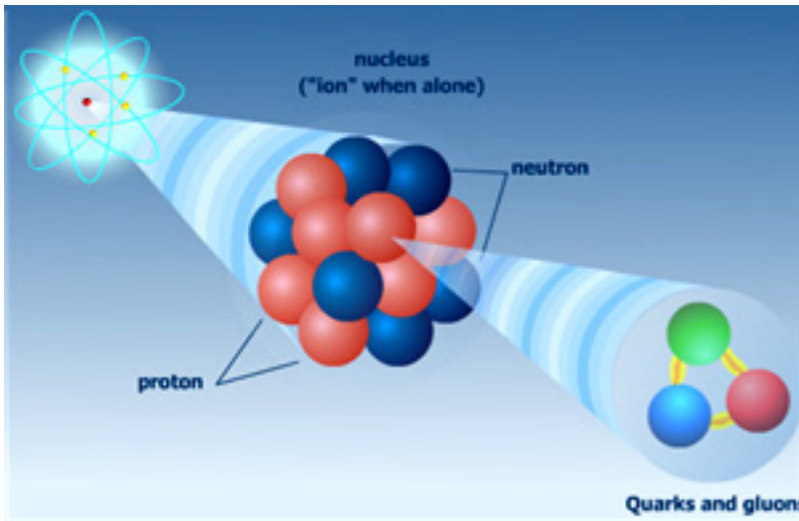


KHuK annual meeting, December 10, 2021



Innovative many-body methods for nuclear structure

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Fakultät für Physik, Universität Bielefeld, Germany*



Goal of nuclear structure theory:

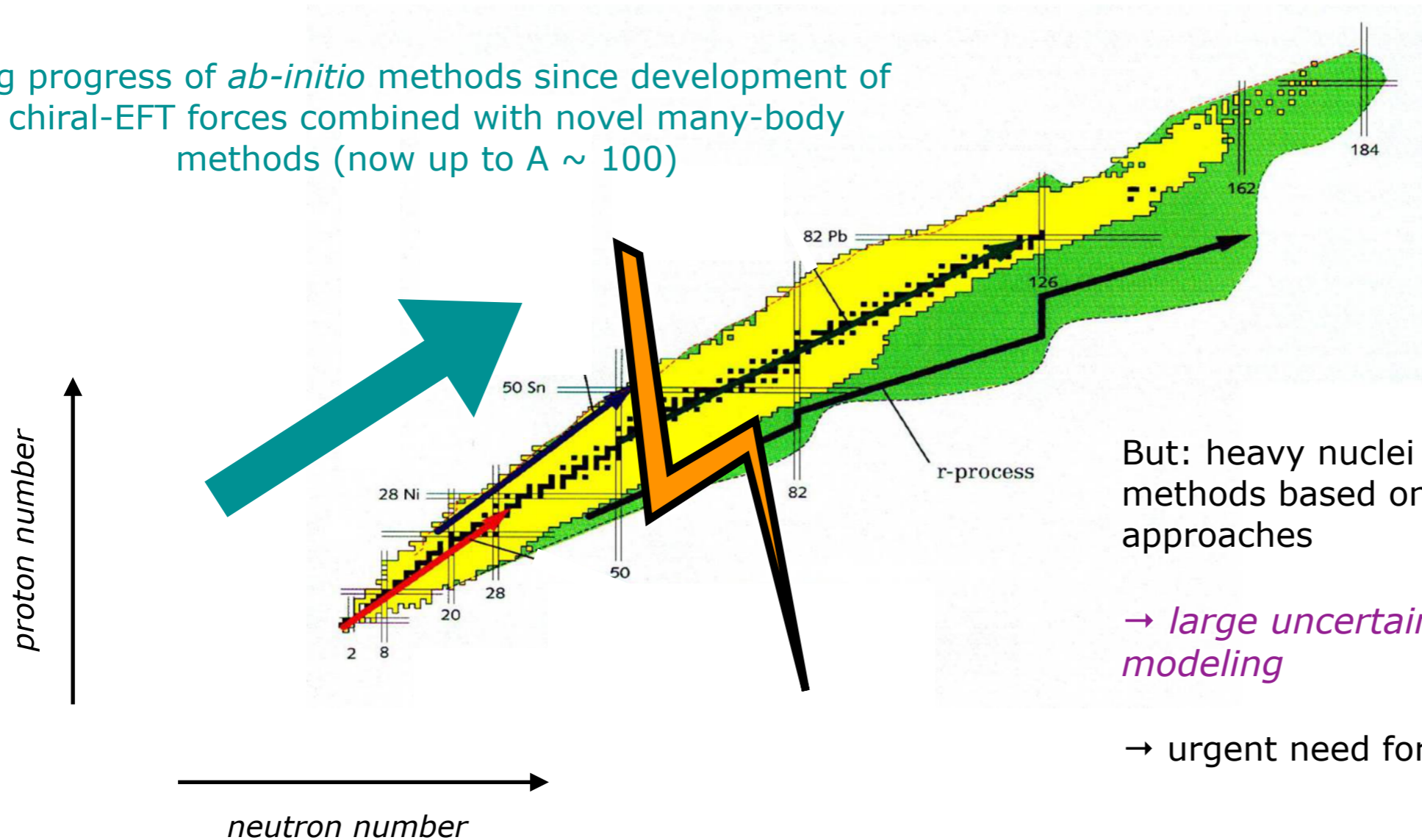
describe properties of all nuclei by solving the nuclear A-body problem

major issues:

nuclear force?

$2 \leq A \leq 300$

Big progress of *ab-initio* methods since development of chiral-EFT forces combined with novel many-body methods (now up to $A \sim 100$)



But: heavy nuclei still only accessible by methods based on "phenomenological" approaches

→ large uncertainties in astrophysical modeling

→ urgent need for improvement

Ab-initio methods for light and mid-mass nuclei

"*ab-initio*" = aim at solving the nuclear many-body problem based on n-nucleon forces derived from chiral EFT and fitted to n-nucleon data

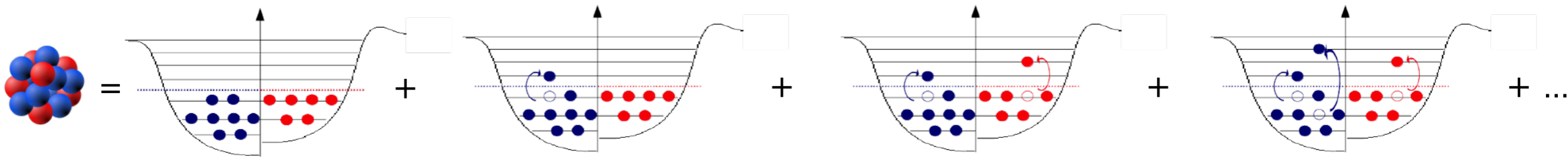
→ controlled approximations, systematically improvable, theoretical uncertainties can be assessed

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO (Q^0)		—	—
NLO (Q^2)		—	—
N ² LO (Q^3)			—
N ³ LO (Q^4)			
N ⁴ LO (Q^5)			

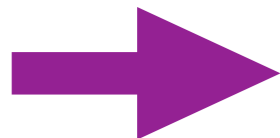
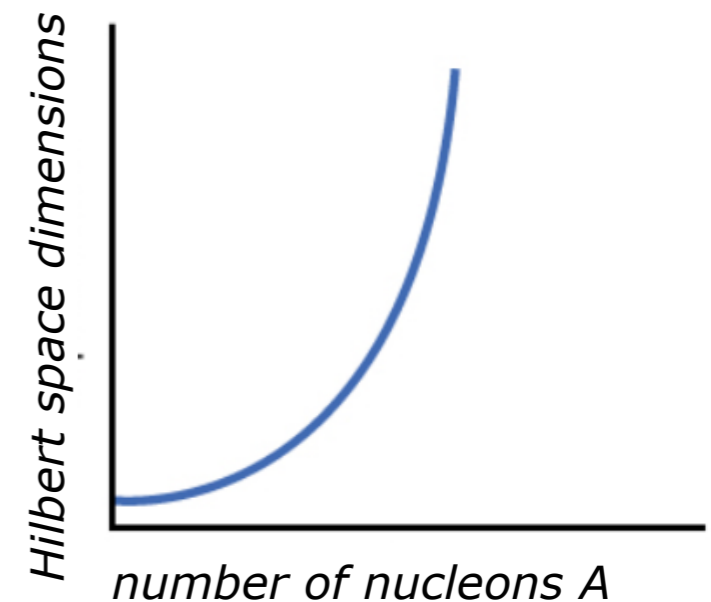
Hierarchy of nuclear forces at increasing orders in chiral expansion
Epelbaum, Krebs, Reinert, Front. Phys. 8, 98 (2020)

Configuration-space methods

$$|\Psi\rangle = A_{0p0h}|\Phi_{0p0h}\rangle + \sum_{1p1h} A_{1p1h}|\Phi_{1p1h}\rangle + \sum_{2p2h} A_{2p2h}|\Phi_{2p2h}\rangle + \sum_{3p3h} A_{3p3h}|\Phi_{3p3h}\rangle + \dots$$



Issue: factorial growth of the number of configurations



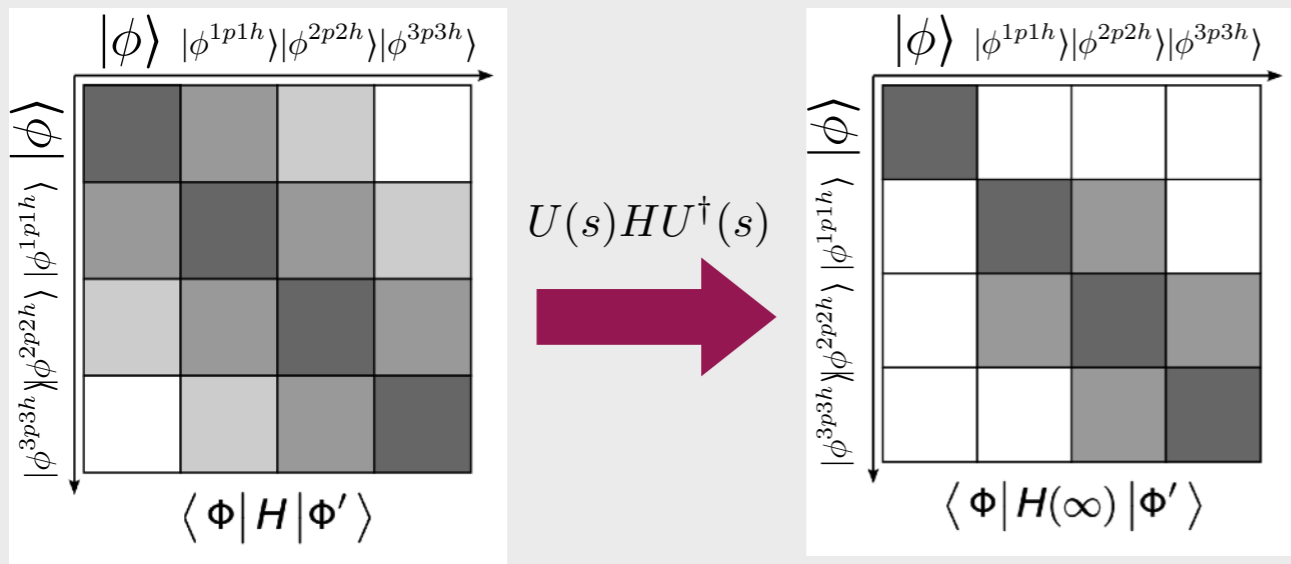
How to account for np-nh correlations, while keeping the problem tractable?

In-Medium Similarity Renormalization group

IMSRG group for nuclei,

K. Tsukiyama, S. K. Bogner, and A. Schwenk, PRL106, 222502 (2011)

decouple reference state $|\phi\rangle$ from excitations:



$$E = \langle \Psi | H | \Psi \rangle = \langle \phi | H(\infty) | \phi \rangle$$

balance between $|\phi\rangle$ and Hamiltonian $H(\infty)$:

- * valence space + core:
Tsukiyama, Bogner, Schwenk PRC 85 C 85, 061304(R) (2012);
Stroberg et al PRL 118, 032502 (2017)
- * no-core: Gebrerufael, Vobig, Hergert, Roth PRL 118, 152503 (2017)

\Rightarrow ground and excitation properties up to \sim Ni

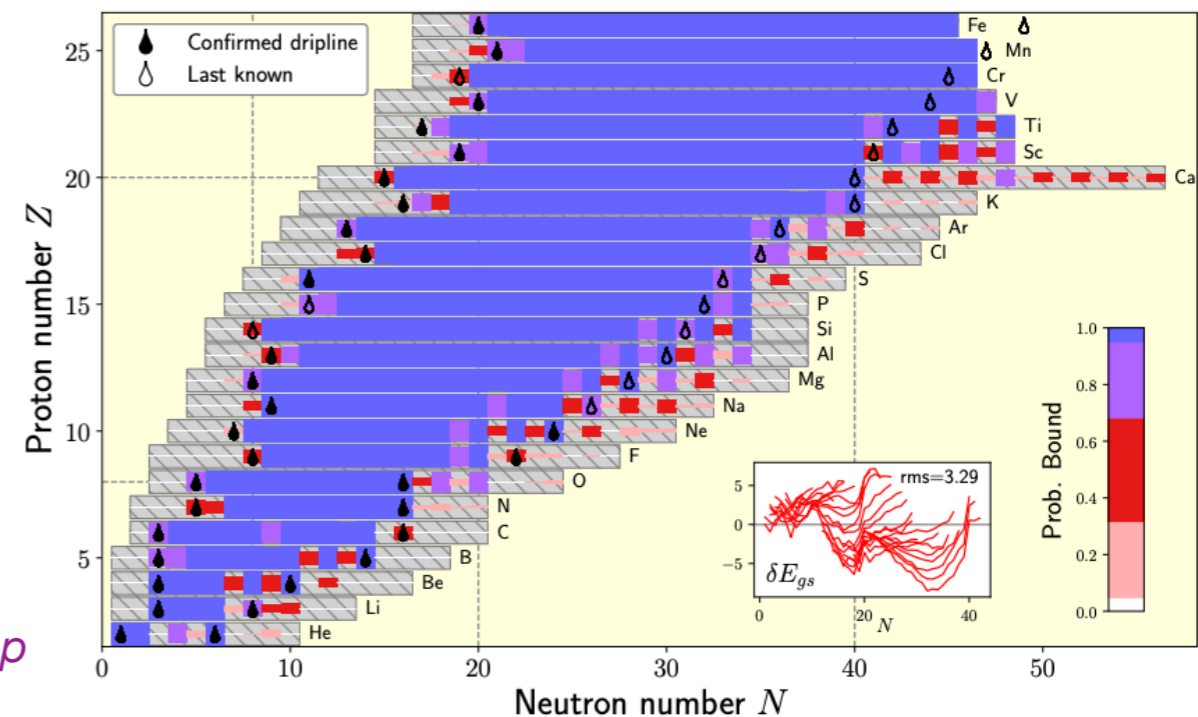
* *Ab initio limits of atomic nuclei*,
Stroberg, Holt, Schwenk, Simonis, PRL 126, 022501 (2021)

* *In-medium similarity renormalization group with three-body operators*
Heinz, Tichai, Hoppe, Hebeler, Schwenk, arXiv:2102.11172 (2021)

to improve precision of calculations

* *Importance truncation for the in-medium similarity renormalization group*
Hoppe, Tichai, Heinz, Hebeler, Schwenk, arXiv:2110.09390 (2021)

to extend the reach of calculations



Calculated probabilities for given isotopes to be bound with respect to one- or two-neutron/proton removal.

Optimization of the single-particle basis

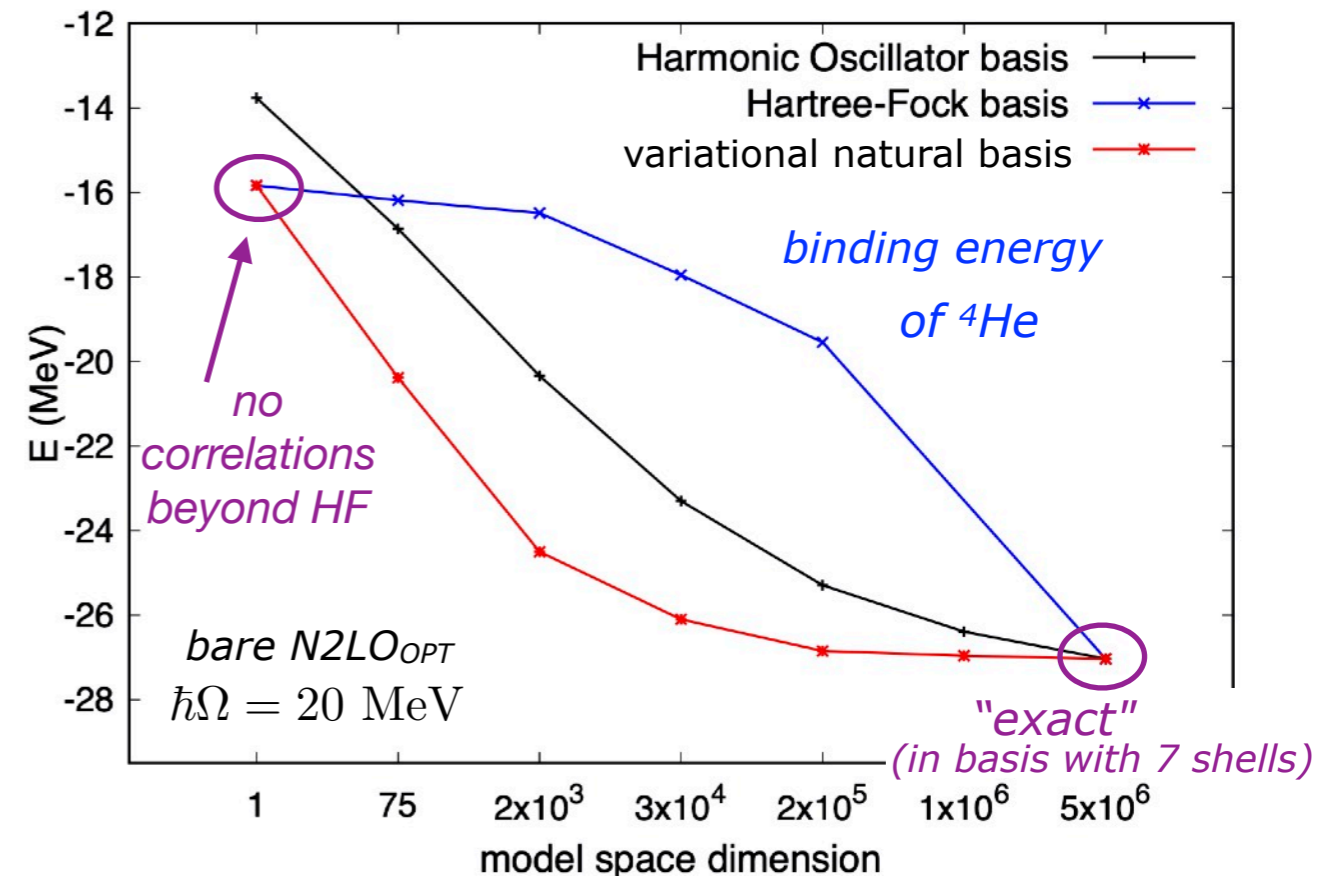
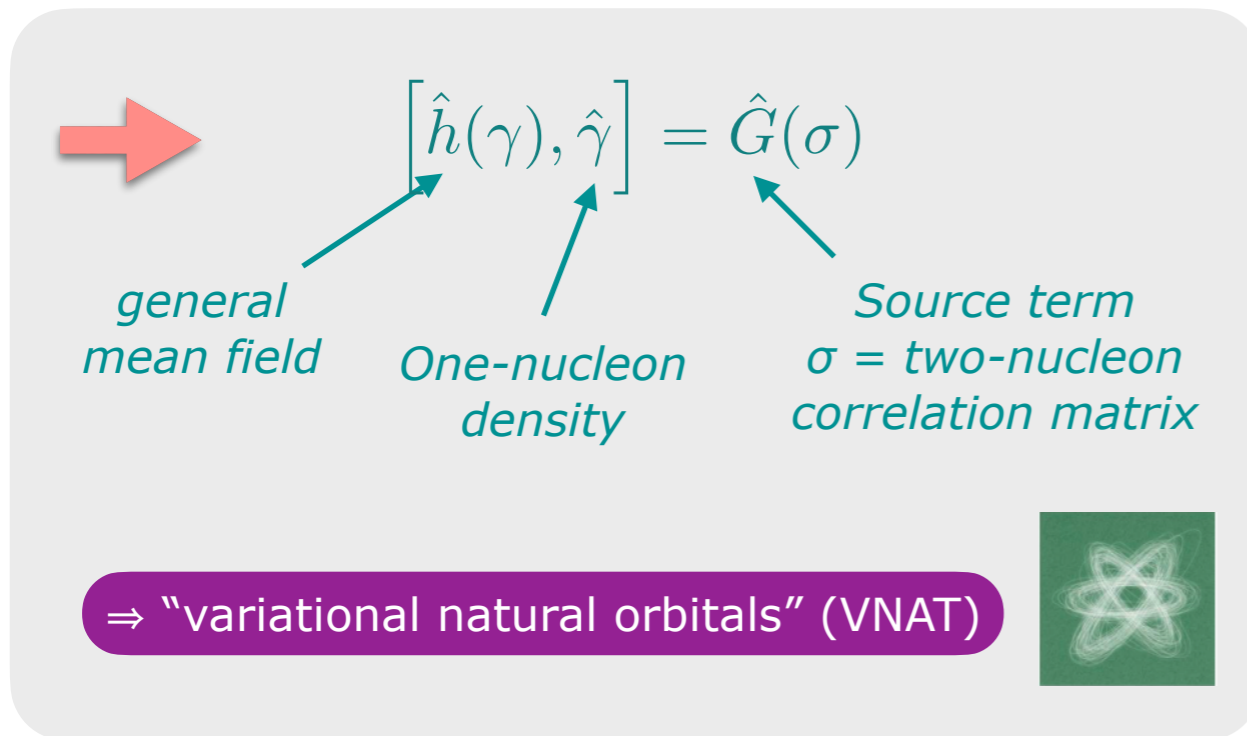
* In a perturbative framework

Natural orbitals for ab initio no-core shell model calculations; Tichai, Müller, Vobig, Roth PRC 99, 034321 (2019)

Natural orbitals for many-body expansion methods; Hoppe, Tichai, Heinz, Hebeler, Schwenk, PRC 103, 014321 (2021)

* From a variational principle

CR et al., PRC 93, 024302 (2016) & PRC 95, 044315 (2017)



Recently studied from the point of view of entanglement:

Entanglement rearrangement in self-consistent nuclear structure calculations

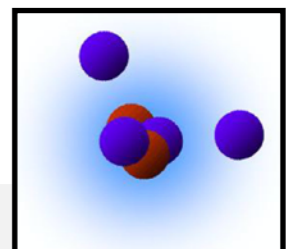
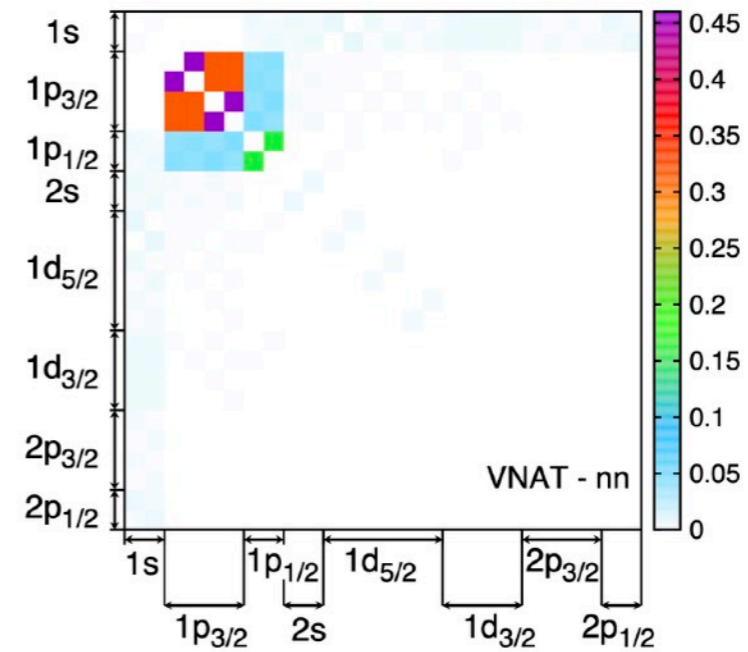
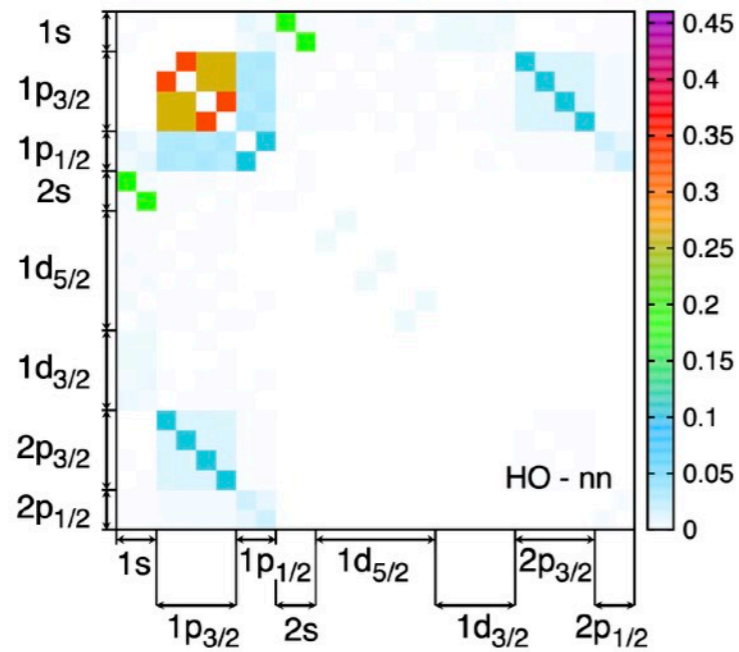
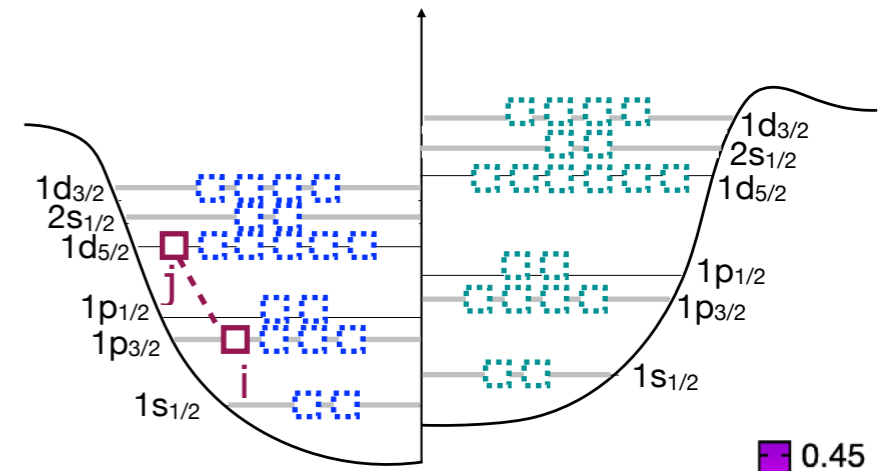
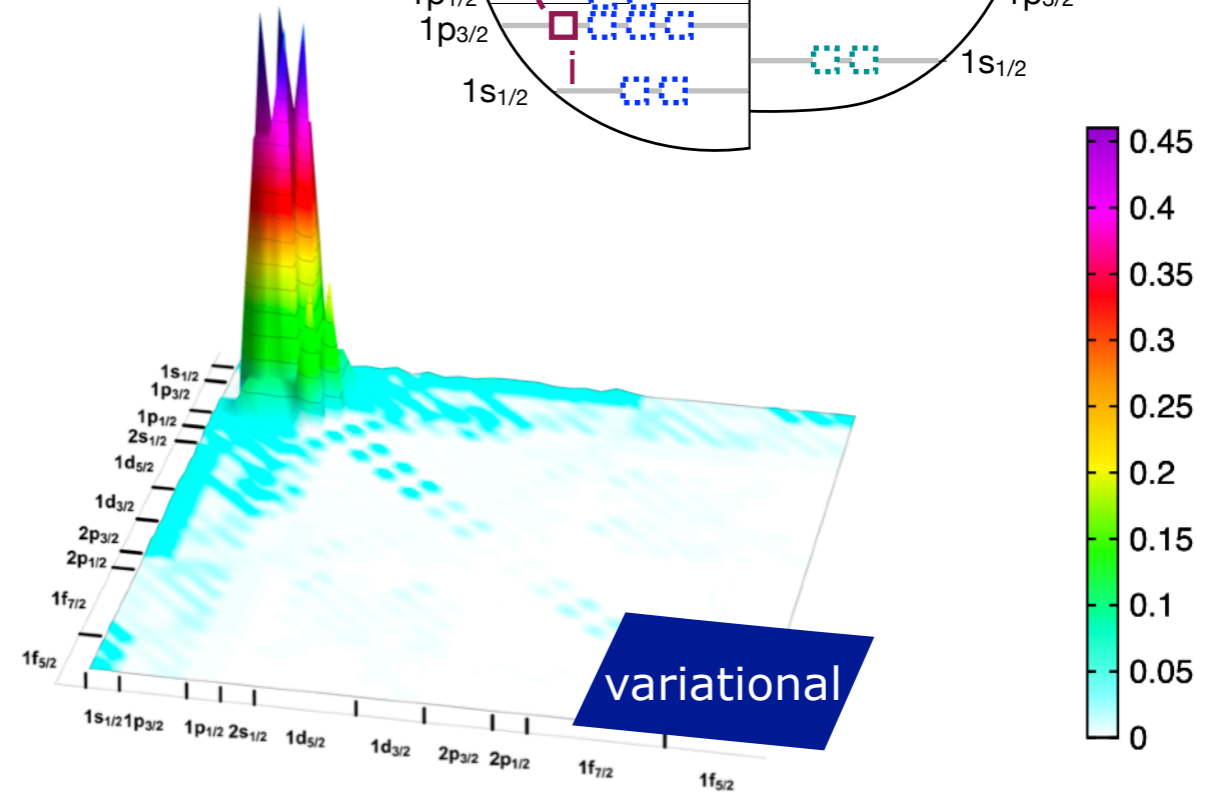
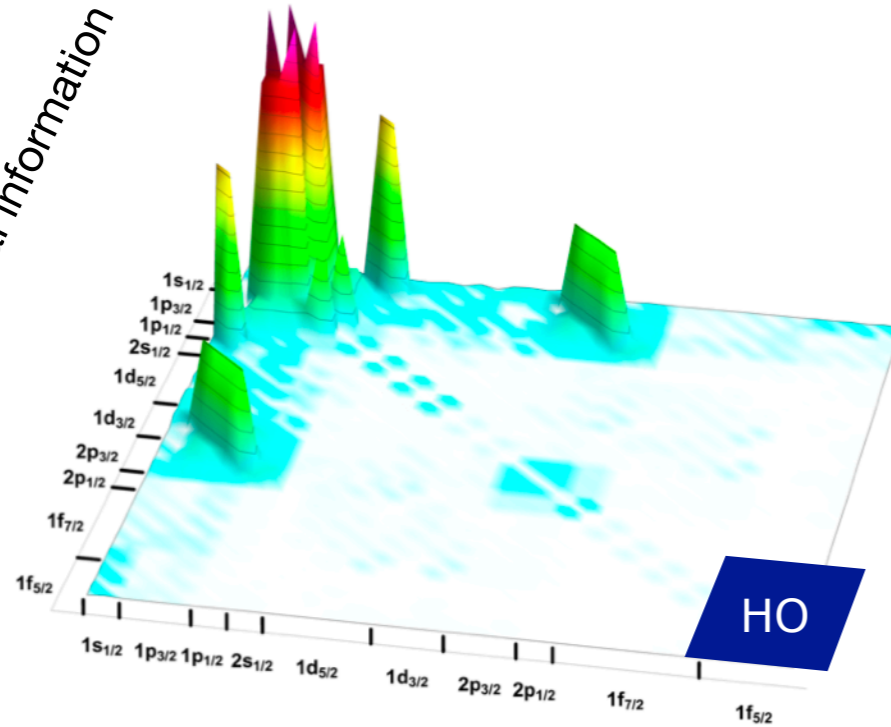
CR, Savage, Pillet, PRC 103, 034325 (2021), arXiv:2007.09157 (2020)

► Mutual information:

= correlations between two orbitals embedded in the system

${}^6\text{He}$

Neutron-neutron mutual information

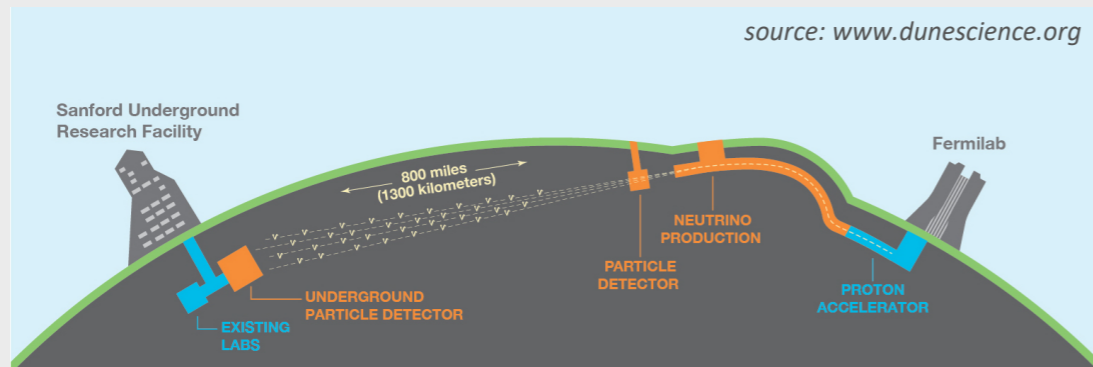


► HO orbitals: entanglement distributed over the basis

► variational orbitals: decoupling of the 1p shell \Rightarrow emergence of ${}^4\text{He}$ -core + nn-valence structure

$$R(\omega, \mathbf{q}) = \sum_f |\langle \Psi_f | O(\mathbf{q}, \omega) | \Psi_i \rangle|^2 \delta(E_f - E_i - \omega)$$

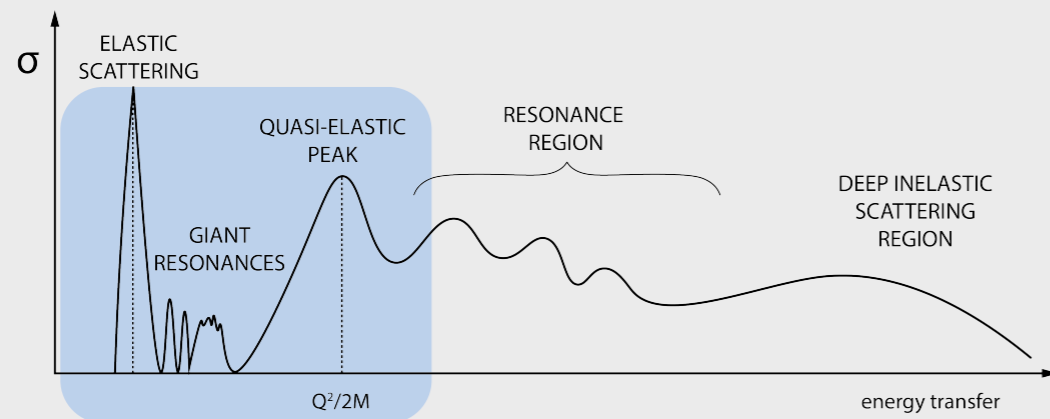
Nuclear theory for neutrino oscillations



Neutrino oscillation experiments require knowledge of neutrino-nucleus cross section for the energy transfers ~hundreds MeV—GeV



Most important nuclei for DUNE and T2HK: ^{12}C , ^{16}O , ^{40}Ar



Possible guidance from *ab initio* nuclear theory

First step: benchmarking with electron data

$$\left. \frac{d\sigma}{d\omega dq} \right|_{\nu\bar{\nu}} = \sigma_0 (v_{CC}R_{CC} + v_{CL}R_{CL} + v_{LL}R_{LL} + v_T R_T \pm v_T R_T')$$

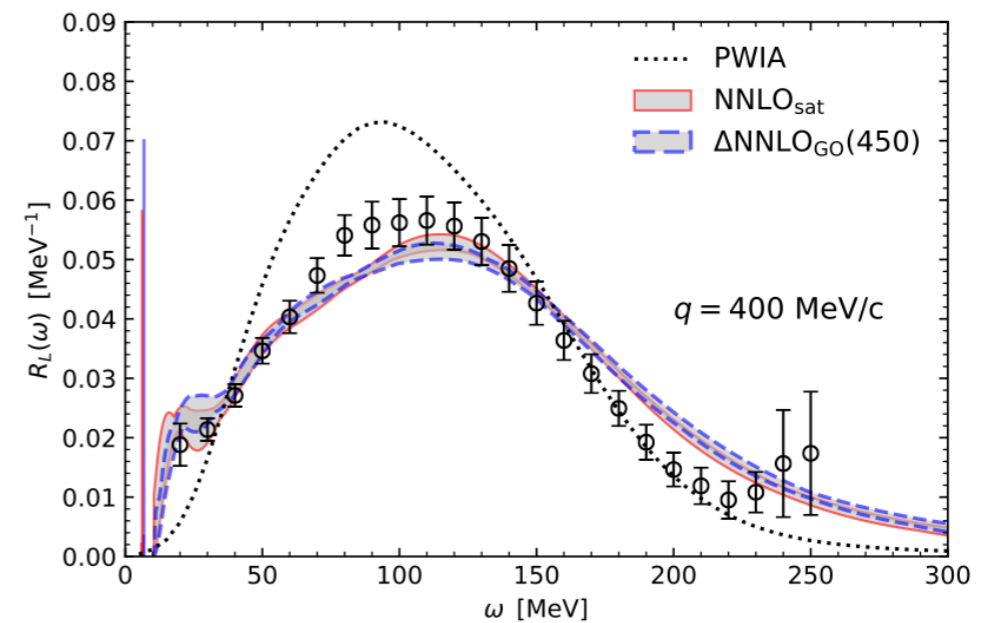
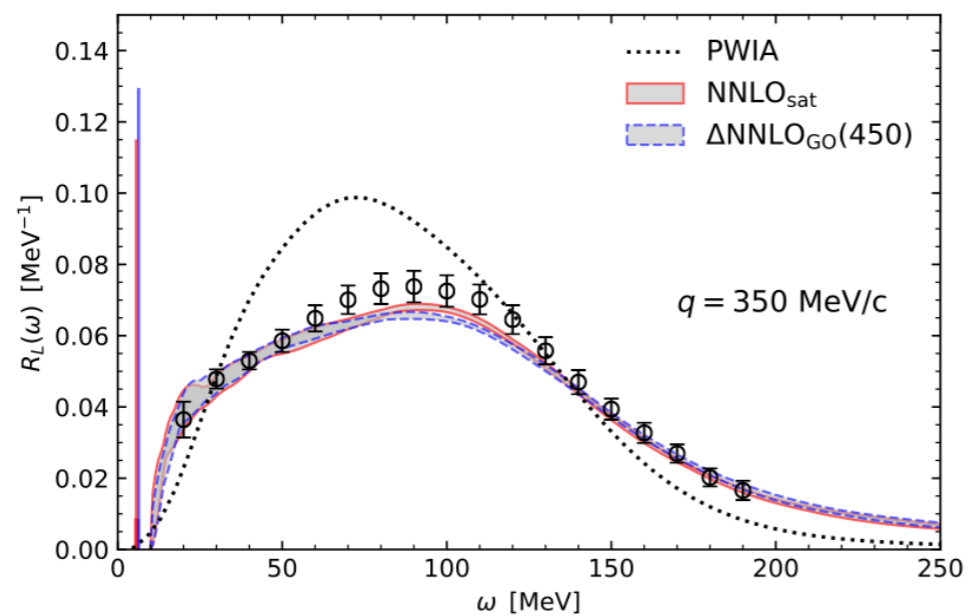
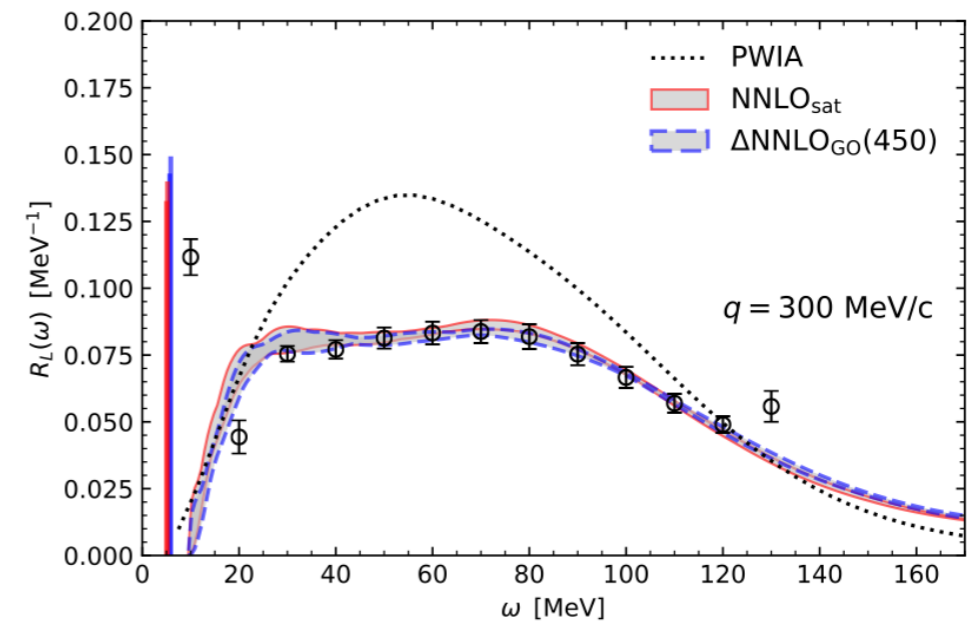
$$\left. \frac{d\sigma}{d\omega dq} \right|_e = \sigma_M (v_L R_L + v_T R_T)$$

Courtesy of Sonia Bacca (Mainz)

First ab initio results for ^{40}Ca

- Lorentz integral transform combined with coupled cluster method
- Consistent treatment of initial and final-state interactions

Physical Review Letters **127** (2021) 7, 072501

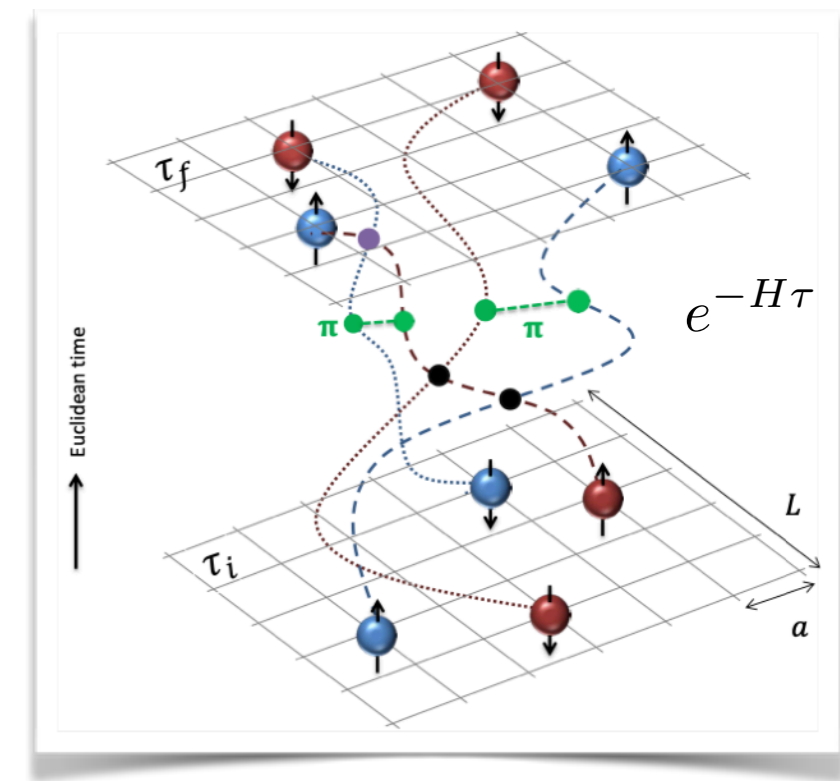


Courtesy of Sonia Bacca (Mainz)

Nuclear Lattice Effective Field Theory (NLEFT)

- Combines chiral EFT and lattice methods
- nucleons on lattice sites interact via chiral EFT forces

description of cluster states



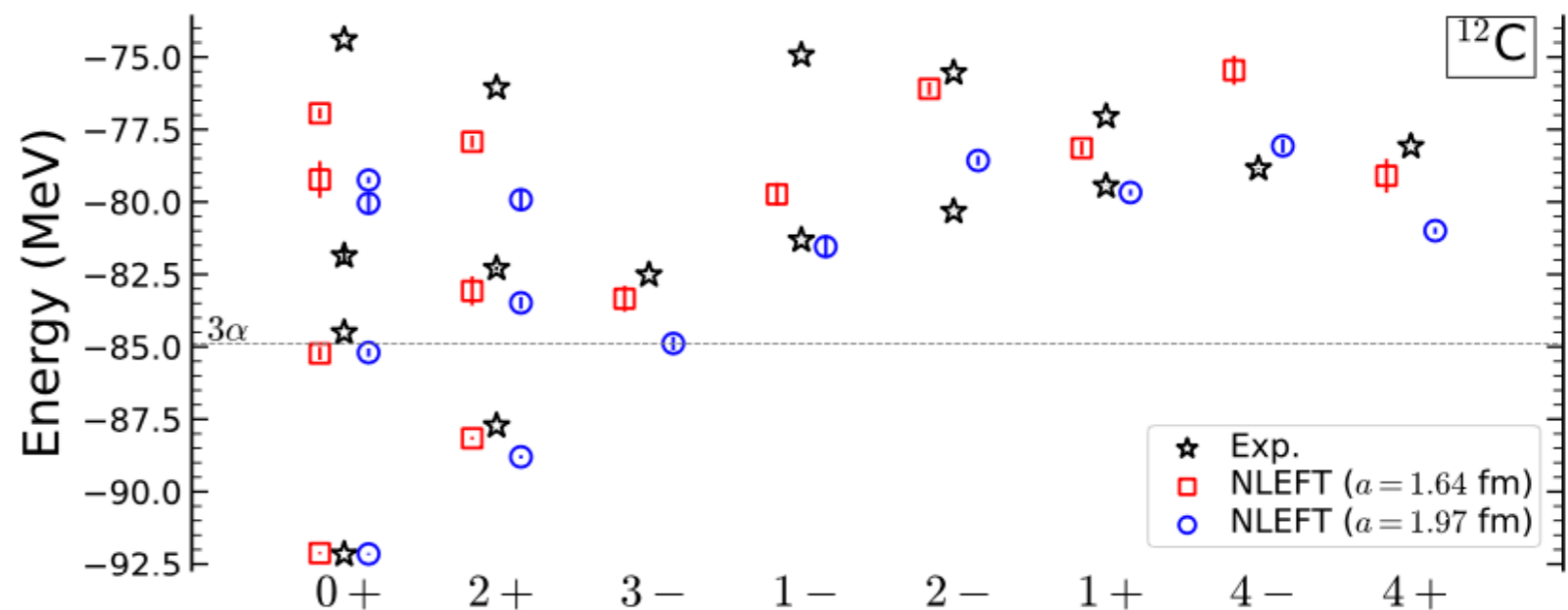
* *Ab initio calculation of the Hoyle state*, Epelbaum, Krebs, Lee, Meißner, PRL 106, 192501 (2011)

* *Wigner SU(4) symmetry, clustering, and the spectrum of ^{12}C* , Shen, Lähde, Lee, Meißner, EPJA 57, 276 (2021)

→ good description of the spectrum up to ~ 15 MeV using

- spin- and isospin-independent NN interaction tuned to ground-state energies of ^4He and ^{12}C
- 3- α cluster states and HO particle-hole states as initial states

⇒ either spin-orbit interactions are weak in ^{12}C , or the effects of α clustering are diminishing their influence.

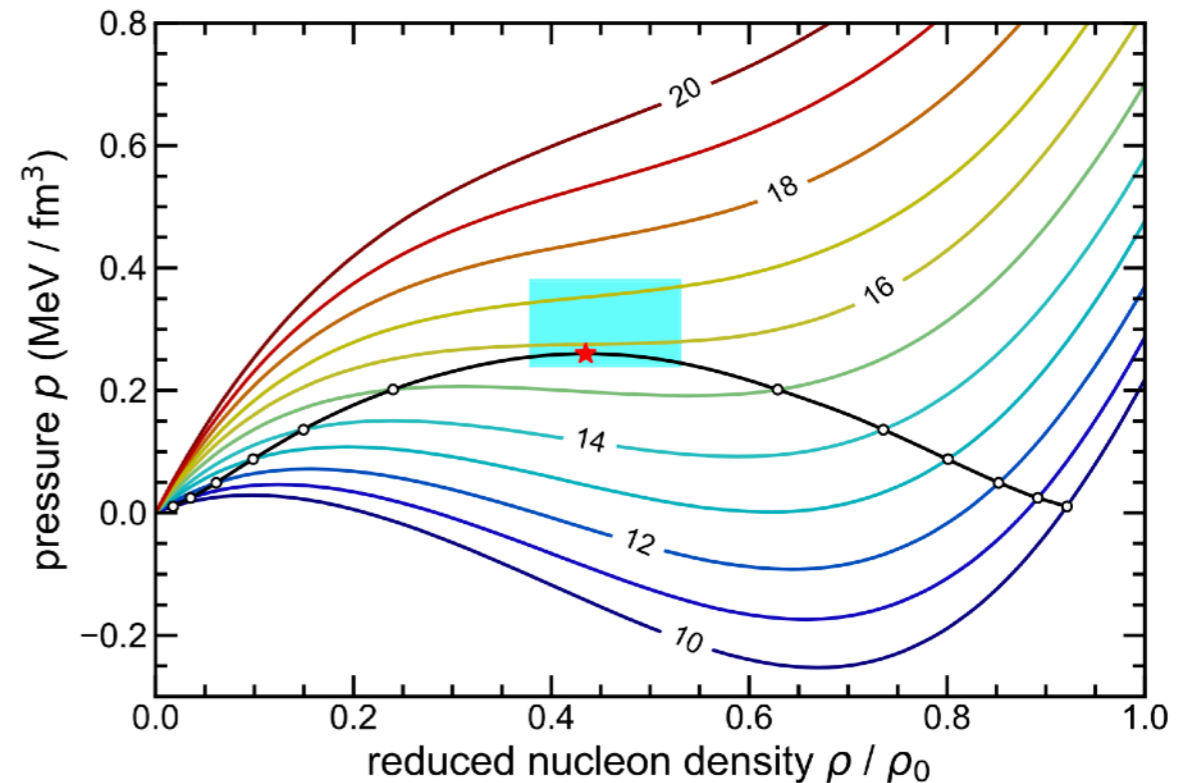


non-zero temperature systems

***ab initio* calculation of the EoS of nuclear matter with the pinhole trace algorithm**

Ab-initio Nuclear Thermodynamics; Lu, Li, Elhatisari, Lee, Drut, Lähde, Epelbaum, Meißner, PRL 125, 192502 (2020)

- * cost savings = V/A
speedup factor ~ 1000
- * good parameter-free description of the liquid-vapor phase transition with leading-order pionless EFT (NN and NNN forces)

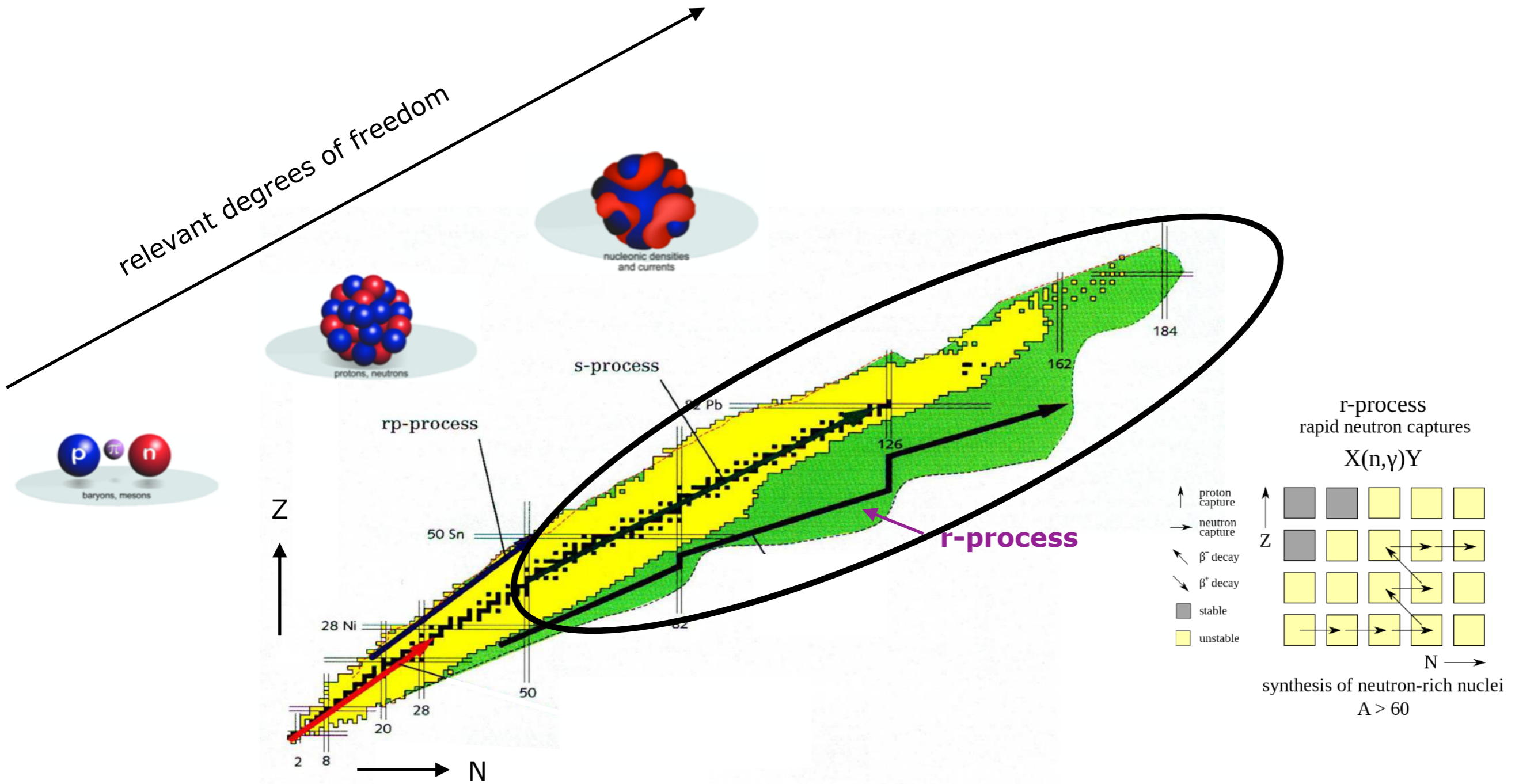


the p - ρ isotherms of symmetric nuclear matter are shown for $L^3 = 6^3$. The black line denotes the liquid-vapor coexistence line, and the red star marks the calculated critical point. The cyan rectangle marks the empirical critical point extracted from heavy-ion collisions [54].

reactions

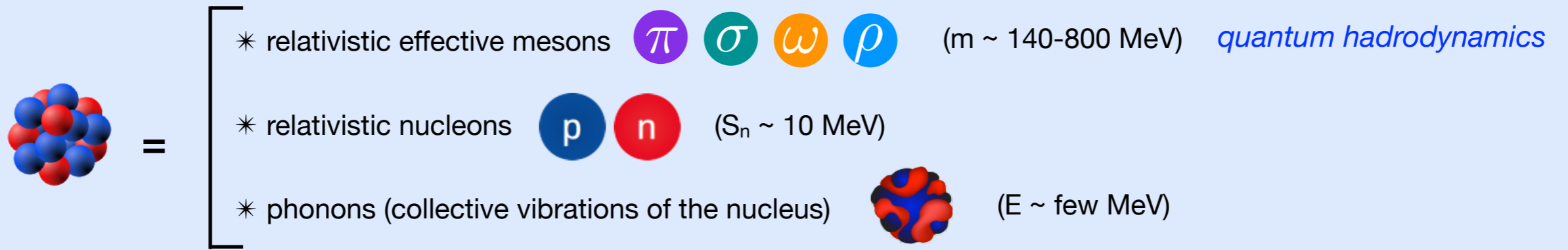
Ab initio alpha-alpha scattering, Elhatisari, Lee, Rupak, Epelbaum, Krebs, Lähde, Luu, Meißner, *Nature*, 528, 111 (2015)

Many-body methods for heavy nuclei

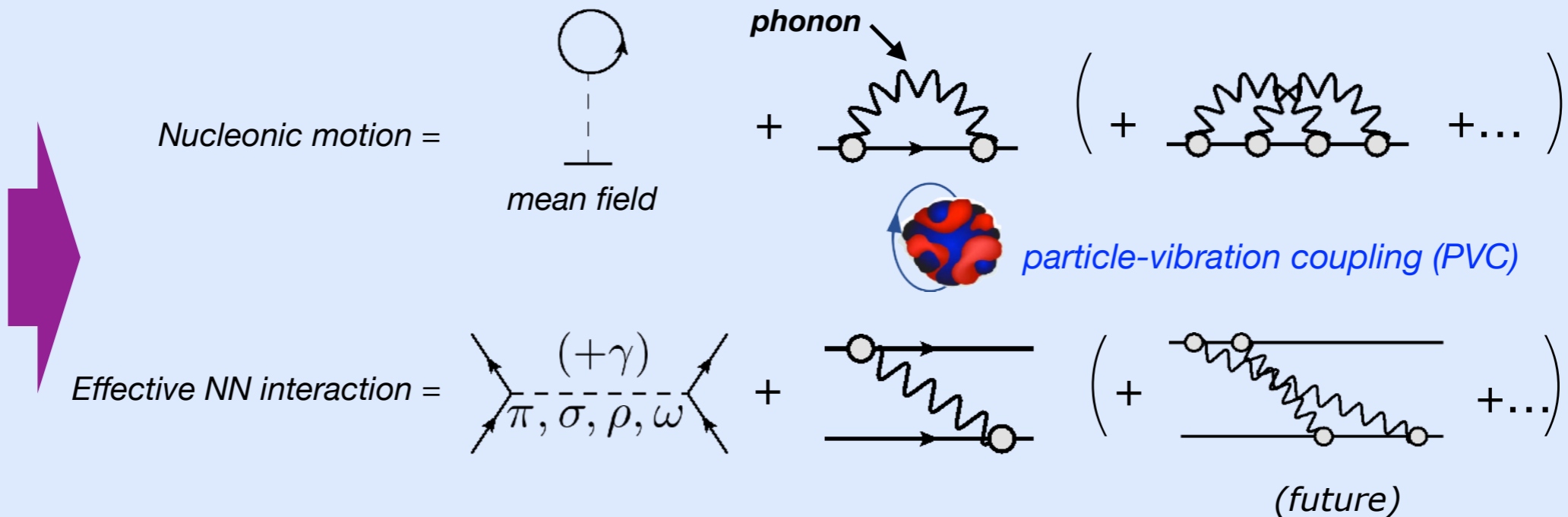


Relativistic Nuclear Field Theory (RNFT)

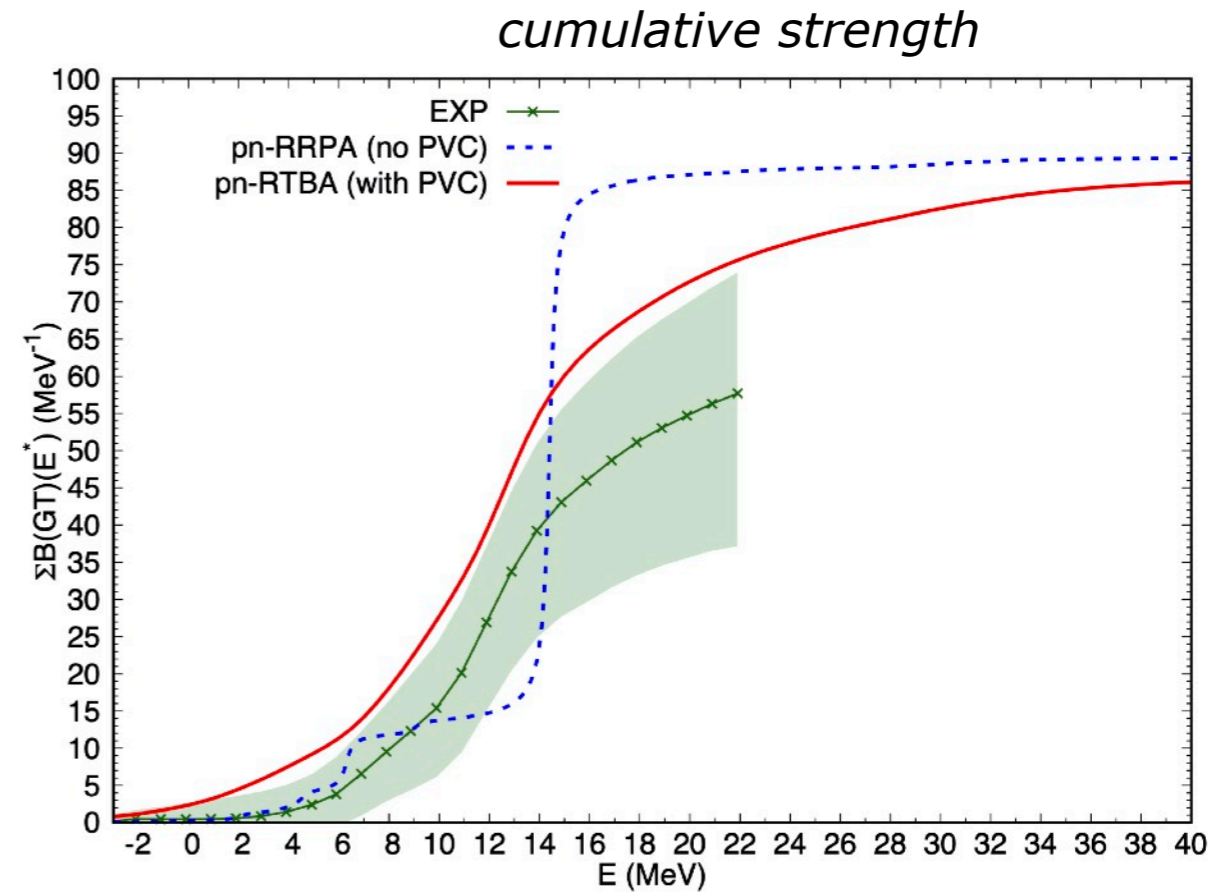
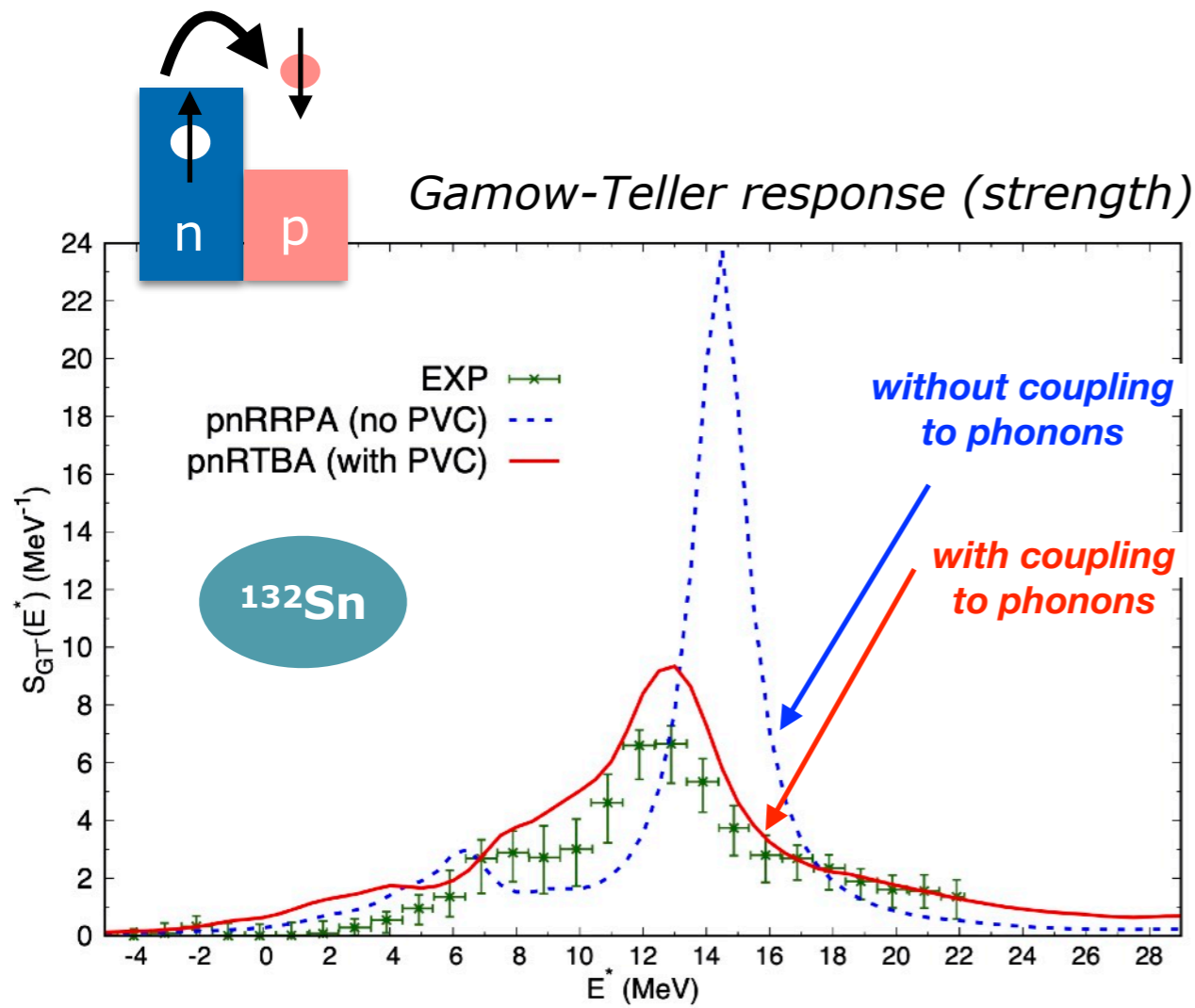
Degrees of freedom in RNFT:



systematic expansion in the PVC vertex



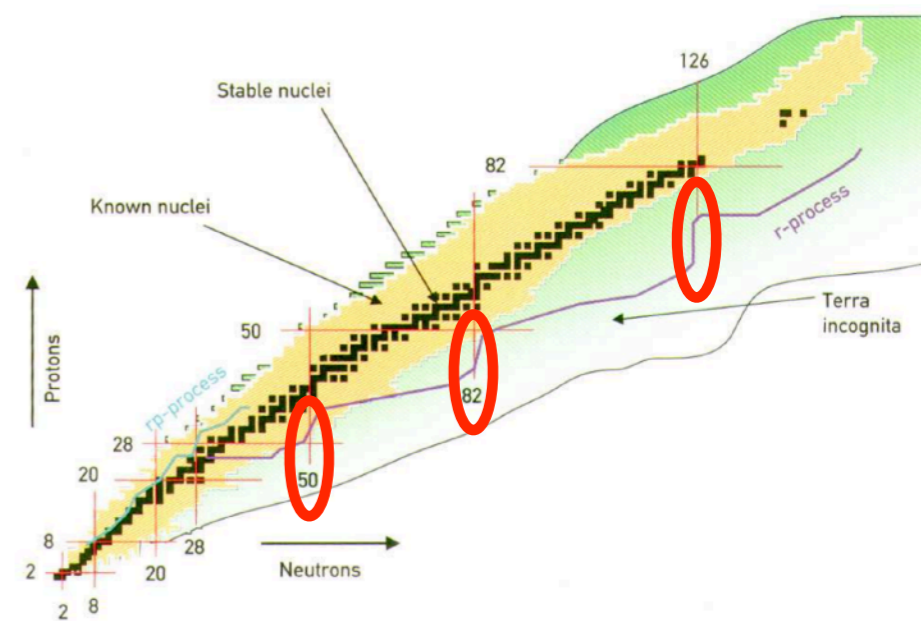
- ◆ Applicability up to heavy/superheavy masses to be useful for astrophysical applications
- ◆ while allowing for a precise description of nuclear phenomena
- ◆ not ab-initio but no new parameters are introduced when going beyond mean field



The particle-vibration coupling provides a natural “quenching mechanism”

β -decay half-lives of *r*-process nuclei including first-forbidden transitions

CR, Litvinova, Martínez-Pinedo, NIC-XVI proceedings, arXiv:2111.14841 (2021)
 CR, Martínez-Pinedo, Litvinova, in preparation

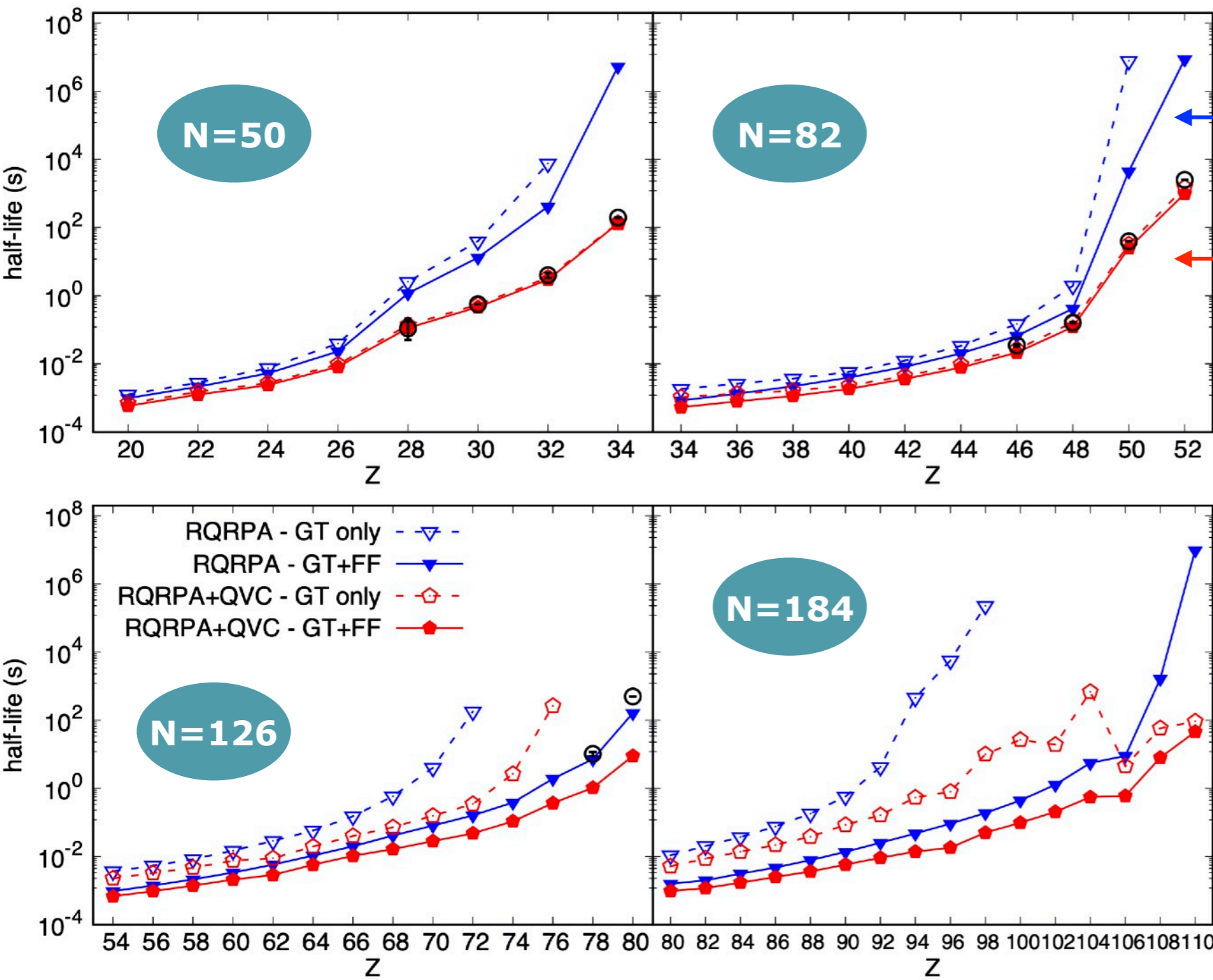


without coupling to phonons

with coupling to phonons

no quenching of GT or FF operators
 ($g_A = -1.276$)

- ▶ large improvement of the half-lives for N=50,82 without extra parameter
- ▶ large impact of the FF modes in N=126,184 chains



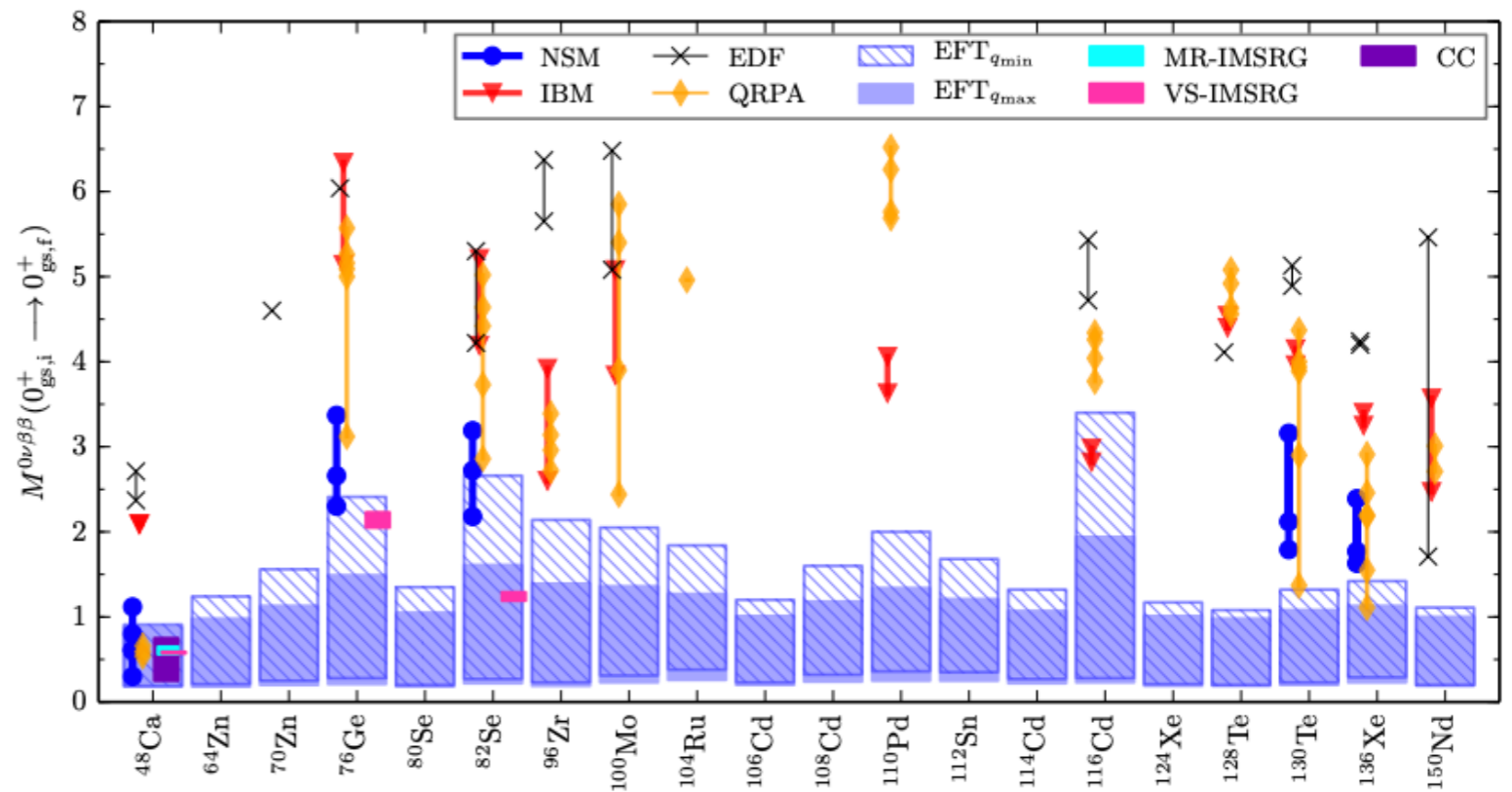
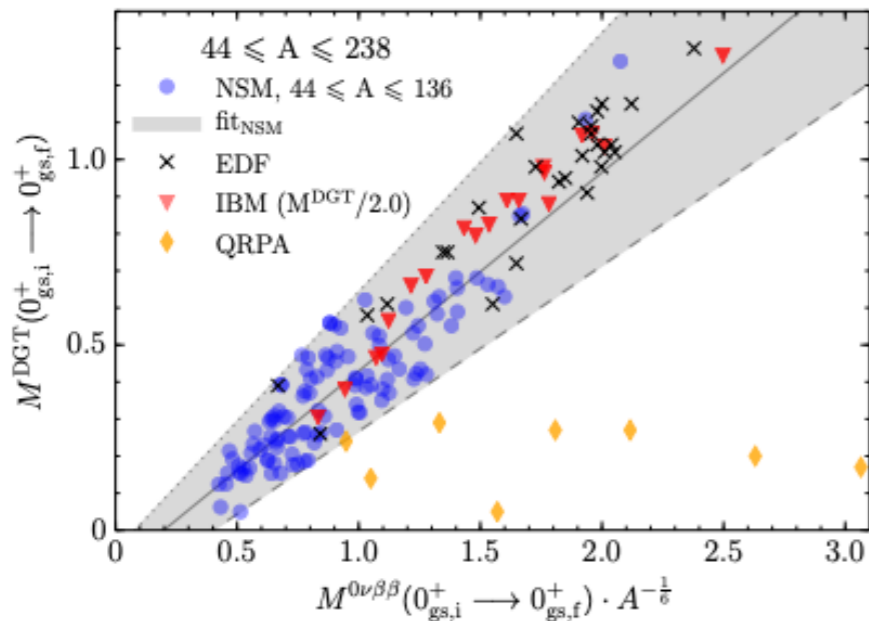
EFT for heavy nuclei

- EFT with collective quadrupole phonons (d) coupled to nucleons (p,n)
- Effective Gamow-Teller (GT) operator with low-energy constants fitted to GT data

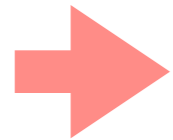
$$\hat{O}_{GT} = C_{\beta}(\tilde{p} \otimes \tilde{n})^{(1)} + \sum_{\ell} C_{\beta\ell} \left[(d^{\dagger} + \tilde{d}) \otimes (\tilde{p} \otimes \tilde{n})^{(\ell)} \right]^{(1)} + \sum_{L\ell} C_{\beta L\ell} \left[(d^{\dagger} \otimes d^{\dagger} + \tilde{d} \otimes \tilde{d})^{(L)} \otimes (\tilde{p} \otimes \tilde{n})^{(\ell)} \right]^{(1)}$$

Neutrinoless double-beta decay from an effective field theory for heavy nuclei,
C. Brase, J. Menéndez, E. A. Coello Pérez, and A. Schwenk, arXiv:2108.11805 (2021)

- used established correlations between double GT and $0\nu\beta\beta$ decay to predict $0\nu\beta\beta$ matrix element with uncertainty estimate



Conclusion, perspectives



Fast progress of *ab-initio* methods based on chiral EFT forces

and continuous effort to extend the reach and precision of the calculations

* theoretical uncertainties

Roth et al. working on a range of methods using concepts from Bayesian statistics as well as Machine Learning techniques to improve uncertainty estimates and model space extrapolations for NCSM and IM-SRG methods (paper coming soon)

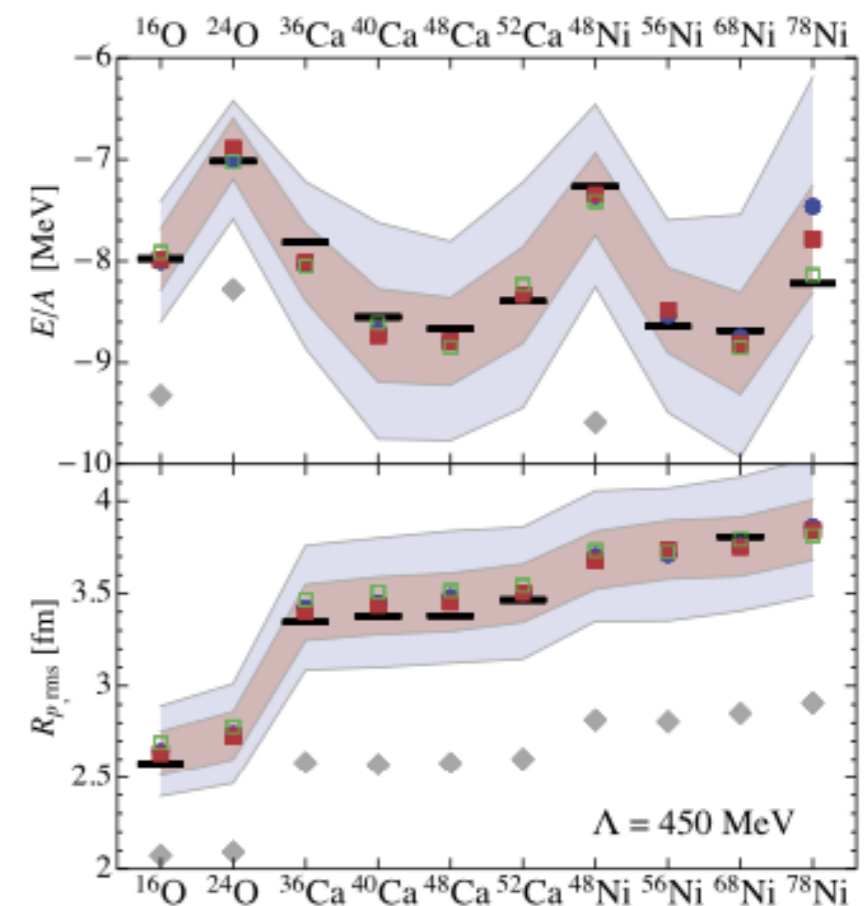
* development of chiral interactions

- *Family of chiral two- plus three-nucleon interactions for accurate nuclear structure studies, Hüther, Vobig, Hebeler, Machleidt, Roth, PLB 808, 135651 (2020).*

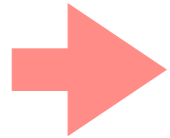
→ accurate description of masses and radii in mid-mass nuclei, and spectroscopy up to sd-shell

- **LENPIC Collaboration** working towards a really consistent formulation of chiral two- plus three-body forces from a fundamental level

Bochum/Bonn/Darmstadt/Jülich: Epelbaum, Krebs, Reinert, Meißner, Hebeler, Hüther, Roth, Vobig, Nogga
+ Iowa, Ohio, Kraków, Japan



Conclusion, perspectives



Heavy nuclei remain out of reach of *ab-initio* methods

* urgent need to improve the nuclear physics inputs for $\beta\beta$ -decay experiments and r-process simulations

→ RNFT and EFT for heavy nuclei are possible ways

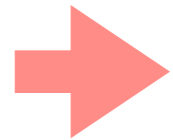
→ works in pair with experimental progress (radioactive-beam facilities)

* in the longer term, would like to link these methods to *ab-initio* concepts

→ RNFT: can one avoid the use of phenomenological functionals?

→ EFT for heavy nuclei: can one derive the parameters?

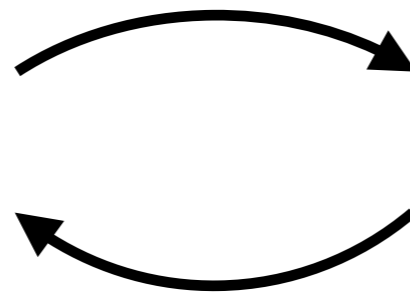
Conclusion, perspectives



Towards Quantum Computing nuclei

- * Quantum Computing holds the promise of exact solutions of the (nuclear) many-body problem
- * Studies of entanglement in nuclei can help design algorithms for near-term devices
 - ▶ example: The variational natural single-particle basis localizes entanglement
 - ⇒ can be useful for **designing workflows for hybrid classical-quantum computations** of nuclei (weakly vs strongly entangled parts of the Hilbert space)

active space diagonalization



orbital optimization (full space)



(new orbitals naturally ordered by entanglement)