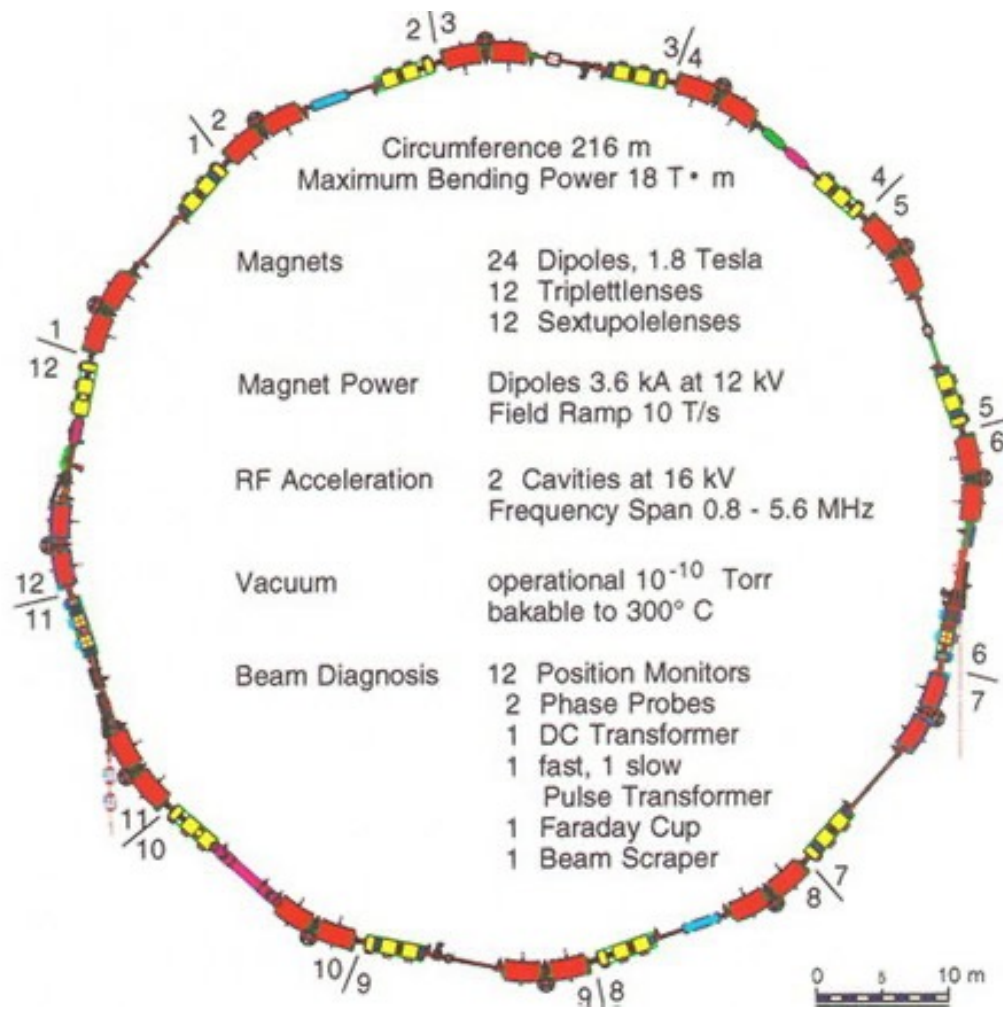


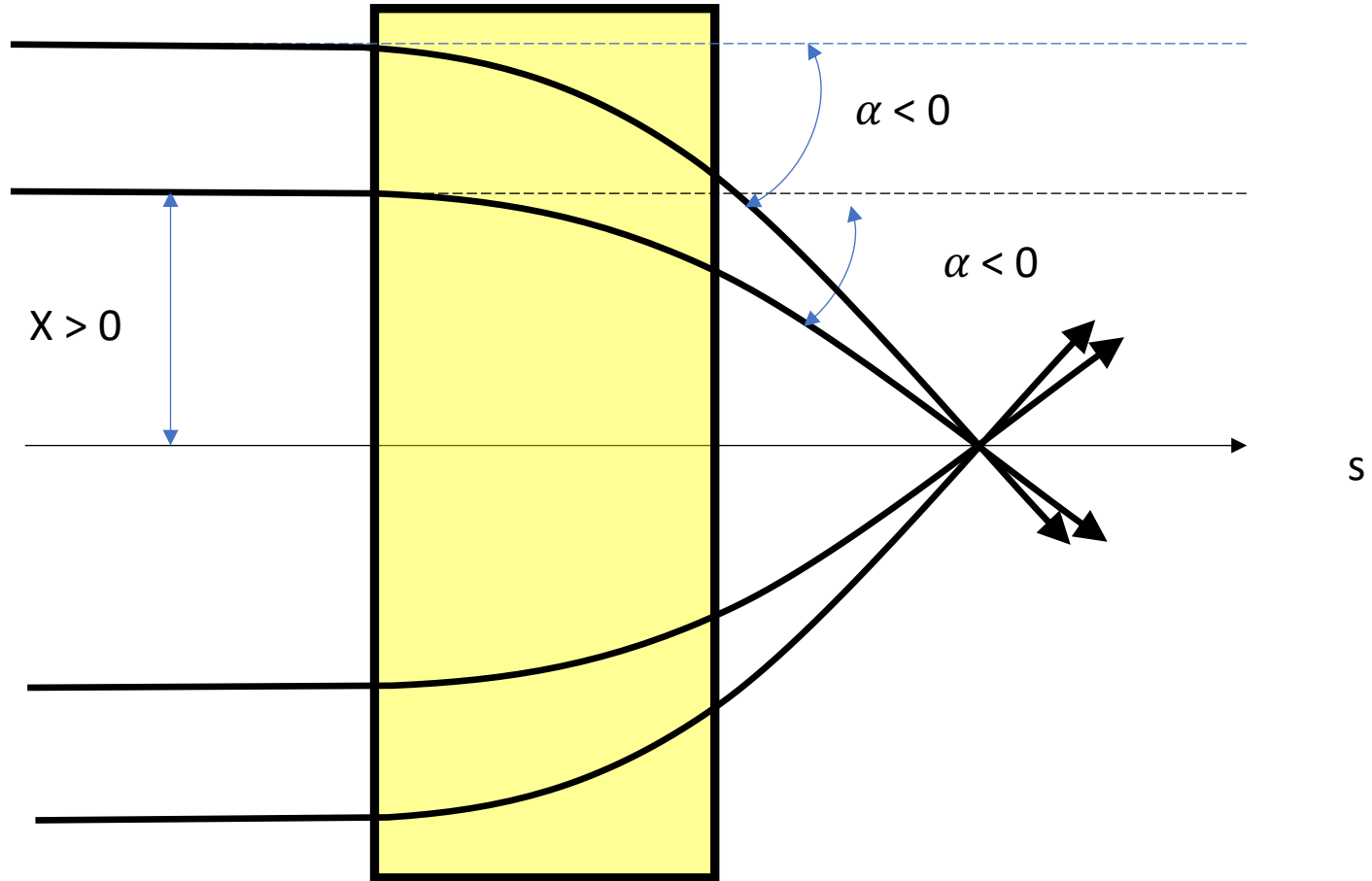
High Intensity Effects in a Circular Accelerators

OP School, 9 November 2021

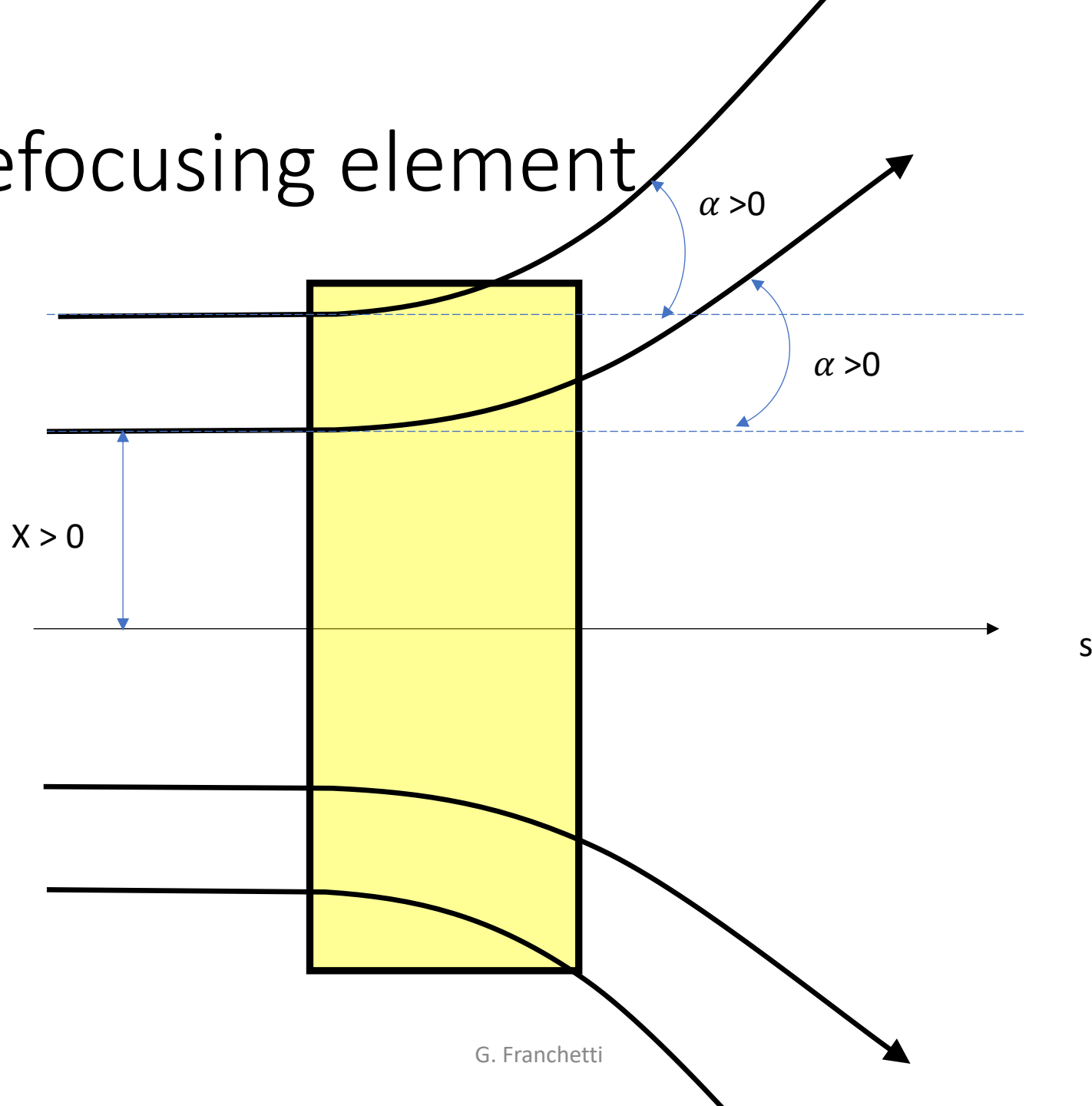
G. Franchetti, GSI



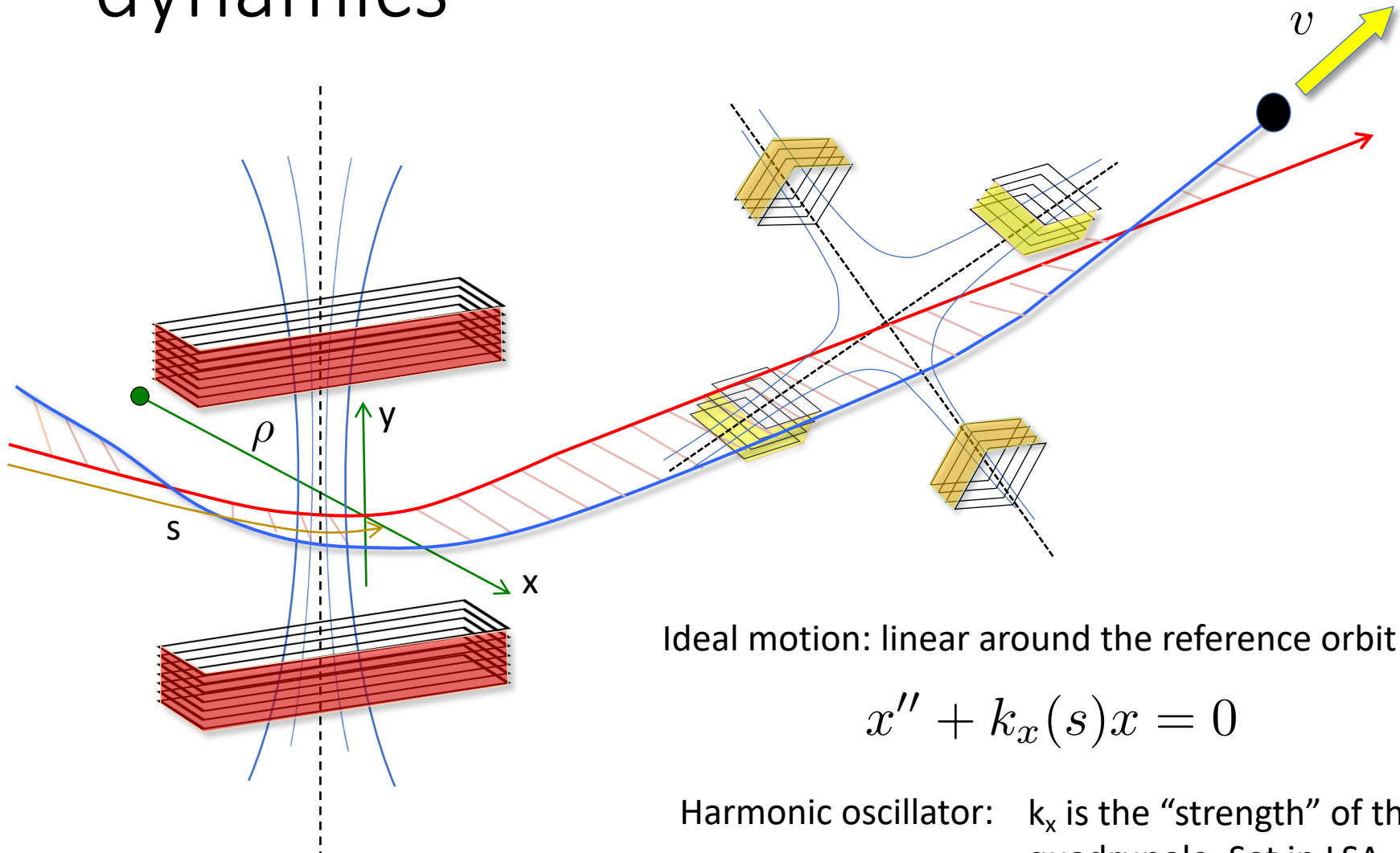
Focusing element



Defocusing element



Reference frame of beam dynamics



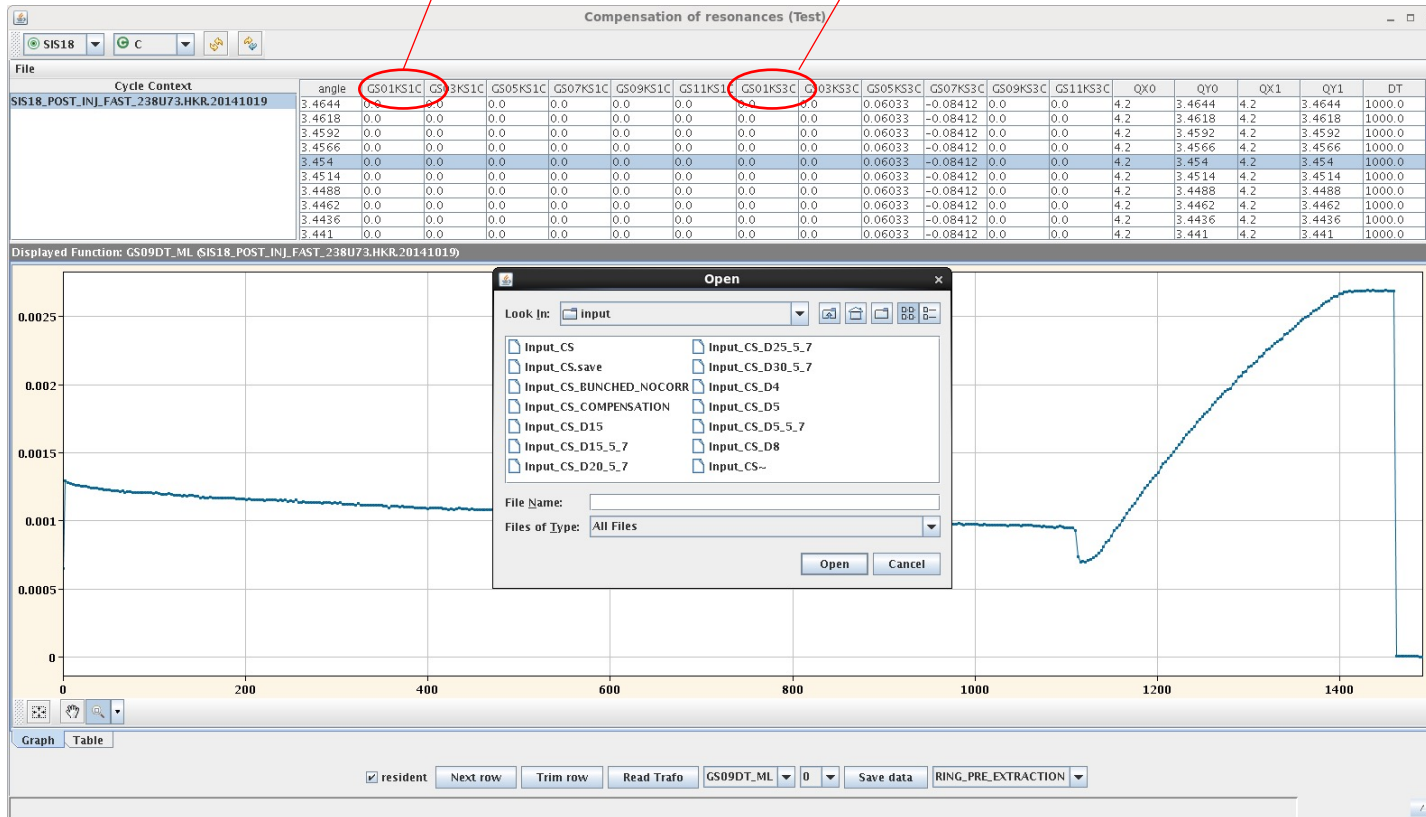
Ideal motion: linear around the reference orbit

$$x'' + k_x(s)x = 0$$

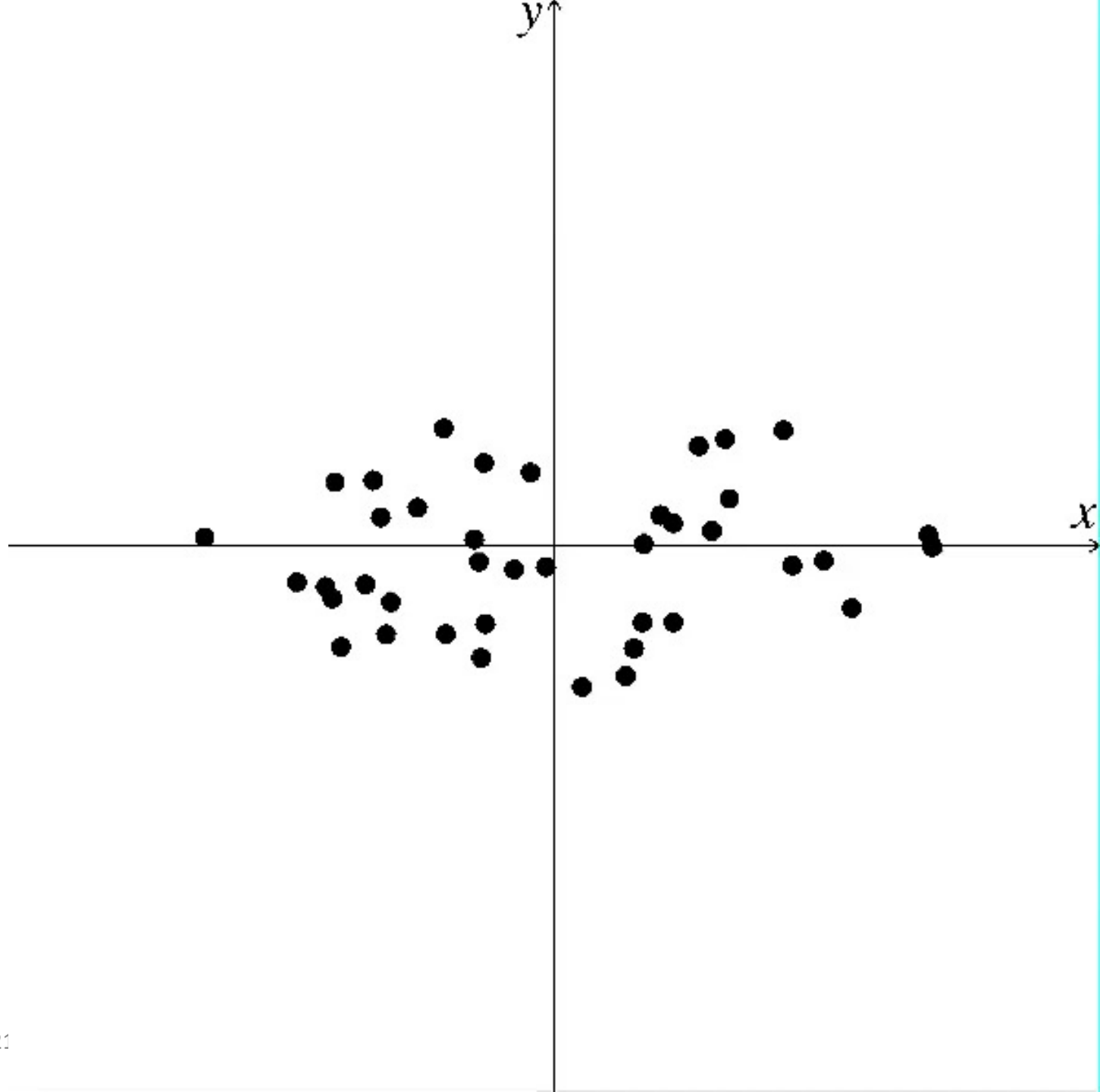
Harmonic oscillator: k_x is the “strength” of the quadrupole. Set in LSA

LSA allows the settings of the elements strength: example

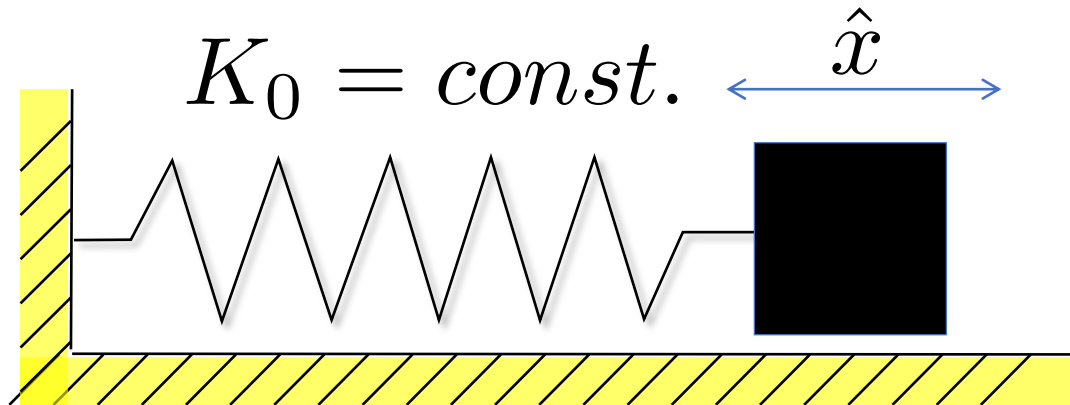
Quadrupole correctors Sextupole correctors



Example from
H. Liebermann



$$x'' + k_x(s)x = 0$$



$$K_0 > 0$$

Stable motion

$$\hat{x} = \sqrt{\beta_x \epsilon_x} \sin(\omega s + \psi_0)$$

$$K_0 < 0$$

Unstable motion

$$\beta_x = 1 / \sqrt{K_0}$$

amplitude

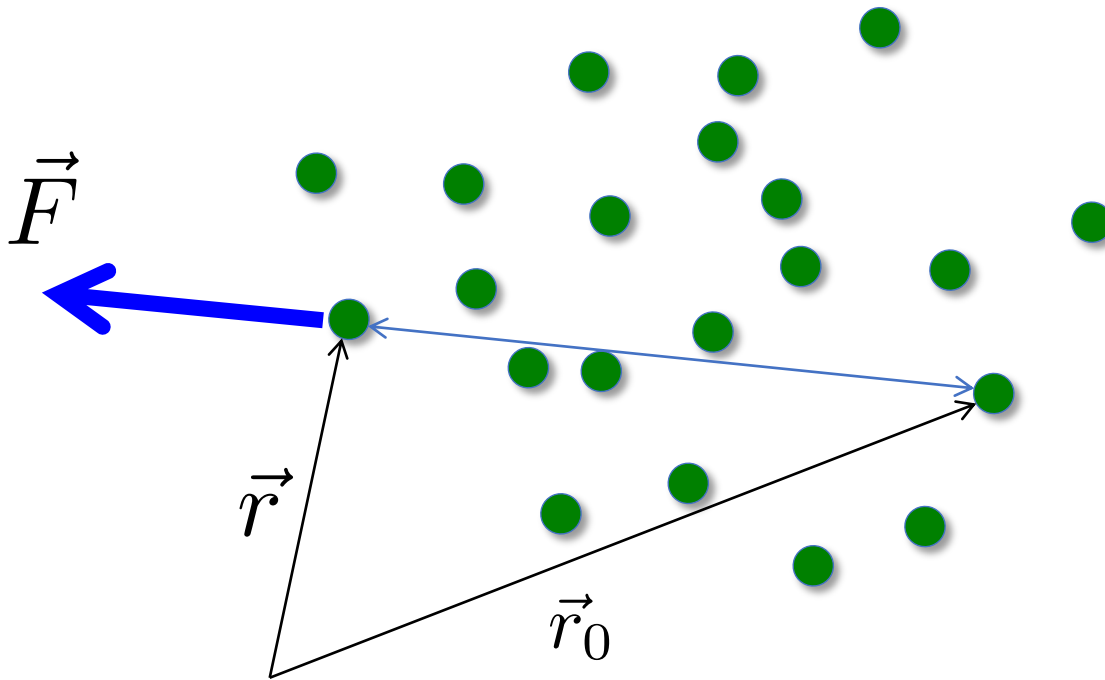
Phase advance

$$\omega = \sqrt{K_0}$$

initial
phase

Coulomb Forces

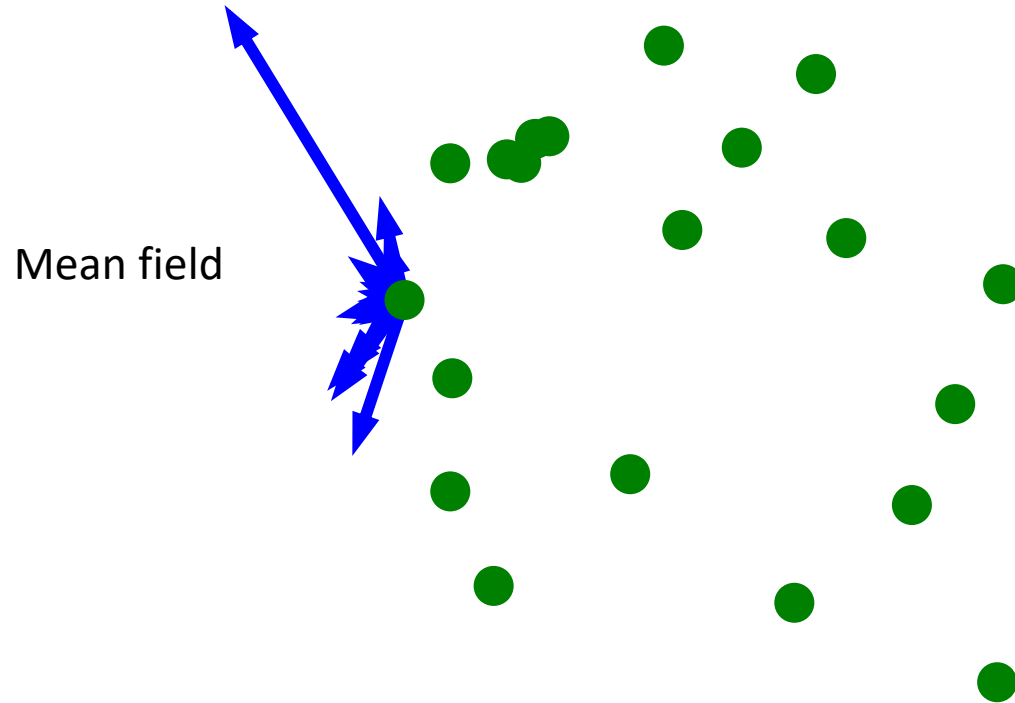
$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q^2}{|\vec{r} - \vec{r}_0|^3} (\vec{r} - \vec{r}_0)$$

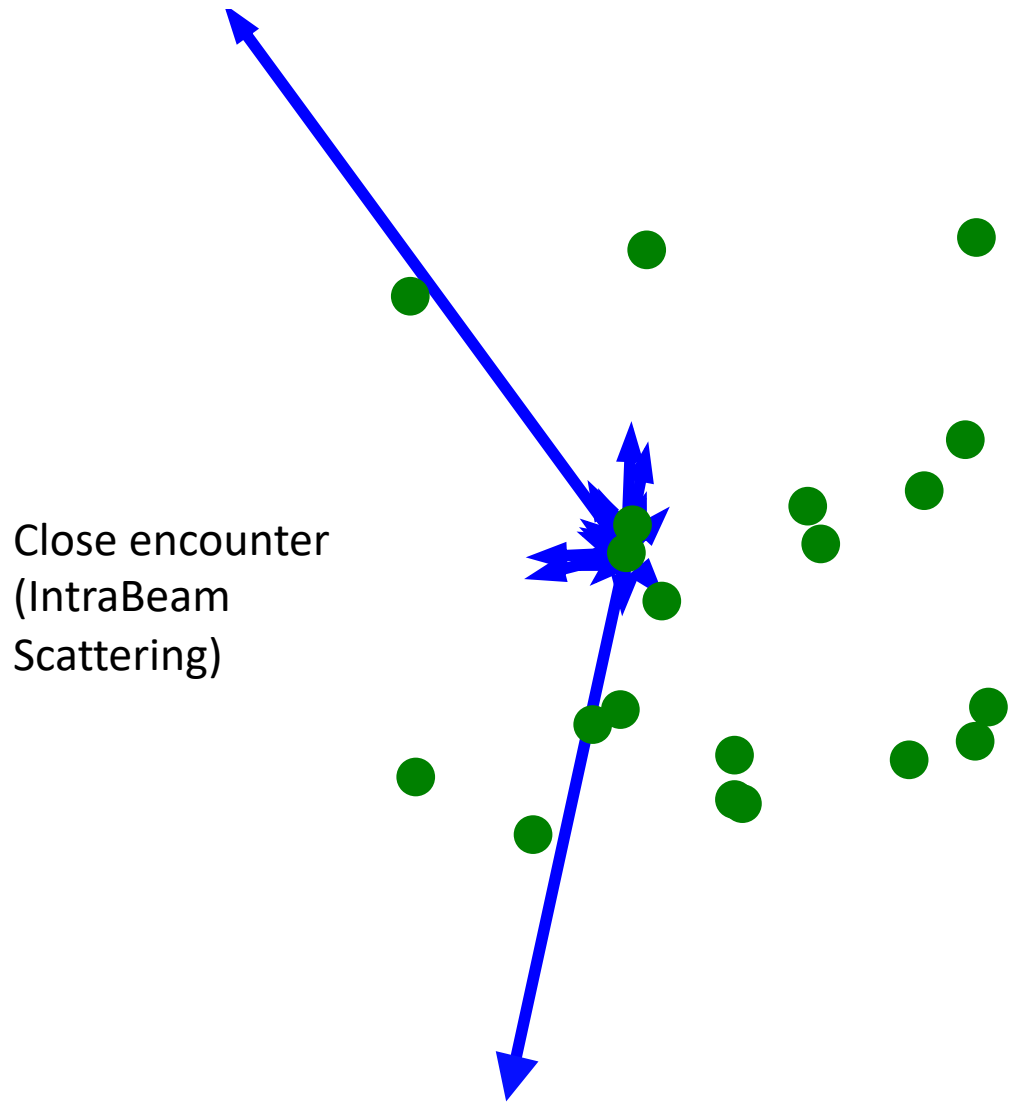


C. Coulomb

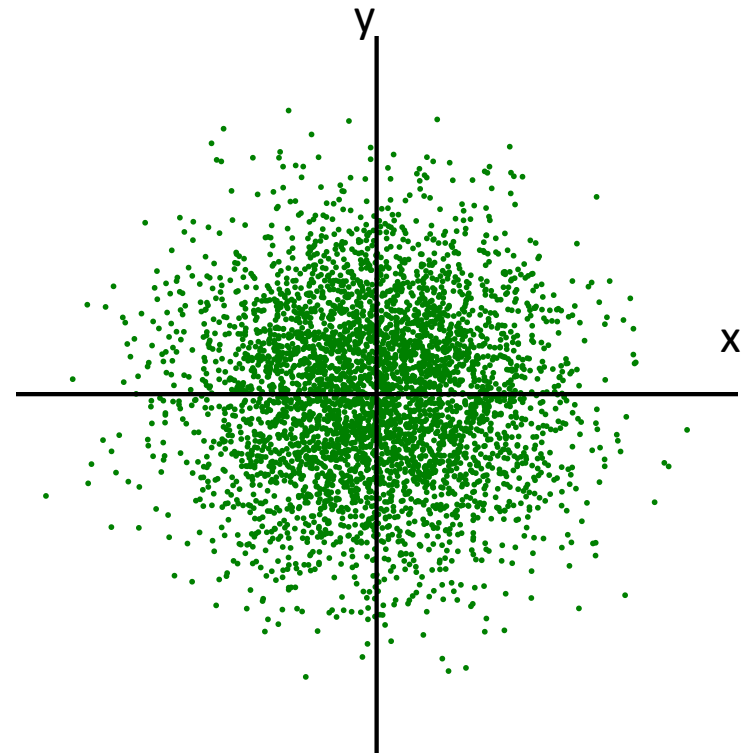
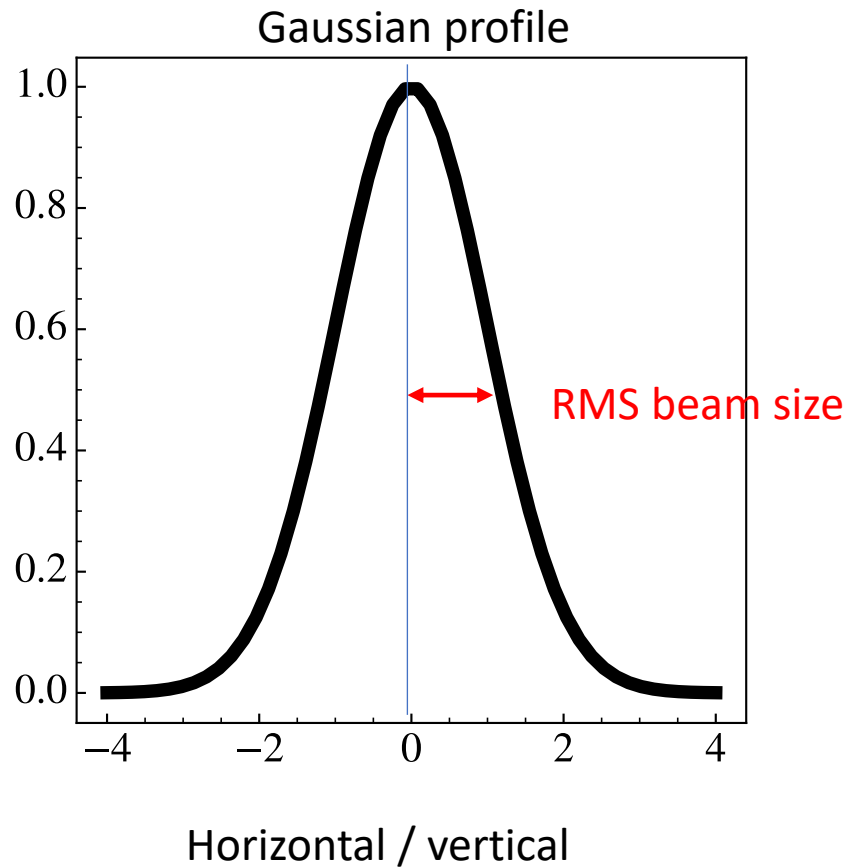


Total Coulomb Forces on one particle



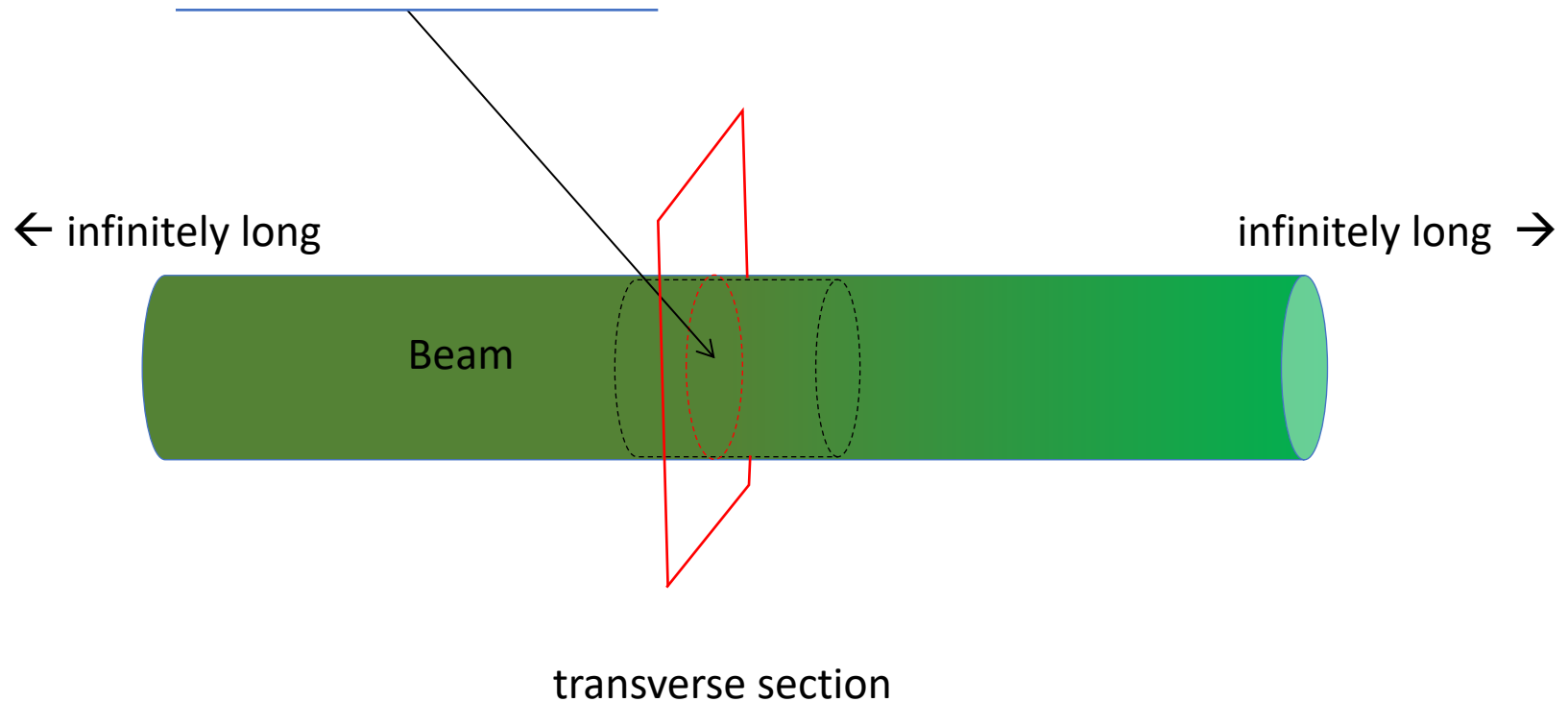


Regular Beam \rightarrow transverse profile

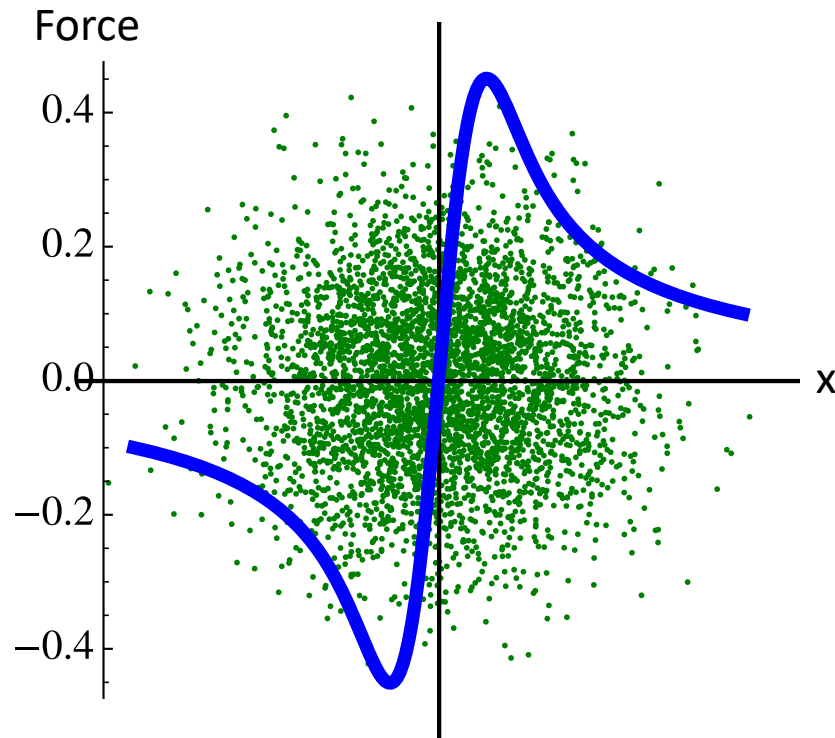


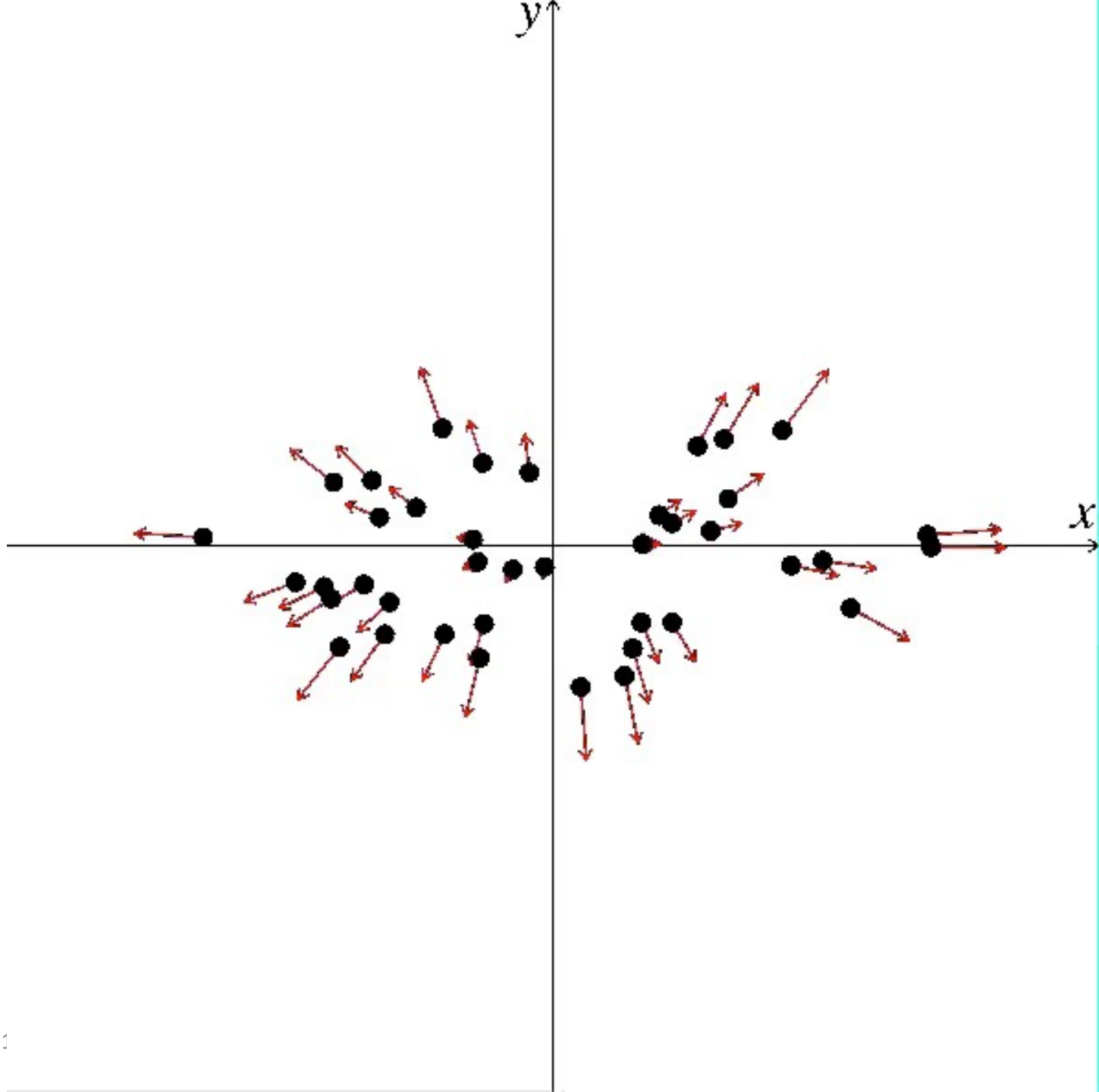
Transverse space charge forces

Space charge forces here are similar to those created by an ideal coasting beam

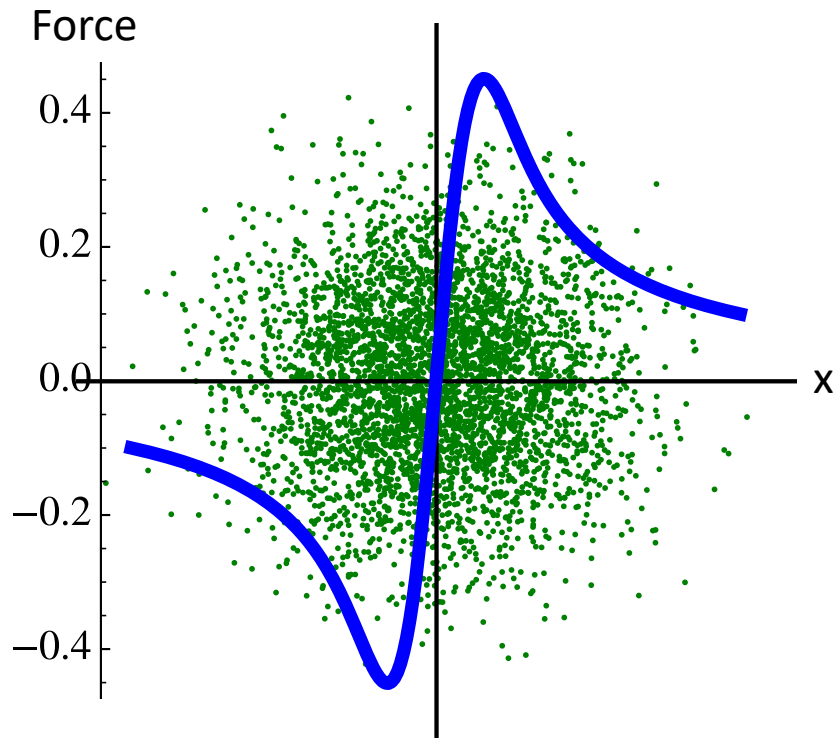


Space charge forces

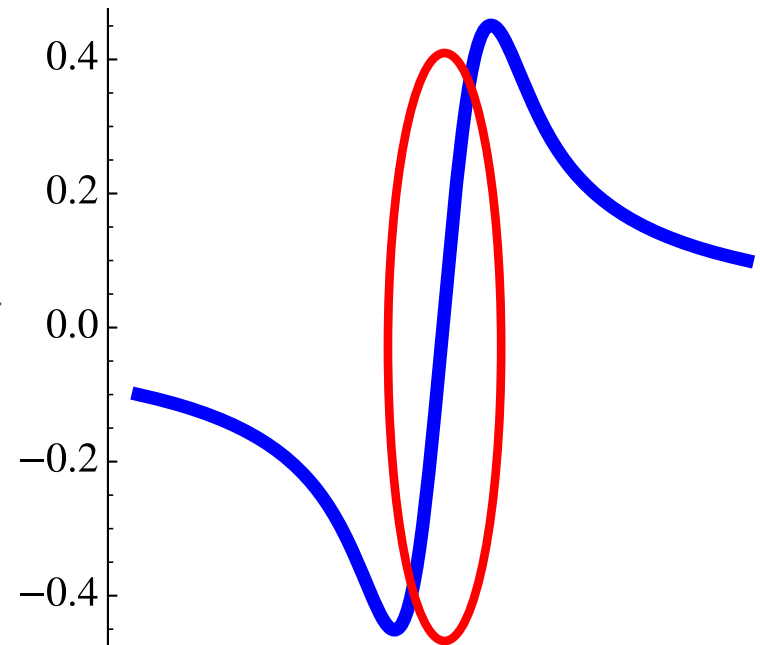




Effect of high intensity on optics

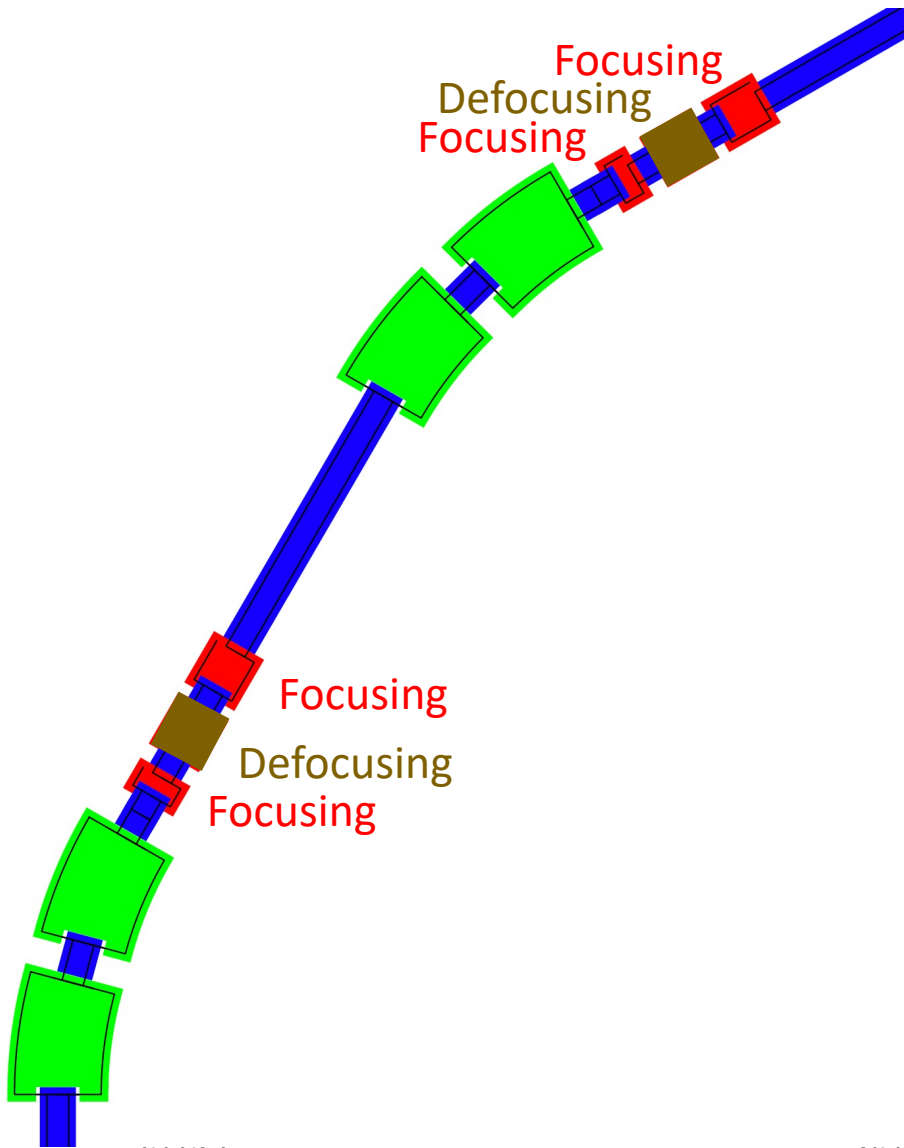


Look only at particles in the beam core

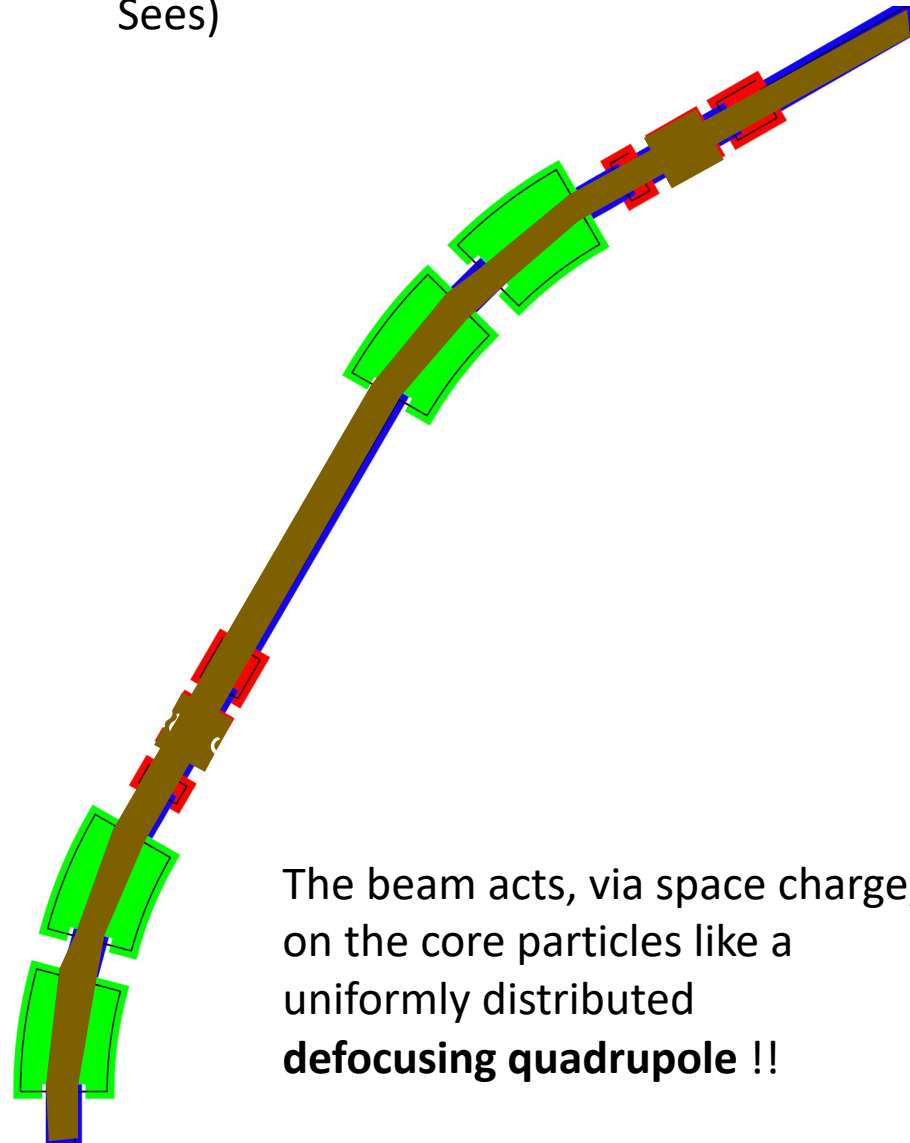


LINEAR DE-FOCUSING FORCE

Naked lattice
(what one ion alone sees)



Lattice with a High intensity beam
(what one particle in the beam center sees)



The beam acts, via space charge, on the core particles like a uniformly distributed **defocusing quadrupole !!**

Effect of high intensity on optics

Naked lattice

$$x'' + k_x(s)x = 0$$



quadrupoles

High intensity beam:
core particles

$$x'' + k_x(s)x - k_{SC,x}(s)x = 0$$

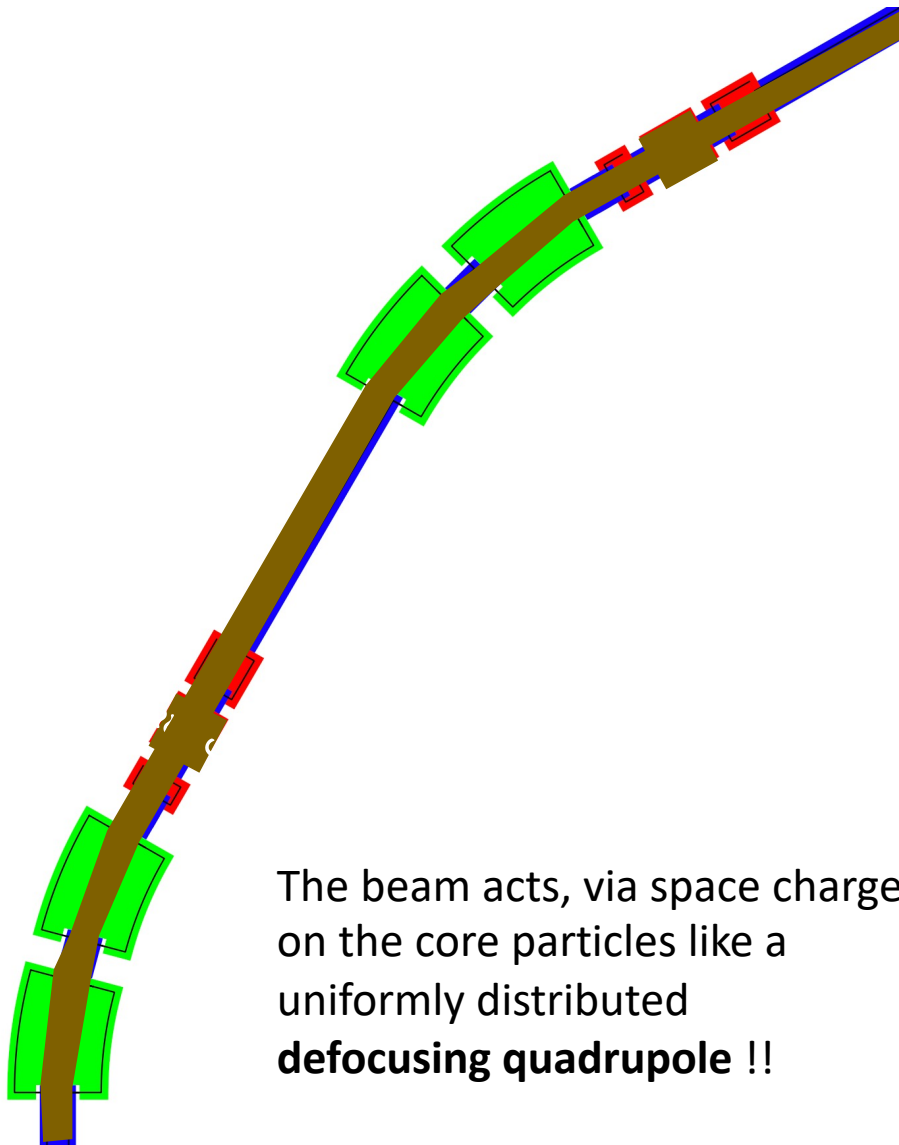


quadrupoles



Defocusing effect of
Space charge

Lattice with a High intensity beam



Modified Optical functions

$$\beta_x$$

$$\alpha_x$$

$$D_x$$

$$D'_x$$

$$Q_x$$

$$\beta_y$$

$$\alpha_y$$

$$Q_y$$

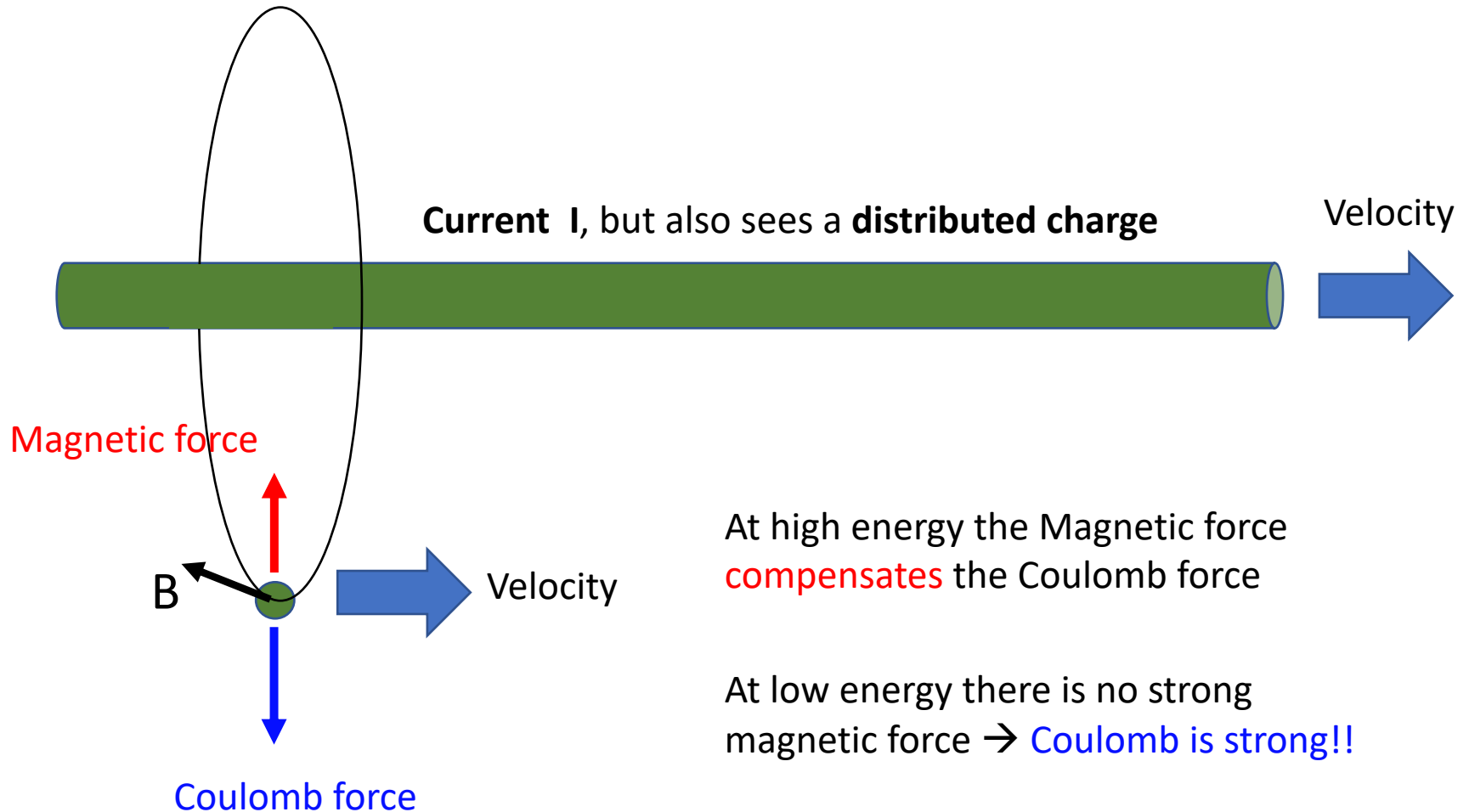
depends on local position "s"

"depressed" tune

depends on local position "s"

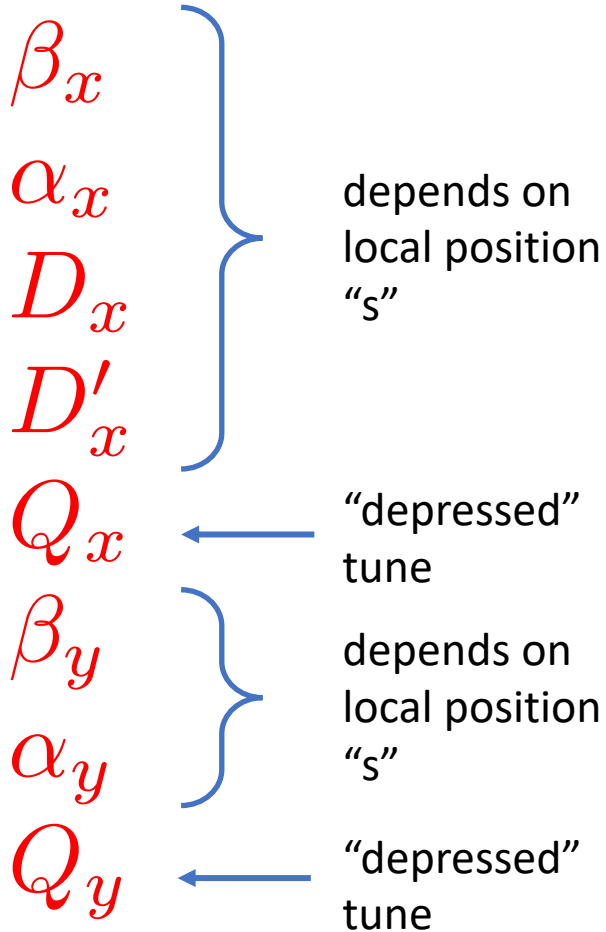
"depressed" tune

The effect of the beam energy

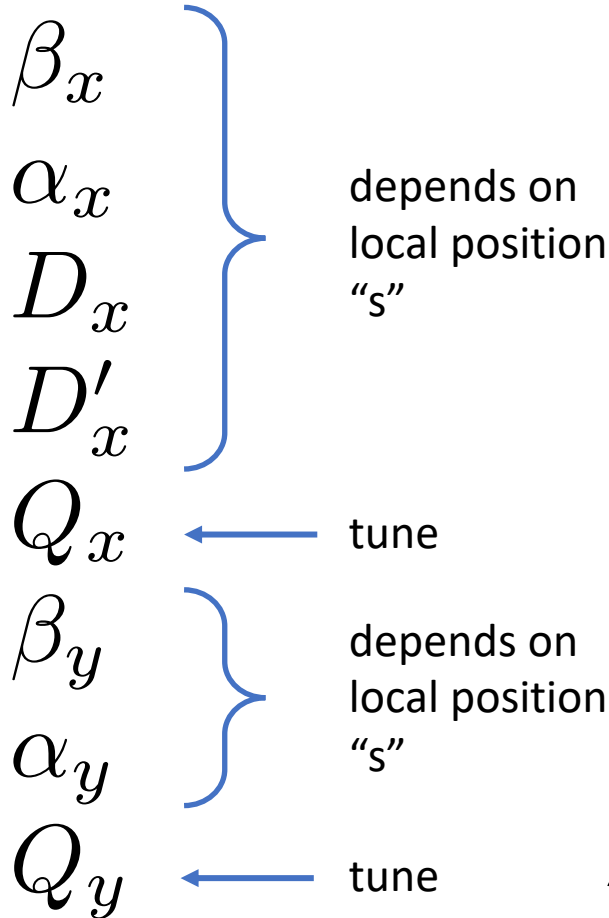


Space charge tuneshift

Modified Optical functions



Optical functions



Space charge tuneshift

$$\Delta Q_x = Q_x - Q_x$$

$$\Delta Q_y = Q_y - Q_y$$

Space Charge tunes shift

$$\Delta Q_{x,sc} = \frac{\bar{\beta}_x}{2} R_{acc} \frac{K}{\sqrt{\bar{\beta}_x \epsilon_x} (\sqrt{\bar{\beta}_x \epsilon_x} + \sqrt{\bar{\beta}_y \epsilon_y})}$$

$$\Delta Q_{y,sc} = \frac{\bar{\beta}_y}{2} R_{acc} \frac{K}{\sqrt{\bar{\beta}_y \epsilon_y} (\sqrt{\bar{\beta}_x \epsilon_x} + \sqrt{\bar{\beta}_y \epsilon_y})}$$

$$K = \frac{qI}{2\pi\epsilon_0 mc^3 \beta^3 \gamma^3}$$

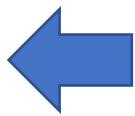
Average rms horizontal beam size

Average rms vertical beam size

K = perveance

ϵ_x, ϵ_y = rms emittance

Space Charge tuneshift

$$K = \frac{qI}{2\pi\epsilon_0 mc^3 \beta^3 \gamma^3}$$


Relativistic gamma:
with high energy is large

The perveance is a combination of

q = charge state

M = ion mass

γ, β \Leftrightarrow the beam energy

I = beam current

Different beams can have the same Perveance !!!

Example

ParamModi (SEPARATE BUNCHING) Deutsch 14. Dezember 2018 06:48 Über

SIS18_SLOW_HADES_20181206_210727

Ring Orbitkorrektur (horizontal)		Ring Orbitkorrektur (vertikal)		Ring Extraktion		Ring langsame Extraktion		Ring Injektion		Ring HF		Ring spezial		SIS18: Modi			
Total	Suche	AEG Tests		Extraktionslinie		Ring Orbitbeule (horizontal)		Ring Orbitbeule (vertikal)									
Multi-Multiturn Injektion																	
Injektionsenergie		11.048	MeV/u			Anzahl Injektionen		1			Extraktion		Targetenergie		1580.0	MeV/u	
Injektionssteifigkeit		92.113997592	Tm			Kühlzeit		16.0	ms			Extraktionssteifigkeit		9.1847826429	Tm		
Injektionsfrequenz		211.1856	kHz			Kühlerbump x (Position)		0.0	mm			Extraktionsfrequenz		1284.6084	kHz		
Hor. Arbeitspunkt QH (Inj.)		4.295				Kühlerbump x (Winkel)		0.0	mrاد			Hor. Arbeitspunkt QH (Flattop)		4.295			
Vert. Arbeitspunkt QV (Inj.)		3.28				Kühlerbump y (Position)		0.0	mm			Hor. Arbeitspunkt QH (Extr.)		4.315			
Radiallage (Injektion)		-1.0	mm			Kühlerbump y (Winkel)		0.0	mrاد			Vert. Arbeitspunkt QV (Flattop)		3.28			
Bumper-Abfallzeit		170	μs			Teilchenzahl		1.0E9				Vert. Arbeitspunkt QV (Extr.)		3.29			
Bumper-Amplitude		85.0	mm			Beam Mode		1				Radiallage (Extraktion)		1.2	mm		
Unilac-Verschiebung		45	μs														
Chopper-Verzoegerung		60	μs														
Chopper-Fenster		100.0	μs														
Chopper Korrekturwinkel		0.0	mrاد														
GTK7MU5 Korrekturwinkel		0.0	mrاد														
GS12MU3I Korrekturwinkel		-0.2	mrاد														
I-Septum Korrekturwinkel		-1.4	mrاد														
Langsame Extraktion																	
E-Septum Korrekturwinkel		-2.5	mrاد			Rampe		Verrundungszeit		32.0, 32.0	ms	Extraktionsbeule		E-Septum-Bump (Anfang)		-20.0	mm
Extraktionszeit		2000.0	ms			Harmonischenzahl		4				E-Septum-Bump (Delta)		3.0	mm		
Spillmitte		0.25				Rampensteilheit		3.0, 3.0	T/s			M-Septum-Bump (Anfang)		14.5	mm		
Spillamplitude		0.65				Einfangzeit		16.0	ms			M-Septum-Bump (Delta)		0.0	mm		
Sextupol Amplitude		0.06				Impulsbreite (DC)		0.09	%			ES/MS Bump (Anfang)		0.0	mm		
Sextupol Phase		20.0	deg			Bucketfill (Bunching)		[0.0, 1.43]				ES/MS Bump (Delta)		0.0	mm		
DQH total		0.031				Bucketfill (Ramp)		[1.43, 1.43]				Bypass Korrekturwinkel		3.0	mrاد		
DQH pre		0.01				Bucketfill (Pre-Extraction)		[1.43, 0.0]				Bypass Korrekturwinkel (Delta)		0.0	mrاد		
DQH spill		0.0032				HF sequentiell		<input checked="" type="checkbox"/>									

⚠ An Geräte schicken ▼
✖ Änderungen verwerfen

$$R_{acc} = 34m$$

$$\text{Energy} = 11.048 \text{ MeV/u} \rightarrow \gamma = 1.01177, \quad \beta = 0.15$$

Silver: Mass 107, Charge +45

$$\text{Number of particles} = 10^9 \rightarrow I = 1.5 \text{ mA}$$

Perveance:

$$K = 1.12 \times 10^{-8} \quad (\text{no units, it is a pure number})$$

Average beta functions

$$\overline{\beta}_x = \frac{R_{acc}}{Q_x} \rightarrow 8 \text{ m}$$



$$\overline{\beta}_y = \frac{R_{acc}}{Q_y} \rightarrow 10.4 \text{ m}$$

Example

N particles = 10^9

ϵ_x	ϵ_y	$\Delta Q_{sc,x}$	$\Delta Q_{sc,y}$
5	5	-0.017	-0.02
30	15	-0.0035	-0.0057

N particles = 10^{11}

ϵ_x	ϵ_y	$\Delta Q_{sc,x}$	$\Delta Q_{sc,y}$	
5	5	-1.7	-2	← 
30	15	-0.35	-0.57	← 

Discussion

Effective tunes of particle in the beam core

$$Q_x = Q_{x0} + \Delta Q_{sc,x}$$

Depressed
Tune

Machine
Tune

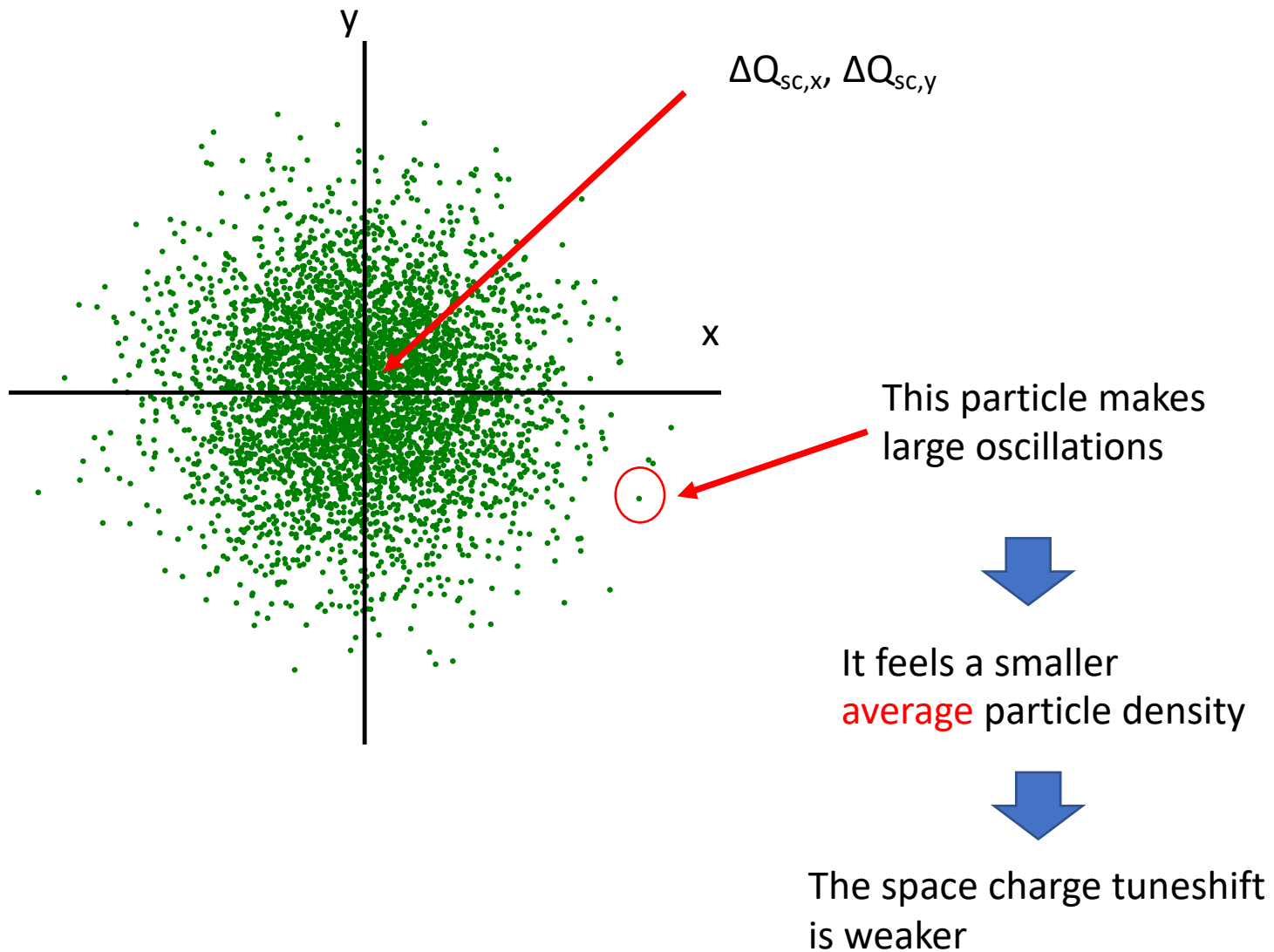
Space charge
tuneshift

If $Q_{x0} = 4.295$

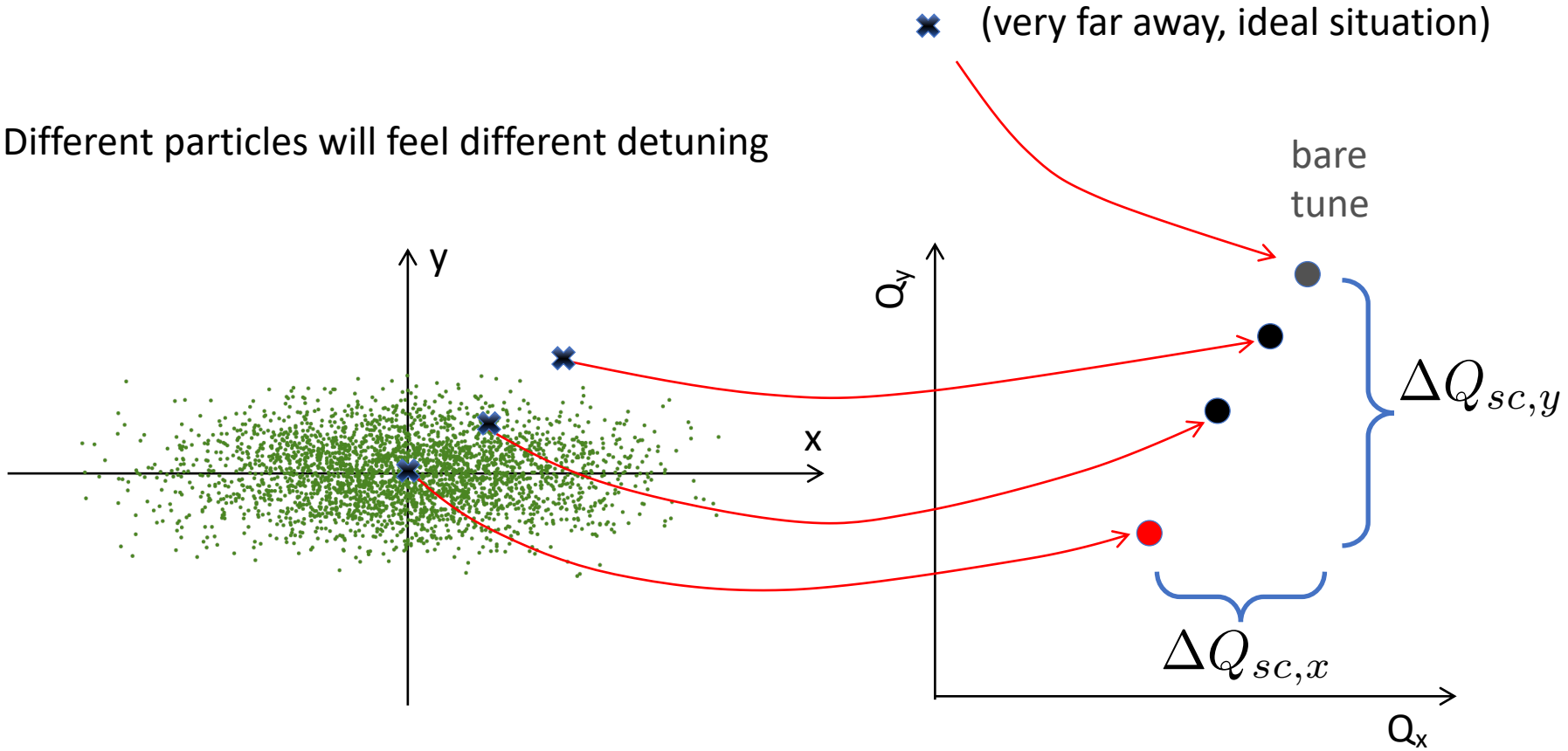
but $\Delta Q_{sc,x} = -0.295$



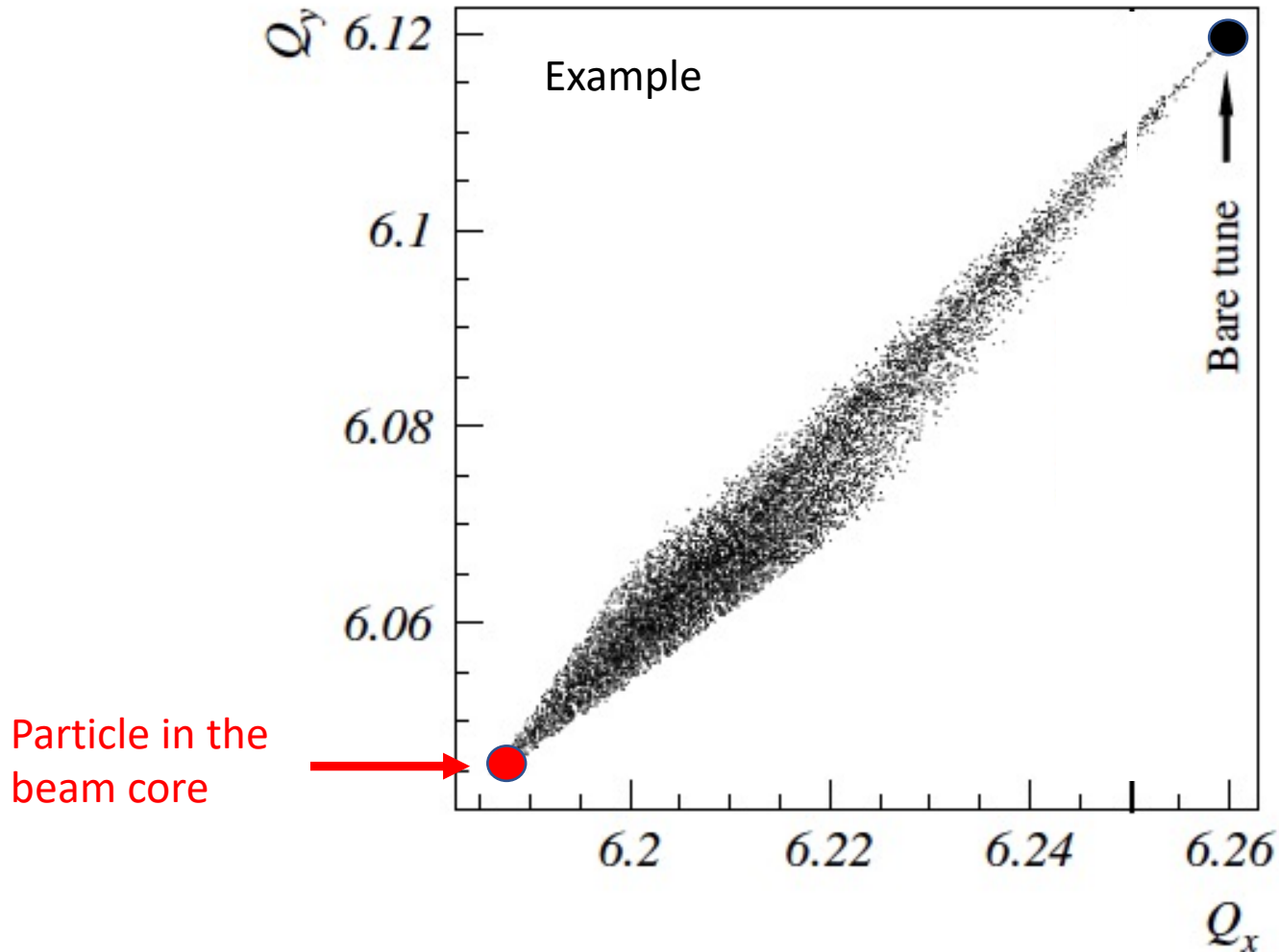
Effective tune in the center of beam $\rightarrow Q_x = 4$!!!!



Different particles will feel different detuning



The space charge tune-spread



Space charge tunespread is a measure of the high intensity

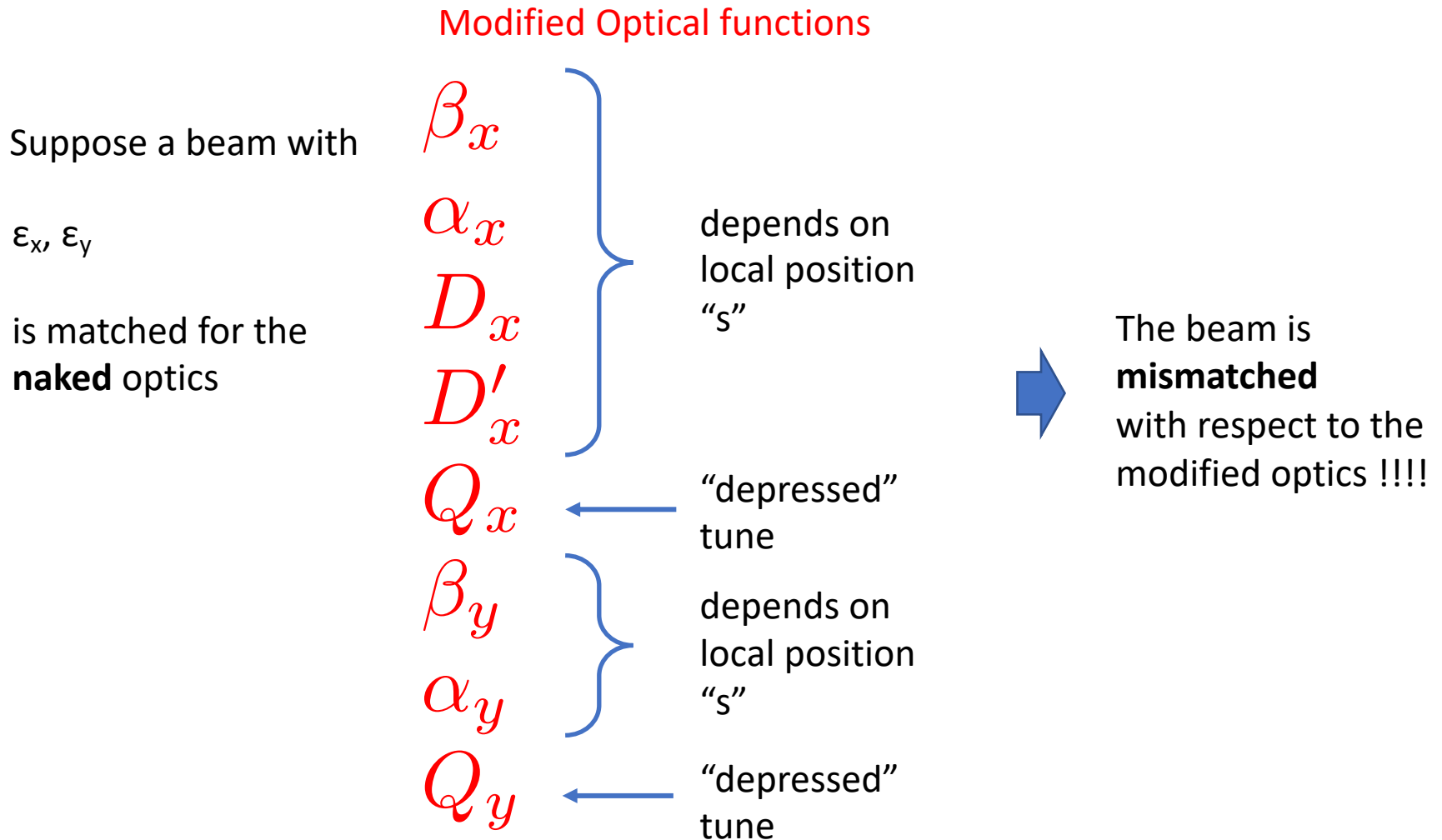
Rings $\Delta Q_{sc,x} = -0.5$  $\frac{Q_x}{Q_{x0}} = 0.9 \div 0.95$

Linacs  $\frac{\psi_x}{\psi_{x0}} \sim 0.6 \div 0.7$

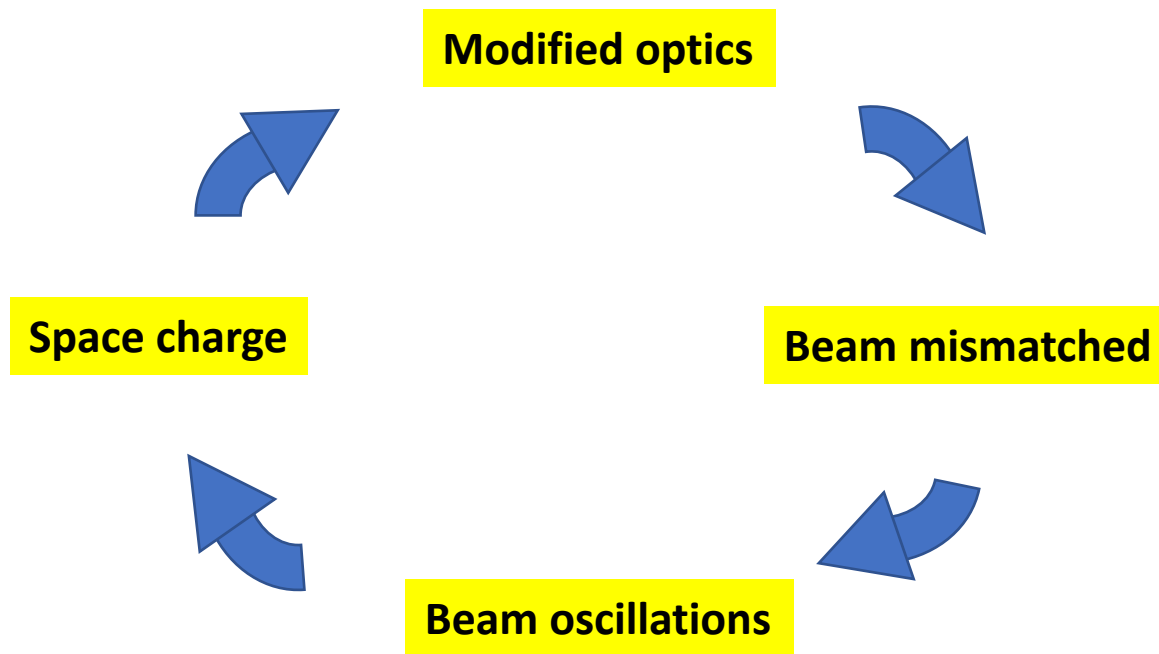


Phase advances rather than tunes

Mismatching issues

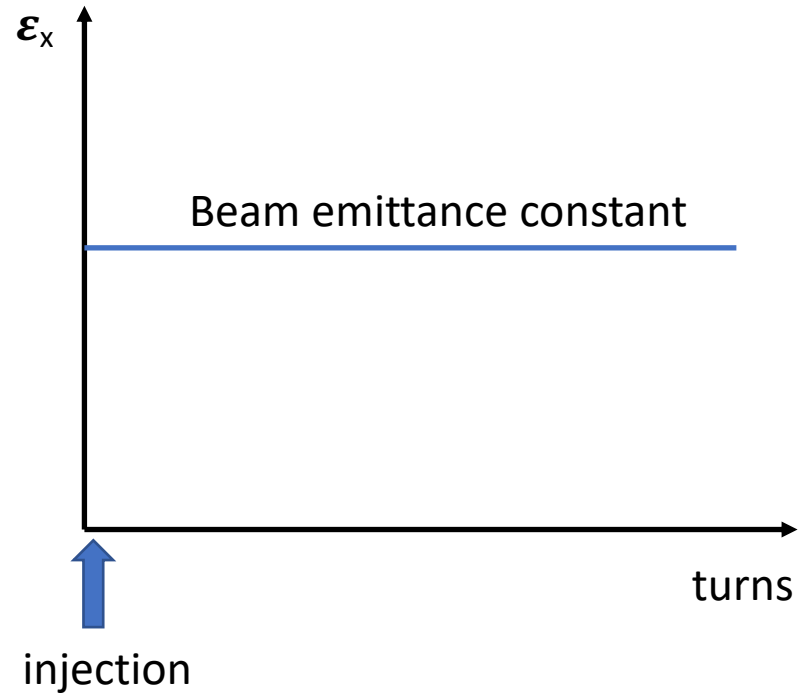
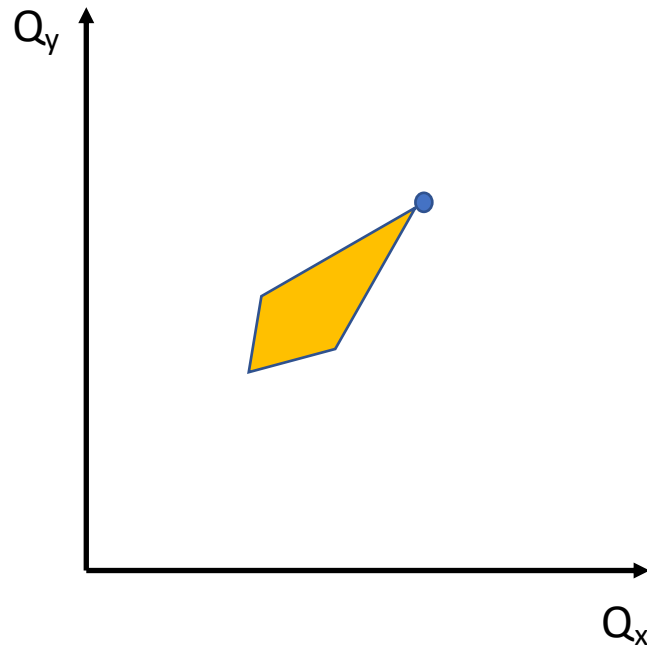


The matching issue



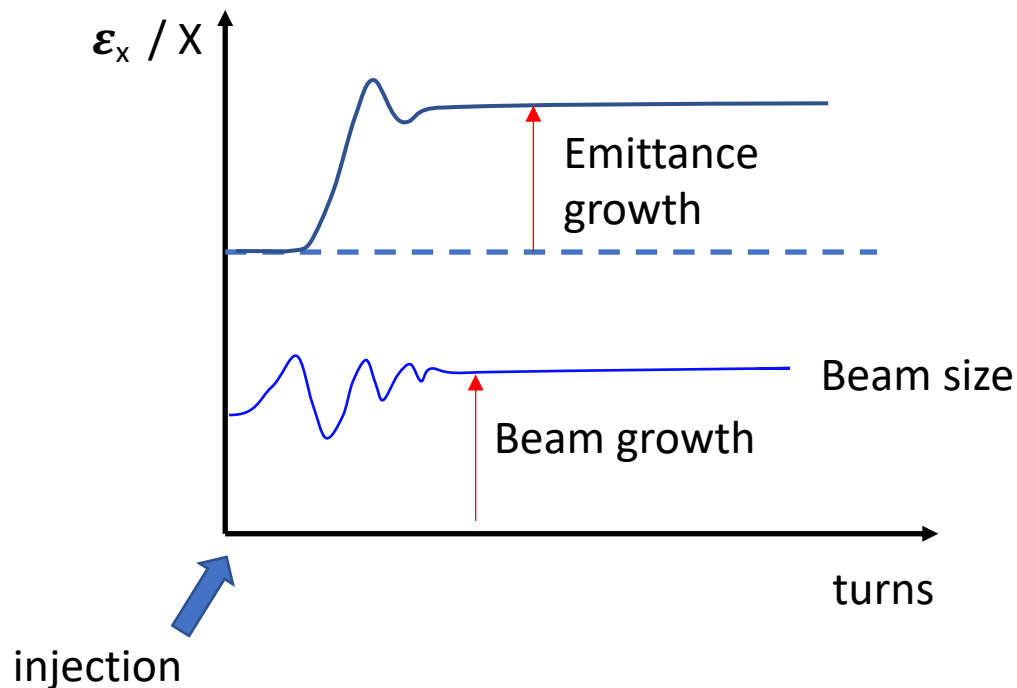
Matched beam and high intensity

Coasting beam



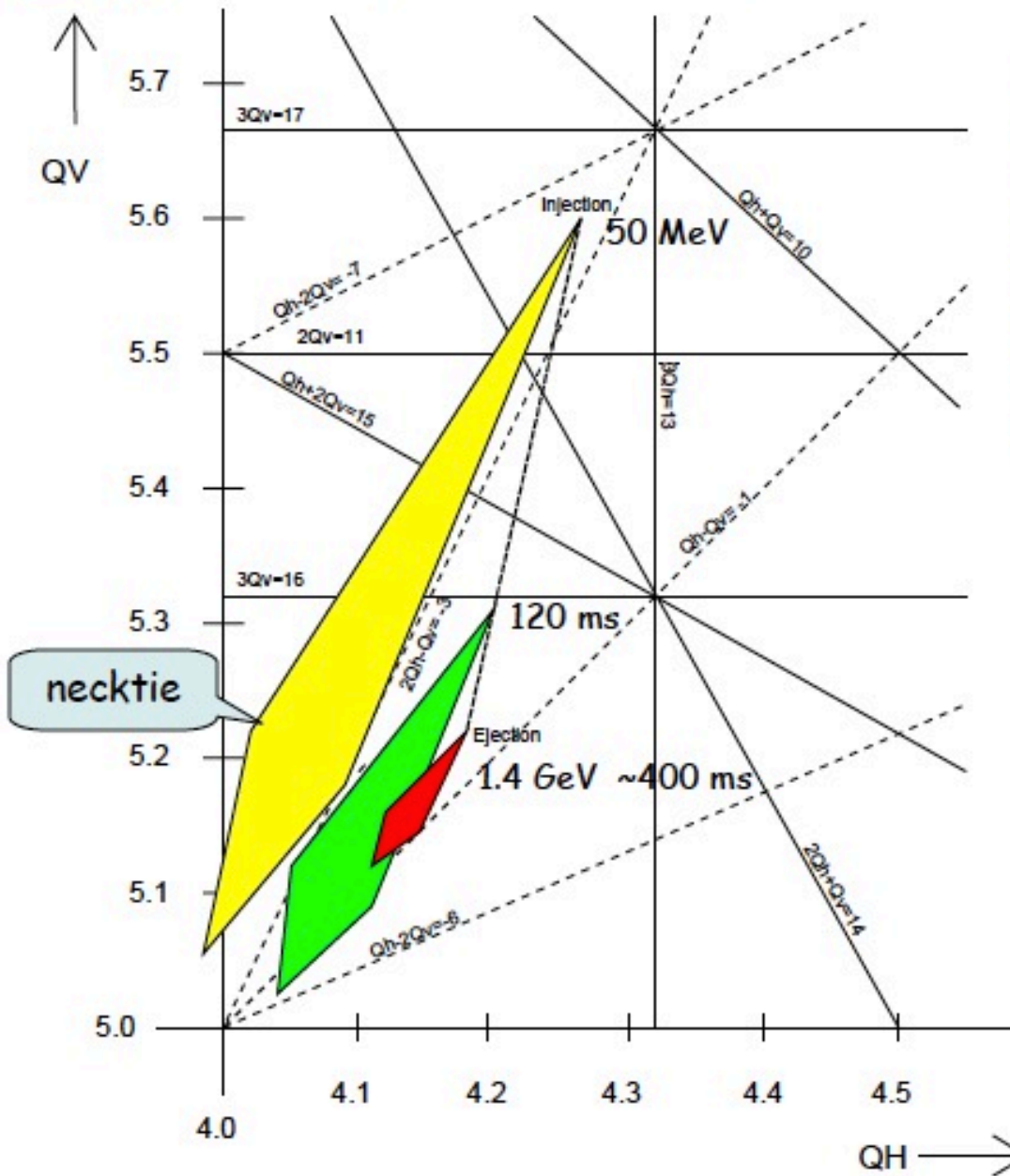
Mismatched beam and high intensity

Coasting beam



Effect is fast and depends on ΔQ_{sc} and the “naked optics”

Resonances

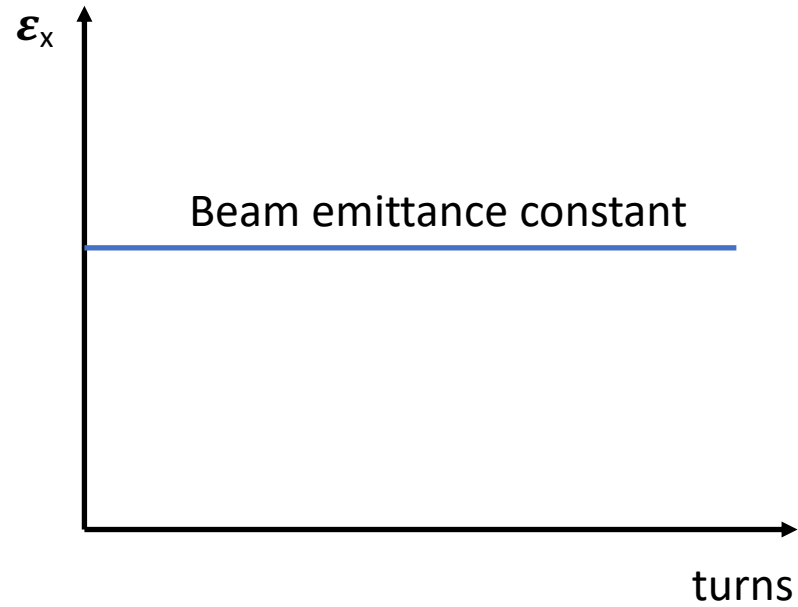
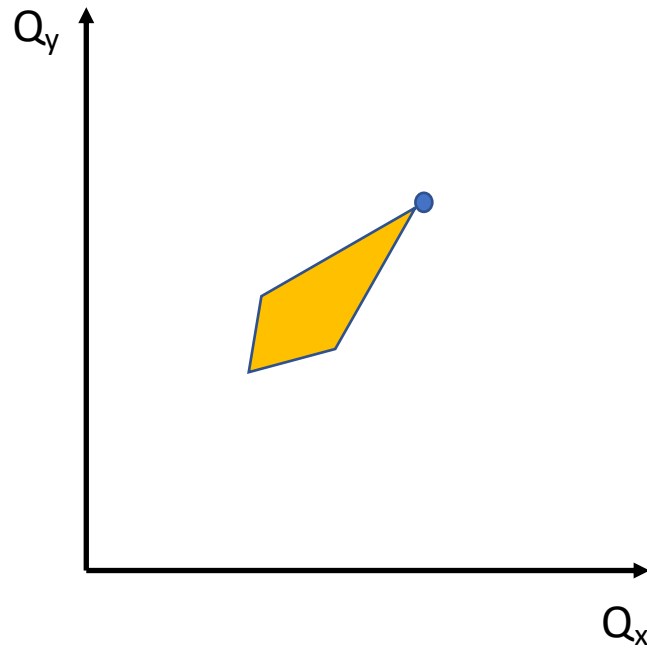


Issues

- 1) Space charge + resonances in coasting beams
- 2) Space charge + resonances in bunched beams
- 3) Collective beam response to direct space charge forces ?

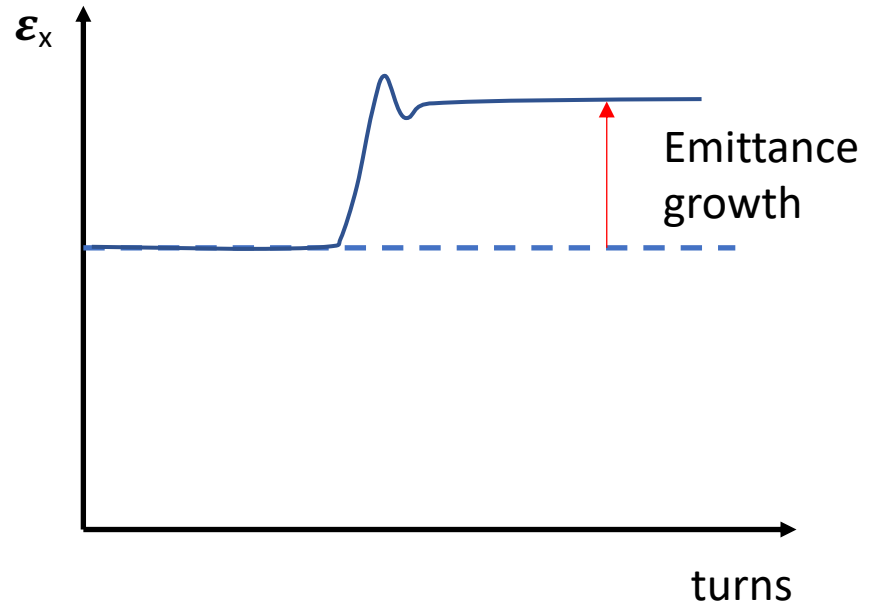
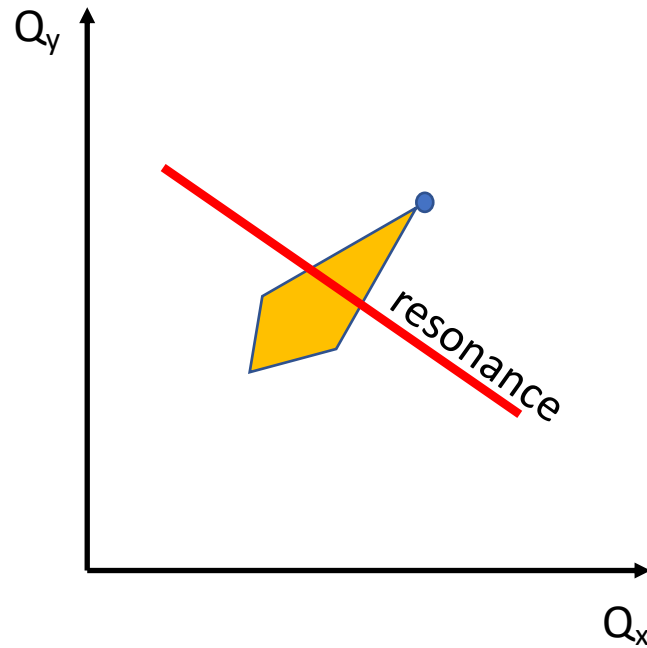
Resonances and high intensity

Coasting beam



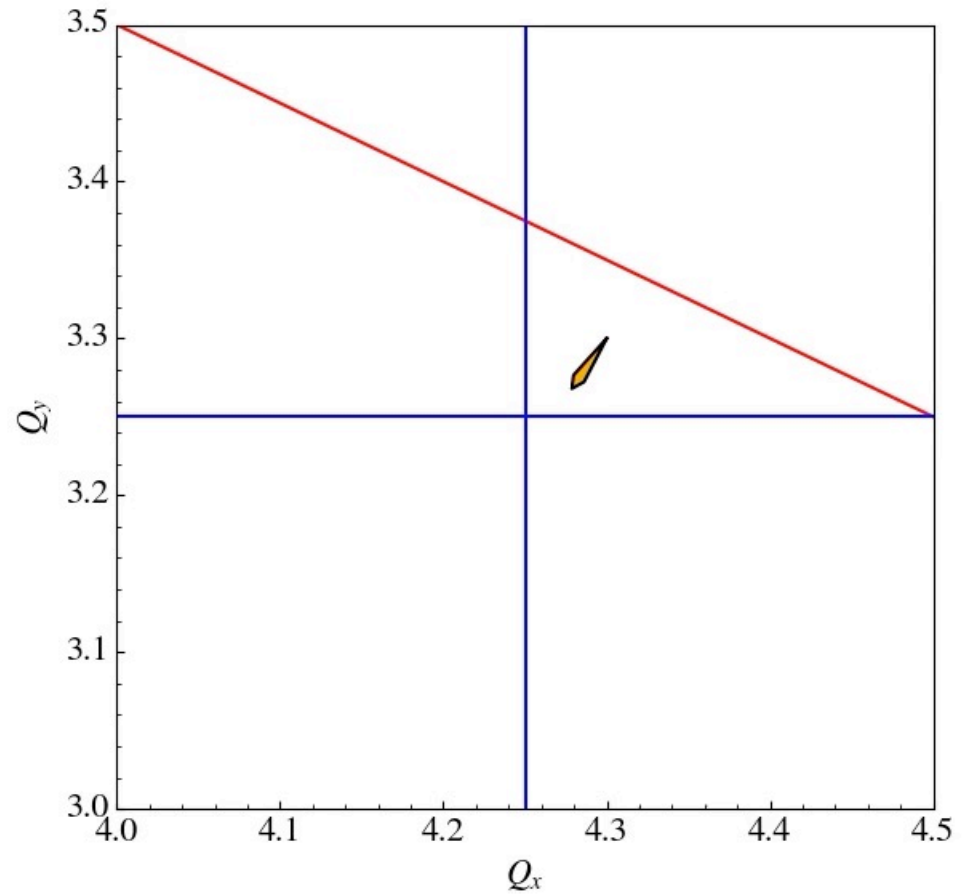
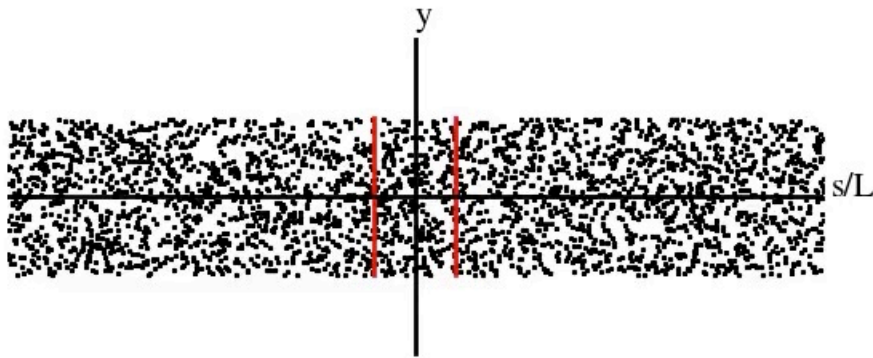
Resonances and high intensity

Coasting beam

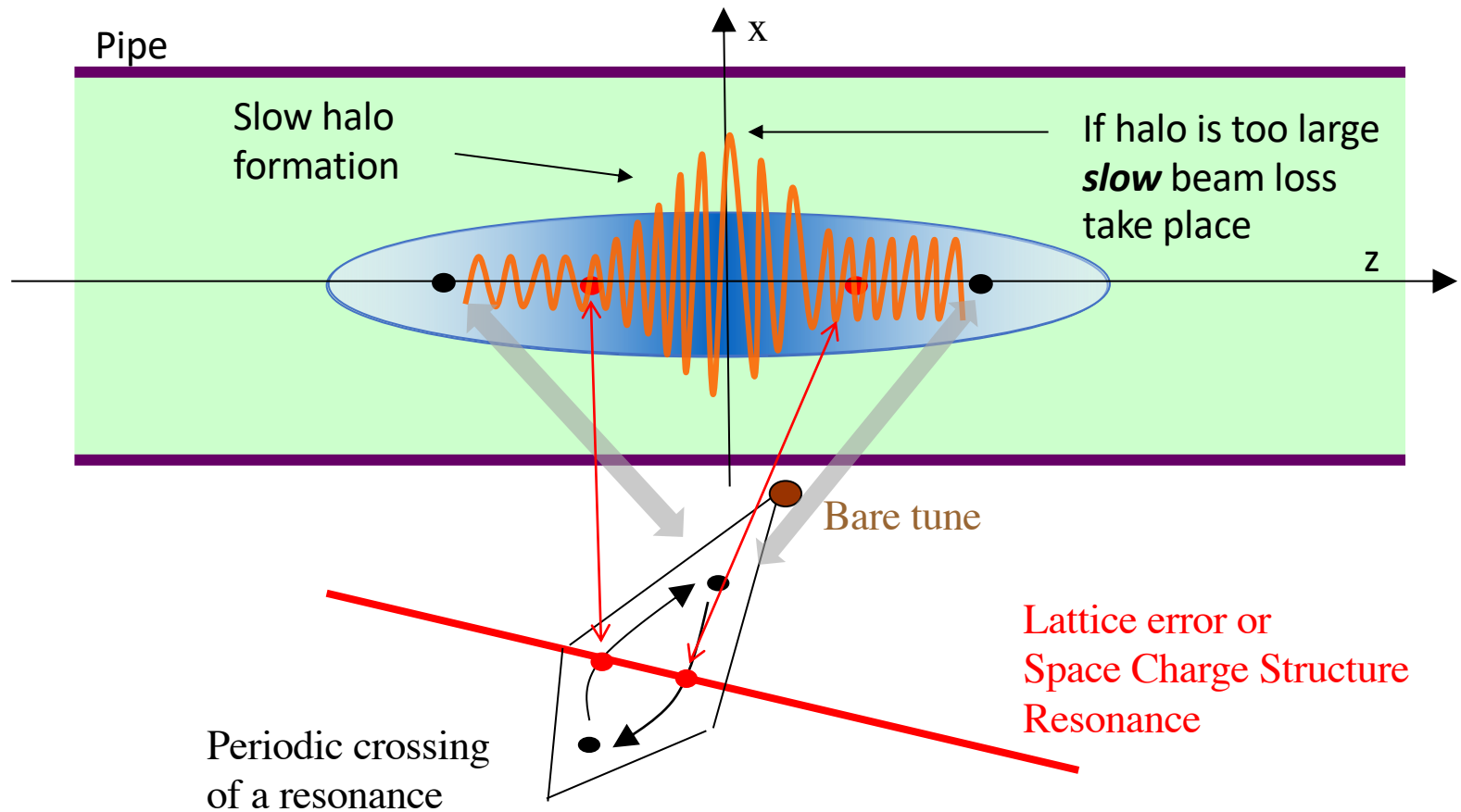


Effect is fast and depends on DQ_{sc} and the "strength" of the resonance

Intensity effect during bunching

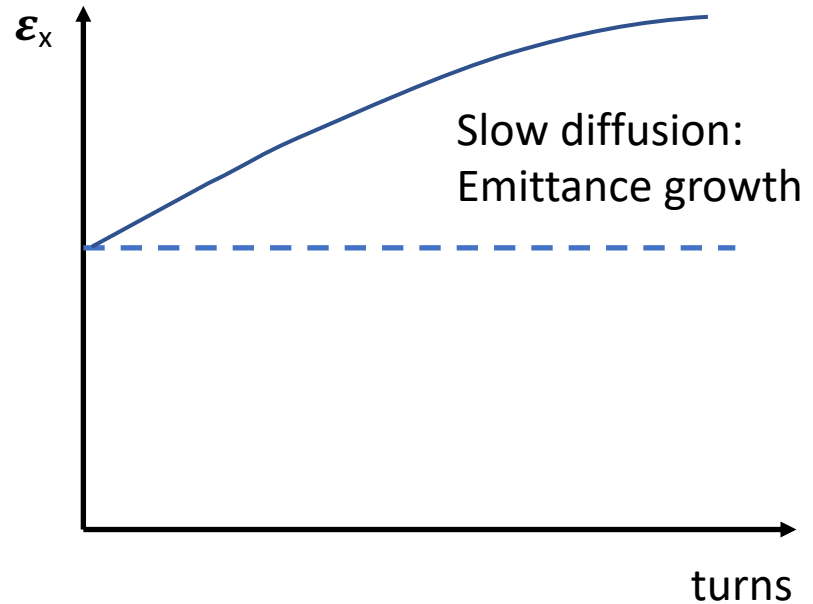
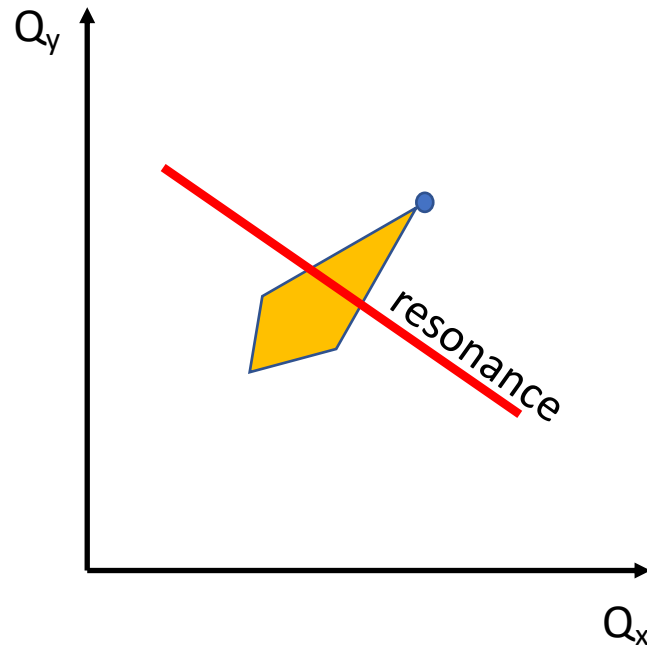


Beam loss mechanism



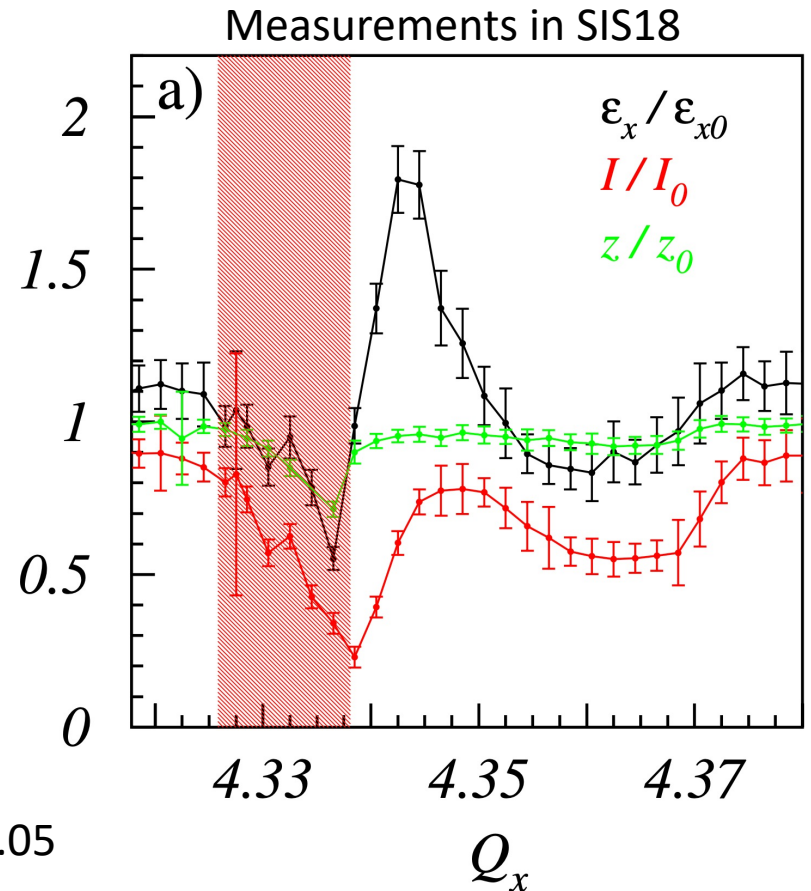
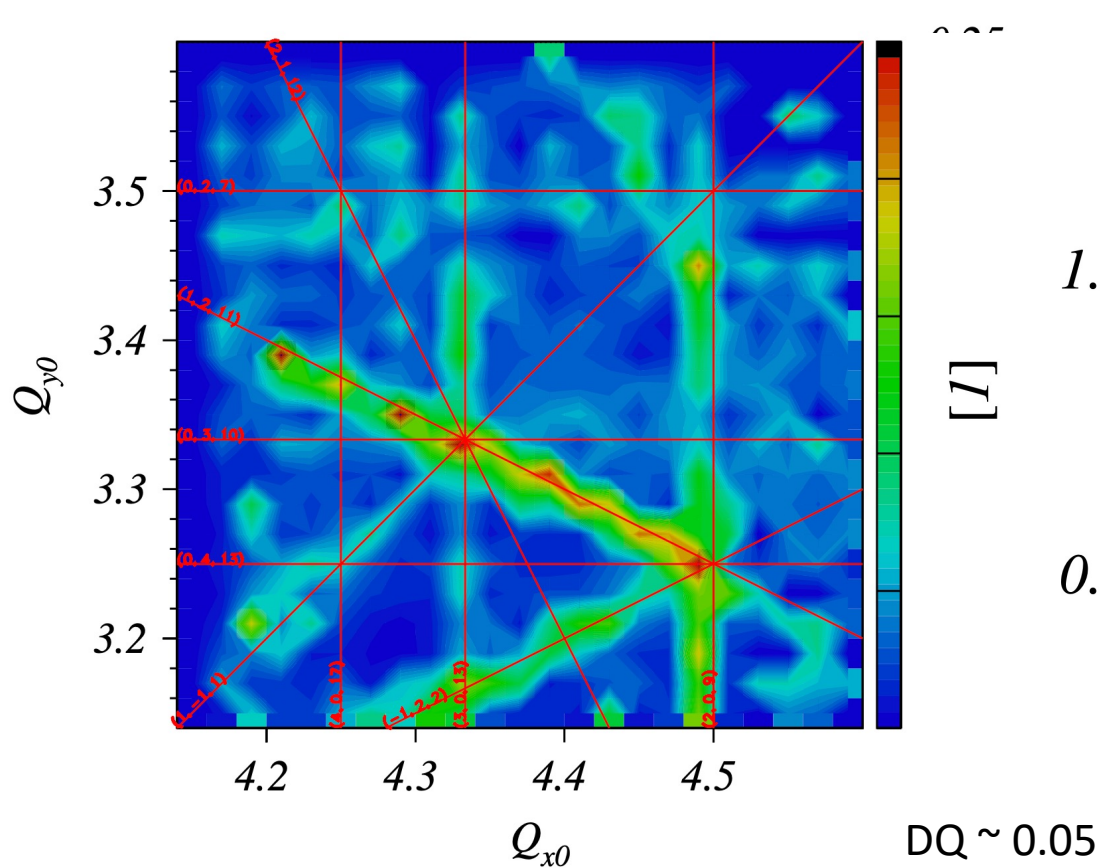
Resonances and high intensity

Bunched Beams

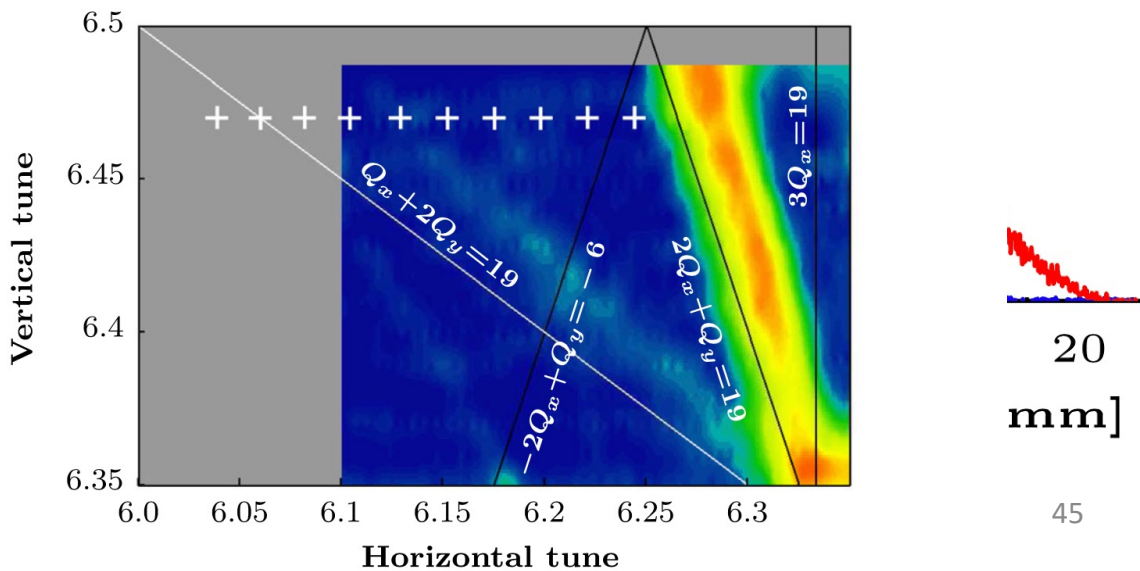
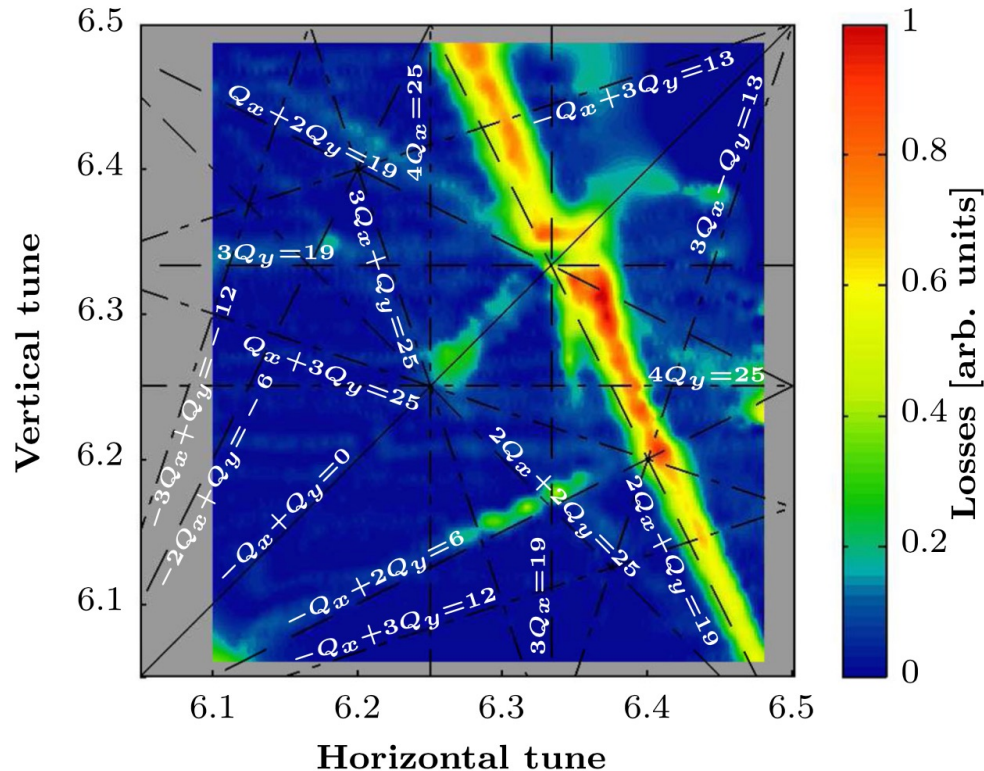
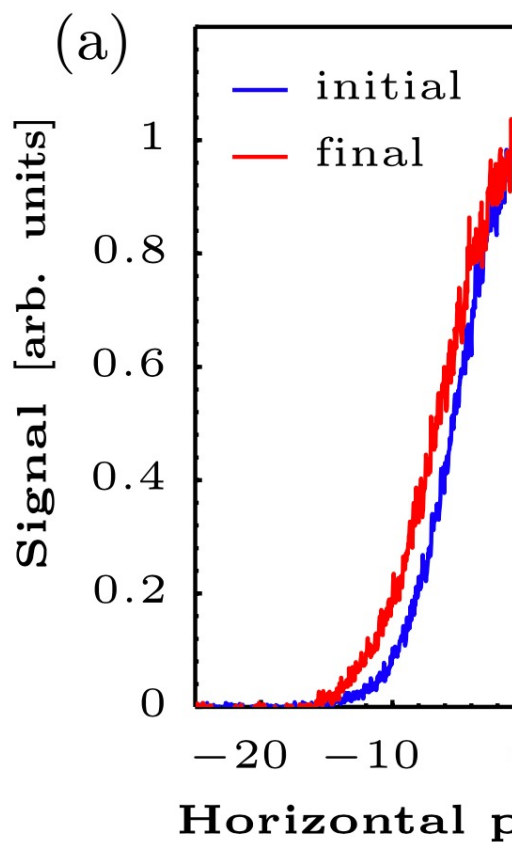


Effect is fast and depends on DQ_{sc} and the "strength" of the resonance
And the position of the tune with respect to the resonance

High intensity bunched beams and resonances @SIS18

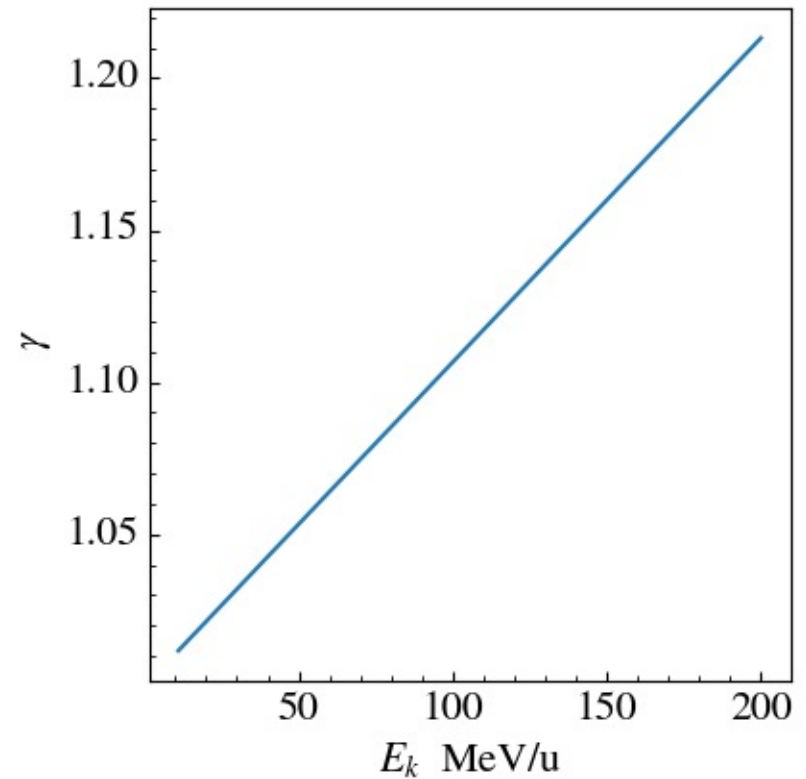
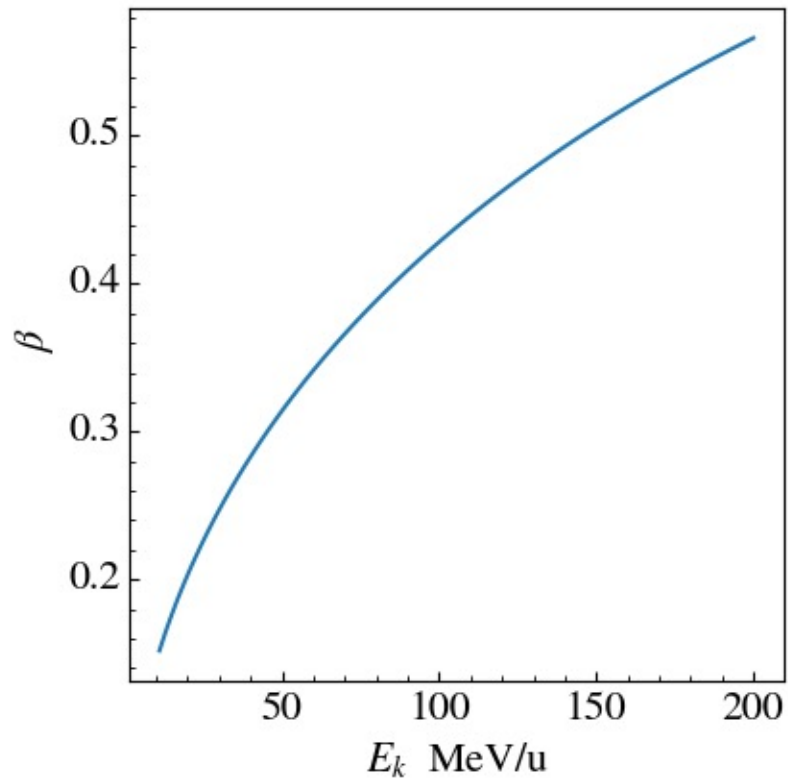


@CERN-PS



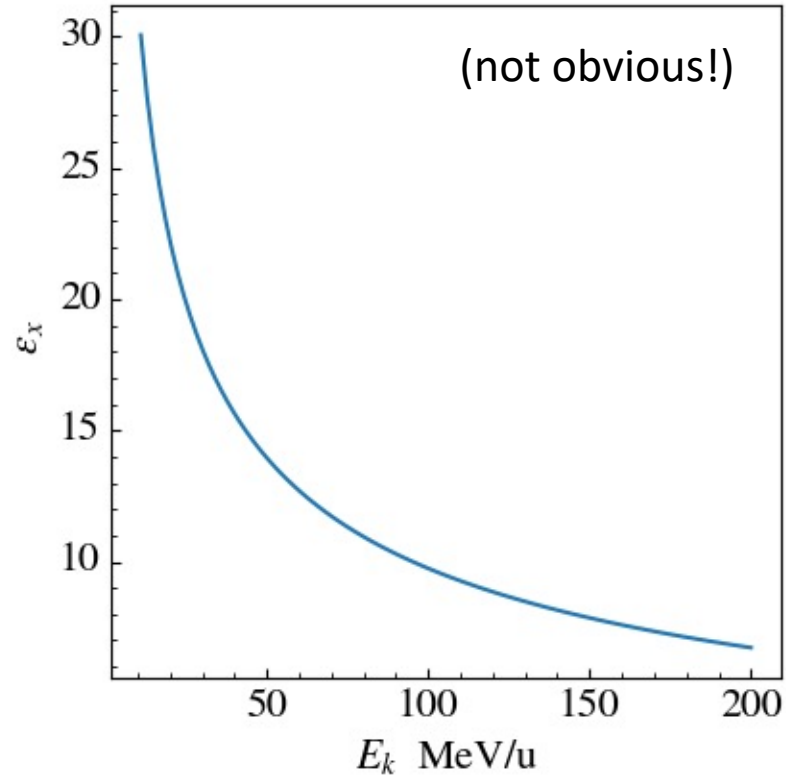
Acceleration

During acceleration

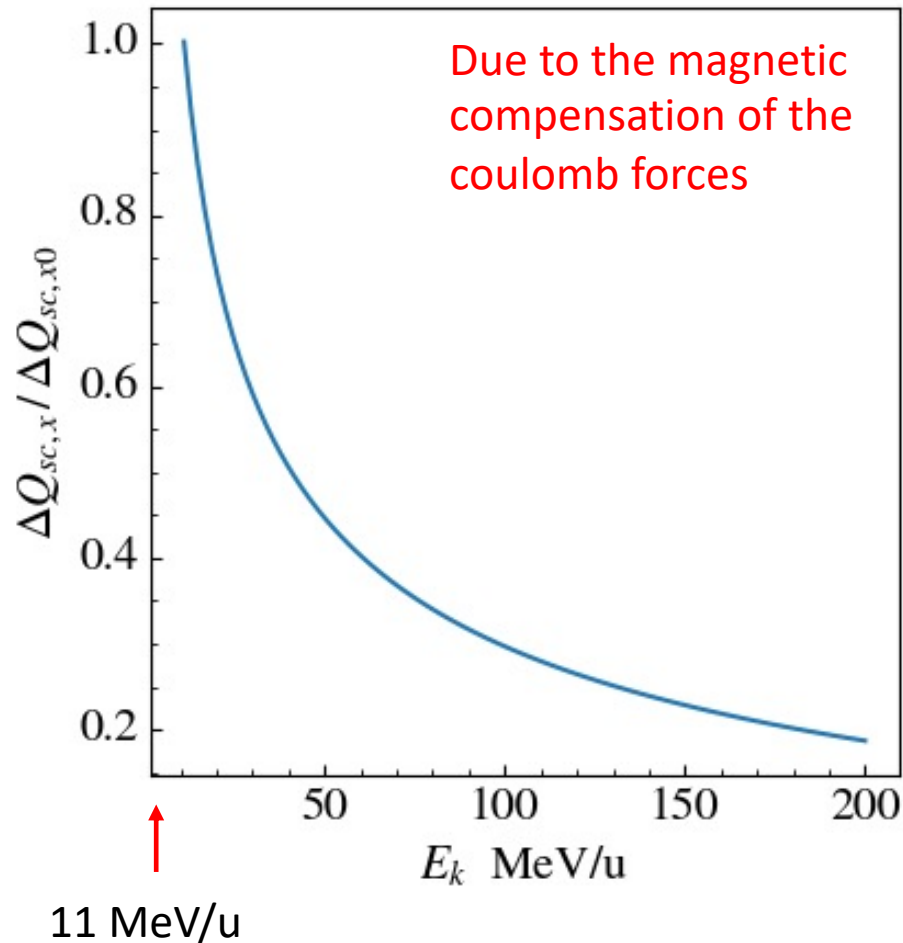


Therefore the current increases

Beam emittance becomes smaller. → the beam becomes smaller



But, the space charge tuneshift become smaller !!



Summary

- Space charge tune-shift is the measure of the “high intensity”;
- High intensity change machine+beam optics;
- A mismatched beam will create emittance increase;
- Overlapping of space charge tunespread will create emittance increase;
- For a bunched beam the overlapping of tunespread with a resonance; will create a diffusional regime → beam loss;
- In a bunch compression the tunespread increases and may lead to resonance overlapping;
- During acceleration space charge tunespread dumps;

Not addressed:

- High intensity on the longitudinal dynamics;
- High intensity, dp/p and bunch compression;
- Negative mass instability;
- Resistive wall instability;