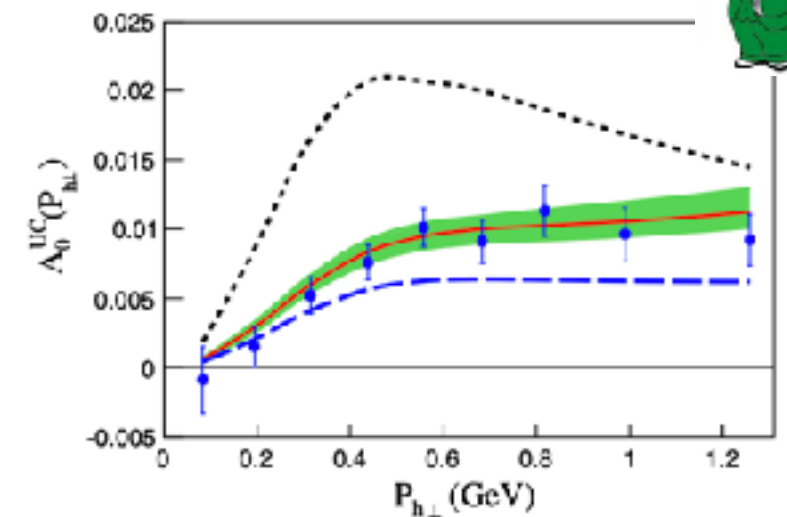
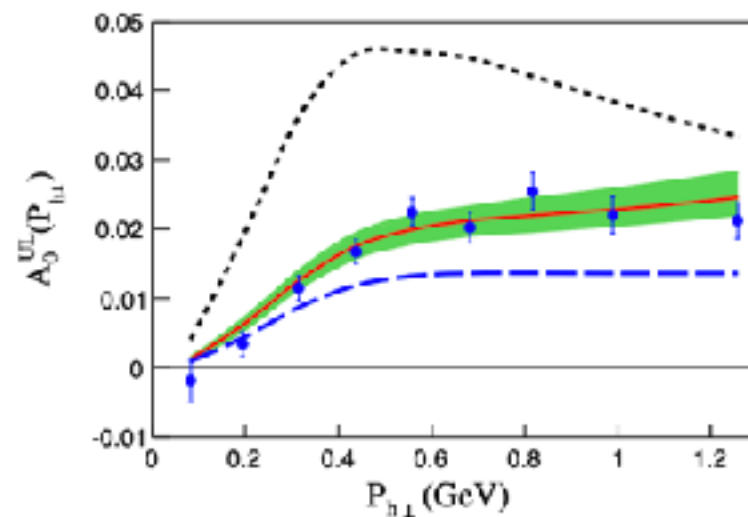
Extraction of Collins FF: data description

PRD93,014009



- NLL' calculations
- - - LL calculation
- no TMD evolution

e^+e^- data are very sensitive to the inclusion of higher order corrections

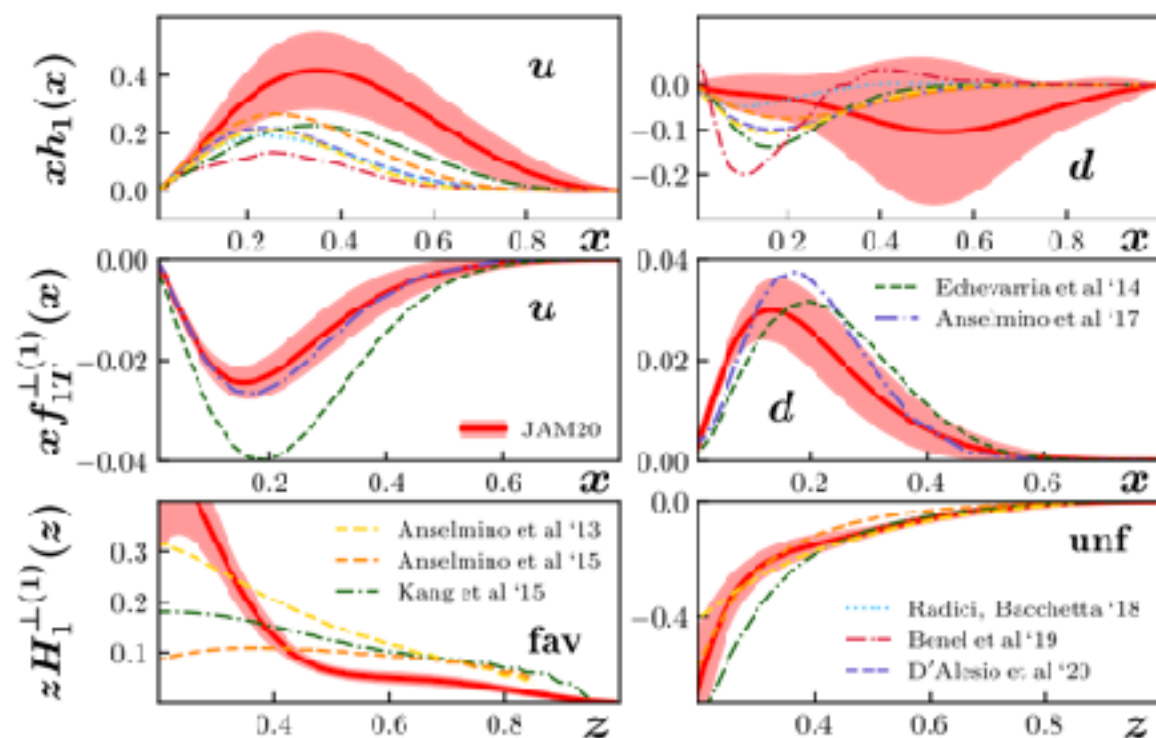


First Global Analysis

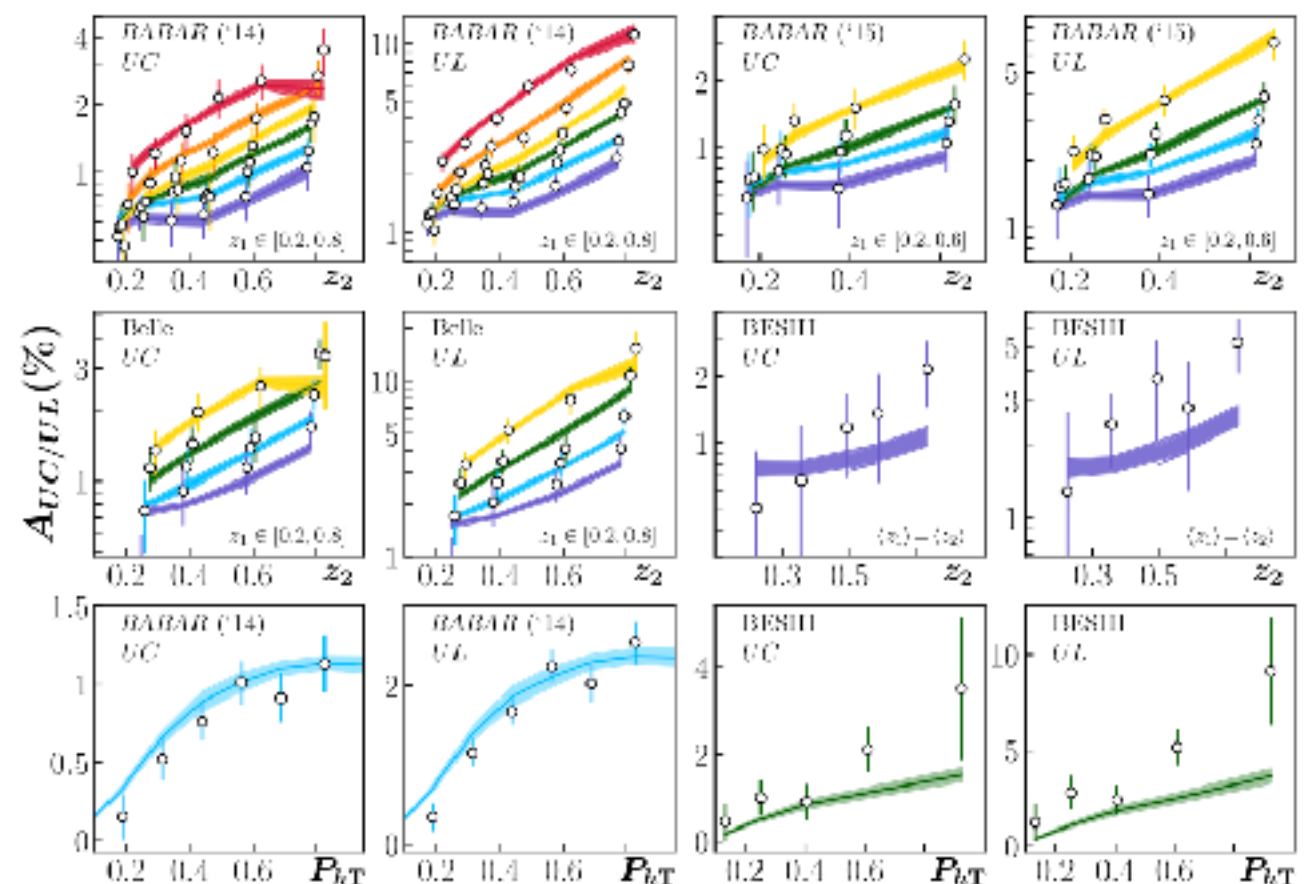
PRD102,054002

First simultaneous QCD global analysis with SIDIS, e+e- annihilation, DY and proton-proton collisions

- Test of universality
- Indication that transverse-spin asymmetries in high-energy collisions have a common origin
- Extracted quark tensor charges are in excellent agreement with lattice QCD



[Anselmino'13](#), [Anselmino'15](#), [Anselmino'17](#), [Benel'19](#),
[D'Alessio'20](#), [Echevarria'14](#), [Kang'15](#), [Radici-Bacchetta'18](#)

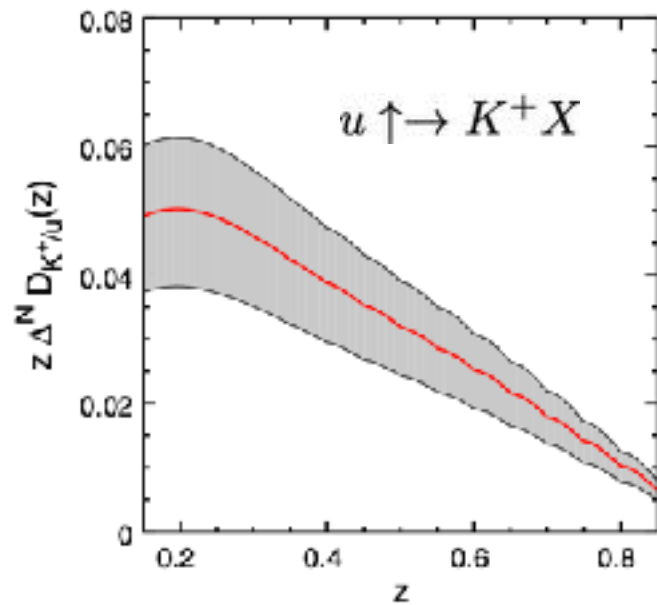


Comparison between theory and data
 (BaBar, Belle, BESIII)

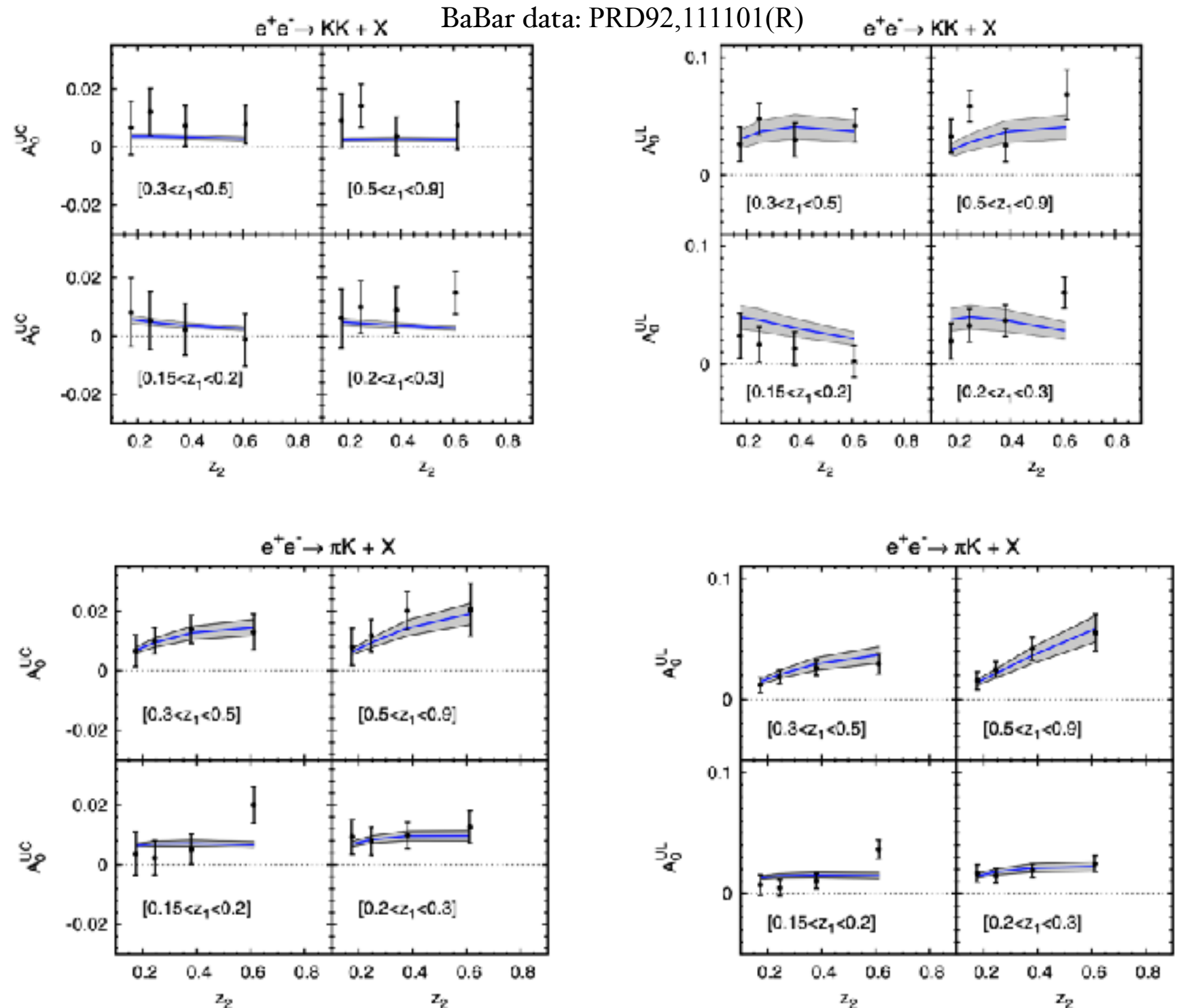
Extraction of Collins FF for Kaons

PRD 93,034025

- Pions FF from previous analysis
- Similar parametrisation for kaon favoured and disfavoured FFs as used for pions

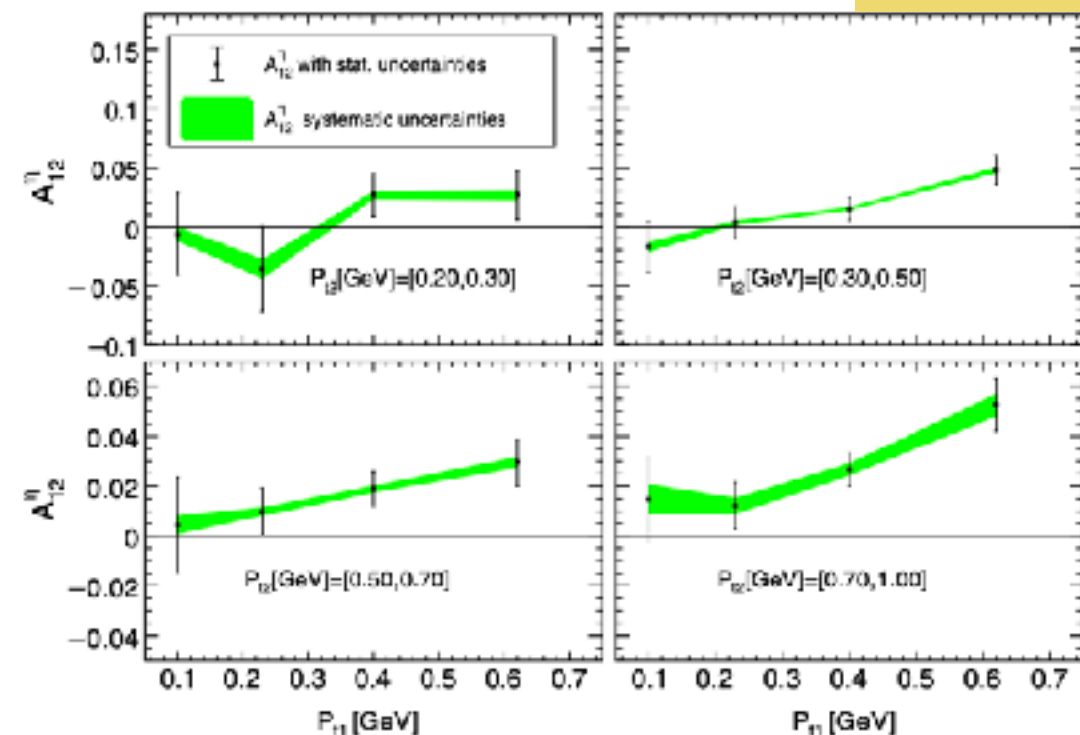
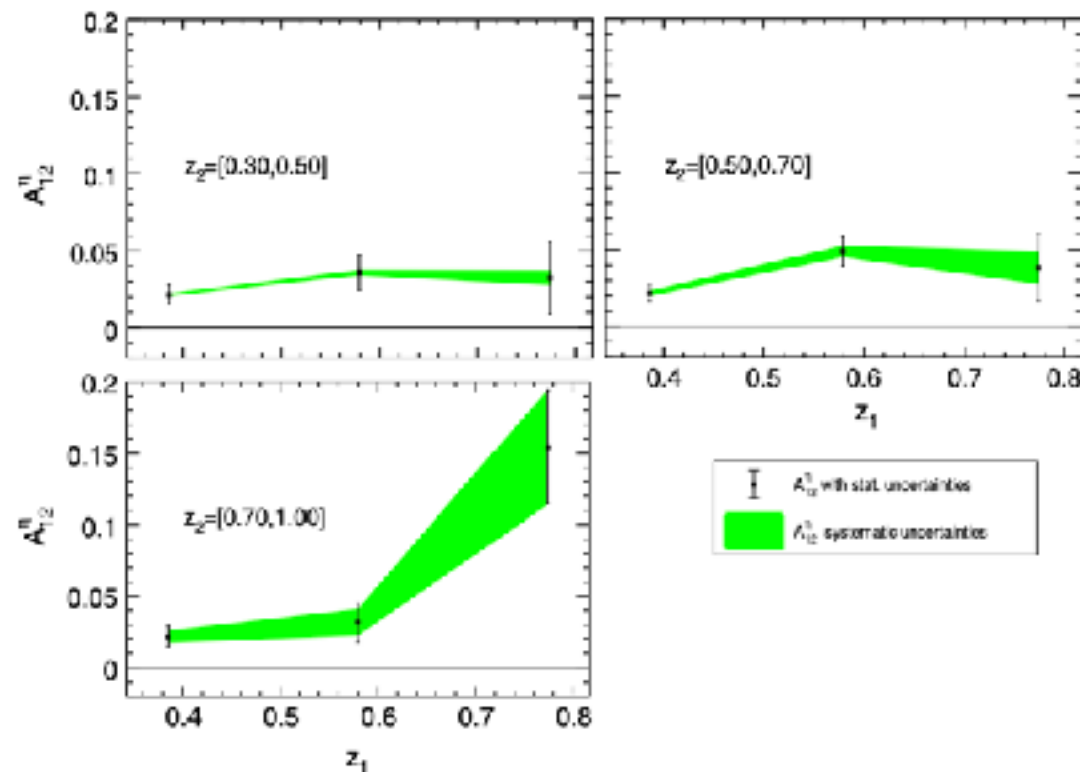


- Positive favored Collins function and a disfavoured contribution compatible with zero
- Large uncertainty
- No definitive conclusions can be extracted for strange Collins FF nor on the disfavoured ones

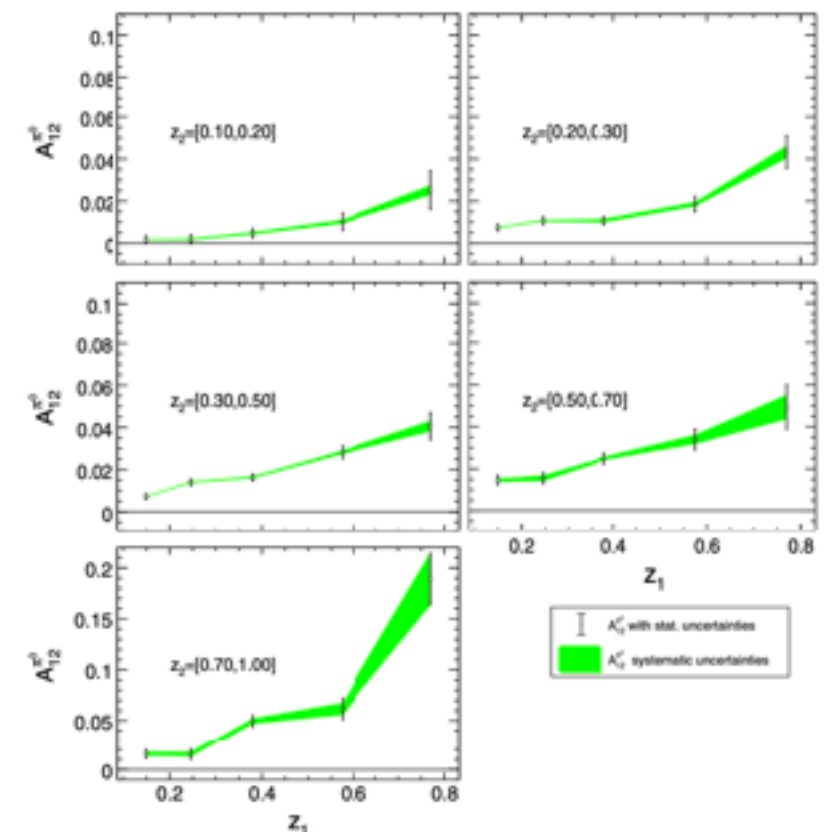


Azimuthal asymmetries of back-to-back $\pi^\pm\eta/\pi^\pm\pi^0$ from Belle

PRD 100,092008



- RF12 (thrust reference frame) only
- π^0 :
 - Significant asymmetries that rise with z
 - Continuous rise with P_{T1} consistent with a linear behaviour
- η :
 - Larger uncertainties
 - The rise with z is much less pronounced
 - Rise of the asymmetry with increasing p_t
 - Charm contribution $\sim 20\text{-}30\%$ larger than neutral pions sample
 - for those bins with similar charm contributions η and π^0 asymmetries are fully consistent



$\Lambda/\bar{\Lambda}$ hyperon polarization at Belle

PRL 122,042001

- $e^+e^- \rightarrow \Lambda(\bar{\Lambda})X$ and $e^+e^- \rightarrow \Lambda(\bar{\Lambda})h^\pm X$ using 800.4 fb^{-1} collected by Belle at or near $\sqrt{s}=10.58 \text{ GeV}$

$$D_{1T}^{\perp \Lambda/q}(z, p_\perp^2)$$

\Rightarrow production of transversely polarized Λ hyperons from unpolarised quark
 \Rightarrow chiral-even function (the sign of this function can be easily determined)

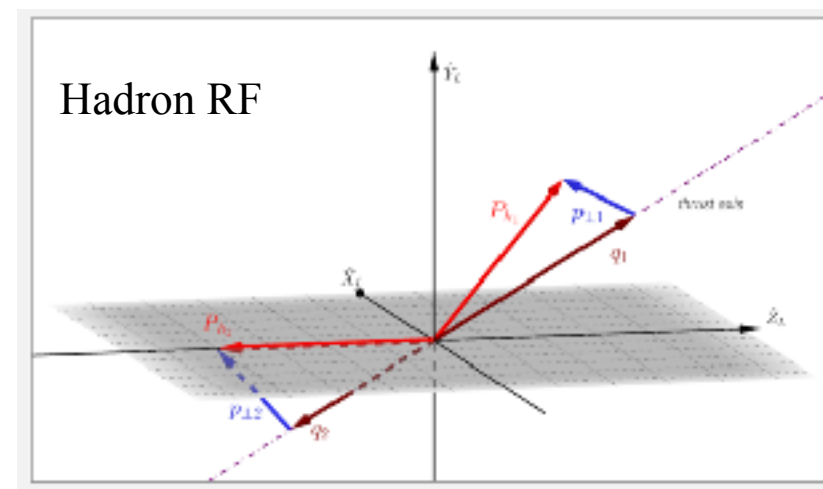
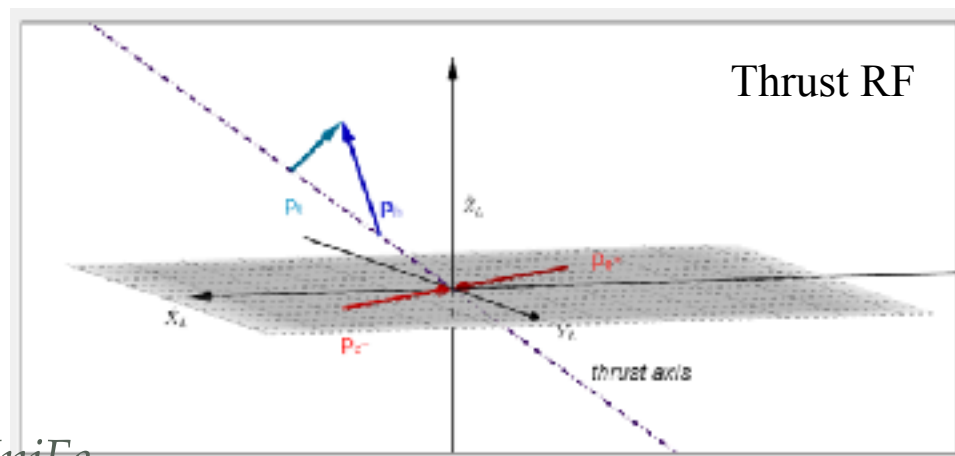
Distribution of protons from Λ decays with a transverse polarisation P :

$$\frac{1}{N} \frac{dN}{d \cos \theta} = 1 + \alpha P \cos \theta,$$

$$P_\Lambda(z, p_\perp) = \frac{\sum_q e_q^2 \frac{4z p_\perp \sqrt{s}}{z^2 s + 4p_\perp^2} \Delta^N D_{\Lambda^+/q}(z, p_\perp)}{\sum_q e_q^2 2\pi D_{\Lambda/q}(z, p_\perp)} \times \frac{\int d(\cos \theta) \sin(2\theta)}{\int d(\cos \theta) (1 + \cos^2 \theta)}$$

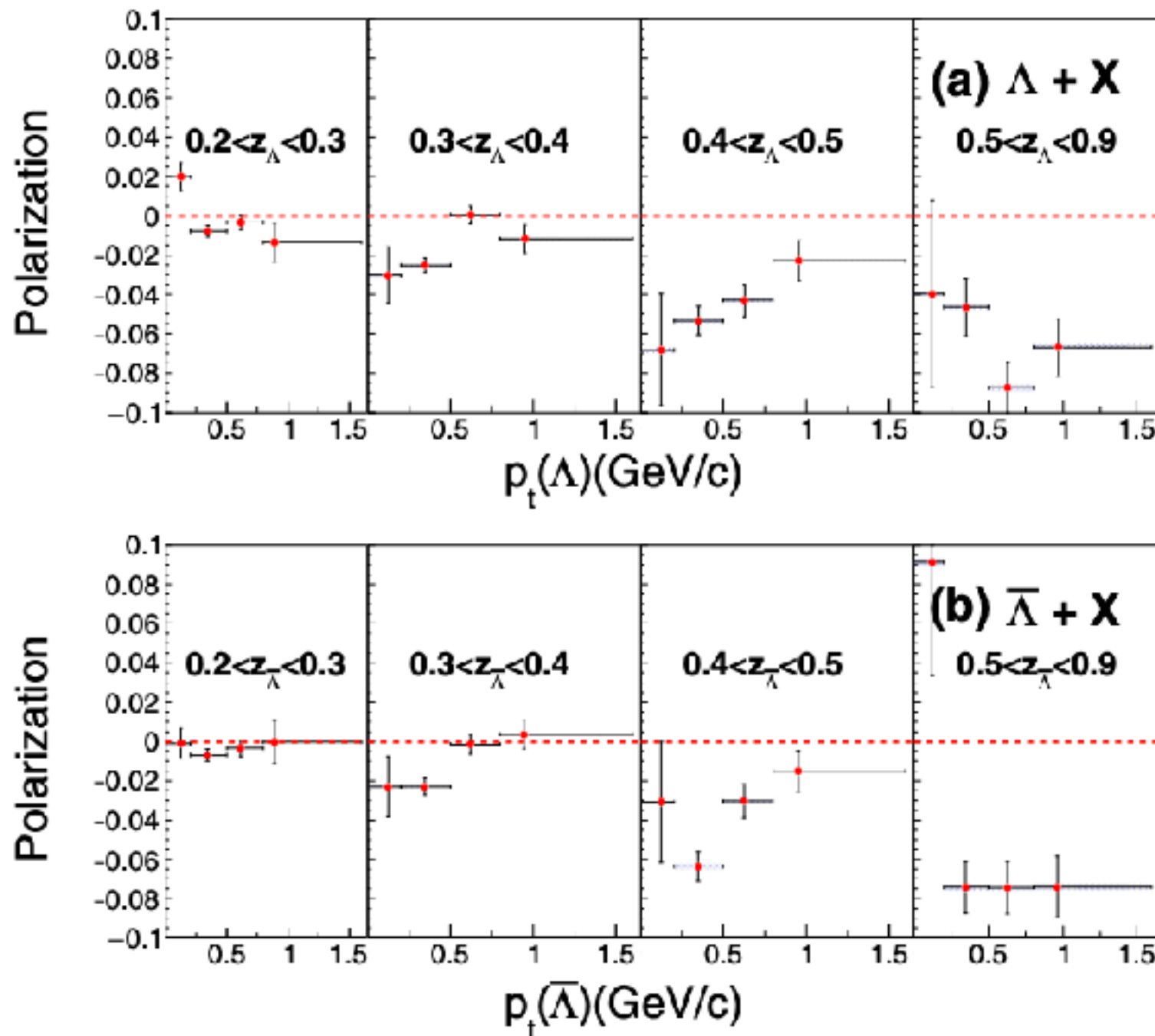
PRD 100,014029

θ : angle between the axis $\hat{n} \propto \hat{T} \times \hat{p}_\Lambda$ and the proton momentum in the Λ rest frame; α is the world average value of the parity-violating decay asymmetry for the Λ (from PDG)
 \Rightarrow Significant transverse polarization is observed



$\Lambda/\bar{\Lambda}$ hyperon polarization at Belle: results

PRL 122,042001

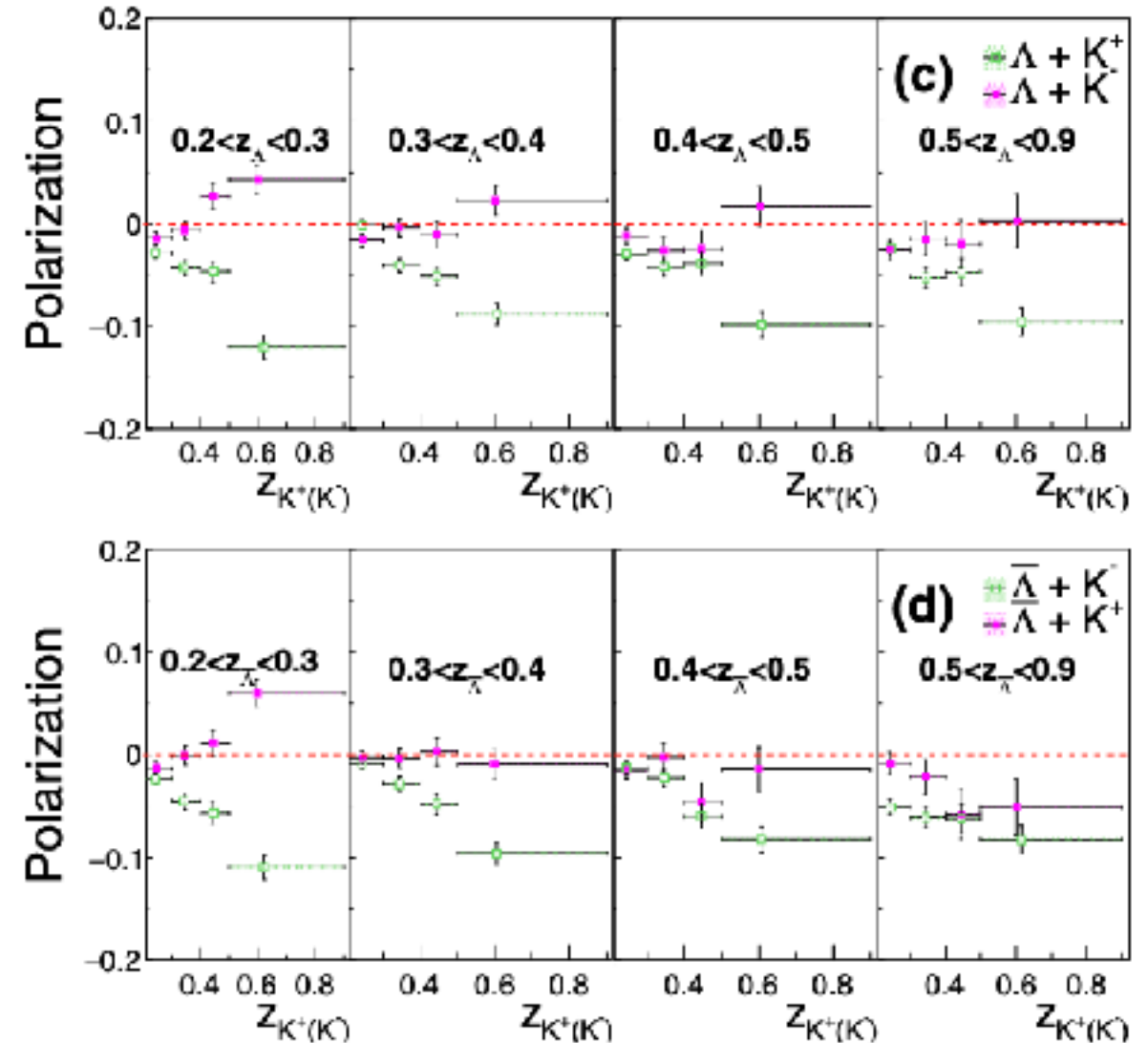
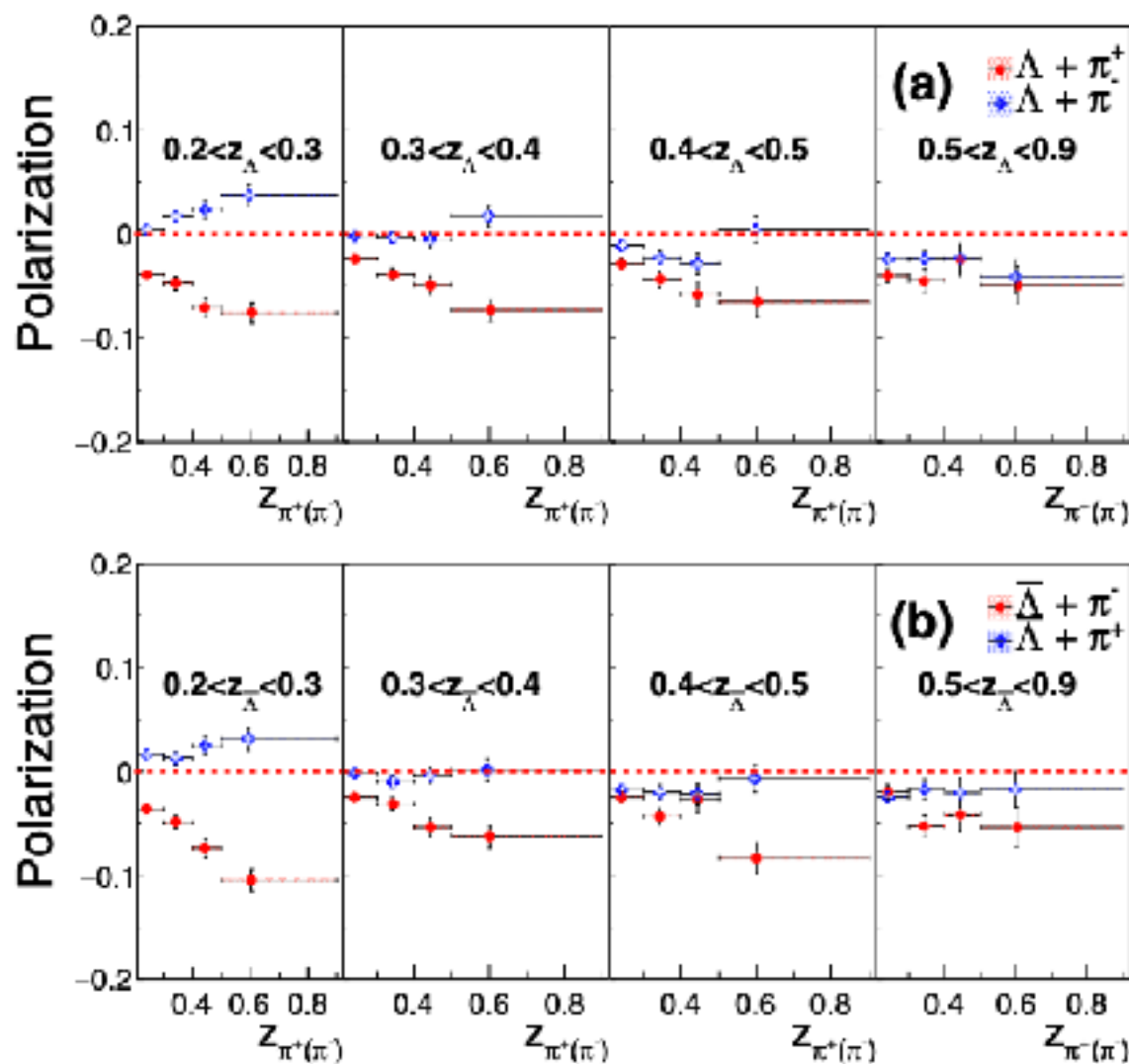


Complex p_t dependence

- $z_\Lambda > 0.5$: increasing asymmetry with p_t ; opposite behaviour for intermediate z_Λ
 - different quark flavour contributions in different kinematic regions (s quark contribution is dominant in the highest z_Λ bin; u quark and charm contribution in the intermediate regions)
- large charm contribution in intermediate z -bins

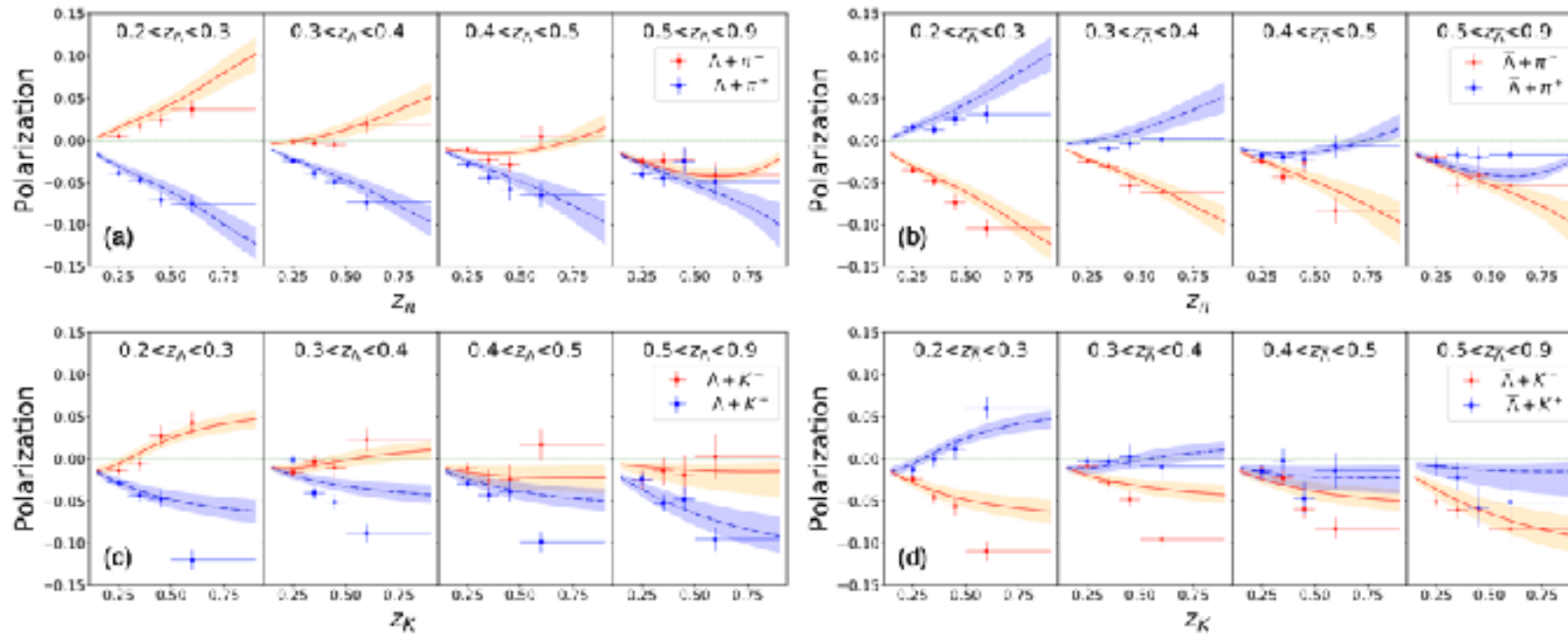
$\Lambda/\bar{\Lambda}$ hyperon polarization at Belle: results

PRL 122,042001



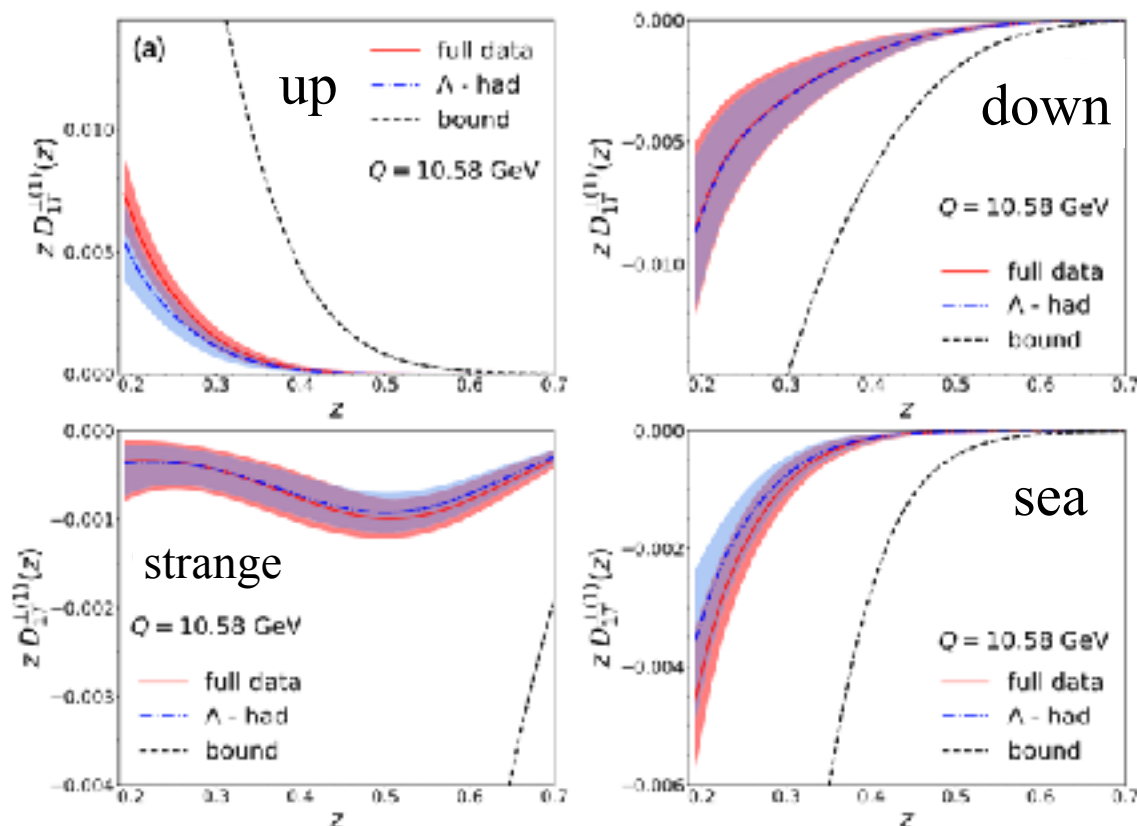
- low z_{Λ} : opposite sign and increasing magnitude
- Sensitivity to the flavour dependence by selecting a light hadron in the opposite hemisphere:
 - Λ polarization from s quark is negative: negative asymmetry observed in $\Lambda K^+ X$ at high z_{Λ} , where s to Λ fragmentation dominates
 - In $\Lambda \pi^- X$ and $\Lambda K^- X$ $u \rightarrow \Lambda$ fragmentation dominates at low z_{Λ} : u fragmentation to Λ is positive

First extraction of Λ polarised FF from Belle data



First extraction from Belle data within a TMD approach:
D'Alessio, Murgia, Zaccheddu

PRD 102,114015



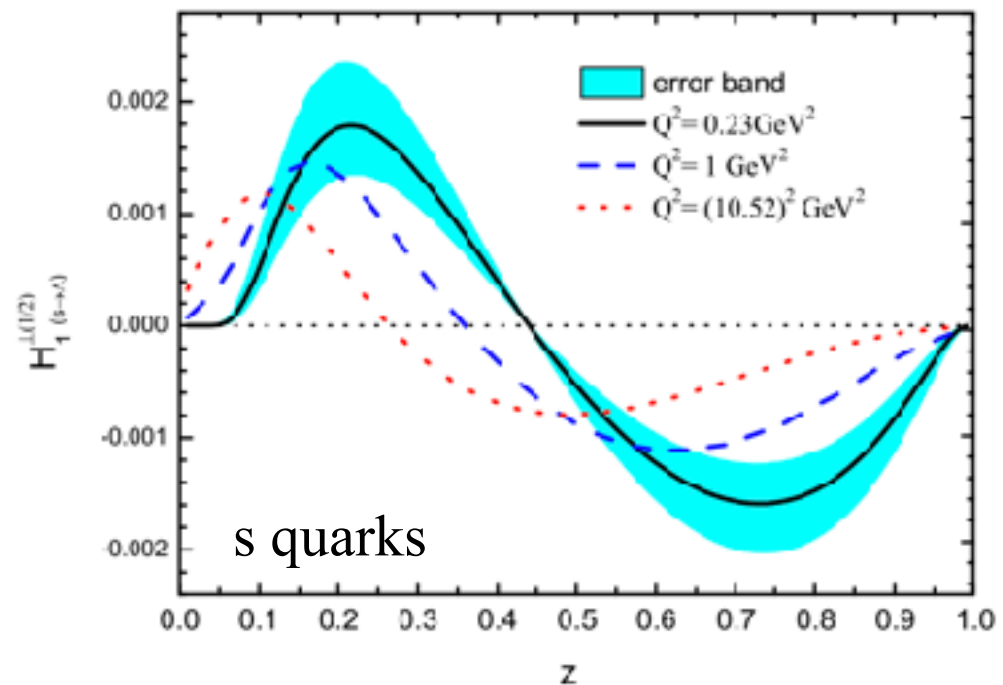
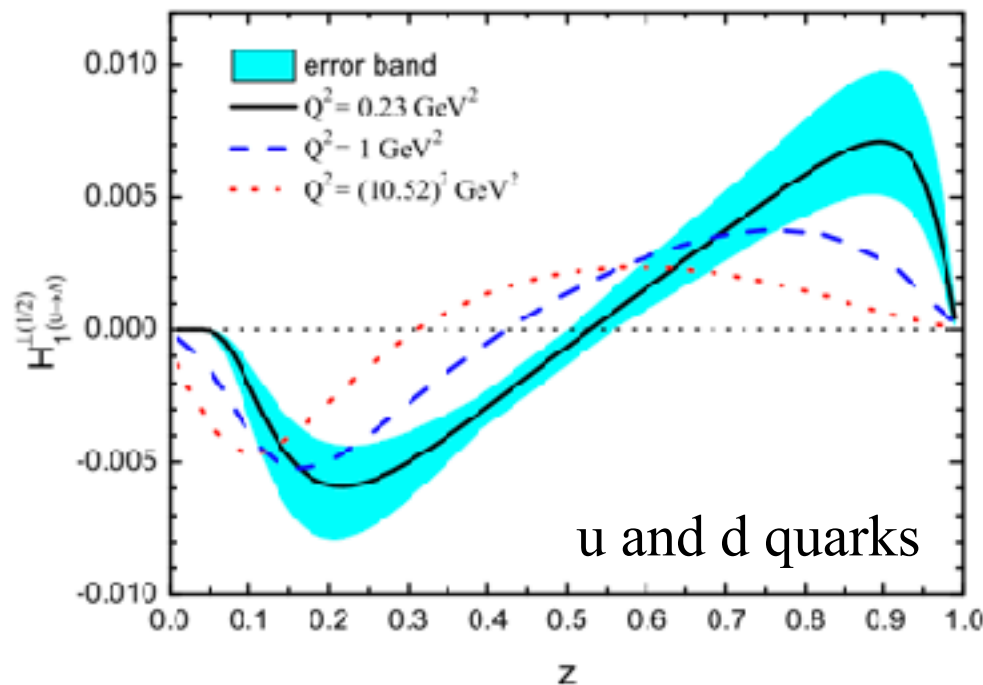
- Clear separation in flavour achieved
- The needed of sea-quark polarised FF is well supported
- Extracted the first pt dependence

Higher statistic and complementary studies in other processes needed for a deeper understanding of this TMD-FF and solving the long-standing puzzle of the observed spontaneous transverse hyperon polarisation

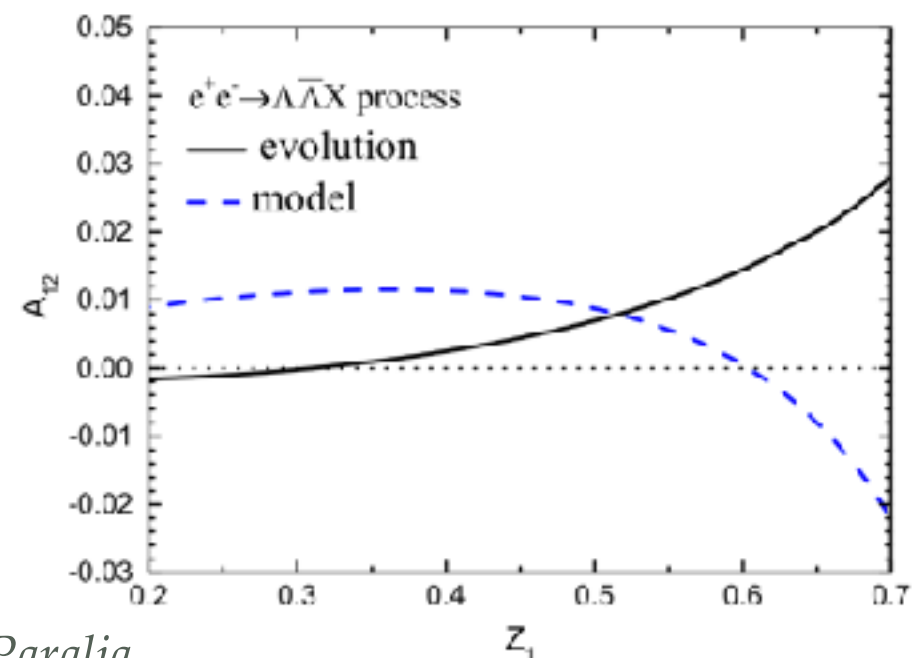
Λ hyperon Collins FFs prediction

PRD97,114015

- Theoretical expectation from Collins FFs for Λ hyperon: $e^+e^- \rightarrow \Lambda \bar{\Lambda} X$
 - Process accessible in e^+e^- facilities
 - Collins FF: complementary information to D_{1T}^\perp



- Azimuthal asymmetry prediction in RF12 frame for $0.2 < z_1 < 0.7$ (integrated over z_2)
- Increasing asymmetry with z , of the order of several percent



Summary and Conclusions

FFs and PDFs are fundamental tools for understanding of hadrons structure

Many other topics not covered in this talk

- Di-hadron FF studies: [PRL 107,072004](#)
- Form Factors
 - Timelike and Spacelike region
- Generalized Parton Distribution functions (GPDs) and Transition Distribution Amplitudes (TDA)

Summary and Conclusions

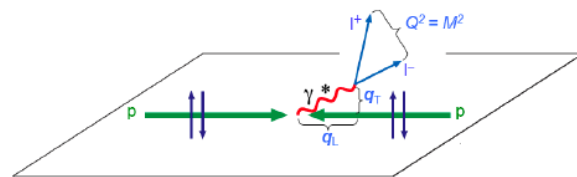
FFs and PDFs are fundamental tools for understanding of hadrons structure

Many other topics not covered in this talk

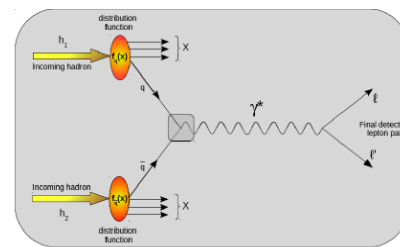
- Di-hadron FF studies: [PRL 107,072004](#)
- Form Factors
 - Timelike and Spacelike region
- Generalized Parton Distribution functions (GPDs) and Transition Distribution Amplitudes (TDA)



TMDs in Drell-Yan Processes



Cross section: it is given by the convolution of two distribution functions



$$\sigma_{\text{Drell-Yan}} = f_q(x, k_{\perp}) \otimes f_{\bar{q}}(x, k_{\perp}) \otimes \hat{\sigma}^{q\bar{q} \rightarrow l^+ l^-}$$

**proton-proton
(nucleon-nucleon)
Drell-Yan**

the total cross section, σ , is suppressed by the antiquark parton distribution function (antiquarks from proton sea).

RICH
FERMILAB

**proton-antiproton
(nucleon-nucleon)
Drell-Yan**

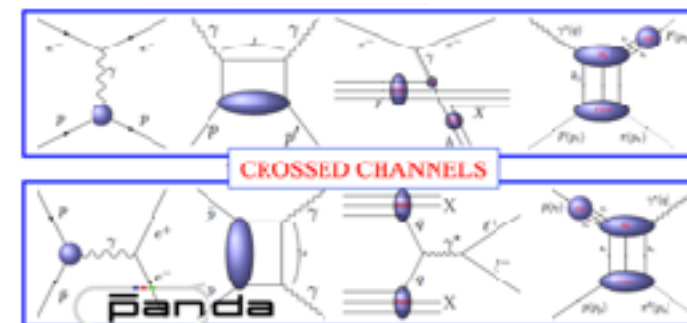
the total cross section, σ , is NOT suppressed by parton distribution functions (antiquarks are valence partons in the antiproton).

PANDA
J-PARK

**proton-pion
(nucleon-pion)
Drell-Yan**

the total cross section, σ , is NOT suppressed by parton distribution functions (antiquarks are valence partons in pions).

COMPASS



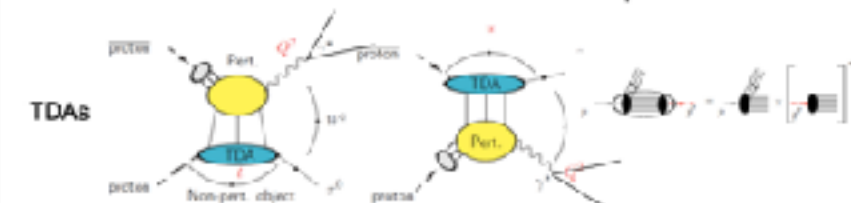
- Time-like Form Factors
- Transition Distribution Amplitudes (TDAs)
- Generalized Distributions Amplitudes (GDAs)
- TMD-Parton Distribution Functions (PDFs)

Transition Distribution Amplitude (TDA)

GPDs: Factorization: Hard limit for forward exclusive processes



TDAs: Factorization: Hard limit for backward exclusive processes



Pbar P → e+ e- n can be studied by PANDA



Summary and Conclusions

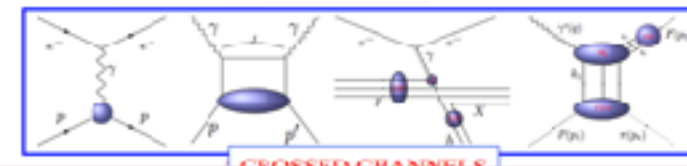
FFs and PDFs are fundamental tools for understanding of hadrons structure

Many other topics not covered in this talk

- Di-hadron FF studies: [PRL 107,072004](#)
- Form Factors
 - Timelike and Spacelike region
- Generalized Parton Distribution functions (GPDs) and Transition Distribution Amplitudes (TDA)



TMDs in Drell-Yan Processes



- Time-like Form Factors
- Transition Distribution Amplitudes (TDAs)

Thanks for your attention

distribution functions

**proton-proton
(nucleon-nucleon)
Drell-Yan**

the total cross section, σ , is suppressed by the antiquark parton distribution function (antiquarks from proton sea).

RICH
FERMILAB

**proton-antiproton
(nucleon-nucleon)
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the total cross section, σ , is NOT suppressed by parton distribution functions (antiquarks are valence partons in the antiproton).

PANDA
J-PARK

**proton-pion
(nucleon-pion)
Drell-Yan**

the total cross section, σ , is NOT suppressed by parton distribution functions (antiquarks are valence partons in pions).

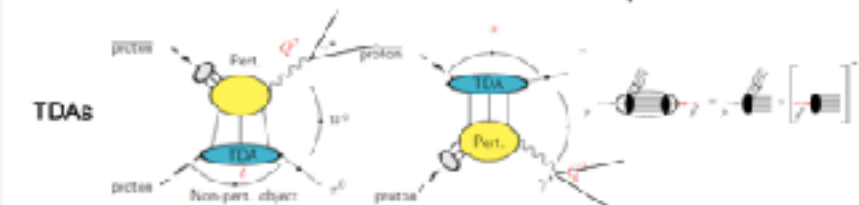
COMPASS

$$\sigma_{\text{Drell-Yan}} = f_q(x, k_\perp) \otimes f_{\bar{q}}(x, k_\perp) \otimes \hat{\sigma}^{q\bar{q} \rightarrow \ell\ell}$$

GPDs: Factorization: Hard limit for forward exclusive processes



TDAs: Factorization: Hard limit for backward exclusive processes



Pbar P \rightarrow e⁺ e⁻ n⁰ can be studied by PANDA





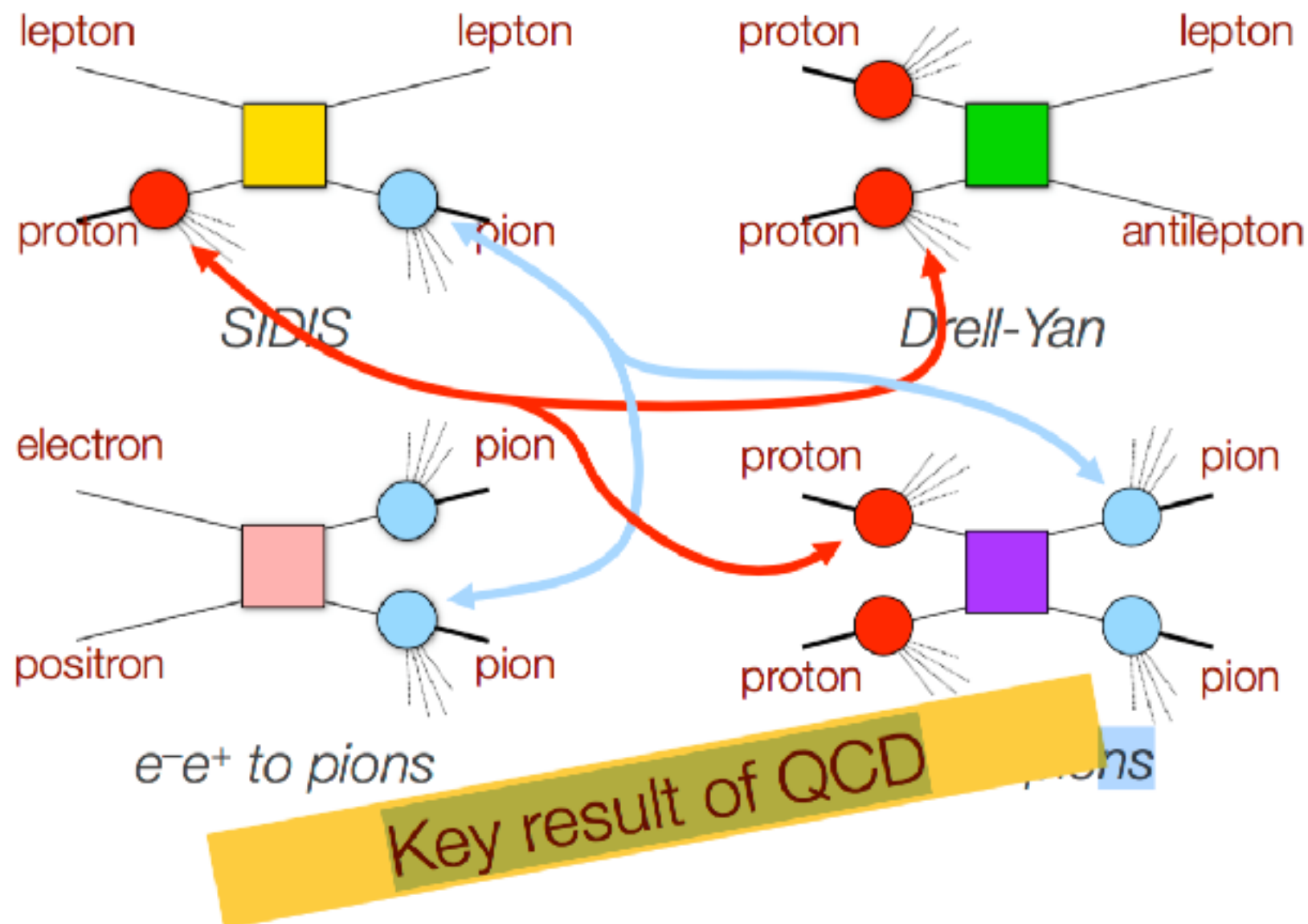
Backup



Universality

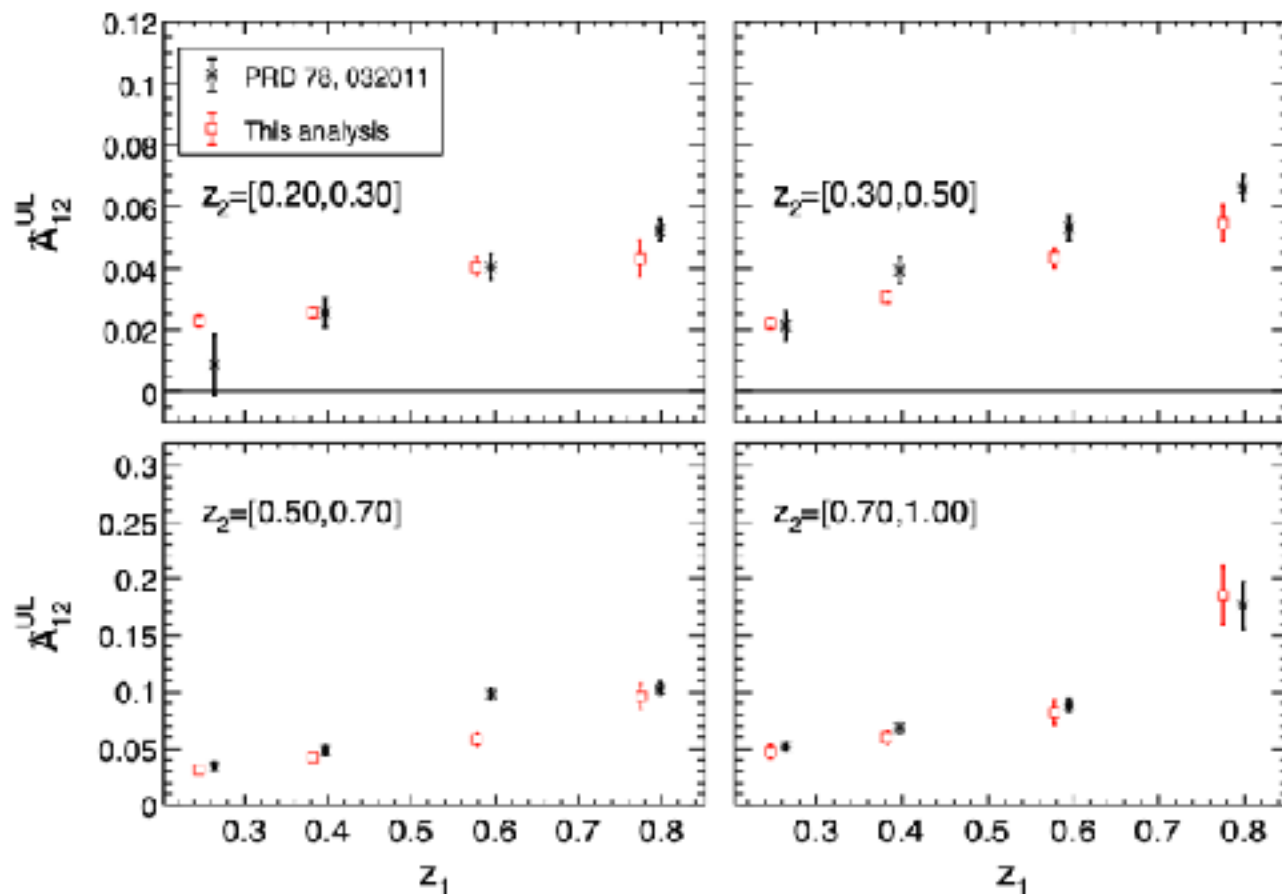
FF: independent of how the parton was produced

PDF: process independent nature



Azimuthal asymmetries of back-to-back π^\pm - (π^0, η, π^\pm) from Belle

PRD100,092008

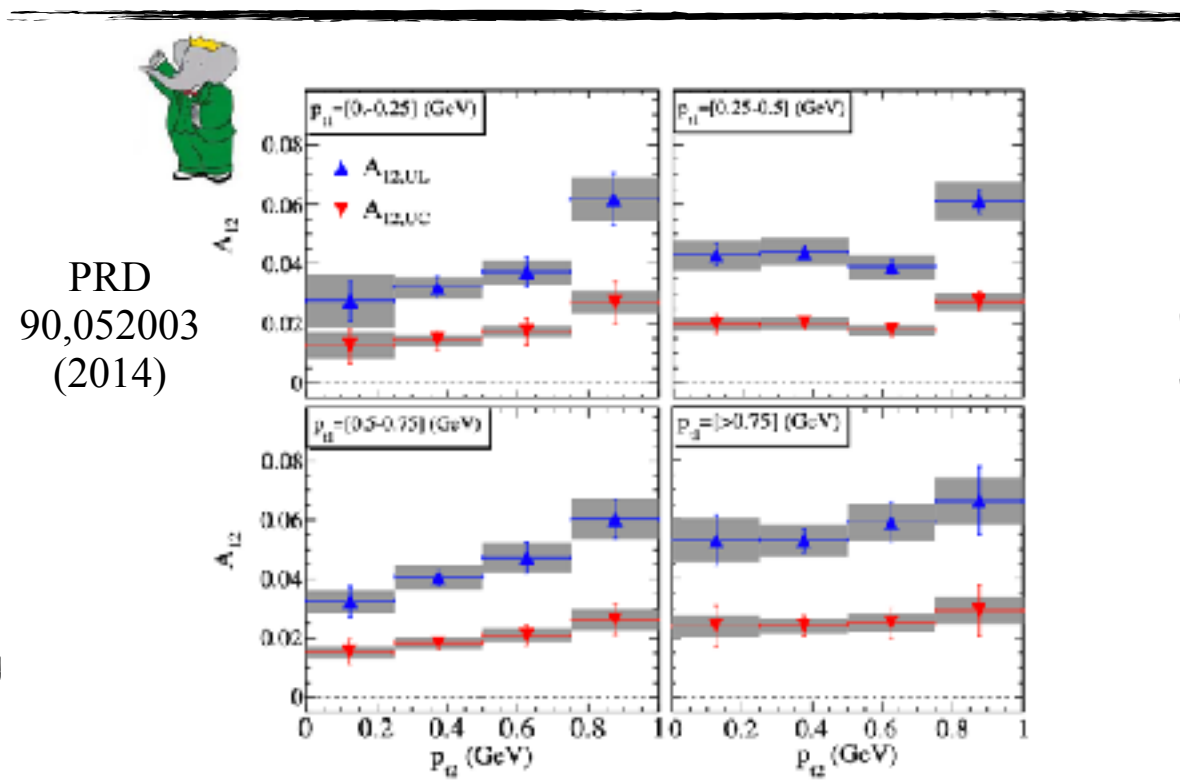


New Belle analysis: RF12 only

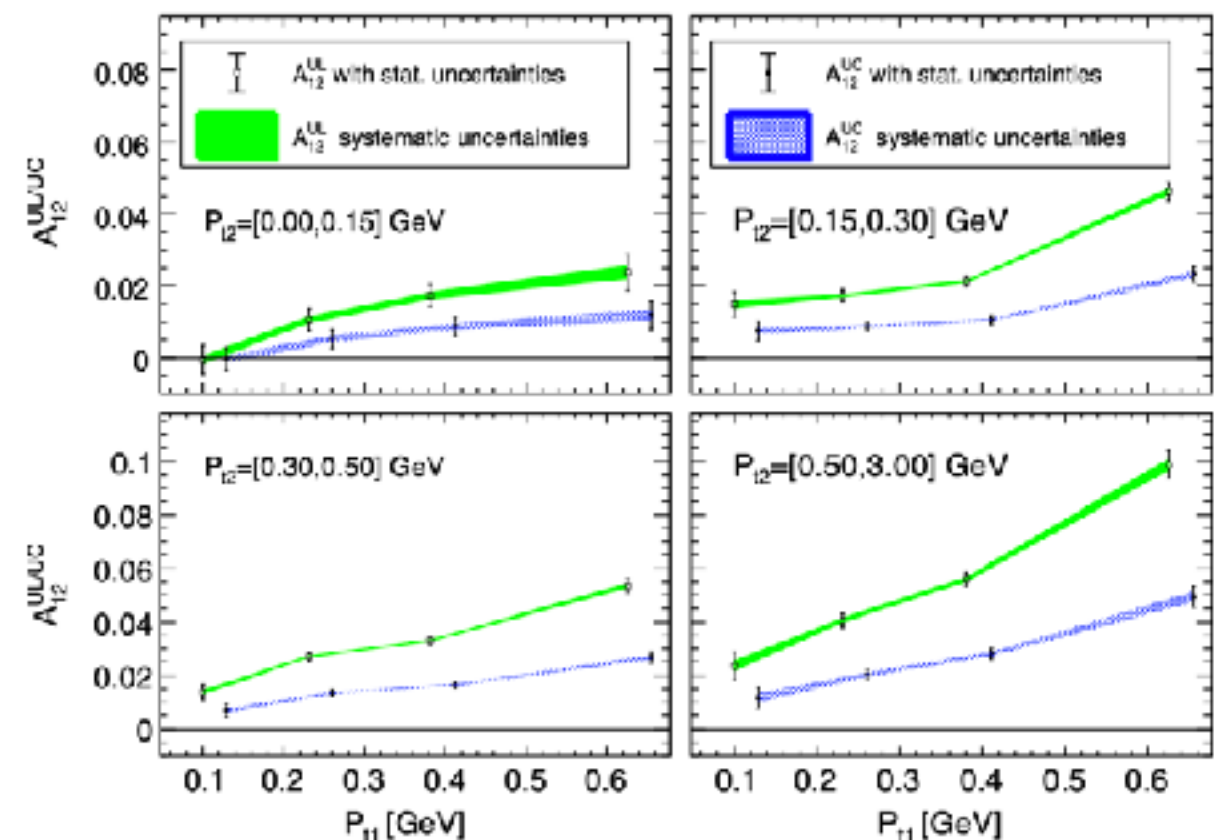
- No charm correction: only the estimated charm fractions are provided
- bin-by-bin generated-thrust axis correction (not the qqbar axis)
- Pt smearing correction
- MC asymmetry subtraction

Only for results comparison:

- Vanishing charm asymmetry
- No thrust axis correction
- Kinematic factors taken into account



PRD
90,052003
(2014)



Results: RF0

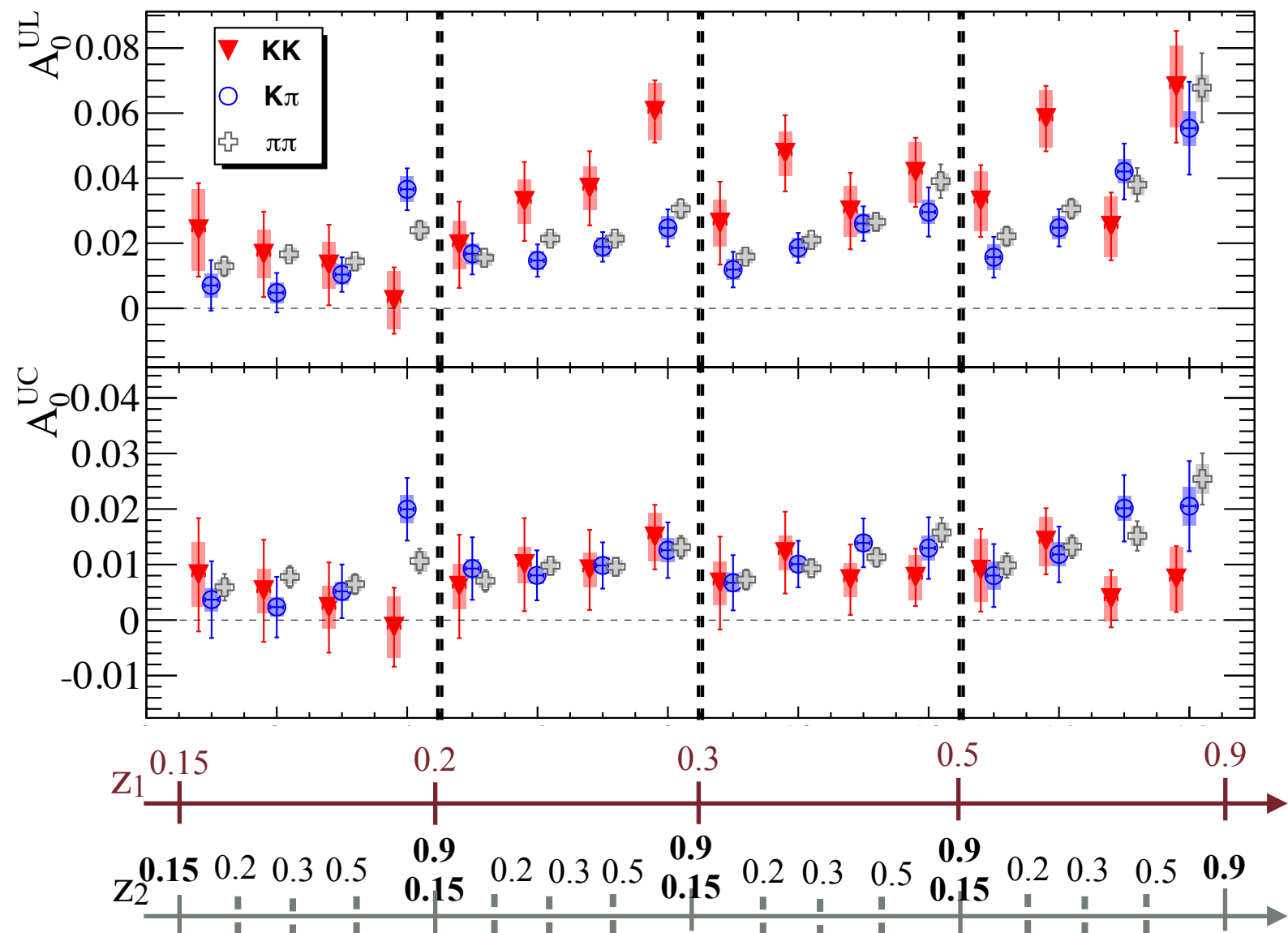
PRD92,111101(R)

Simultaneous measurement of KK , $K\pi$ and $\pi\pi$ Collins asymmetries

- all corrections are applied

- ✧ Rising of the asymmetry as a function of z :
 - ✧ more pronounced for U/L
- ✧ A_0^{UL} KK asymmetry slightly higher than pion asymmetry for high z
- ✧ KK asymmetry consistent with zero at lower z

Note that A_0^{UL} and A_0^{UC} asymmetries are obtained using the same data sample, and are strongly correlated

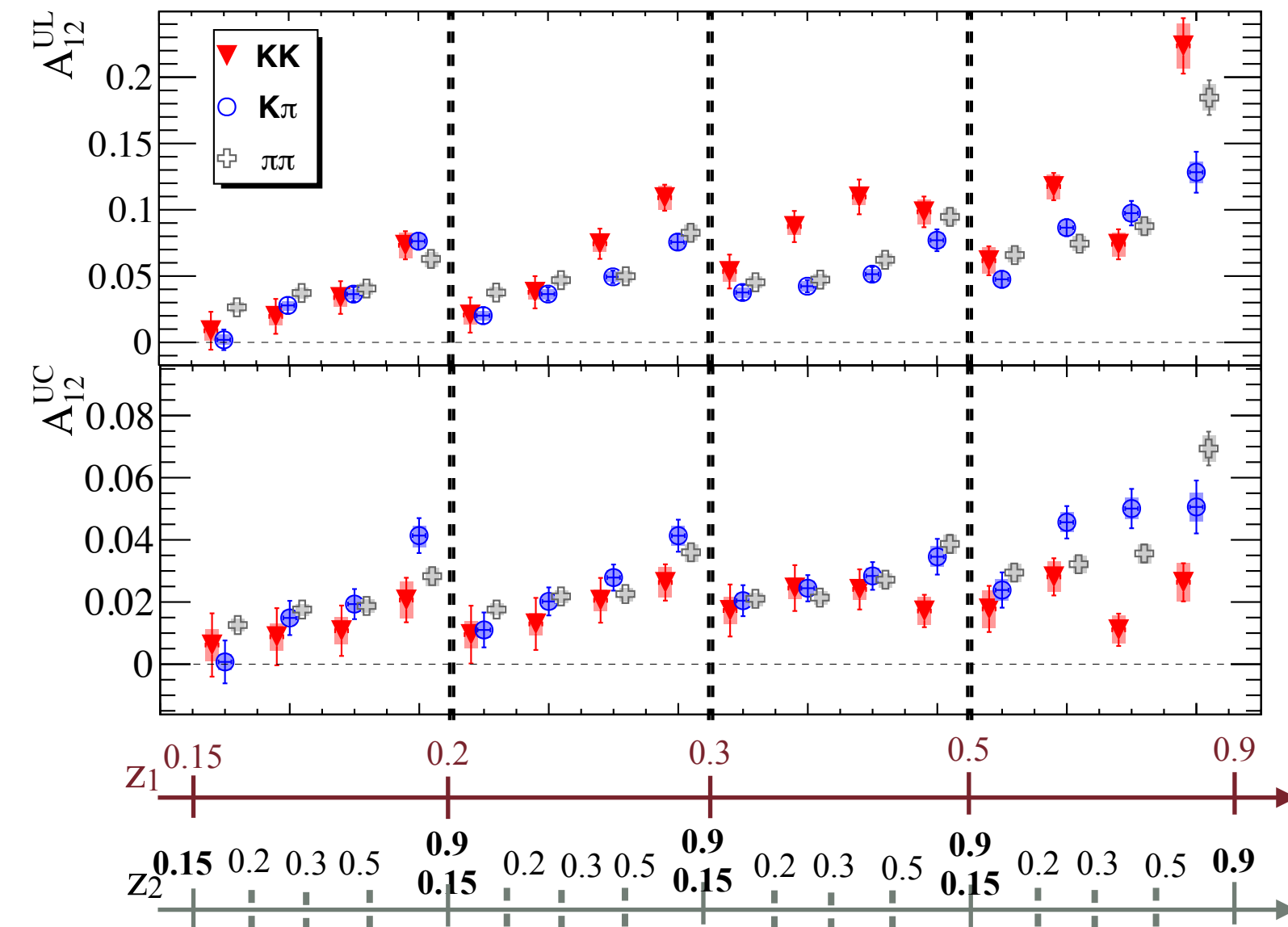


Results: RF12

PRD92,111101(R)

Simultaneous measurement of KK , $K\pi$ and $\pi\pi$ Collins asymmetries

- all corrections are applied



- ◈ Rising of the asymmetry as a function of z :
 - ◈ more pronounced for U/L
- ◈ A_{12}^{UL} KK asymmetry slightly higher than pion asymmetry for high z
- ◈ KK asymmetry consistent with zero at lower z

Note that A_{12}^{UL} and A_{12}^{UC} asymmetries are obtained using the same data sample, and are strongly correlated

Extraction of Collins FF: data description

PRD93,014009

HERMES

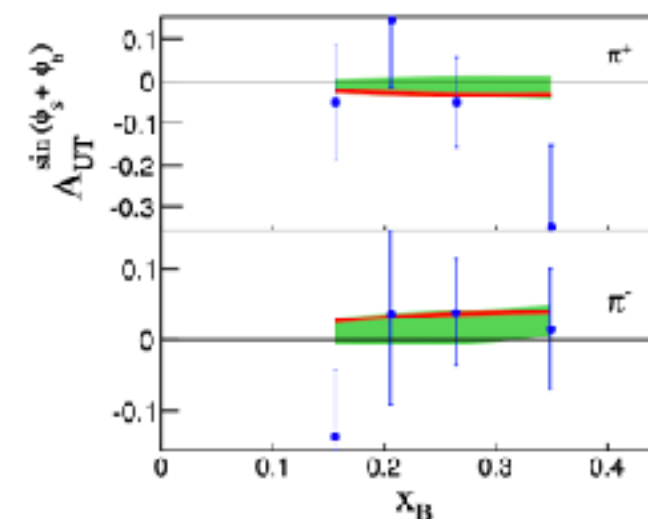
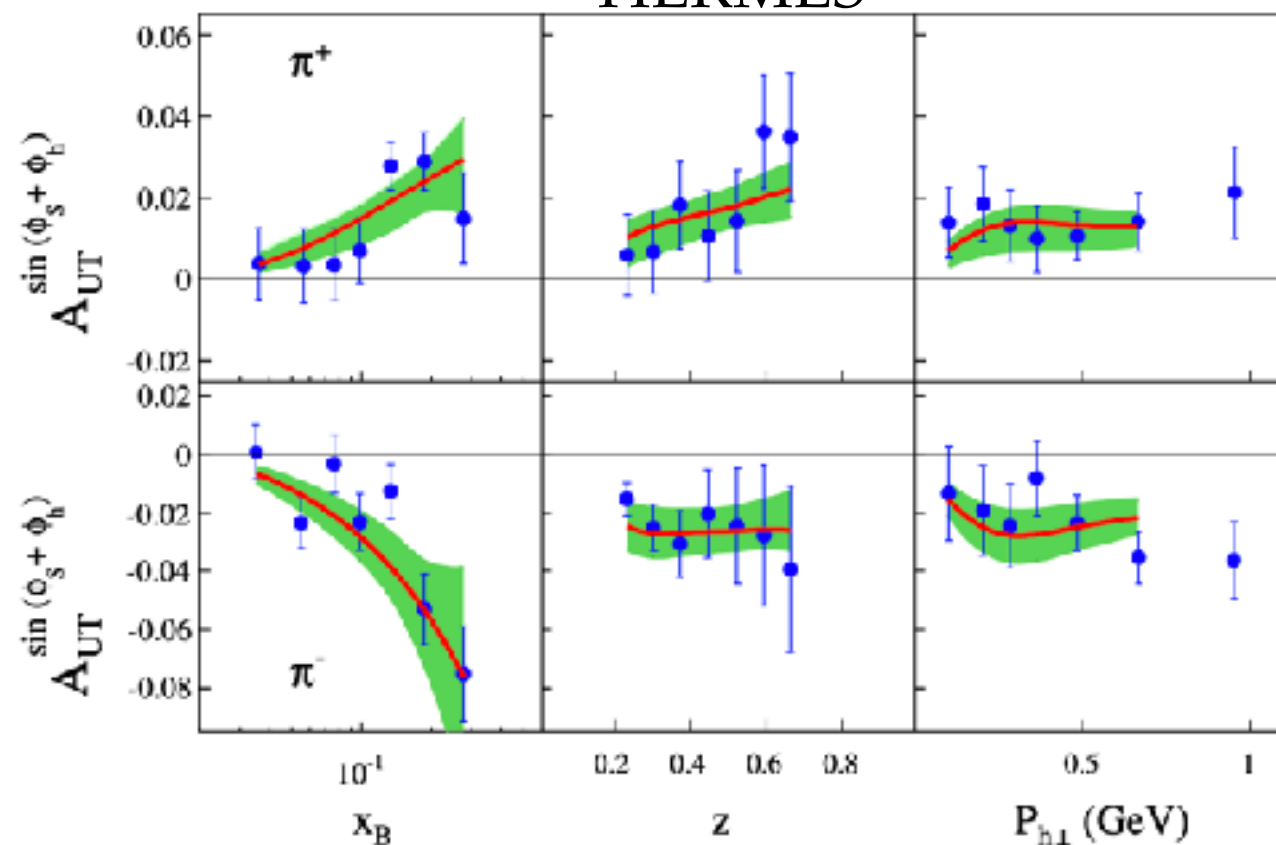
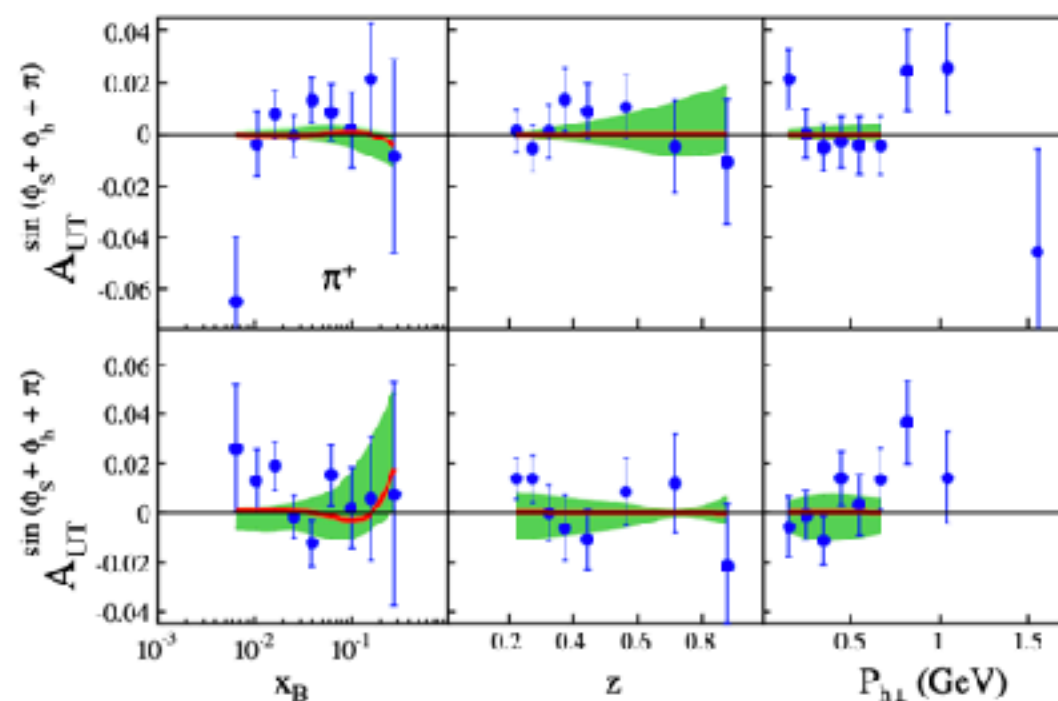
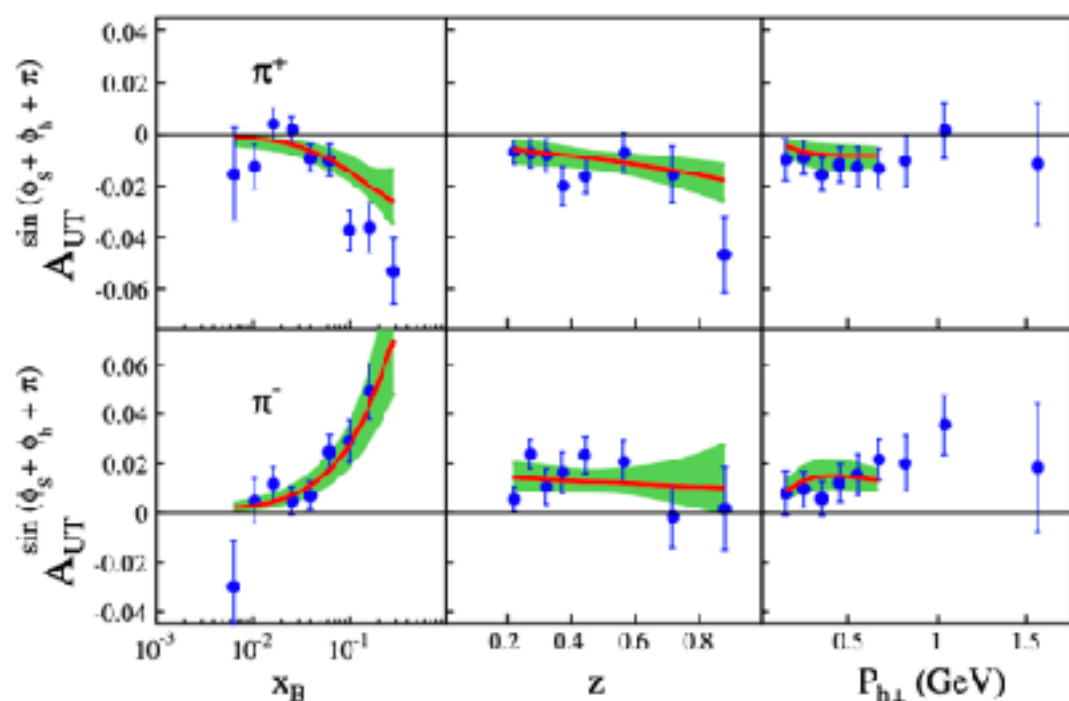


FIG. 14. Description of Collins asymmetries measured by the JLab's HALL A [9] as a function of x_B on the effective neutron target. The shaded region corresponds to our estimate of a 90% C.L. error band.



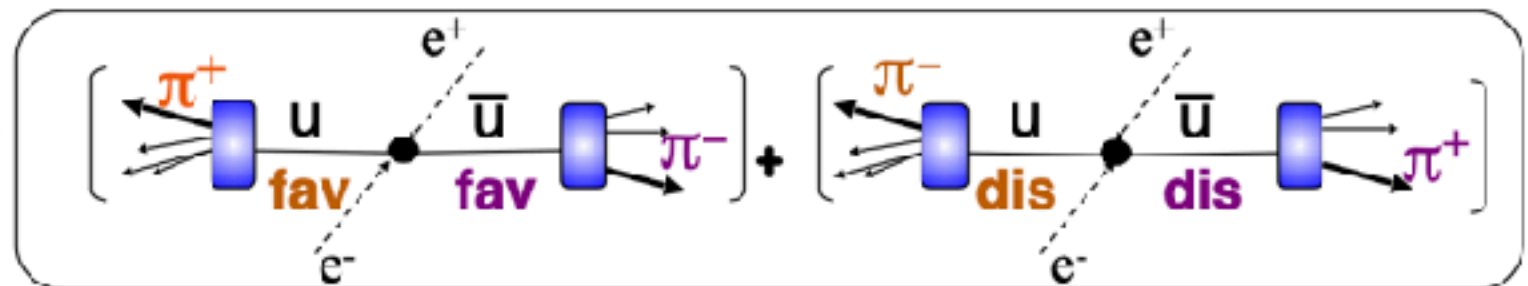
COMPASS

Favored and Disfavored processes

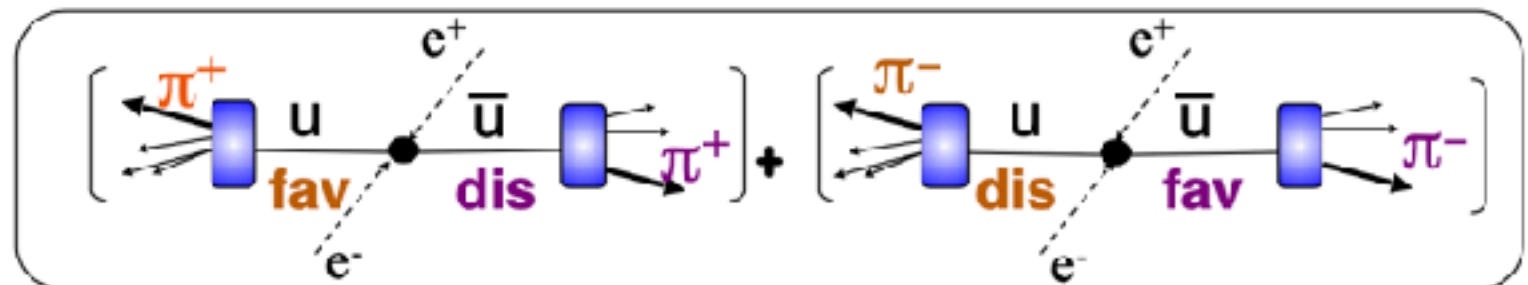
Different combinations of charged pions \Rightarrow sensitivity to **favored** or **disfavored** FFs

- **favored** process: fragmentation of a quark of flavor q into a hadron with a valence quark of the same flavor: i.e.: $u \rightarrow \pi^+$, $d \rightarrow \pi^-$
- **disfavored** for $d \rightarrow \pi^+$, $u \rightarrow \pi^-$, and $s \rightarrow \pi^\pm$

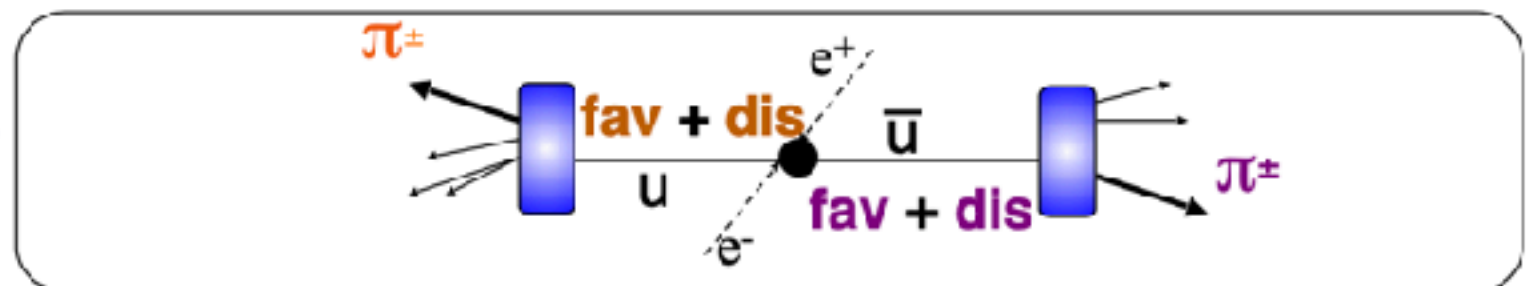
Unlike-sign pion pair = **U**:
 $\pi^+\pi^-$: (**fav** x **fav**) + (**dis** x **dis**)



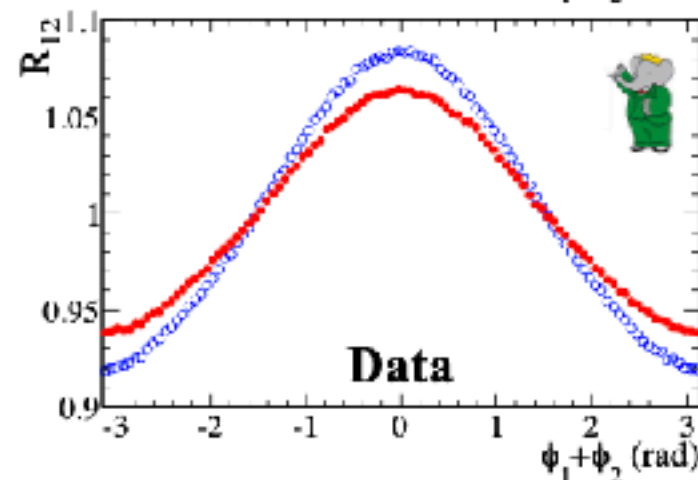
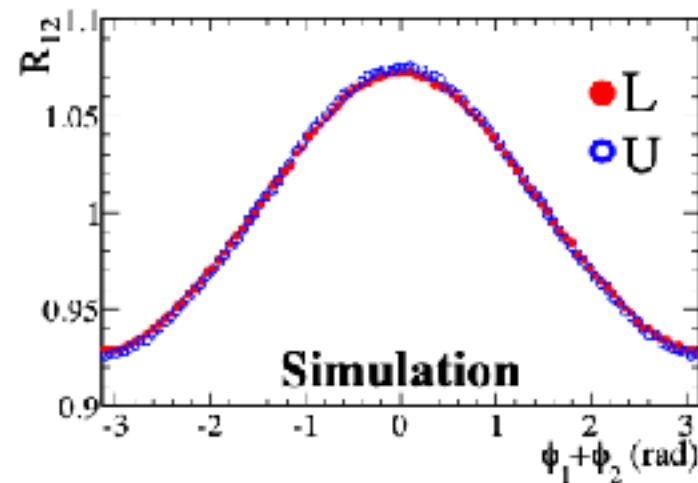
Like-sign pion pair = **L**:
 $\pi^+\pi^+$: (**fav** x **dis**) + (**dis** x **fav**)



Charged pion pair = **C (U+L)**:
 $\pi\pi$: (**fav** + **dis**) x (**fav** + **dis**)
 $\pi=\pi^\pm$



Raw asymmetries and Double Ratios



- **Collins asymmetry:**

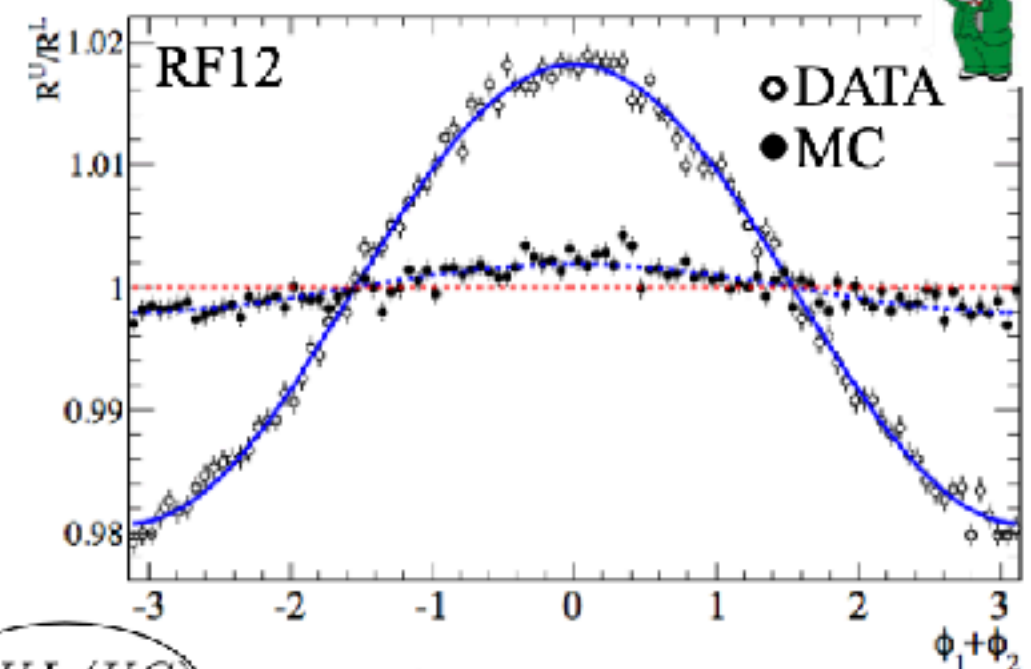
- consider all the **U** (unlike) and **L** (like) sign pion pairs
- make histograms of $\phi_\alpha = \phi_1 + \phi_2$ or $2\phi_0$ ($\alpha=12,0$)

- **The MC generator (JETSET) does not include the Collins effect**

→ flat distribution is expected

- strong modulation due to acceptance of the detector
- but similar distribution for **U** and **L** pairs

- Data shows difference between U and L distributions, that can be ascribed to the **Collins effect**



Acceptance effects can be reduced by performing the ratio of **U/L** sign pion pairs (or **U/C**):

- MC: consistent with a flat distribution
- Data: cosine modulation clearly visible

$$\frac{R_\alpha^U}{R_\alpha^{L(C)}} = \frac{N^U(\phi_\alpha) / \langle N^U(\phi_\alpha) \rangle}{N^{L(C)}(\phi_\alpha) / \langle N^{L(C)}(\phi_\alpha) \rangle} \rightarrow B_\alpha^{UL(UC)} + A_\alpha^{UL(UC)} \cdot \cos(\phi_\alpha)$$

Backgrounds: contributions and corrections

- In each bin, the data sample includes pairs from
 - signal uds events
 - $B\bar{B}$ events (small, mostly at low z)
 - $c\bar{c}$ events (important at low/medium z)
 - $\tau^+\tau^-$ events (important at high z)

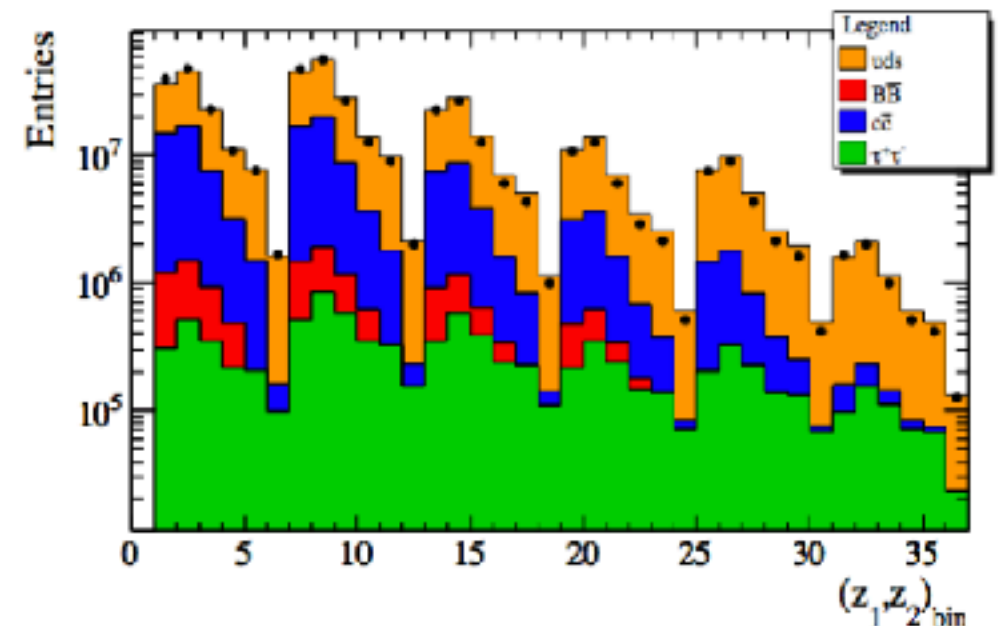
Fraction of $\pi\pi$ due to the i^{th} bkg process

True asymmetry

$$A_{\alpha}^{meas} = (1 - \sum_i F_i) \cdot A_{\alpha} + \sum_i F_i \cdot A_{\alpha}^i$$

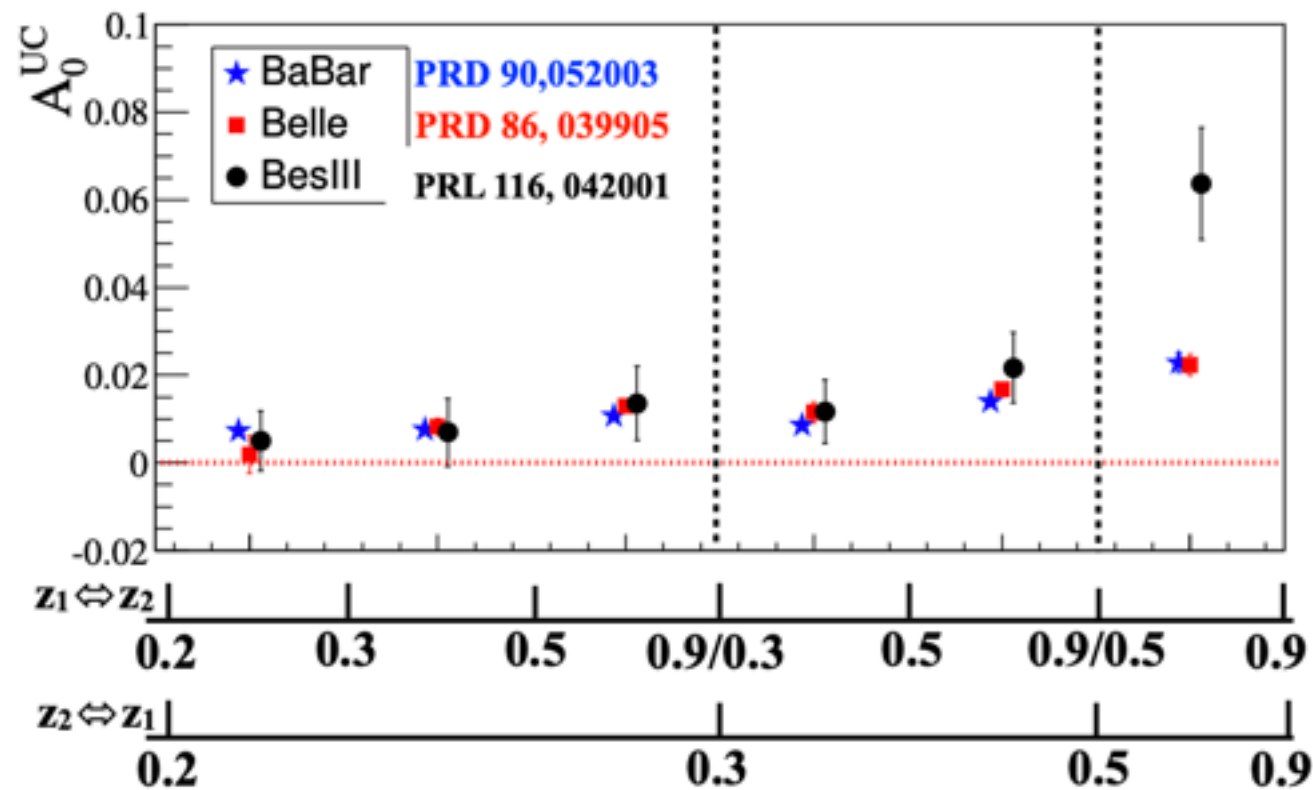
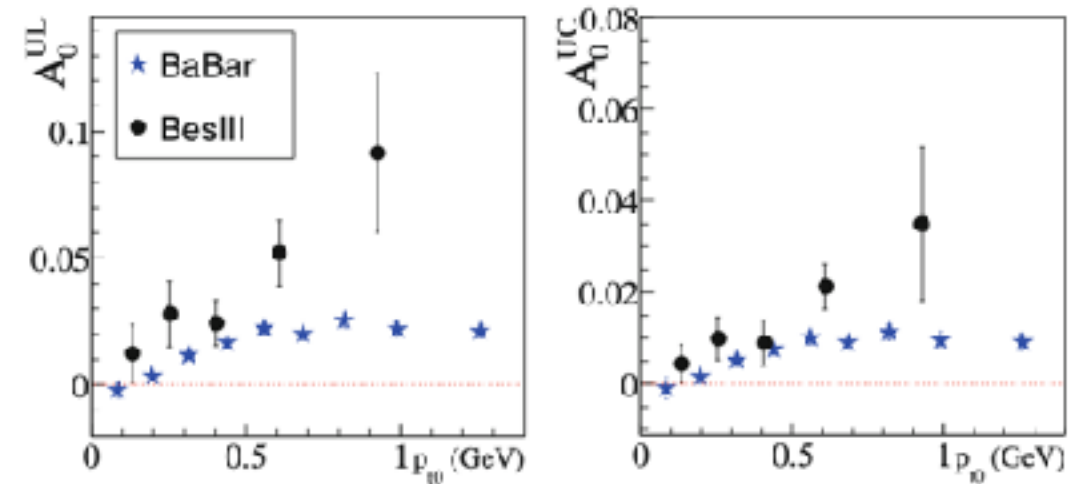
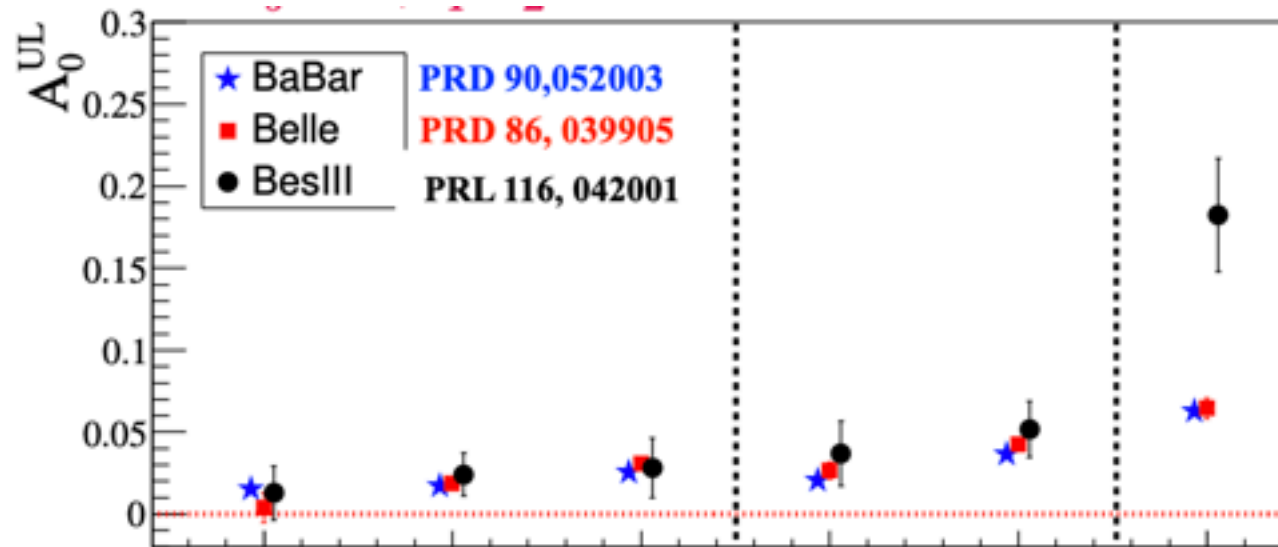
Bkg asymmetry

- We must calculate these quantities:
 - F_i using MC sample; we assign MC-data difference in each bin as systematic error
 - $A^{B\bar{B}}$ must be zero; we set $A^{B\bar{B}} = 0$
 - A^{τ} small in simulation; checked in data; we set $A^{\tau} = 0$
- **Charm** background contribution is about 30% on average
 - Both fragmentation processes and weak decays can introduce azimuthal asymmetries
 - We used a **$D^{*\pm}$ -enhanced control sample** to estimate its effect

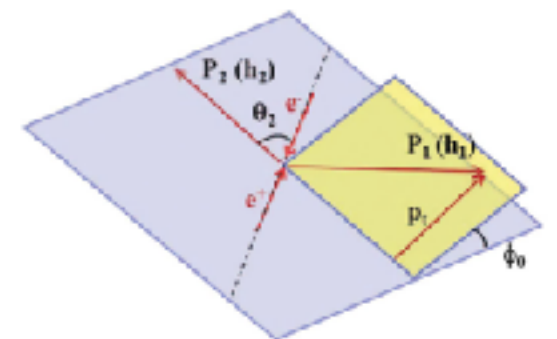


$$\begin{cases} A_{\alpha}^{meas} &= (1 - F_c - F_B - F_{\tau}) \cdot A_{\alpha} + F_c \cdot A_{\alpha}^{ch} \\ A_{\alpha}^{D^*} &= f_c \cdot A_{\alpha}^{ch} + (1 - f_c - f_B) \cdot A_{\alpha} \end{cases}$$

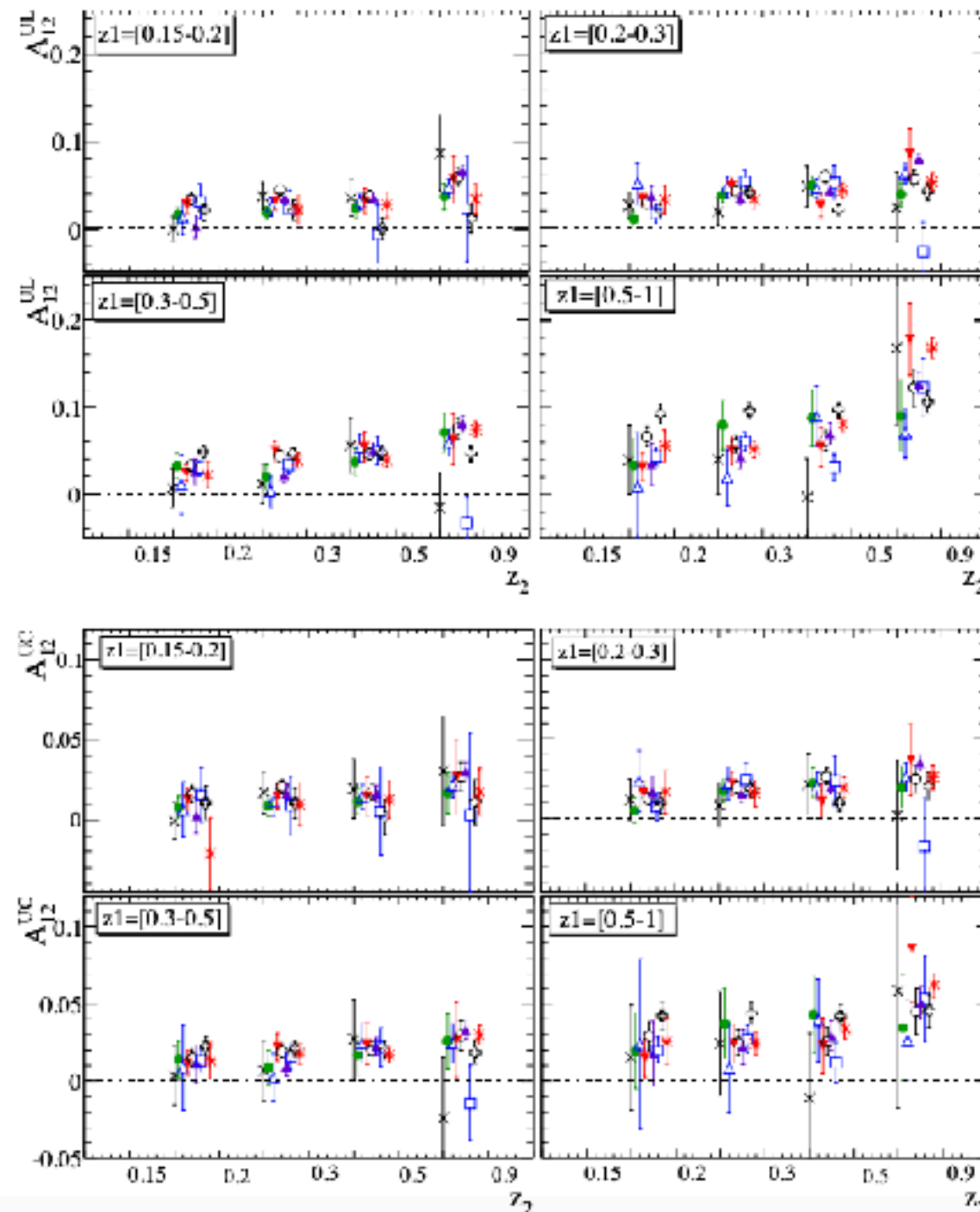
Collins FFs Results Comparison



- Larger asymmetry from BESIII data, in agreement with the prediction (PRD93,014009)



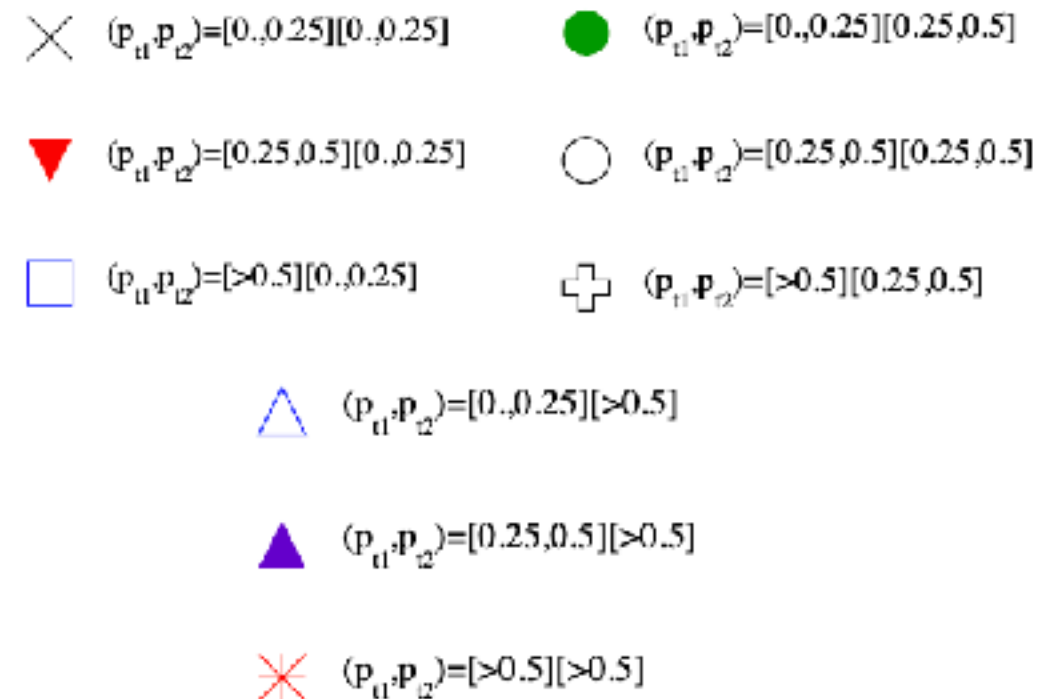
4-D: asymmetry vs. $(z_1, z_2) \times (p_{t1}, p_{t2})$



We study the asymmetries in the RF12 frame in a four-dimensional space:

$$(z_1, z_2, p_{t1}, p_{t2})$$

- We use 4 z_i and 3 p_t intervals
- Test to probe the factorization of the Collins fragmentation functions
- Powerful tools to access p_t - z correlation



Azimuthal asymmetries of back-to-back π^\pm - (π^0, η, π^\pm) from Belle

PRD100,092008

$$\mathcal{R}_{12}^{UL} \approx 1 + \cos(\phi_{12}) \frac{\sin^2(\theta)}{1 + \cos^2(\theta)} \times \left\{ \frac{5(H_1^{\perp, \text{fav}} \otimes H_1^{\perp, \text{fav}} + H_1^{\perp, \text{dis}} \otimes H_1^{\perp, \text{dis}}) + 2H_{1,s \rightarrow \pi}^{\perp, \text{dis}} \otimes H_{1,s \rightarrow \pi}^{\perp, \text{dis}}}{5(D_1^{\text{fav}} \otimes D_1^{\text{fav}} + D_1^{\text{dis}} \otimes D_1^{\text{dis}}) + 2D_{1,s \rightarrow \pi}^{\text{dis}} \otimes D_{1,s \rightarrow \pi}^{\text{dis}}} - \frac{10H_1^{\perp, \text{fav}} \otimes H_1^{\perp, \text{dis}} + 2H_{1,s \rightarrow \pi}^{\perp, \text{dis}} \otimes H_{1,s \rightarrow \pi}^{\perp, \text{dis}}}{10D_1^{\text{fav}} \otimes D_1^{\text{dis}} + 2D_{1,s \rightarrow \pi}^{\text{dis}} \otimes D_{1,s \rightarrow \pi}^{\text{dis}}} \right\},$$

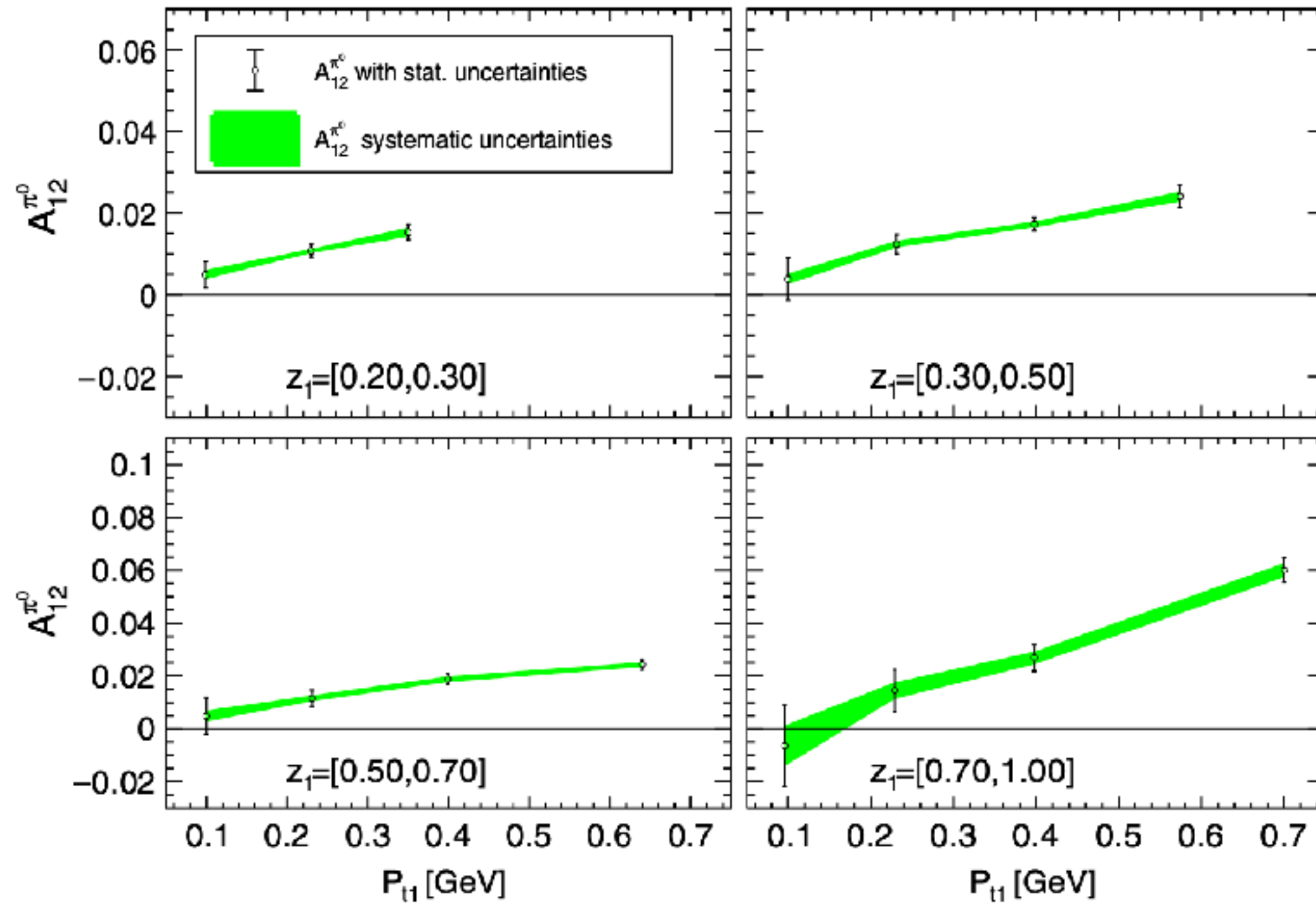
$$\mathcal{R}_{12}^{UC} \approx 1 + \cos(\phi_{12}) \frac{\sin^2(\theta)}{1 + \cos^2(\theta)} \times \left\{ \frac{5(H_1^{\perp, \text{fav}} \otimes H_1^{\perp, \text{fav}} + H_1^{\perp, \text{dis}} \otimes H_1^{\perp, \text{dis}}) + 2H_{1,s \rightarrow \pi}^{\perp, \text{dis}} \otimes H_{1,s \rightarrow \pi}^{\perp, \text{dis}}}{5(D_1^{\text{fav}} \otimes D_1^{\text{fav}} + D_1^{\text{dis}} \otimes D_1^{\text{dis}}) + 2D_{1,s \rightarrow \pi}^{\text{dis}} \otimes D_{1,s \rightarrow \pi}^{\text{dis}}} - \frac{5(H_1^{\perp, \text{fav}} + H_1^{\perp, \text{dis}}) \otimes (H_1^{\perp, \text{fav}} + H_1^{\perp, \text{dis}}) + 4H_{1,s \rightarrow \pi}^{\perp, \text{dis}} \otimes H_{1,s \rightarrow \pi}^{\perp, \text{dis}}}{5(D_1^{\text{fav}} + D_1^{\text{dis}}) \otimes (D_1^{\text{fav}} + D_1^{\text{dis}}) + 4D_{1,s \rightarrow \pi}^{\text{dis}} \otimes D_{1,s \rightarrow \pi}^{\text{dis}}} \right\},$$

$$\mathcal{R}_{12}^{\pi^0} = \frac{R_{12}^{0\pm}}{R_{12}^L} \approx 1 + \cos(\phi_{12}) \frac{\sin^2(\theta)}{1 + \cos^2(\theta)} \times \left\{ \frac{5(H_1^{\perp, \text{fav}} + H_1^{\perp, \text{dis}}) \otimes (H_1^{\perp, \text{fav}} + H_1^{\perp, \text{dis}}) + 4H_{1,s \rightarrow \pi}^{\perp, \text{dis}} \otimes H_{1,s \rightarrow \pi}^{\perp, \text{dis}}}{5(D_1^{\text{fav}} + D_1^{\text{dis}}) \otimes (D_1^{\text{fav}} + D_1^{\text{dis}}) + 4D_{1,s \rightarrow \pi}^{\text{dis}} \otimes D_{1,s \rightarrow \pi}^{\text{dis}}} - \frac{10H_1^{\perp, \text{fav}} \otimes H_1^{\perp, \text{dis}} + 2H_{1,s \rightarrow \pi}^{\perp, \text{dis}} \otimes H_{1,s \rightarrow \pi}^{\perp, \text{dis}}}{10D_1^{\text{fav}} \otimes D_1^{\text{dis}} + 2D_{1,s \rightarrow \pi}^{\text{dis}} \otimes D_{1,s \rightarrow \pi}^{\text{dis}}} \right\}.$$

$$\mathcal{R}_{12}^\eta = \frac{R_{12}^{\eta\pm}}{R_{12}^L} \approx 1 + \cos(\phi_{12}) \frac{\sin^2(\theta)}{1 + \cos^2(\theta)} \times \left\{ \frac{5(H_1^{\perp, \text{fav}_\eta} + H_1^{\perp, \text{dis}_\eta}) \otimes (H_1^{\perp, \text{dis}} + H_1^{\perp, \text{fav}}) + 4H_{1,s \rightarrow \eta}^{\perp, \text{dis}} \otimes H_{1,s \rightarrow \pi}^{\perp, \text{dis}}}{5(D_1^{\perp, \text{fav}_\eta} + D_1^{\perp, \text{dis}_\eta}) \otimes (D_1^{\perp, \text{dis}} + D_1^{\perp, \text{fav}}) + 4D_{1,s \rightarrow \eta}^{\perp, \text{dis}} \otimes D_{1,s \rightarrow \pi}^{\perp, \text{dis}}} - \frac{10H_1^{\perp, \text{fav}} \otimes H_1^{\perp, \text{dis}} + 2H_{1,s \rightarrow \pi}^{\perp, \text{dis}} \otimes H_{1,s \rightarrow \pi}^{\perp, \text{dis}}}{10D_1^{\perp, \text{fav}} \otimes D_1^{\perp, \text{dis}} + 2D_{1,s \rightarrow \pi}^{\perp, \text{dis}} \otimes D_{1,s \rightarrow \pi}^{\perp, \text{dis}}} \right\}.$$

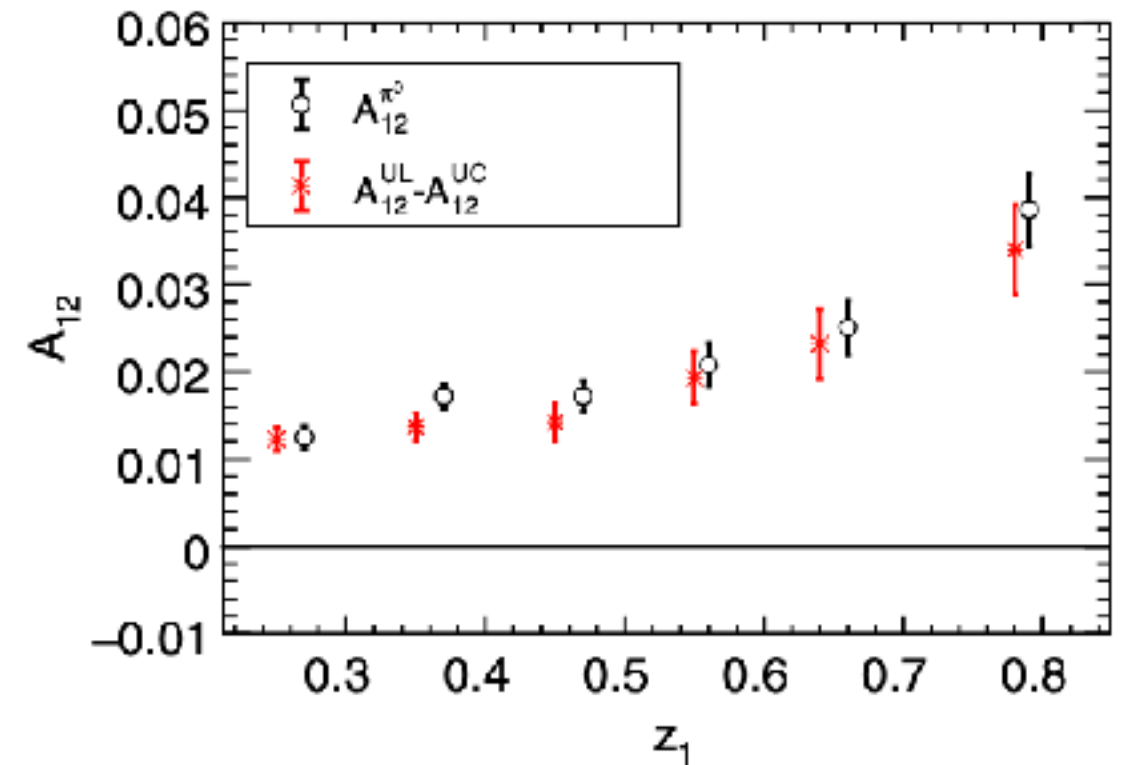
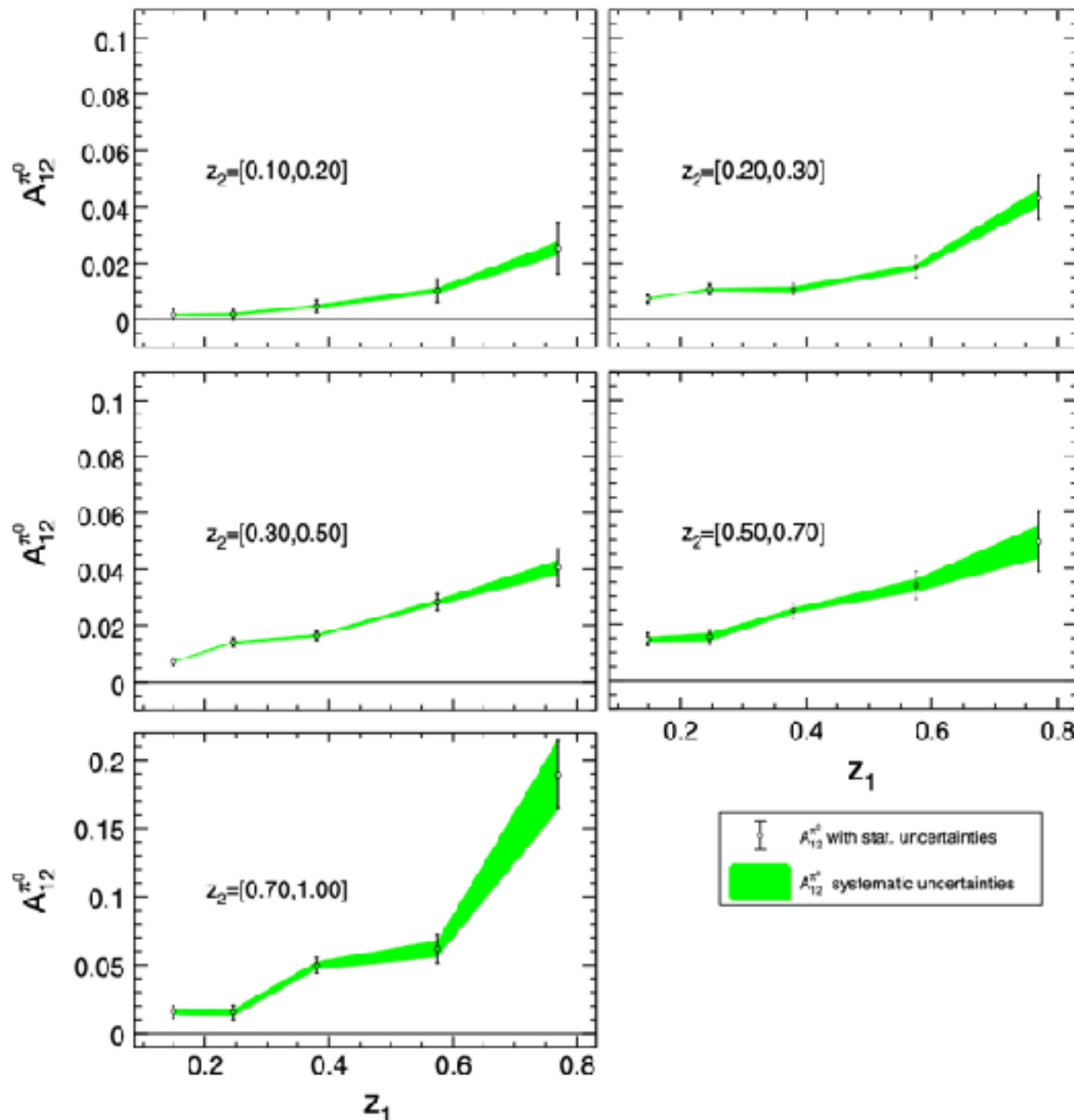
Azimuthal asymmetries of back-to-back $\pi^\pm - (\pi^0, \eta, \pi^\pm)$ from Belle

PRD100,092008



Azimuthal asymmetries of back-to-back $\pi^\pm\pi^0$ from Belle

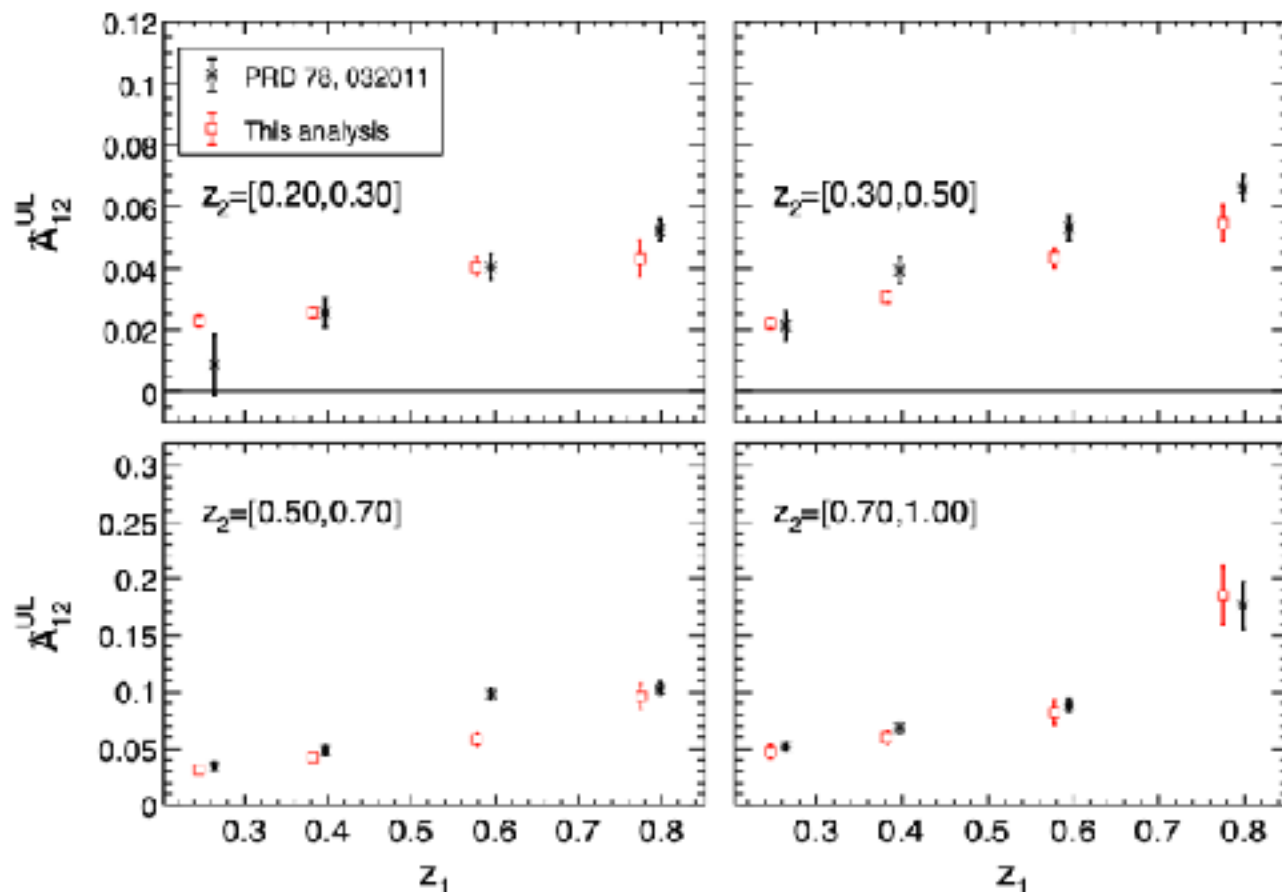
PRD100,092008



- Good agreement between $A_{12}^{UL} - A_{12}^{UC}$ and $A_{12}^{\pi^0}$ as expected from isospin relation (taking into account statistical and systematic uncertainties)

Azimuthal asymmetries of back-to-back π^\pm - (π^0, η, π^\pm) from Belle

PRD100,092008

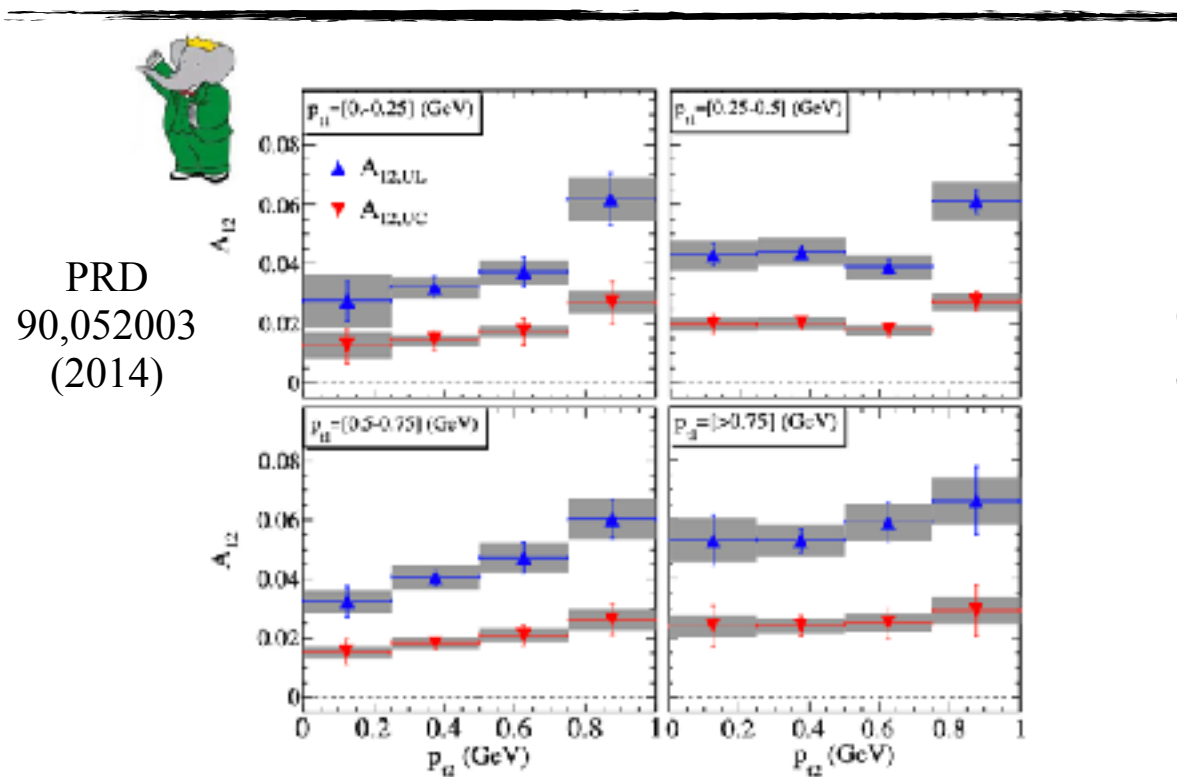


New Belle analysis: RF12 only

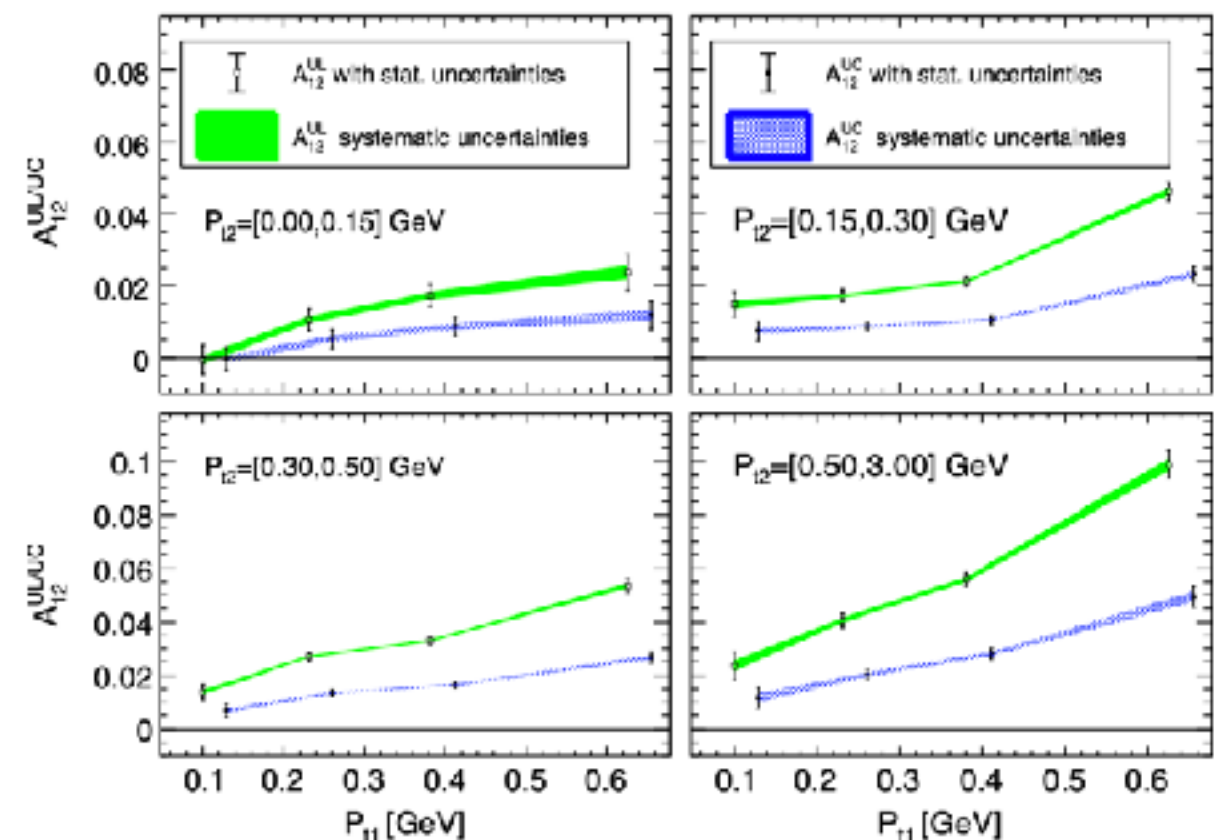
- No charm correction: only the estimated charm fractions are provided
- bin-by-bin generated-thrust axis correction (not the qqbar axis)
- Pt smearing correction
- MC asymmetry subtraction

Only for results comparison:

- Vanishing charm asymmetry
- No thrust axis correction
- Kinematic factors taken into account

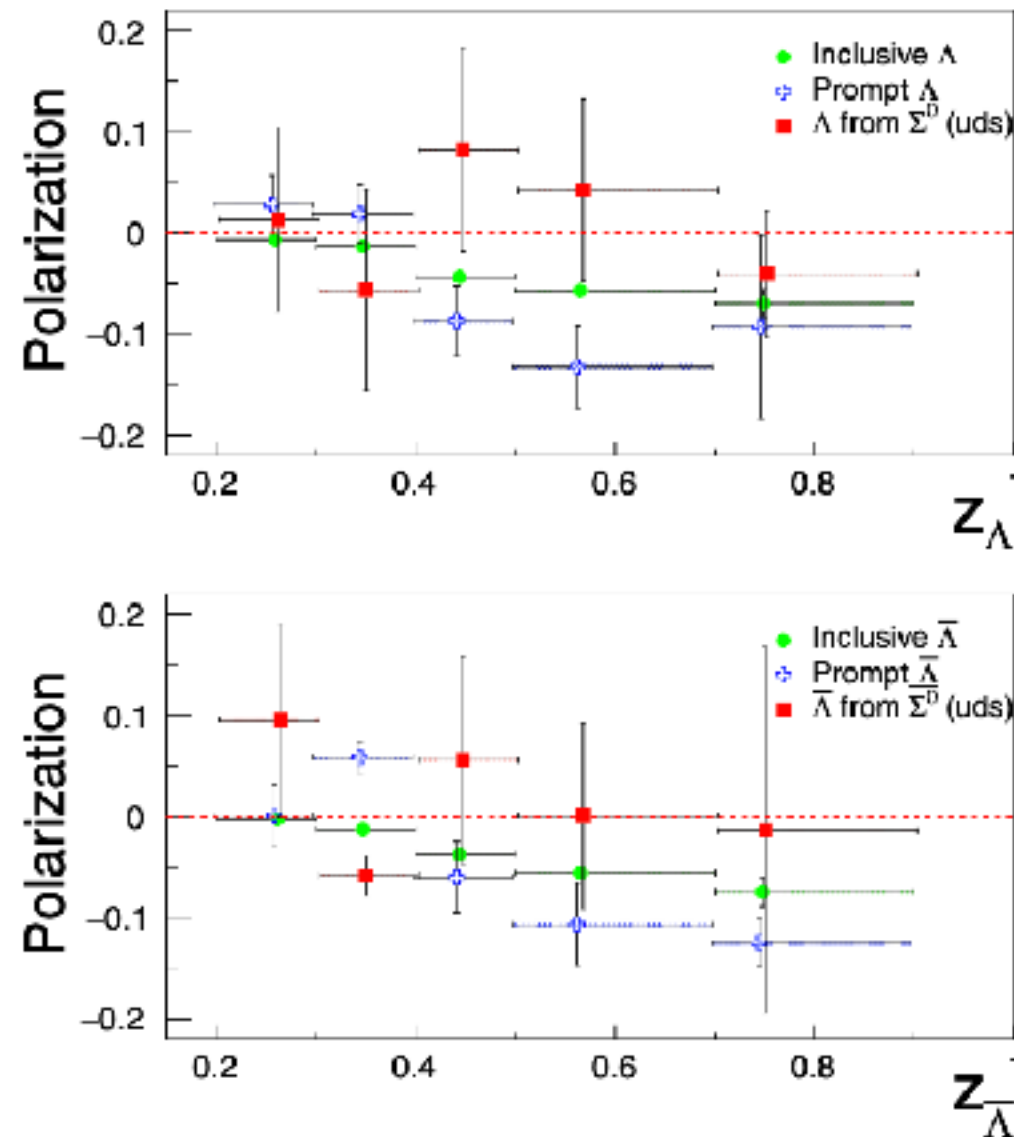


PRD
90,052003
(2014)



$\Lambda/\bar{\Lambda}$ hyperon polarization at Belle

PRL122,042001



- Charm corrected unfolded transverse polarisation also obtained ==> large statistical uncertainties

$\bar{\Lambda}/\Lambda$ hyperon polarization

PRD 100,014029

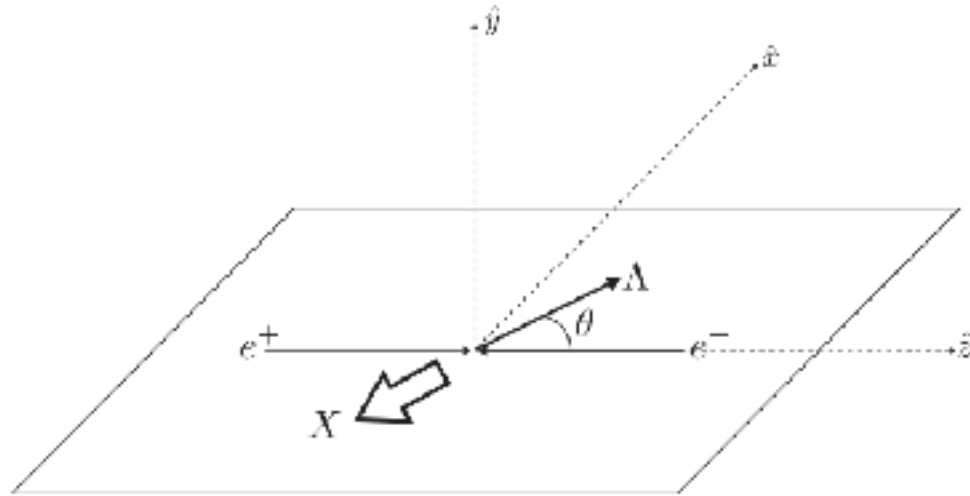
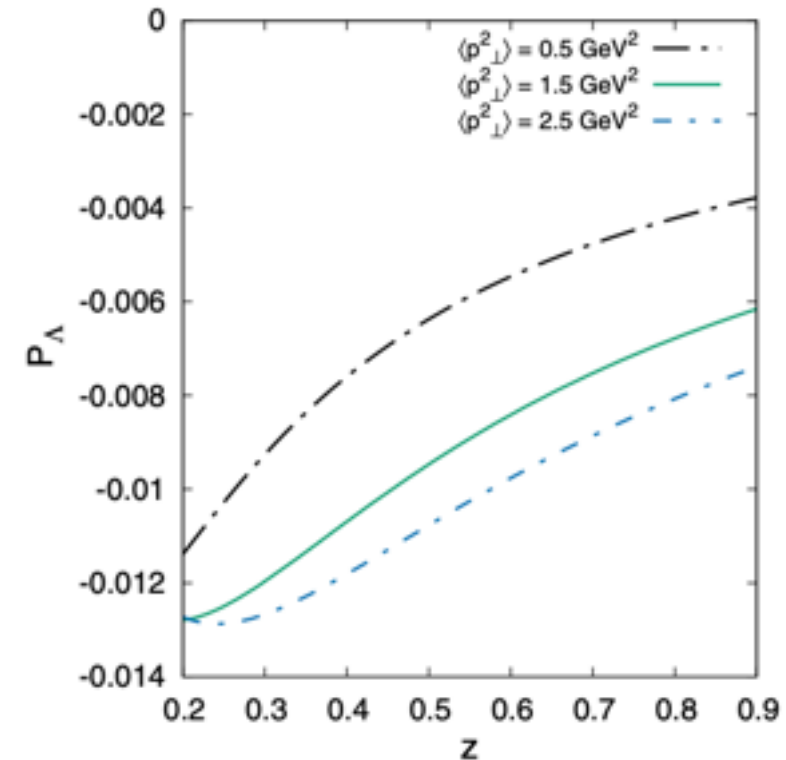
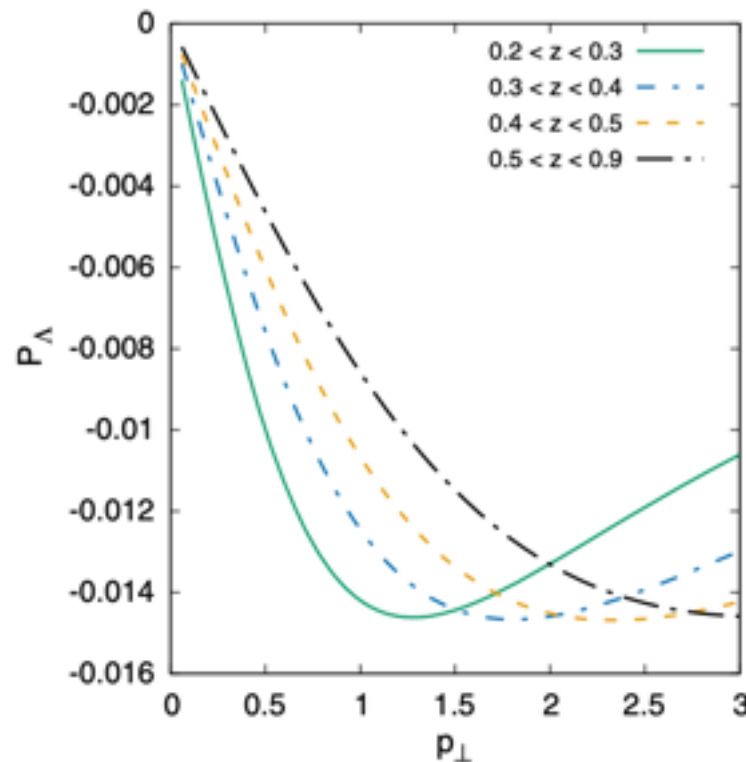


FIG. 1. Definition of the c.m. reference frame for the process $e^+e^- \rightarrow \Lambda X$.

$$P_{\Lambda}(z, \cos \theta) = \frac{\sum_q e_q^2 \int p_{\perp} dp_{\perp} \frac{4z p_{\perp} \sqrt{s}}{z^2 s + 4p_{\perp}^2} \Delta^N D_{\Lambda^{\dagger}/q}(z, p_{\perp})}{\sum_q e_q^2 D_{\Lambda/q}(z)} \times \frac{\sin(2\theta)}{1 + \cos^2 \theta} \quad (21)$$

$$P_{\Lambda}(z, p_{\perp}) = \frac{\sum_q e_q^2 \frac{4z p_{\perp} \sqrt{s}}{z^2 s + 4p_{\perp}^2} \Delta^N D_{\Lambda^{\dagger}/q}(z, p_{\perp})}{\sum_q e_q^2 2\pi D_{\Lambda/q}(z, p_{\perp})} \times \frac{\int d(\cos \theta) \sin(2\theta)}{\int d(\cos \theta) (1 + \cos^2 \theta)} \quad (22)$$



Helicity Formalism: TMD Fragmentation Functions for quarks

Quark Polarisation States determine the Hadron ones

$$\rho_{\lambda_h, \lambda'_h}^{h, S_h} \hat{D}_{h/q, s_q}(z, \mathbf{k}_{\perp h}) = \sum_{\lambda_q, \lambda'_q} \rho_{\lambda_q, \lambda'_q}^{q, s_q} \hat{D}_{\lambda_q, \lambda'_q}^{\lambda_h, \lambda'_h}(z, \mathbf{k}_{\perp h})$$

$$P_j^h \hat{D}_{h/q, s_q} = \hat{D}_{S_j/q, s_q}^h - \hat{D}_{-S_j/q, s_q}^h = \Delta \hat{D}_{S_j/s_q}^h$$

Helicity density matrix

$$\rho_{\lambda_i, \lambda'_i}^{i, s_i} = \frac{1}{2} \begin{pmatrix} 1 + P_z^i & P_x^i - iP_y^i \\ P_x^i + iP_y^i & 1 - P_z^i \end{pmatrix}$$

Amsterdam notation:

Collins $\Delta^N D_{h/q^\uparrow} \longleftrightarrow H_1^\perp$

Polarizing $\Delta D_{S_Y/q}^h \longleftrightarrow D_{1T}^\perp$

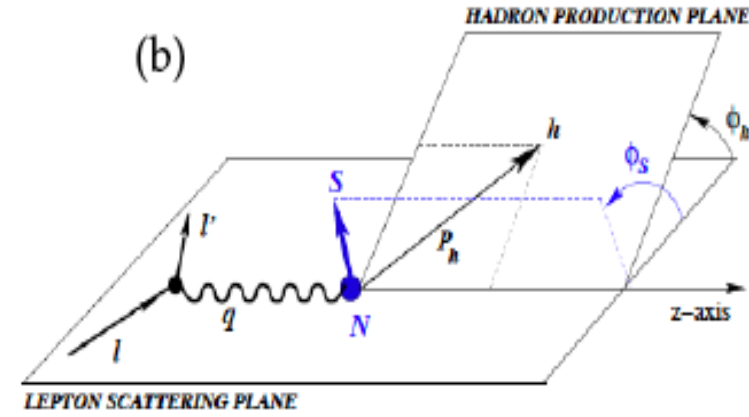
$$\begin{cases} \Delta^- D_{S_Y/s_T}^{h_1} \\ \Delta D_{S_X/s_T}^{h_1} \end{cases} \leftrightarrow \begin{cases} H_1 \\ H_{1T}^\perp \end{cases}$$

8 independent Fragmentation Functions

		Hadron		
		U	L	T
Q u a r k	U	$\hat{D}_{h/q}$		$\Delta \hat{D}_{S_Y/q}^h$
	L		$\Delta \hat{D}_{S_Z/s_L}^{h/q}$	$\Delta \hat{D}_{S_X/s_L}^{h/q}$
	T	$\Delta^N D_{h/q^\uparrow}$	$\Delta \hat{D}_{S_Z/s_T}^{h/q}$	$\Delta \hat{D}_{S_X/s_T}^{h/q} / \Delta^- \hat{D}_{S_Y/s_T}^{h/q}$

SIDIS cross section

$$\frac{d\sigma^h}{dx dy d\phi_S dz d\phi d\mathbf{P}_{h\perp}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x} \right) \left\{ \begin{aligned} & \left[F_{UU,T} + \epsilon F_{UU,L} \right. \\ & \quad \left. + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \right] \\ & + \lambda_l \left[\sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right] \\ & + S_L \left[\sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right] \\ & + S_L \lambda_l \left[\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right] \\ & + S_T \left[\sin(\phi - \phi_S) \left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) \right. \\ & \quad + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \\ & \quad + \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \\ & \quad \left. + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \right] \\ & + S_T \lambda_l \left[\sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} \right. \\ & \quad + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \\ & \quad \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \right] \end{aligned} \right\}$$



Unpolarized proton and lepton


Longitudinally polarized proton, unpolarized lepton

Longitudinally polarized proton, polarized lepton

Transversely polarized proton, unpolarized lepton

Transversely polarized proton, polarized lepton

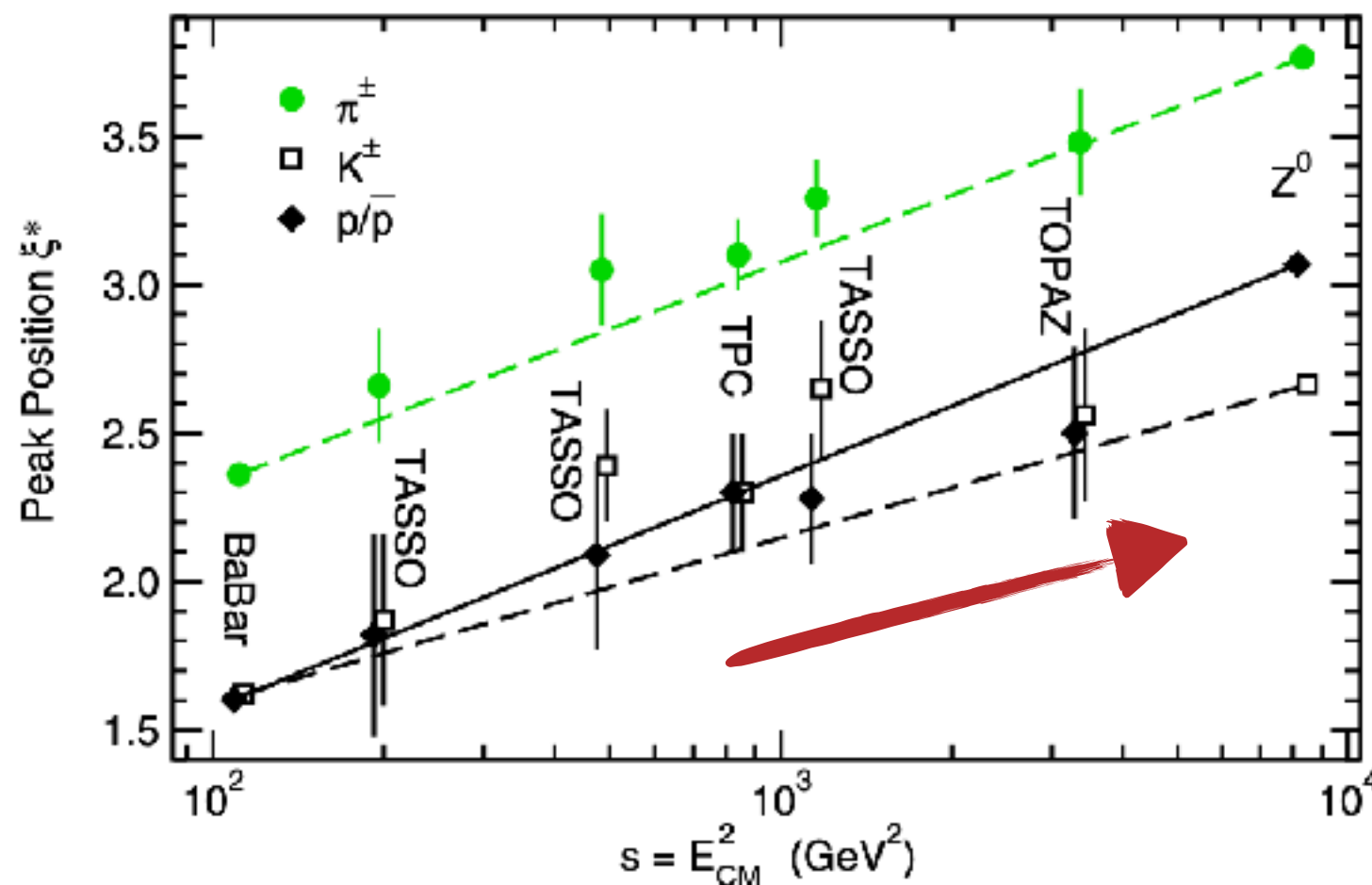
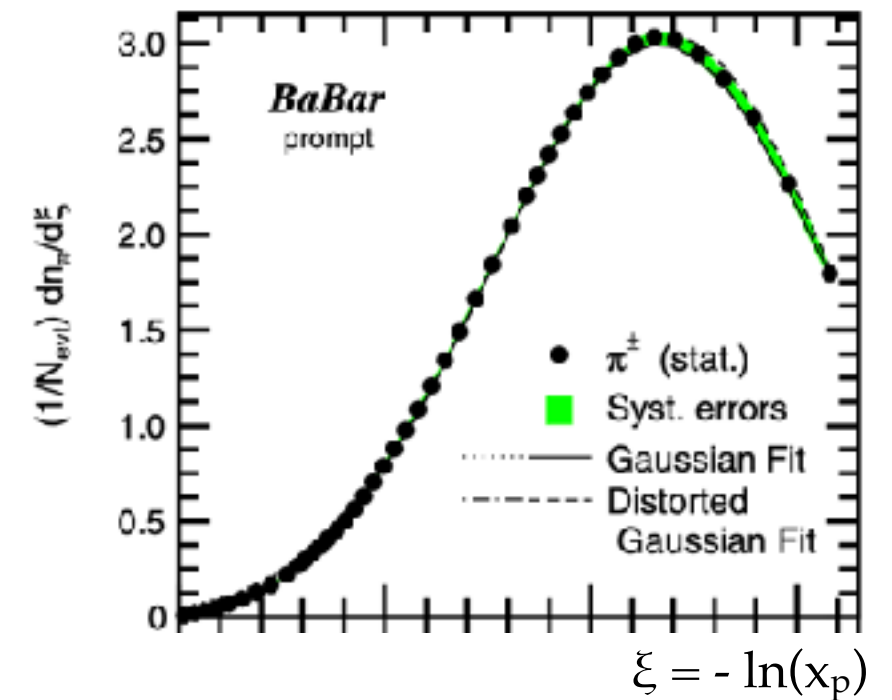
JHEP093(07)

Single hadron FF		
Unpolarized ingredients	Polarized ingredients	Flavor sensitivity
Single hadron cross sections: $e^+e^- \rightarrow hX$ $D_{1,q}^h(z, Q^2)$ PRL111 (2013) 062002 PRD101(2020) 092004	Azimuthal asymmetries: $e^+e^- \rightarrow (h)(h)X,$ $\cos(\phi_1 + \phi_2)$ $H_{1,q}^{\perp(1)h}(z, Q^2)$ PRL 96 (2006) 232002 PRD 78 (2008) 032011	Unpol SIDIS, pp: $\frac{d\sigma}{dz}$ $e^+e^- \rightarrow (h)(h)X$ PRD92 (2015) 092007 PRD101(2020) 092004 and scale dependence
Transverse momentum dependent FFs: $e^+e^- \rightarrow (h)X$ $D_{1,q}^h(z, k_T, Q^2)$ PRD 99 (2019) 112006	Transverse momentum dependent asymmetries $e^+e^- \rightarrow (h)(h)X,$ $\cos(\phi_1 + \phi_2), Q_t$ $H_{1,q}^{\perp h}(z, k_T, Q^2)$ PRD100 (2019) 92008	Polarizing Λ fragmentation PRL 122 (2019), 042001 $D_{1,q}^{\perp h}(z, k_T, Q^2)$
		
Dihadron FF (IFF)		
Unpolarized ingredients	Polarized ingredients	Flavor sensitivity
Dihadron cross sections $e^+e^- \rightarrow (hh)X$ $D_{1,q}^{h_1 h_2}(z, m, Q^2)$ PRD96 (2017) 032005	Azimuthal asymmetries: $e^+e^- \rightarrow (hh)(hh)X,$ $\cos(\phi_1 + \phi_2),$ $H_{1,q}^{h_1, h_2, \triangleleft}(z, Q^2, M_h)$ PRL107 (2011) 072004	Unpol SIDIS, pp: $\frac{d^2\sigma}{dzdm}$

Test of MLLA+LHPD QCD predictions

Modified Leading Logarithmic Approximation (MLLA) with Local Parton-Hadron Duality (LHPD) ansatz (Z.Phys.C27,65):

1) the multiplicity distributions vs $\xi = -\ln(x_p)$ should be Gaussian near the peak



2) the peak position should decrease exponentially with increasing hadron mass at a given E_{cm}

- observed that $\xi_K \approx \xi_p$

3) ξ^* should increase logarithmically with E_{mc} for a given hadron type

- similar slope for pions and protons, different for kaons
- possibly due to flavour composition changing with E_{cm}

$\Lambda/\bar{\Lambda}$ hyperon polarization at Belle

PRL 122,042001

- $e^+e^- \rightarrow \Lambda(\bar{\Lambda})X$ and $e^+e^- \rightarrow \Lambda(\bar{\Lambda})h^\pm X$ using 800.4 fb^{-1} collected by Belle at or near $\sqrt{s}=10.58 \text{ GeV}$

$$D_{1T}^{\perp \Lambda/q}(z, p_\perp^2)$$

\Rightarrow production of transversely polarized Λ hyperons from unpolarised quark
 \Rightarrow chiral-even function (the sign of this function can be easily determined)

Distribution of protons from Λ decays with a transverse polarisation P :

$$\frac{1}{N} \frac{dN}{d \cos \theta} = 1 + \alpha P \cos \theta,$$

$$\hat{D}_{h^\uparrow/q}(z, k_\perp) = \frac{1}{2} \hat{D}_{h/q}(z, k_\perp)$$

$$+ \frac{1}{2} \Delta^N D_{h^\uparrow/q}(z, k_\perp) \frac{\hat{P}_h \cdot (p_q \times k_\perp)}{|p_q \times k_\perp|}$$

hadron polarisation vector along \uparrow direction

PRD 100,014029

θ : angle between the axis $\hat{n} \propto \hat{T} \times \hat{p}_\Lambda$ and the proton momentum in the Λ rest frame; α is the world average value of the parity-violating decay asymmetry for the Λ (from PDG)
 \Rightarrow Significant transverse polarization is observed

