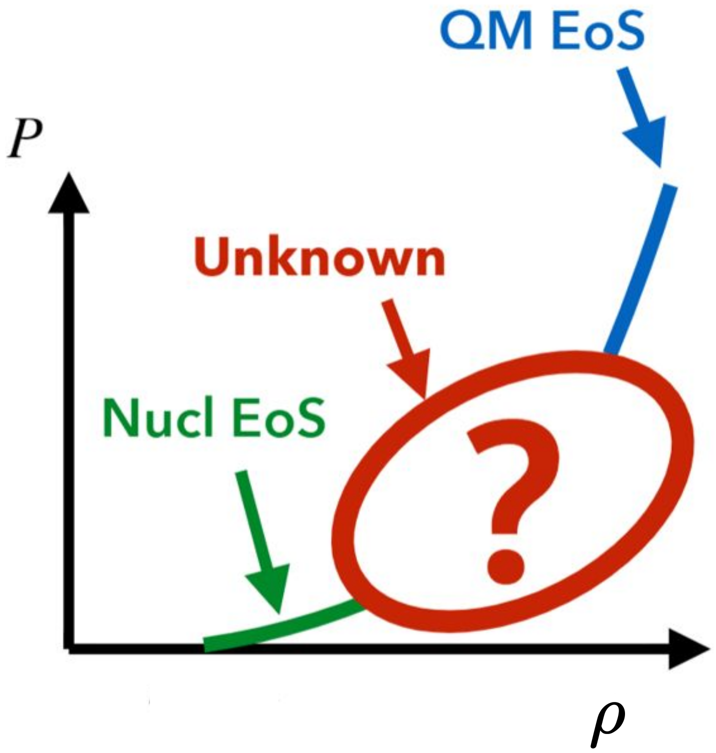
A Neural Network Reconstruction of the Dense Matter Equation of State

Shriya Soma, Shuzhe Shi, Lingxiao Wang, Horst Stöcker, Kai Zhou

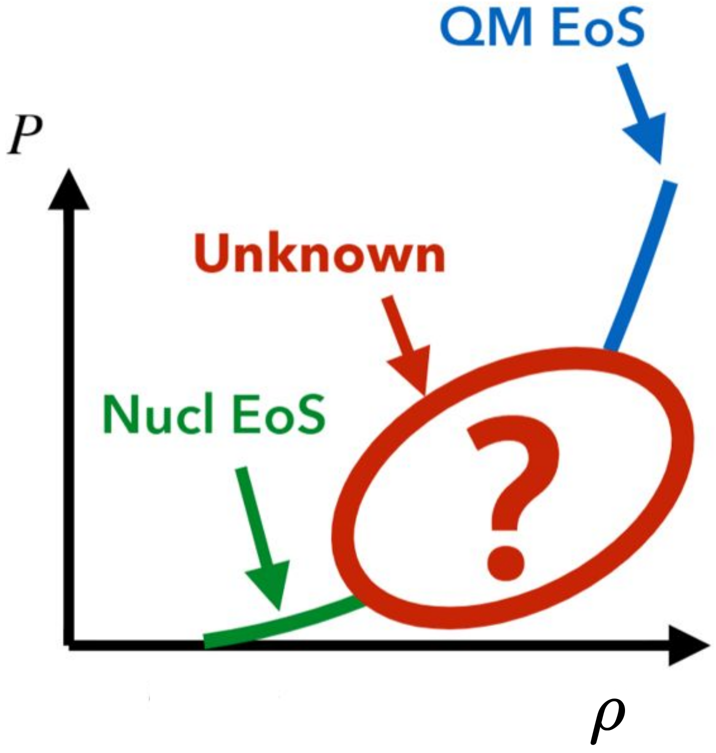
FAIR Next Generation Scientists - 7th Edition Workshop
24 May, 2022

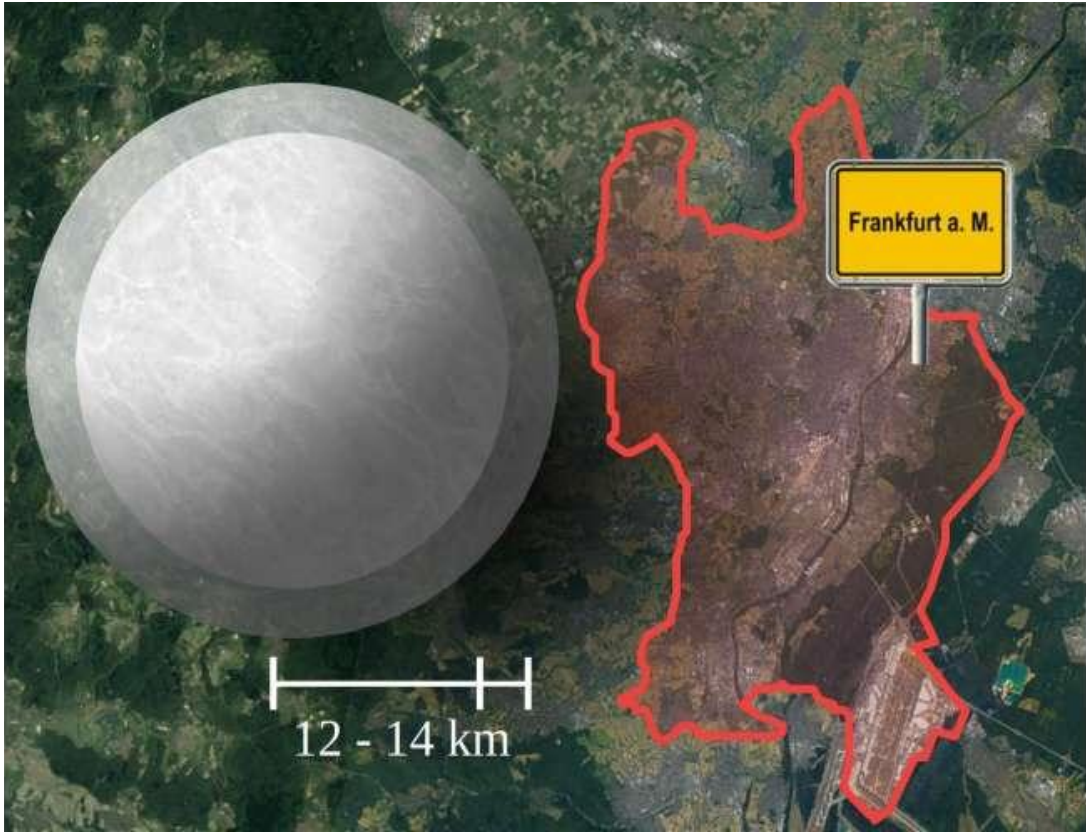
DAAD Deutscher Akademischer Austauschdienst German Academic Exchange Service
HGS-HIRe for FAIR Helmholtz Graduate School for Hadron and Ion Research
GOETHE UNIVERSITÄT FRANKFURT AM MAIN
FIAS Frankfurt Institute for Advanced Studies

Dense Matter EoS



Dense Matter EoS



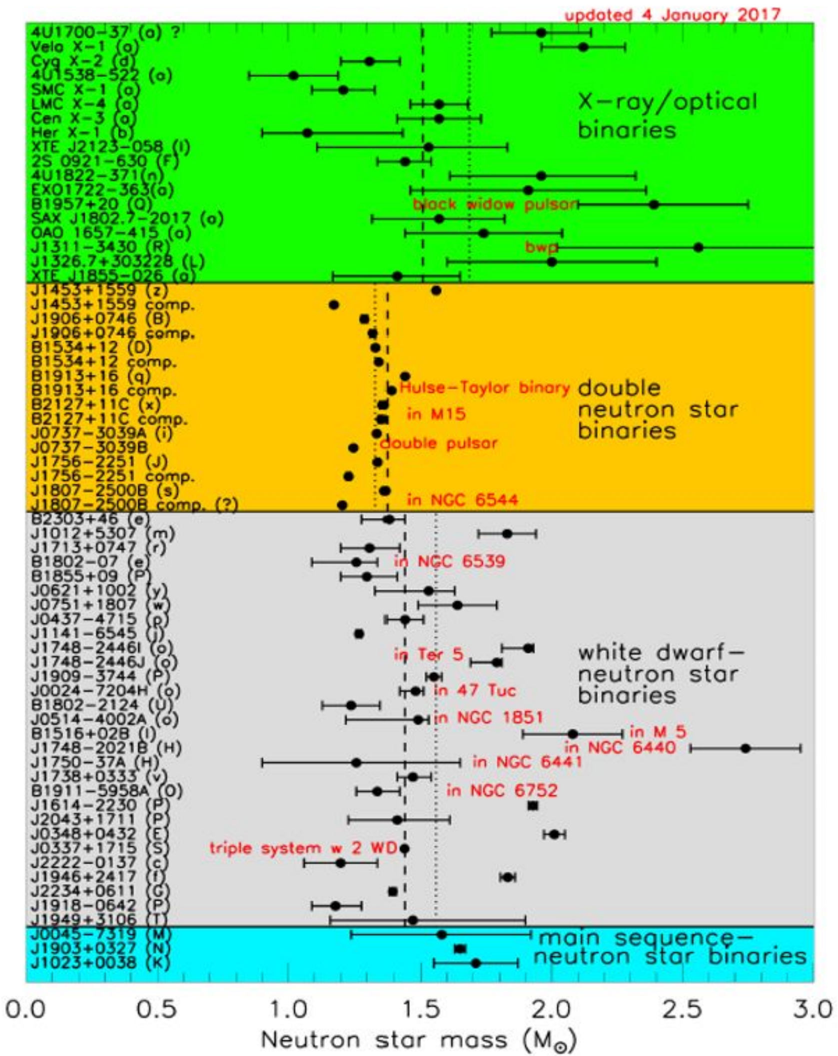


NS Observables

- Mass
- Radius
- Tidal Deformability

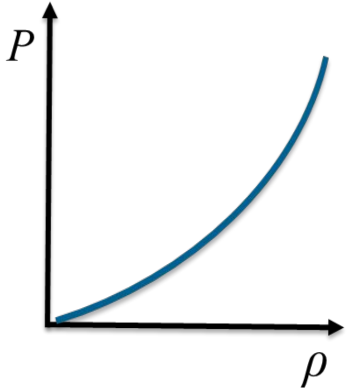
Antoniadis *et al.*, Science **340** (2013)
Cromartie *et al.*, NatAs **4** (2019) 72
Riley *et al.*, ApJL **887** (2019) L21
Riley *et al.*, ApJL **918** (2021) L27
Abbott *et al.*, PRX **9** (2019) 011001
Coughlin *et al.*, **480** (2018) 3

[Lattimer]



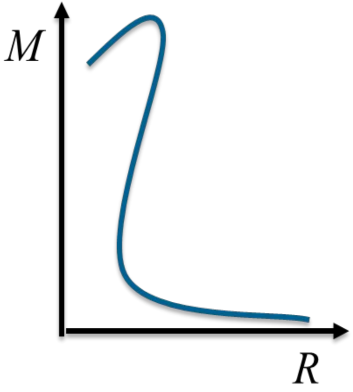
TOV Equations: From EoS to Stellar Structure

EoS → TOV Equations → MR Curve

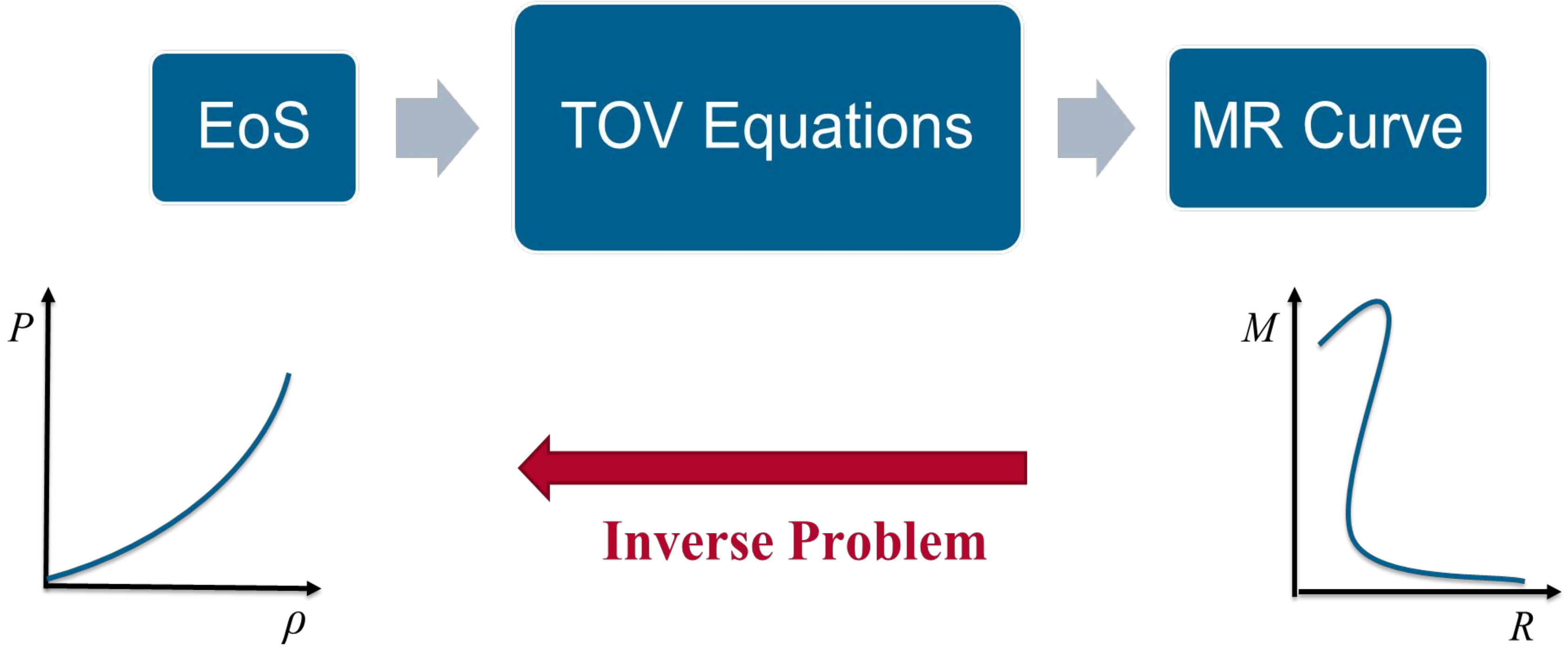


$$-\frac{dP}{dr} = \frac{\left[\epsilon(r) + P(r)\right]\left[M(r) + 4\pi r^3 P(r)\right]}{r[r - 2M(r)]}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \epsilon(r),$$



MR Observables to EoS: An Inverse Problem



The Bayesian Approach

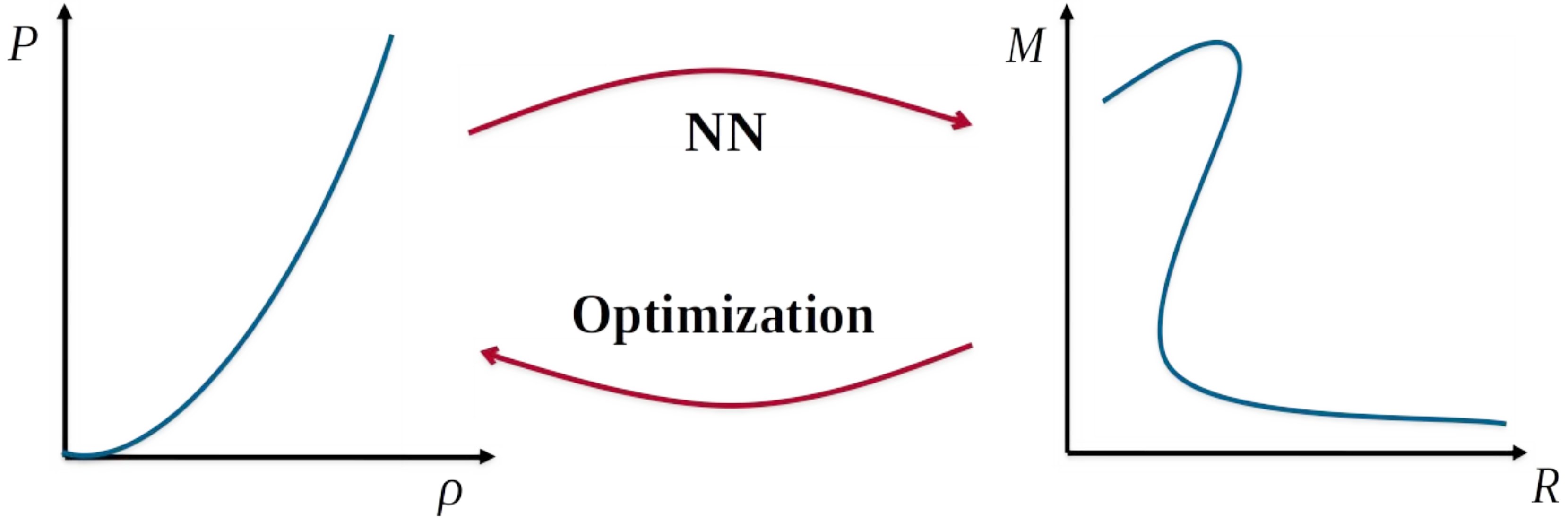
$$P(\mathrm{EoS}\,|\,M\text{-}R) = \frac{P(M\text{-}R\,|\,\mathrm{EoS})\,P(\mathrm{EoS})}{P(M\text{-}R)}$$

Steiner *et al.*, ApJL **765** (2013) L5

Raithel *et al.*, ApJ **844** (2017) 156

Automatic Differentiation

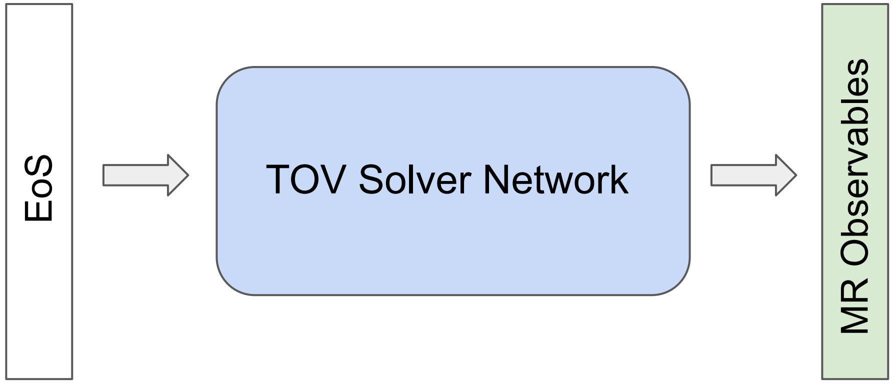
- Train a neural network (NN) to output the MR curve from an EoS



- Optimize the input (EoS) to obtain the desired output (MR curve)

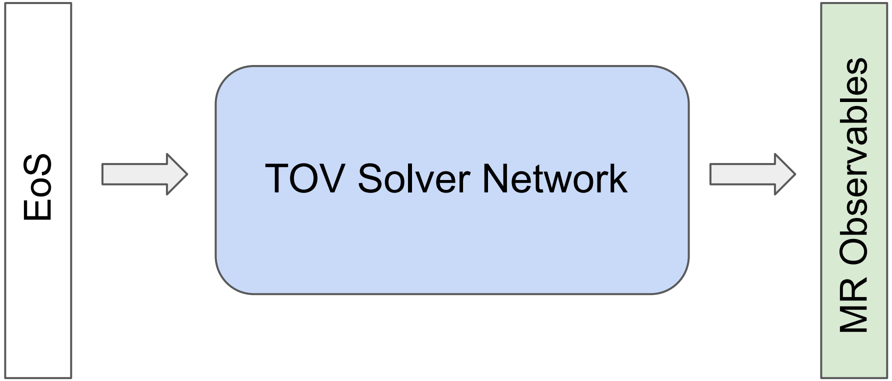
Procedure

1. Train a NN model to solve TOV Equations – TOV Solver Network



Procedure

1. Train a NN model to solve TOV Equations – TOV Solver Network

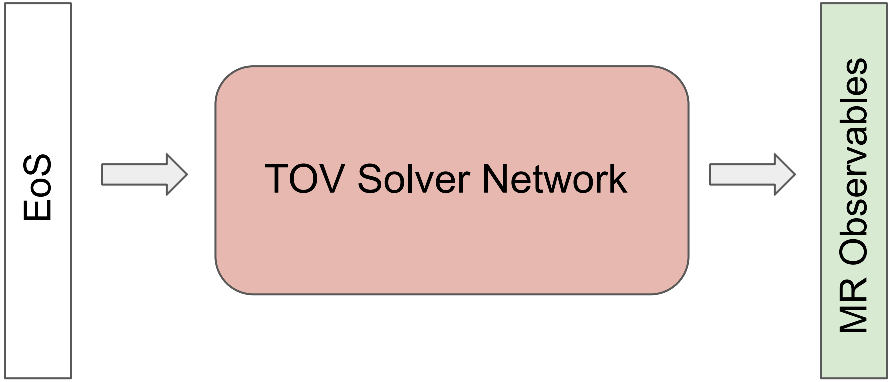


$$P(\mathrm{EoS} \mid M\text{-}R) = \frac{\boxed{P(M\text{-}R \mid \mathrm{EoS})}\, P(\mathrm{EoS})}{P(M\text{-}R)}$$

Procedure

1. Train a NN model to solve TOV Equations – TOV Solver Network

2. Fix the weights of TOV Solver (freeze training)

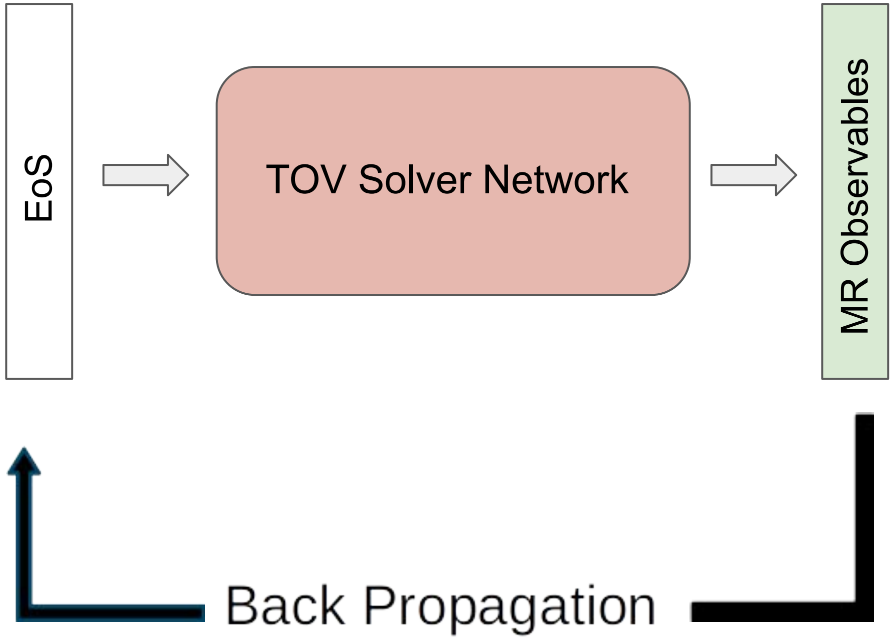


Procedure

1. Train a NN model to solve TOV Equations – TOV Solver Network

2. Fix the weights of TOV Solver (freeze training)

3. Optimize the input layer (EoS)

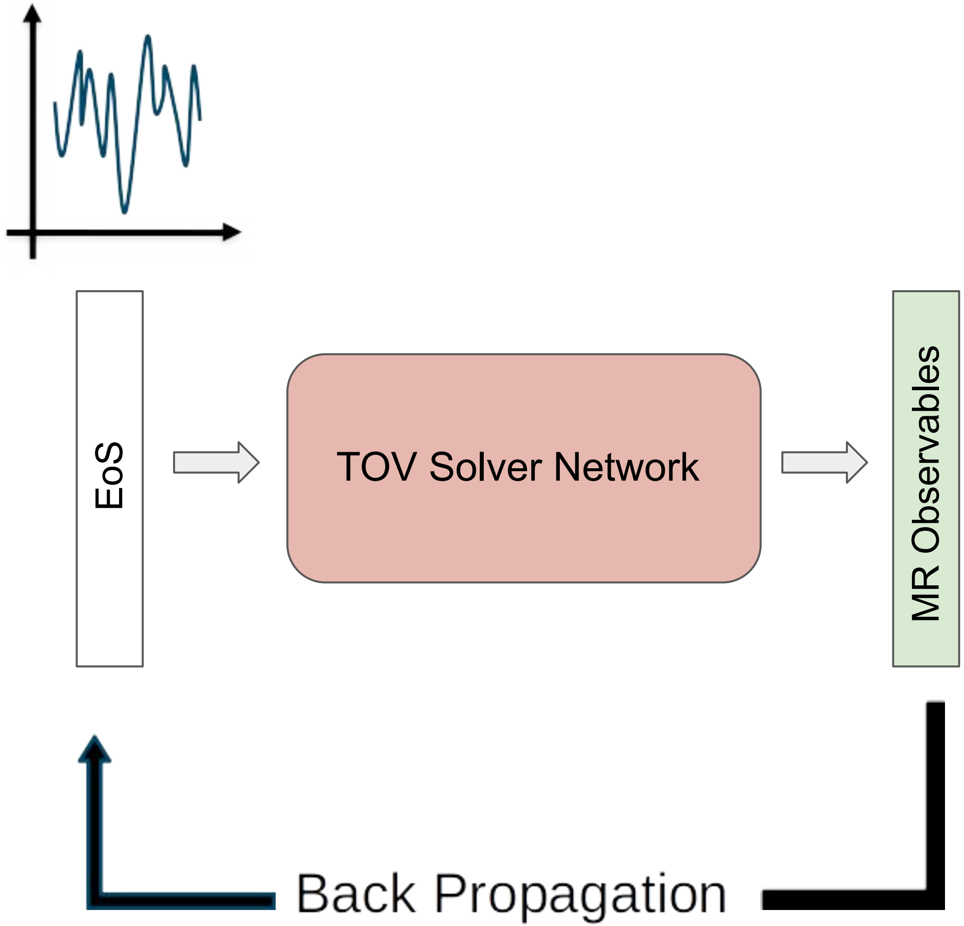


Procedure

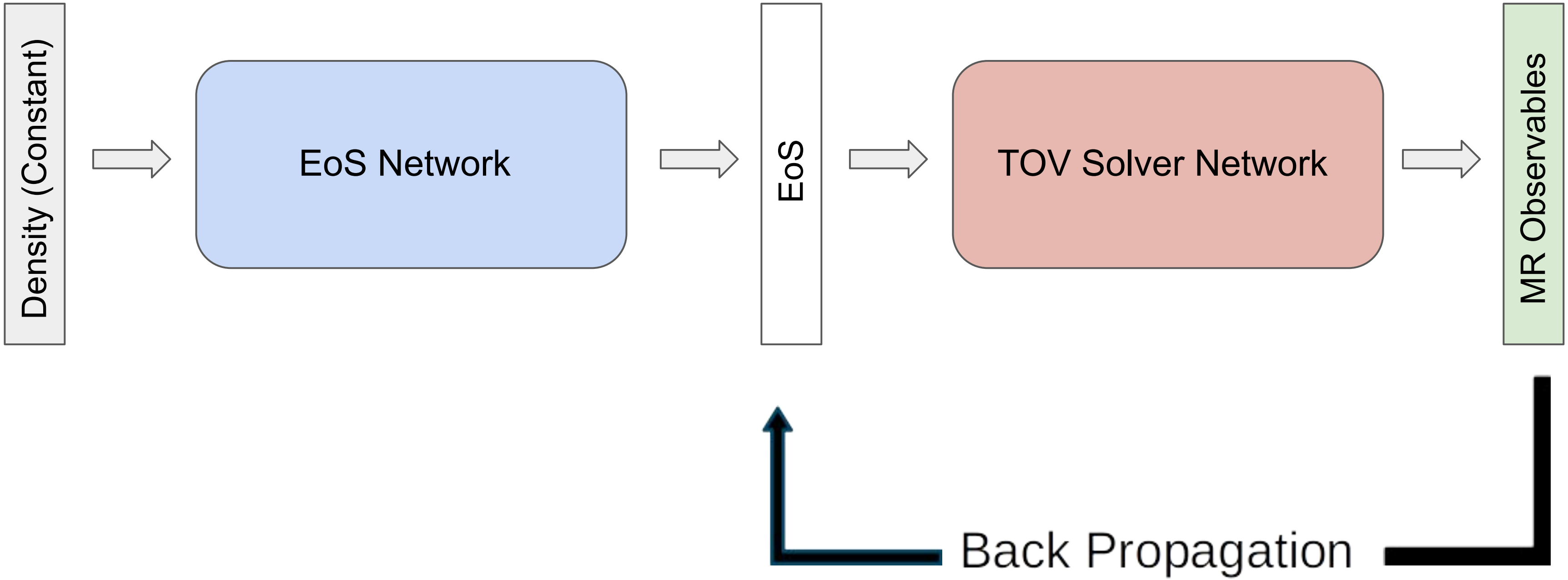
1. Train a NN model to solve TOV Equations – TOV Solver Network

2. Fix the weights of TOV Solver (freeze training)

3. Optimize the input layer (EoS)



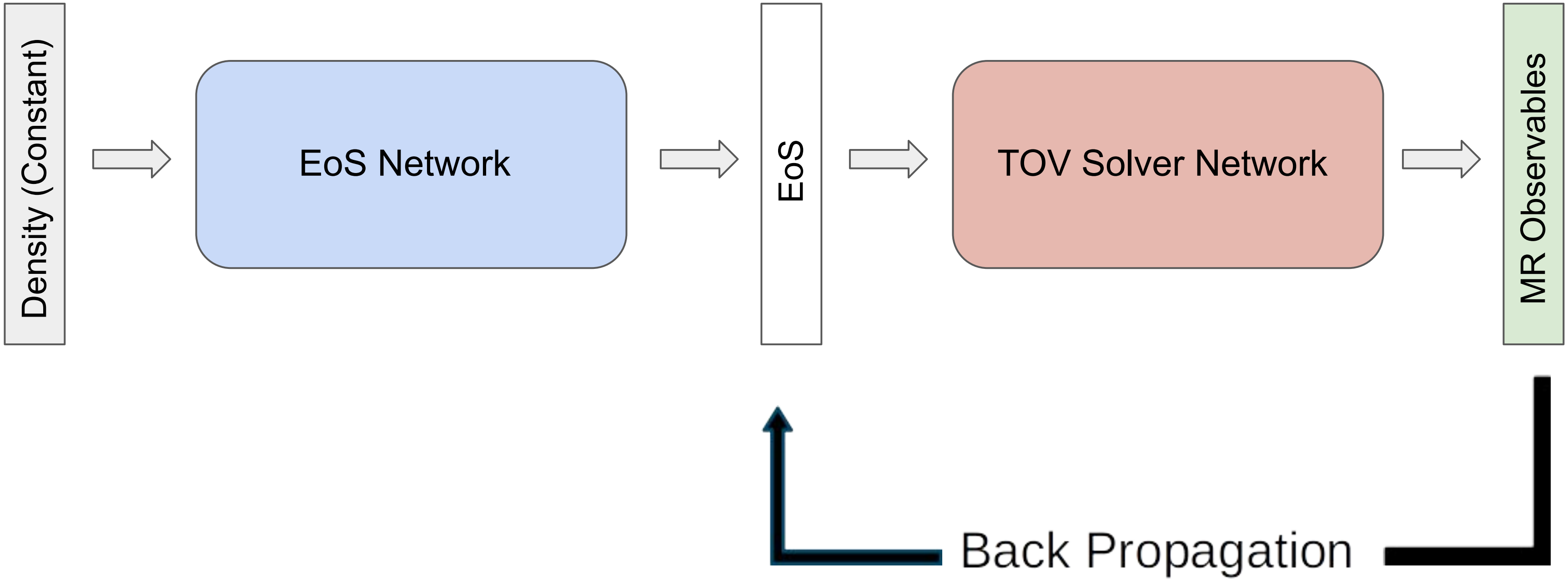
Procedure



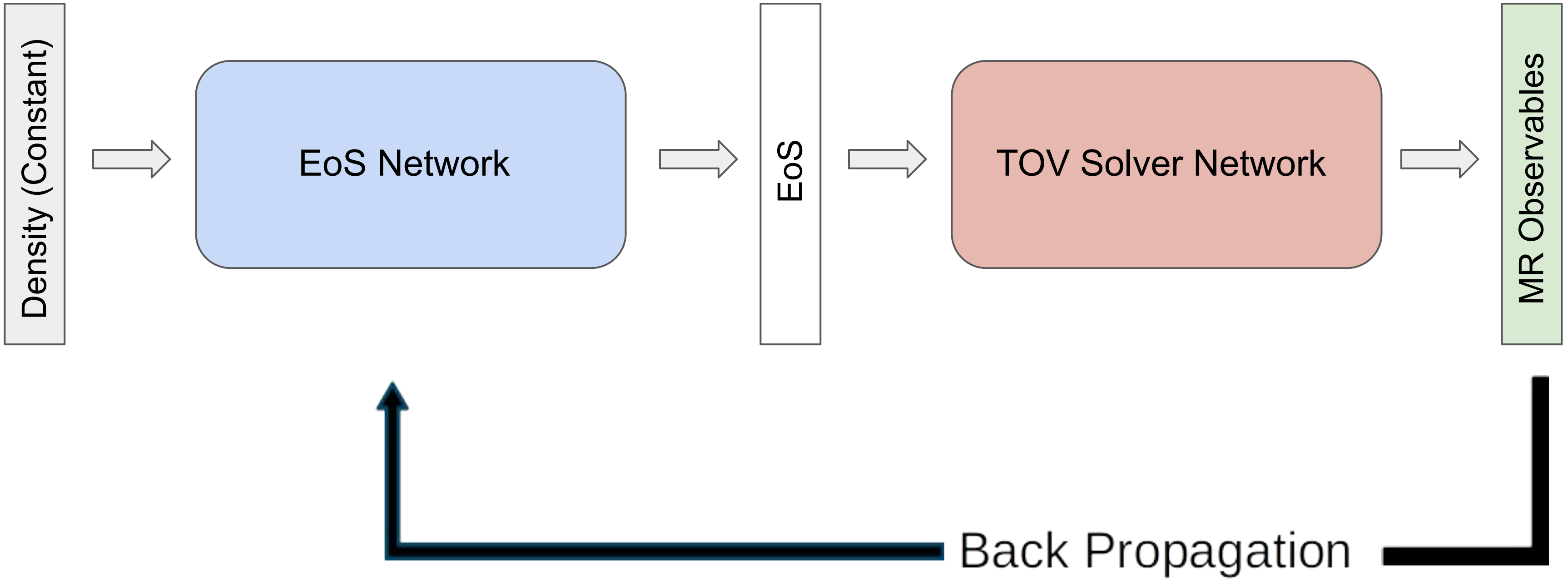
Density (Constant)
EoS Network
EoS
TOV Solver Network
MR Observables
Back Propagation

Procedure

$$P(\mathrm{EoS} \mid M\text{-}R) = \frac{P(M\text{-}R \mid \mathrm{EoS})\,\boxed{P(\mathrm{EoS})}}{P(M\text{-}R)}$$



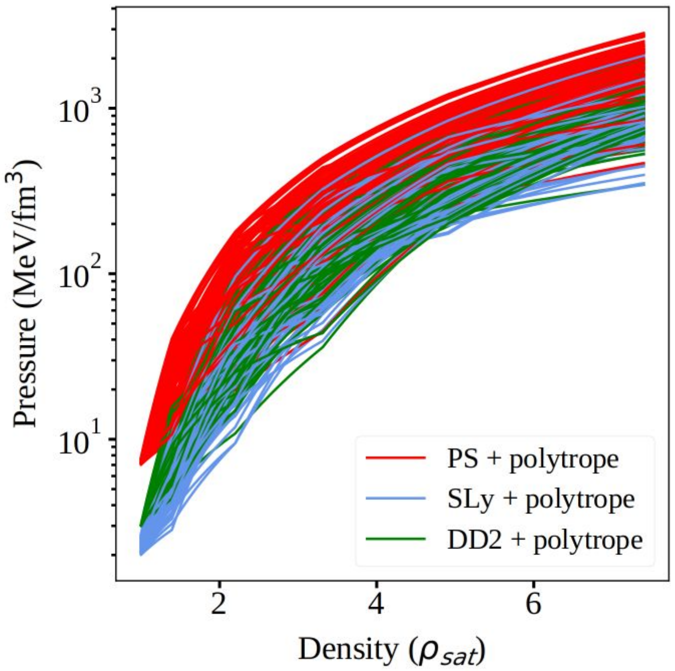
Procedure

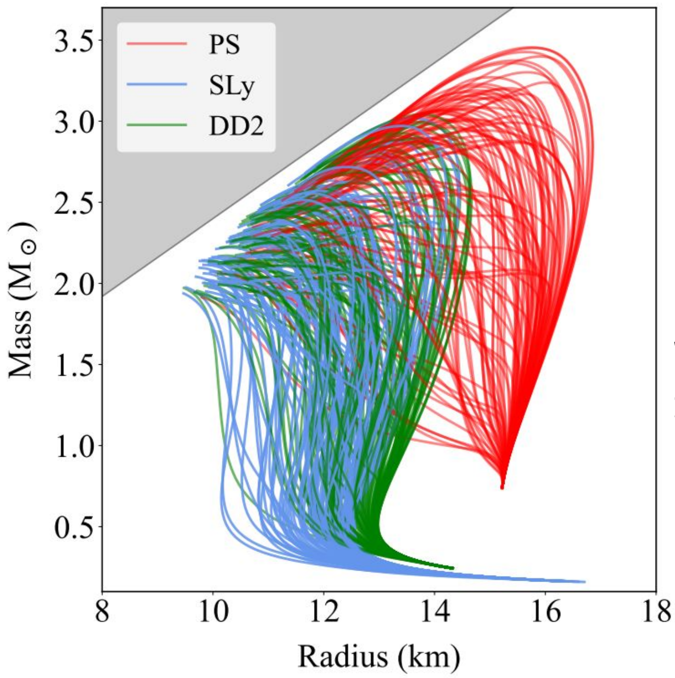


Density (Constant)
EoS Network
EoS
TOV Solver Network
MR Observables
Back Propagation

TOV Solver Network: Training

- $\rho < \rho_0$: SLy / PS / DD2
- $\rho > \rho_0$: Piecewise Polytropes at (1.0, 1.4, 2.2, 3.3, 4.9, 7.4) ρ_0 [Raithel *et al.*, *ApJ* **831** (2016) 44]

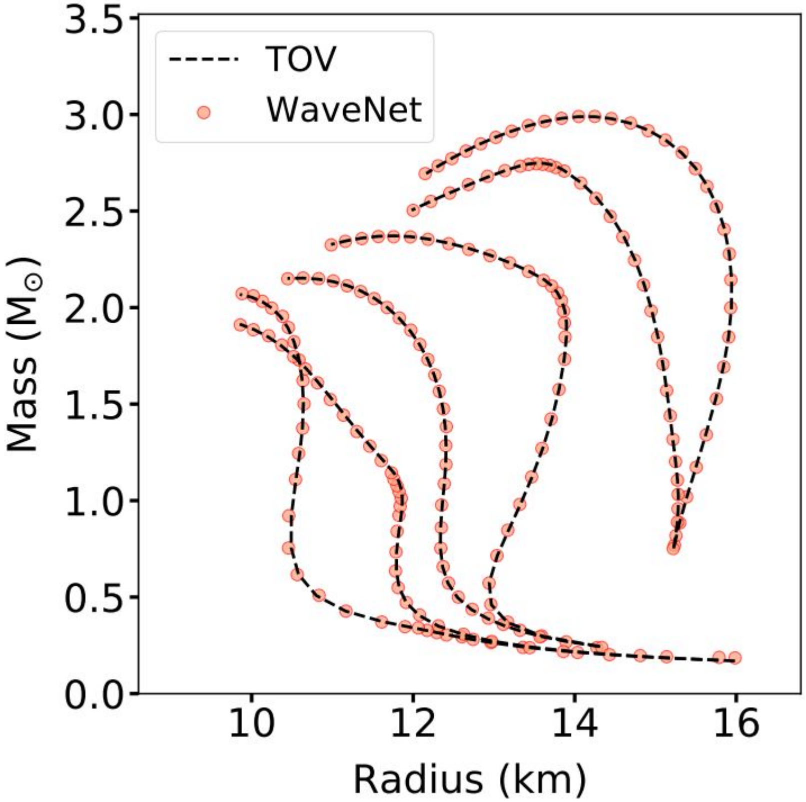




EoSs Generated: 3 x 100,000

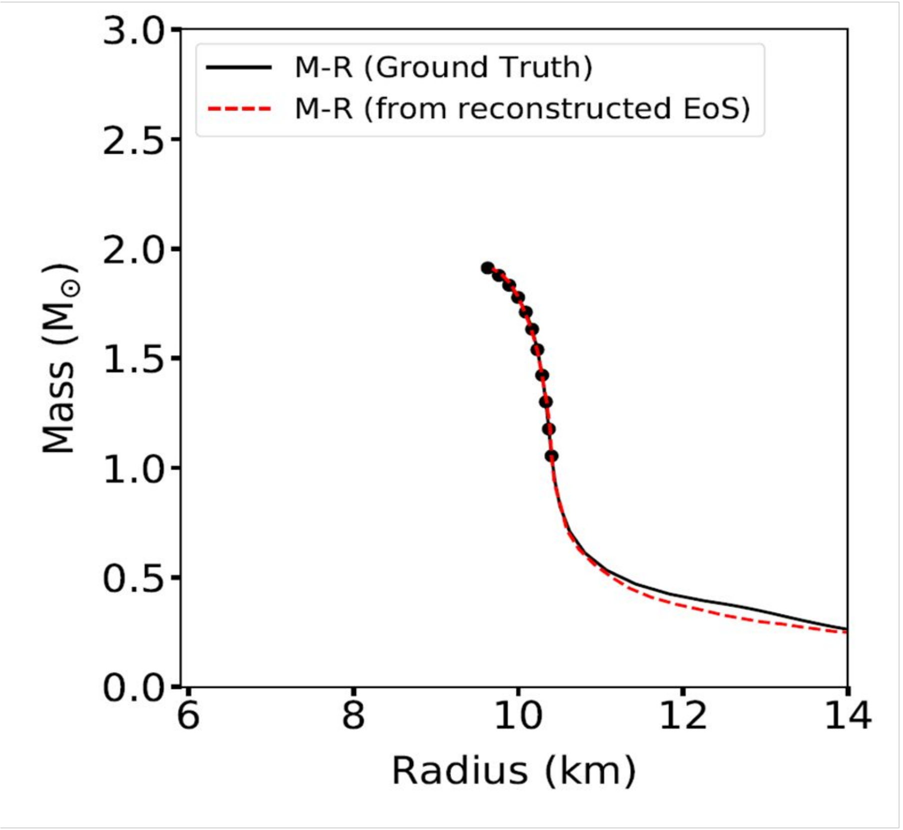
On exclusion of MR curves with maximum mass < 1.9 Solar Mass: 228,569 EoSs

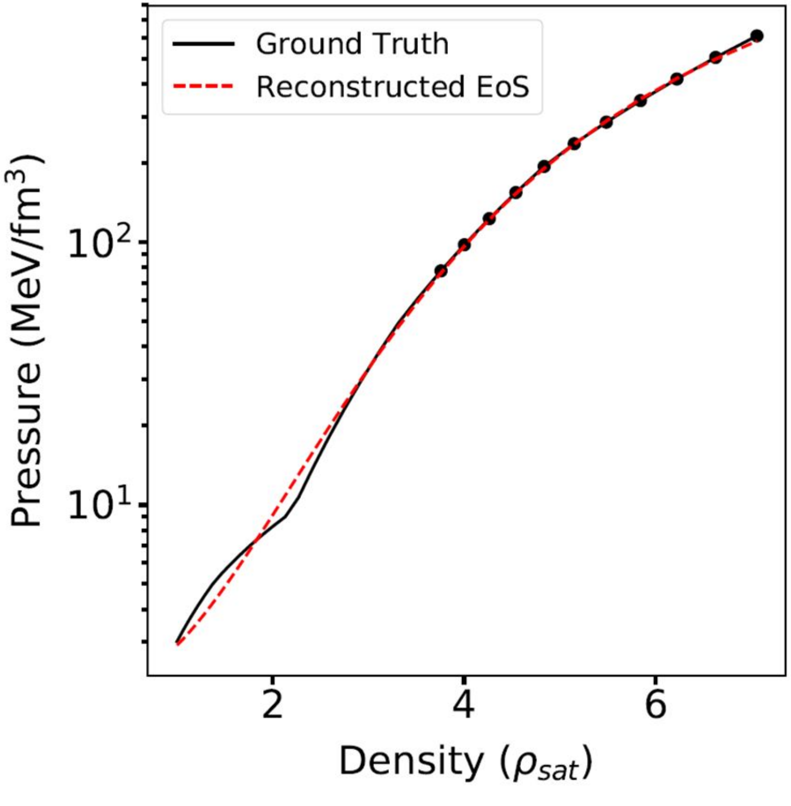
TOV Solver Network: Performance



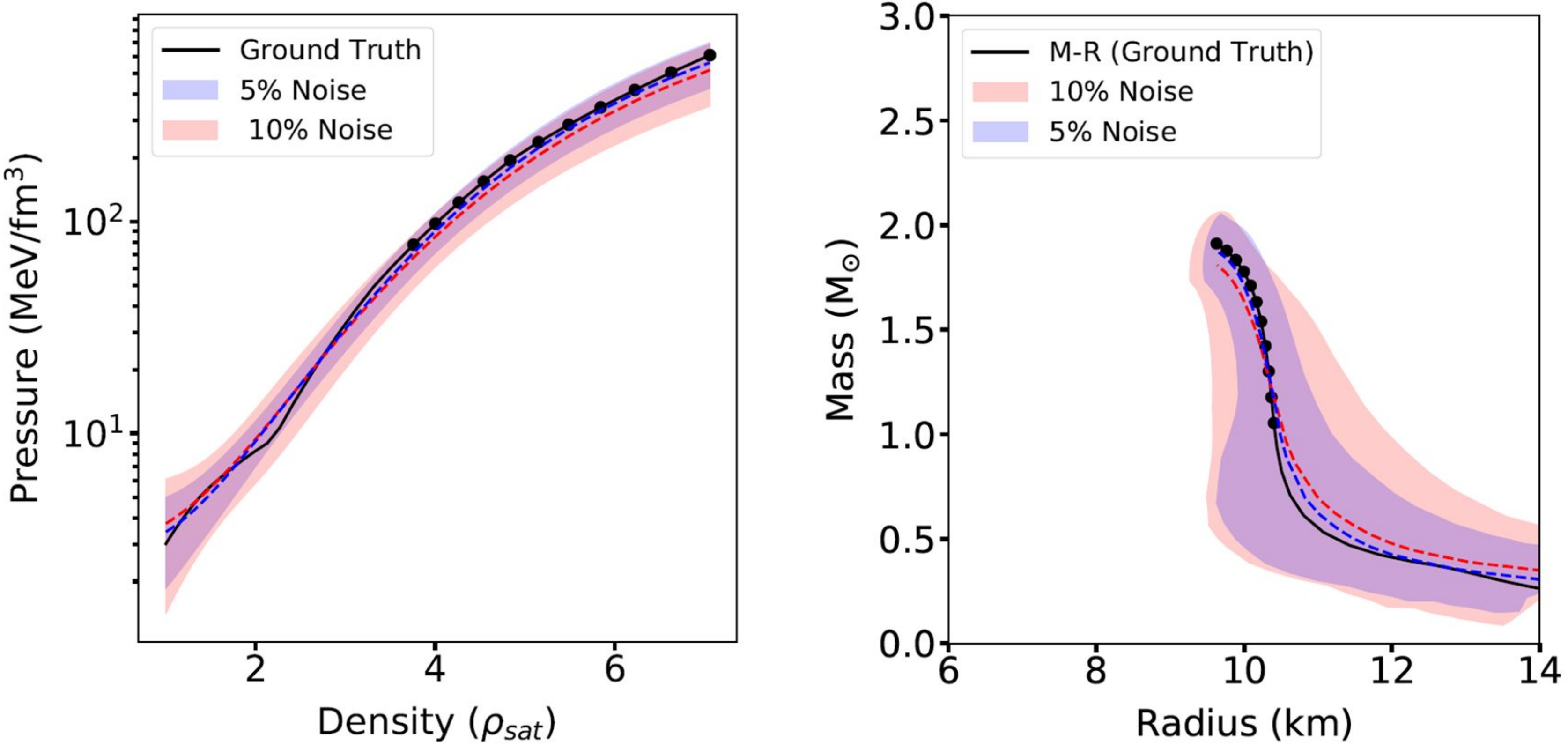
Accuracy : 99.9%

Mock data: An Ideal Case



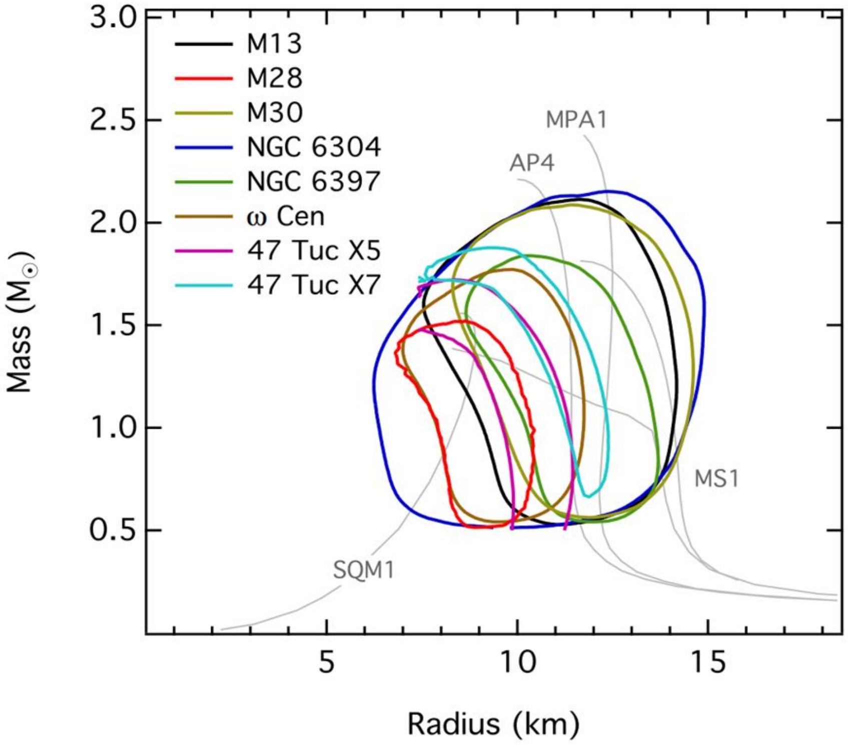


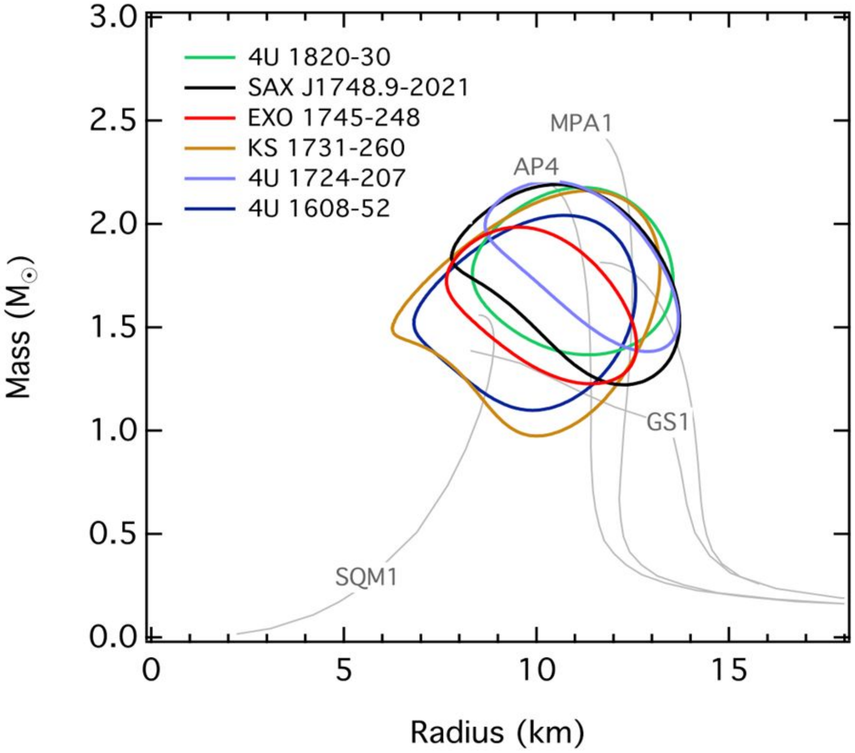
Mock data: A Realistic Scenario



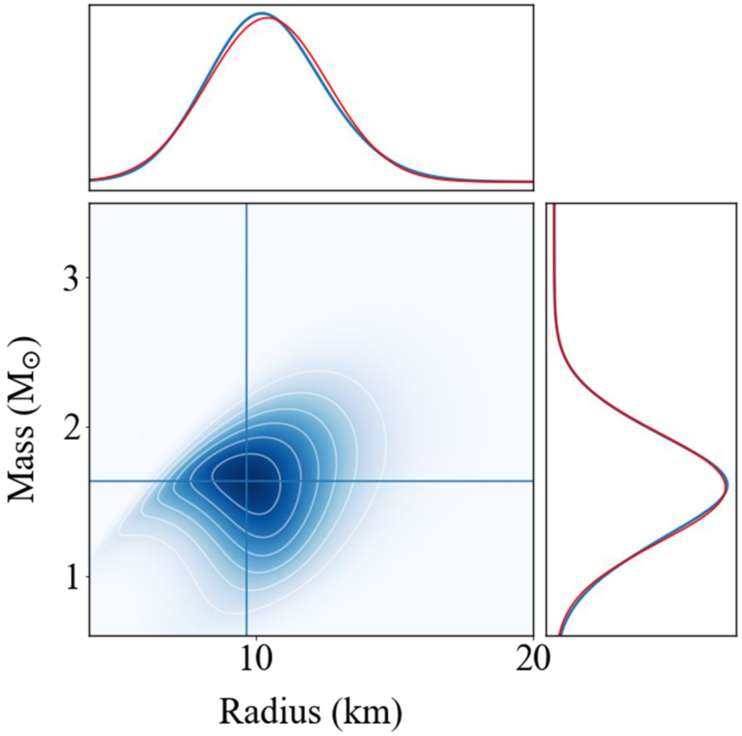
NS Radius and Mass measurements

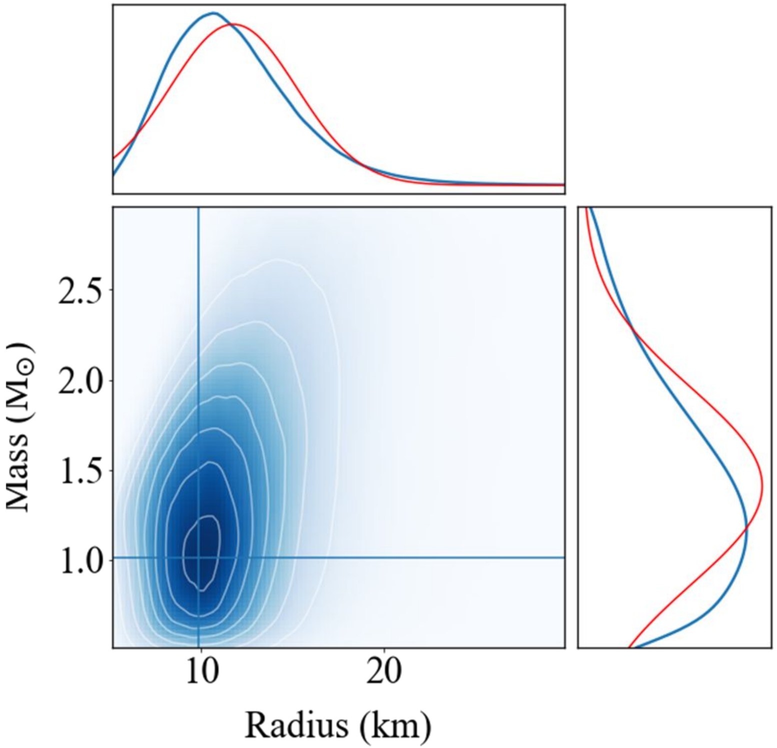
Özel *et al.*, ApJ **820** (2016) 28
Bogdanov *et al.*, ApJ **831** (2016) 184
Riley *et al.*, ApJL **887** (2019) L21
Riley *et al.*, ApJL **918** (2021) L27



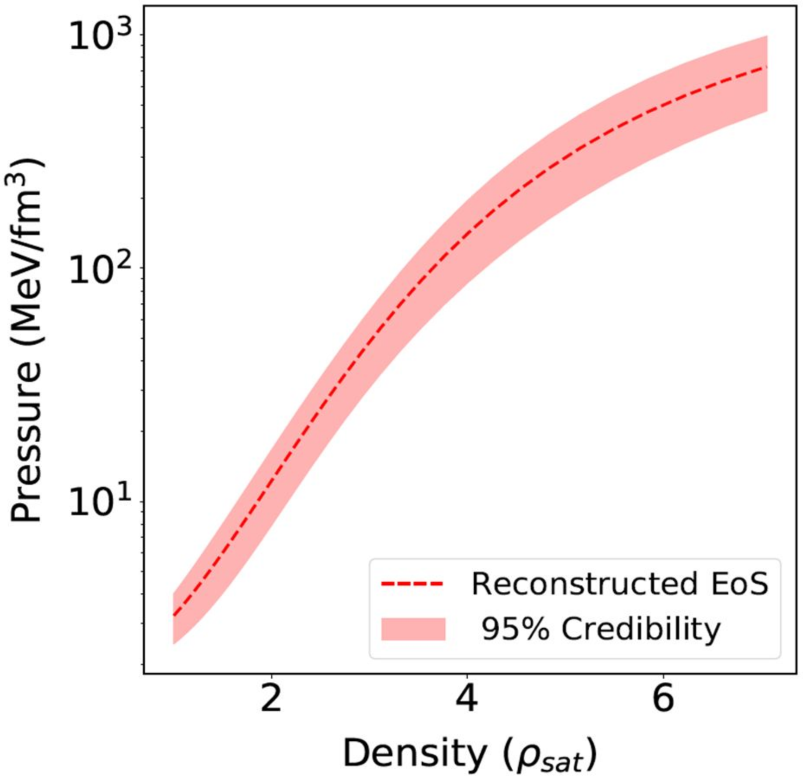


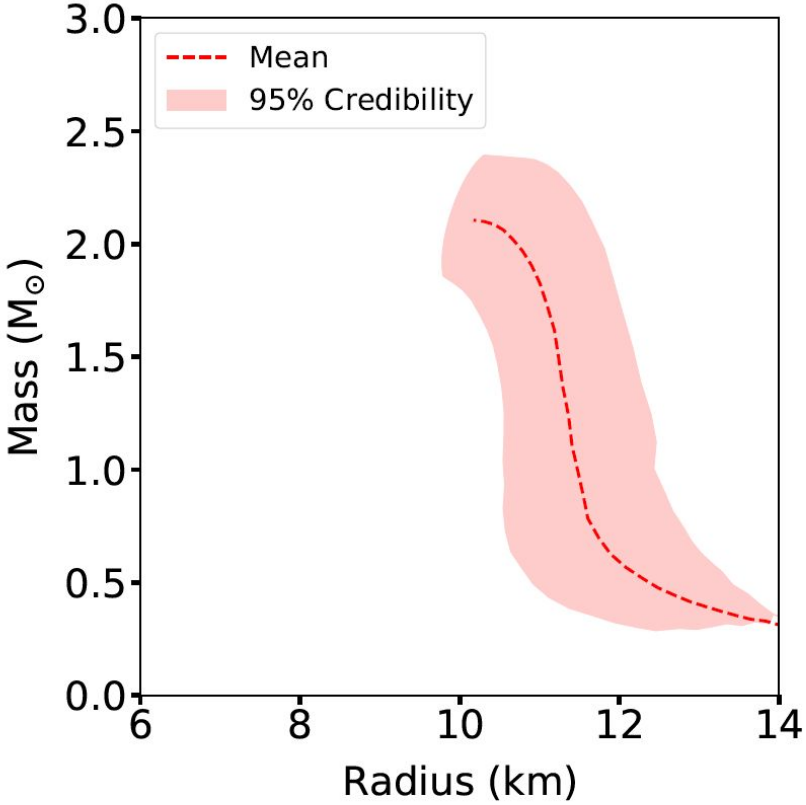
Normal Distribution Fits to MR data



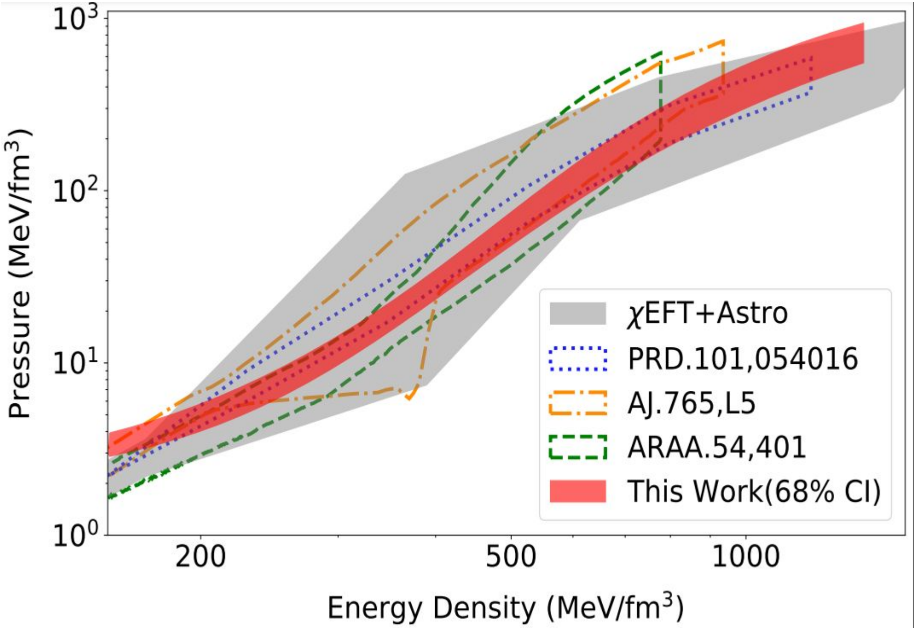


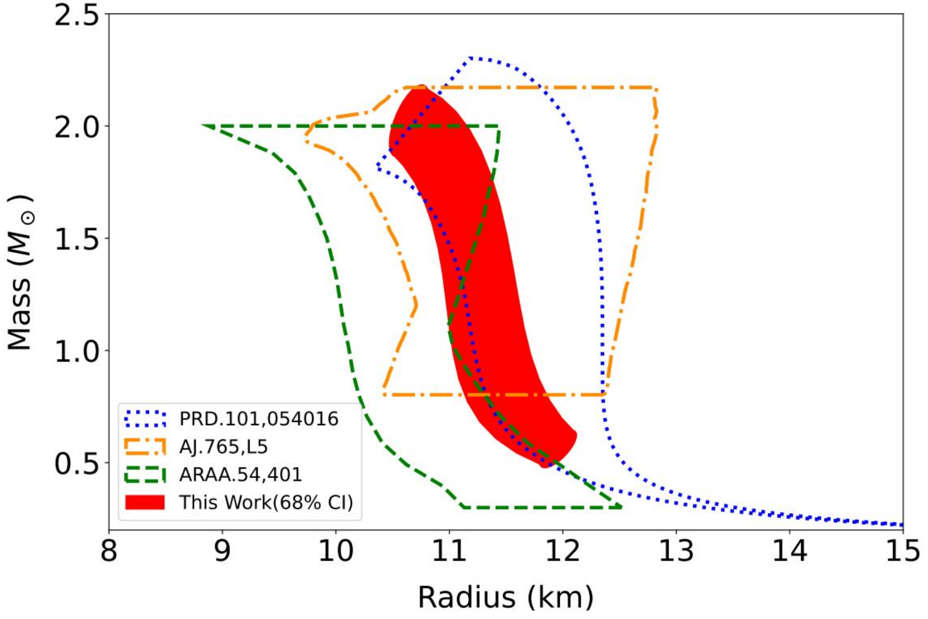
Reconstructed EoS from real MR Observables





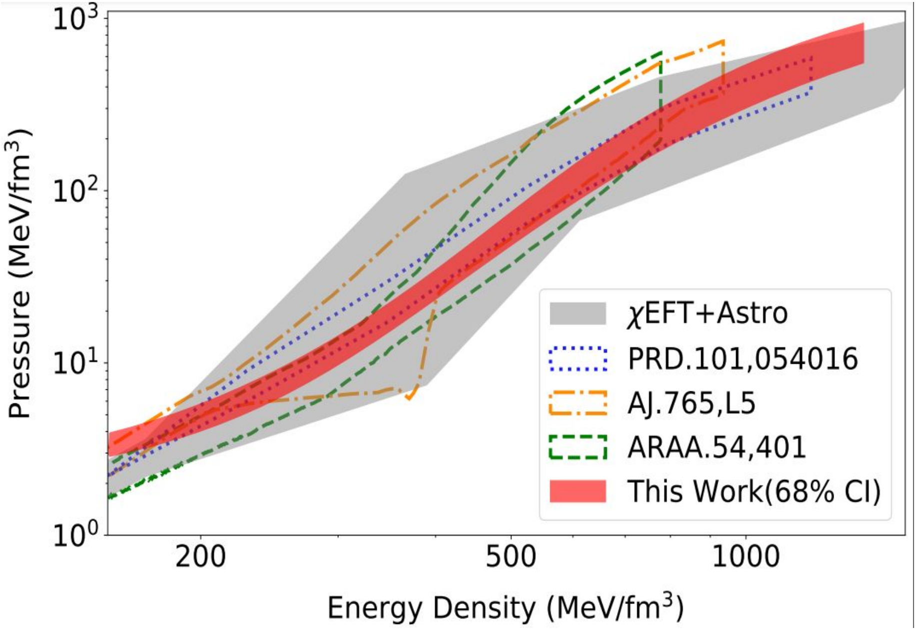
Comparison with Previous Works

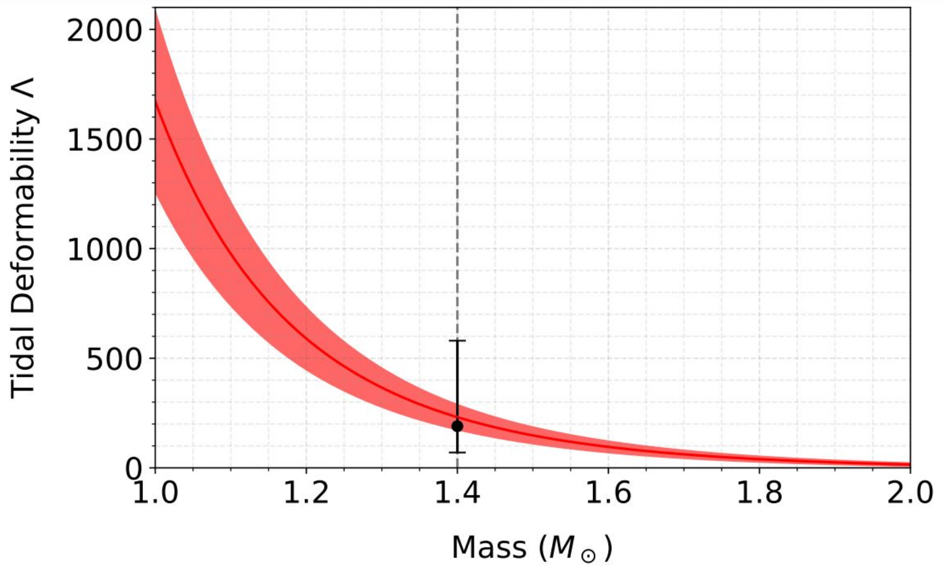




PRD : Fujimoto *et al.*
AJ : Steiner *et al.*
ARAA : Özel *et al.*

Comparison with Previous Works





$$\Lambda_{1.4} = 190^{+390}_{-120} \quad \text{at the 90\% level}$$

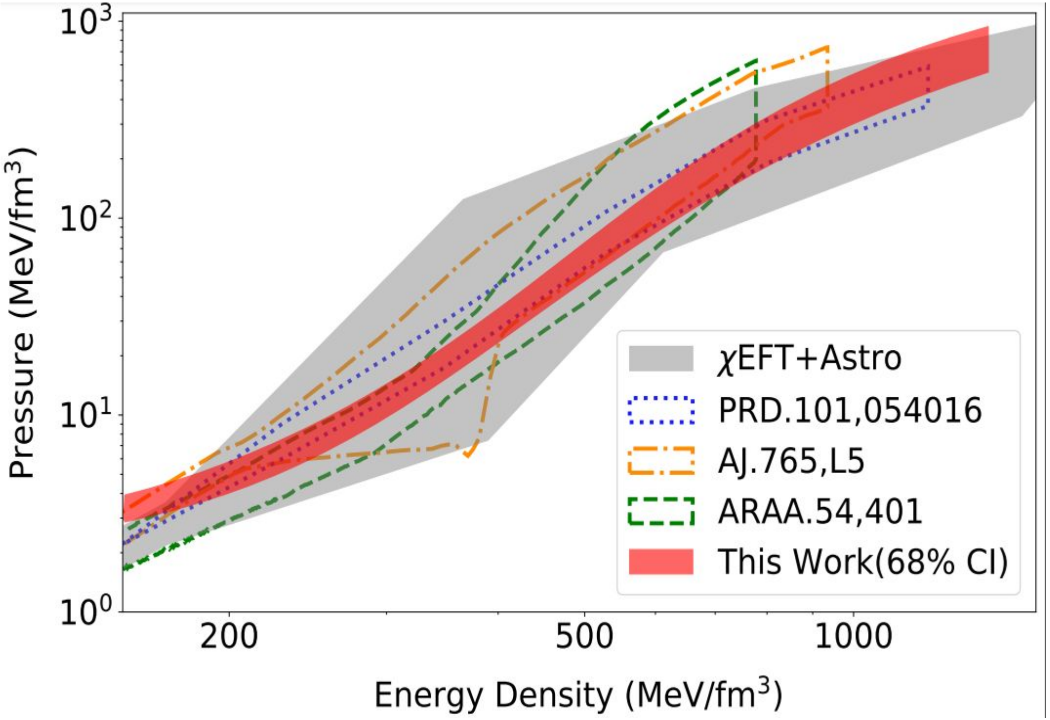
Abbott *et al.*, PRL **121** (2018) 161101

PRD : Fujimoto *et al.*

AJ : Steiner *et al.*

ARAA : Özel *et al.*

Summary

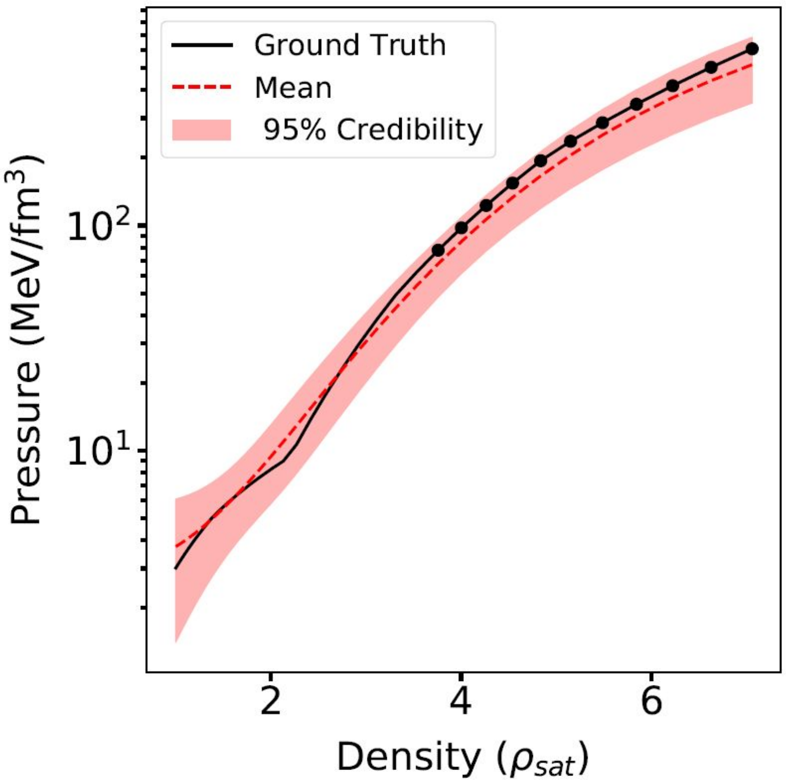


- Trained a NN to replace the TOV Equations
- Inverted the NN to optimize the input layer (EoS)
- Reconstructed the EoS from Real Observations (post successful tests on mock data)
- Consistent with Λ limits from GW170817

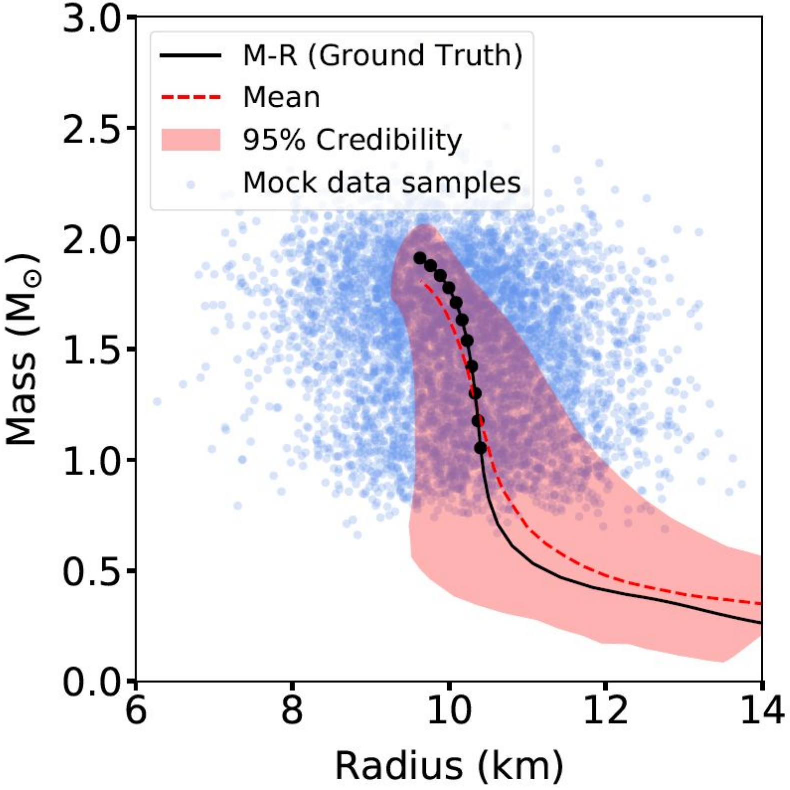
Thank you.

Email: soma@fias.uni-frankfurt.de

Mock data

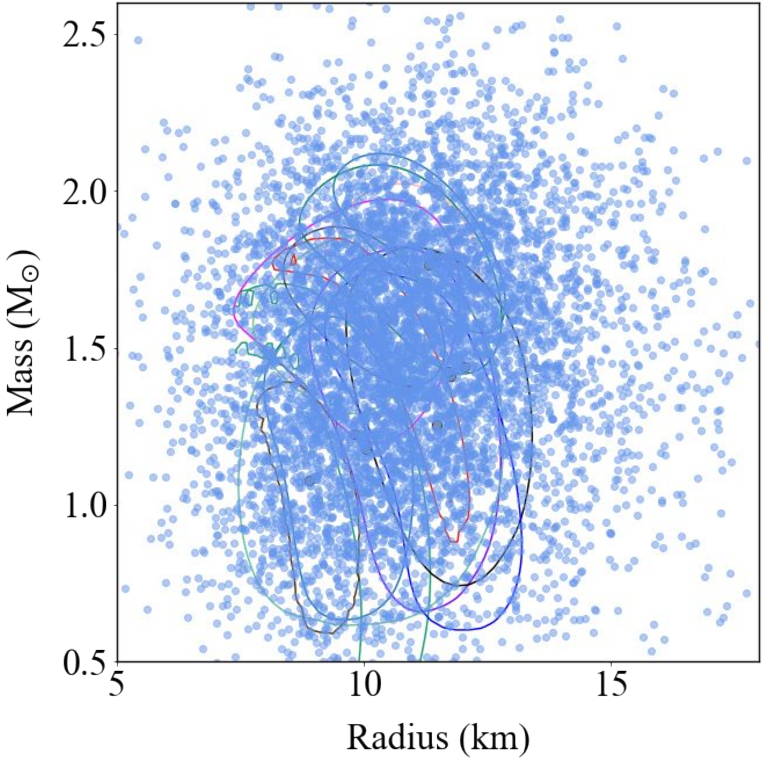


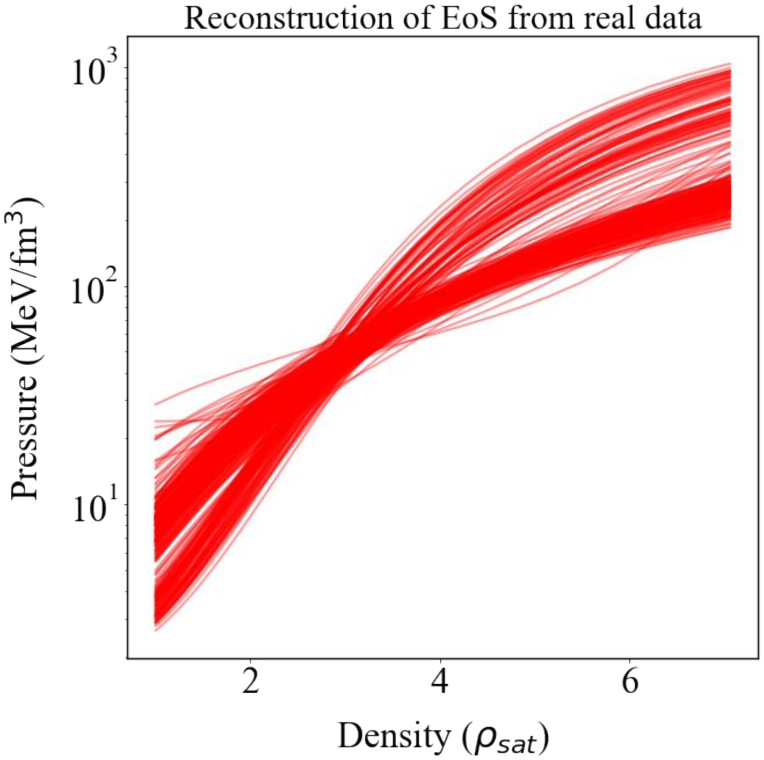
Ground Truth
Mean
95% Credibility
Pressure (MeV/fm^3)
10^2
10^1
2
4
6
Density (ρ_{sat})



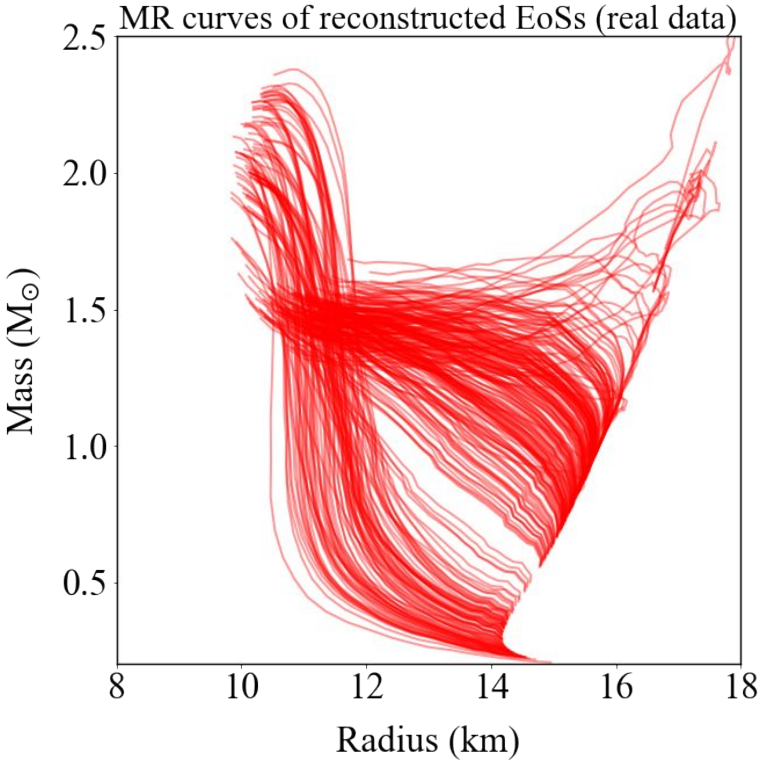
M-R (Ground Truth)
Mean
95% Credibility
Mock data samples
Mass ($M_\odot$)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
6
8
10
12
14
Radius (km)

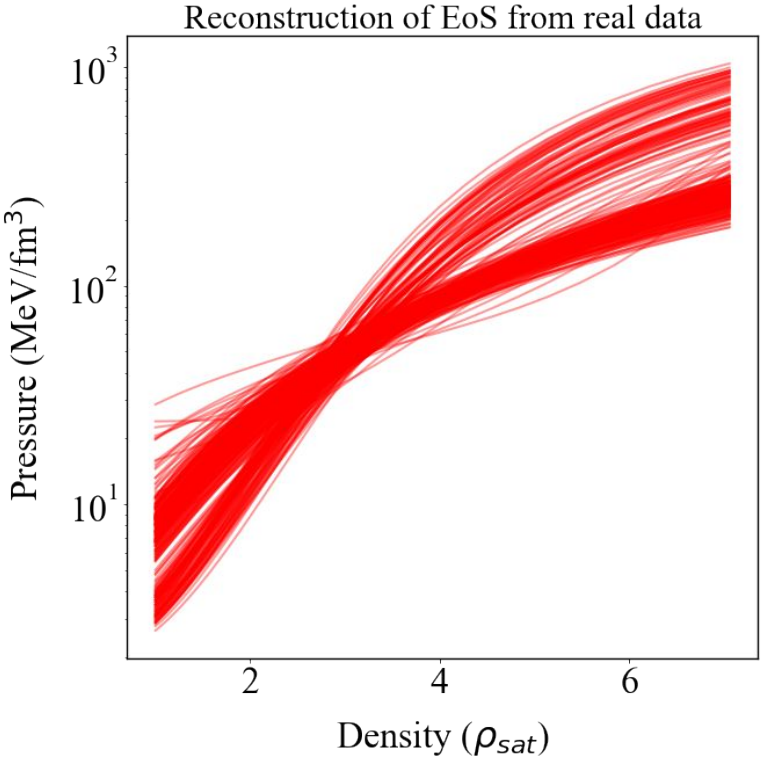
Behind the Scenes



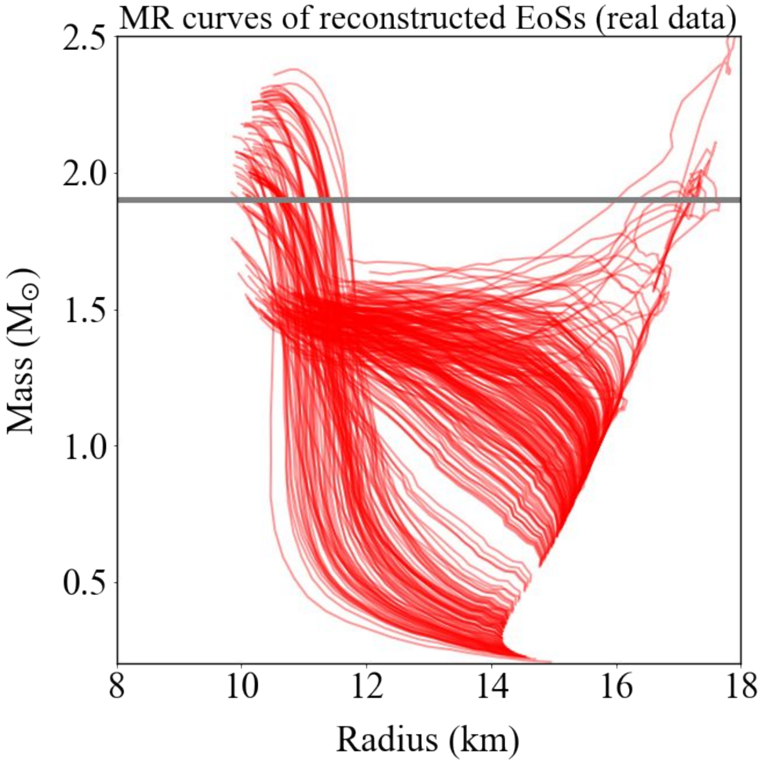


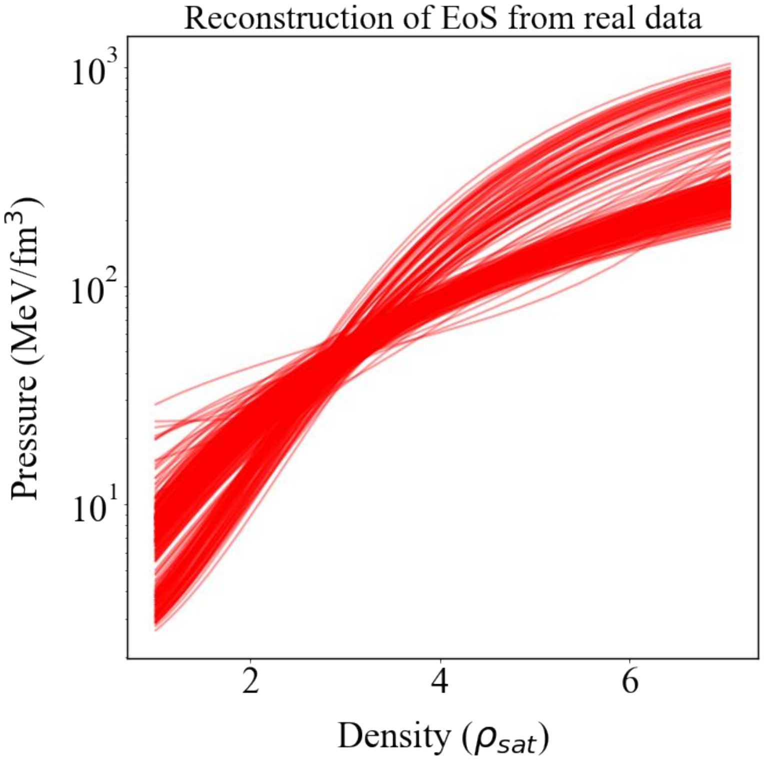
Behind the Scenes



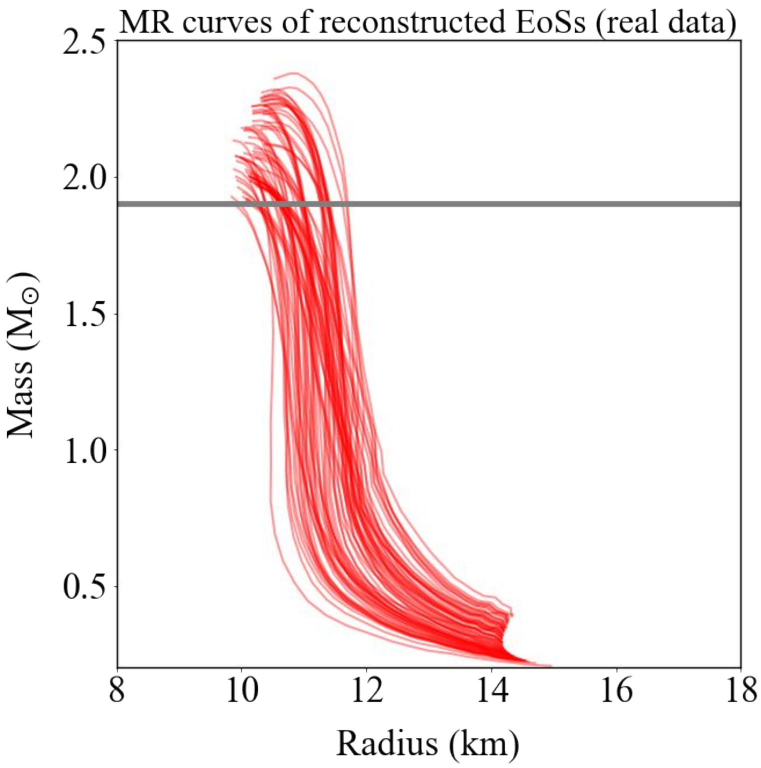


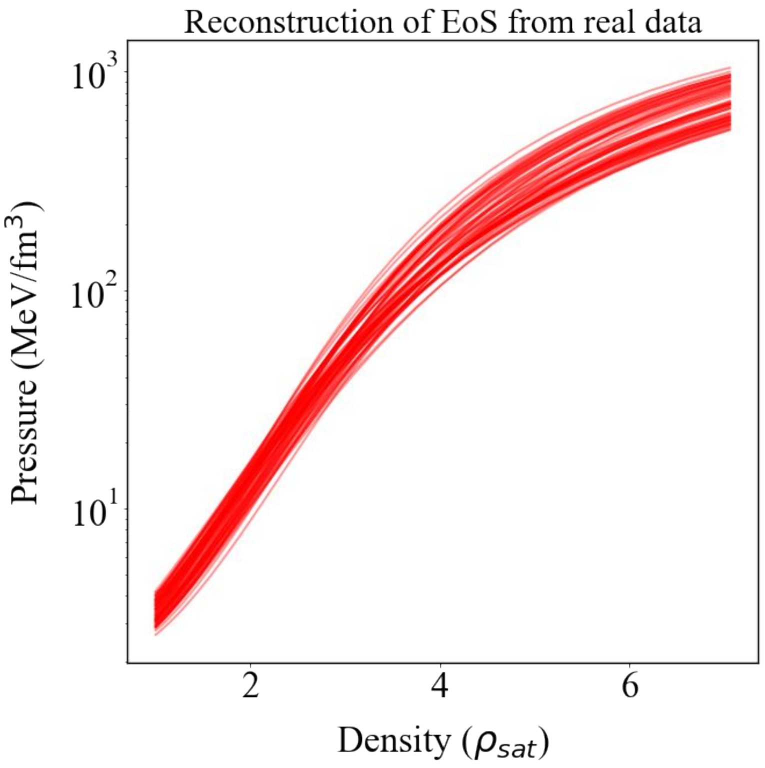
Behind the Scenes



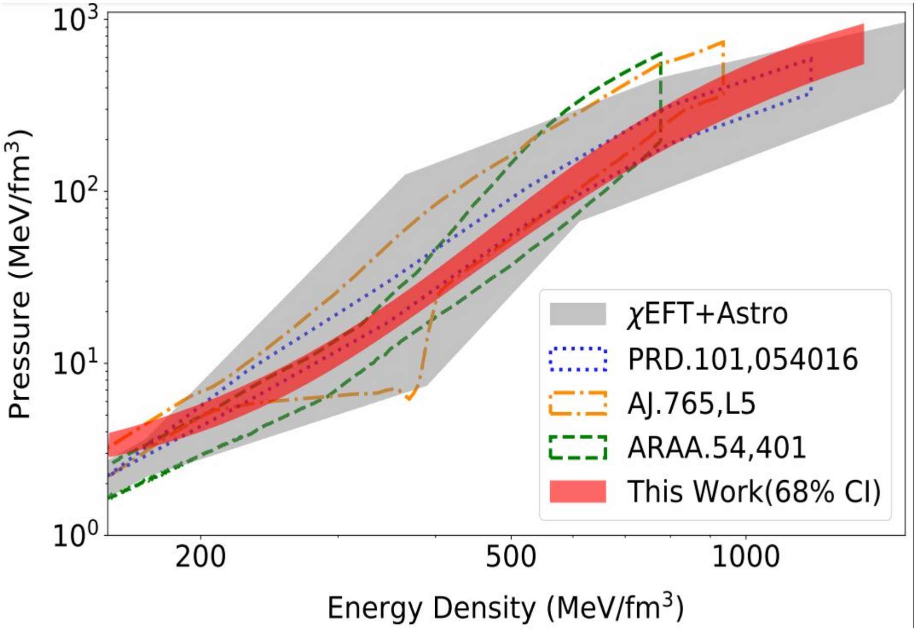


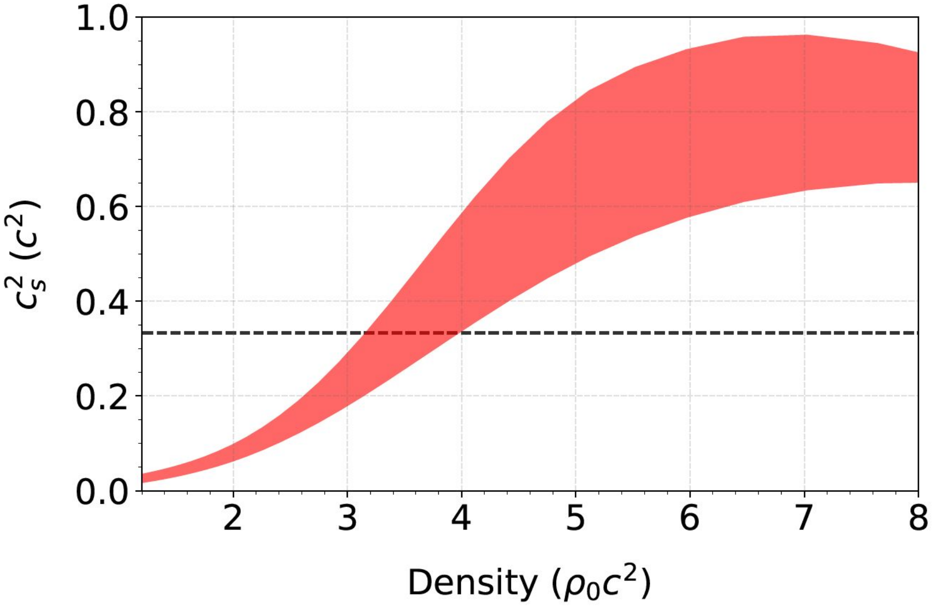
Behind the Scenes





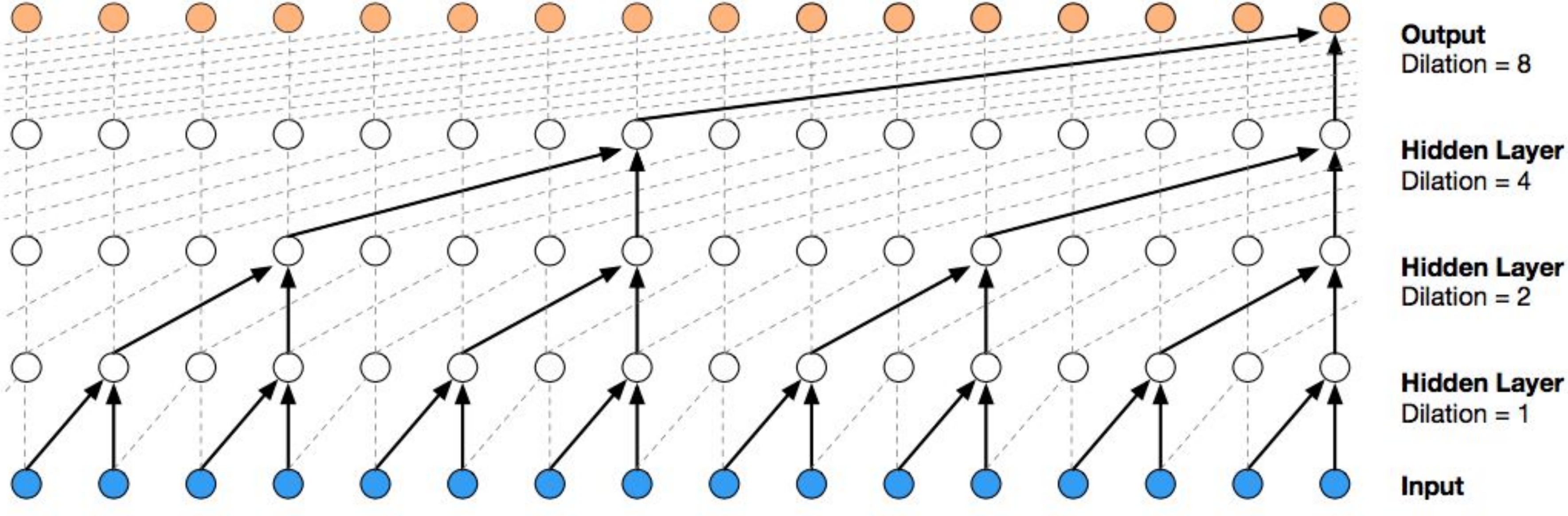
Speed of Sound



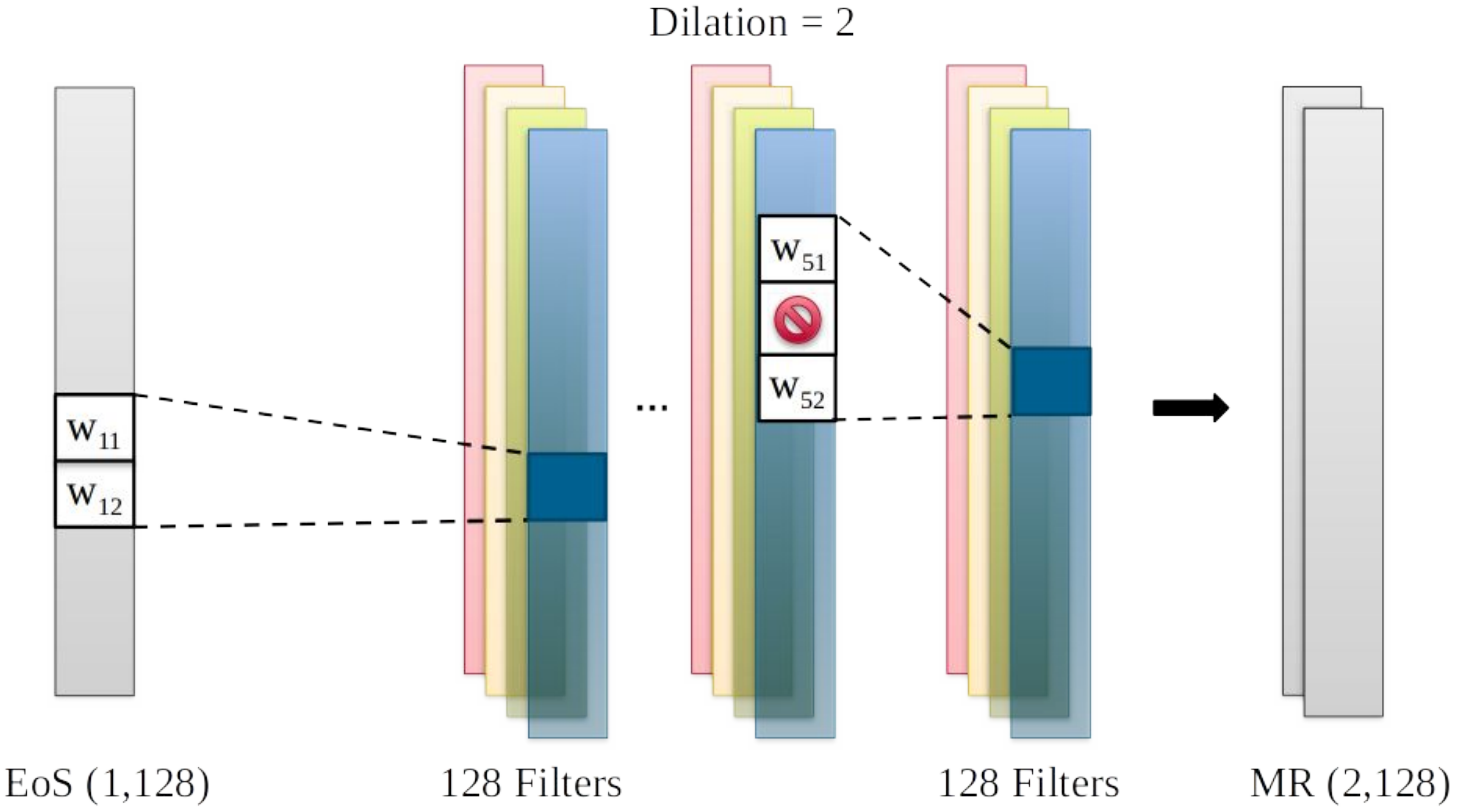


WaveNet - An Autoregressive Network

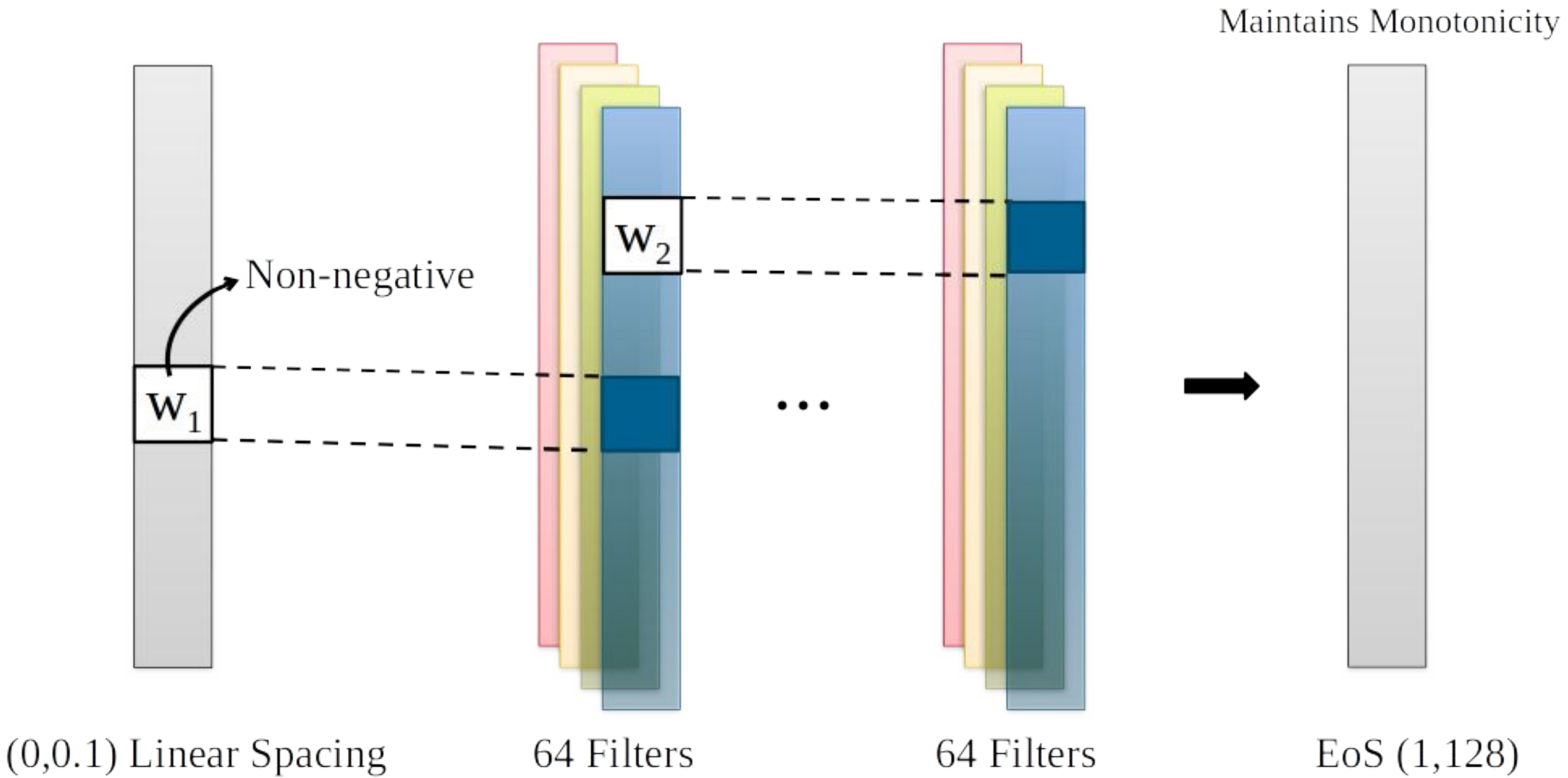
Causal Series



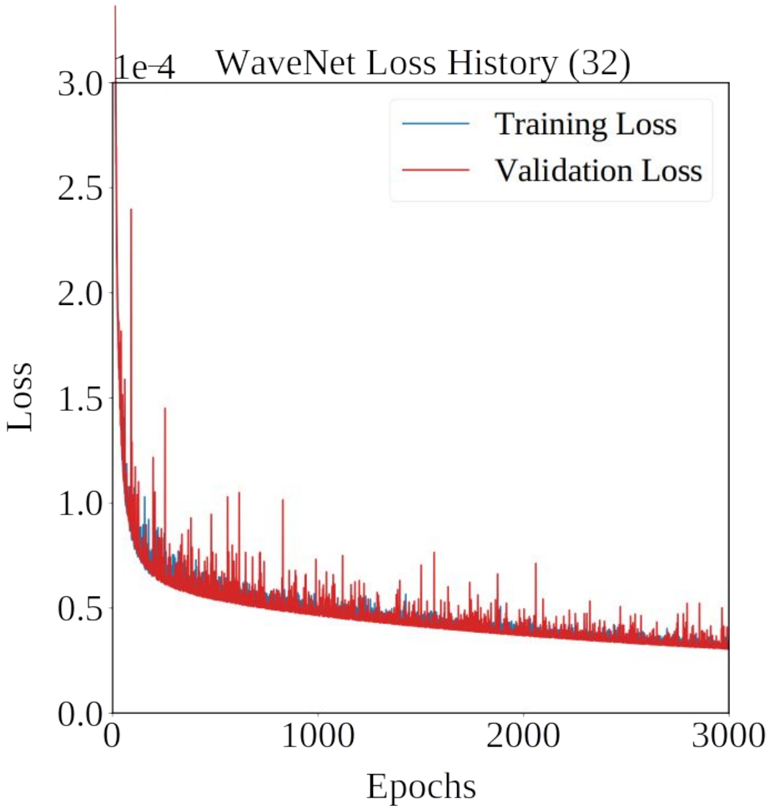
WaveNet - An Autoregressive Network



1D Convolutions - Preserving Order



Learning Curves

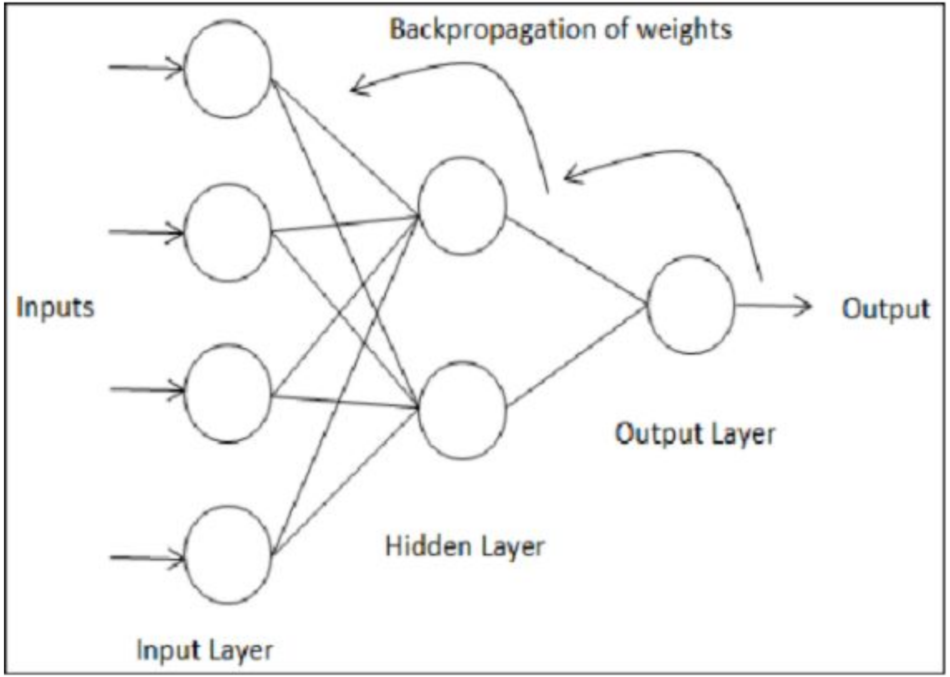


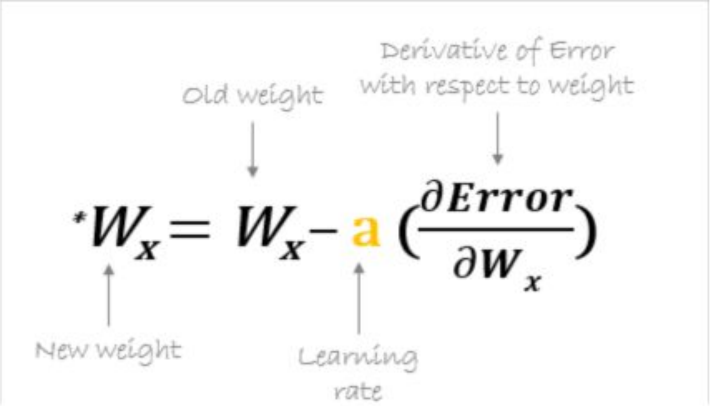
Number of Epochs : 3000

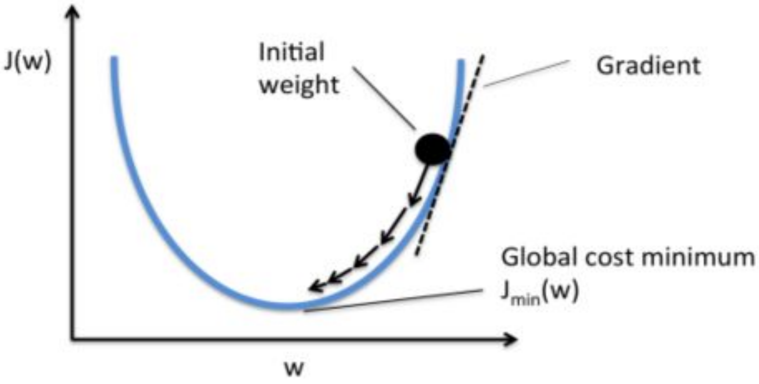
Number of Layers : 10
Dilations : 1, 2, 4, 8, 16, 32, 16, 32, 64

Padding : 'causal'
Activation function : 'elu' (last layer - sigmoid)

Back Propagation







Data Preparation

- $\rho < \rho_0$: SLy / PS / DD2
- $\rho > \rho_0$: Piecewise Polytropes at (1.0, 1.4, 2.2, 3.3, 4.9, 7.4) ρ_0

$$P = K_i \rho^{\Gamma_i} \quad ; \quad d\frac{\epsilon}{\rho} = -P d\frac{1}{\rho}$$

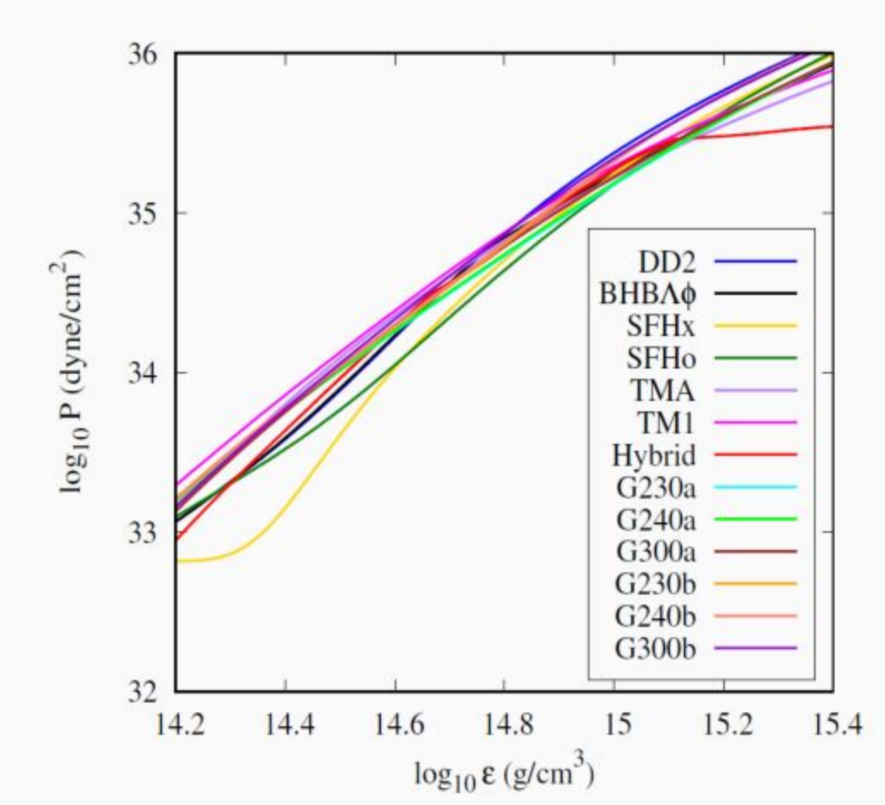
where,

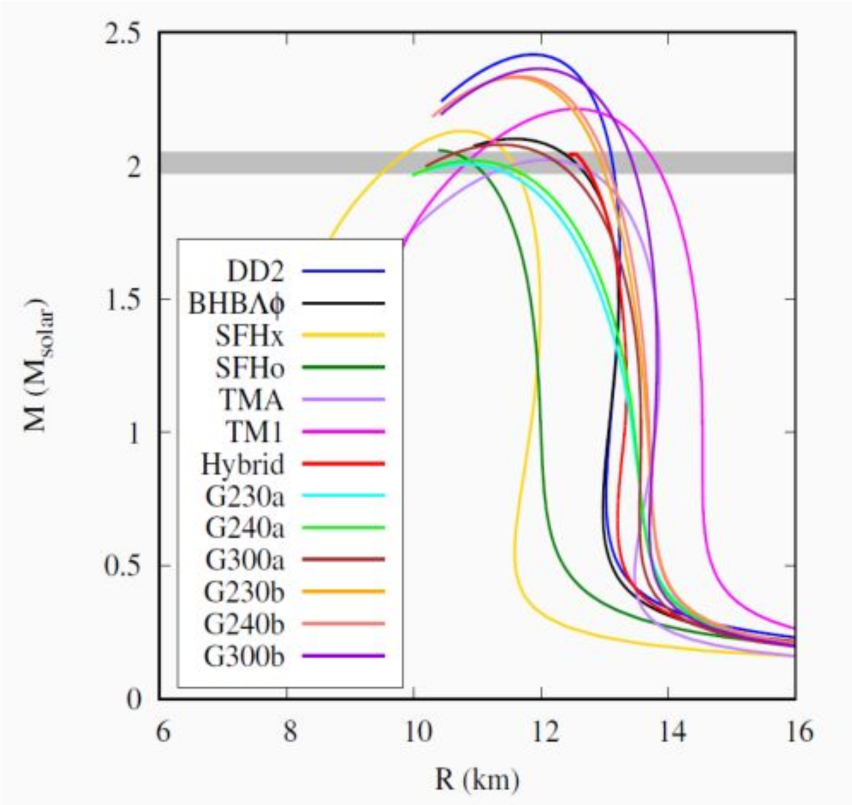
$$K_i = \frac{P_{i-1}}{\rho_{i-1}^{\Gamma_i}}$$

and $\Gamma_i \in [1,\ \min\{5, \Gamma_{luminal}\}]$;

$$\frac{dP}{d\epsilon} \leq 1 \quad ; \quad \Gamma = \Gamma_{luminal} \quad when \quad \frac{dP}{d\epsilon} = 1$$

EoSs and corresponding MR curves





EoS Parameters

EoS	n_0 (fm^{-3})	m^*/m	BE (MeV)	K (MeV)	S (MeV)	L (MeV)	M_{max} $(M_\odot)$	M_B $(M_\odot)$
DD2	0.1491	0.56	16.02	243.0	31.67	55.04	2.42	2.89
BHB$\Lambda\phi$	0.1491	0.56	16.02	243.0	31.67	55.04	2.1	2.43
SFHo	0.1583	0.76	16.19	245.4	31.57	47.10	2.06	2.43
SFHx	0.1602	0.72	16.16	238.8	28.67	23.18	2.13	2.53
TM1	0.1455	0.63	16.31	281.6	36.95	110.99	2.21	2.30
TMA	0.1472	0.64	16.03	318.2	30.66	90.14	2.02	2.30
G230a	0.153	0.78	16.30	230.0	32.50	89.76	2.01	2.31
G230b	0.153	0.70	16.30	230.0	32.50	94.46	2.33	2.75
G240a	0.153	0.78	16.30	240.0	32.50	89.70	2.02	2.75
G240b	0.153	0.70	16.30	240.0	32.50	94.39	2.34	2.75
G300a	0.153	0.78	16.30	300.0	32.50	89.33	2.08	2.40
G300b	0.153	0.70	16.30	300.0	32.50	93.94	2.36	2.78
Hybrid	0.1491	0.56	16.02	243.0	31.67	55.04	2.05	2.39
Exp.	0.15-0.16	0.55-0.75	16.00	220-315	29.00-31.70	45.00-61.90	-	-