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# Production of $\Xi^-$ hypernuclei and properties of $\Xi$ -nucleus potentials

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# Outline

## 1. Introduction

- Theoretical studies of  $\Xi^-$  s.p. potentials
- Experimental information on  $\Xi^-$  hypernuclei

## 2. Production of $\Xi^-$ hypernuclei

- Distorted-wave Impulse approximation (DWIA)
- Medium effects on  $\Xi^-$  production [Optimal Fermi averaging \(OFA\)](#)
- Analysis of  $\Xi^-$  QF spectra of [the  \${}^9\text{Be}\(\text{K}^-, \text{K}^+\)\$  reaction](#)
- Properties of the  $\Xi^-$ - ${}^8\text{Li}$  potential

## 3. Properties of $\Xi$ -nucleus potentials

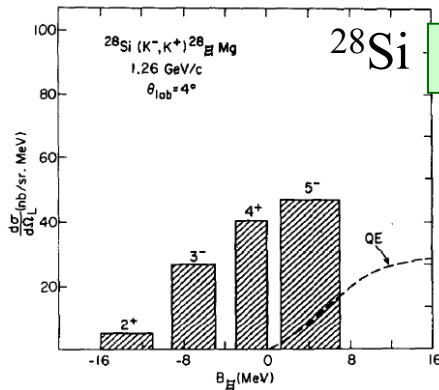
- $\Xi^-$ - ${}^{12}\text{C}$  vs.  $\Xi^-$ - ${}^{14}\text{N}$  potentials
- $\Xi^-$ - ${}^{56}\text{Fe}$  potentials and  $\Xi^-$ -atomic states
- Discussion

## 4. Summary

# 1. Introduction

# Theoretical studies of $\Xi^-$ s.p. potentials

$V_{\Xi} ?$



## $\Xi$ -hypernuclei via (K-,K+) reactions

C.B. Dover, A.Gal, Ann. Phys. 146 (1989) 309.

$$V_{\Xi}^0 = -24 \pm 4 \text{ MeV for } r_0 = 1.1 \text{ fm } (W_{\Xi}^0 \simeq -1 \text{ MeV})$$

## DWIA analysis of $^{12}\text{C}(K^-,K^+)$ spectra from BNL exp.

T.Iijima et al., NPA546(1992) 588.

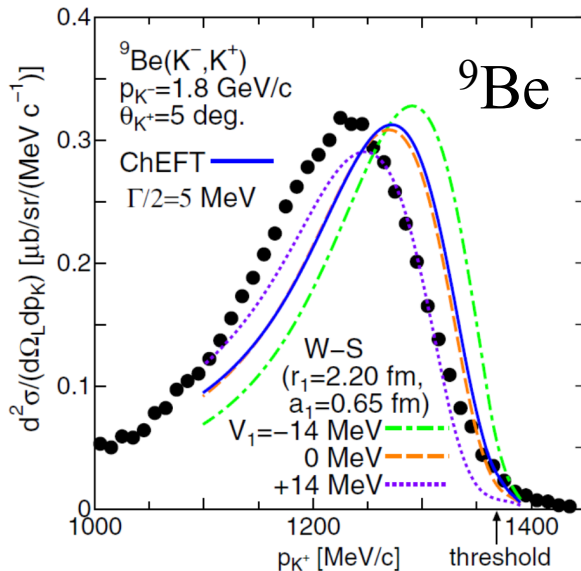
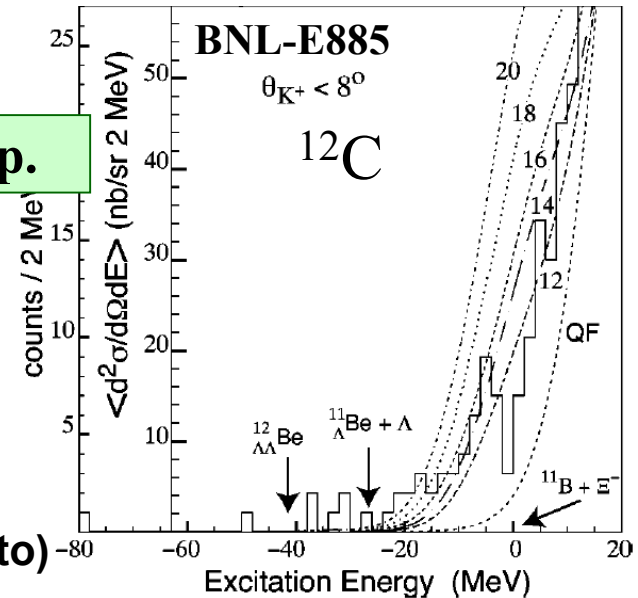
Tadokoro et al., PRC51(1995) 2656

P.Khaustov et al., PRC61(2000) 054603

$$V_{\Xi}^0 \simeq -16 \text{ MeV}$$

$$V_{\Xi}^0 \simeq -14 \text{ MeV}$$

Ehime, ESC16a/b, HAL  
(Yamamoto)



## Semi-Classical Distorted Wave Model Analysis

M. Kohno et al., PTP123(2010)157; NPA835(2010)358.

$$V_{\Xi}^0 = -20, -10, 0, +10, +20 \text{ MeV (fss2)}$$

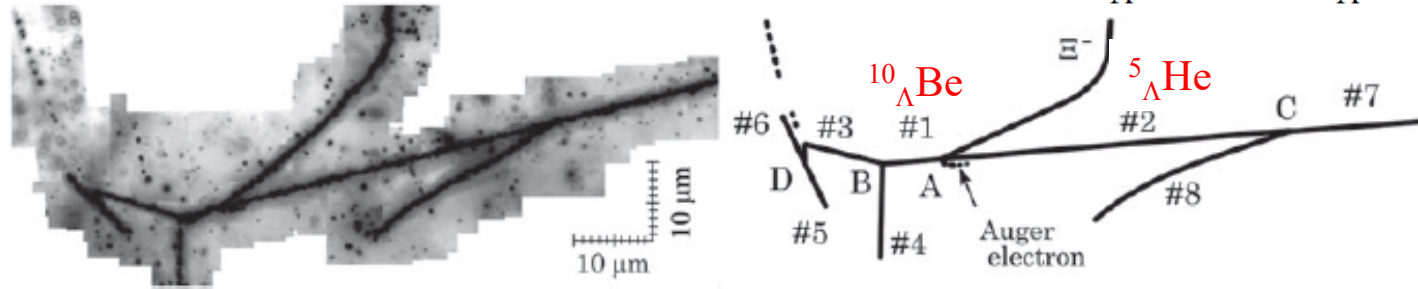
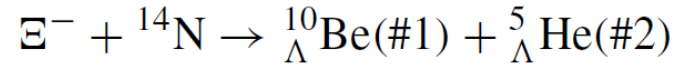
M. Kohno, PRC100 (2019) 024313

$$V_{\Xi}^0 = -14, 0, +14 \text{ MeV (Chiral NLO)}$$

# Experimental Information on $\Xi^-$ Hypernuclei

## “KISO” event from emulsion

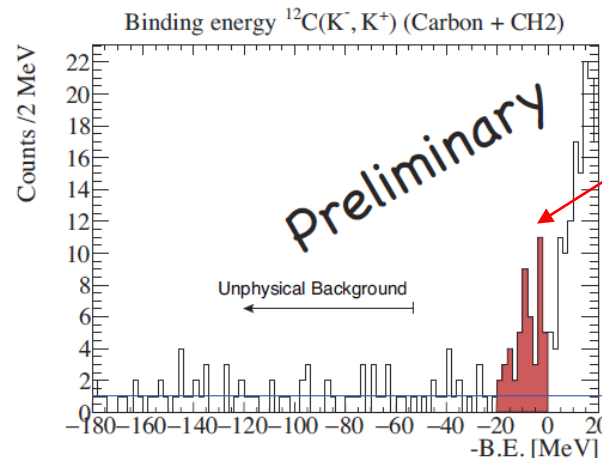
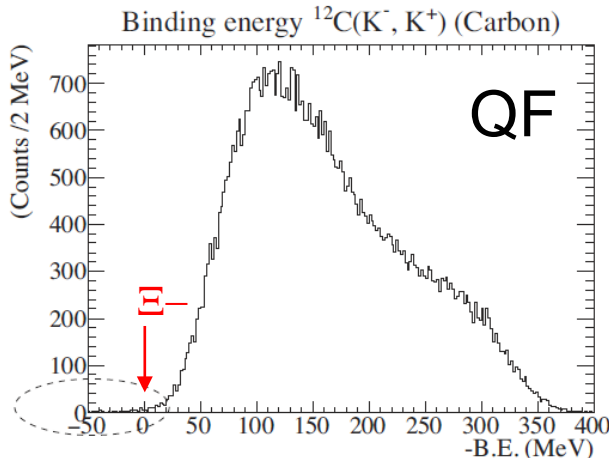
Nakazawa, et al., PTEP, 2015, 033D02



$B_{\Xi}(2p) = 1.03 \pm 0.18 \text{ MeV}$  or  $3.87 \pm 0.21 \text{ MeV}$ , which suggested to form a Coulomb-assisted nuclear 2p bound state for  $\Xi^-$ .

## ${}^{12}\text{C}(K^-, K^+){}^{12}_{\Xi}\text{Be}$ spectrum in J-PARC E05

T. Nagae et al., AIP Conf. Proc. 2130, 020015 (2019).



5.4 MeV FWHM

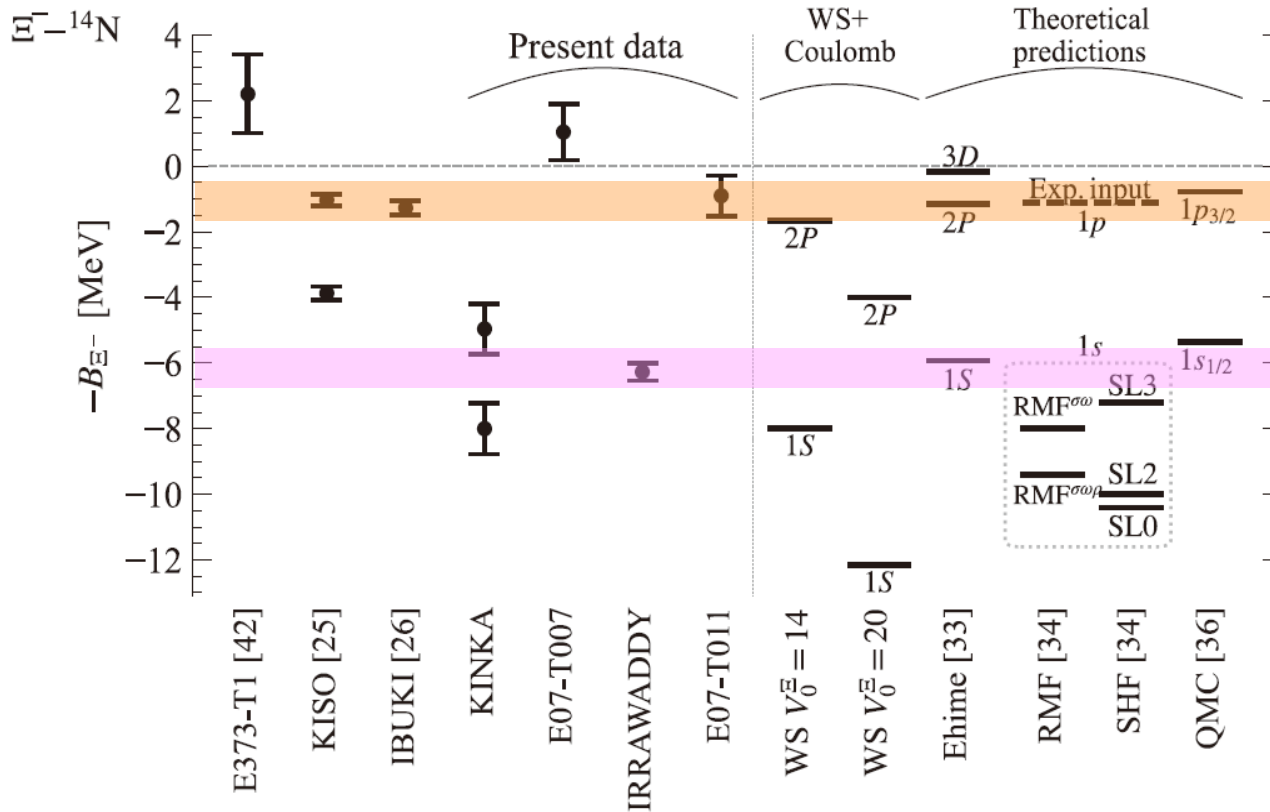
✓ Accurate observation of  $\Xi^-$  production: Their analysis is now ongoing.

# Recent observations of $\Xi^-$ hypernuclei from emulsion

compared with theoretical predictions



M. Yoshimoto, Prog. Theor. Exp. Phys. **2021**, 073D02.



**2P capture**

**1s?**

Coulomb only in  $\Xi^-{}^{14}\text{N}$ :  
 $B_{\Xi^-}(2P) = 0.39$  MeV  
 $B_{\Xi^-}(1s) = 1.21$  MeV

- ✓  $\Xi^-$  capture from the 2P state:  $B_{\Xi^-}(2P) = 1.03$  MeV(KISO)-1.27 MeV (IBUKI)
- The  $\Xi$ -nucleus potential is attractive in the real part.
- The 2P capture rate (4%) obtained from cascade cal.
- $\Xi\text{N}-\Lambda\Lambda$  coupling is weak (consistent with HAL-QCD).

## In this talk,

We study theoretically production of  $\Xi^-$  hypernuclei in the nuclear  $(K^-, K^+)$  reaction and properties of the  $\Xi$ -nucleus potential.

- We evaluate the in-medium cross sections of the  $K^-p \rightarrow K^+\Xi^-$  reaction, using [the optimal Fermi-averaging procedure \(OFA\)](#). T. Harada and Y. Hirabayashi, PRC102 (2020) 024618.
- We demonstrate the  $\Xi^-$  QF spectrum produced via the  ${}^9\text{Be}(K^-, K^+)$  reaction at 1.8 GeV/c [to extract useful information on the  \$\Xi\$ -nucleus potential for  \$\Xi^-\$  - \${}^8\text{Li}\$](#)  from the data of the BNL-E906 experiment.  
T. Harada and Y. Hirabayashi, PRC103 (2021) 024605.
- We discuss the properties of  $\Xi$ -nucleus potentials in comparison with the recent data from emulsion.

## 2. Production of $\Xi^-$ hypernuclei



## 2.1

# Distorted wave impulse approximation (DWIA)

# Distorted-wave impulse approximation (DWIA)

- Inclusive double-differential cross sections

$$\frac{d^2\sigma}{dE_{K^+}d\Omega_{K^+}} = \beta \frac{1}{[J_A]} \sum_{m_A} \sum_{B, m_B} |\langle \Psi_B | \hat{F} | \Psi_A \rangle|^2 \delta(\omega - E_B + E_A)$$

- Energy transfer and momentum transfer in lab. system

$$\omega = E_{K^-} - E_{K^+}, \quad \mathbf{q} = \mathbf{p}_{K^-} - \mathbf{p}_{K^+}$$

- Kinematical factor for translating from K-N to K-A.

$$\beta = \left( 1 + \frac{E_{K^+}^{(0)} p_{K^+}^{(0)} - p_{K^-}^{(0)} \cos \theta_{\text{lab}}}{E_B^{(0)} p_{K^+}^{(0)}} \right) \frac{p_{K^+} E_{K^+}}{p_{K^+}^{(0)} E_{K^+}^{(0)}}$$

- External operator for the production reaction

$$\hat{F} = \int d\mathbf{r} \chi_{p_{K^+}}^{(-)*}(\mathbf{r}) \chi_{p_{K^-}}^{(+)}(\mathbf{r}) \sum_{j=1}^A \bar{f}_{K^- p \rightarrow K^+ \Xi^-} \delta(\mathbf{r} - \mathbf{r}_j) \hat{O}_j$$

# Distorted-waves for outgoing $K^+$ and incoming $K^-$

## ■ Eikonal approximation

C. B. Dover, et al., PRC22, 2073 (1980).

$$\chi_{K^-}^{(-)*}(\mathbf{r}) \chi_{K^-}^{(+)}(\mathbf{r}) = \exp(i\mathbf{q} \cdot \mathbf{r}) D(\mathbf{b}, z)$$

$$D(\mathbf{b}, z) = \exp\left(-\frac{\sigma_{K^-}(1 - i\alpha_{K^-})}{2} \int_{-\infty}^z \rho(\mathbf{b}, z') dz' - \frac{\sigma_K(1 + i\alpha_K)}{2} \int_z^{\infty} \rho(\mathbf{b}, z') dz'\right)$$

$$\sigma_m = (Z/A)\sigma_{mp}^{\text{tot}} + (N/A)\sigma_{mn}^{\text{tot}} \quad \alpha_{K^-} = \alpha_K = 0$$

$$\sigma_{K^-} = 28.9 \text{ mb} \quad \sigma_{K^+} = 19.4 \text{ mb}$$

T. Motoba, et al., PRC38, 1322 (1988).

S. Tadokoro, et al., PRC51, 2656 (1995).

P. Khaustov et al., PRC61, 054603 (2000).

## ■ Partial wave expansion

$$\chi_{K^-}^{(-)*}(\mathbf{r}) \chi_{K^-}^{(+)}(\mathbf{r}) = \sum_L \sqrt{4\pi(2L+1)} i^L \tilde{j}_L(r) Y_L^0(\hat{\mathbf{r}})$$

$$\tilde{j}_L(r) = \sum_{\ell\ell'} \sqrt{\frac{2\ell'+1}{4\pi} \frac{2\ell+1}{2L+1}} i^{\ell-L} (\ell 0 \ell' 0 | L 0)^2 j_\ell(r) D_{\ell'}(r)$$

$$\ell \leq 30$$

$$\text{Recoil effect: } \mathbf{r} \rightarrow \frac{M_C}{M_A} \mathbf{r}$$

# Production cross sections in $A(a,b)_Y B$ reactions

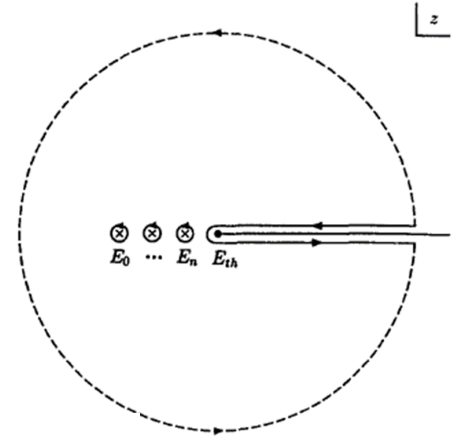
Morimatsu, Yazaki, NPA483 (1988) 493.

## Green's function method

### Strength function

$$S(E) = \sum_{B, m_B} |\langle \Psi_B | \hat{F} | \Psi_A \rangle|^2 \delta(\omega - E_B + E_A)$$

$$= -\frac{1}{\pi} \text{Im} \sum_{\alpha\alpha'} \int dr dr' F_{\Xi}^{\alpha\prime\dagger}(\mathbf{r}) G_{\Xi}^{\alpha\alpha'}(E; \mathbf{r}, \mathbf{r}') F_{\Xi}^{\alpha'}(\mathbf{r}')$$



$$\sum_B |\Psi_B\rangle \delta(E - E_B) \langle \Psi_B| = (-) \frac{1}{\pi} \text{Im} \left[ \frac{1}{E - H_B + i\epsilon} \right]$$

### Completeness relation

$$G^{(+)}(E; \mathbf{r}, \mathbf{r}') = \sum_n \frac{\varphi_n(\mathbf{r})(\tilde{\varphi}_n(\mathbf{r}'))^*}{E - E_n + i\epsilon} + \frac{2}{\pi} \int_0^\infty dk \frac{k^2 S(k) u(k, \mathbf{r})(\tilde{u}(k, \mathbf{r}'))^*}{E - E_k + i\epsilon}$$

bound states,  
quasibound states

Continuum states,  
resonance states

## 2.2

# Medium effects on $\Xi^-$ production

# Distorted-wave impulse approximation (DWIA)

- Inclusive double-differential cross sections

$$\frac{d^2\sigma}{dE_{K^+}d\Omega_{K^+}} = \beta \frac{1}{[J_A]} \sum_{m_A} \sum_{B, m_B} |\langle \Psi_B | \hat{F} | \Psi_A \rangle|^2 \delta(\omega - E_B + E_A)$$

- Energy transfer and momentum transfer in lab. system

$$\omega = E_{K^-} - E_{K^+}, \quad \mathbf{q} = \mathbf{p}_{K^-} - \mathbf{p}_{K^+}$$

- Kinematical factor for translating from K-N to K-A.

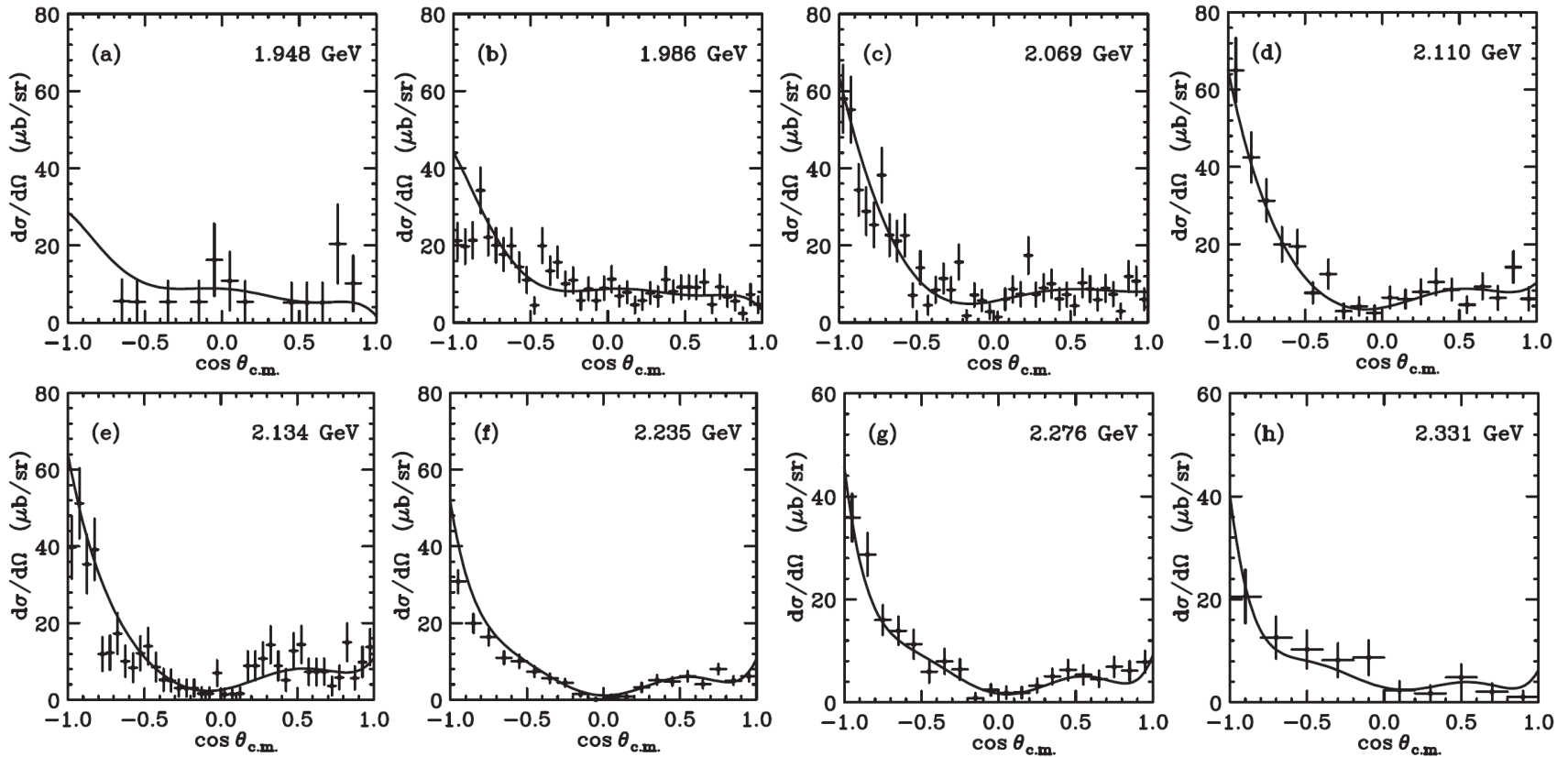
$$\beta = \left( 1 + \frac{E_{K^+}^{(0)} p_{K^+}^{(0)} - p_{K^-}^{(0)} \cos \theta_{\text{lab}}}{E_B^{(0)} p_{K^+}^{(0)}} \right) \frac{p_{K^+} E_{K^+}}{p_{K^+}^{(0)} E_{K^+}^{(0)}}$$

- External operator for the production reaction

$$\hat{F} = \int d\mathbf{r} \chi_{p_{K^+}}^{(-)*}(\mathbf{r}) \chi_{p_{K^-}}^{(+)}(\mathbf{r}) \sum_{j=1}^A \boxed{\bar{f}_{K^- p \rightarrow K^+ \Xi^-}} \delta(\mathbf{r} - \mathbf{r}_j) \hat{O}_j$$

↖ In-medium  $K^- p \rightarrow K^+ \Xi^-$  amplitude

# Differential cross sections for the $K^-p \rightarrow K^+\Xi^-$ reactions



**Experimental data :**

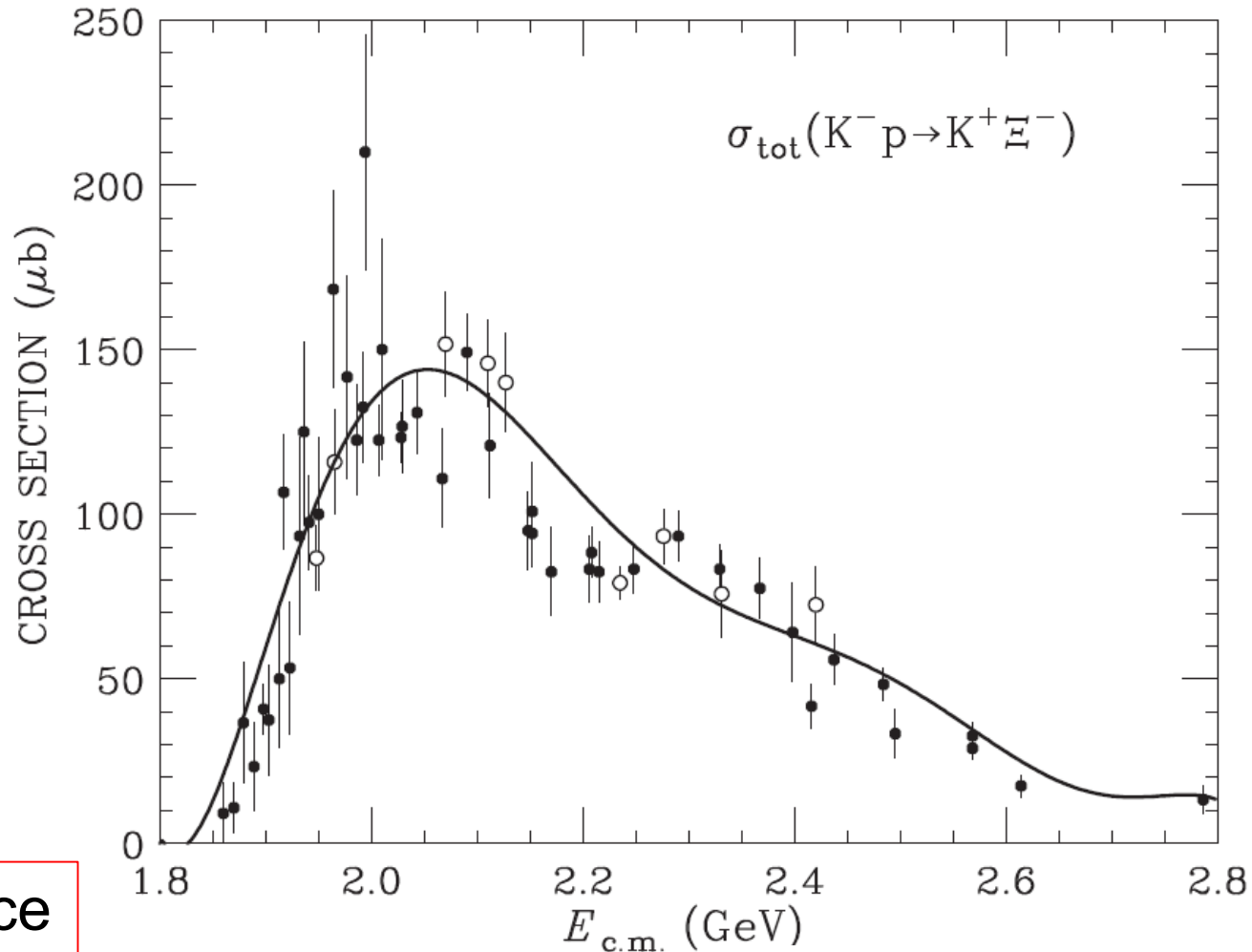
**free space**

- W. P. Trower, et al., PR170, 1207 (1968).
- G. Burgun et al., NPB 8, 447 (1968).
- P. M. Dauber, et al., PR179, 1262 (1969).
- T. G. Trippe and P. E. Schlein, PR158, 1334 (1967).
- G. W. London, et al., PR143, 1034 (1966).
- V. Flaminio, et al., CERN-HERA Report 79-02 (1979).

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{c.m.}}^{\text{elem}} = \lambda^2 \sum_{\ell=0}^{\ell_{\max}} A_{\ell}(E_{\text{c.m.}}) P_{\ell}(\cos \theta_{\text{c.m.}})$$

Parameters ( $\ell \leq 6$ )  
for fits to the data

# Total cross section $\sigma_{\text{tot}}$ for the $K^-p \rightarrow K^+\Xi^-$ reaction



free space

Data taken from

V. Flaminio, et al., CERN-HERA Report  
79-02 (1979).

$$\begin{aligned}\sigma_{\text{tot}}(E_{\text{c.m.}}) &= \int d\Omega \left( \frac{d\sigma}{d\Omega} \right)_{\text{c.m.}}^{\text{elem}} \\ &= 4\pi \tilde{\chi}^2 A_0(E_{\text{c.m.}})\end{aligned}$$



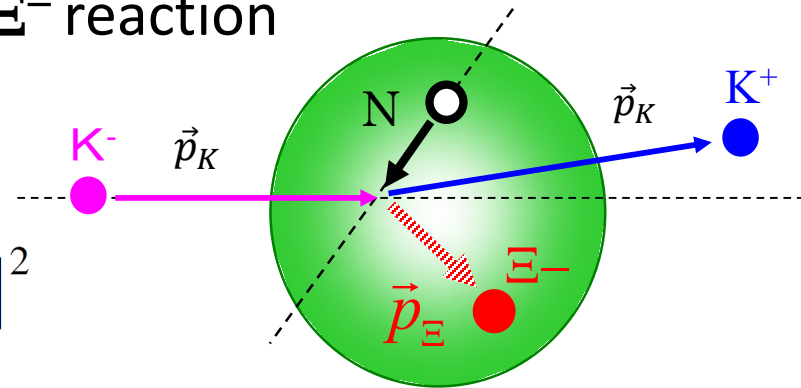
# Optimal Fermi-averaging (OFA) procedure

T. Harada and Y.Hirabayashi, NPA744 (2004) 323.

## ■ "Optimal" cross section for $K^-p \rightarrow K^+\Xi^-$ reaction

$$\left(\frac{d\sigma}{d\Omega}\right)_{\theta_{\text{lab}}}^{\text{opt}} \equiv \beta |\bar{f}_{K^-p \rightarrow K^+\Xi^-}|^2$$

$$= \frac{p_{K^+} E_{K^+}}{(2\pi)^2 v_{K^-}} |t_{\bar{K}N, K\Xi}^{\text{opt}}(p_{K^-}; \omega, \mathbf{q})|^2$$



## ■ Optimal Fermi-averaged $KN \rightarrow K\Xi$ $t$ -matrix

$$t_{\bar{K}N, K\Xi}^{\text{opt}}(p_{\bar{K}}; \omega, \mathbf{q})$$

Elementary  $t$ -matrix

$$= \frac{\int_0^\pi \sin \theta_N d\theta_N \int_0^\infty dp_N p_N^2 \rho(p_N) t_{\bar{K}N, K\Xi}(E_2; \mathbf{p}_{\bar{K}}, \mathbf{p}_N)}{\int_0^\pi \sin \theta_N d\theta_N \int_0^\infty dp_N p_N^2 \rho(p_N)} \Big|_{p_N = p_N^*}$$

*momentum dist.*

$$\cos \theta_N = \hat{\mathbf{p}}_{\bar{K}} \cdot \hat{\mathbf{p}}_N$$

## ■ On-energy-shell equation for a struck proton momentum: $\mathbf{p}_N^*$

$$\sqrt{(\mathbf{p}_N^* + \mathbf{q})^2 + m_\Xi^2} - \sqrt{(\mathbf{p}_N^*)^2 + m_N^2} = \omega \quad \text{including the binding effects.}$$

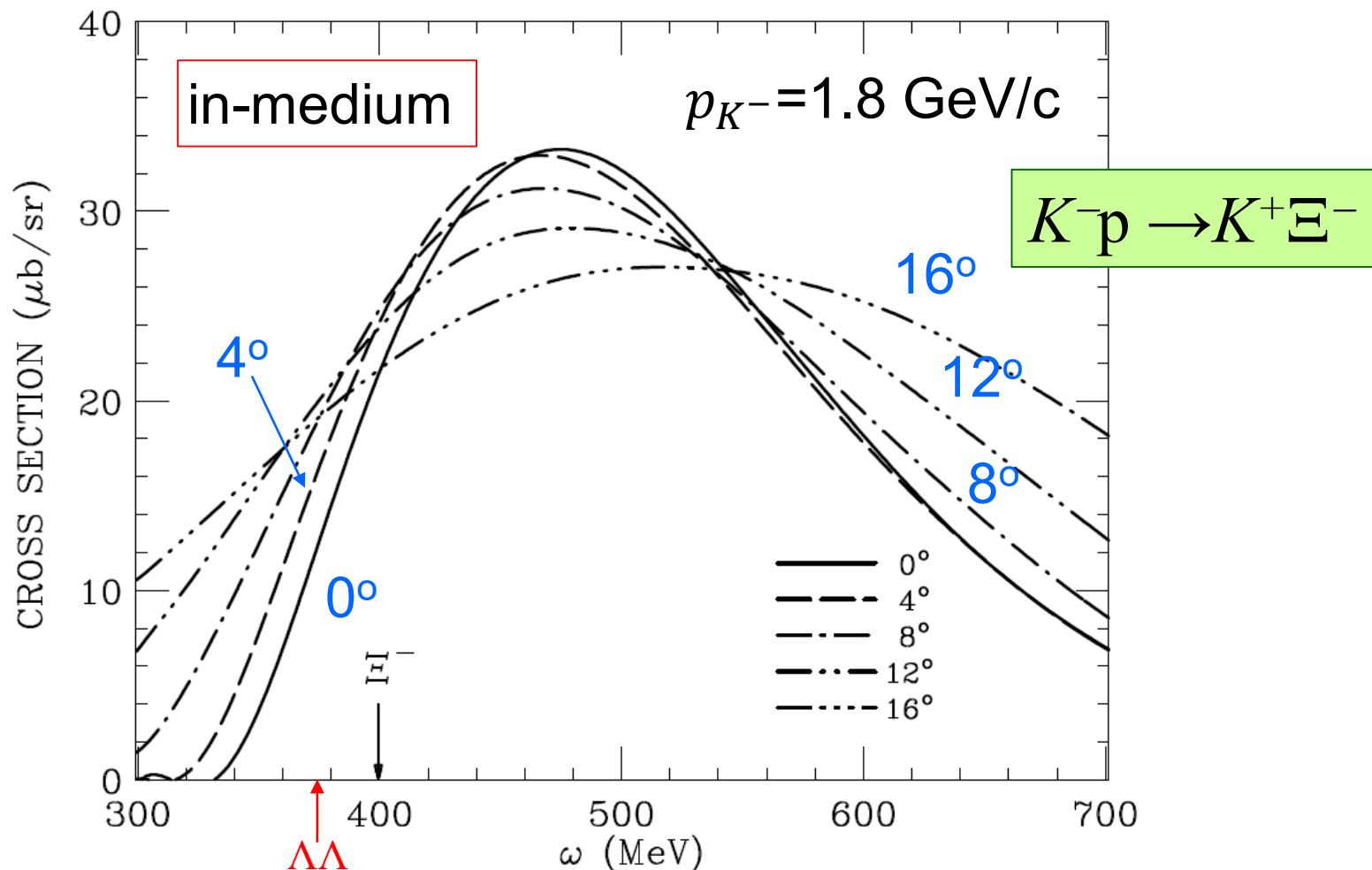
Optimal momentum approximation:  $\tau = t_a + t_a G_a h G_a t_a + t_a G_a h (G_a + G_a t_a G_a) h G_a t_a + \dots$

S. A. Gurvitz, PRC33 (1986) 422.

$h = G_a^{-1} - G^{-1}$  vanishes

# “Optimal” cross section for the $K^-p \rightarrow K^+\Xi^-$ reaction

$$(d\sigma/d\Omega)^{\text{opt}} \quad \left(\frac{d\sigma}{d\Omega}\right)_{\text{lab}}^{\text{opt}} \equiv \beta |\bar{f}_{K^-p \rightarrow K^+\Xi^-}|^2$$

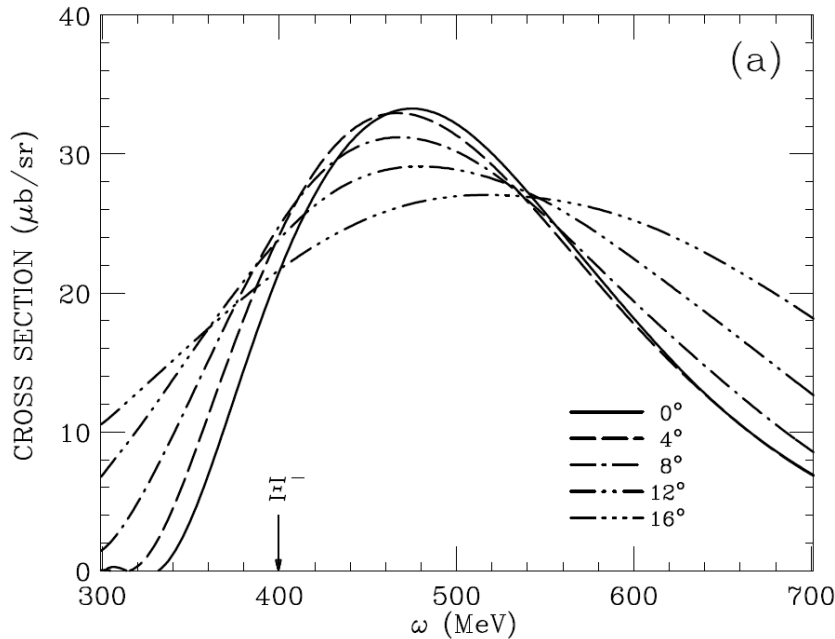


✓ Strong energy and angular dependencies of the cross sections.

# Energy dependence of *in-medium* $K^-p \rightarrow K^+\Xi^-$ cross sections

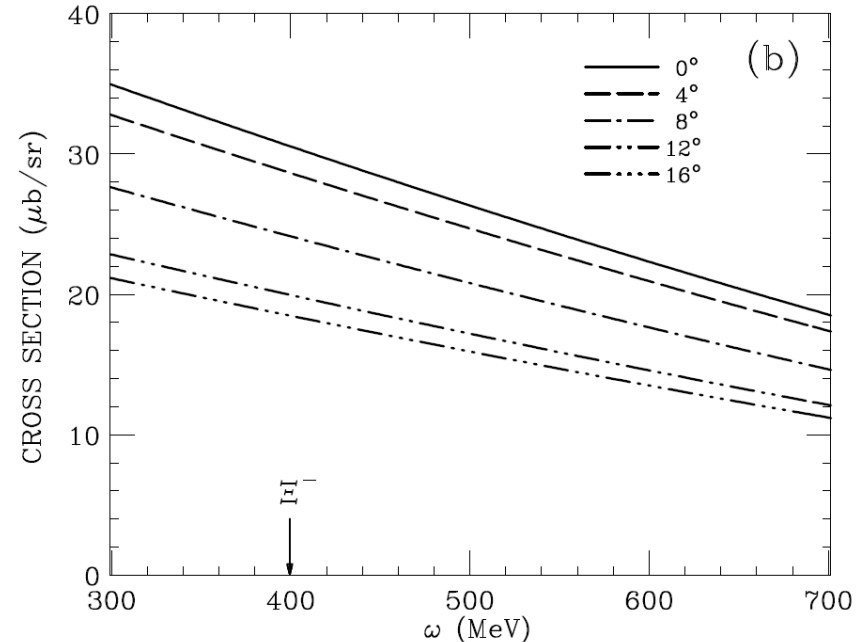
$$(d\sigma/d\Omega)^{\text{opt}} \quad p_{K^-} = 1.8 \text{ GeV}/c \quad \beta(d\sigma/d\Omega)^{\text{av}}$$

## Optimal Fermi averaging (OFA)



$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{lab}}^{\text{opt}} \equiv \beta |\bar{f}_{K^-p \rightarrow K^+\Xi^-}|^2$$

## Ordinary Fermi average in DWIA



$$\left(\frac{d\sigma}{d\Omega}\right)_{\theta_{\text{lab}}}^{\text{av}} = \int d\mathbf{p}_N \rho(p_N) \left(\frac{d\sigma}{d\Omega}\right)^{\text{elem}}$$

Const.

- ✓ The behavior of OFA is very different from that of the ordinary Fermi-averaging.

# Application of Optimal Fermi-averaging (OFA) procedure (1)

## $\Lambda$ production

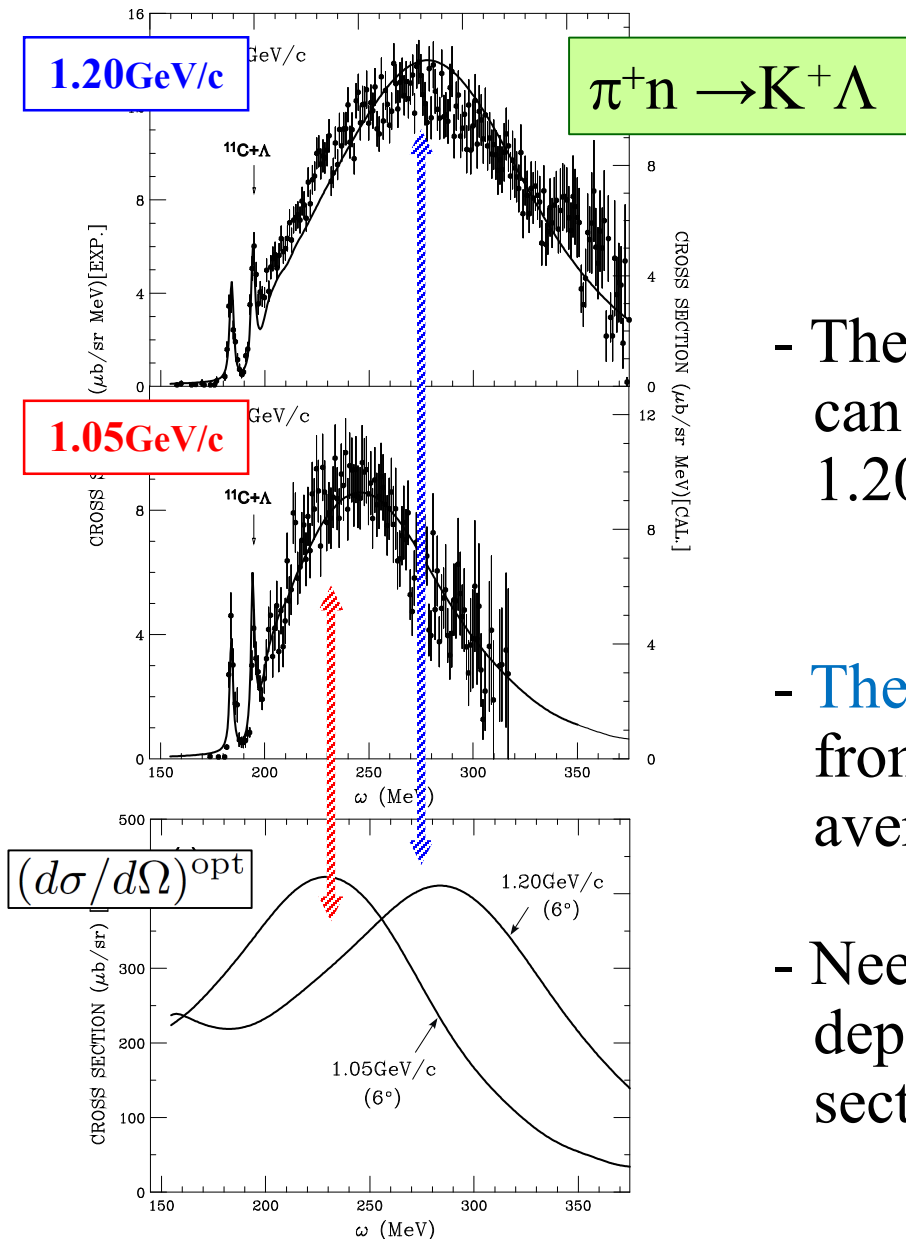
### $^{12}\text{C}(\pi^+, \text{K}^+)$ reactions

- The calculated spectra in the QF region can explain the experimental data at 1.20 and 1.05 GeV/c.

**This makes the width look narrow.**

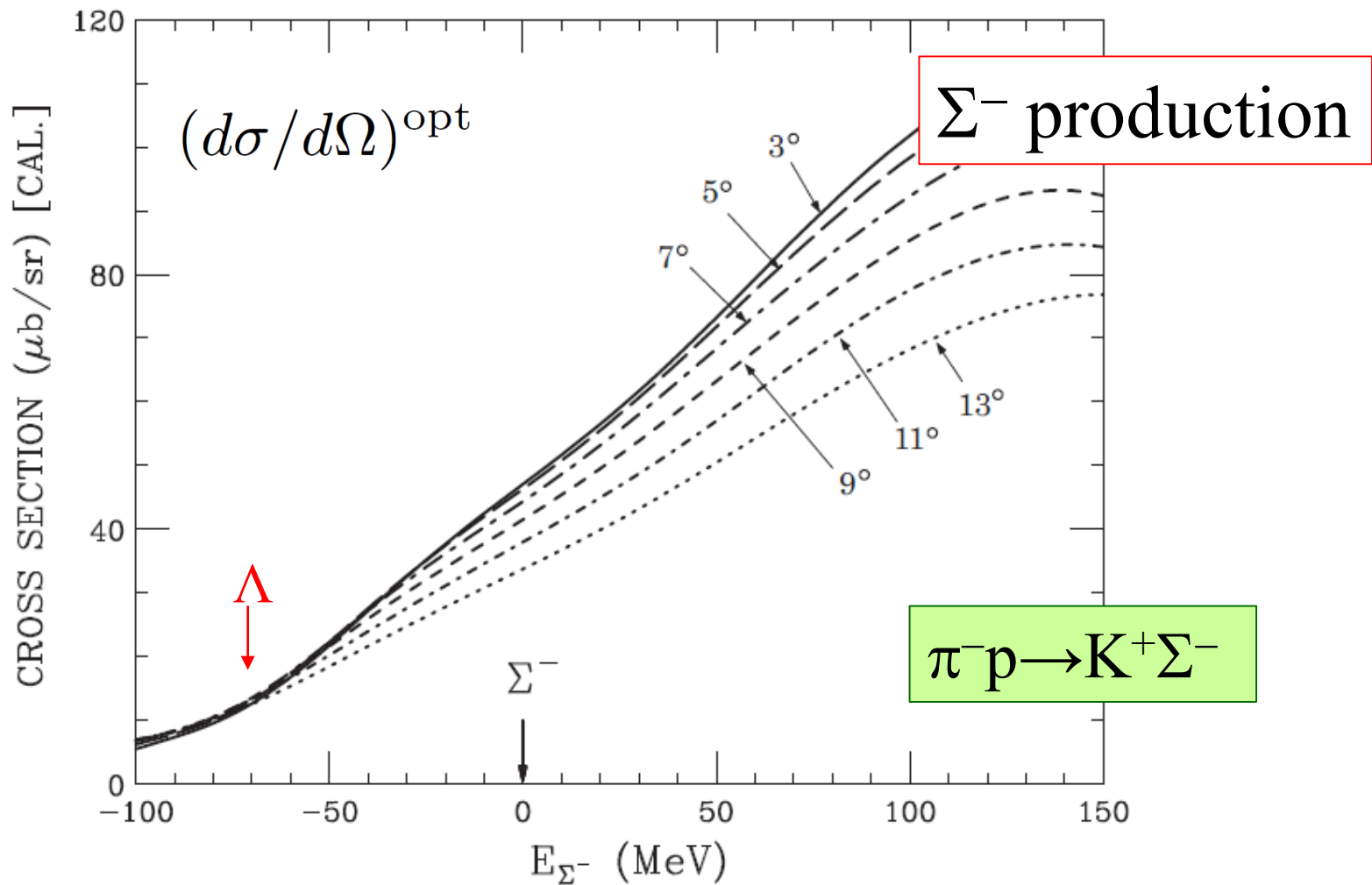
- The  $\omega$  energy-dependence originates from the nature of the “optimal Fermi-averaging” t-matrix.

- Need careful consideration for energy-dependent of the elementary cross section.



# Application of Optimal Fermi-averaging (OFA) procedure (2)

“ $\pi^-p \rightarrow K^+\Sigma^-$  reactions” on the nucleus ( $^{28}\text{Si}$ ,  $^{209}\text{Bi}$ ,  $^6\text{Li}$ )



- ✓ There exists a strong angular and energy dependence in the OFA amplitudes.

# Application of Optimal Fermi-averaging (OFA) procedure (3)

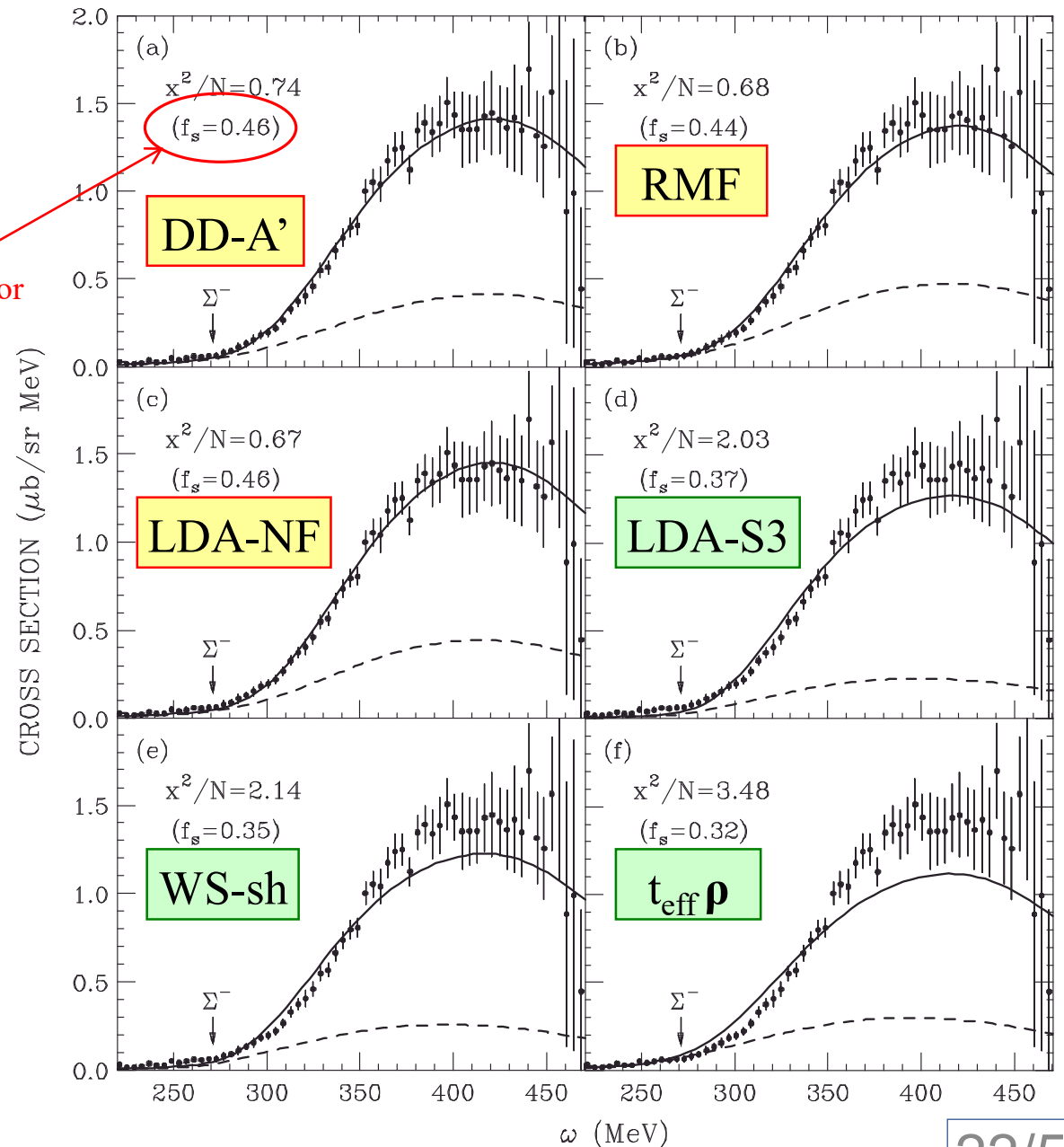
$^{28}\text{Si}(\pi^-, \text{K}^+)$  reaction  
at 1.2 GeV/c

$\Sigma^-$

- Calculated spectra with OFA can explain the data of the  $(\pi^-, \text{K}^+)$  spectra, by using the  $\Sigma$ -nucleus potentials for fits to the  $\Sigma^-$  atomic X-ray data.

T. Harada, Y. Hirabayashi,  
NPA759 (2005) 143

Normalization factor

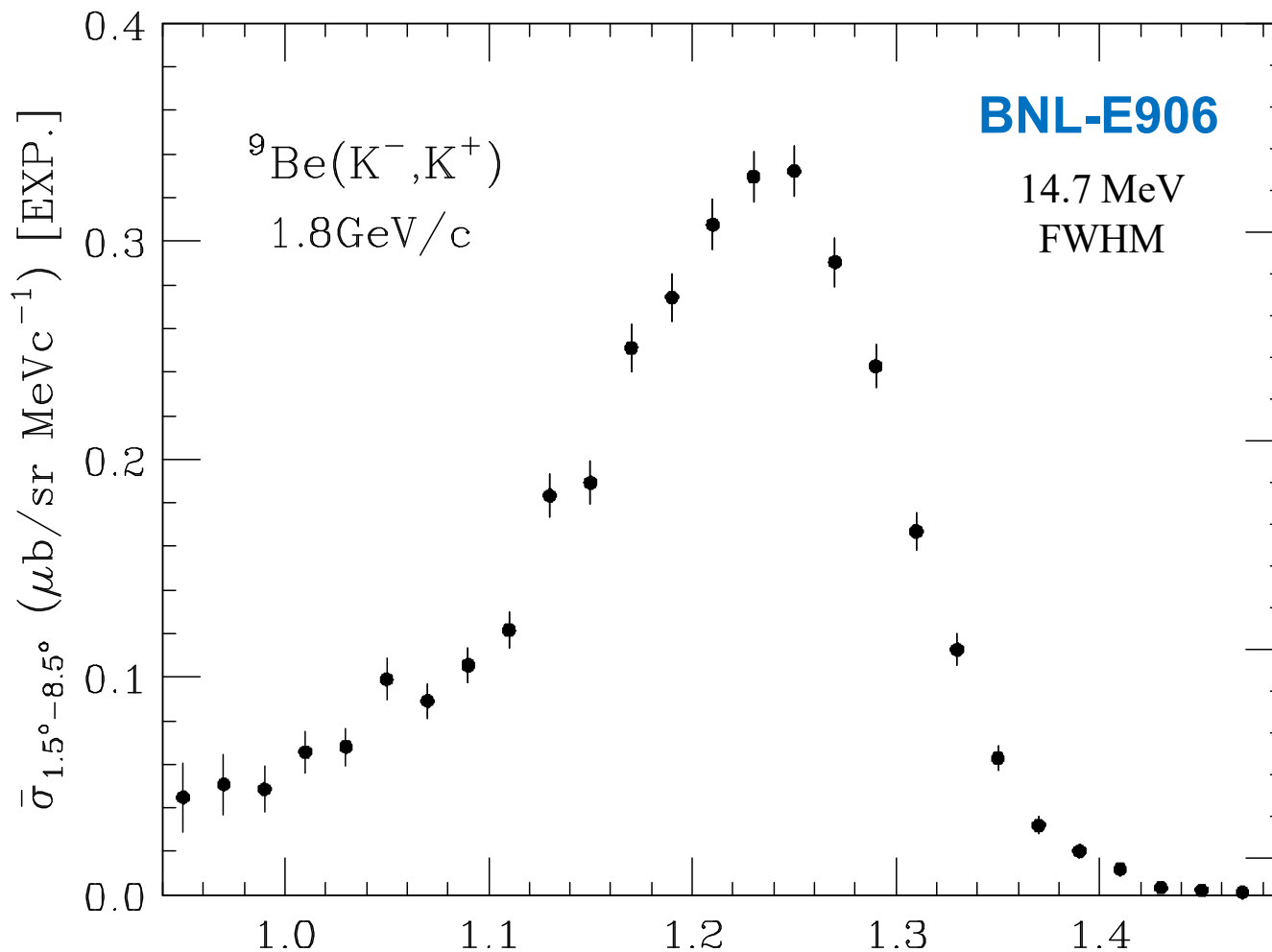


## 2.3

# Analysis of $\Xi^-$ QF spectra of the ${}^9\text{Be}(K^-, K^+)$ reaction at BNL-E906

# Quasifree $\Xi^-$ production in the ${}^9\text{Be}(K^-, K^+)$ reaction

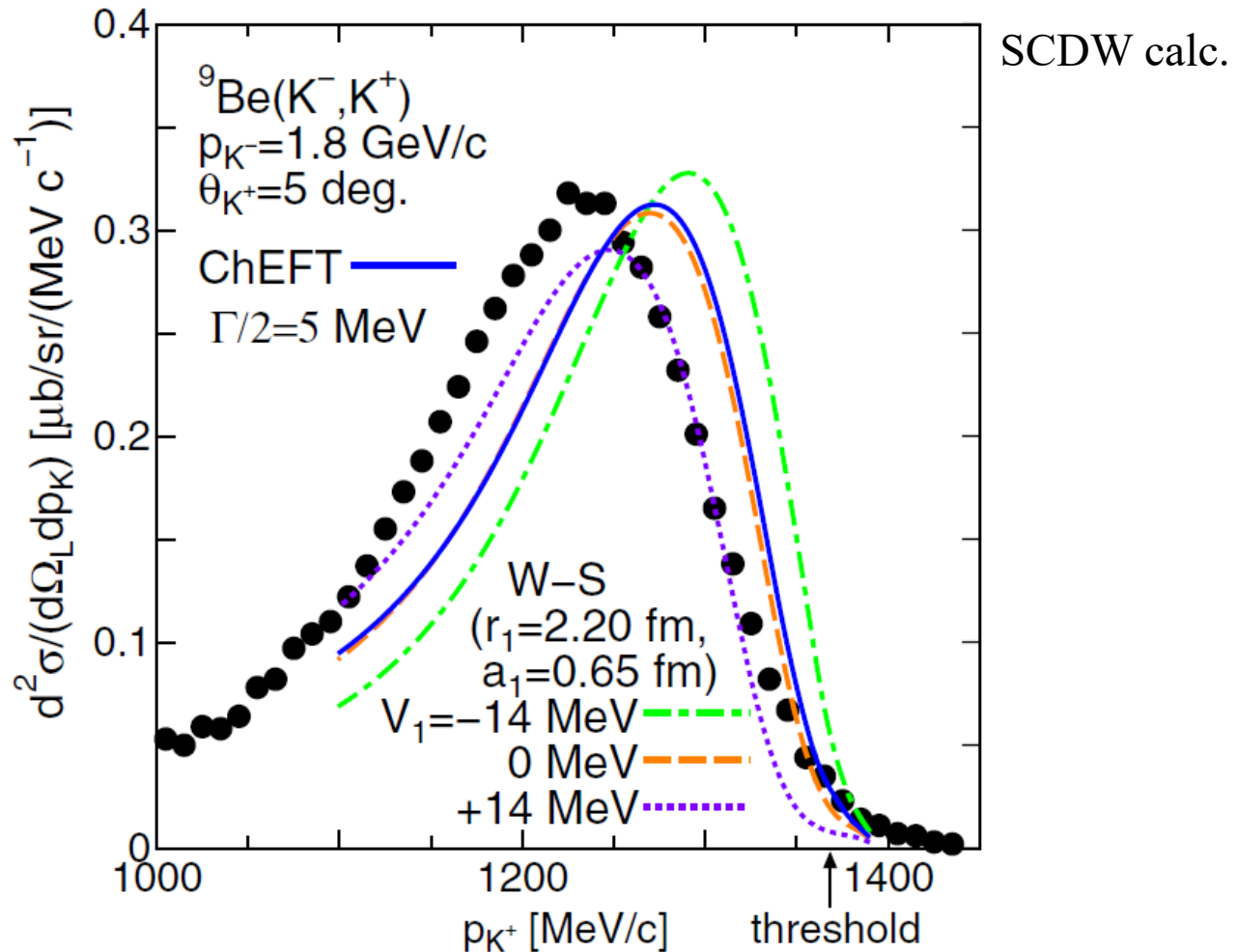
T. Tamagawa, Ph.D. thesis, Univ. of Tokyo, 2000 (unpublished).



Angular averaged cross sections  $p_{K^+}$  (GeV/c)

$$\bar{\sigma}_{1.5^\circ-8.5^\circ} \equiv \int_{\theta_{\text{lab}}=1.5^\circ}^{\theta_{\text{lab}}=8.5^\circ} \left( \frac{d^2\sigma}{dp_{K^+}d\Omega_{K^+}} \right) d\Omega \bigg/ \int_{\theta_{\text{lab}}=1.5^\circ}^{\theta_{\text{lab}}=8.5^\circ} d\Omega.$$

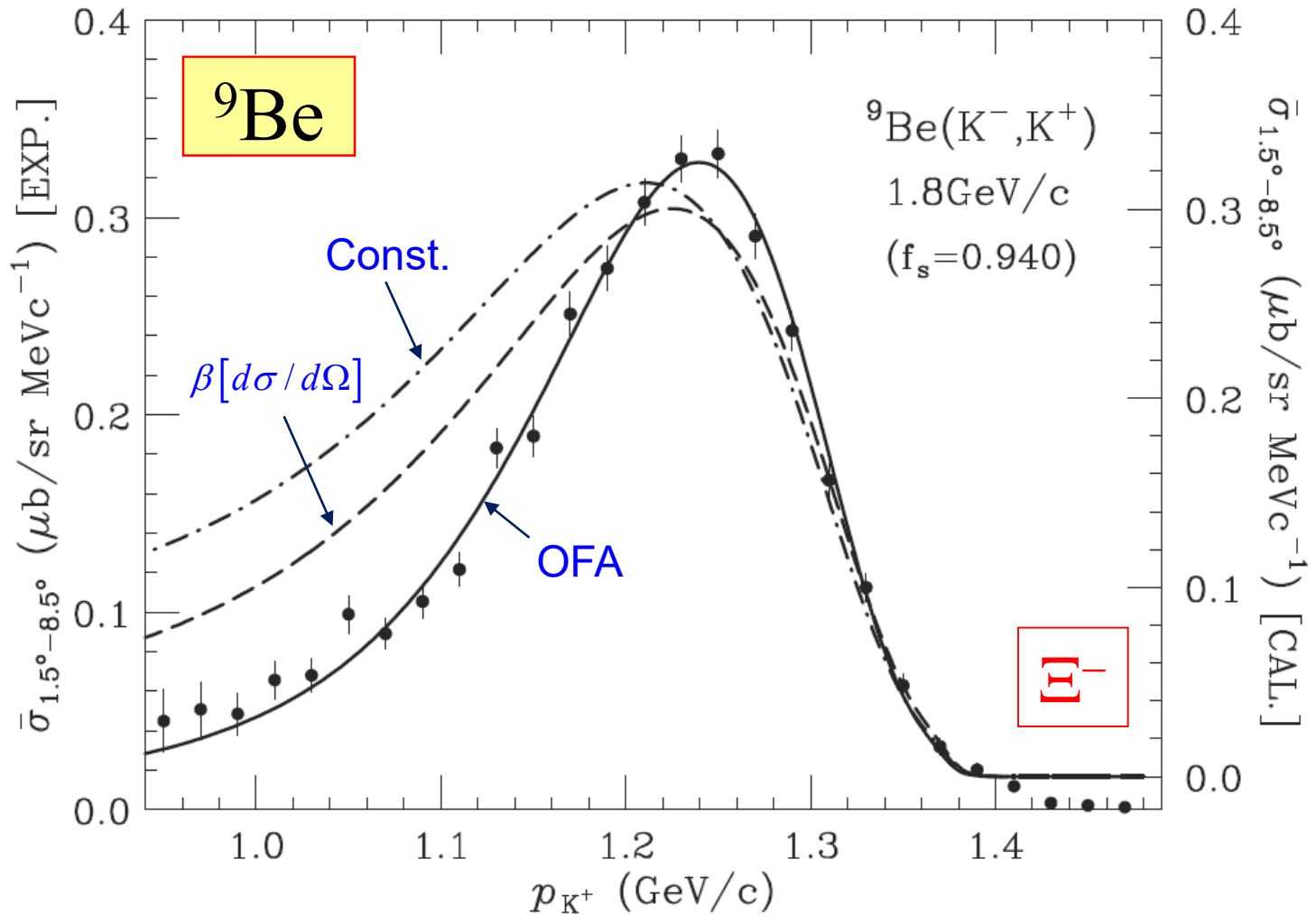




- ✓ It seems challenging to extract information on the  $\Xi$ -nucleus potential because these calculated spectra cannot sufficiently reproduce the data.

# Verification of the optimal Fermi-averaged $K^- p \rightarrow K^+ \Xi^-$ ampl.

Data: Tamagawa (BNL-E906 collaboration)



- ✓ This calculated spectrum with OFA improves to reproduce the data of the  ${}^9\text{Be}(K^-, K^+)$  reaction at BNL-E906.

# Remarks

- We show the strong energy and angular dependencies of the in-medium  $K^-p \rightarrow K^+\Xi^-$  production cross section, which are important to describe the shape and magnitude of the  $\Xi^-$  production spectrum in the  $(K^-, K^+)$  reaction on the nuclear target.

T. Harada and Y. Hirabayashi, PRC102 (2020) 024618.

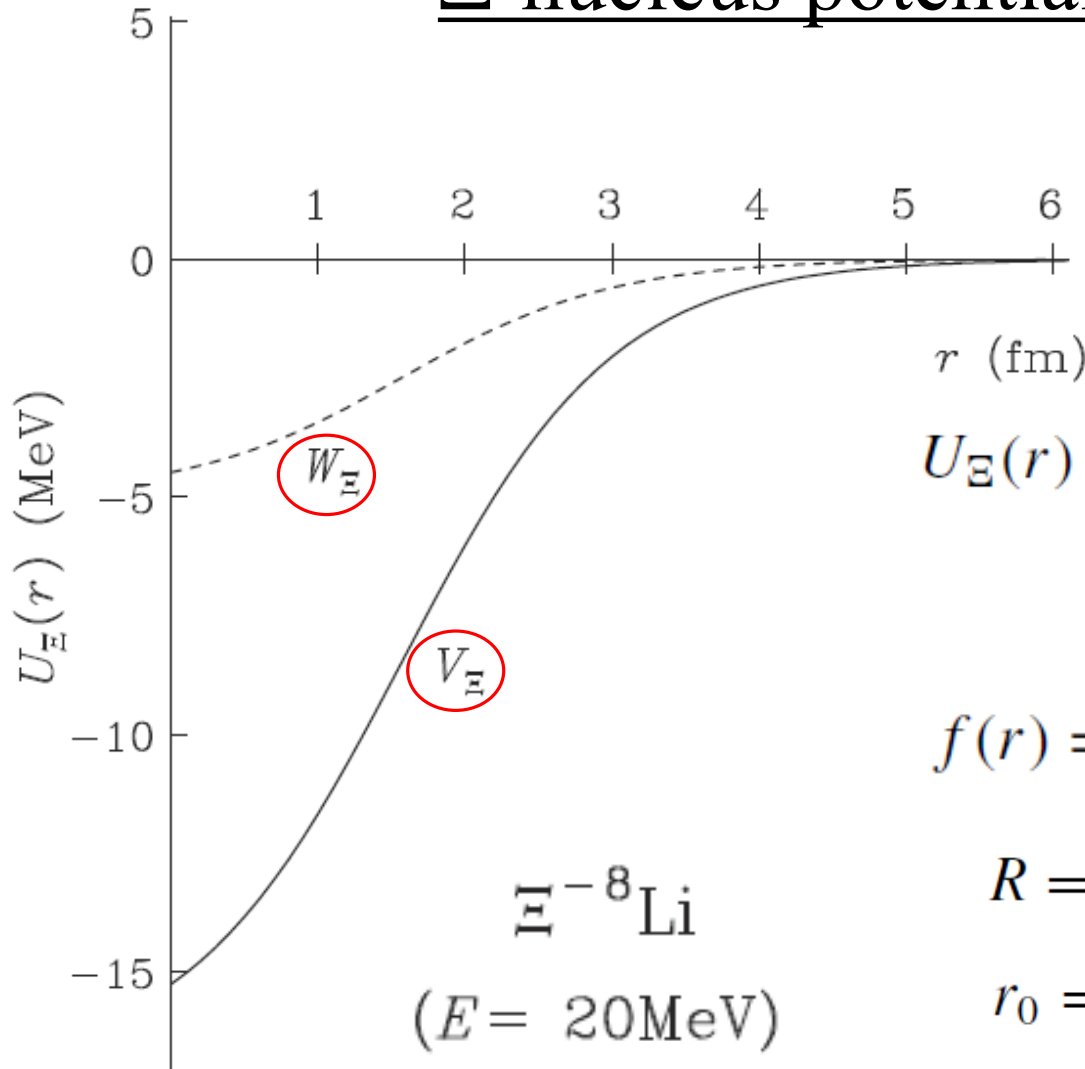
- This result may be a basis for the study extracting the properties of the  $\Xi$ -nucleus potential from the QF data.
- We expect that an analysis of  $\Xi^-$  QF spectrum produced via the  ${}^9\text{Be}(K^-, K^+)$  reaction at 1.8 GeV/c can **extract useful information on the  $\Xi$ -nucleus potential for  $\Xi^-$  - ${}^8\text{Li}$**  from the data of the BNL-E906 experiment.

## 2.4

# Properties of the $\Xi^-$ - $^8\text{Li}$ potential

# $\Xi$ -nucleus potential for $\Xi^-$ - $^8\text{Li}$

Woods-Saxon pot.



$$U_{\Xi}(r) = V_{\Xi}(r) + iW_{\Xi}(E, r)$$

$$= [V_0^{\Xi} + iW_0^{\Xi}g(E)]f(r)$$

fitting parameters

$$f(r) = [1 + \exp \{(r - R)/a\}]^{-1}$$

$$R = r_0 A_{\text{core}}^{1/3} = 1.57 \text{ fm}$$

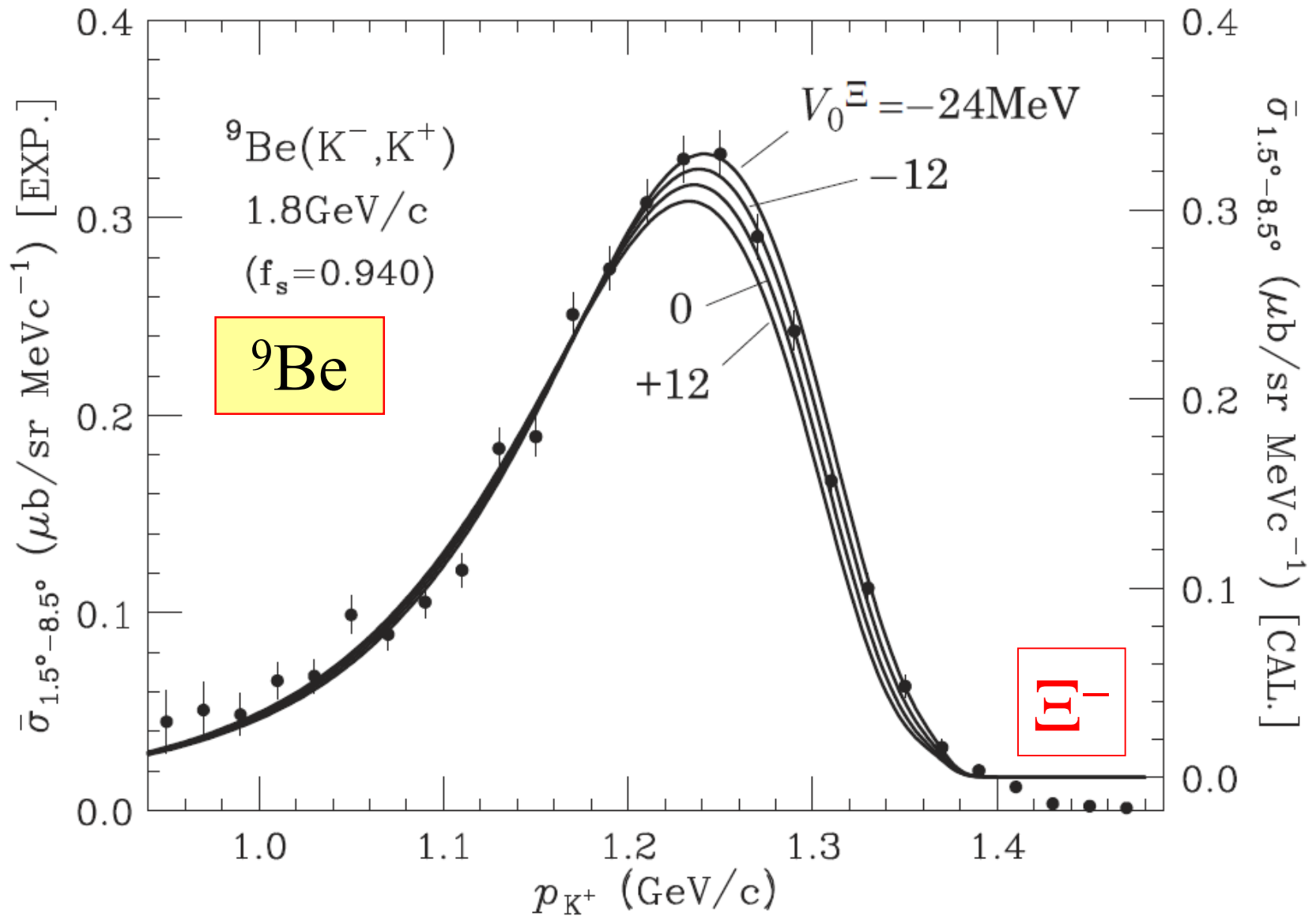
$$r_0 = 0.783 \text{ fm} \quad a = 0.722 \text{ fm}$$

$$\langle r^2 \rangle_V^{1/2} = \left[ \int r^2 V_{\Xi}(r) dr / \int V_{\Xi}(r) dr \right]^{1/2} = 2.81 \text{ fm}$$

$$V_{\text{so}}^{\Xi}(1/r)[df(r)/dr]\sigma \cdot \mathbf{L}$$

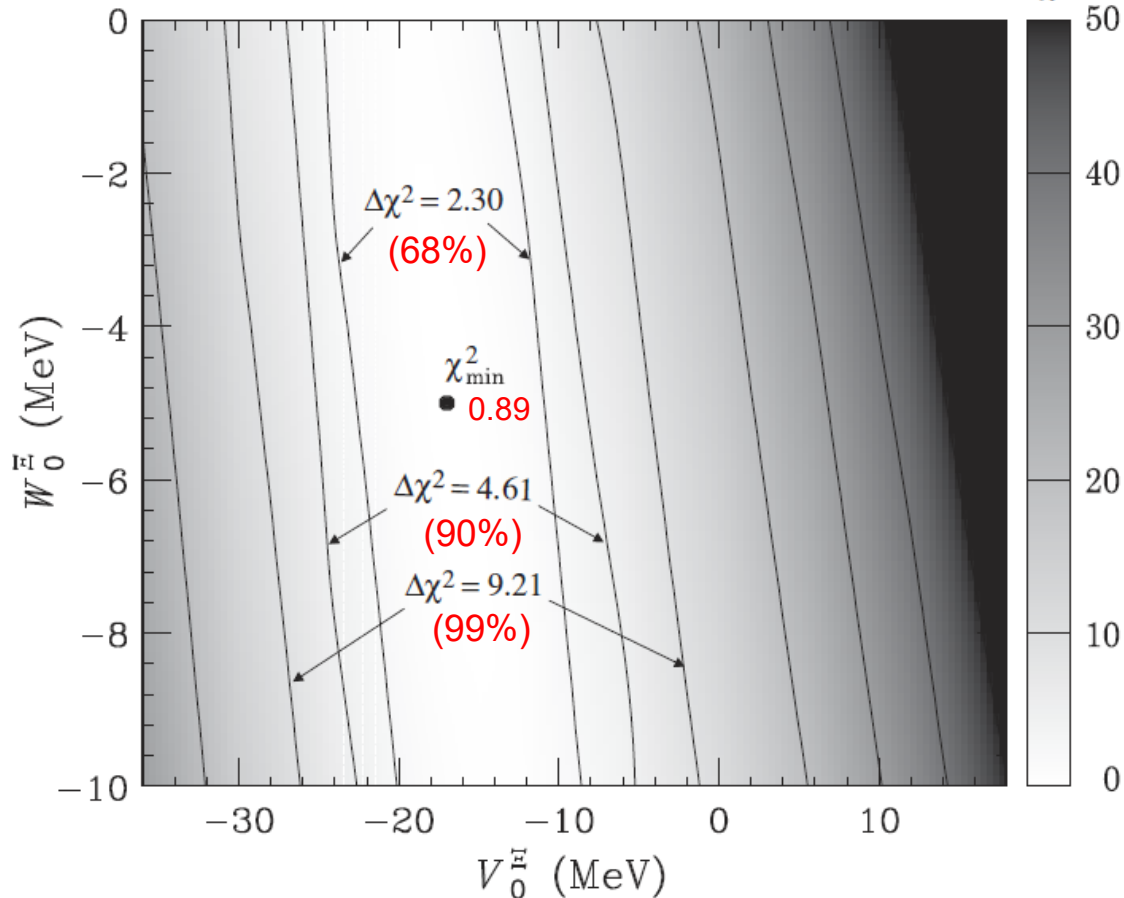
$$V_{\text{so}}^{\Xi} \simeq \frac{1}{10} V_{\text{so}}^N \simeq 2 \text{ MeV}$$

# Effects of the real part of the $\Xi$ -nucleus potential



# Contour plots of the $\chi^2$ -value distribution in $\{V_0, W_0\}$ plane

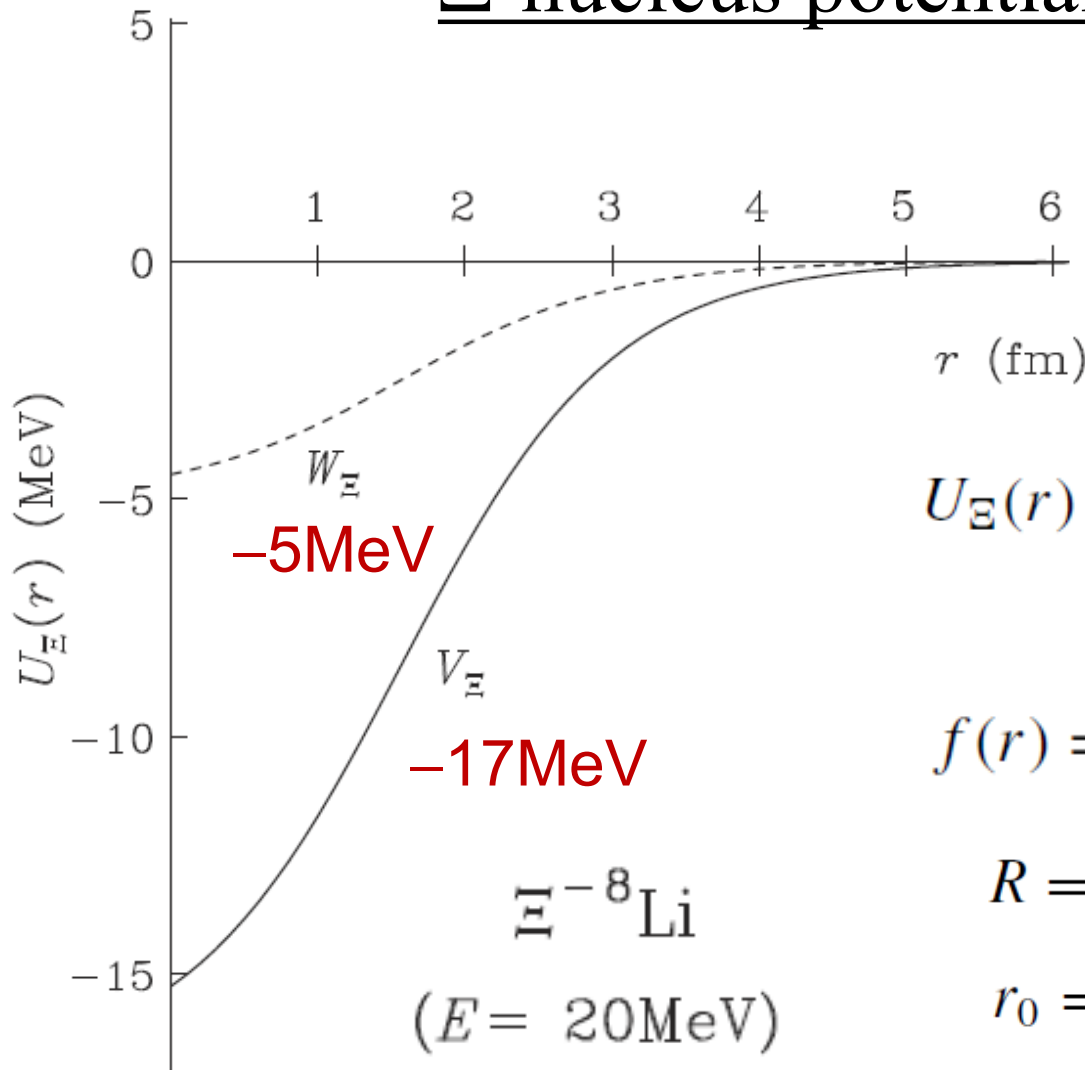
$$U_{\Xi}(r) = (V_0 + iW_0) / [1 + \exp\{(r - R)/a\}]$$



- ✓ The minimum position of  $\chi_{\min}^2/N = 15.2/17 = 0.89$ , and  $\Delta\chi^2 = 2.30, 4.61$ , and  $9.21$  correspond to 68%, 90%, and 99% confidence levels for two parameters, respectively.
- ✓ The value of  $\chi^2$  is almost insensitive to  $W_0$ .

# $\Xi$ -nucleus potential for $\Xi^-$ - $^8\text{Li}$

Woods-Saxon pot.



$$U_{\Xi}(r) = V_{\Xi}(r) + iW_{\Xi}(E, r)$$

$$= [V_0^{\Xi} + iW_0^{\Xi}g(E)]f(r)$$

$$f(r) = [1 + \exp \{(r - R)/a\}]^{-1}$$

$$R = r_0 A_{\text{core}}^{1/3} = 1.57 \text{ fm}$$

$$r_0 = 0.783 \text{ fm} \quad a = 0.722 \text{ fm}$$

$$\langle r^2 \rangle_V^{1/2} = \left[ \int r^2 V_{\Xi}(r) dr / \int V_{\Xi}(r) dr \right]^{1/2} = 2.81 \text{ fm}$$

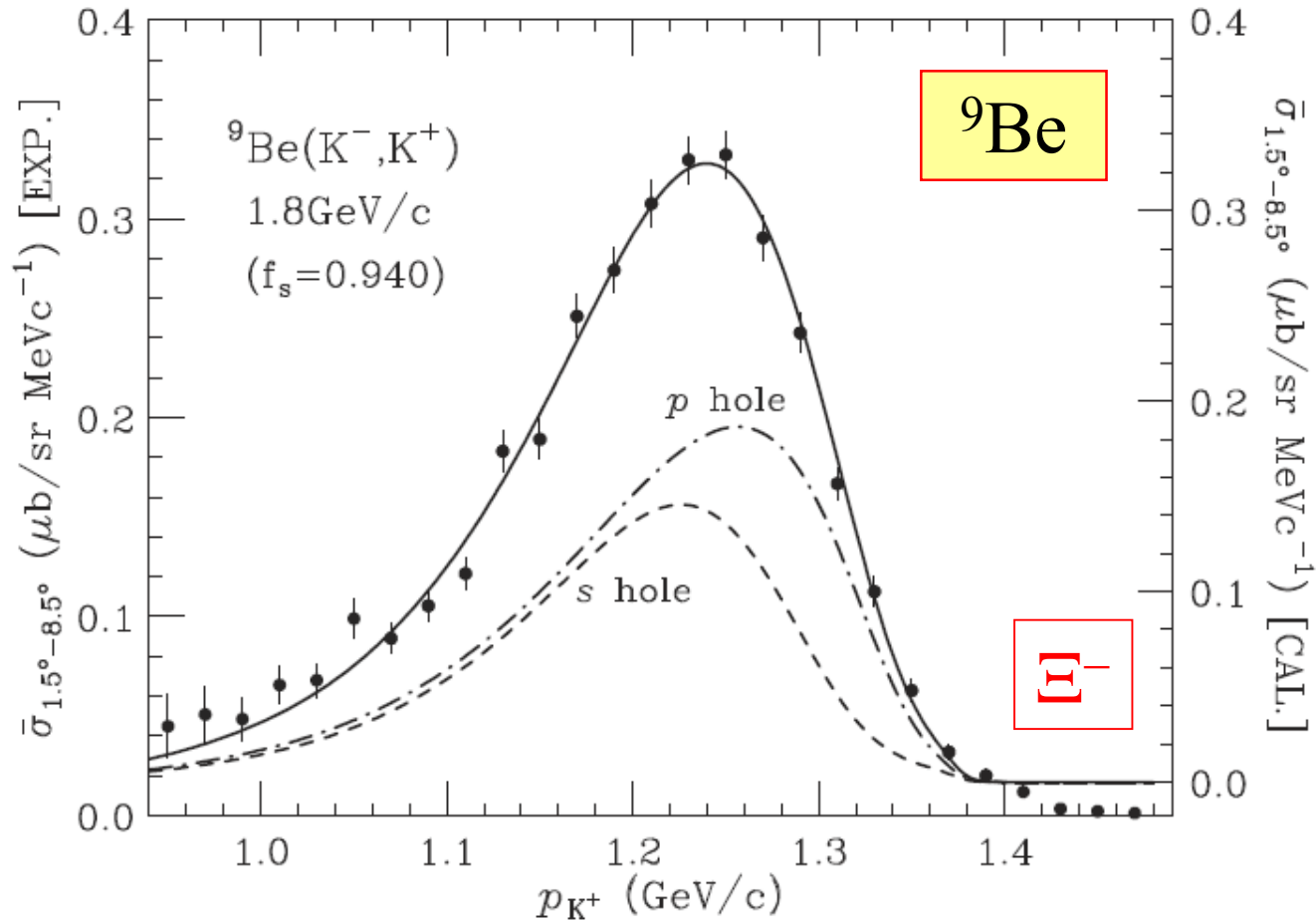
$$V_{\text{so}}^{\Xi}(1/r)[df(r)/dr]\sigma \cdot \mathbf{L}$$

$$V_{\text{so}}^{\Xi} \simeq \frac{1}{10} V_{\text{so}}^N \simeq 2 \text{ MeV}$$



# $\Xi^-$ QF spectrum by ${}^9\text{Be}(K^-,K^+)$ reaction at 1.8 GeV/c

Data: Tamagawa (BNL-E906 collaboration)



$$V_0^{\Xi} = -17 \text{ MeV}, W_0^{\Xi} = -5 \text{ MeV}$$

Reduced  $\chi^2 = 15.2/17$

✓ The calculated spectrum can explain the data very well.

# Validity of the imaginary part of the $\Xi$ -nucleus potential

## ■ The first-order optical potential ( $t\rho$ )

$$U_{\Xi}^{(1)}(r) = t_{\Xi-p}\rho_p(r) + t_{\Xi-n}\rho_n(r)$$

## ■ The imaginary part of $U_{\Xi}^{(1)}$

The optical theorem

$$4\pi \text{Im}f_{\Xi N}(0) = k_{\Xi}\sigma_{\text{tot}}$$

$$W_{\Xi}^{(1)}(r) = -\langle v_{\Xi-p}\sigma(\Xi^-p \rightarrow \Xi^0n, \Lambda\Lambda) \rangle \rho_p(r)/2$$

Gal, Toker, Alexander, Ann. Phys. **137** (1981) 341.

### - In-medium cross section

(Pauli correction + C.M. correction + Fermi-averaging)

$$\langle v_{\Xi-p}\sigma(\Xi^-p \rightarrow \Xi^0p, \Lambda\Lambda) \rangle \quad \langle v_{\Xi-p}\sigma(\Xi^-p \rightarrow \Lambda\Lambda) \rangle$$

## ■ $\Xi^-p$ reaction cross sections

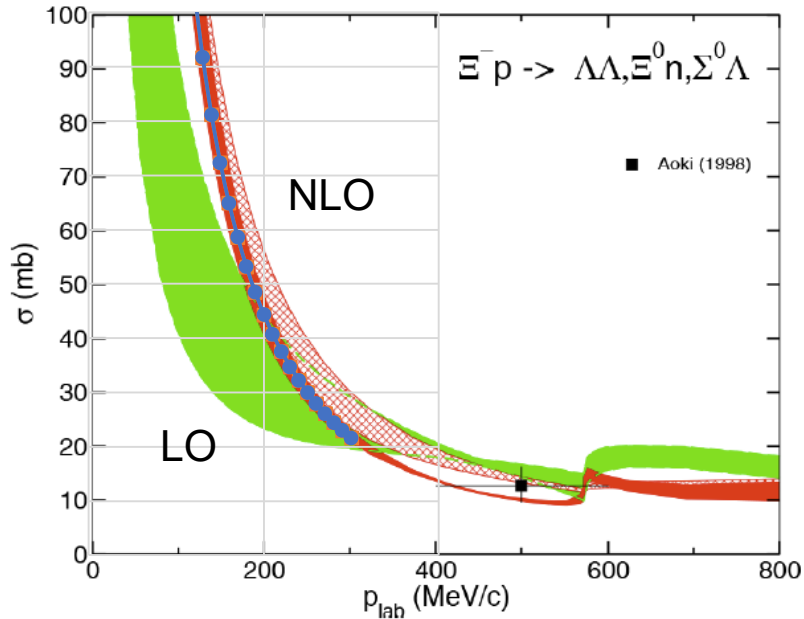
$$v_{\Xi-p}\sigma = (v_{\Xi-p}\sigma)_0 / (1 + \alpha v)$$

velocity:  $v$  parameters:  $(v_{\Xi-p}\sigma)_0, \alpha$

# $\Xi^- p$ reaction cross sections in ChiEFT

## $\Xi^- p \rightarrow \Xi^0 p, \Lambda\Lambda$ reactions

Haidenbauer and Meiner, EPJ. A **55**, 23 (2019).



$$\square (v_{\Xi^- p} \sigma)_0 = 25 \text{ mb}, \quad \alpha = 18$$

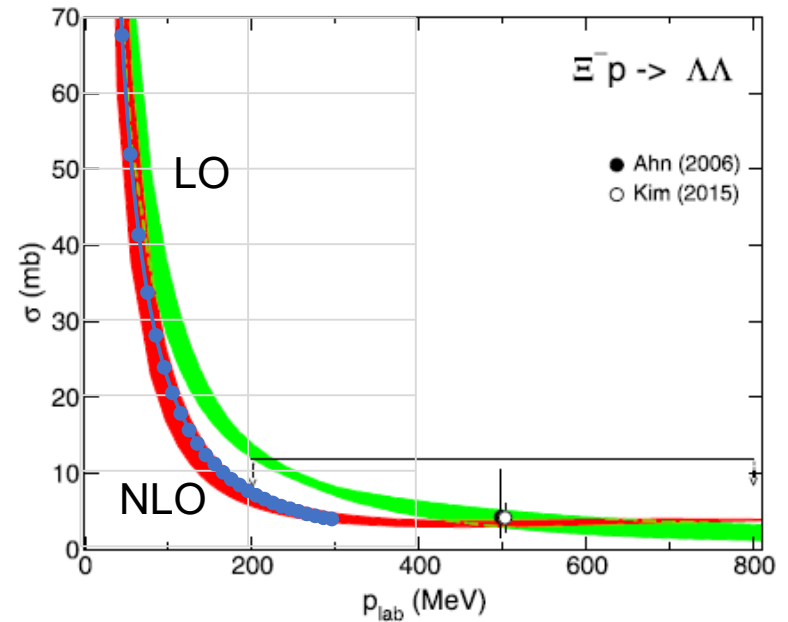
$$\langle v_{\Xi^- p} \sigma \rangle = 7.02 \text{ mb}$$

$$\text{Im}b = \mu \langle v \sigma \rangle / 8\pi = 0.078 \text{ fm}$$

$$W_0 = -6.2 \text{ MeV}$$

## $\Xi^- p \rightarrow \Lambda\Lambda$ reactions

Haidenbauer et al, NPA **954**, 273 (2016).



$$\square (v_{\Xi^- p} \sigma)_0 = 4.5 \text{ mb}, \quad \alpha = 20$$

$$\text{Im}b = 0.018 \text{ fm}$$

$$W_0 = -1.5 \text{ MeV}$$

## Remarks

- We have studied phenomenologically the  $\Xi^-$  production spectrum of the  ${}^9\text{Be}(\text{K}^-, \text{K}^+)$  reaction at 1.8 GeV/c within the DWIA using the optimal Fermi-averaged  $\text{K}^- \text{p} \rightarrow \text{K}^+ \Xi^-$  amplitude.
- The weak attraction in the  $\Xi$ -nucleus potential for  $\Xi^-$ - ${}^8\text{Li}$  provides the ability to explain the BNL-E906 data, consistent with analyses for previous experiments:

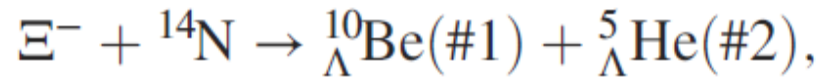
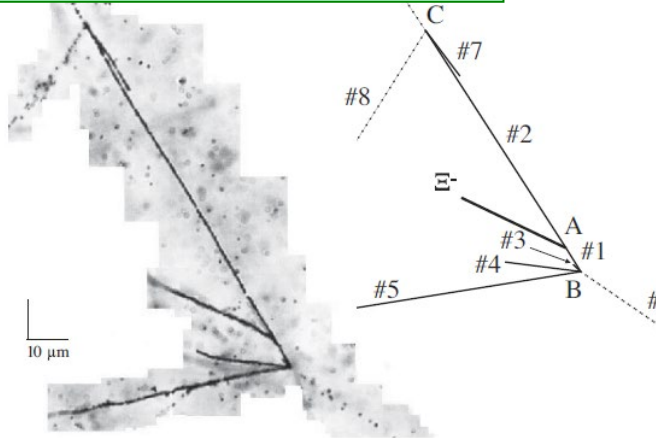
$$V_0 = -17 \pm 6 \text{ MeV} \quad \text{for } W_0 = -5 \text{ MeV}$$

It is difficult to determine the value of  $W_0$  from the data due to the insufficient resolution of 14.7 MeV FWHM.

T. Harada and Y. Hirabayashi, PRC103 (2021) 024605.

# 3. Properties of $\Xi$ -nucleus potentials

## IBUKI (J-PARC E07)



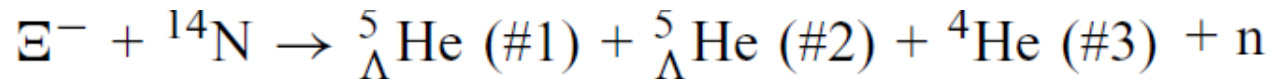
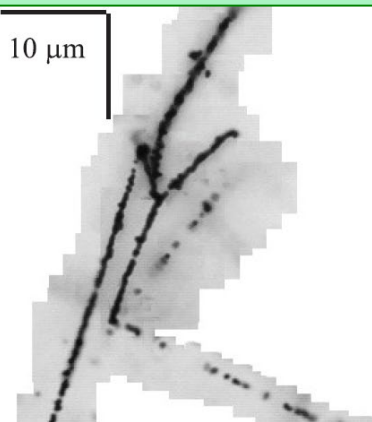
$$B_{\Xi^-} = 1.27 \pm 0.21 \text{ MeV}$$

Hayakawa, et al., PRL **126**, 062501 (2021).

Event	Target	Decay mode	$B_{\Xi^-}$ [MeV]	
KISO [9,10]	${}^{14}\text{N}$	${}_{\Lambda}^{10}\text{Be}$	${}_{\Lambda}^5\text{He}$	$3.87 \pm 0.21$
	${}^{14}\text{N}$	${}_{\Lambda}^{10}\text{Be}^*$	${}_{\Lambda}^5\text{He}$	$1.03 \pm 0.18$
IBUKI (present data)	${}^{14}\text{N}$	${}_{\Lambda}^{10}\text{Be}$	${}_{\Lambda}^5\text{He}$	$1.27 \pm 0.21$

$B_{\Xi^-}(2p) = 1.03 \pm 0.18 \text{ MeV}$  (KISO) and  $1.27 \pm 0.21 \text{ MeV}$  (IBUKI), which suggested to form a Coulomb-assisted nuclear  $2p$  bound state for  $\Xi^-$ .

## IRRAWADDY (J-PARC E07)



$$B_{\Xi^-} = 6.27 \pm 0.27 \text{ MeV}$$

Yoshimoto, et al., PTEP **2021**, 073D02(2021).

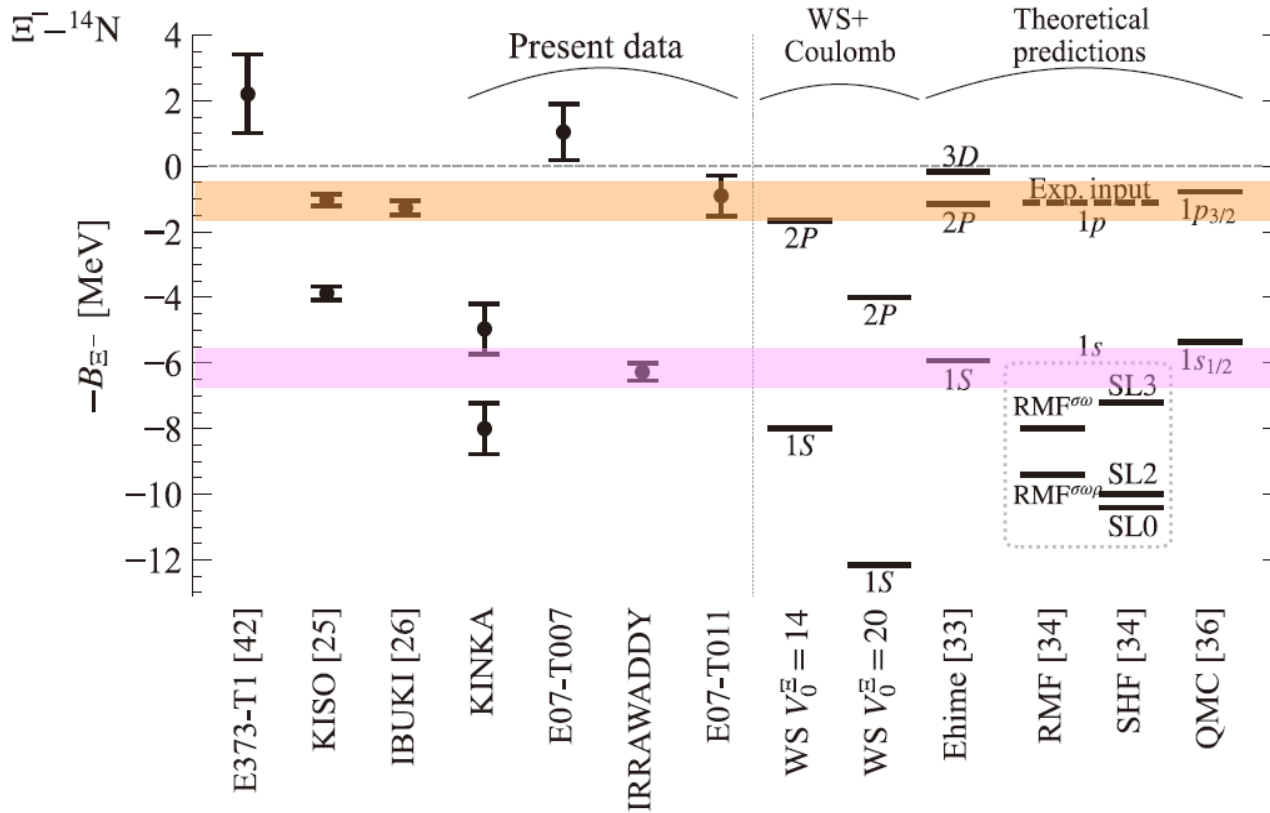
New events give the first indication of the nuclear  $1s$  state of  $\Xi^-{}^{14}\text{N}$ , and suggest that the  $\Xi\text{N}-\Lambda\Lambda$  coupling is weak.

# Recent observations of $\Xi^-$ hypernuclei from emulsion

compared with theoretical predictions



M. Yoshimoto, Prog. Theor. Exp. Phys. **2021**, 073D02.



2P capture

1s?

Coulomb only in  $\Xi^-{}^{14}\text{N}$ :  
 $B_{\Xi}(2P) = 0.39$  MeV  
 $B_{\Xi}(1s) = 1.21$  MeV

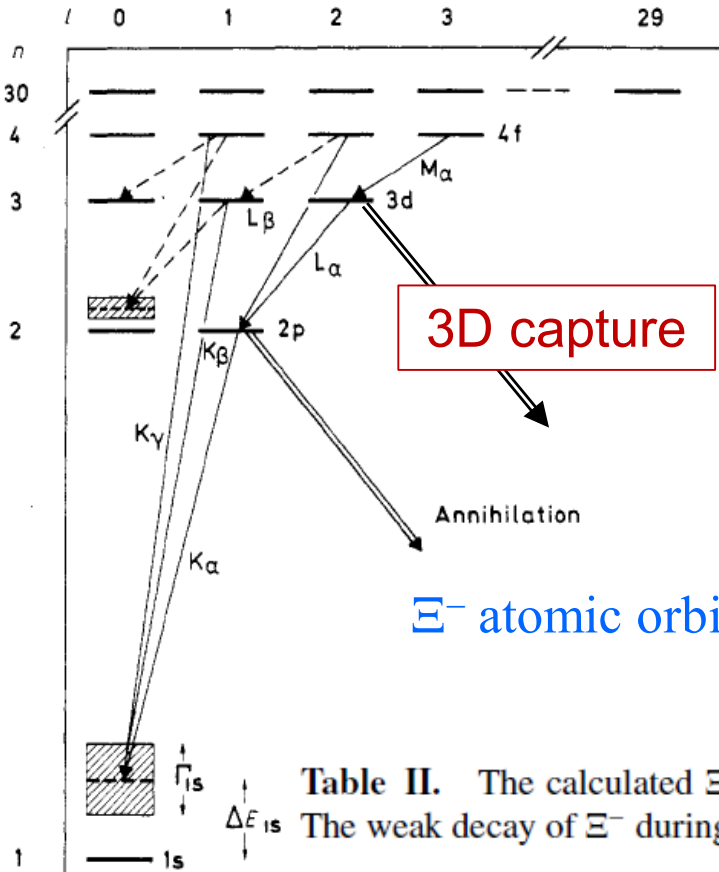
- ✓  $\Xi^-$  capture from the 2P state:  $B_{\Xi}(2P) = 1.03$  MeV(KISO)-1.27 MeV (IBUKI)
- The  $\Xi$ -nucleus potential is attractive in the real part.
- The 2P capture rate (4%) obtained from cascade cal.
- $\Xi\text{N}-\Lambda\Lambda$  coupling is weak (consistent with HAL-QCD).

# $\Xi^-$ absorption cascade process in $\Xi^-$ - $^{14}\text{N}$ atom

D. Zhu, et al., PRL 67 (1991) 2268.

TABLE I. Calculated  $\Xi^-$  capture probabilities (in %) from  $s$ ,  $p$ ,  $d$ , and  $f$  atomic orbits,<sup>a</sup> for various choices of  $\Xi^-$ -nucleus real and imaginary potential-well depths<sup>b</sup>  $V_0$  and  $W_0$ .

Target	$V_0$ (MeV)	$W_0$ (MeV)	$s$	$p$	$d$	$f$
$^6\text{Li}$	23.7	6.4	0.04	18.6	80.6	...
	23.7	3.2	0.04	30.3	68.9	...
	0	3.2	0.07	35.7	63.4	...
$^{14}\text{N}$	28.6	7.7	0.00	0.2	54.1	45.6
	28.6	3.9	0.03	0.4	69.9	29.6
	0	3.9	0.03	1.3	75.7	22.9



$\Xi^-$  atomic orbits  $(nl)_{\Xi}$  with  $l_{\Xi} = l_p$  due to  $\Xi^- p \rightarrow \Lambda \Lambda$   $^1S_0$  channel.

T. Koike, JPS Conf. Proc. 17, 033011 (2017).

Table II. The calculated  $\Xi^-$  nuclear absorption probability (in % per stopped  $\Xi^-$ ) from each atomic state. The weak decay of  $\Xi^-$  during the cascade is 8.2% for any potentials.

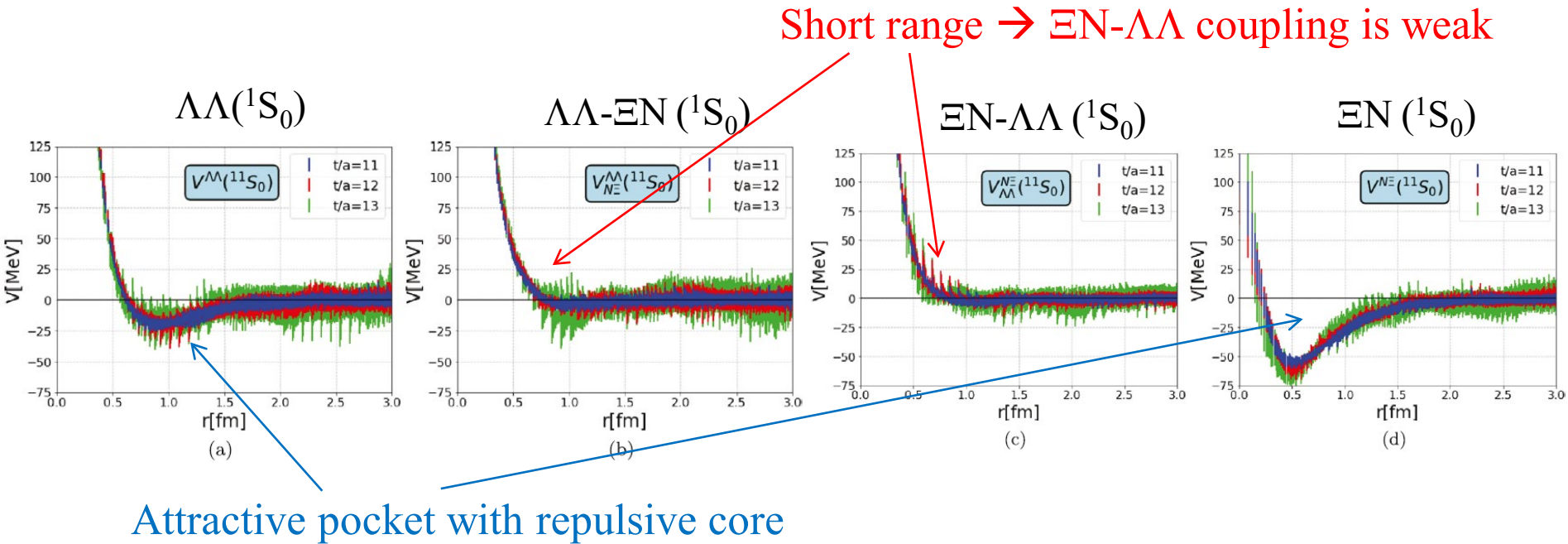
state	ND	ESC04d	ESC08c	ESC08c-A	state	ND	ESC04d	ESC08c	ESC08c-A
1s	0.02	$3 \times 10^{-4}$	$1 \times 10^{-4}$	0.01	total s	0.07	0.04	0.06	0.06
2p	3.9	0.25	3.8	2.4	total p	5.7	0.88	5.7	4.0
3d	35.7	23.5	34.7	34.9	total d	67.1	47.9	65.3	65.3
4f	7.8	19.0	8.6	9.4	total f	18.9	42.9	20.8	22.5
5g	0.01	0.03	0.01	0.01	total g	0.03	0.10	0.04	0.04

$\Xi^-$ - $^{14}\text{N}$  absorption: 3D ~ 35%, 2P ~ 4%, 1S < 0.1%



# $\Lambda\Lambda$ and $\Xi N$ interactions from lattice QCD near the physical point

K. Sasaki et al.(HAL QCD Collaboration), NP998 (2020) 121737.

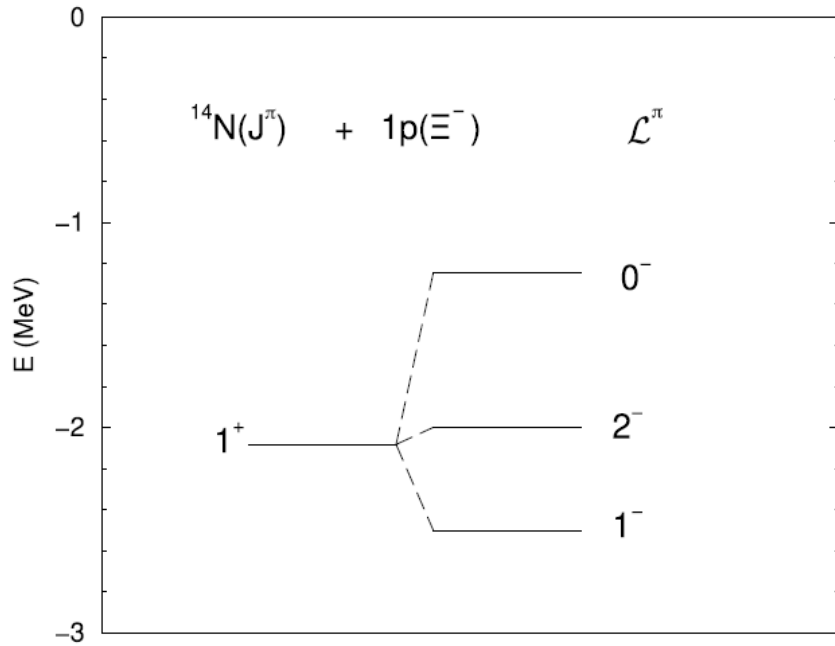


which implies that the leading-order truncation of the derivative expansion is reasonable. The diagonal potentials,  $V^{\Lambda\Lambda}$  and  $V^{N\Xi}$  in Fig. 1 (a, d), have attractive pocket with a long-range tail together with a short-range repulsive core. From the meson exchange picture, the one-pion exchange is allowed only in  $N\Xi$ - $N\Xi$  channel. One interesting feature is that the overall attraction in  $V^{N\Xi}$  is substantially larger than that in  $V^{\Lambda\Lambda}$ . The off-diagonal potentials shown in Fig. 1 (b, c) are found to be non-zero only at short distance, which suggests that the  $\Lambda\Lambda$ - $N\Xi$  coupling is weak at low energies.

# Remarks

- KEK-E224 and BNL-E885: Fukuda et al., (1998)  
Khaustov et al. (2000)  
 $-V_0^{\Xi} = 14 \text{ MeV}, < 20 \text{ MeV}$
- BNL-E906:  
 $-V_0^{\Xi} = 17 \pm 6 \text{ MeV}$  Harada-Hirabayashi (2021)
- Density dependence of  $V_0^{\Xi}(\rho_0)$  :  
 $-V_0^{\Xi}(\rho_0) = 21.9 \pm 0.7 \text{ MeV}$  Friedman-Gal (2021)
- Microscopic calculations +  $\Xi\text{N}$  G-matrix  
Lattice QCD, ChEFT:  $-V_0^{\Xi} < 10 \text{ MeV}$   
Ehime, NHD, NSC08, NSC16, ...
- Contributions of  $\Xi\text{N} \rightarrow \Lambda\Lambda$  coupling and  $\Xi\text{NN}$  force  
weak (from HAL-QCD)

# Interpretation of $\Xi^-$ binding energies $B_{\Xi}$ for $\Xi^-$ - $^{14}\text{N}$



Friedman, Gal, PLB 820 (2021) 136555.

Residual  $\Xi\text{N}$  interaction

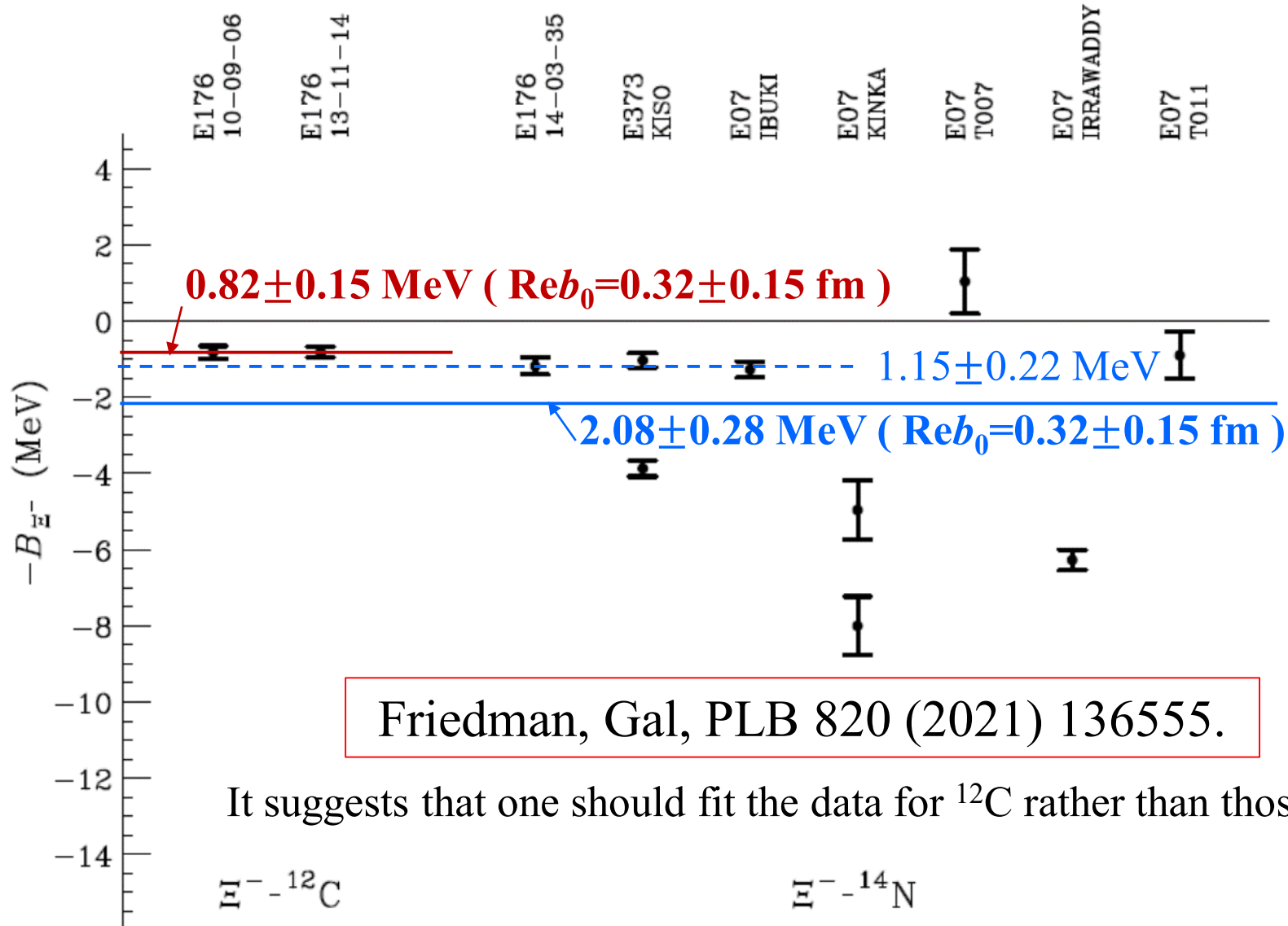
$$\mathcal{V}_{\Xi\text{N}} = F_{\Xi\text{N}}^{(2)} Q_{\text{N}} \cdot Q_{\Xi},$$

$$Q_B = \sqrt{\frac{4\pi}{5}} Y_2(\hat{r}_B)$$

$$V_0^{\Xi} = -21.9 \pm 0.7 \text{ MeV}$$

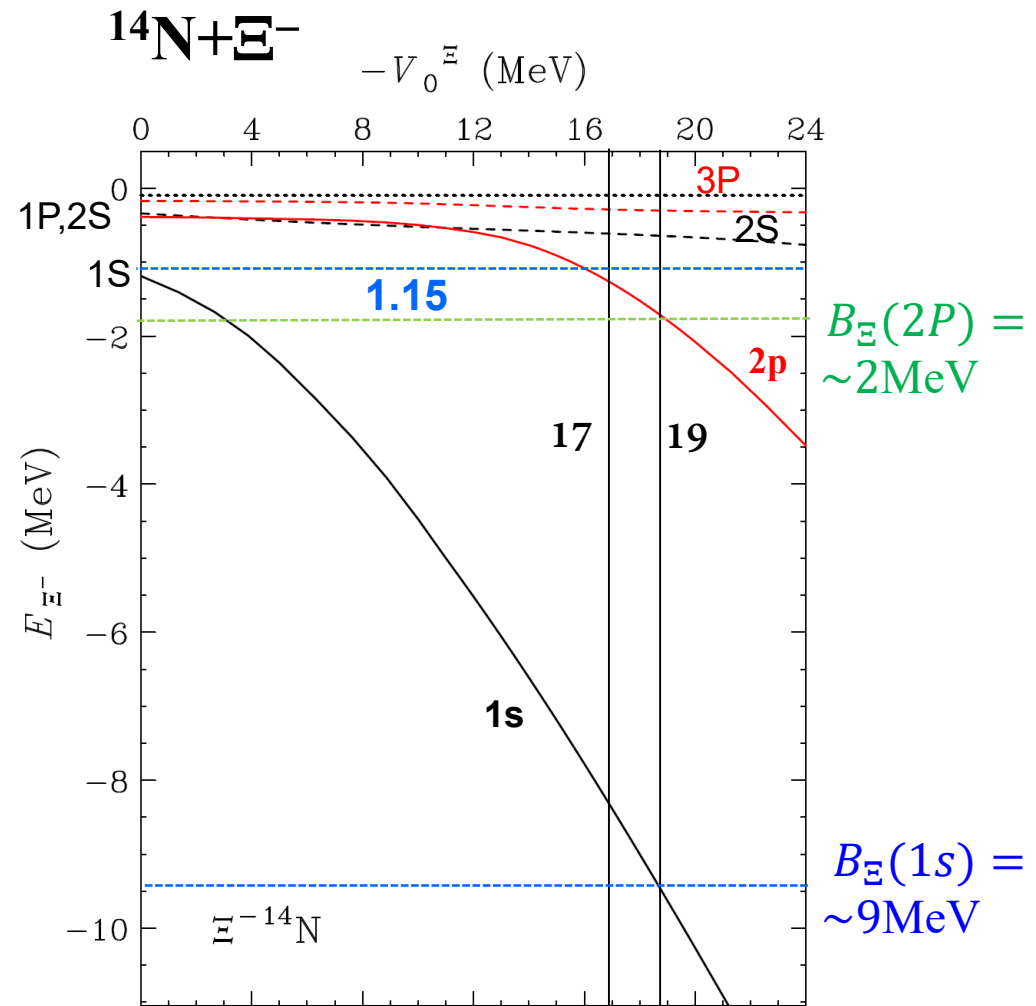
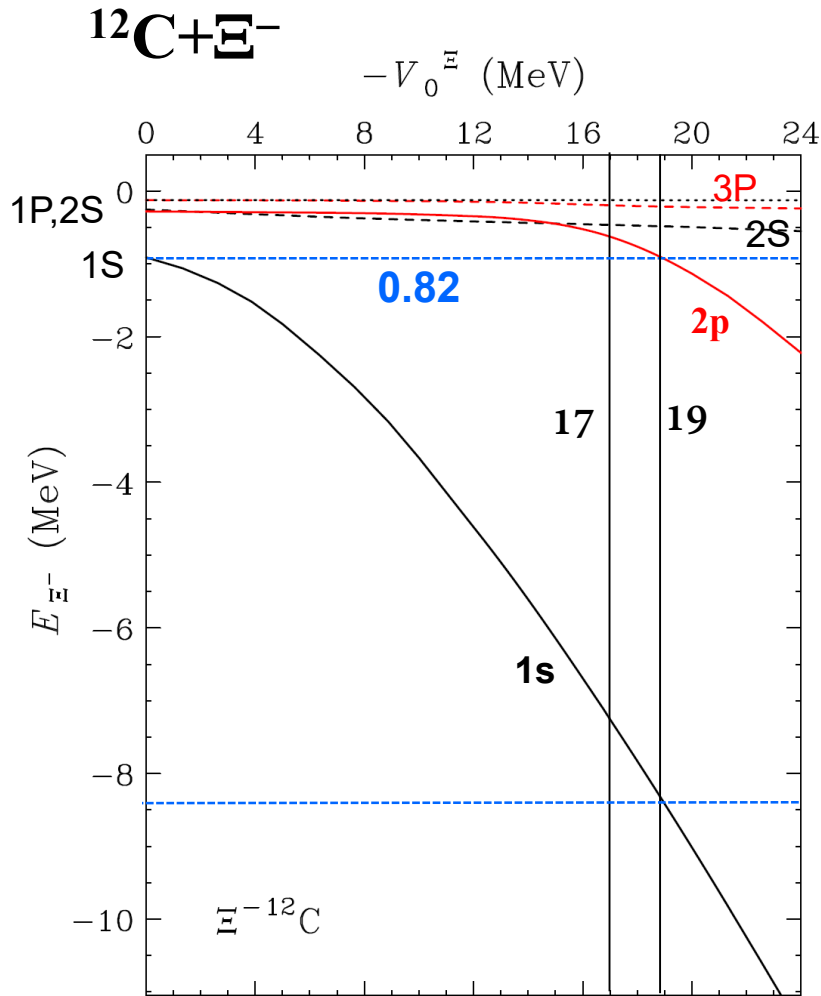
capture from  $1p_{\Xi^-}$  Coulomb-assisted bound states. This involved using just one common strength parameter of a density dependent optical potential. Long-range  $\Xi\text{N}$  shell-model correlations were essential in making the  $^{14}\text{N}$  events consistent with the  $^{12}\text{C}$  events. Earlier attempts to explain these data overlooked this point, therefore reaching quite different conclusions [50–54]. Predicted then are  $1s_{\Xi^-}$  bound states with  $B_{\Xi^-}^{1s} \sim 10$  MeV in  $^{12}\text{C}$  and somewhat larger in  $^{14}\text{N}$ , deeper by 4–5 MeV than the  $1s_{\Xi^-}$  states claimed by

# $\Xi^-$ Binding energies $B_{\Xi^-}$ for twin $\Lambda$ hypernuclei in emulsion (E176/E373/E07)



$E^{-12}\text{C}$  vs.  $E^{-14}\text{N}$

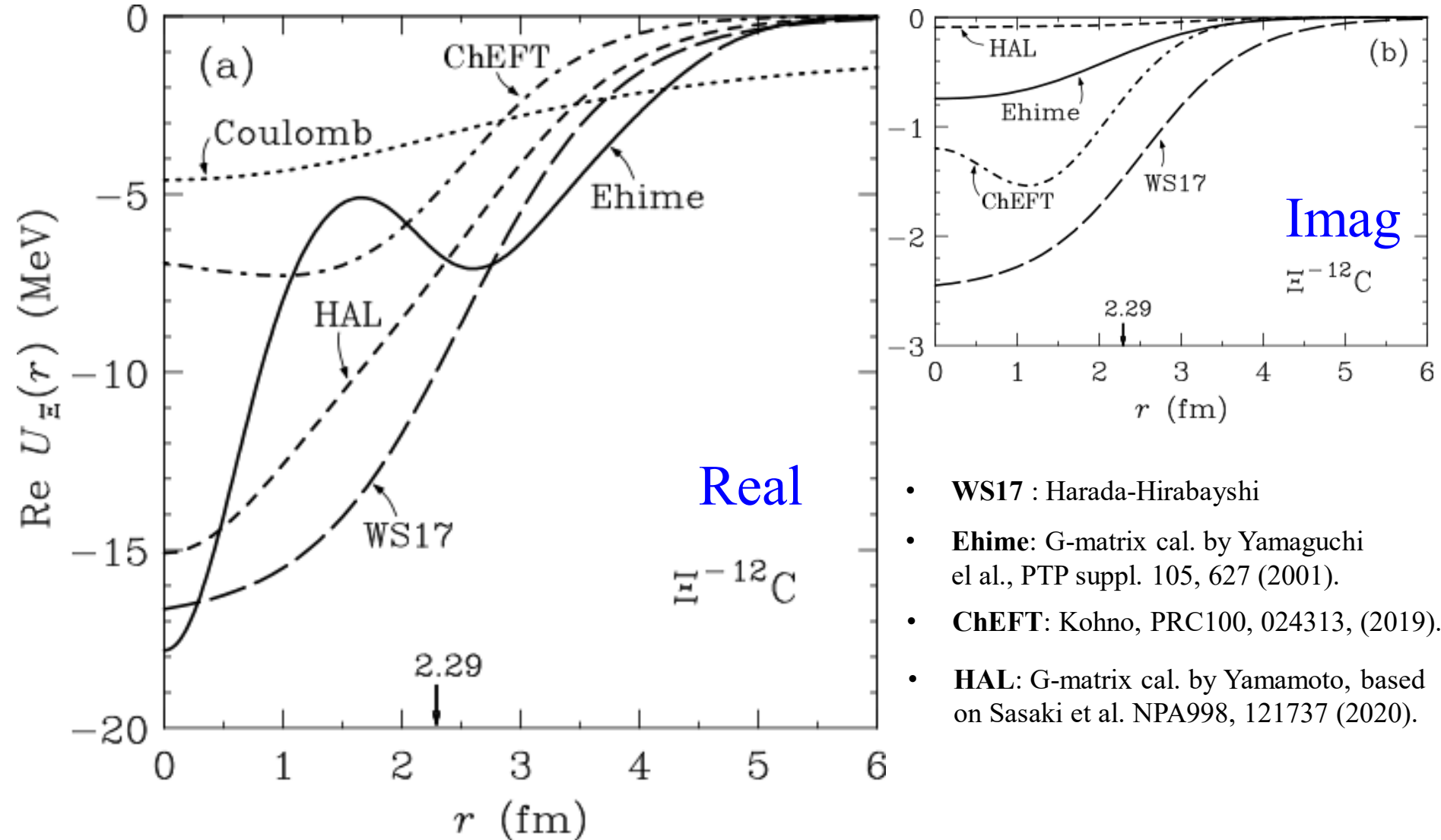
# Dependence of $V_0^\Xi$ strength in the $\Xi$ -nucleus potentials



- ✓ The  $\Xi$ -nucleus WS potential for  $^{12}\text{C}$  is inconsistent with that for  $^{14}\text{N}$  for reproducing the data;  $-V_0 = 17\text{ MeV} \rightarrow 19\text{ MeV}$ , and we confirmed FG.

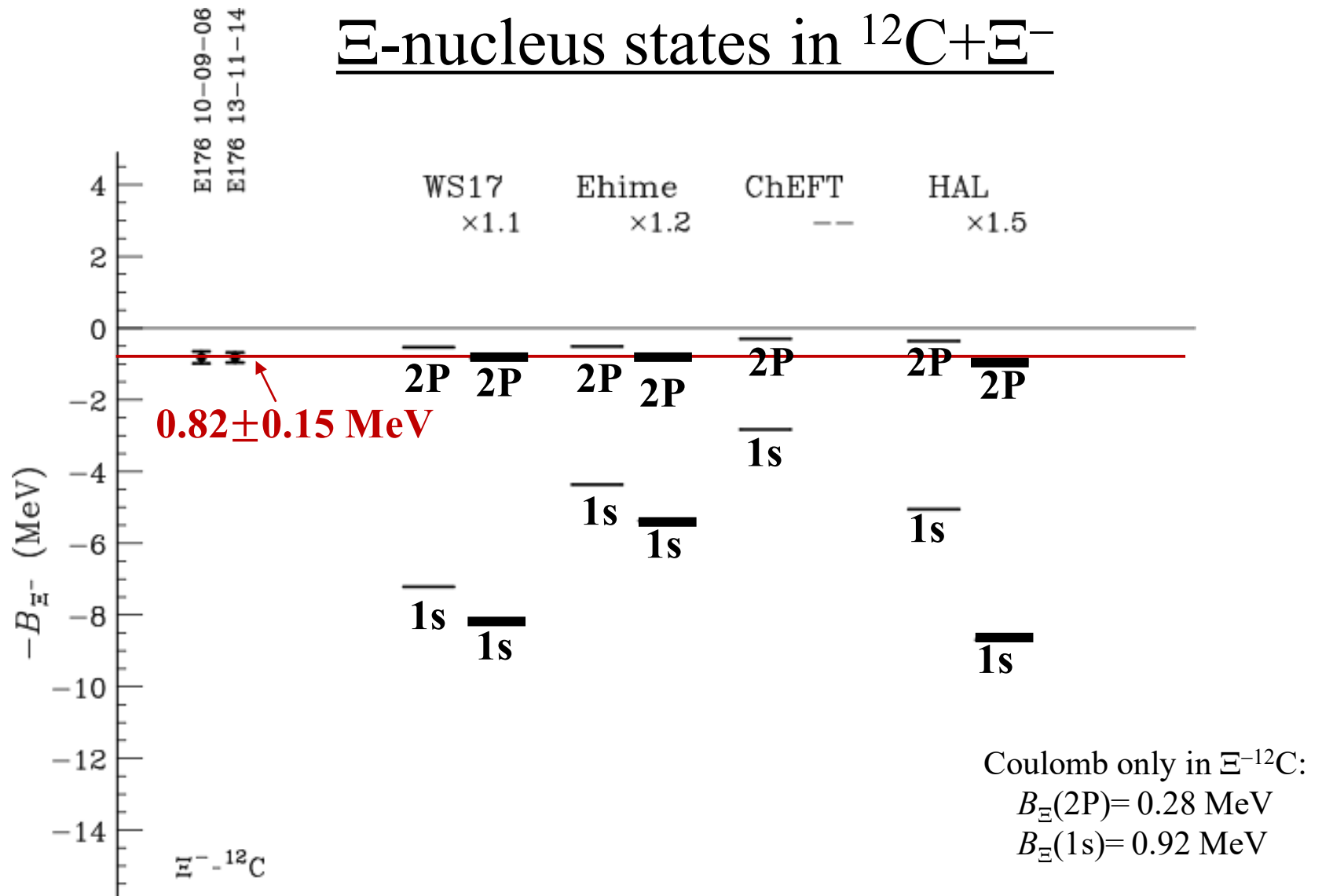
# $\Xi^-$ -nucleus optical potential for $^{12}\text{C}+\Xi^-$

WS and theoretical potentials by microscopic calculations



- **WS17** : Harada-Hirabayashi
- **Ehime**: G-matrix cal. by Yamaguchi et al., PTP suppl. 105, 627 (2001).
- **ChEFT**: Kohno, PRC100, 024313, (2019).
- **HAL**: G-matrix cal. by Yamamoto, based on Sasaki et al. NPA998, 121737 (2020).

# $\Xi$ -nucleus states in $^{12}\text{C}+\Xi^-$

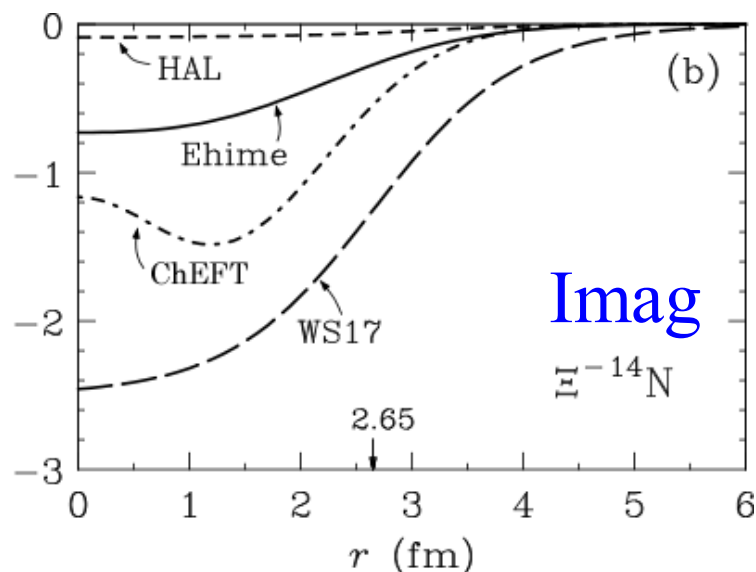
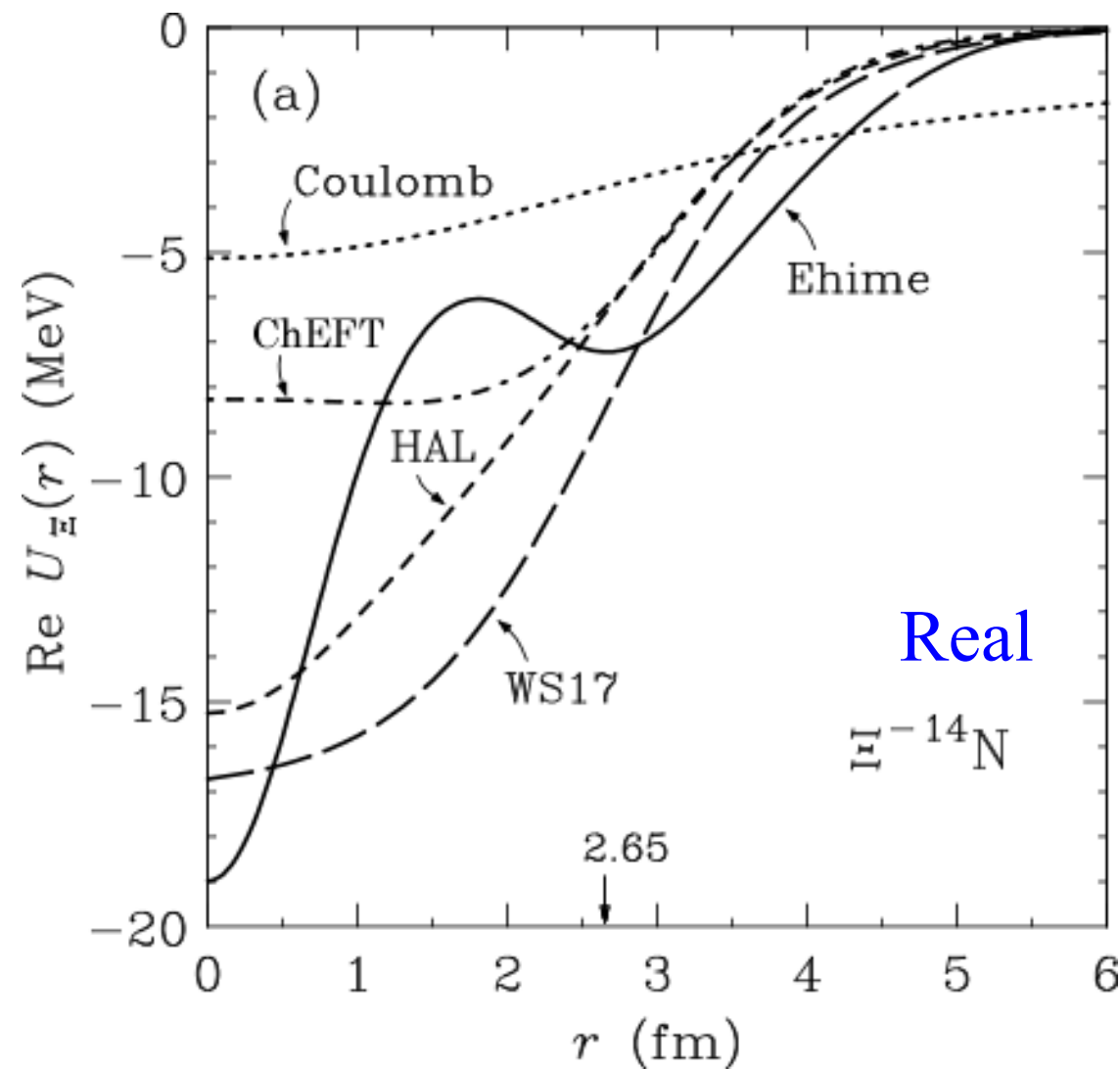


- ✓ If we fit the experimental value of  $B_{\Xi}$  for  $^{12}\text{C}-\Xi^-$  2P state by introducing artificial factors, it suggests that the potential strengths are more attractive.



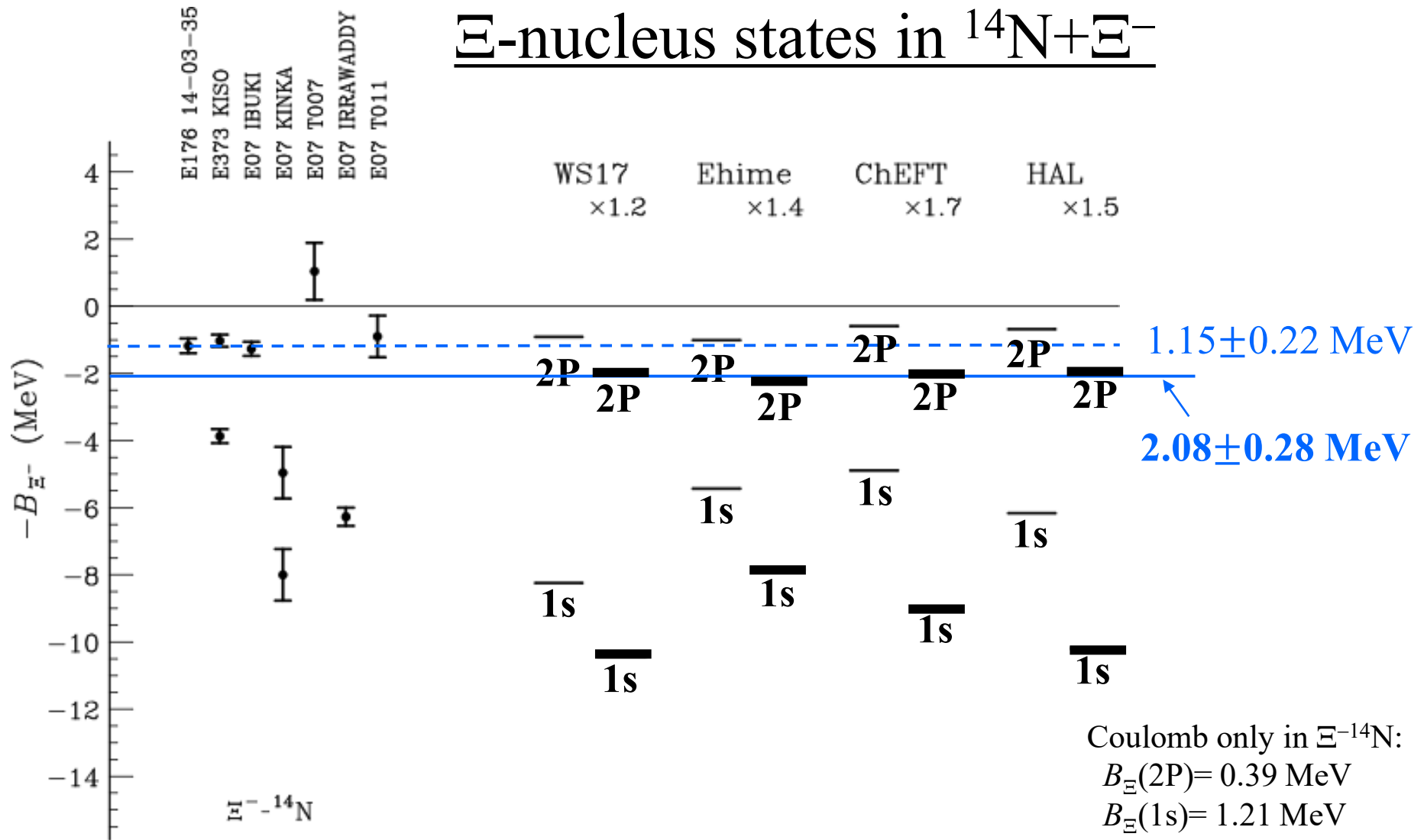
# $\Xi^-$ -nucleus optical potential for $^{14}\text{N}+\Xi^-$

WS and theoretical potentials by microscopic calculations



- **WS17** : Harada-Hirabayashi
- **Ehime**: G-matrix cal. by Yamaguchi et al., PTP suppl. 105, 627 (2001).
- **ChEFT**: Kohno, PRC100, 024313, (2019).
- **HAL**: G-matrix cal. by Yamamoto, based on Sasaki et al. NPA998, 121737 (2020).

# $\Xi$ -nucleus states in $^{14}\text{N}+\Xi^-$

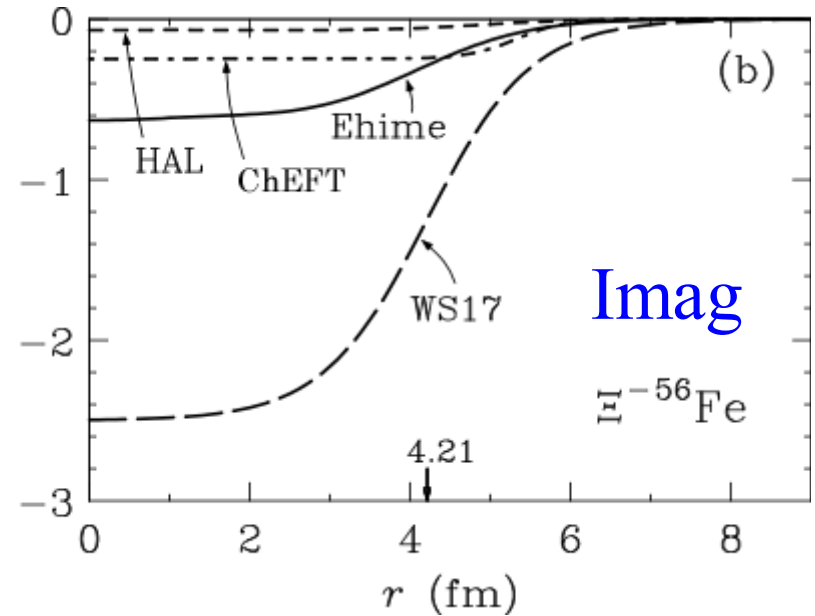
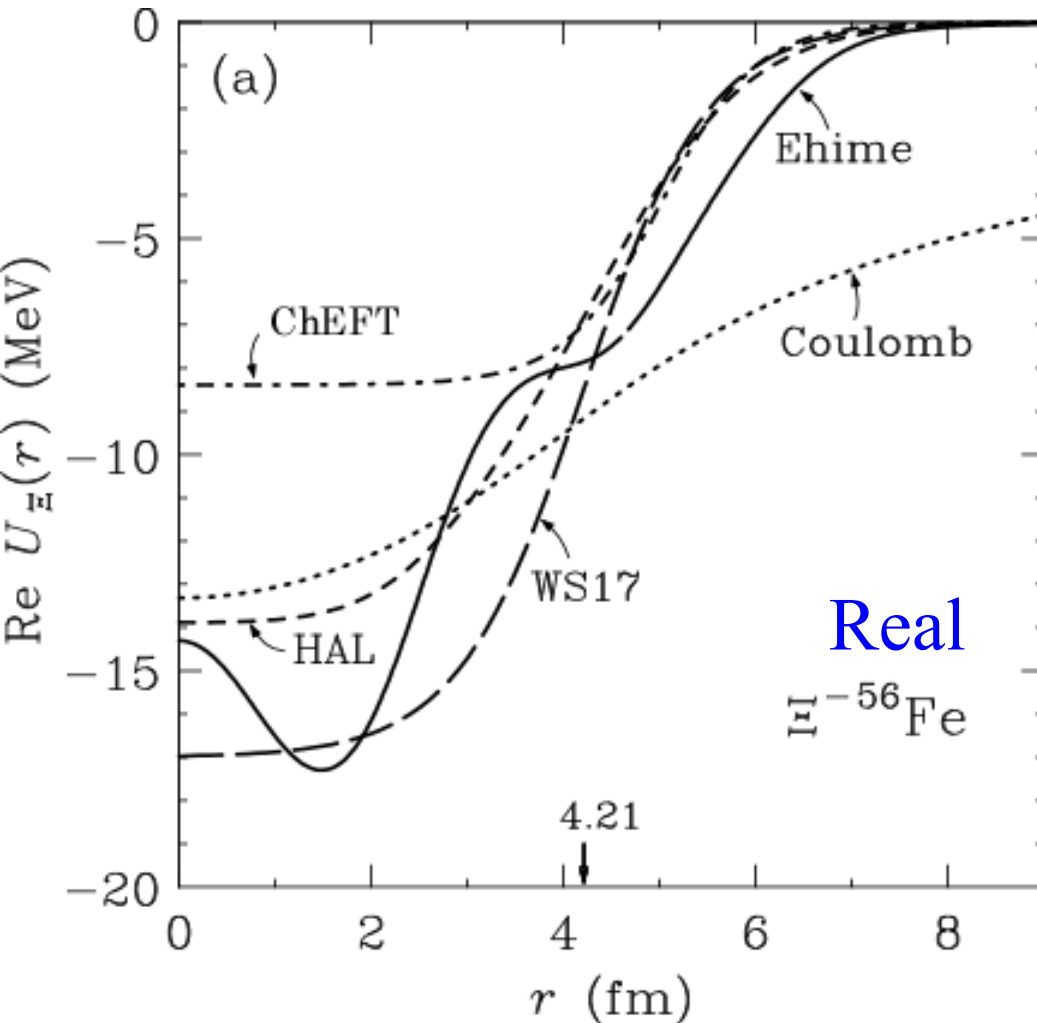


- ✓ If we try to explain the experimental value of  $B_{\Xi^-}(2P) = 2$  MeV for  $^{14}\text{N}-\Xi^-$  by introducing artificial factors, we need much more attractive in these potentials.

$\Xi^-$ - $^{56}\text{Fe}$  potentials  
for  $\Xi^-$ -atomic states

# $\Xi^-$ -nucleus optical potentials in $^{56}\text{Fe}+\Xi^-$

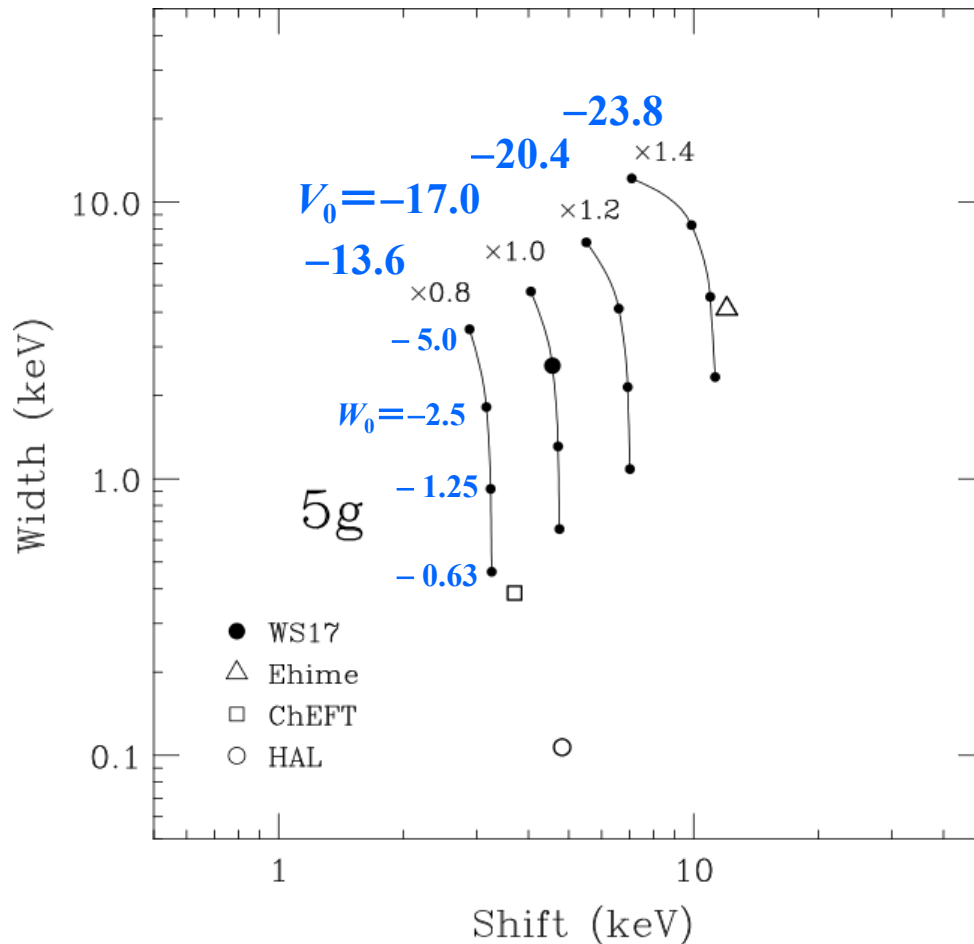
WS and theoretical potentials by microscopic calculations



- **WS17** : Harada-Hirabayashi
- **Ehime**: G-matrix cal. by Yamaguchi et al., PTP suppl. 105, 627 (2001).
- **ChEFT**: Kohno-Miyagawa, PTEP 2021, 103D04
- **HAL**: G-matrix cal. by Yamamoto, based on Sasaki et al. NPA998, 121737 (2020).

J-PARC E03:  $\Xi^-$  atomic X-ray on  $^{56}\text{Fe}$

# Strong-shifts and widths on $\Xi^-$ atoms



$\Xi^-^{56}\text{Fe}$

Dependence of  $V_0$  and  $W_0$ :

$V_0 = -13.6, -17.0, -20.4, -23.8$  MeV

$W_0 = -0.63, -1.25, -2.50, -5.00$  MeV

- ✓ Strong-shifts and widths on  $\Xi^-^{56}\text{Fe}$  indicate useful information on the properties of the  $\Xi$ -nucleus potential in the surface region, especially the imaginary parts.

# Strong-shifts and widths on $\Xi^-$ atoms

07 Feb. 2022

$\Xi^-56\text{Fe}$

		<b>5g</b>	<b>6h</b>	(keV)
WS17	E	4.565	0.0285	
	Width	2.564	0.0098	
Ehime	E	12.016	0.0837	
	Width	4.112	0.0086	
ChEFT	E	3.702	0.0207	
	Width	0.386	0.0012	
HAL	E	4.819	0.0337	
	Width	0.107	0.0004	
fss2	E	1.281	0.012	(Kohno)
	Width	0.088	0.001	

# Summary

We have discussed theoretically production of  $\Xi^-$  hypernuclei in the nuclear ( $K^-$ ,  $K^+$ ) reaction and properties of the  $\Xi$ -nucleus potentials.

- We have studied the  $\Xi^-$  production spectrum of the  ${}^9\text{Be}(K^-, K^+)$  reaction at 1.8 GeV/c within the DWIA using the **optimal Fermi-averaged  $K^-p \rightarrow K^+\Xi^-$  amplitude**.

T. Harada and Y. Hirabayashi, PRC102 (2020) 024618.

- The weak attraction in the  $\Xi$ -nucleus potential for  $\Xi^-$ - ${}^8\text{Li}$  provides the ability to explain the BNL-E906 data, consistent with analyses for previous experiments:  **$V_0 = -17 \pm 6 \text{ MeV}$  for  $W_0 = -5 \text{ MeV}$**

T. Harada and Y. Hirabayashi, PRC103 (2021) 024605.

- We discuss the properties of  $\Xi$ -nucleus potentials compared to the recent emulsion data.
- Future subjects and prospects  **$\rightarrow$   $\Xi\text{NN}$  force and  $\Xi\text{N}-\Lambda\Lambda$  coupling**

**More information on  $\Xi^-$  bound states and widths is needed.**

**Thank you very much  
for your attention.**