

X-ray Emission Study Performed for H-like Lead Ions at the Electron Cooler of CRYRING@ESR

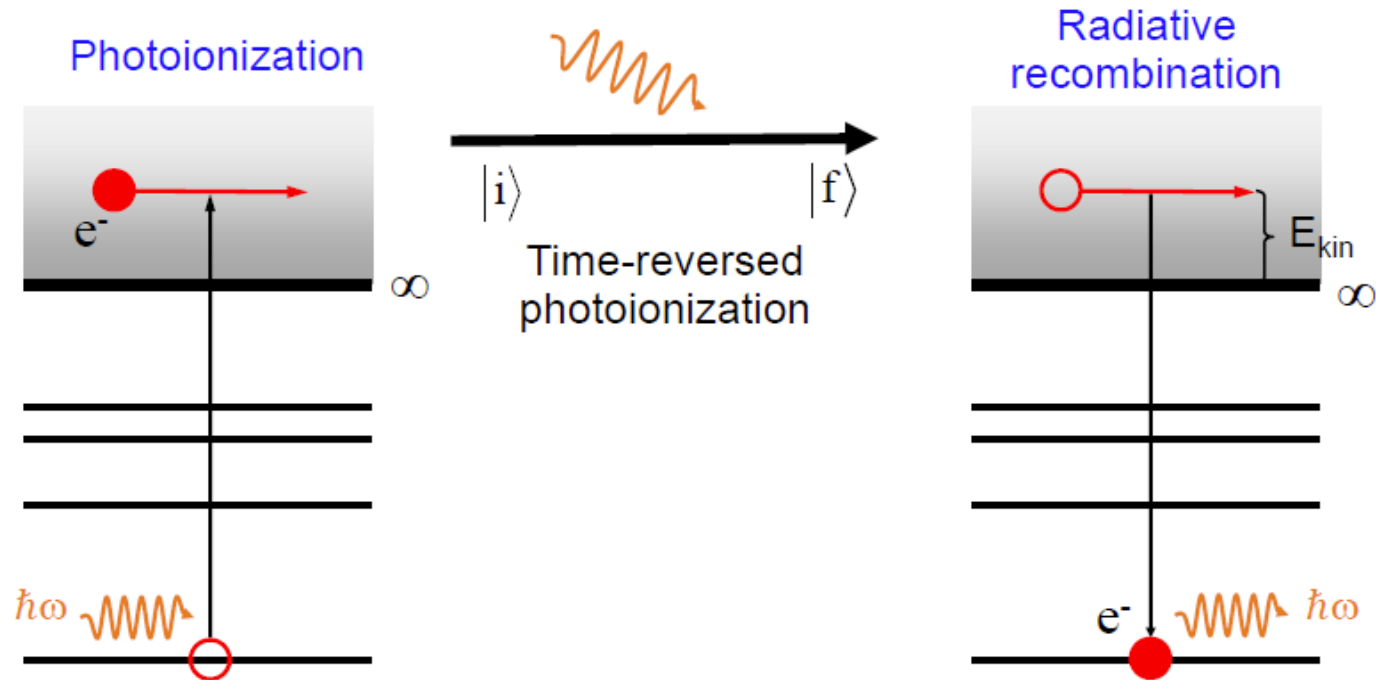
Binghui Zhu

Helmholtz Institute Jena
IOQ, Friedrich-Schiller-Universität Jena

23/05/2022

1. Introduction

Radiative recombination (RR)



- Electron capture into a bound ionic state by emission of a photon.

$$\hbar\omega = E_B + E_{\text{KIN}}$$

- Recombination process populates preferentially high (n,l) states at low relative electron-ion beam energies.

The diagram illustrates the energy levels of a hydrogen atom, showing the progression from the Bohr model to the inclusion of fine structure and the Lamb shift. The vertical axis represents energy, with labels 0 , $-\frac{Z^2}{9}R_y$, $-\frac{Z^2}{4}R_y$, and $-Z^2R_y$ corresponding to principal quantum numbers $n=3$, $n=2$, and $n=1$ respectively.

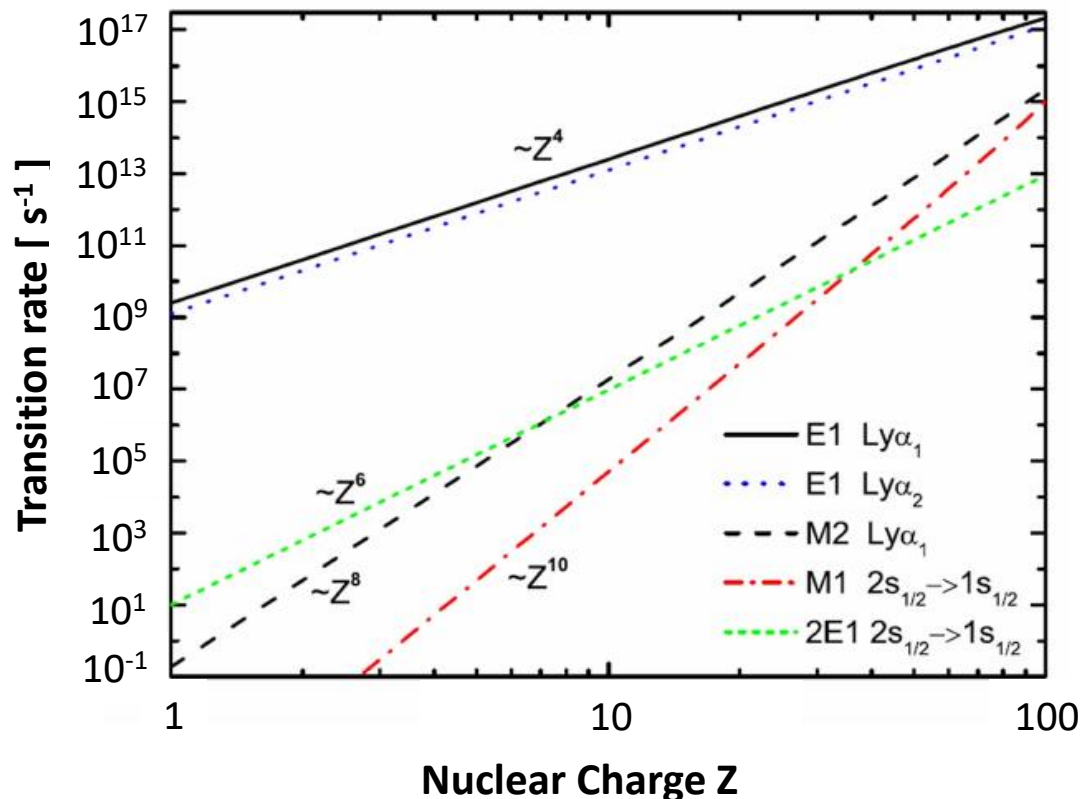
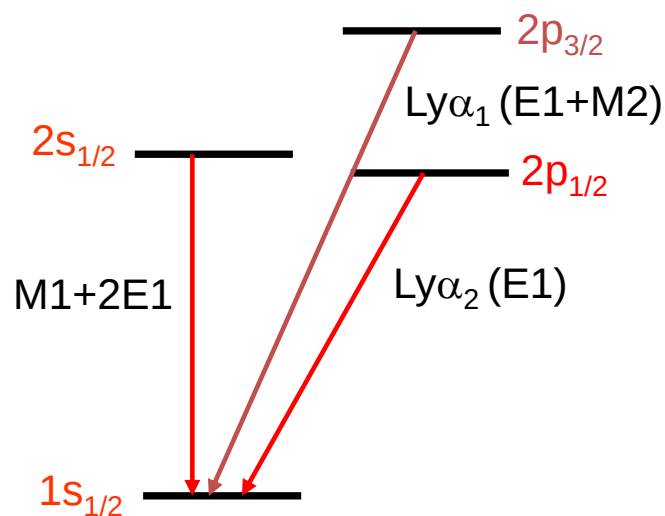
On the left, the Bohr energy levels E_2^{Bohr} are shown as a single line for $n=2$. This is then refined by adding the fine structure splitting ΔE_2^{FS} , which splits the $n=2$ level into $2s$ and $2p$ states. Finally, the Lamb shift ΔE_2^{Lamb} is added, further splitting the $2s$ and $2p$ states into $2s_{1/2}$ and $2p_{1/2}$ states. The energy difference between $2s_{1/2}$ and $2p_{1/2}$ is labeled as the Lamb shift, with a note $\sim Z^4$ indicating its dependence on the atomic number.

Transitions are shown with red arrows for the Balmer- α transition (from $n=3$ to $n=2$) and purple arrows for the Lyman- α transition (from $n=2$ to $n=1$). The Lyman- α transition is shown as a doublet, reflecting the fine structure splitting of the $n=2$ level.

Lamb shift: $\Delta E = \frac{\alpha}{\pi} \frac{(Z\alpha)^4}{n^3} F(Z\alpha) m_e c^2$

The structure of one-electron system: transition rate

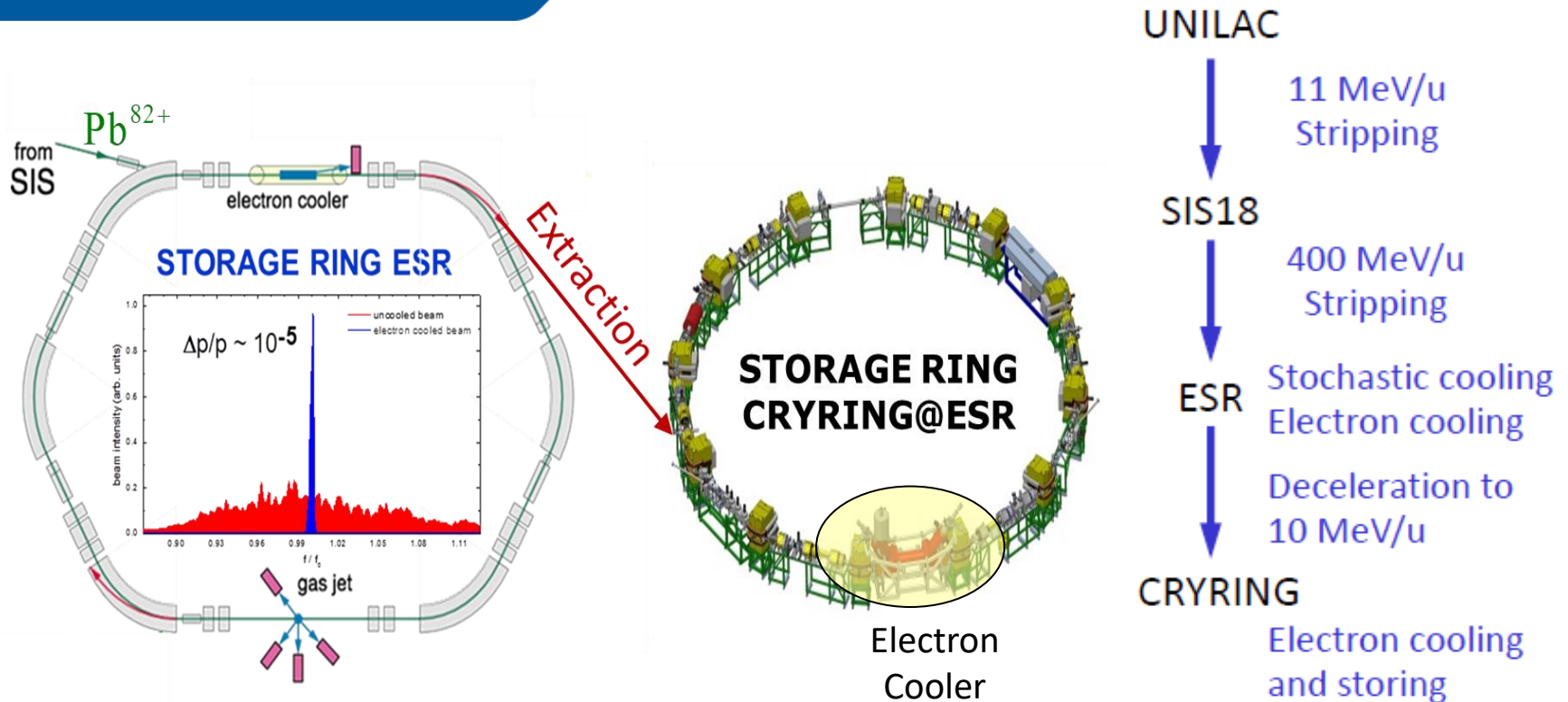
Jitrik and Bunge., *Phys. Scr.* 69, 196 (2004)



- The transitions rates of Γ_{M1} and Γ_{2E1} for $2s_{1/2} \rightarrow 1s$ are enhanced by 2-6 orders of magnitude as compared to Γ_{E1} of $2p_{1/2} \rightarrow 1s$.
- $\Gamma_{M1} / \Gamma_{2E1}$ scales approximately as Z^2 , and M1 represent the main contribution to the radiative decay of 2s state at high Z .

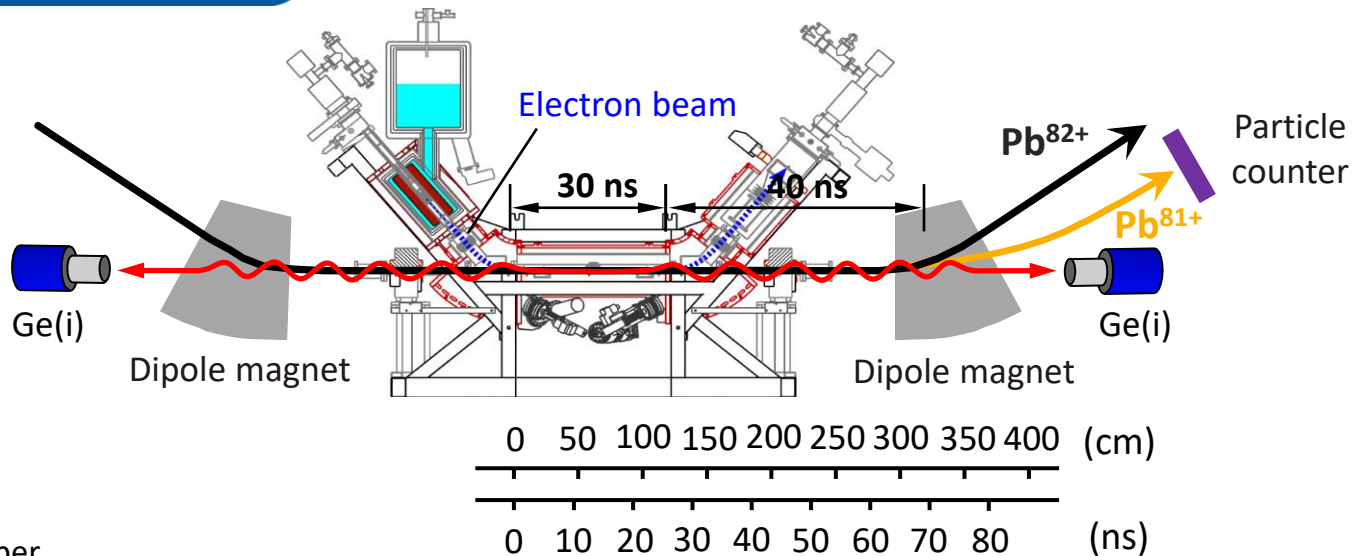
2. Experiment

Experimental environment at GSI



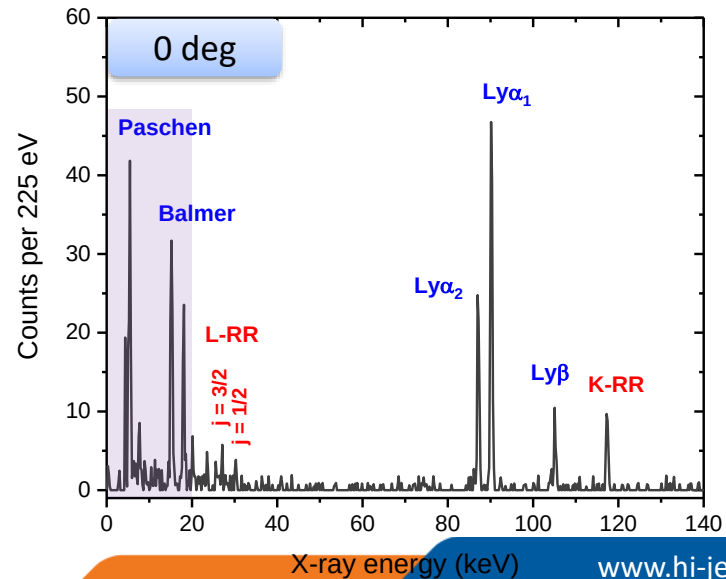
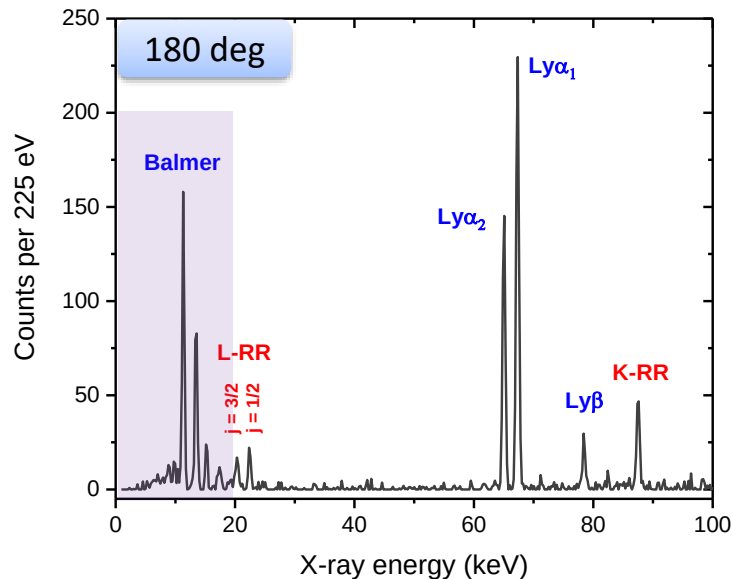
For the experiment, bare Pb ions were decelerated in the ESR to an energy of 10 MeV/u and injected into the CRYRING@ESR for radiative recombination study.

Experimental setup at the electron cooler



New x-ray viewport

Beryllium window chamber



HI JENA
Helmholtz Institute Jena

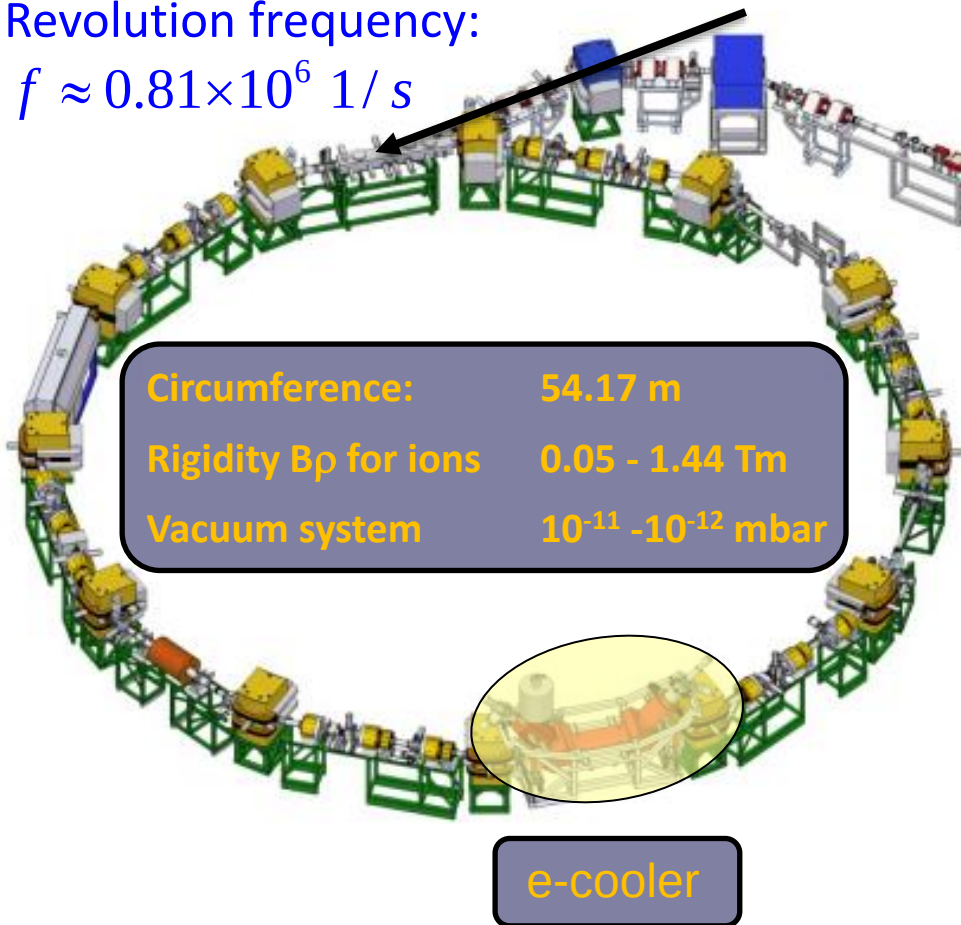
www.hi-jena.de

Field ionization effect in magnetic fields

Pb⁸²⁺ @10MeV/u

Revolution frequency:

$$f \approx 0.81 \times 10^6 \text{ 1/s}$$



Charge state of Pb⁸¹⁺ : $q = 81$

Reduced velocity: $\beta = \frac{v}{c} = 0.14$

Dipole magnet radius: $\rho = 1.2 \text{ m}$

Magnetic field: $B = \frac{m \cdot v}{q \cdot \rho} = 0.98 \text{ T}$

Field ionization limit n_{dip} derived from the empirical formula:

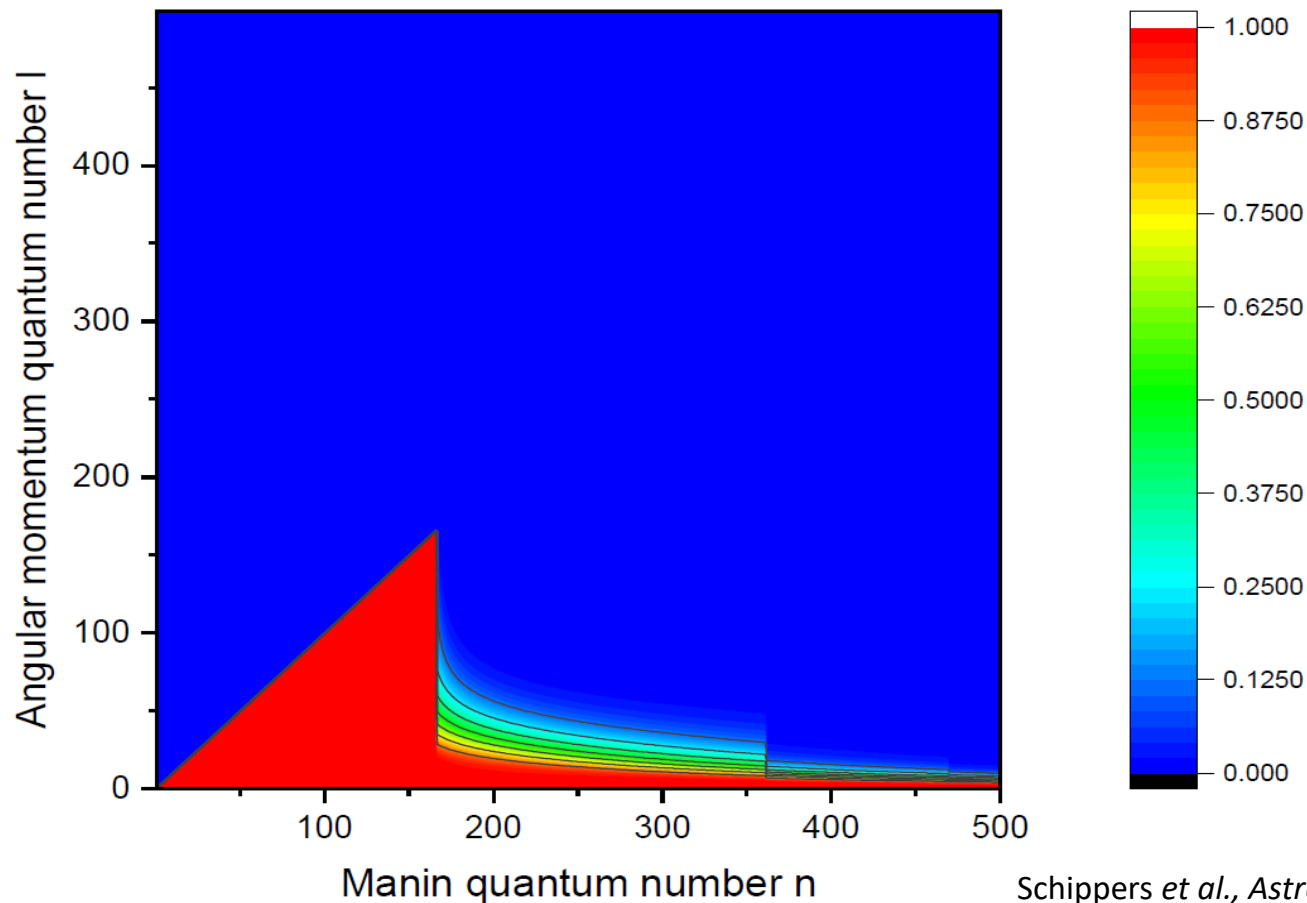
$$n_{\text{dip}} = \left(6.2 \times 10^{10} q^3 / v \cdot B \right)^{1/4} \\ \approx 165$$

Zong et al., J. Phys. B. 31, 3729 (1998)

HI JENA
Helmholtz Institute Jena

www.hi-jena.de

Field ionization effect in magnetic fields

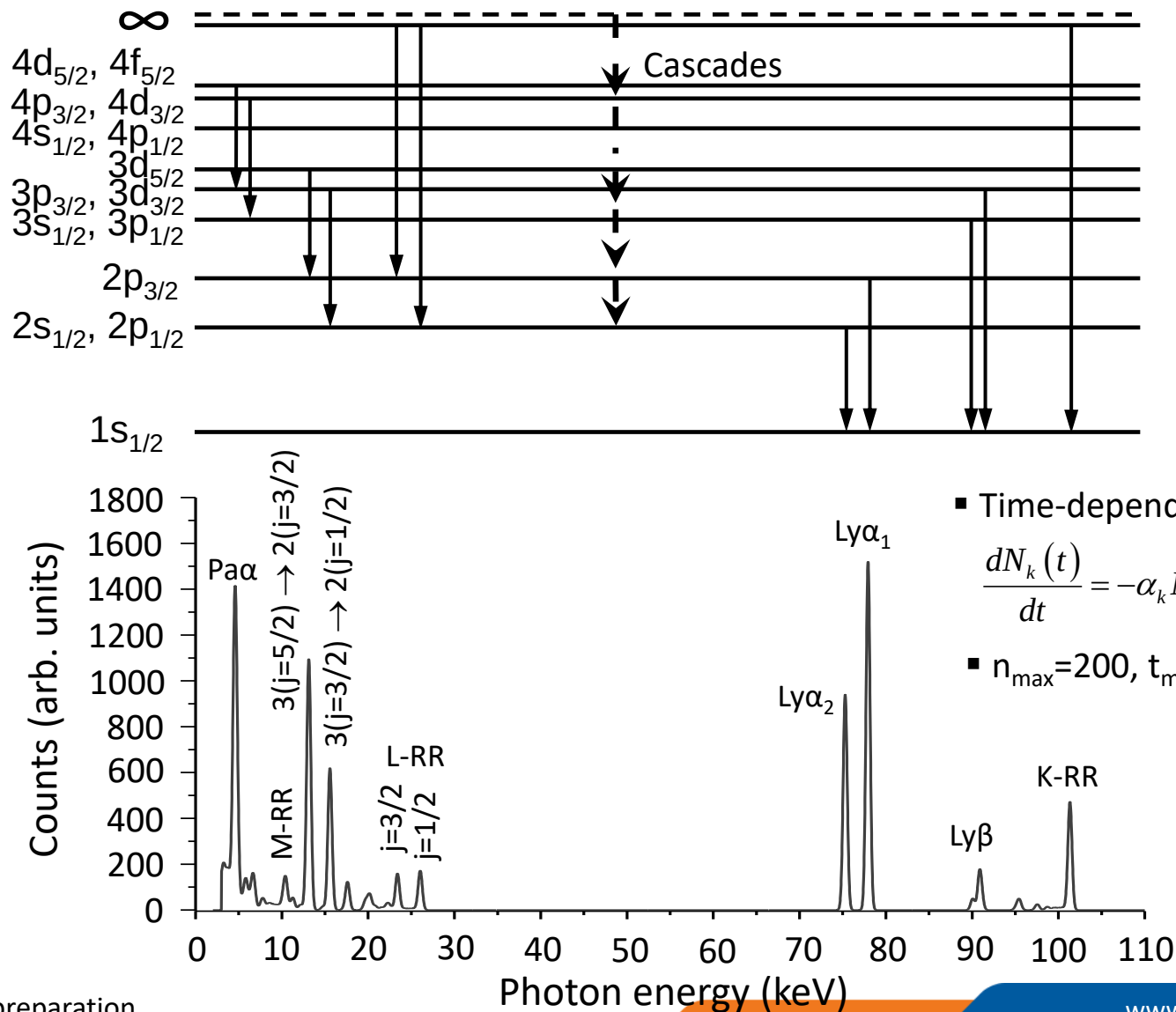


Schippers *et al.*, *Astrophys. J.* 555, 1027 (2001)
Böhm *et al.*, *Phys. Rev. A* 65, 052728 (2002)

Rydberg states with even higher n but low l are typically not field ionized in magnetic fields, and contribute via fast radiative cascades.

3. Theory

Simulation of radiative cascades



- Time-dependent rate equation:

$$\frac{dN_k(t)}{dt} = -\alpha_k N_k(t) + \sum_{m(>k)} \Gamma_{m \rightarrow k} N_m(t)$$

- $n_{\max}=200, t_{\max}=70$ ns

Atomic structure calculations: FAC code

M. F. Gu, *Can. J. Phys.* 86, 675 (2008)

- ❖ The atomic structure calculations are based on the relativistic configuration interaction with independent particle basis wavefunctions.
- ❖ The basis wavefunctions are derived from a local central potential, which is self-consistently determined to represent electronic screening of the nuclear potential.
- ❖ Relativistic effects are fully taken into account using the Dirac Coulomb Hamiltonian.
- ❖ Higher order QED effects are included: self-energy, finite nuclear size and vacuum polarization effects.
- ❖ Fast and reasonable accurate.

For hydrogen-like lead:

- ❑ $\sim 10^{-4}$ in level energies as compared to the results from Yerokhin and Shabaev [1].
- ❑ The transition rates are accurate to 99% when compared with tabulated data from Pal'chikov [2].

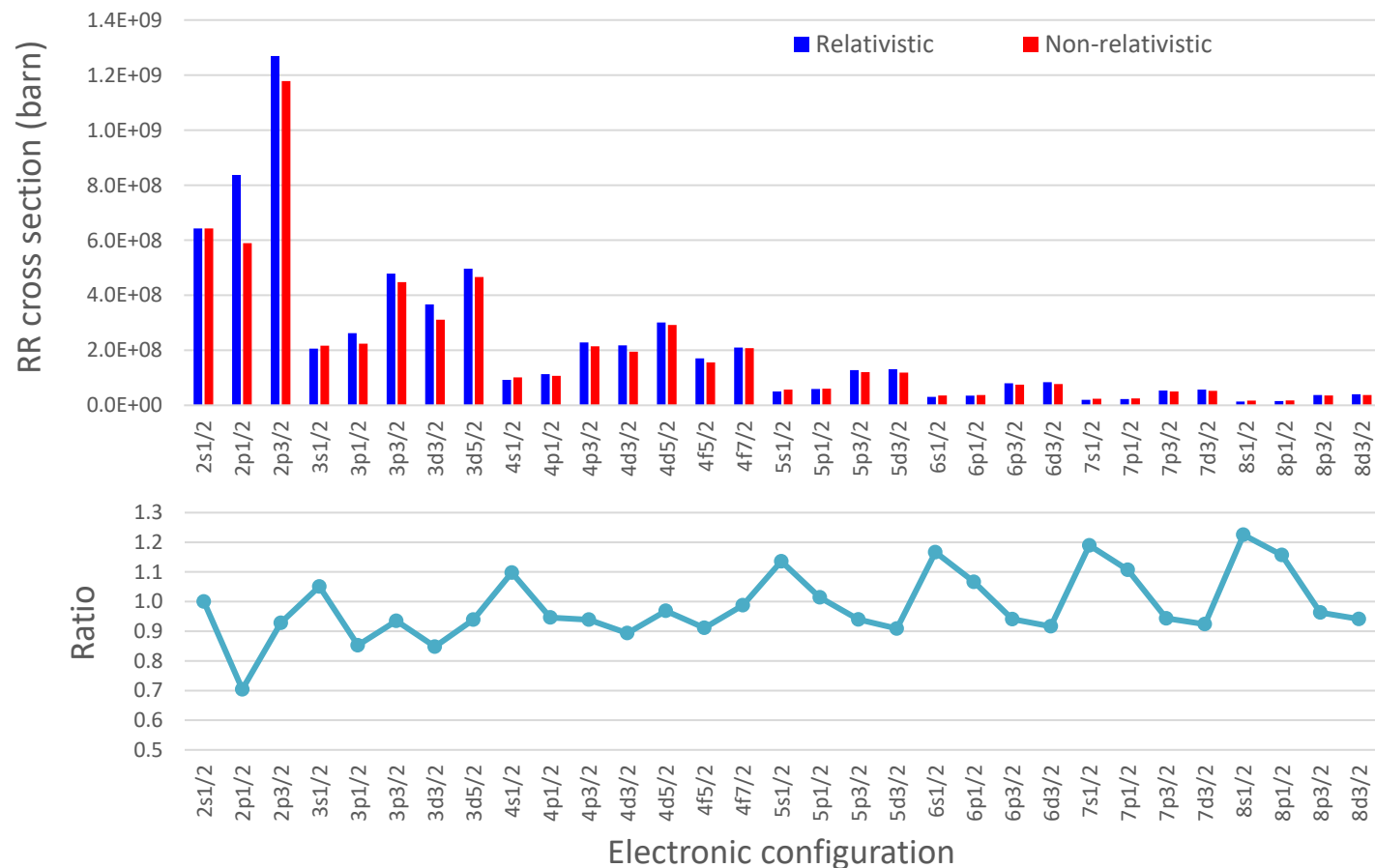
[1] Yerokhin and Shabaev, *J. Phys. Chem. Ref. Data* 44, 033103 (2015)

[2] Pal'chikov, *Phys. Scr.* 57, 581 (1998)

RR cross sections: nonrelativistic versus relativistic theory

Parameters setting:

- Nuclear charge Z : 82
- Electron-ion collision energy: 2.6 meV



RR rate coefficients at the electron cooler

Parameters setting:

- Projectile ions: Pb^{82+}
- Maximum main quantum number $n=200$.
- Electron velocity distribution at an electron cooler:

$$f(v) = \left(\frac{m_e}{2\pi}\right)^{3/2} \frac{1}{kT_{\perp}(kT_{\parallel})^{1/2}} \exp\left[-\left(\frac{m_e v_{\perp}^2}{2kT_{\perp}} + \frac{m_e v_{\parallel}}{2kT_{\parallel}}\right)\right]$$

- Transverse temperature:
 $kT_{\perp} = 2.59 \text{ meV}$
- Longitudinal temperature:
 $kT_{\parallel} = 51.8 \text{ } \mu\text{eV}$

RR rate coefficient:

$$\alpha_{n,l} = \langle v \sigma_{n,l}(v) \rangle = \int v \sigma_{n,l}(v) f(v) d^3v$$

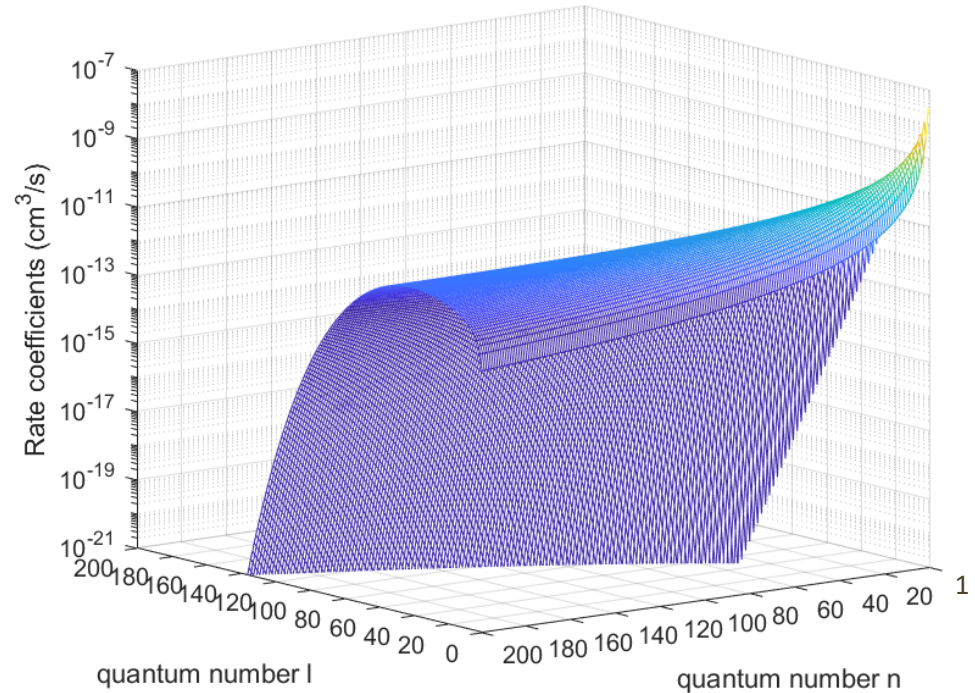
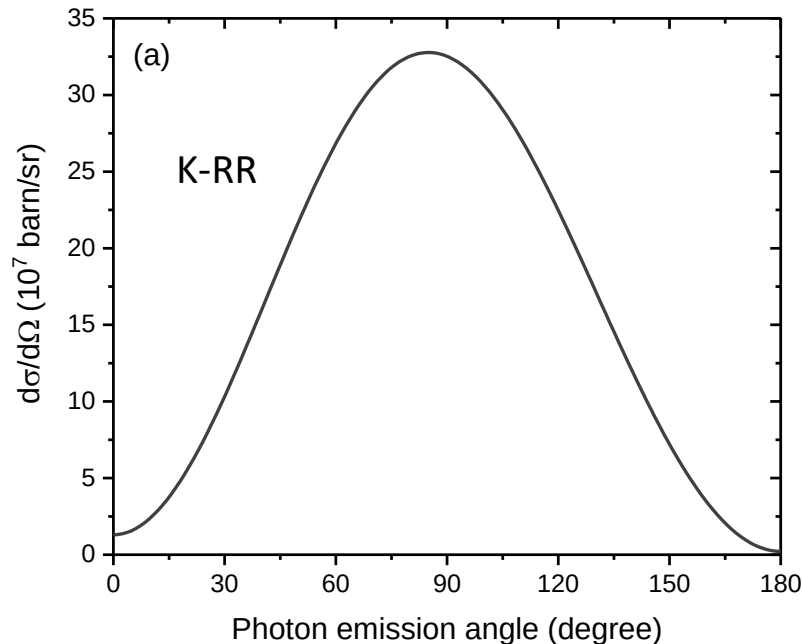


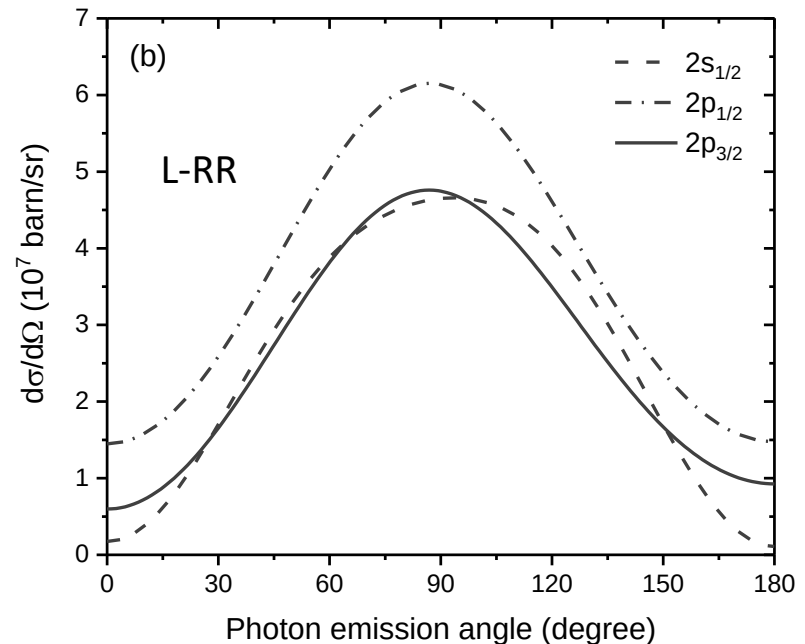
Figure: State-selective (n,l) rate coefficients calculated for radiative recombination into bare lead ions, with flattened free electron velocity distribution convolution.

Angular Distributions

■ K-RR and L-RR



Eichler and Stöhlker, *Phys. Rep.* 439, 1 (2007)



Hint: At electron cooler, we regard the direction of transverse electron beam as collisional axis ($kT_{\perp} \gg kT_{\parallel}$), thus the radiative recombination events registered by Ge(i) detectors at angle at 0 deg and 180 deg would correspond to $\theta_{lab} = 90^\circ$ in laboratory frame.

■ Characteristic x-rays

Recombination into high Rydberg states and following radiative cascades will substantially attenuate any potential initial alignment of excited states produced via RR, as a consequence, an isotropic emission pattern in the projectile frame for subsequent characteristic projectile transitions is expected.

HI JENA
Helmholtz Institute Jena

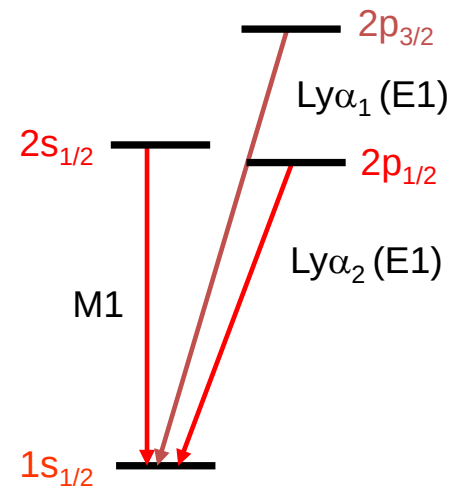
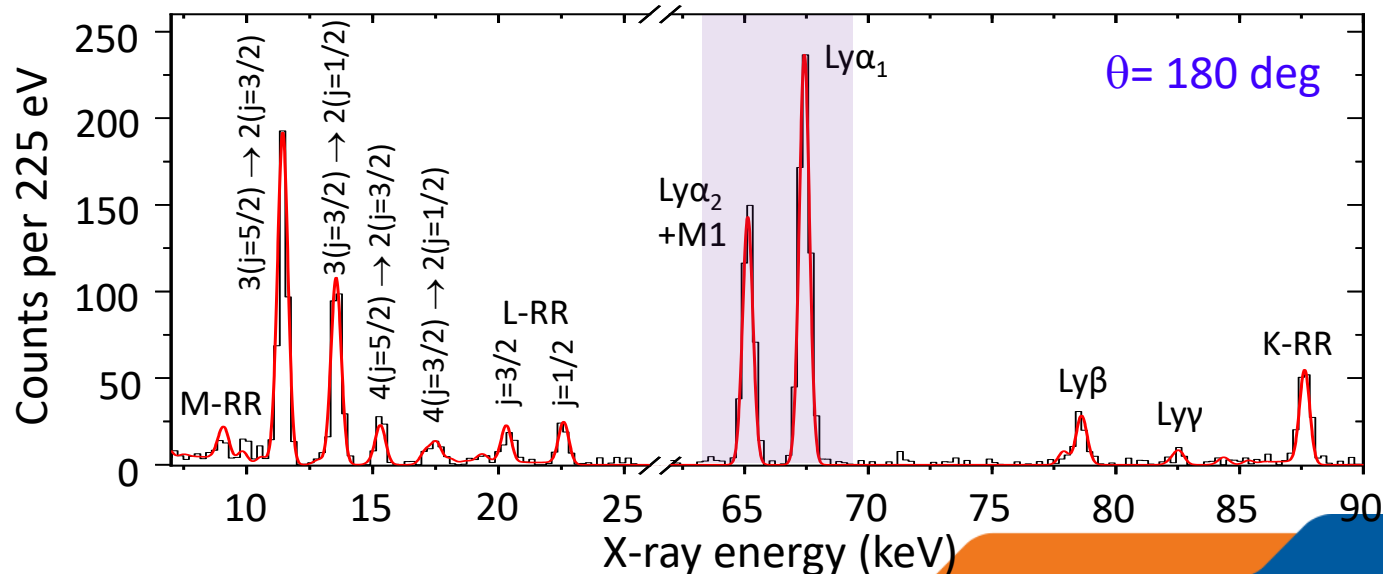
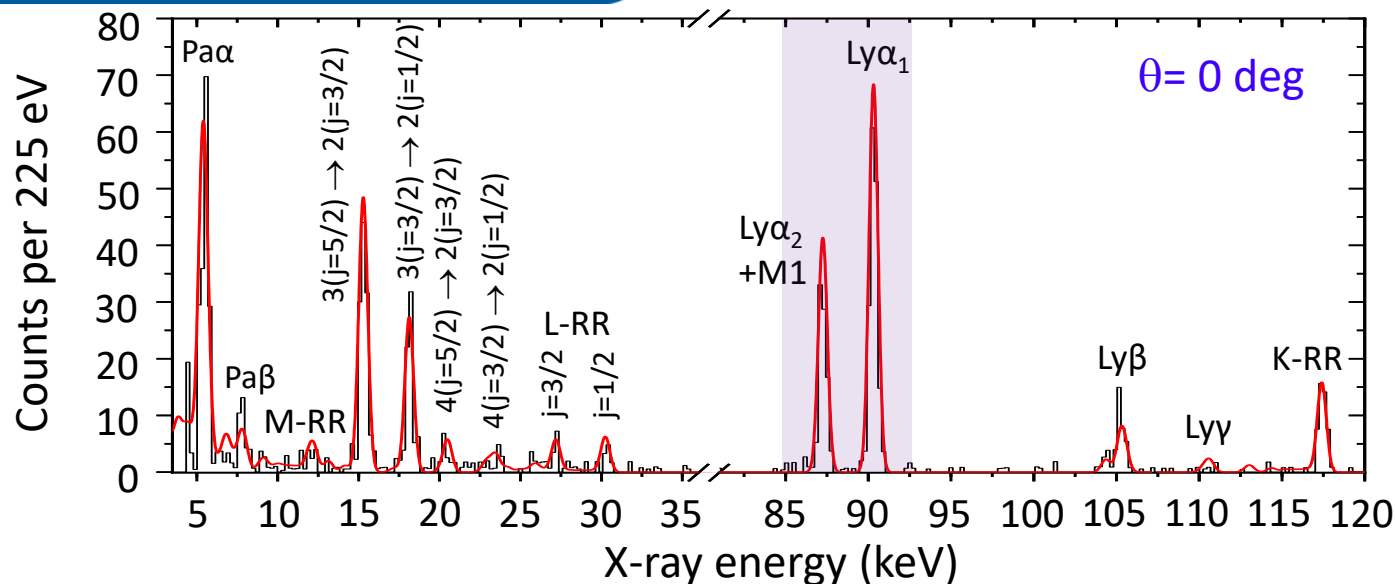
Stöhlker *et al.*, *Phys. Rev. Lett.* 79, 3270 (1997)

www.hi-jena.de

4. Results and discussions

Experimental results and comparison with theory

Zhu et al., Phys. Rev. A 105, 052804 (2022)



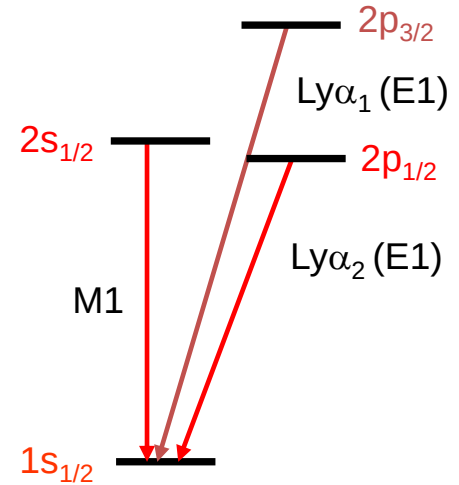
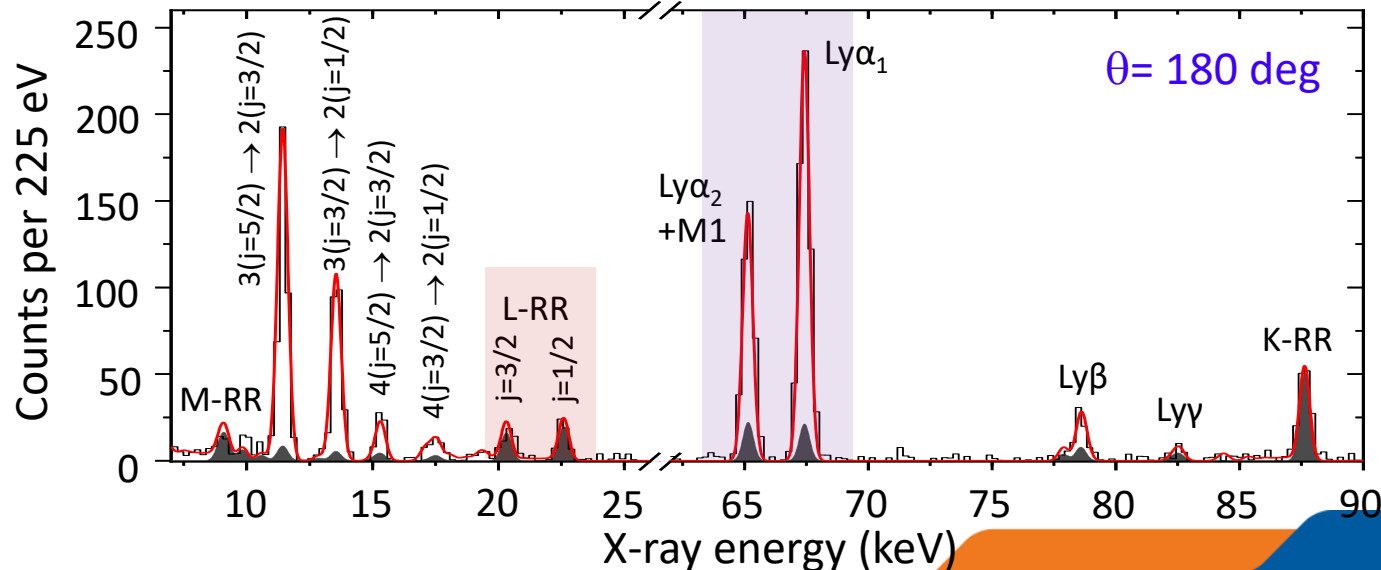
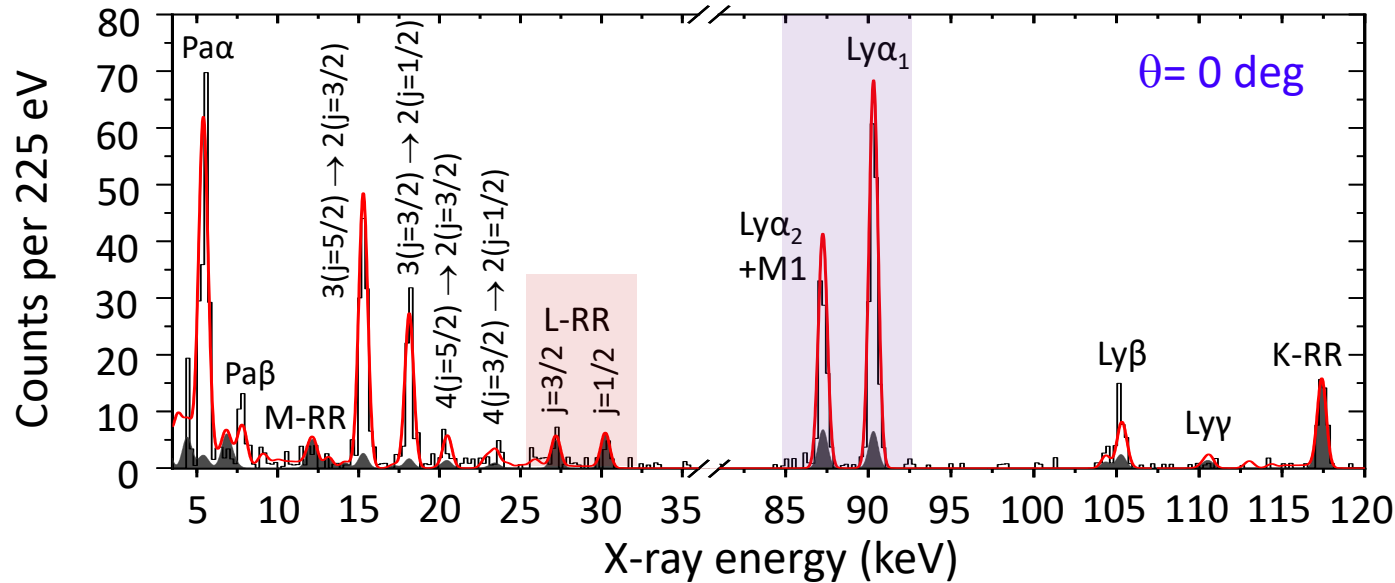
- $2s_{1/2} \rightarrow 1s_{1/2}$:
 $\Gamma_{M1} = 5.32 \times 10^{13} \text{ s}^{-1}$
- $2p_{1/2} \rightarrow 1s_{1/2}$:
 $\Gamma_{E1} = 2.93 \times 10^{16} \text{ s}^{-1}$
- $2p_{3/2} \rightarrow 1s_{1/2}$:
 $\Gamma_{E1} = 2.57 \times 10^{16} \text{ s}^{-1}$

HI JENA
Helmholtz Institute Jena

www.hi-jena.de

Experimental results and comparison with theory

Zhu et al., Phys. Rev. A 105, 052804 (2022)



- $2s_{1/2} \rightarrow 1s_{1/2}$:
 $\Gamma_{M1} = 5.32 \times 10^{13} \text{ s}^{-1}$
- $2p_{1/2} \rightarrow 1s_{1/2}$:
 $\Gamma_{E1} = 2.93 \times 10^{16} \text{ s}^{-1}$
- $2p_{3/2} \rightarrow 1s_{1/2}$:
 $\Gamma_{E1} = 2.57 \times 10^{16} \text{ s}^{-1}$

HI JENA
Helmholtz Institute Jena

www.hi-jena.de

Experimental results and comparison with theory

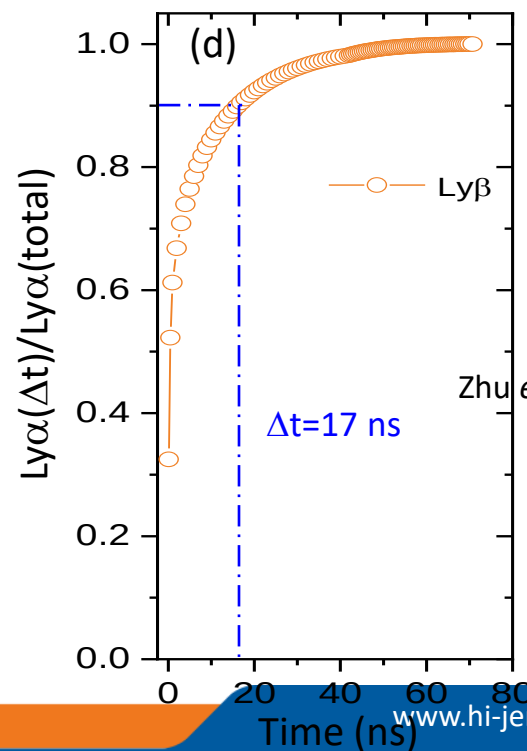
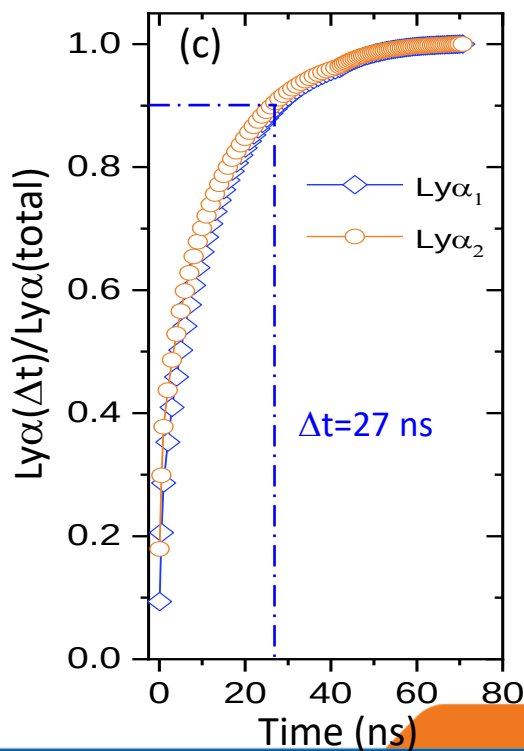
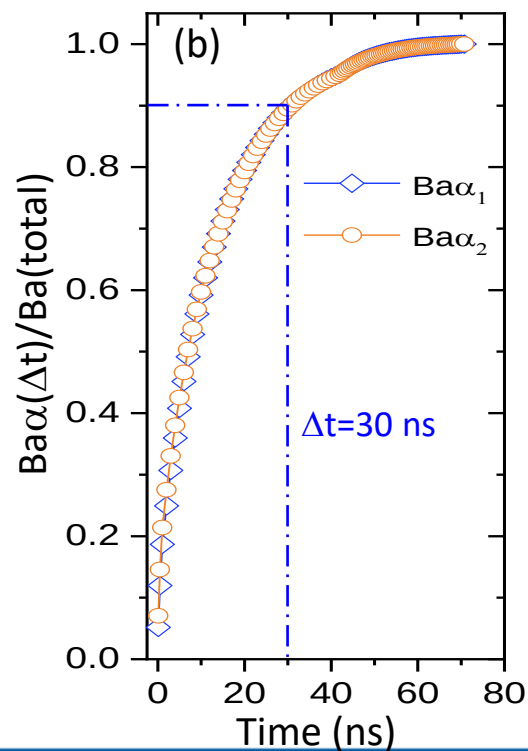
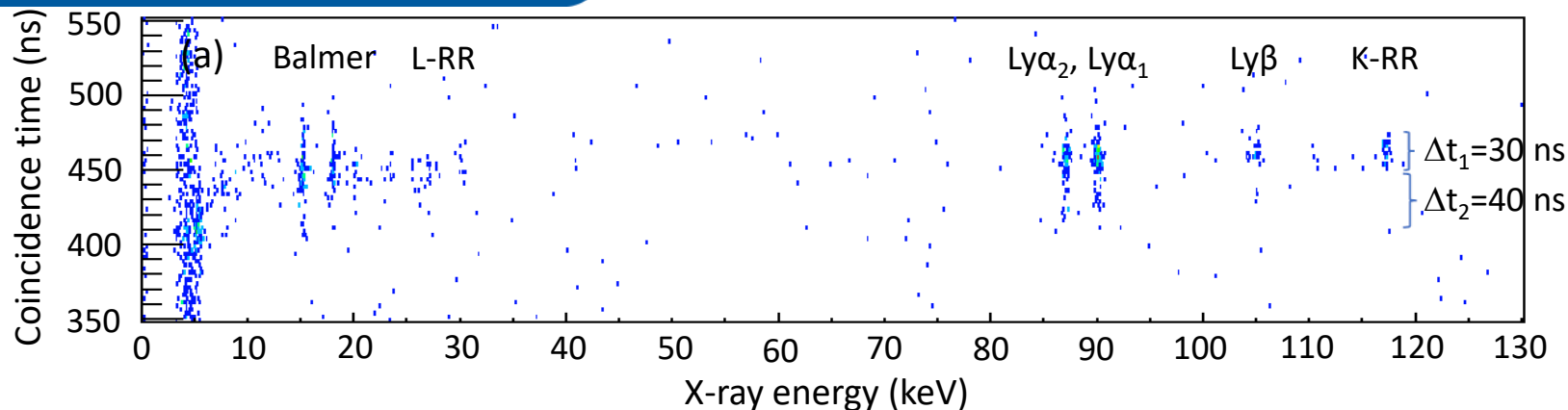
Table: Contribution from feeding cascades to the intensity of Lyman- α_1 and Lyman- α_2 lines. Uncertainties shown here are due to counting statistics whereas systematic uncertainties are neglected.

Characteristic x-rays	0 deg.	180 deg.	Theory
Ly α_1	93.6% $\pm$ 12.0%	94.0% $\pm$ 5.5%	93.9%
Ly α_2	86.3% $\pm$ 15.6%	87.4% $\pm$ 6.5%	86.8%

More than 90% from feeding cascades contribute to the intensities of Lyman- α radiation.

Time evolution of radiative cascades

Zhu et al., Phys. Rev. A 105, 052804 (2022)

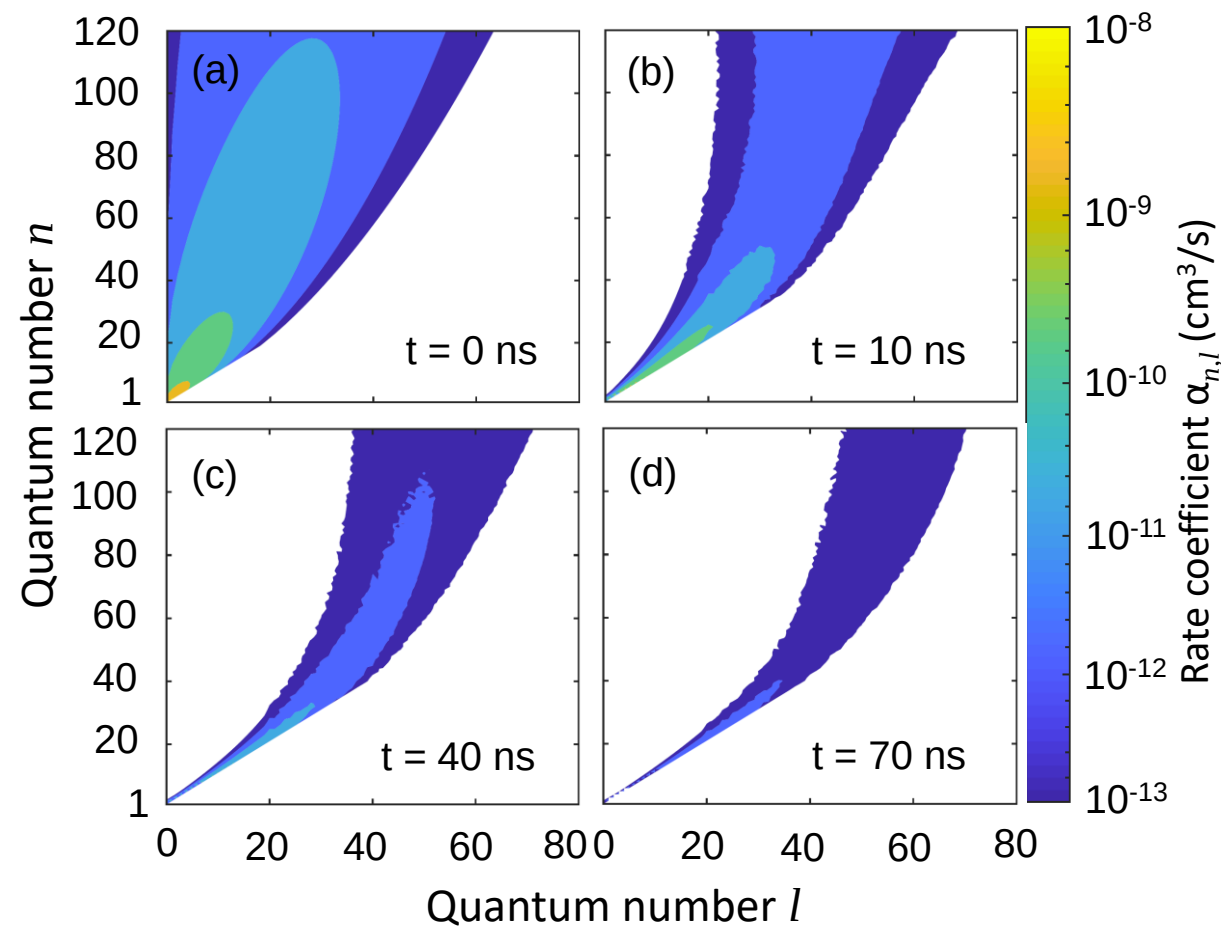


Zhu et al., in preparation

HI JENA
Helmholtz Institute Jena

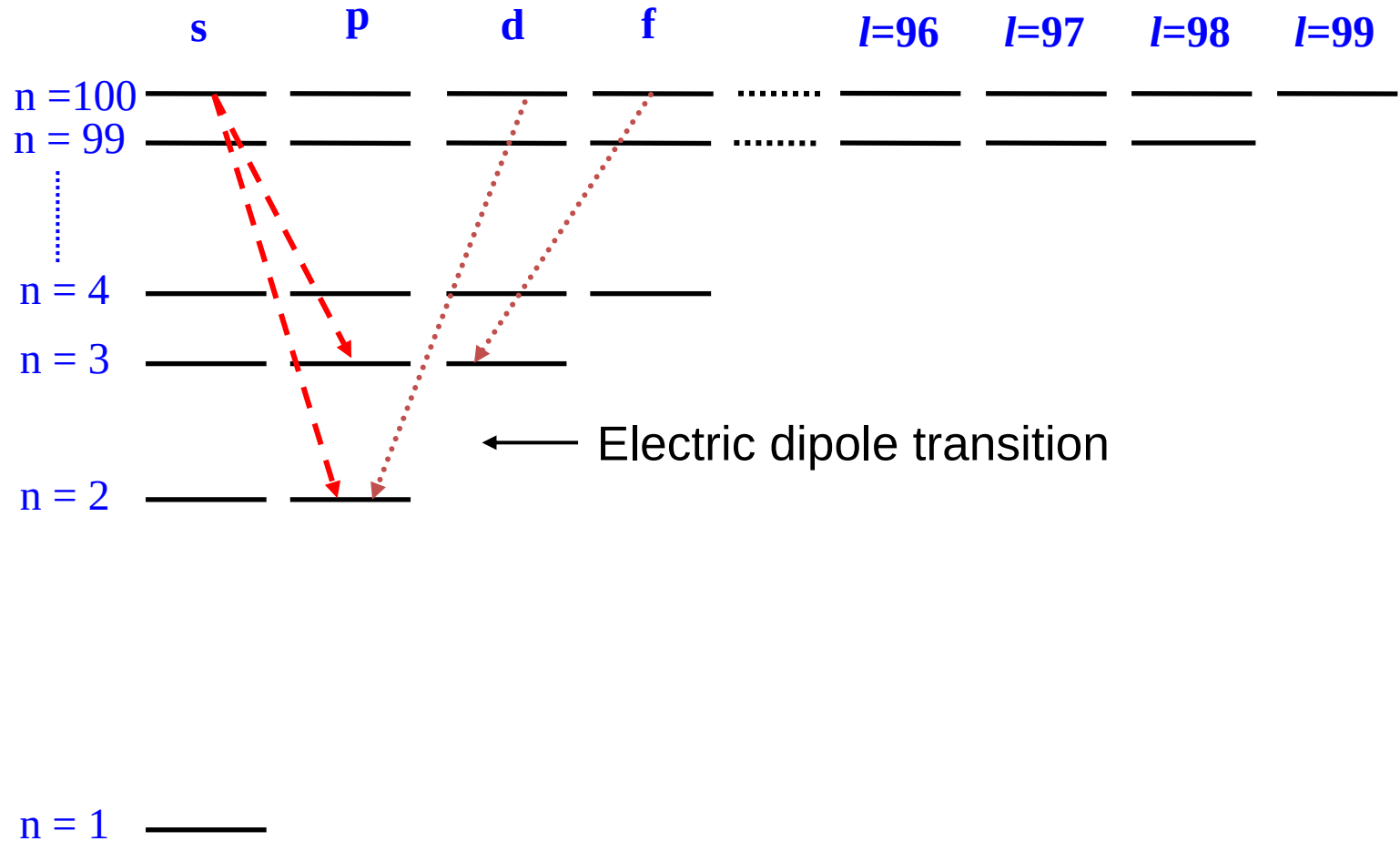
www.hi-jena.de

Calculated (n,l) -population rate coefficients versus time

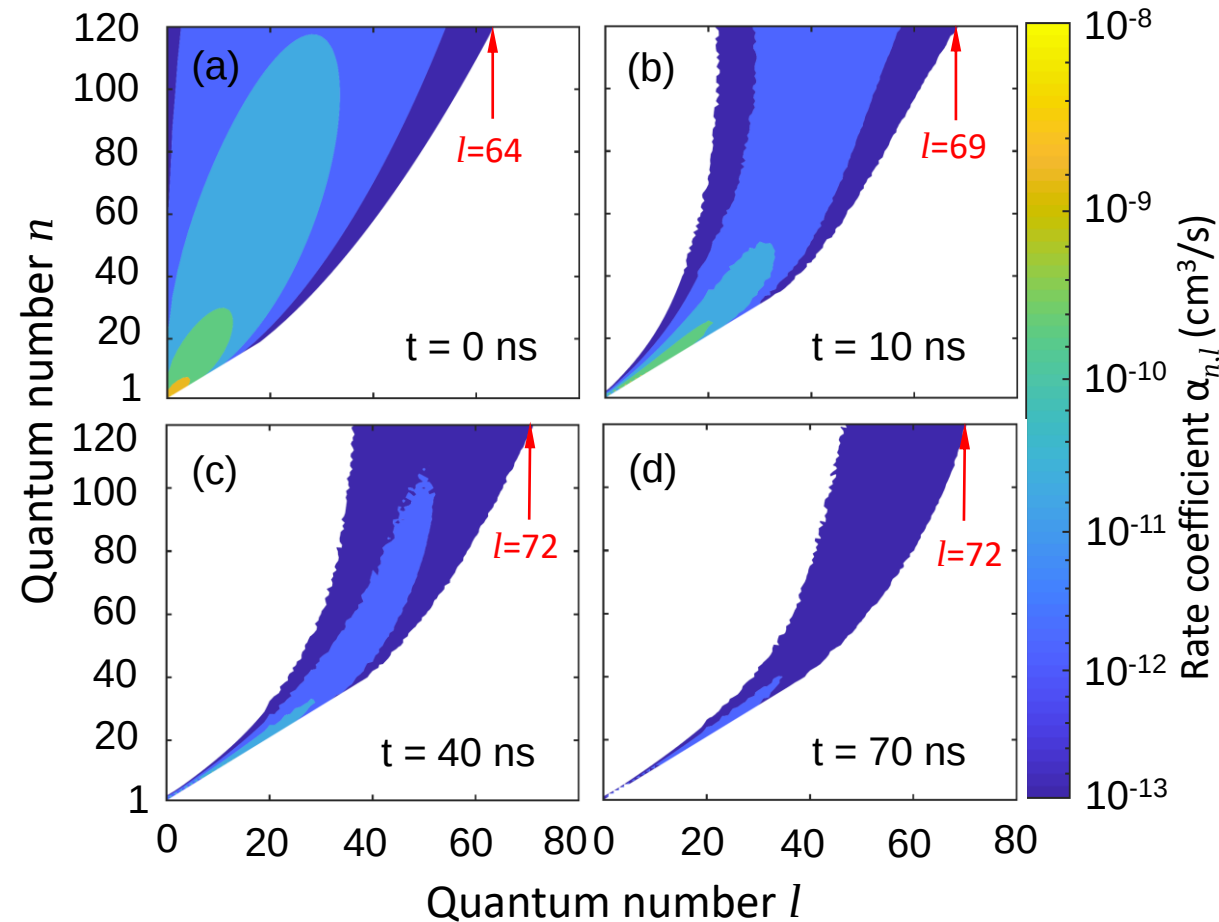


- The (n, l) states having a high n but a low l will decay directly to the inner shells via fast E1 transitions.

Level scheme of radiative cascades

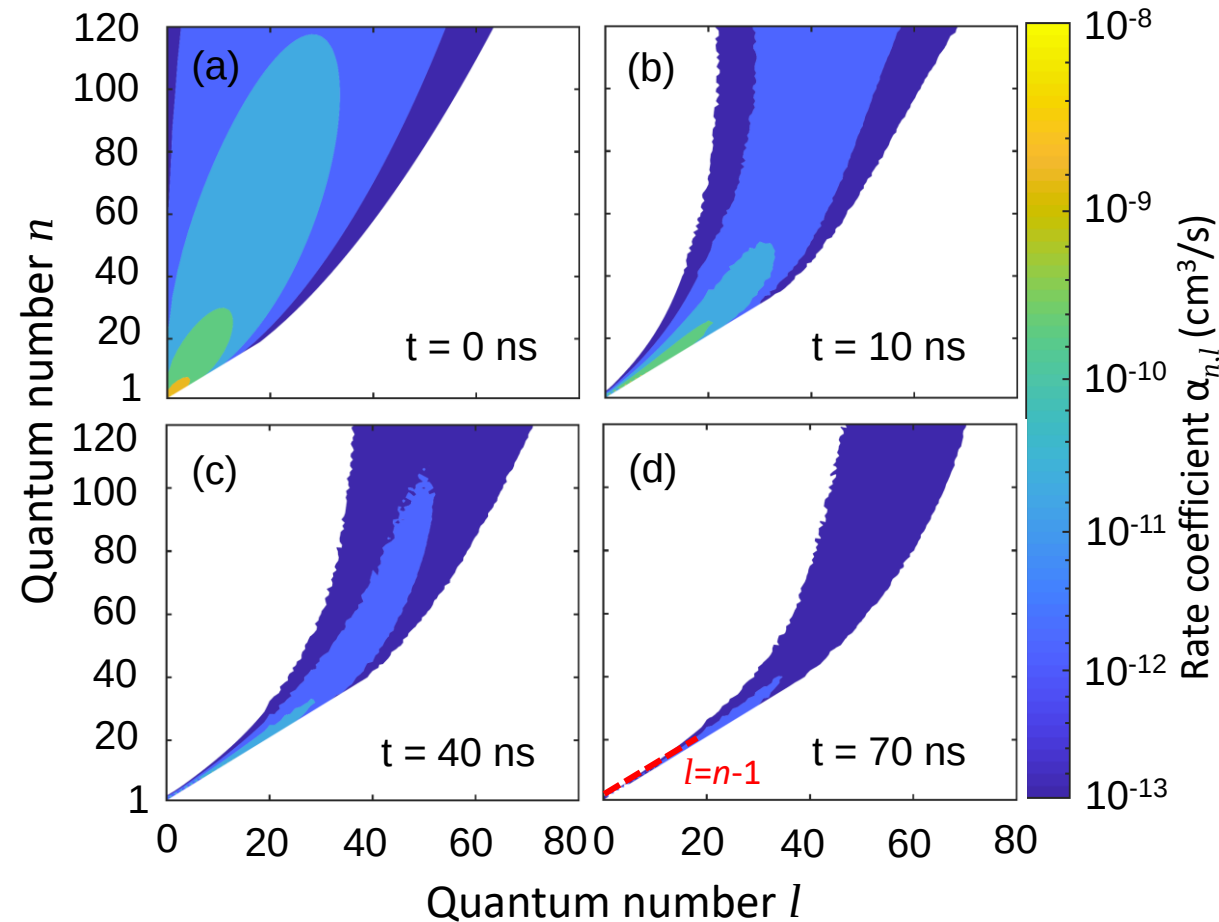


Calculated (n,l) -population rate coefficients versus time



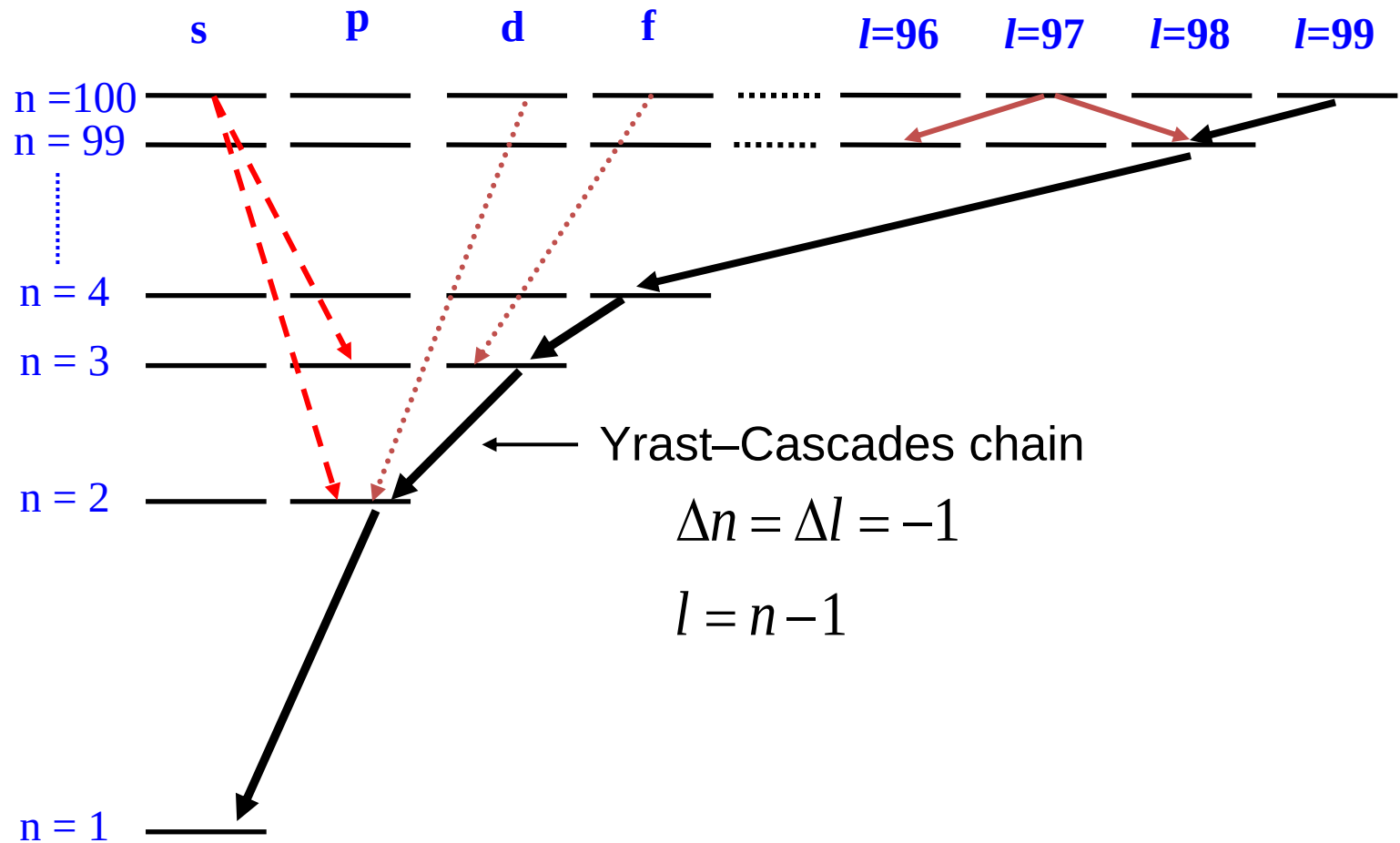
- The (n, l) states having a high n but a low l will decay directly to the inner shells via fast E1 transitions.
- The (n, l) states having a high n and a high l will decay step by step ($l \rightarrow l \pm 1$) to lower lying levels.

Calculated (n,l) -population rate coefficients versus time

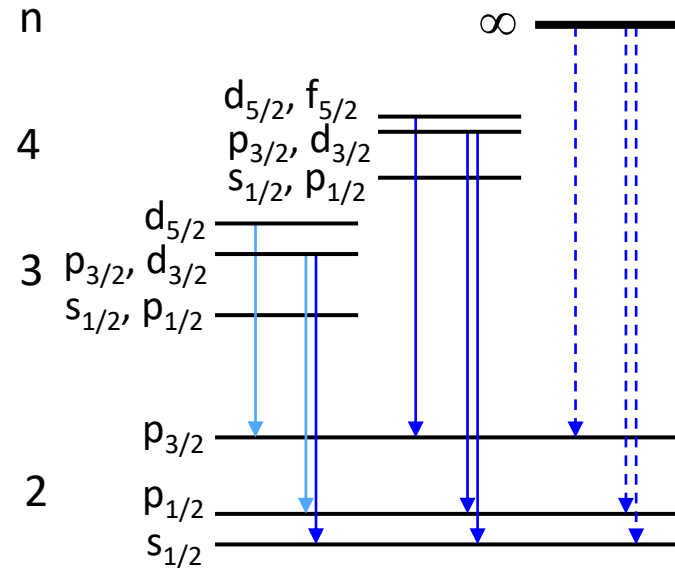
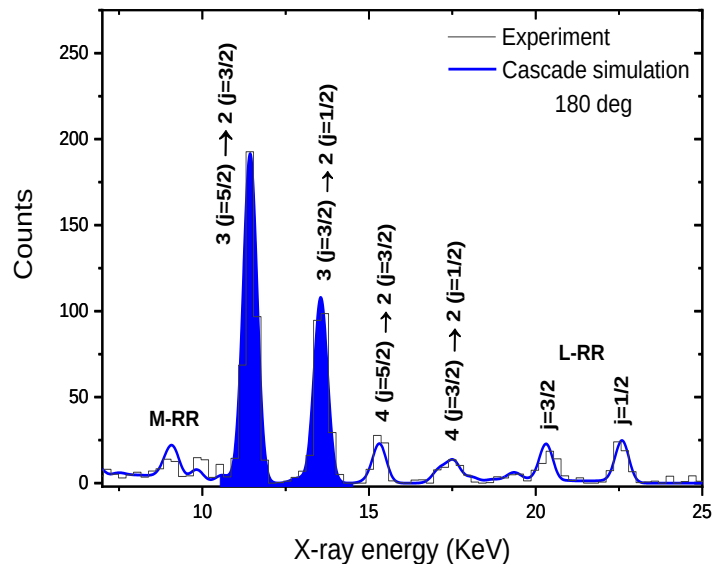
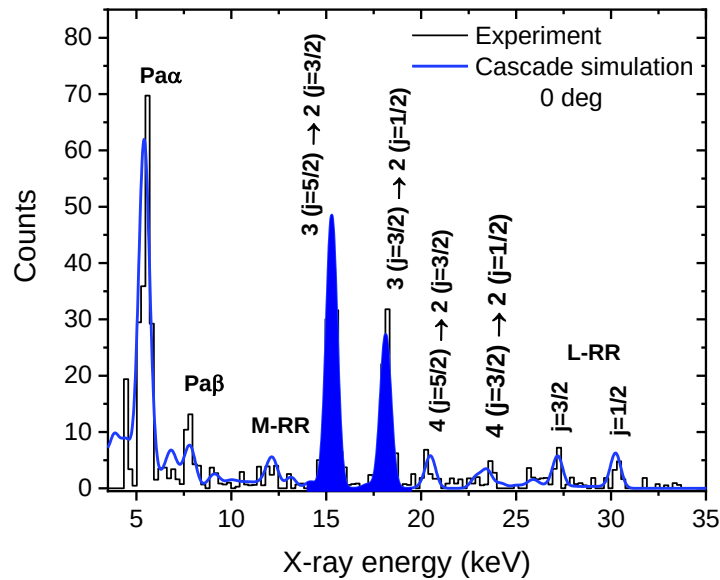


- The (n, l) states having a high n but a low l will decay directly to the inner shells via fast E1 transitions.
- The (n, l) states having a high n and a high l will decay step by step ($l \rightarrow l \pm 1$) to lower lying levels.
- The Yrast-cascades chain in inner shells forms the dominant features of characteristic projectile x-rays.

Level scheme of radiative cascades



Balmer spectra: state selective study



Zhu et al., Phys. Rev. A 105, 052804 (2022)

HI JENA
Helmholtz Institute Jena

www.hi-jena.de

Summary/Conclusion

- ❑ Capability of RR experiment for highly charged heavy ions has been fully commissioned at the CRYRING@ESR, and allows for the first physics production runs.
- ❑ Fine structure resolved level populations are studied in comparison with rigorous relativistic theory.
- ❑ Population pattern of characteristic x-ray emission is well revealed by our elaborate theoretical model calculating time-dependent cascade feeding processes.

Acknowledgement

A. Gumberidze
T. Over
Z. Andelkovic
A. Bräuning-Demian
R. J. Chen
D. Dmytriiev
O. Forstner
C. Hahn
F. Herfurth
M. O. Herdrich
P. -M. Hillenbrand
A. Kalinin
F. M. Kröger
M. Lestinsky
Yu. A. Litvinov
E. B. Menz
W. Middents
T. Morgenroth
N. Petridis
Ph. Pfäfflein
M. S. Sanjari
R. S. Sidhu
U. Spillmann
S. Trotsenko
L. Varga
G. Vorobyev

G. Weber
S. Schippers
R. Schuch
Th. Stöhlker*



HI JENA
Helmholtz Institute Jena



中国科学院近代物理研究所
Institute of Modern Physics, Chinese Academy of Sciences



SPONSORED BY THE



Federal Ministry
of Education
and Research

Thanks for your attention!

HI JENA
Helmholtz Institute Jena

www.hi-jena.de