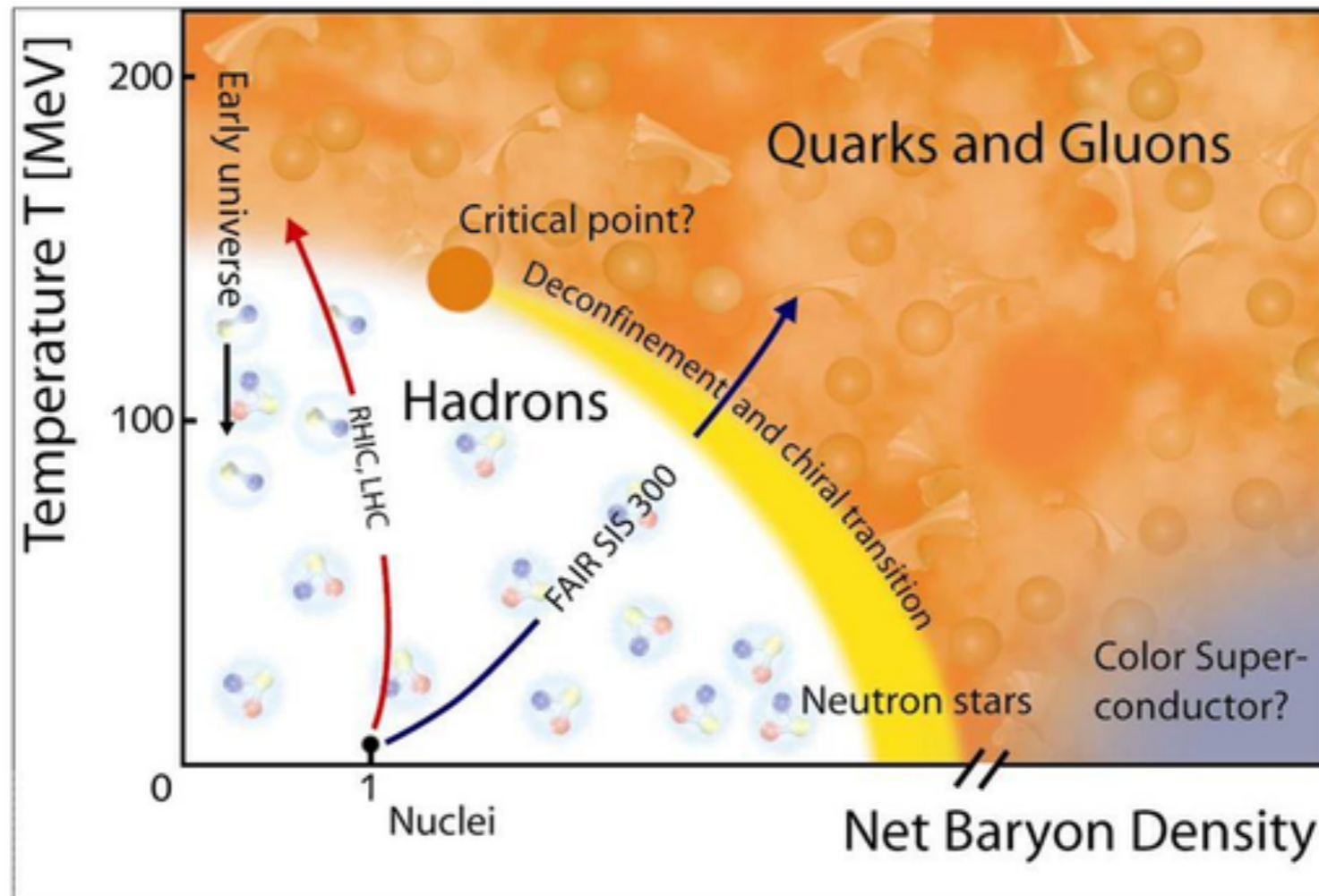


The QCD phase diagram: from heavy ion collisions to neutron star mergers

Owe Philipsen

- Three exciting years: 2019-2021
- Breakthrough for chiral transition in the massless limit ?
- Increased overlap + cooperation between lattice and functional communities

The QCD phase diagram

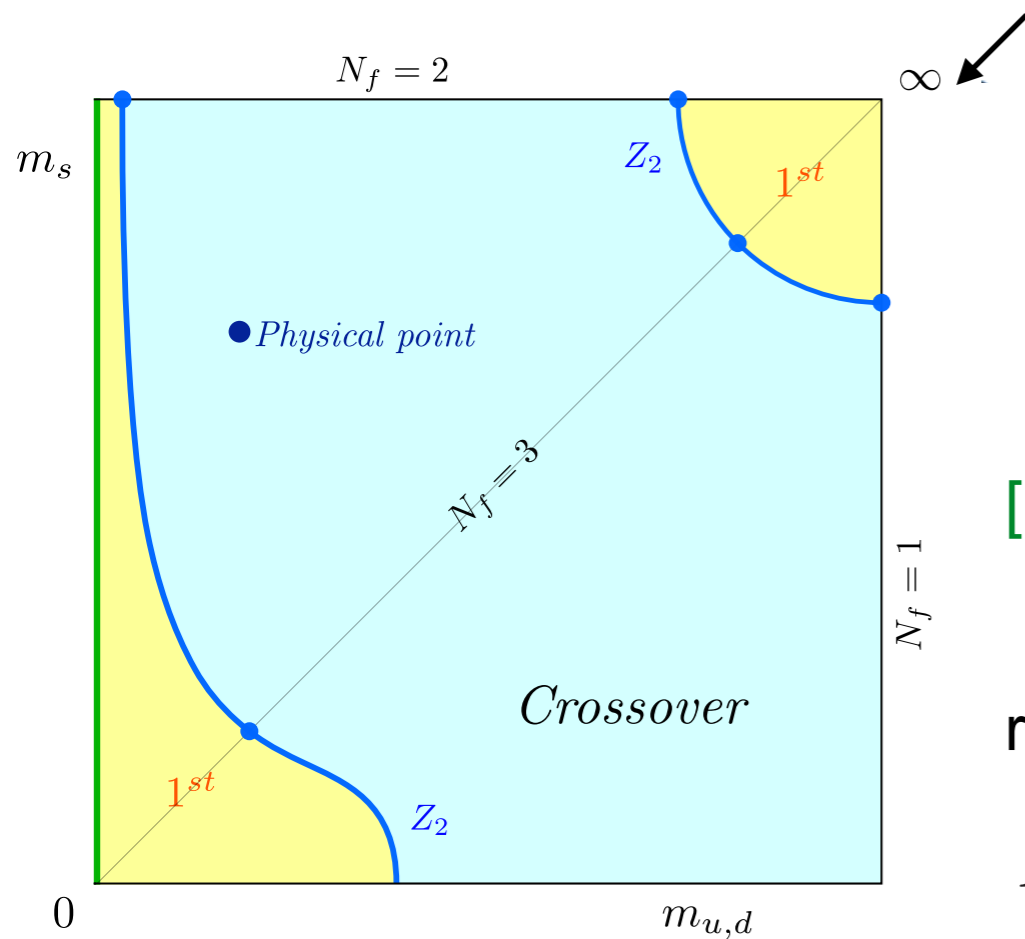


from GSI

- Fundamental for particle-, nuclear-, astro- physics and cosmology, textbook knowledge!
- Non-perturbative nature/confinement prevents perturbative solution
- “Sign problem” prevents Monte Carlo simulation (NP-hard problem?)

The nature of the QCD thermal transition at zero density

deconfinement p.t.:
breaking of global $Z(3)$ symmetry

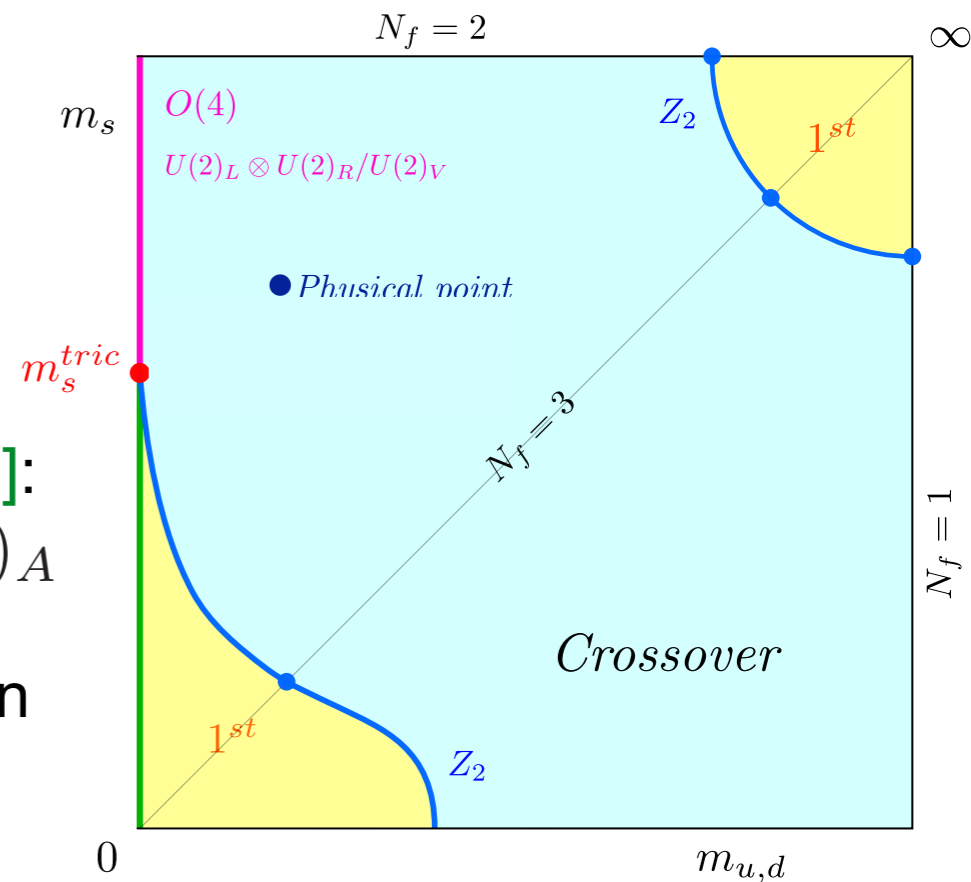


[Pisarski, Wilczek, PRD 84]:
 $N_f = 2$ depends on $U(1)_A$

restored

broken

$N_f \geq 3$ 1st order



chiral p.t.

restoration of global symmetry in flavour space

$$SU(2)_L \times SU(2)_R \times U(1)_A$$

↑
anomalous

Can a trace of the chiral transition
(scaling) be detected experimentally?

Heavy mass corner: bench mark for effective theories

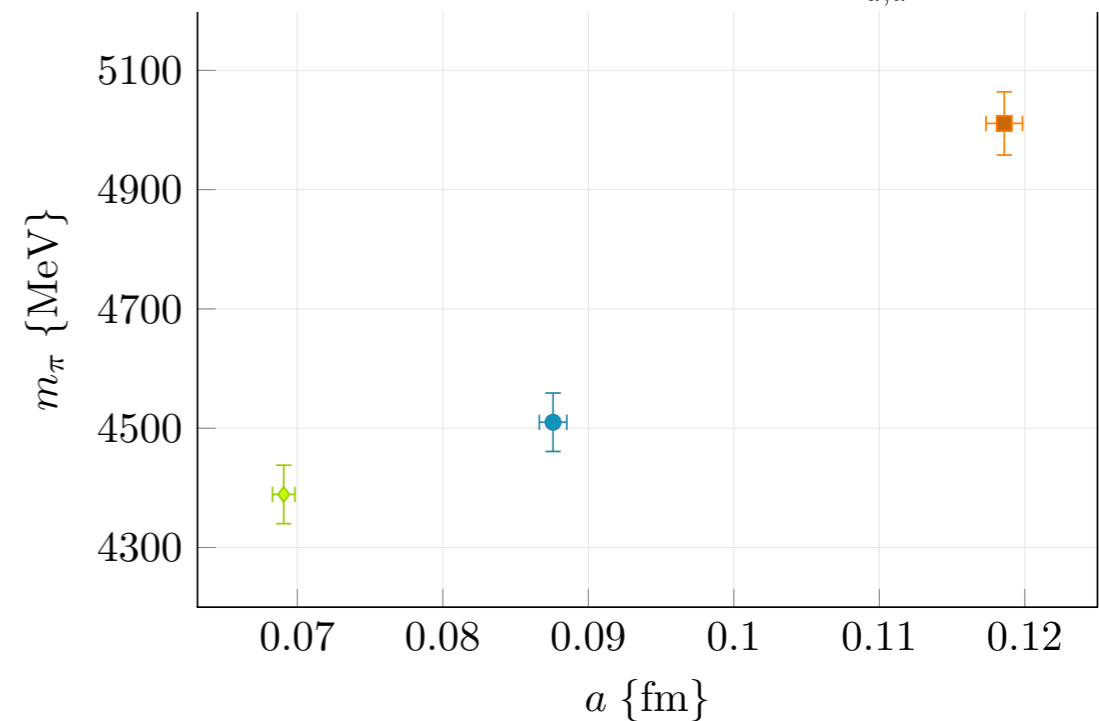
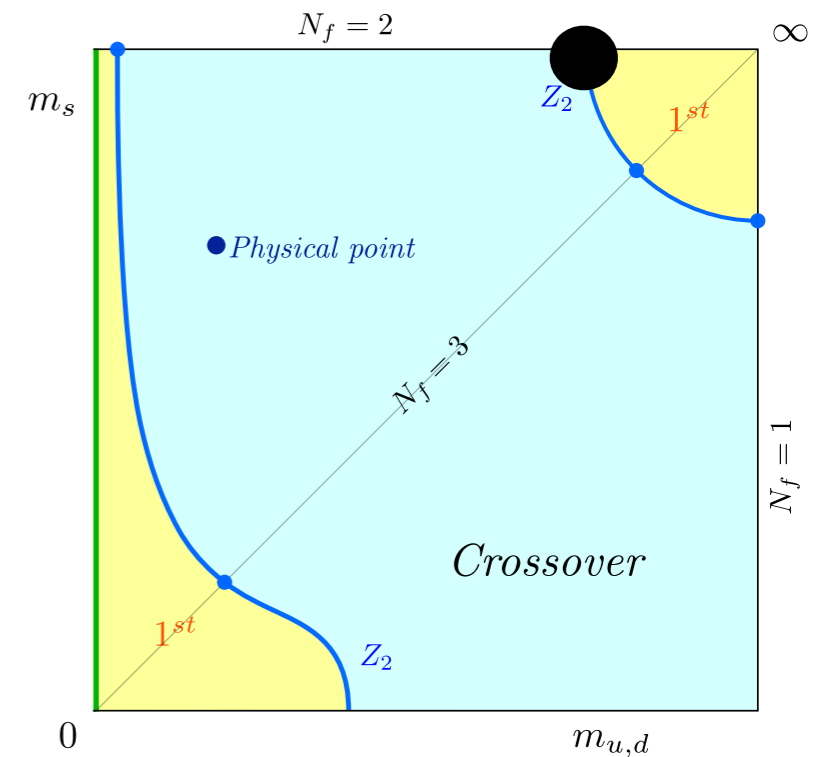
- Z(3) symmetry explicitly broken by $\frac{1}{m_q}$
- Deconfinement transition weakens, disappears at $\frac{1}{m_q^c} \Leftrightarrow m_\pi^c$
- Lattice determination in progress: $m_\pi^c \approx 4 \text{ GeV}$ [WHOT, Frankfurt]

● Dyson-Schwinger study $m_q^c \approx 460 \text{ MeV}$ [Fischer, Luecker, Pawłowski, PRD 15]

● Effective lattice theory for heavy quarks within 12% of full result [Fromm et al., JHEP 12; Kim et al. LAT 21]

Also applicable to finite μ_B

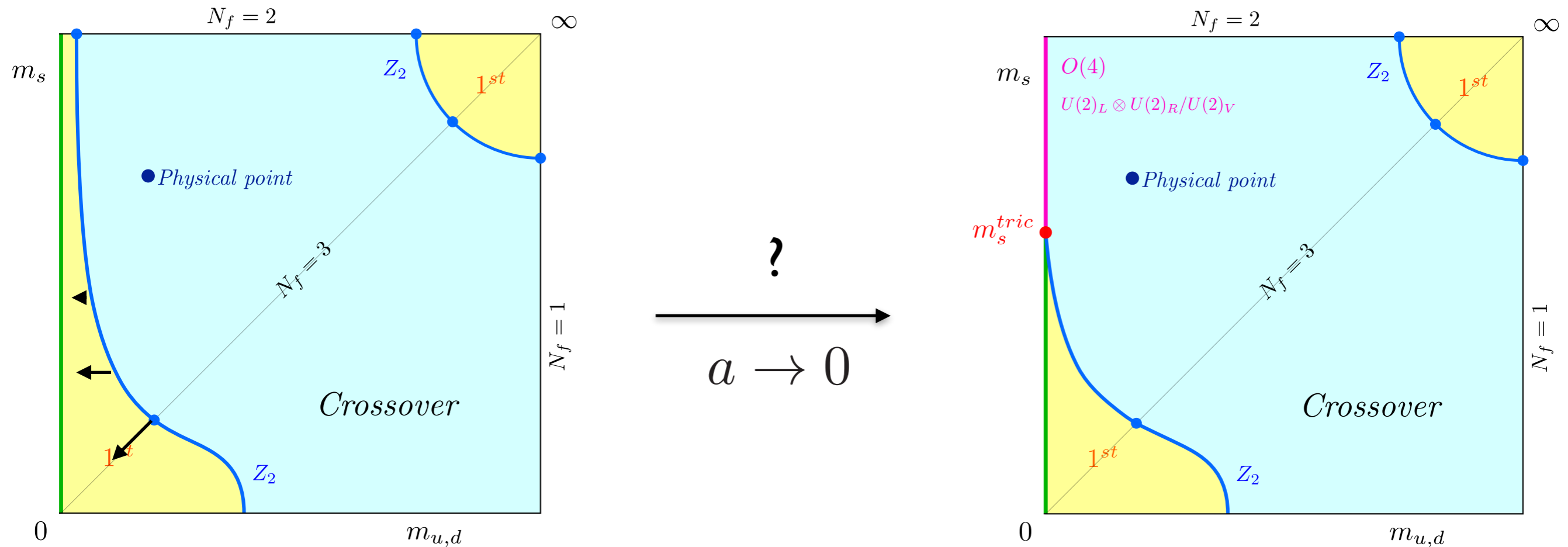
Nuclear liquid gas transition from QCD! [Fromm et al., PRL 12]



Cuteri, O.P., Schön, Sciarra, PRD 21

The nature of the QCD chiral transition

...is elusive, massless limit not simulable!

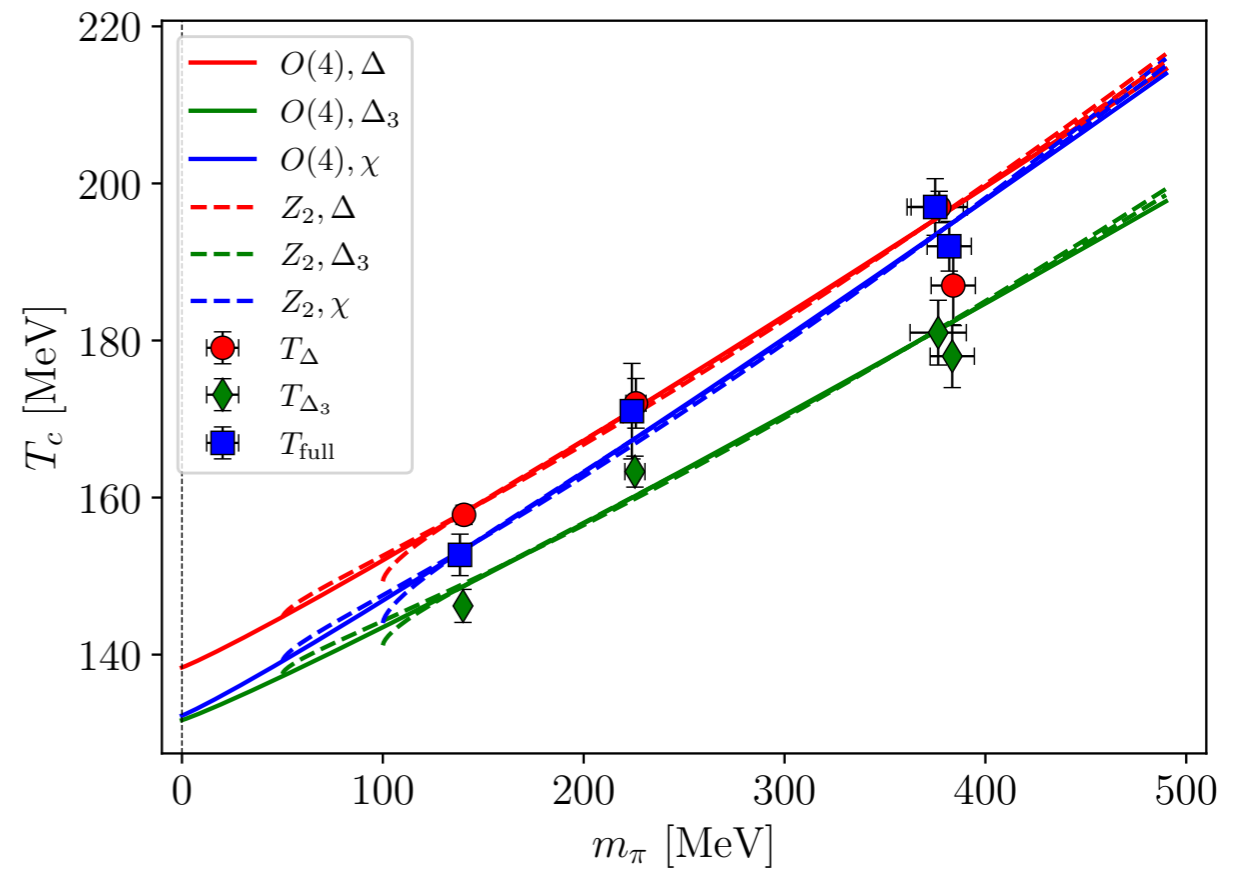
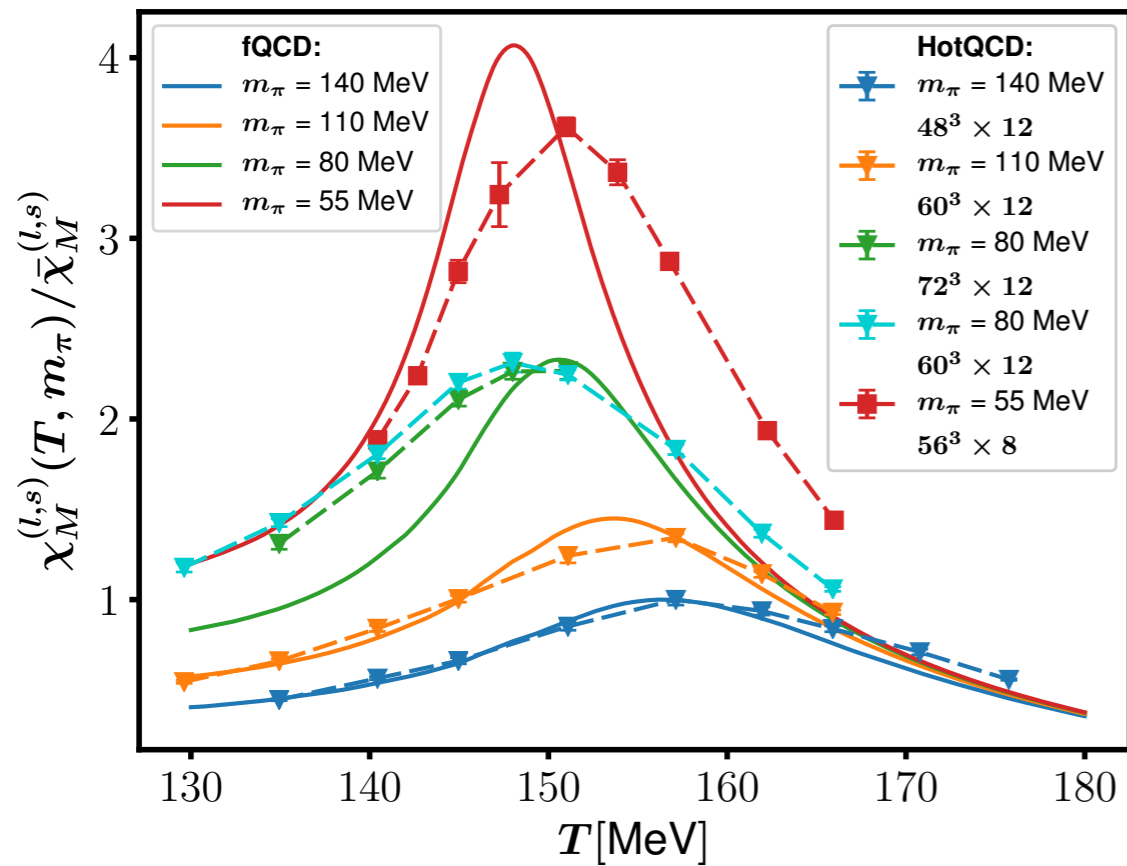


- Coarse lattices or unimproved actions: 1st order for $N_f = 2, 3$
- 1st order region shrinks rapidly as $a \rightarrow 0$
- Improved staggered actions: no 1st order region so far, even for $N_f = 3$ $m_{PS} > 45\text{MeV}$

Details and references: [O.P., Symmetry 13, 2021]

From the physical point to the chiral limit

arXiv:2012.06231



[HotQCD, PRL 19] HISQ (staggered)

[Kotov, Lombardo, Trunin, PLB 21] Wilson twisted mass

$$T_c^0 = 132_{-6}^{+3} \text{ MeV}$$

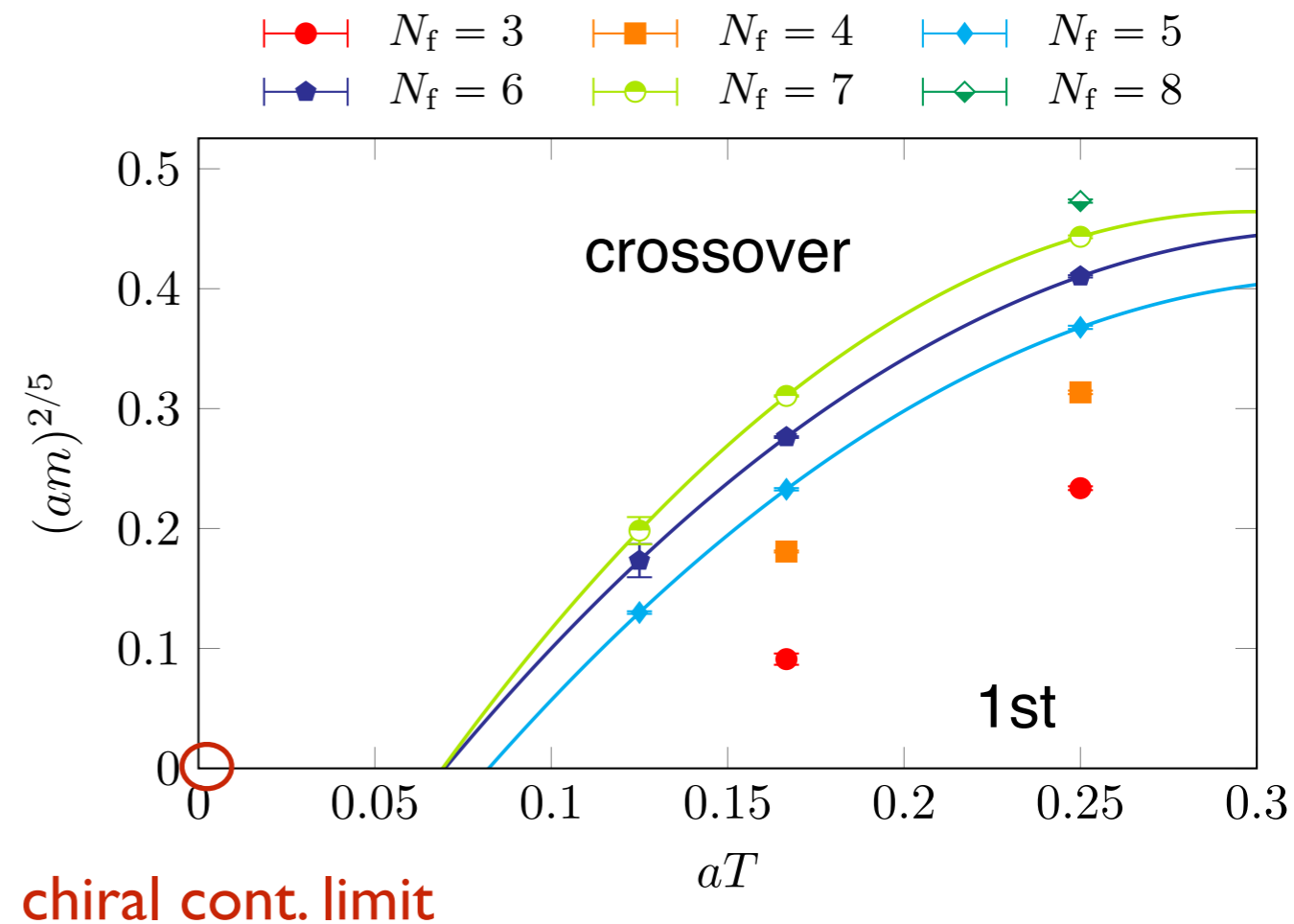
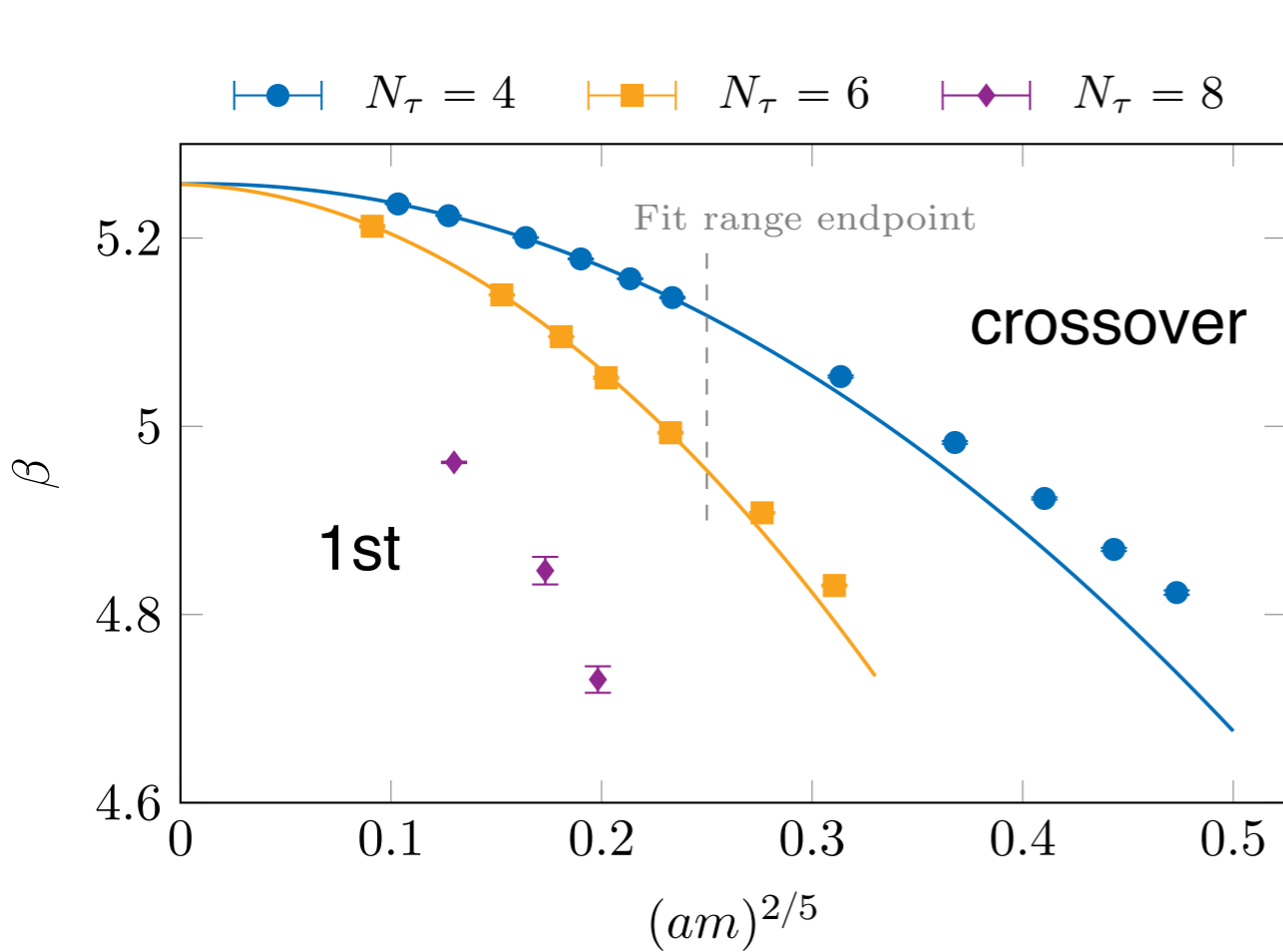
$$T_{pc}(m_l) = T_c^0 + K m_l^{1/\beta\delta}$$

$$T_c^0 = 134_{-4}^{+6} \text{ MeV}$$

- Keep strange quark mass fixed, crossover gets stronger as chiral limit approached
- Cannot distinguish between $Z(2)$ vs. $O(4)$ exponents, need exponential accuracy!
- Determination of chiral critical temperature possible, but not the order of the transition
- Comparison with fRG: $T_c^0 \approx 142 \text{ MeV}$, "most likely $O(4)$ " [Braun et al., PRD 20,21]

Nature of chiral transition as function of N_f

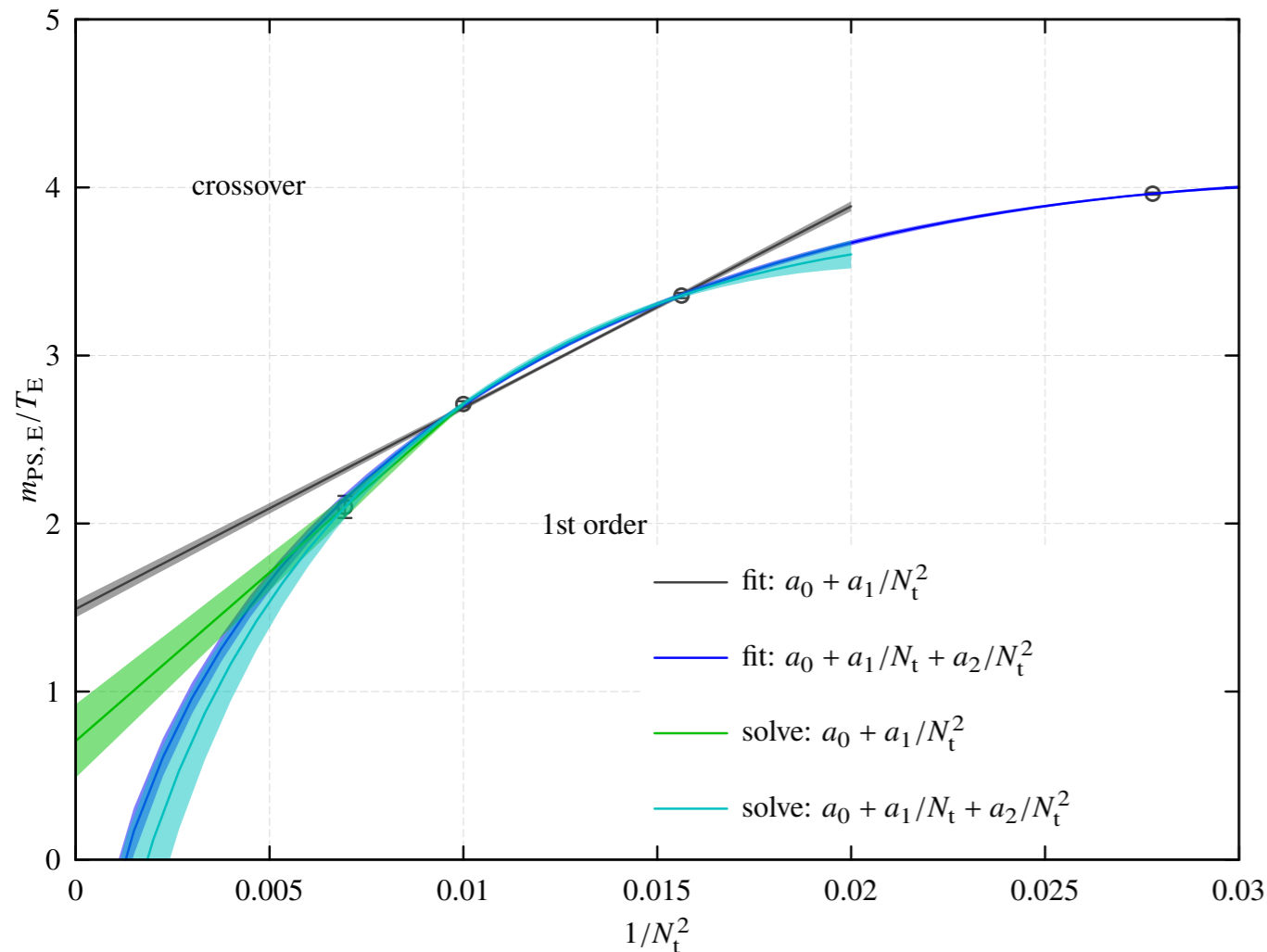
[Cuteri, O.P., Sciarra, JHEP 21] $N_f \in [2, 8]$ standard staggered



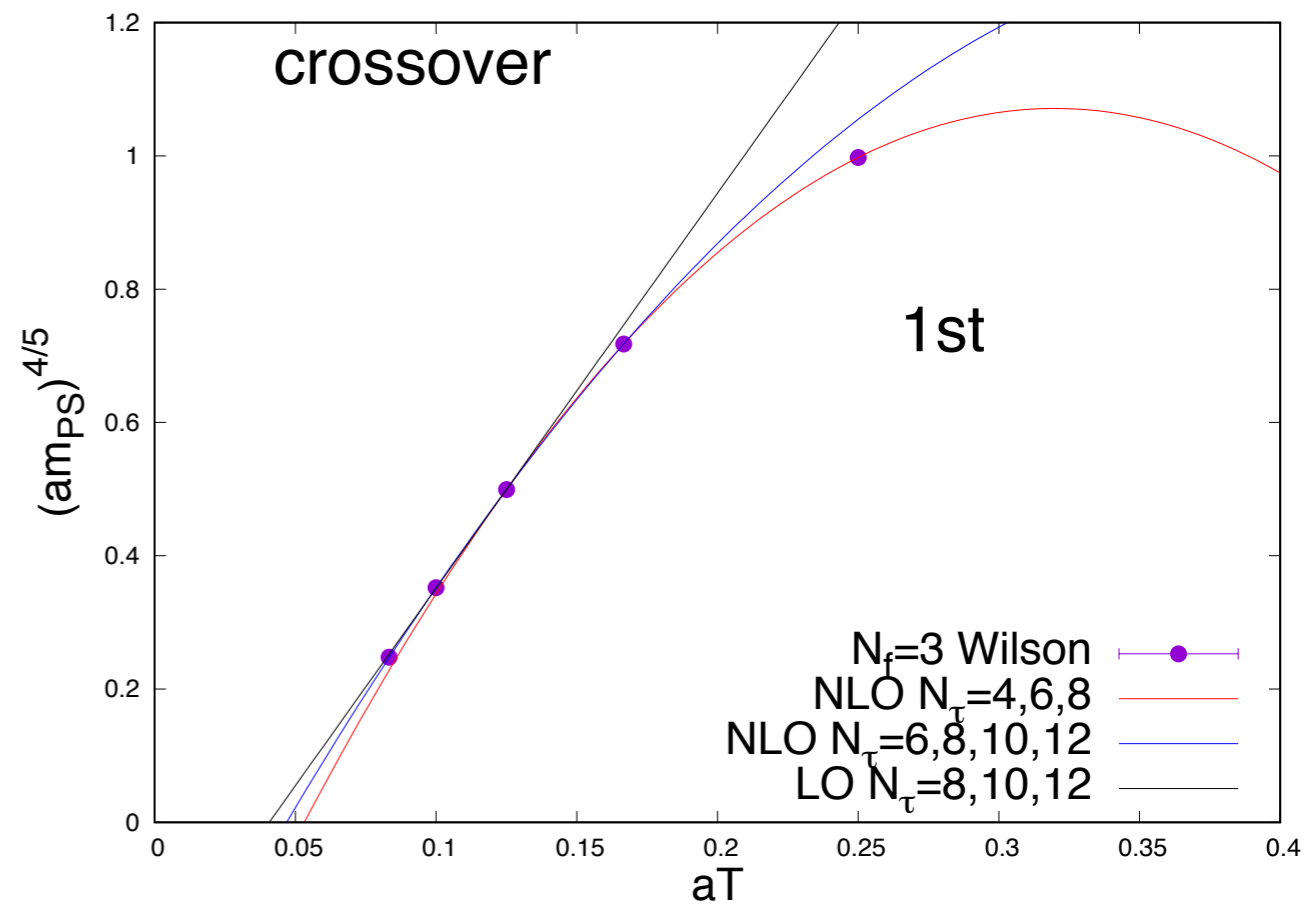
- **Tricritical endpoints+scaling** of chiral critical boundary
- Known exponents, i.e. chiral extrapolation is possible!
- Finite $N_\tau^{\text{tric}}(N_f)$ implies **second-order transition in chiral continuum limit!**

Nf=3 O(a)-improved Wilson fermions

[Kuramashi et al. PRD 20] $m_\pi^c \leq 110$ MeV $N_\tau = 4, 6, 8, 10, 12$



Re-analysis using: $am_{PS}^2 \propto am_q$

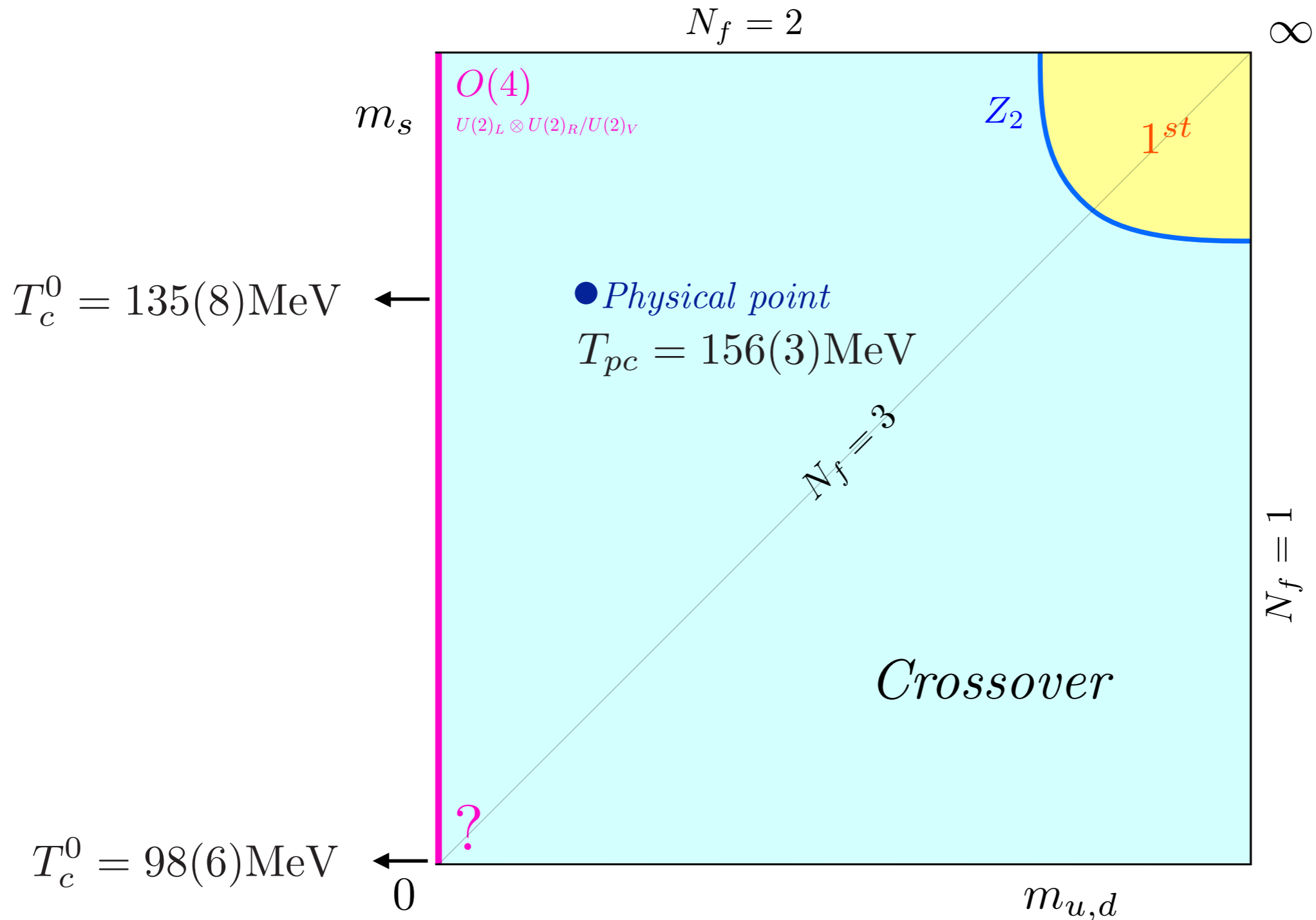


[Cuteri, O.P., Sciarra, JHEP 21]

Tricritical scaling, Nf=3 consistent with staggered, 2nd order in continuum!

The emerging final(?) Columbia plot

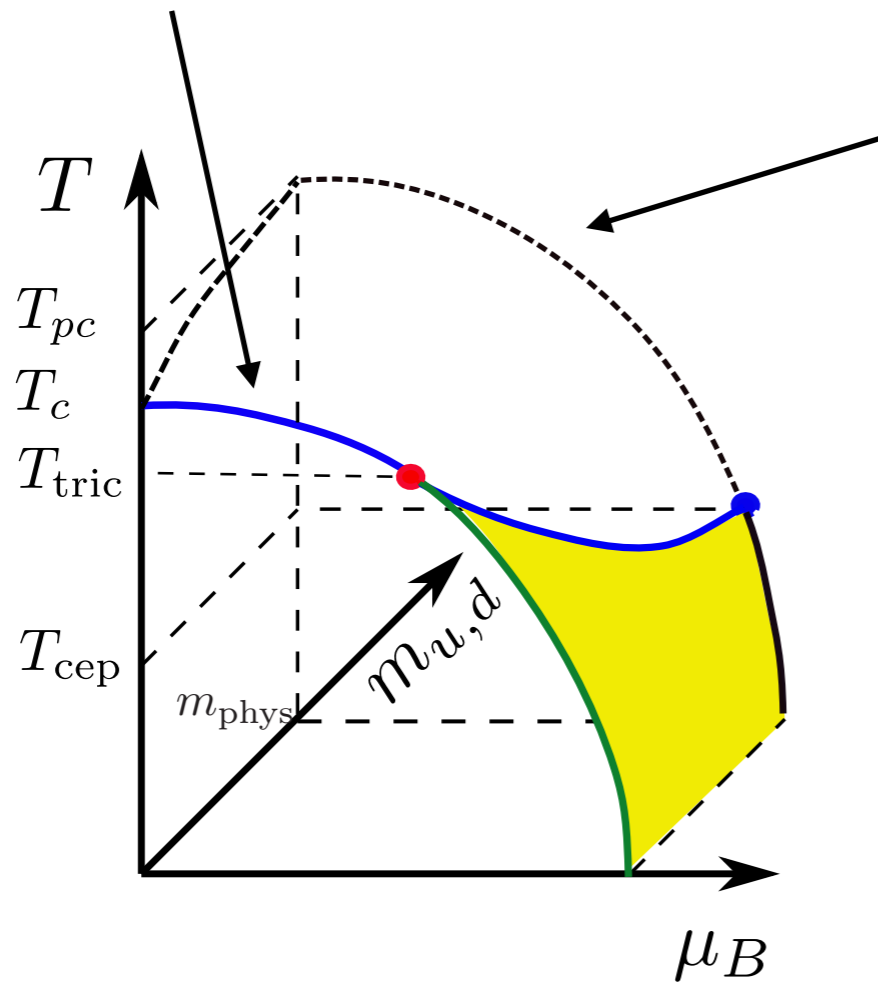
$$m_\pi^c \approx 4.0(4) \text{ GeV}, T_c \approx 285(10) \text{ MeV}$$



Chiral limit and the physical point

The “standard scenario”: [Halasz et al., PRD 98; Hatta, Ikeda, PRD 03...]

Importance of the chiral limit!



$$\frac{T_{pc}(\mu_B)}{T_{pc}(0)} = 1 - \kappa_2 \left(\frac{\mu_B}{T_{pc}(0)} \right)^2 + \dots$$

κ_2	Action
0.0158(13)	imag. μ , stout-smearred staggered
0.0135(20)	imag. μ , stout-smearred staggered
0.0145(25)	Taylor, stout-smearred staggered
0.016(5)	Taylor, HISQ

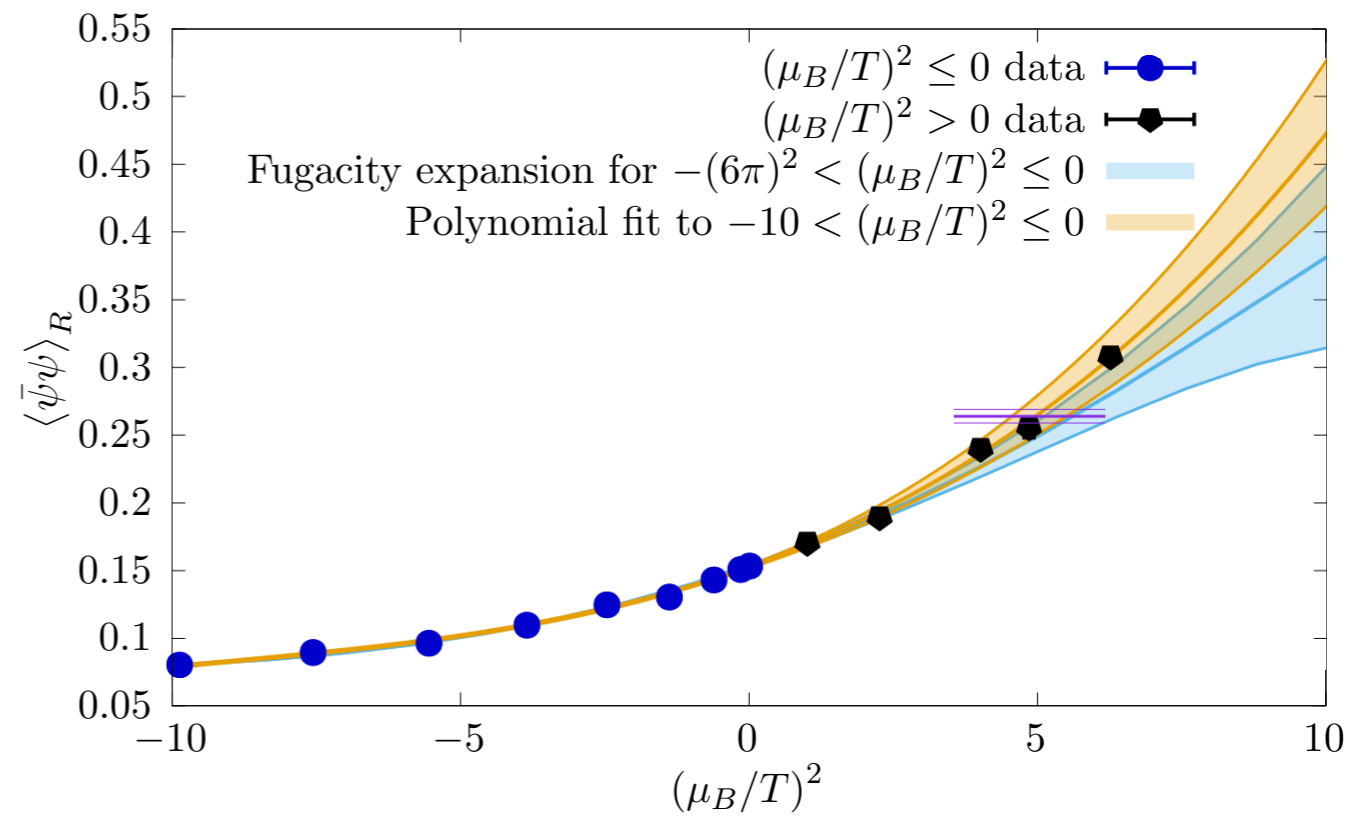
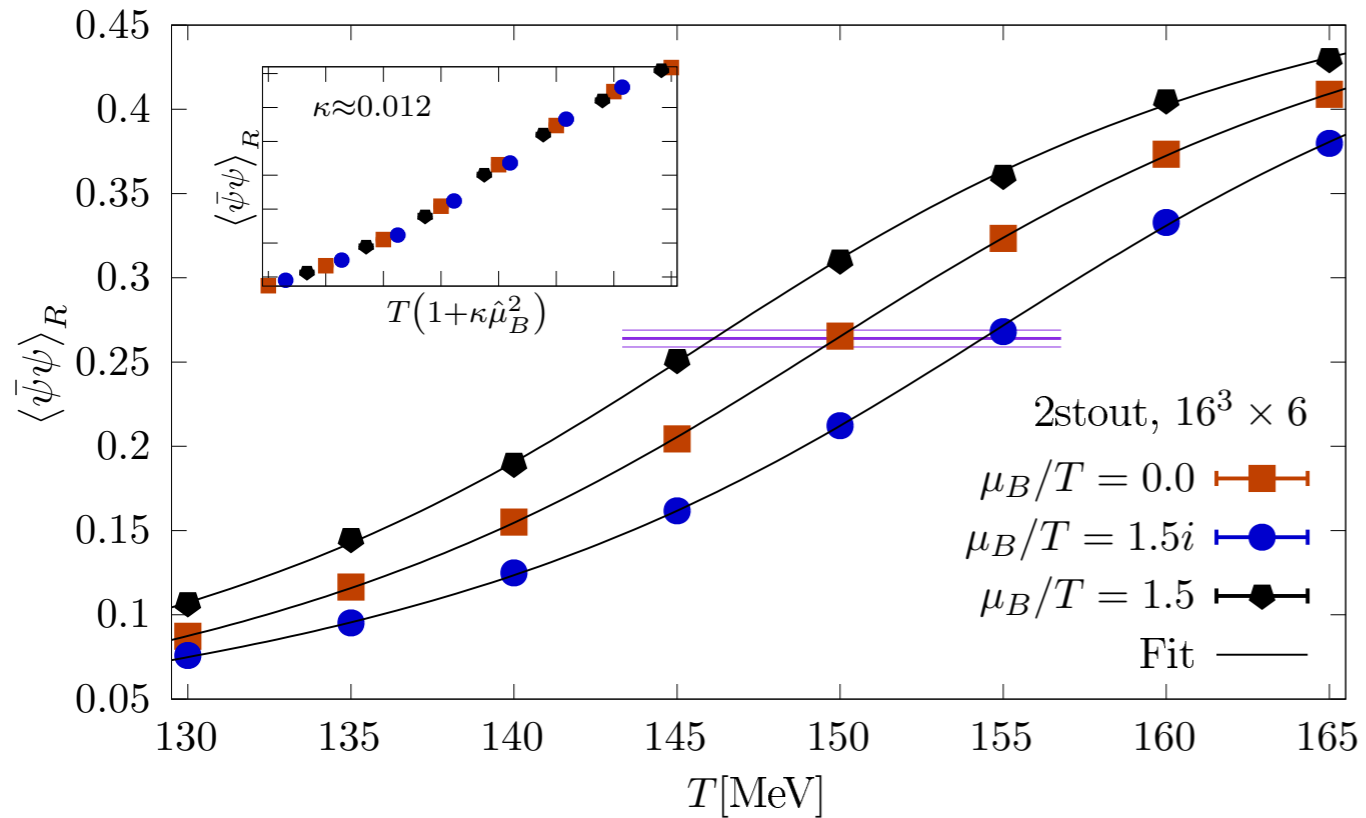
[Bellwied et al, PLB 15]
 [Bonati et al, NPA 19]
 [Bonati et al, PRD 18]
 [HotQCD, PLB 19]

$$T_{pc} > T_c > T_{\text{tric}} > T_{\text{cep}}$$



$$\mu_B^{\text{cep}} > 3.1 T_{pc}(0) \approx 485 \text{ MeV}$$

Critical endpoint: reweighting LQCD revisited

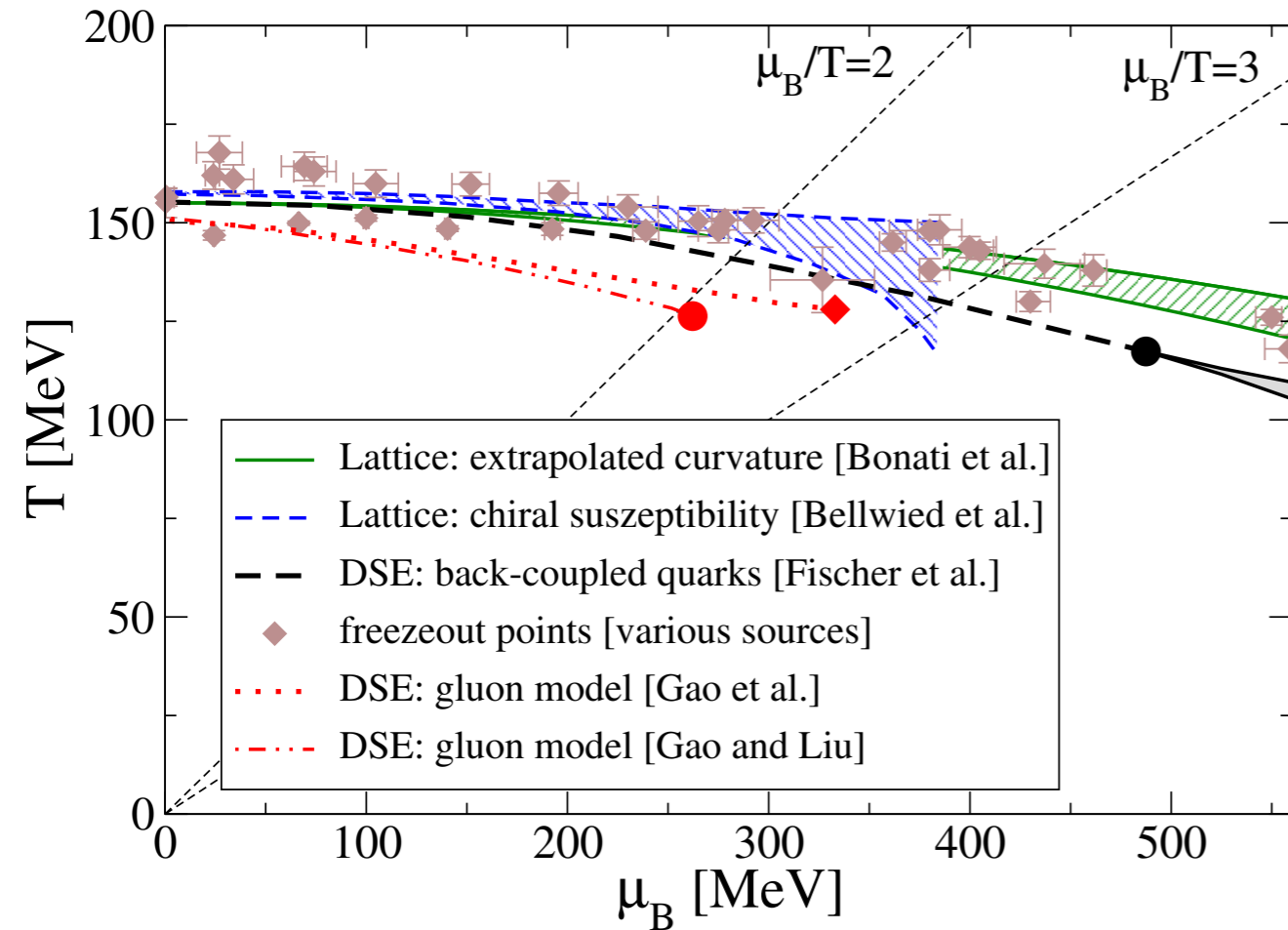


[Borsanyi et al., arXiv:2108.09213]

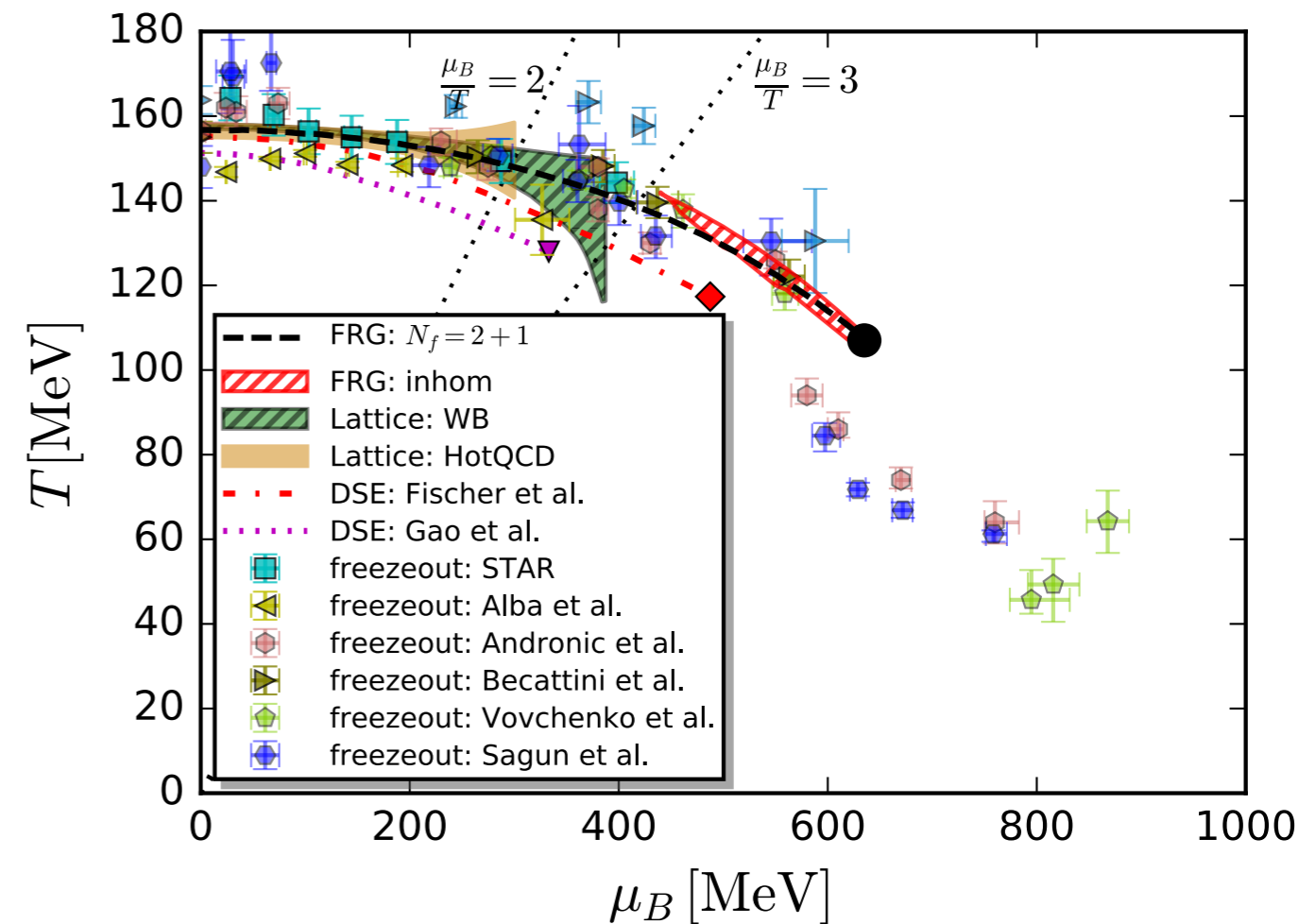
- Fodor, Katz result from 2001 shows gap of rooted staggered fermions, not phase transition [Giordano et al., PRD 20]
- New treatment rooted determinant + reweighting in sign only [Giordano et al. JHEP 20]
- Simulation with stout-sm. staggered action, $N_\tau = 6$: no sign of criticality for $\mu_B < 2.5T$

Critical endpoint: DSE and fRG

[Fischer, PPNP 19]

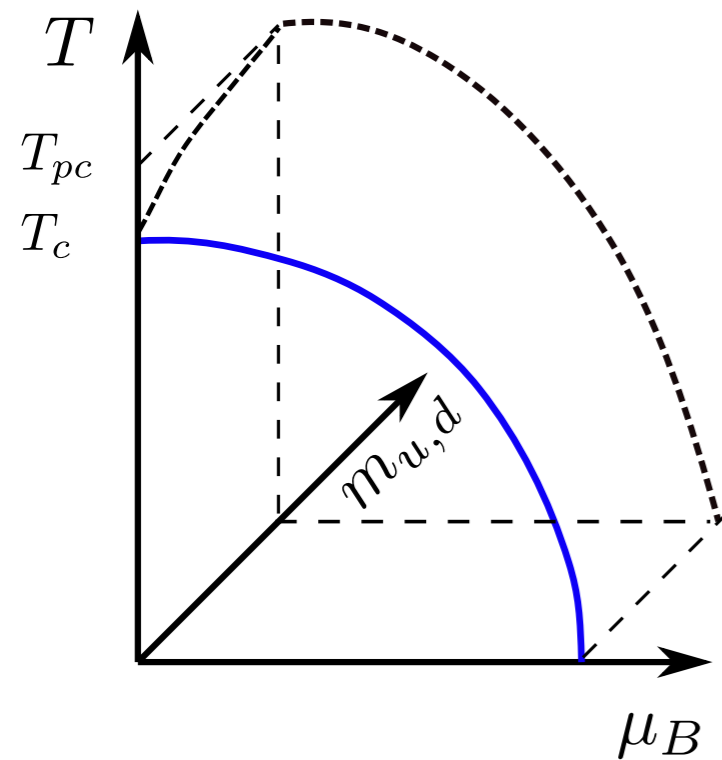


[Fu, Pawłowski, Rennecke, PRD 20]



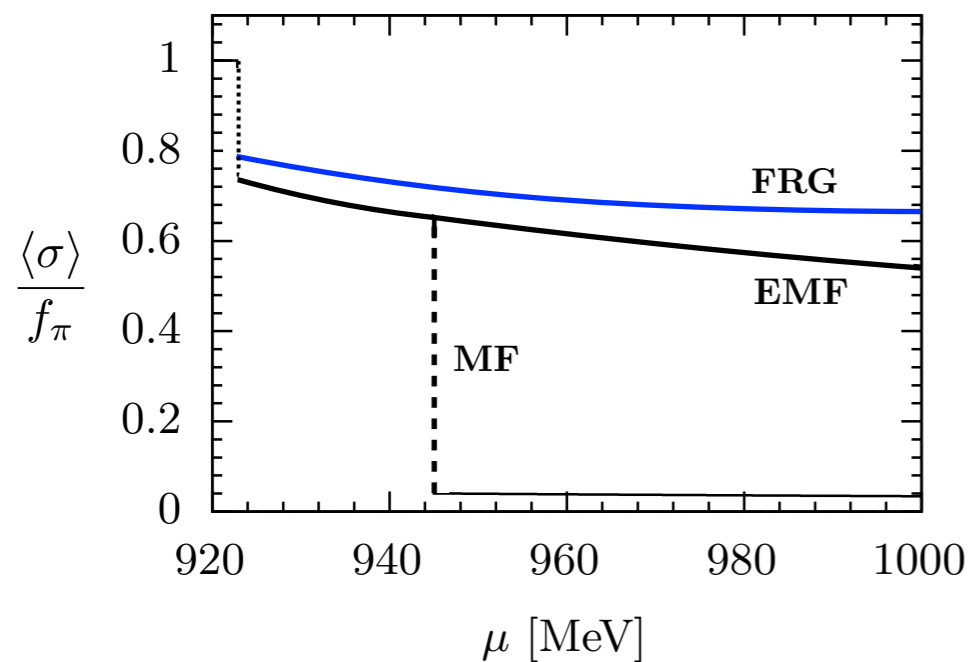
- Quantitative agreement with lattice for curvature $T_{pc}(\mu_B)$
- Critical endpoint seen in current truncations, location consistent with lattice bounds
- Checked for stability under meson backcoupling [Bernhardt et al. PRD 21]
- Same result with hybrid approach DSE+fRG [Gao, Pawłowski, arXiv:2112.01395]

Another possible scenario



- Chiral limit: all second order
- Physical masses: all crossover
- Consistent with **all** available lattice results
- Predicted by some models:

[Brandes, Kaiser, Weise, arXiv:2103.06096]



Chiral nucleon meson model, also chiral quark meson model

-First-order chiral transitions unstable against fluctuations

-Second-order transition in chiral limit, crossover otherwise

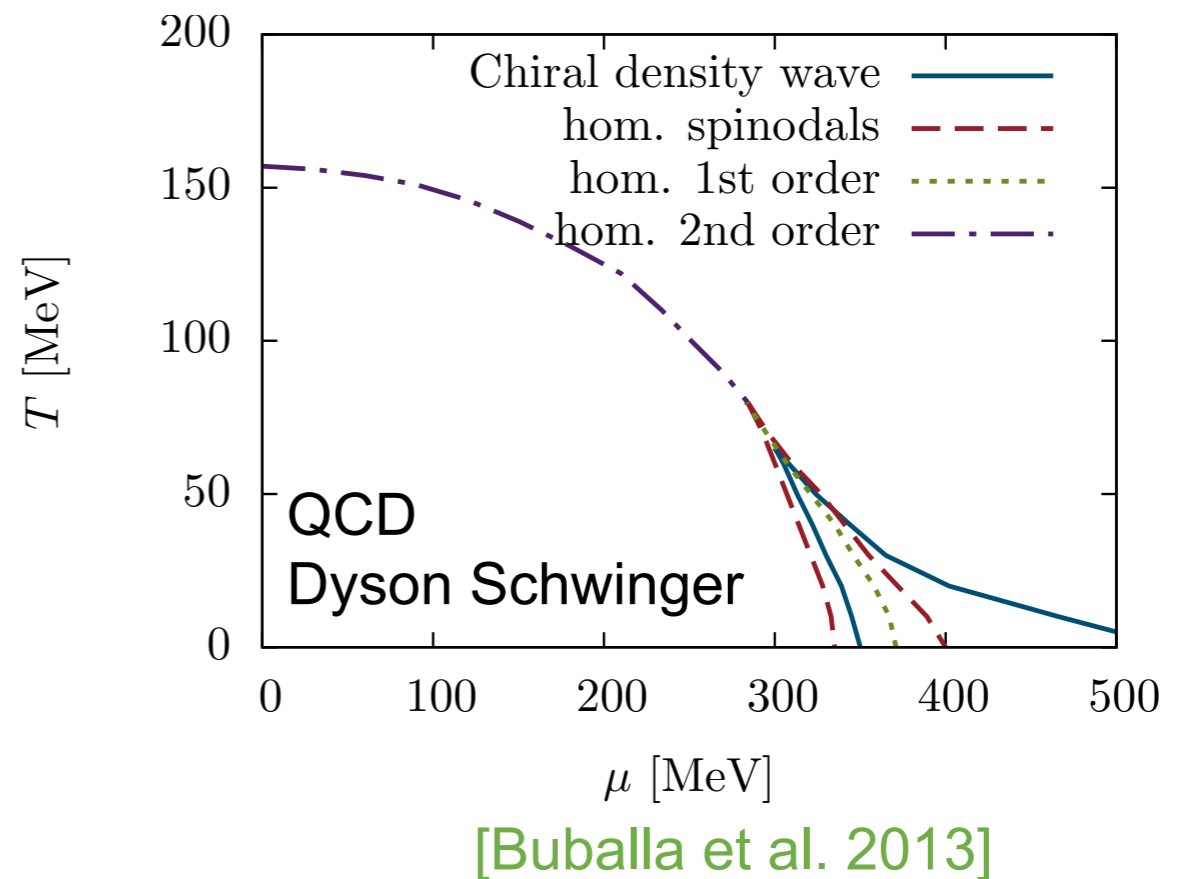
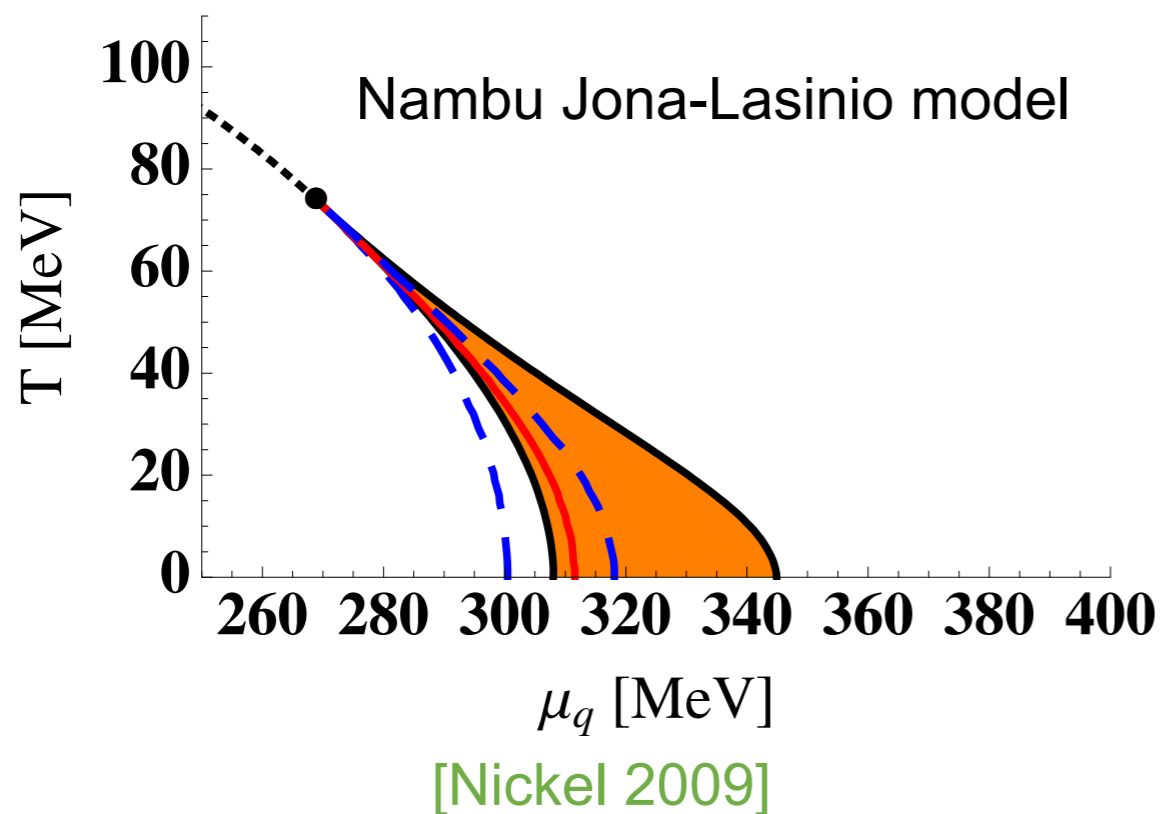
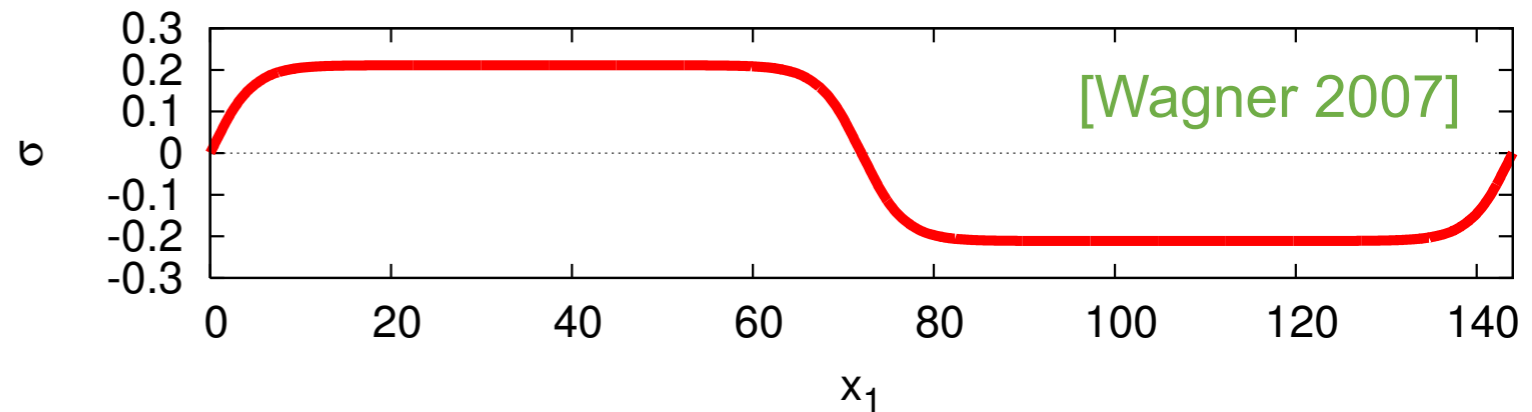
Inhomogeneous phases

QCD-inspired (symmetry!) models exhibit inhomogeneous phases, what about...QCD?

NJL, Gross-Neveu (1+1d), $N_f = \infty$:

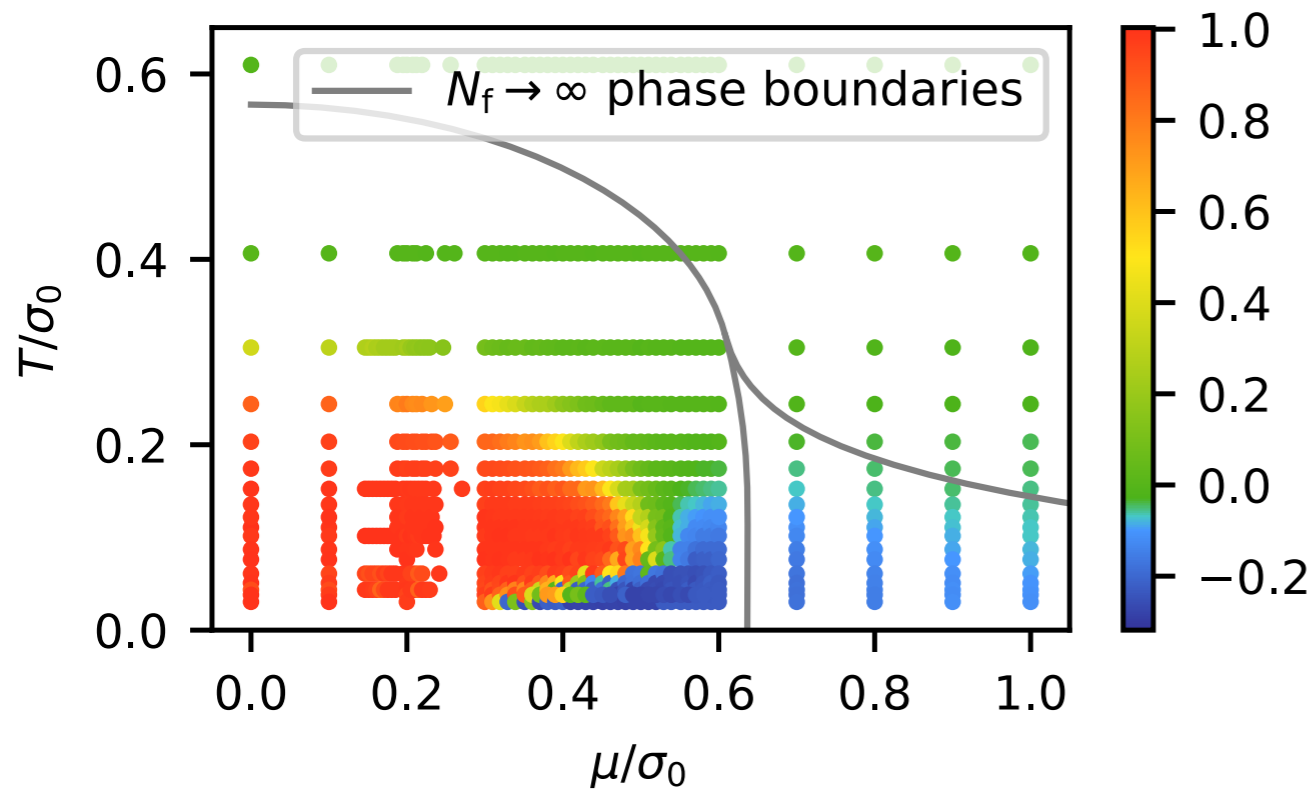
Chiral condensate varies in space

Robust under change of model details



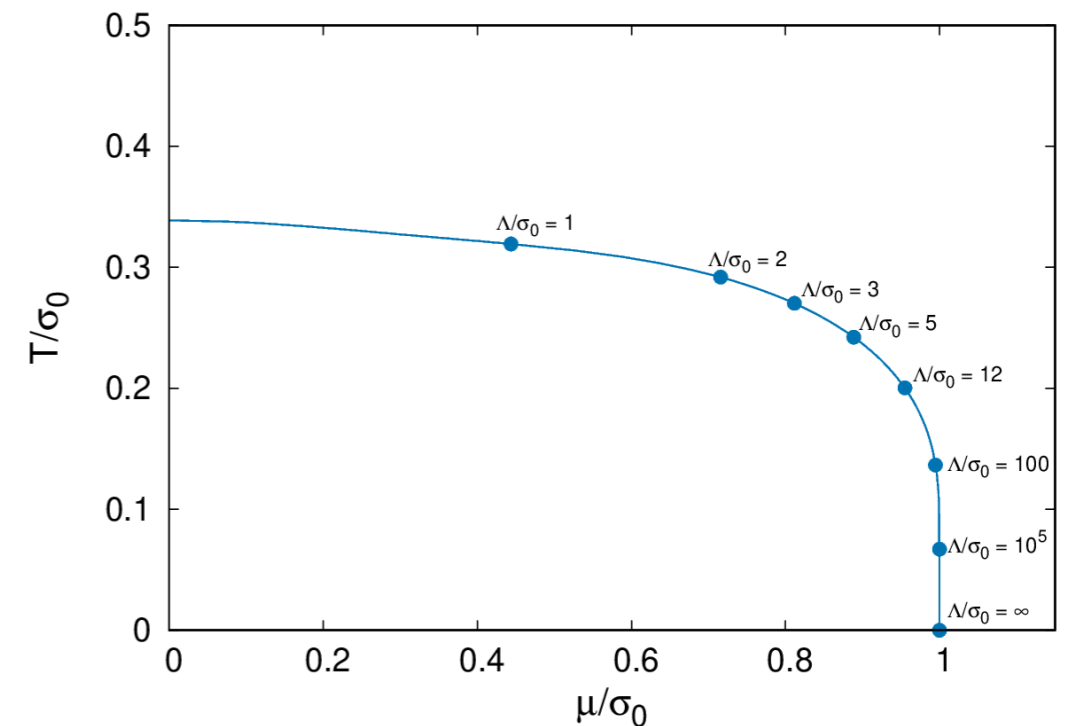
Gross-Neveu, systematic investigations

1 + 1d, $N_f = 8$ [Lenz et al., PRD 20]



First fully non-perturbative lattice observation of an inhomogeneous phase!

2 + 1d, $N_f = \infty$ [Buballa et al., PRD 21]

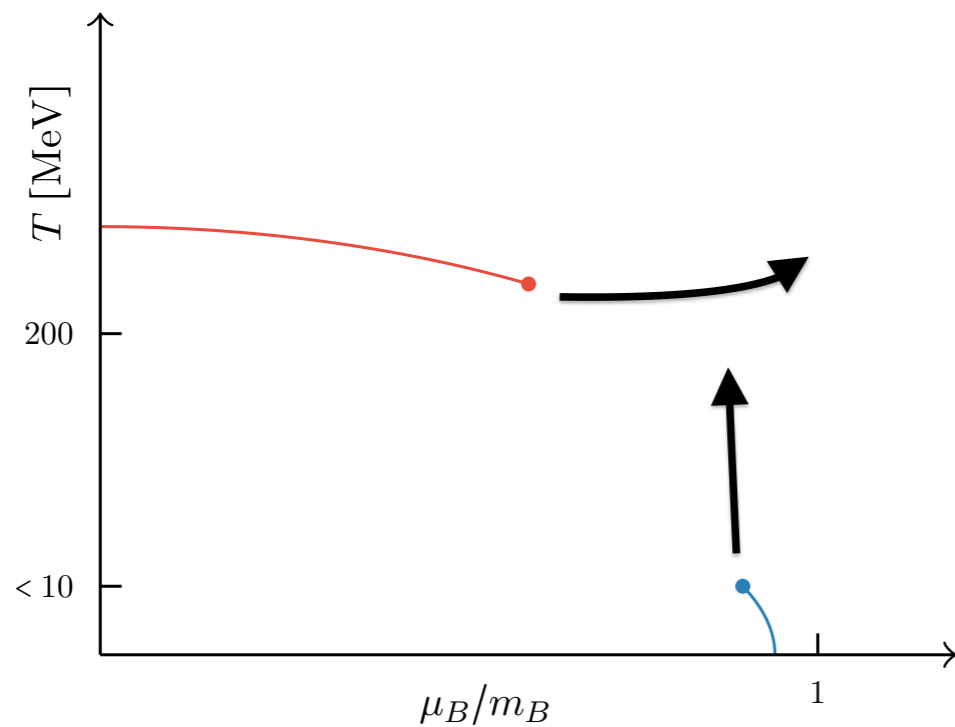


Stability analysis continuum:
Inhomogeneous phase depends on UV cutoff

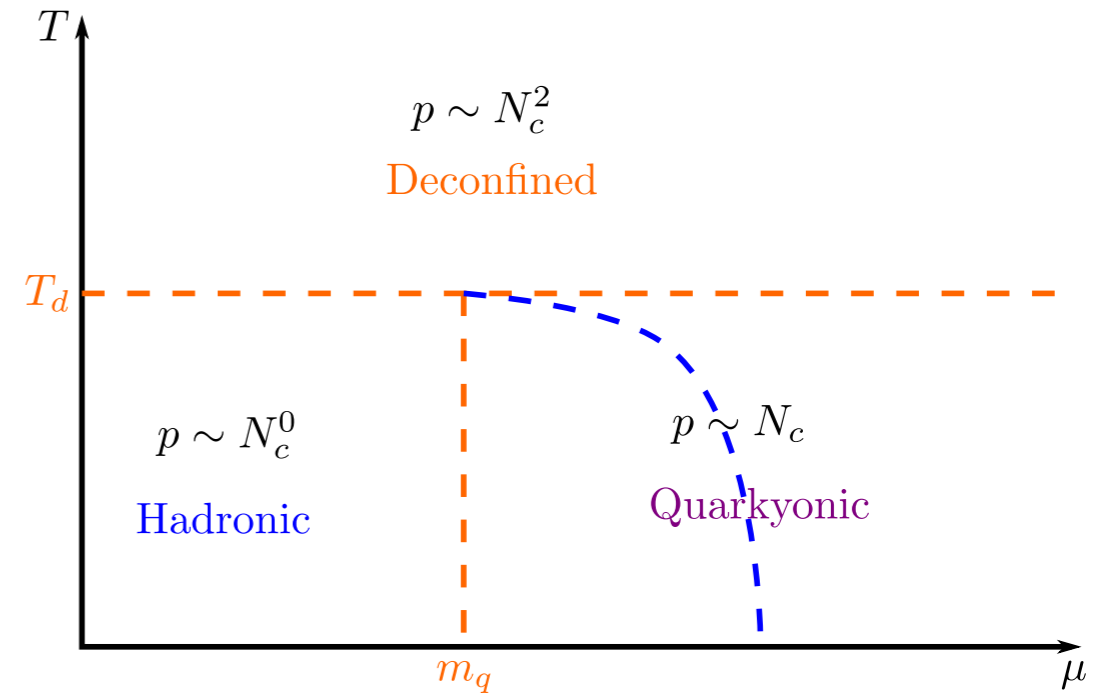
Lattice:
Inhom. phase discretisation-dependent,
so far no sign of it when cutoff is removed

General N_c and quarkyonic matter in 3+1d QCD

Wilson LQCD with heavy quarks [O.P., Scheunert, JHEP 19]

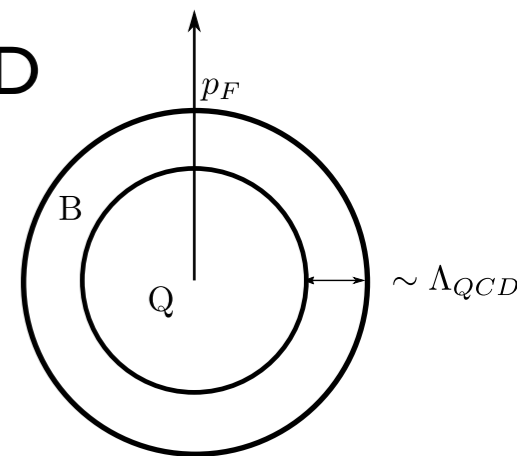


large N_c



[Pisarski, McLerran, NPA 07]

- Conjectured large N_c phase diagram emerges smoothly in heavy QCD
- Baryon matter consistent with quarkyonic matter: $p \sim N_c$ (through three orders in $\frac{1}{m_q}$ expansion)
- No phase transition to quarkyonic matter besides nuclear liquid gas!



New emerging chiral spin symmetry

[Glozman, EPJA 15]

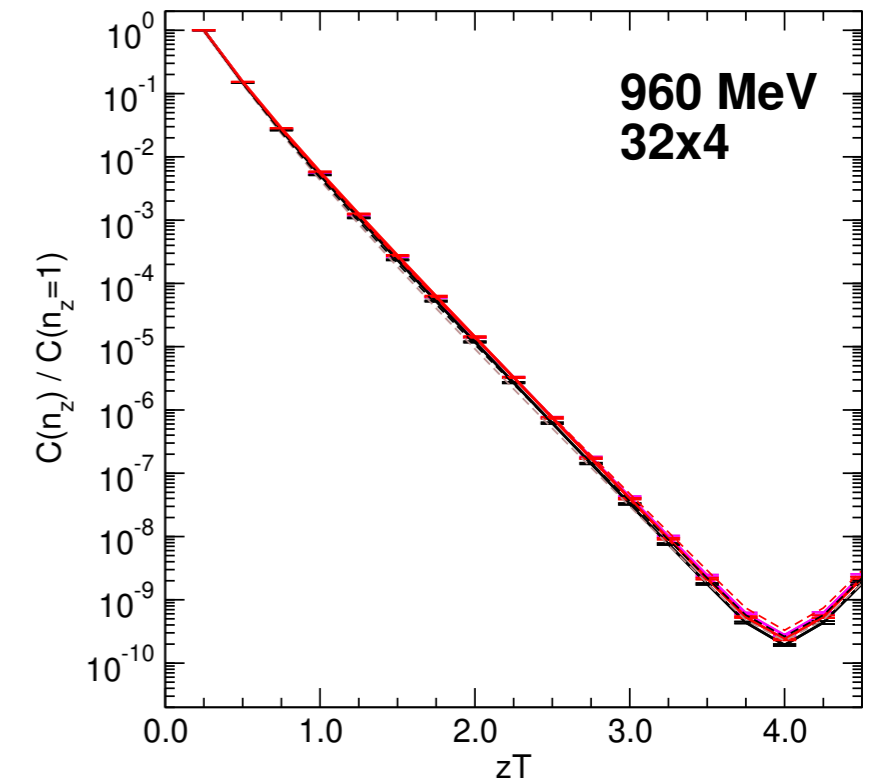
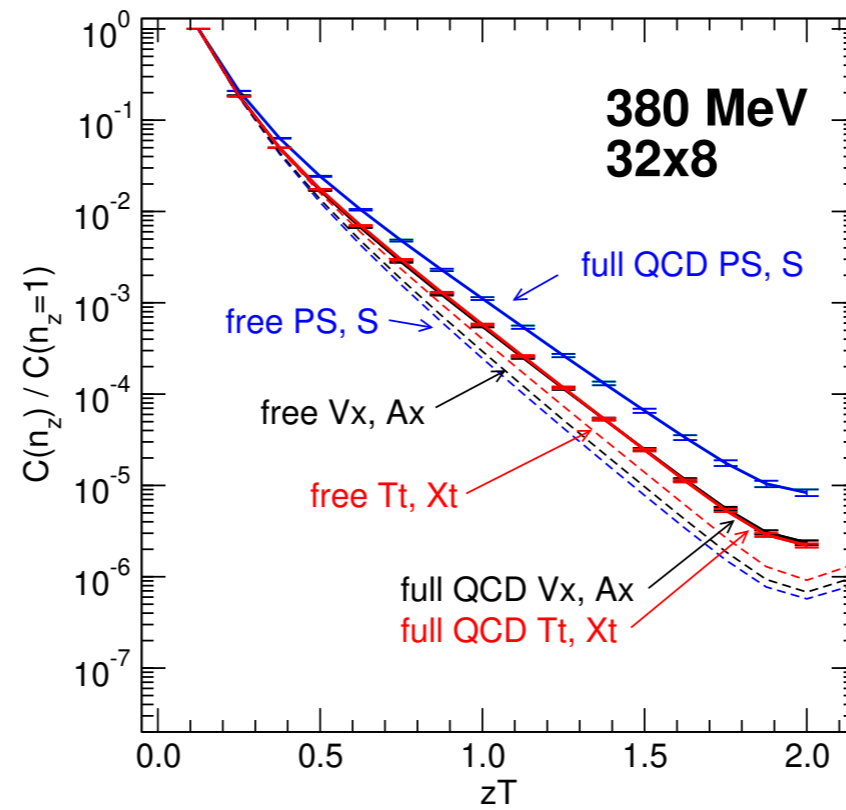
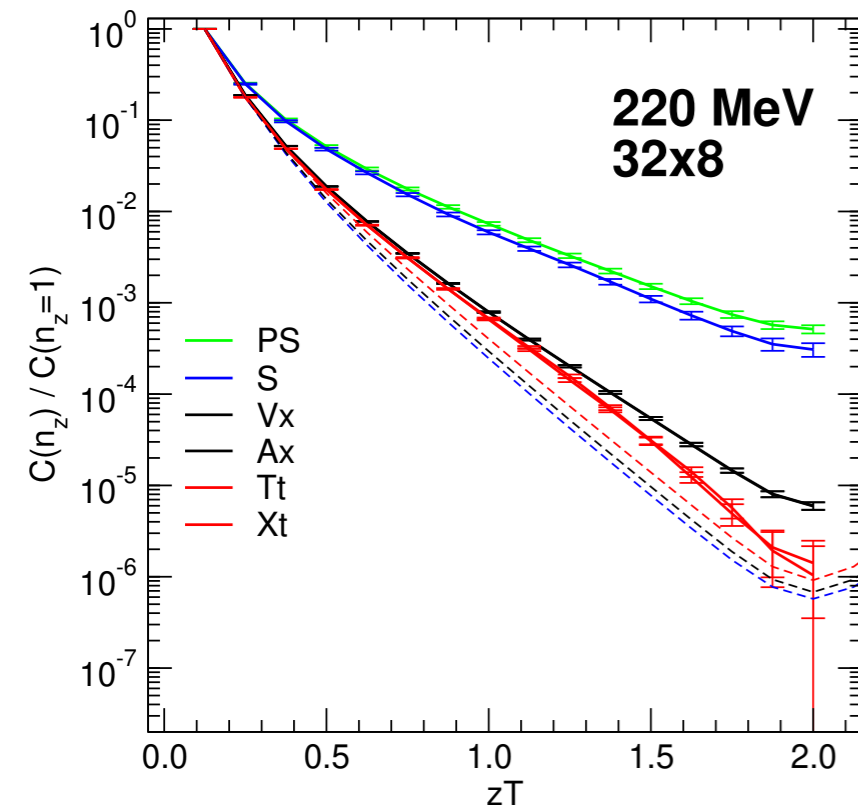
$$SU(4) \supset SU(2)_{CS} \otimes SU(2) \supset SU(2)_L \times SU(2)_R \times U(1)_A$$

CS larger than, and contains, chiral symmetry

No symmetry of action:
E-interactions invariant
B-interactions not invariant

Test with spatial correlators (screening masses)

$$C_\Gamma(n_z) = \sum_{n_x, n_y, n_t} \langle \mathcal{O}_\Gamma(n_x, n_y, n_z, n_t) \mathcal{O}_\Gamma(\mathbf{0}, 0)^\dagger \rangle$$



[Rohrhofer et al., PRD 19]

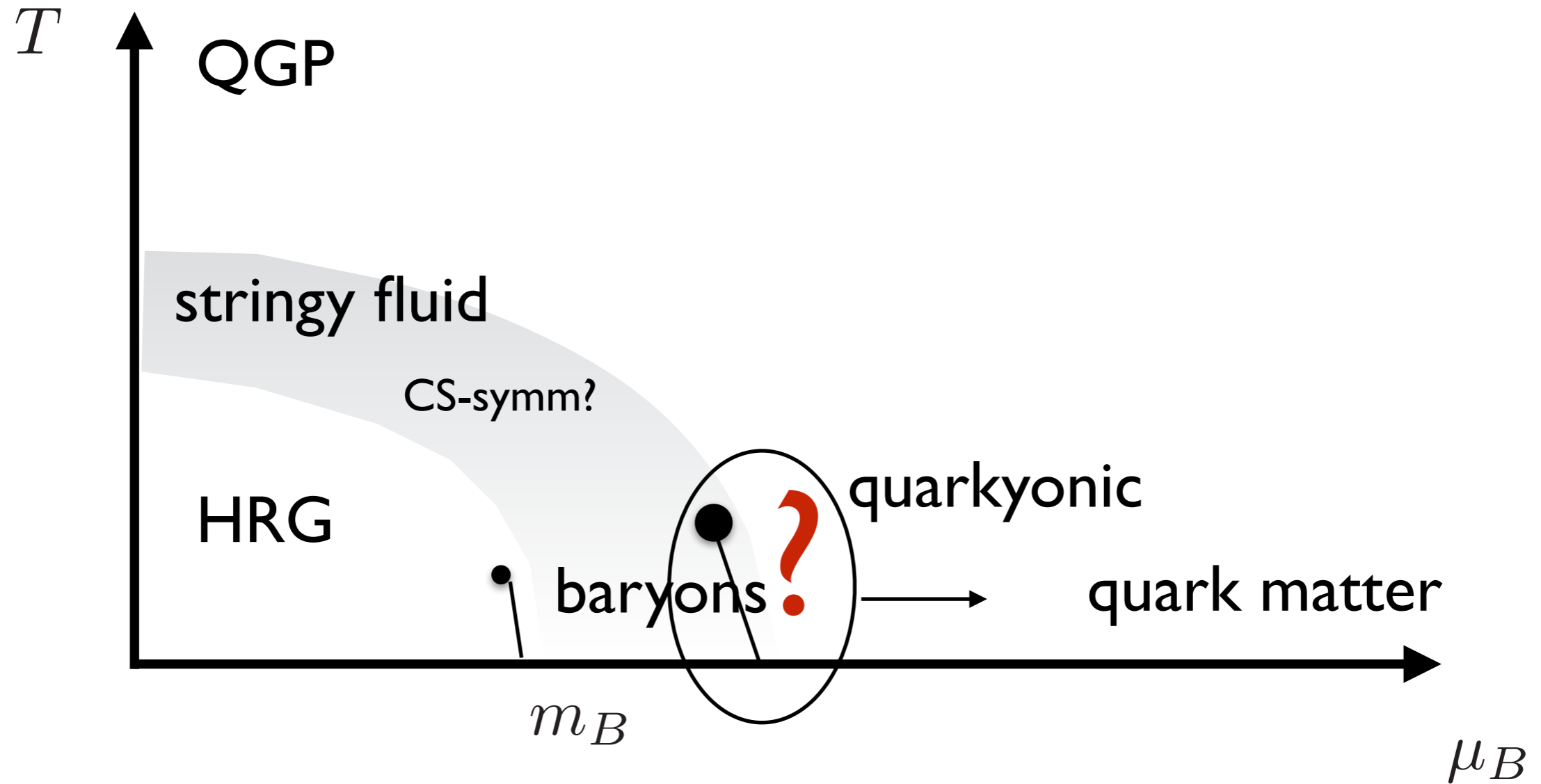
JLQCD configurations

“stringy fluid” for $T_{pc} < T < 900$ MeV

physical light quark masses

$N_f = 2$ DW, $N_\tau = 4, 6, 8, 12$

Conclusions



Conclusions

