

Strangeness in Heavy Ion Collisions

J. Aichelin, C. Hartnack, M. Winn,
(Subatech, Nantes)

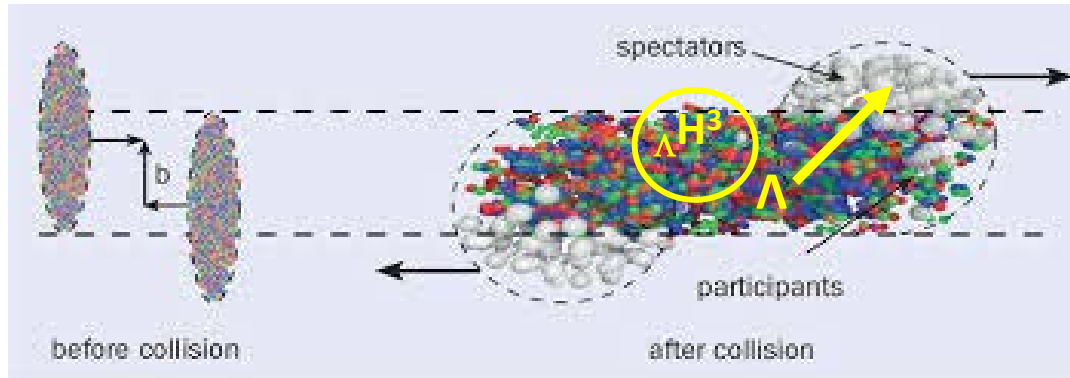
E. Bratkovskaya, Y. Leifels, A. LeFevre, T. Song
(GSI Helmholtzzentrum für Schwerionenforschung, Darmstadt)

C. Blume, S. Glässel
(Goethe Universität, Frankfurt)

L. Tolos
(Universitat Autònoma, Barcelona)

V. Kireyeu, V. Voronyuk, V. Kolesnikov
(Joint Institut for Nuclear Research, Dubna)

Why should we study strangeness in HI Collisions ?



Access to elementary strangeness production mechanism:

In HI collisions strangeness can be produced differently than in NN collisions (three body reaction)

Access to in medium properties of strange hadrons and $\Lambda(K)N$ potential

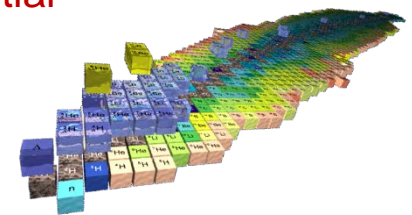
(test of the theoretical predictions, -> neutron stars?)

Access to the reaction dynamics

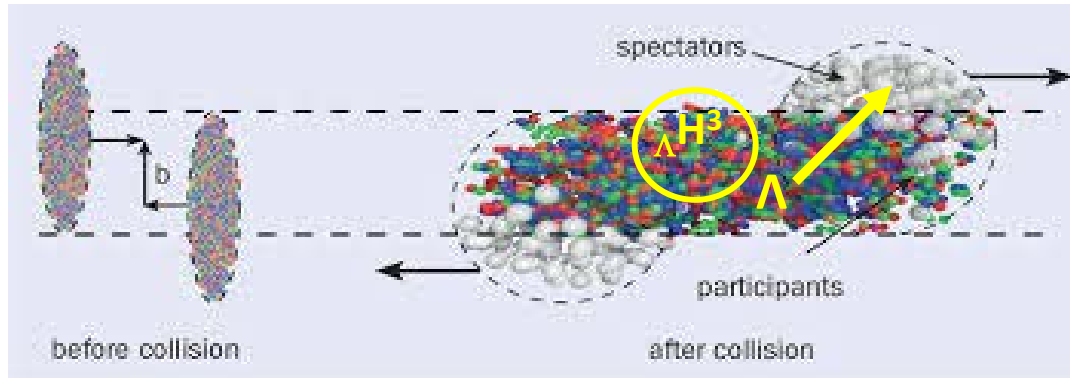
(strange particles come exclusively from the participant zone -> different v_2)

Access to quark gluon plasma properties

(different T_c for strange and nonstrange hadrons?)



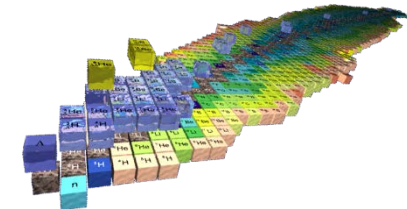
Why should we study strangeness in HI Collisions ?



Access to hypernuclei formed mostly from spectators

dynamics of interface between participants and spectators
formation and properties of hypernuclei

offers the possibility to hypernucleus spectroscopy



Access to midrapidity hypernuclei

observed up to LHC energies

phase space distribution of baryons and ΛN interaction at different densities

Problem: for many details (equation of state) no observable

→ one has to simulate the collision on a computer

How to model Heavy Ion Collisions on a Computer

Transport approaches

Present microscopic approaches:

- ❑ VUU(1985), BUU(1985), HSD(1996), PHSD(2008), SMASH(2016) solve the time evolution of the one-body phase-space density of nucleons in a mean field
→ good for studies of the EOS but **not for fragments (2 body correlations)**
- ❑ UrQMD is a n-body model but makes clusterization via coalescence and a statistical fragmentation model
- ❑ (I)QMD is a n-body model but is limited to energies < 1.5 AGeV
→ describes fragments at SIS energies,
but conceptually not adapted for NICA/FAIR energies and higher
- ❑ PHQMD is a new n-body model not limited to low beam energies
→ includes the formation of a quark gluon plasma et higher energies
at low energies similar to (I)QMD

All these models propagate nucleons, no clusters:

In order to understand the **microscopic origin of cluster formation** one needs:

- a realistic model for the dynamical time evolution of HICs
- **dynamical modelling of cluster formation** based on interactions

Transport eqs. for N-body theories like (PH)QMD, AMD, FMD

Roots in Quantum Mechanics

Remember QM courses when you faced the problem:

- we have a Hamiltonian $\hat{H} = -\frac{\hbar^2 \nabla^2}{2m} + V$
- the Schrödinger eq.

$$\hat{H}|\psi_j\rangle = E_j|\psi_j\rangle$$

has no analytical solution

- we look for the ground state energy

Ritz variational principle:

Assume a **trial function** $\psi(q, \alpha)$ which contains one **adjustable parameter** α , which is varied to find the lowest energy expectation value:

$$\frac{d}{d\alpha} \langle \psi | \hat{H} | \psi \rangle = 0 \rightarrow \alpha_{min}$$

determines α for which $\psi(q, \alpha)$ is **closest to the true ground state w.f.** and $\langle \psi(\alpha_{min}) | \hat{H} | \psi(\alpha_{min}) \rangle = E_0(\alpha_{min})$ **closest to the true ground state energy**



Walther Ritz

Extended Ritz variational principle (Koonin, TDHF)

Take **trial wavefct** with **time dependent** parameters and solve

$$\delta \int_{t_1}^{t_2} dt \langle \psi(t) | i \frac{d}{dt} - H | \psi(t) \rangle = 0. \quad (1)$$

QMD **trial wavefct** for particle i with $p_{oi}(t)$ and $q_{oi}(t)$

$$\psi_i(q_i, q_{oi}, p_{oi}) = C \exp[-(q_i - q_{oi} - \frac{p_{oi}}{m}t)^2 / 4L] \cdot \exp[ip_{oi}(q_i - q_{oi}) - i \frac{p_{oi}^2}{2m}t]$$

For N particles: $\psi_N = \prod_{i=1}^N \psi_i(q_i, q_{oi}, p_{oi})$

QMD

$$\psi_N^F = \text{Slaterdet} \left[\prod_{i=1}^N \psi_i(q_i, q_{oi}, p_{oi}) \right]$$

AMD/FMD

For the QMD trial wavefct eq. (1) yields

$$\frac{dq}{dt} = \frac{\partial \langle H \rangle}{\partial p} \quad ; \quad \frac{dp}{dt} = - \frac{\partial \langle H \rangle}{\partial q}$$

For Gaussian wavefct
eq. of motion very similar
to Hamilton's eqs.
(but only for Gaussians !!)

Quantum Molecular Dynamics (QMD)

- nucleon-nucleon density dependent two body potential:

$$\begin{aligned} V^{ij} &= G^{ij} + V_{\text{Coul}}^{ij} \\ &= V_{\text{Skyrme}}^{ij} + V_{\text{Yuk}}^{ij} + V_{\text{mdi}}^{ij} + V_{\text{Coul}}^{ij} + V_{\text{sym}}^{ij} \\ &= t_1 \delta(\vec{x}_i - \vec{x}_j) + t_2 \delta(\vec{x}_i - \vec{x}_j) \rho^{\gamma-1}(\vec{x}_i) + t_3 \frac{\exp\{-|\vec{x}_i - \vec{x}_j|/\mu\}}{|\vec{x}_i - \vec{x}_j|/\mu} + \\ &\quad t_4 \ln^2(1 + t_5 (\vec{p}_i - \vec{p}_j)^2) \delta(\vec{x}_i - \vec{x}_j) + \frac{Z_i Z_j e^2}{|\vec{x}_i - \vec{x}_j|} + \\ &\quad t_6 \frac{1}{\rho_0} T_3^i T_3^j \delta(\vec{r}_i - \vec{r}_j) \end{aligned}$$

$t_1 - t_4$ depend on the EoS

t_4 contains the momentum dependence of the potential

- In addition cross sections: NN elastic, $NN \leftrightarrow N\Delta$, $\Delta \rightarrow N\pi$

PHQMD

The goal: to develop a unified n-body microscopic transport approach for the description of heavy-ion dynamics and dynamical cluster formation from low to ultra-relativistic energies

Realization: combined model **PHQMD** = (PHSD & QMD) & SACA

Parton-Hadron-Quantum-Molecular Dynamics

Initialization → propagation of baryons:
QMD (Quantum-Molecular Dynamics)

Propagation of partons (quarks, gluons) and mesons
+ **collision integral** = interactions of hadrons and partons (QGP)
from **PHSD** (Parton-Hadron-String Dynamics)

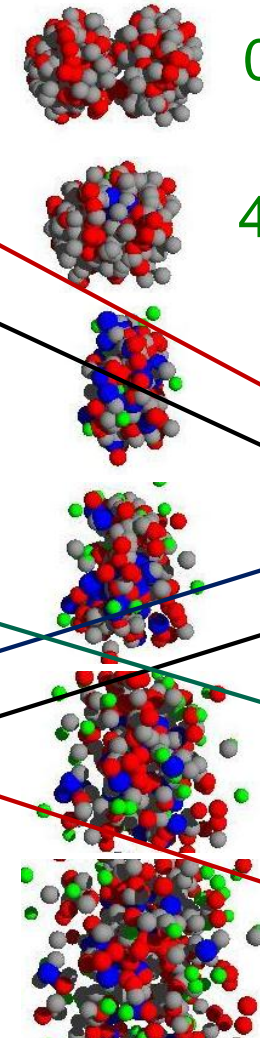
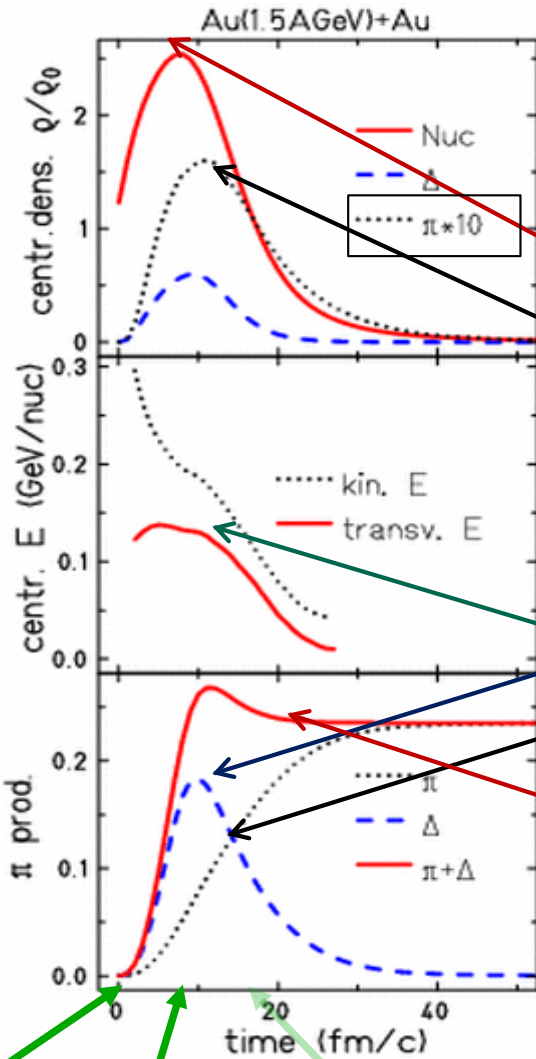
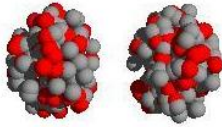
Clusters recognition:
SACA (Simulated Annealing Clusterization Algorithm)
vs. **MST** (Minimum Spanning Tree)

SACA: R. K. Puri, J. Aichelin, J.Comput.Phys. 162 (2000) 245-266

PHSD: W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215



Time evolution (IQMD)



0 fm/c : start of the reaction

4 fm/c : rising of resonance prod.

8 fm/c: max. central $\rho(\text{nuc.})$

10 fm/c max central $\rho(\pi)$

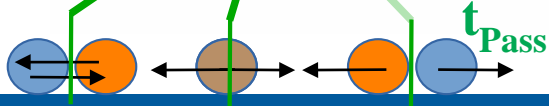
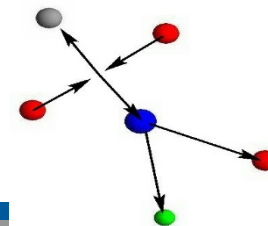
12 fm/c: max number of Δ

14 fm/c: π dominate over Δ

16-20 fm/c: nucleon spectra become 'thermal'

20 fm/c: π number stabilizes

Proton
Neutron
Delta
Pion



Strange hadrons as a tool to study the
nuclear reaction dynamics
and the nuclear equation of state

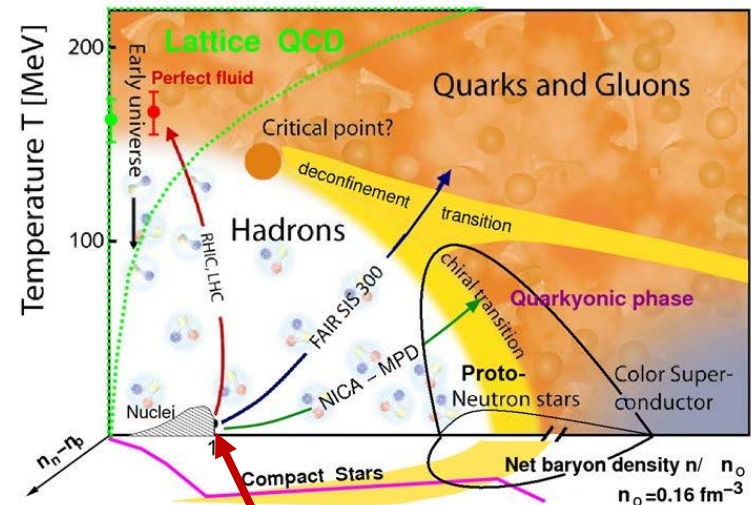
Nuclear equation of state

- Experimentally one can explore the region of large (T, μ) but the EoS is not an observable

Strategy:

- develop **transport approaches** which simulate the heavy ion collisions
- vary EoS** in the simulation
- identify **observables which are sensitive** to the EOS
- compare** experimental measurements with theory

The Phase Diagram of Strongly Interacting Matter



Equation of State (EOS): relationship between Energy, Pressure, Temperature, Density and Isospin Asymmetry of Nuclear Matter

Only known point
(cold nuclei)

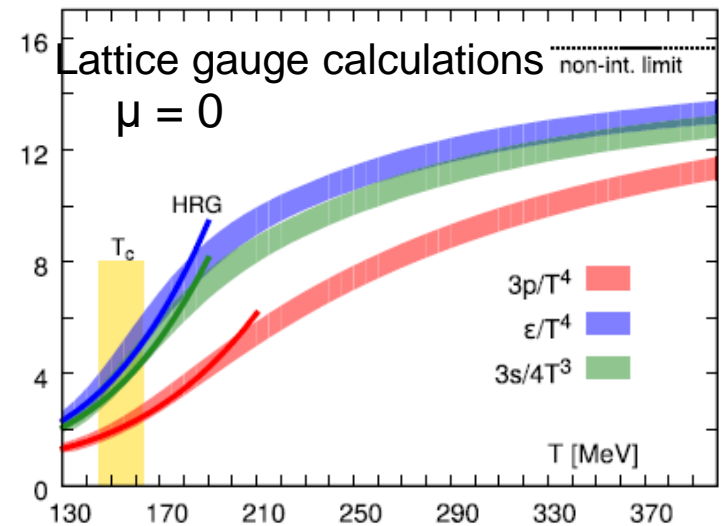
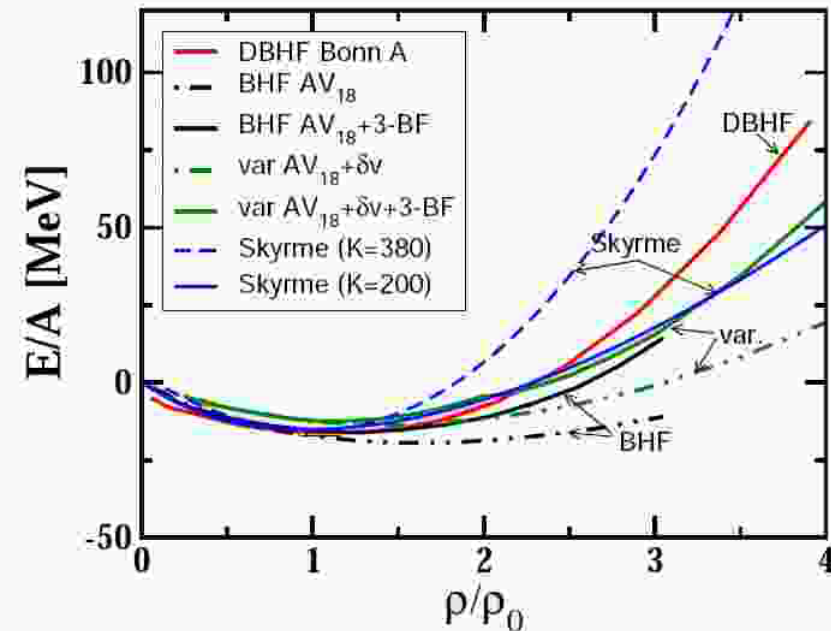
Nuclear equation of state (theory)

theory is limited to

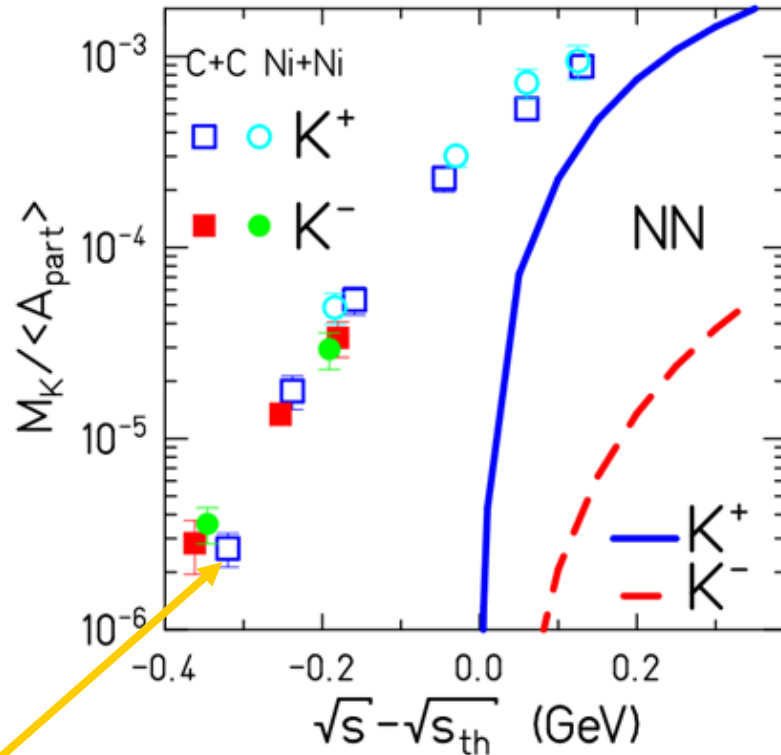
to low temperature and $\rho < 1.5 - 2\rho_0$
 Brückner G –matrix
 (hole line expansion)

high temperatures and $\mu \approx 0$
 lattice gauge calculations

At low energies $E/A(T,\rho)$ instead $P(T,\rho(\mu))$ or $\epsilon(T,\rho(\mu))$



Strangeness and the nuclear equation of state



□ AA collisions:
experimental observation of K^+, K^-
production below the NN-threshold

- NN: Excitation function of K^+ and K^- quite different
- AA: Excitation function of K^+ and K^- quite similar
- Fermi motion cannot explain very subthreshold production
- Conclusion
AA: new mechanisms for strangeness production

Near threshold strangeness production in AA

I. Strangeness production channels at low energies

baryon-baryon collisions:



$$K = (K, K^0)$$

$$\bar{K} = (K^-, \bar{K}^0)$$

$$B = (N, \Delta, \dots)$$

$$Y = (\Lambda, \Sigma)$$



dominant channel for low energy K⁻ production!

meson-baryon collisions:

• meson-meson collisions:

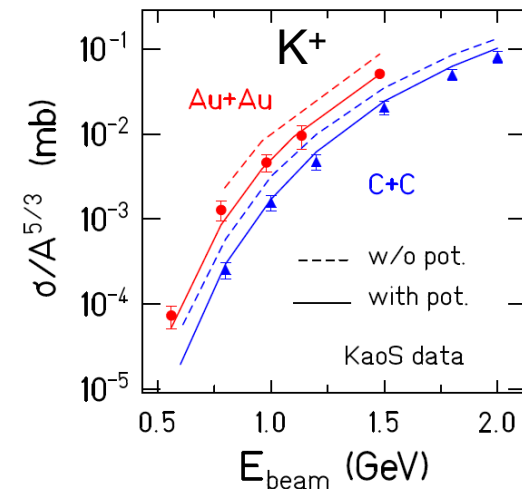
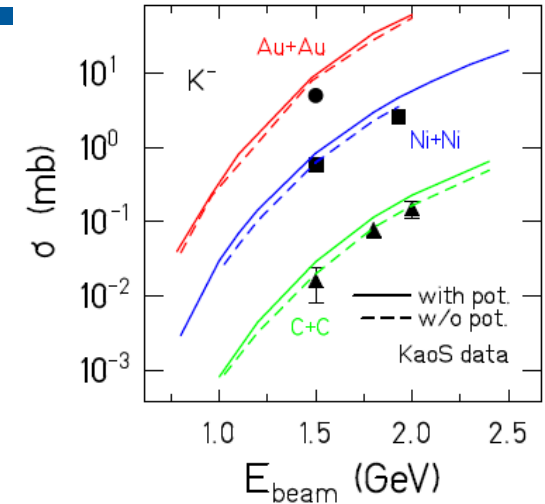


• resonance decays: $K^* \rightarrow \pi + K, \dots, \phi \rightarrow K + \bar{K}$

II. Strangeness rescattering

= (quasi-)elastic scattering with baryons and mesons

III. K(K⁻)-Nucleus potential V(ρ)



Origin of the different excitation functions

Dominant for K^+ in AA: Two step process $NN \rightarrow N\Delta$ $N\Delta \rightarrow K^+\Lambda N$

lowers the effective threshold
enhances K^+ below NN threshold

two step process more probable in central collision

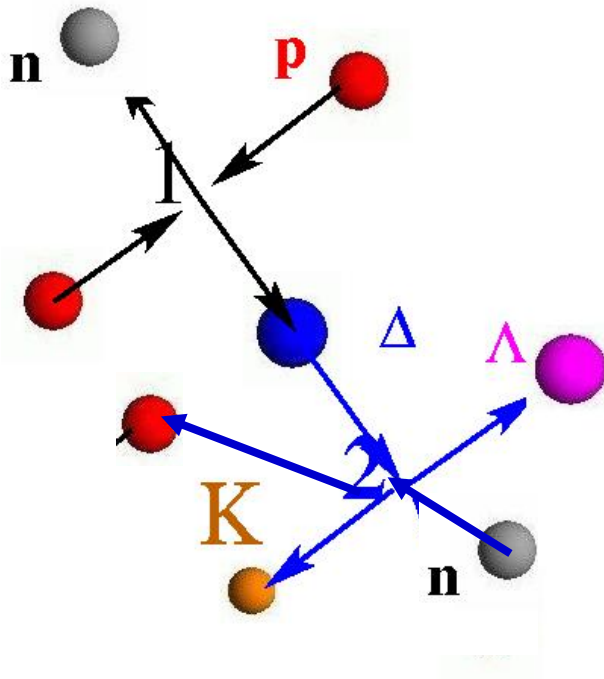
Theory and simulations:

soft EoS: system gets to higher densities
 \rightarrow mean free path for $N\Delta \rightarrow K^+\Lambda N$ shorter

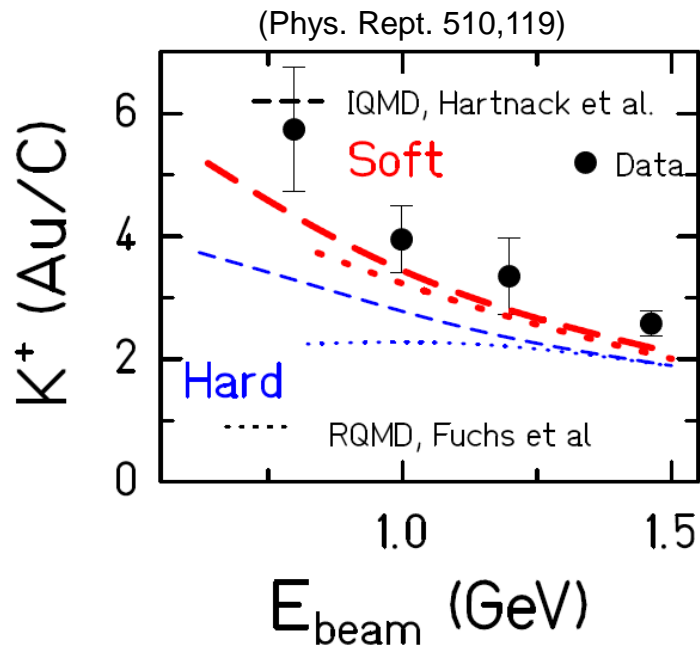
$N\Delta \rightarrow K^+\Lambda N$ competes with Δ decay

\rightarrow for a soft EOS we expect more $N\Delta \rightarrow K^+\Lambda N$ collisions

and hence more K^+



Strangeness production and the nuclear EoS



Comparison with experiment

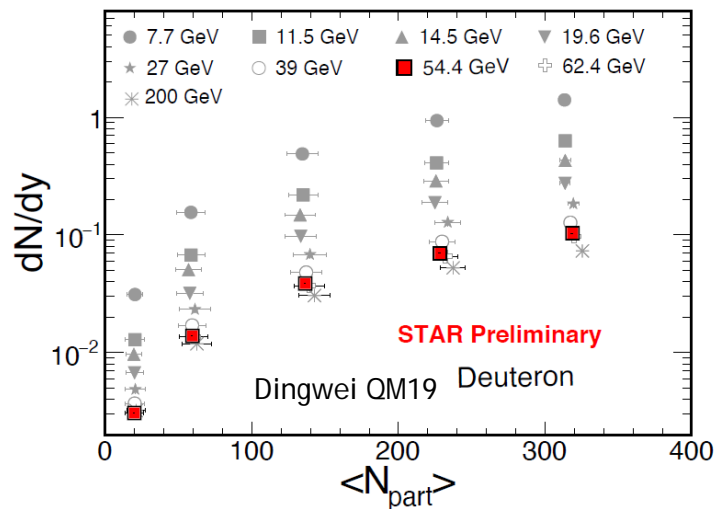
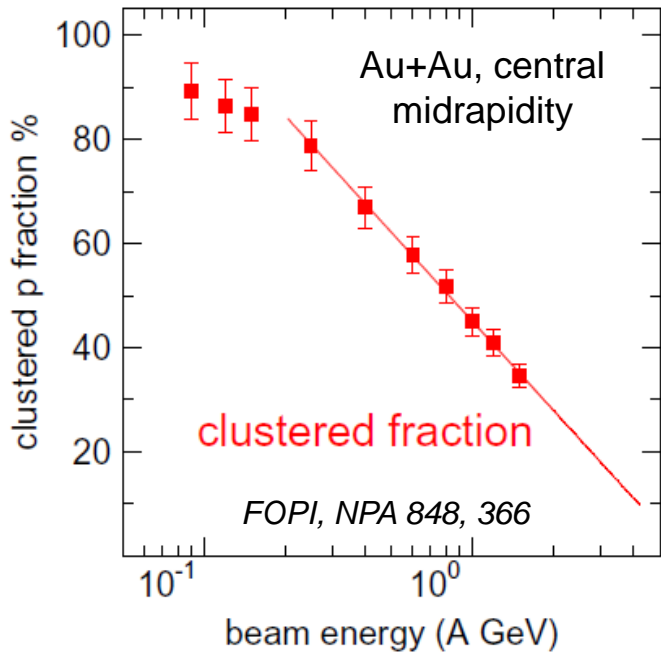
- confirms the EoS dependence of K^+ yield
- **soft EoS: best agreement with data**

Up to today the observable
which shows the strongest EoS dependence

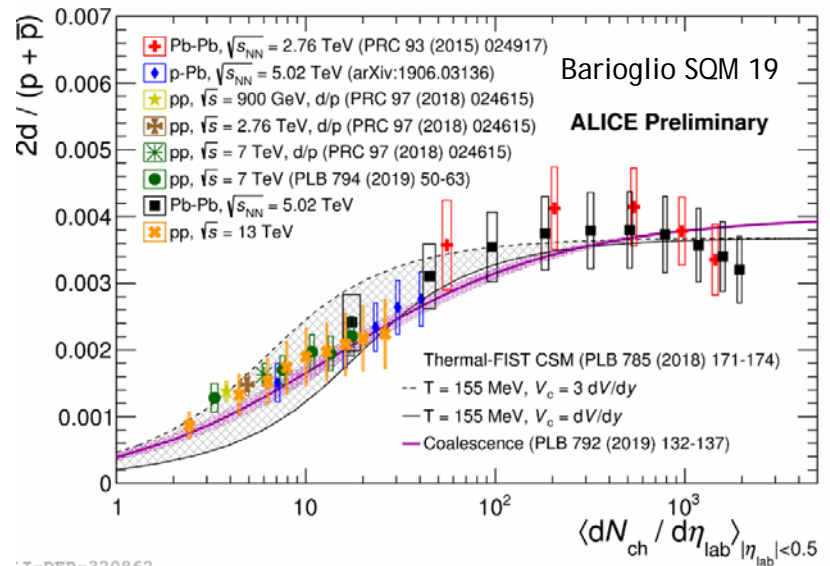
- Perspectives: FAIR and NICA (Russia) have higher beam energies
excitation functions of Ξ and Ω become available
sensitive probes for studying the reaction mechanism

Cluster and Hypercluster in Heavy Ion Collisions

Clusters in HICs



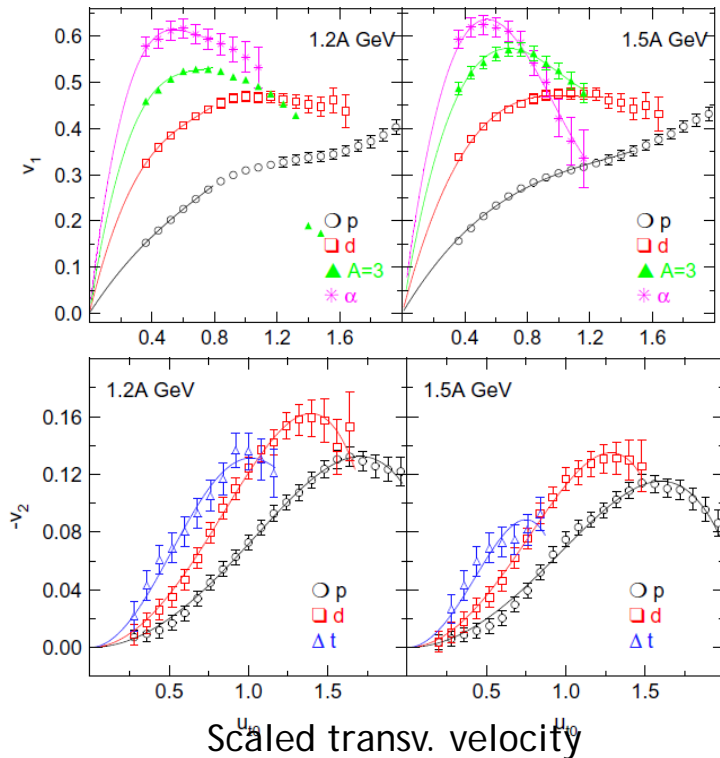
- Clusters are very abundant at low energy;
 - at 3 AGeV in central Au+Au collisions ~20% of the baryons are in clusters!
 - cluster production continues to STAR energies
1% - 0.3% of the nucleons are bound in d at y_{cm}
 - decrease slightly up to LHC energies
- midrapidity clusters exist at all beam energies where fireball temperatures $T > 100$ MeV



SI-DER-320862

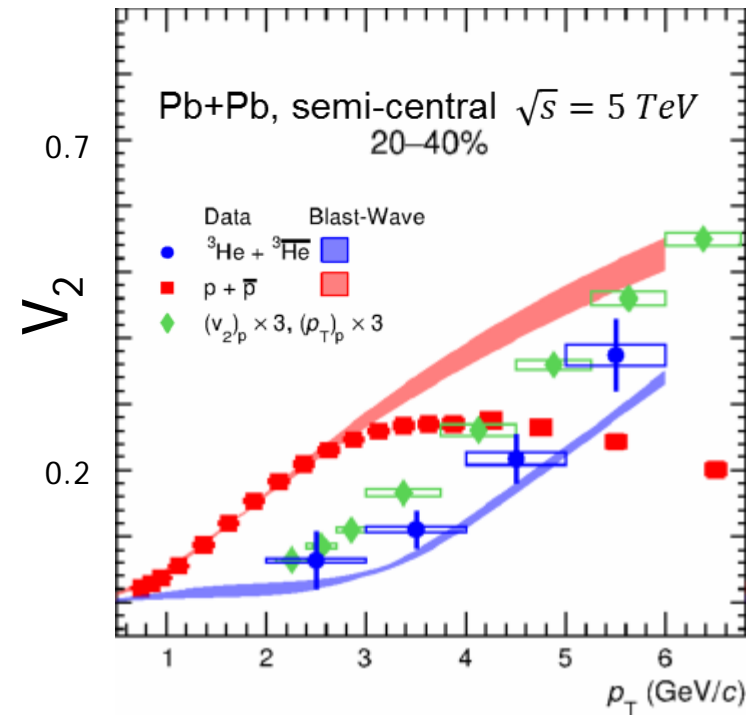
There is more than multiplicity of clusters

Au+Au, semi-central FOPI, NPA 876,1



Baryons in clusters have quite different properties ($v_1, v_2, dn/dp_T$)

and explore therefore different phase space regions:



In addition, cluster open new physics opportunities

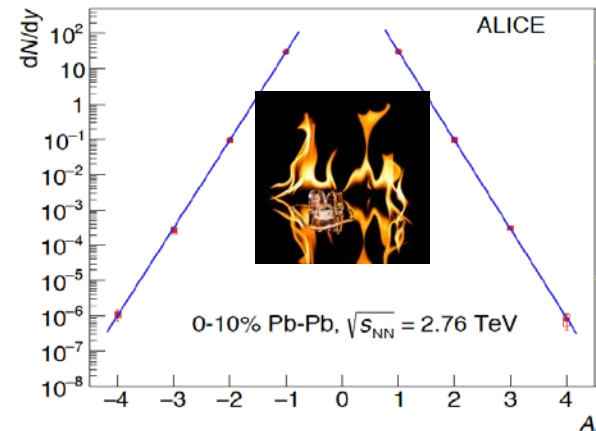
- possible signals of a 1st order phase transition at finite μ
- fluctuations of the phase space densities of nucleons
- hyper-nucleus formation at mid as well as target/proj. rapidities

Last but not least: How it is possible that clusters survive ?

- Freeze out temperature of hadrons: 120 - 158 MeV
Binding energy of clusters: around 5 MeV/N
- Clusters cannot survive a heat bath of more than 120 MeV.
- The first collision with a heat bath constituent would destroy them
- But they exist!!!!

Ice in a fire' puzzle:

how a weakly bound objects (cluster) can be formed and survive in a hot environment ?!



ALICE, NPA 971, 1 (2018)

Methods to identify clusters in models which propagate nucleons:

Static approaches:

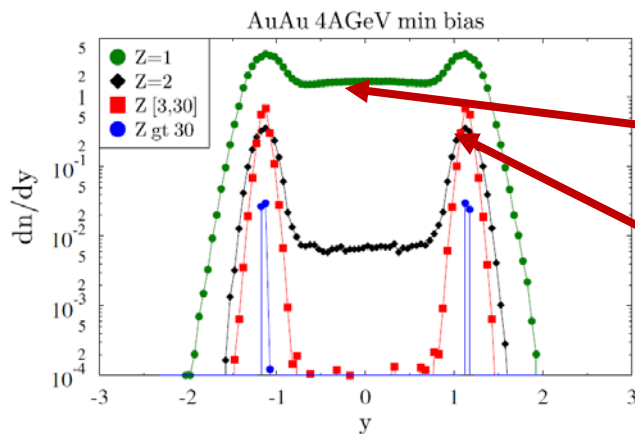
cluster multiplicity determined at a fixed time or temp

- coalescence (early, assumption: no coll. later)
- statistical model (V,T,N) very late $\rho \ll \rho_0$

Dynamical approaches:

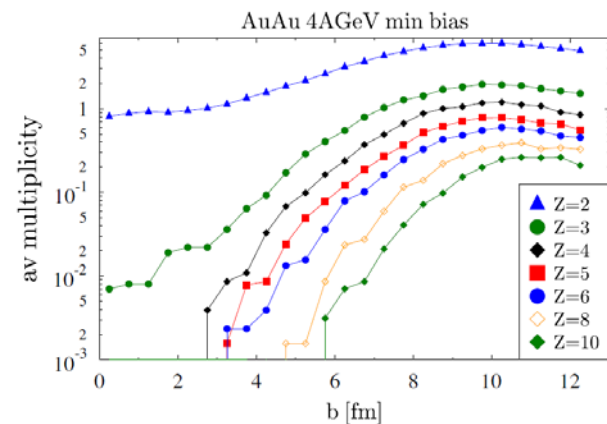
cluster multiplicity is a function of time

- minimum spanning tree (correlation in coord. space)
- simulated annealing (correlation in mom and coord. space)
- time dep. perturbation theory using Wigner densities



Midrapidity clusters

Proj/Target clusters
smooth transition

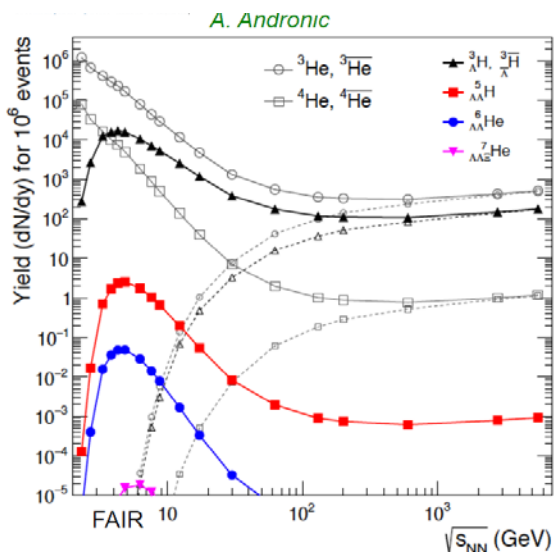


Sudden Formation of Clusters

Statistical model:

Sudden freeze out of a strongly Interacting system

Describes the multiplicities but not the spectra (yield V, T, μ)

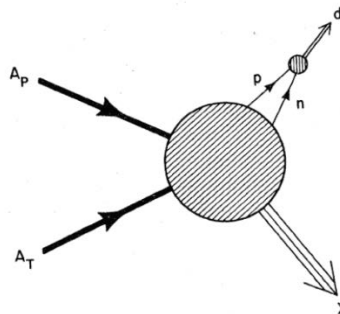


Coalescence:

Butler and Pearson PR129,836 (p+A)
based of QM treatment of the production
results in

$$n_d(\mathbf{p}) \sim \frac{1}{p^2} n_n^2\left(\frac{\mathbf{p}}{2}\right)$$

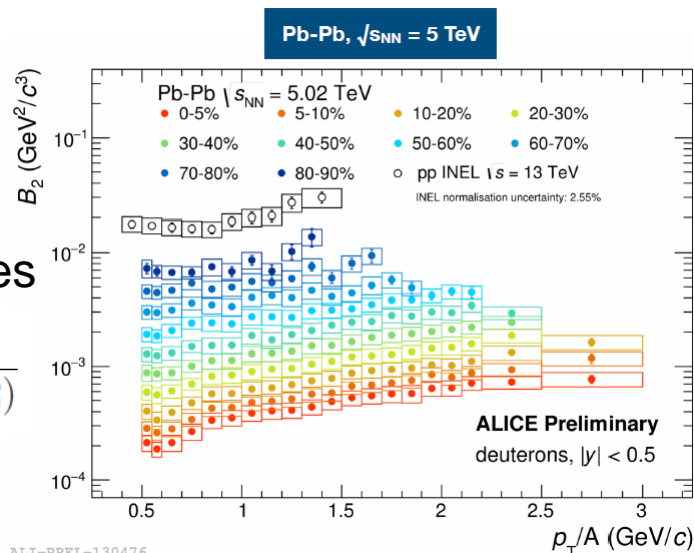
despite production is a
3-body process



Kapusta PRC41,1493 $n_d(\mathbf{p}) \sim V n_n^2\left(\frac{\mathbf{p}}{2}\right)$

Today one investigates

$$B(\mathbf{p}, V, p_0, V_{NN}) = \frac{n_d(\mathbf{p})}{n_n^2(\mathbf{p}/2)}$$



Dynamical cluster production



I. Minimum Spanning Tree

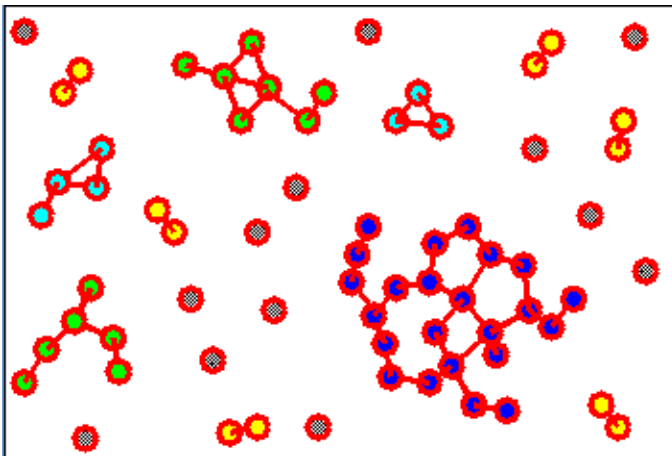
I. **Minimum Spanning Tree (MST)** is a **cluster recognition** method applicable for the (asymptotic) **final state** where coordinate space **correlations may only survive for bound states**.

The MST algorithm searches for accumulations of particles in coordinate space:

1. Two particles are **bound** if their distance in coordinate space fulfills

$$|\vec{r}_i - \vec{r}_j| \leq 4 \text{ fm (range of NN potential)}$$

2. A particle is **bound to a cluster** if it is **bound with at least one particle** of the cluster.



Additional momentum cuts (coalescence) change little:

large relative momentum
-> finally not at the same position

II.SACA or ECRA now FRIGA

If we want to identify fragments earlier one has to use momentum space info as well as coordinate space info

Idea by Dorso et al. (Phys.Lett.B301:328,1993) :

- a) Take the positions and momenta of all nucleons at time t .
- b) Combine them in all possible ways into all kinds of fragments or leave them as single nucleons
- c) Neglect the interaction among clusters
- d) Choose that configuration which has the highest binding Energy

Simulations have shown that the most bound configuration is the precursor of the final fragment distribution
(large persistent coefficient)

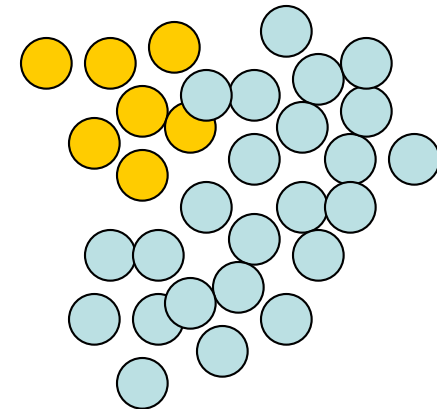
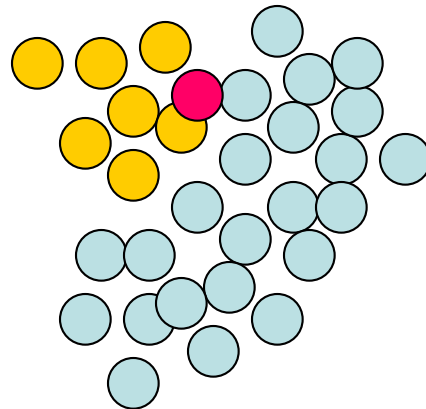
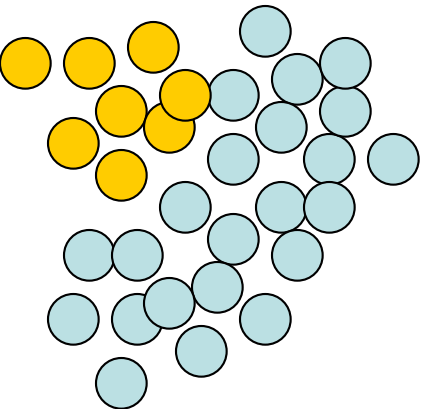
How does Simulated Annealing work?

SACA: PLB301,328; J.Comp.Phys.162,245, NPA619,375

now FRIGA :Nuovo Cim. C39,399 (including symmetry and pairing energy)

Take randomly 1 nucleon
out of a fragment

Add it randomly to another
fragment



$$E = E_{kin}^1 + E_{kin}^2 + V^1 + V^2$$

$$E' = E_{kin}^{1'} + E_{kin}^{2'} + V^{1'} + V^{2'}$$

There is no interaction between clusters-> no energy conserv.

If $E' < E$ take the new configuration

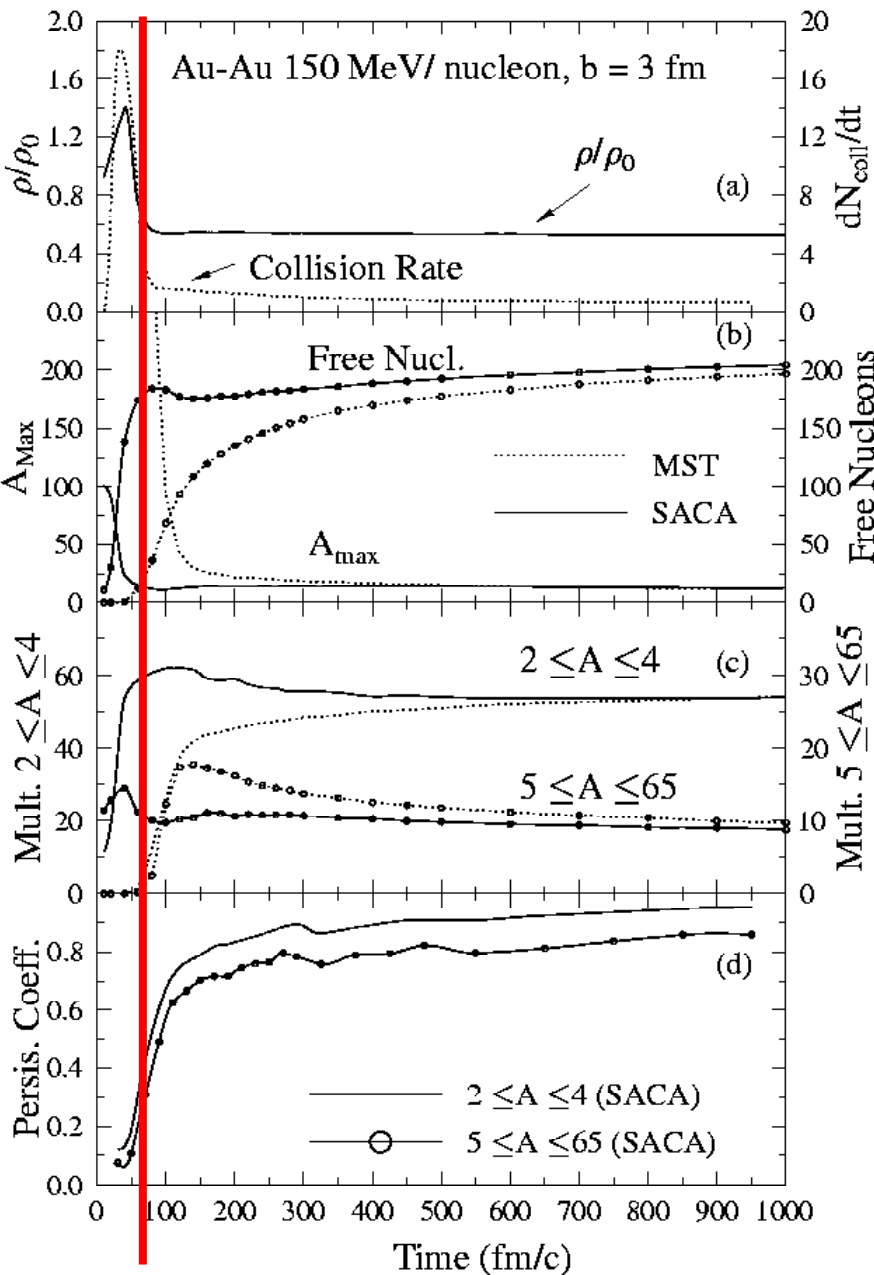
If $E' > E$ take the old with a probability depending on $E' - E$

Repeat this procedure very many times

→ Leads automatically to the most bound configuration

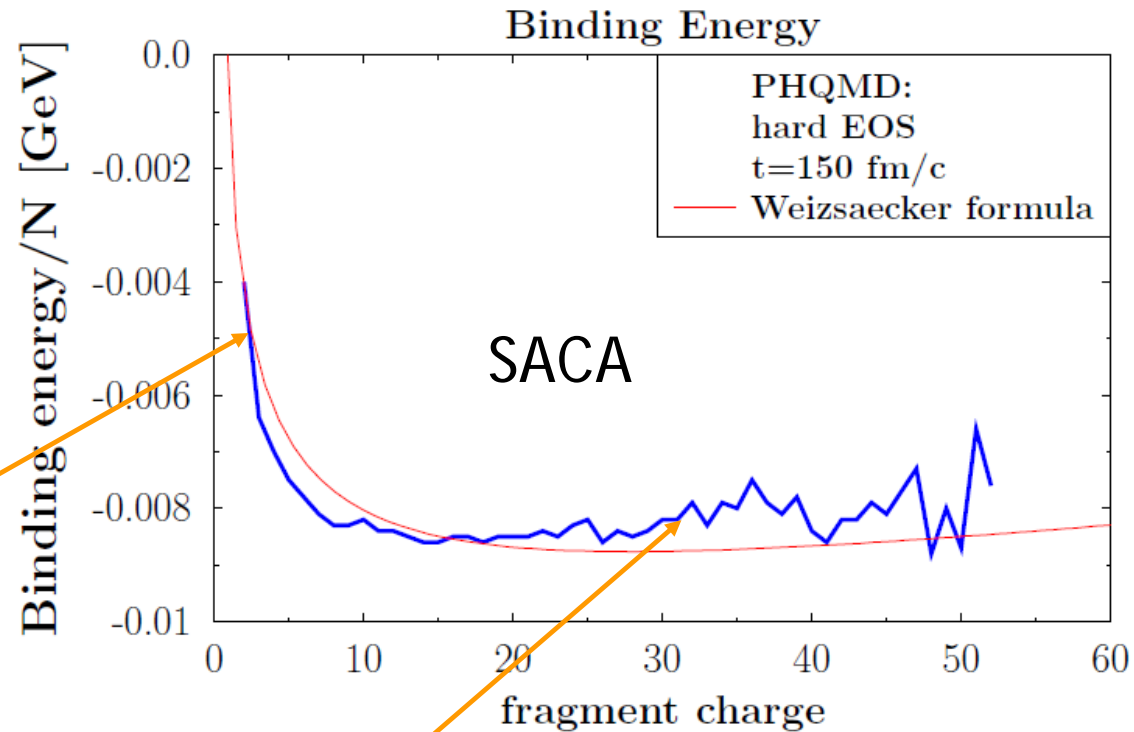
IQMD

SACA can really identify the fragment pattern very early as compared to the Minimum Spanning Tree (MST) which assumes that two nucleons form a fragment if they are closed than r_{\max} .



At $1.5t_{\text{pass}}$ A_{max} and multiplicities of intermediate mass fragments are determined

N-body models can produce cluster with the right E_{bind}



There are two kinds of fragments

- formed from **participant matter** created during the expansion of the fireball “ice” ($E_{\text{bind}} \approx 8 \text{ MeV/N}$) in “fire” ($T \geq 100 \text{ MeV}$)

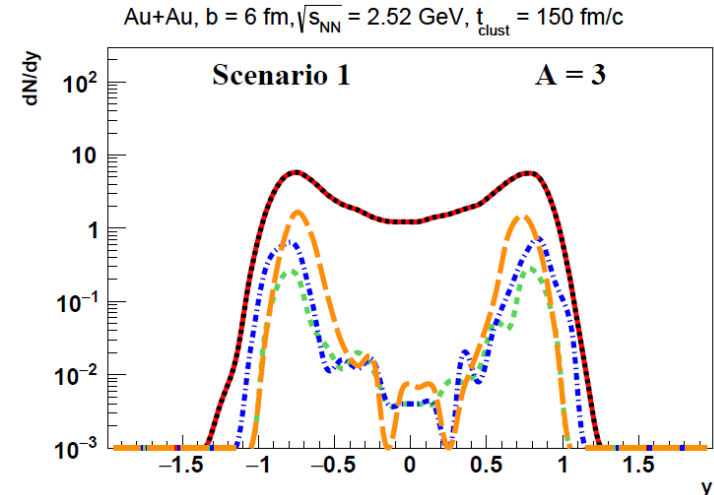
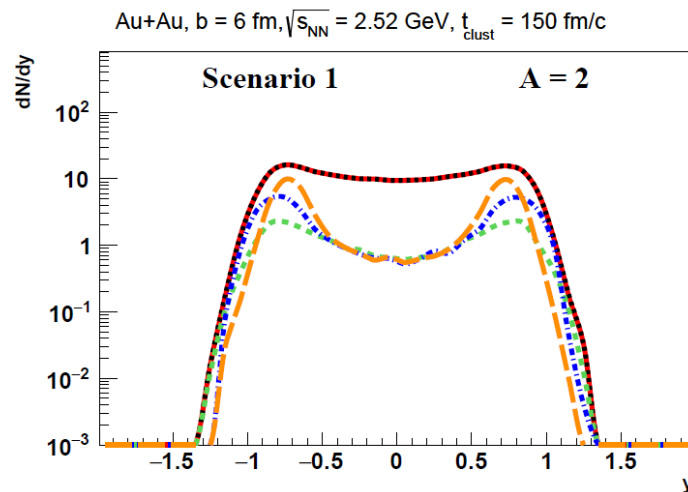
- formed from **spectator matter** close to beam and target rapidity initial-final state correlations HI reaction makes spectator matter unstable

- Cluster formation is sensitive to **nucleon dynamics**
- ➔ One needs to **keep the nucleon correlations (initial and final)** by realistic **nucleon-nucleon interactions** in transport models:
 - **QMD** (quantum-molecular dynamics) – allows to keep correlations
 - **MF** (mean-field based models) – correlations are smeared out
 - **Cascade** (no potential) – correlations kept but nucleons do not stay together

Cluster stability over time:

V. Kireyeu, PRC 103, 044905

- QMD:**
- PHQMD + psMST
- MF:**
- PHSD + psMST
- Cascade:**
- SMASH + psMST
 - UrQMD + psMST

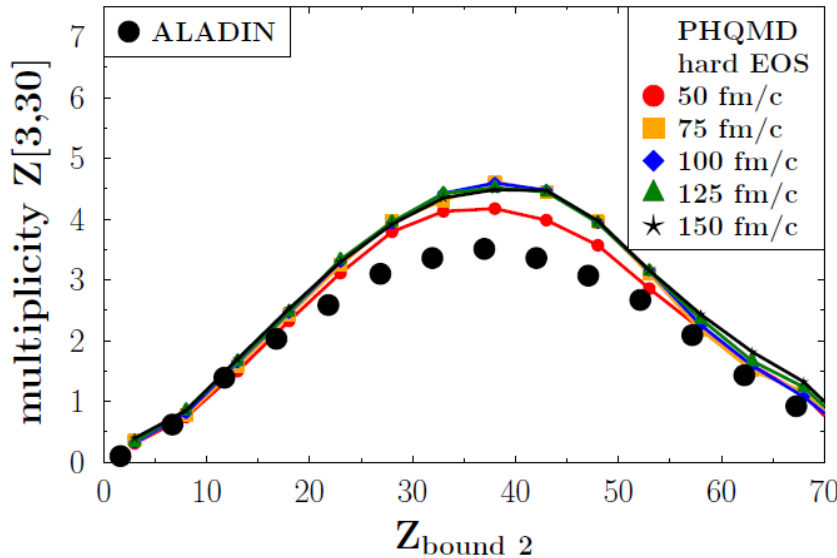


Heavy clusters (spectator fragments): experim. measured up to $E_{\text{beam}} = 1$ AGeV (ALADIN Collab.)

PHQMD with SACA shows an agreement with ALADIN data for very complex cluster observables as

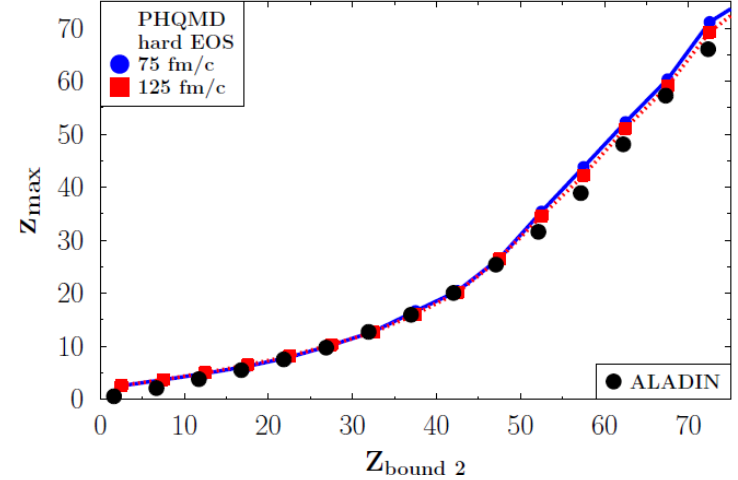
- Largest clusters (Z_{bound})
- Energy independent 'rise and fall'
- Rms p_T^2

Au+Au, 600 AMeV, min bias, SACA

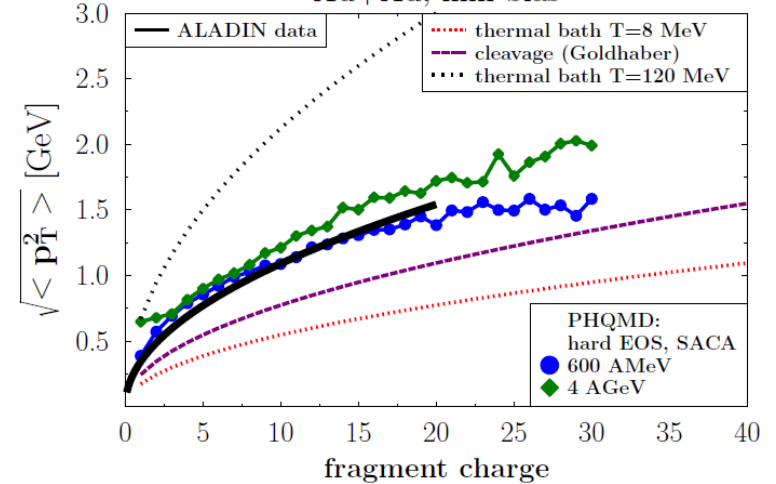


$$Z_{\text{bound } 2} = \sum_i Z_i \Theta(Z_i - (1 + \epsilon)) ; (\epsilon < 1)$$

Au+Au, 600 AMeV, min bias, SACA



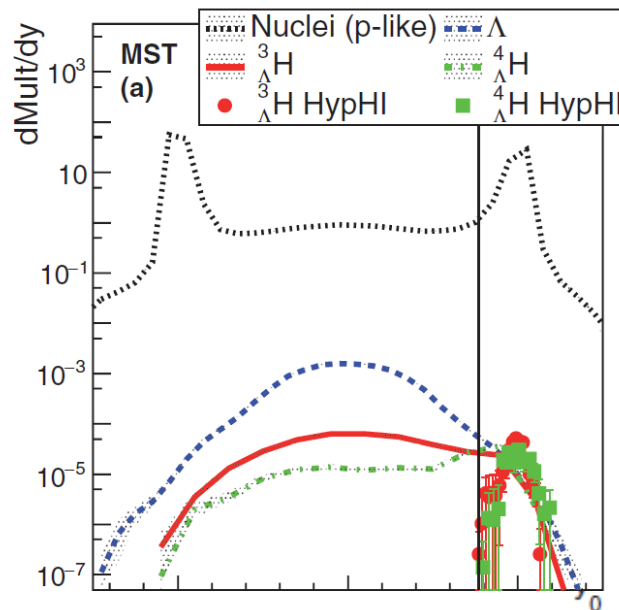
Au+Au, min bias



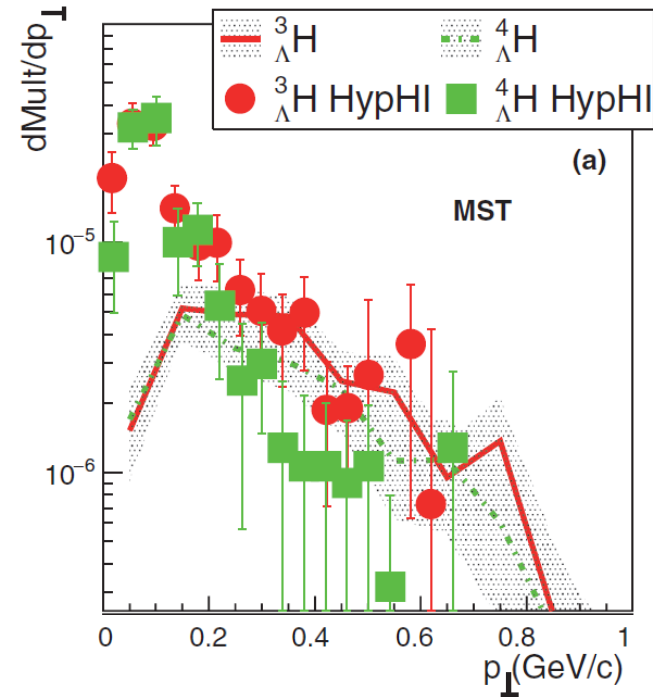
PHQMD shows $\sqrt{p_T^2(Z)} \propto \sqrt{Z}$. dependence as exp. data

hyper-nuclei of HypHI

hyper-nuclei of HypHI (PLB747,129) ${}^6\text{Li} + {}^{12}\text{C}$ at 2A GeV (PLB747,129)



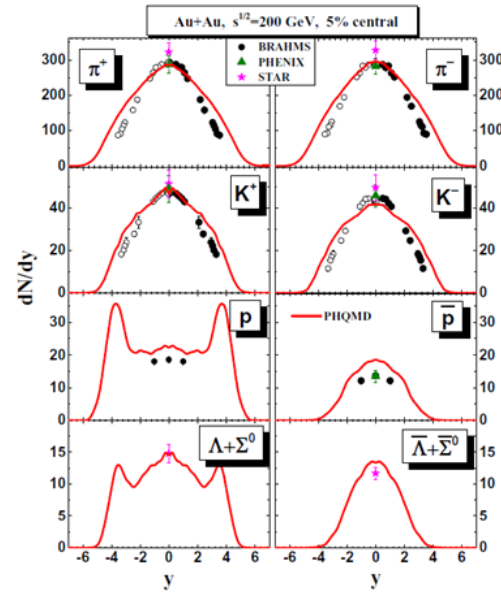
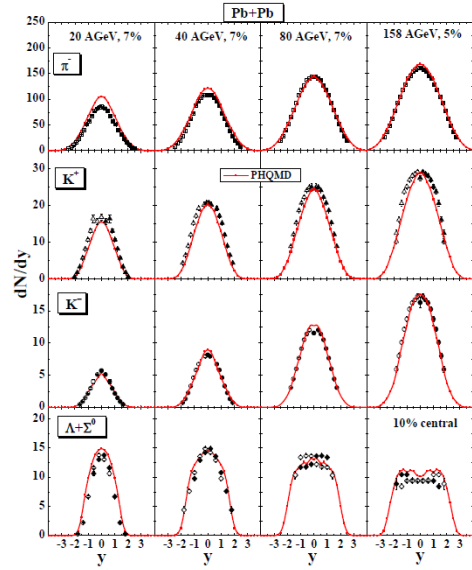
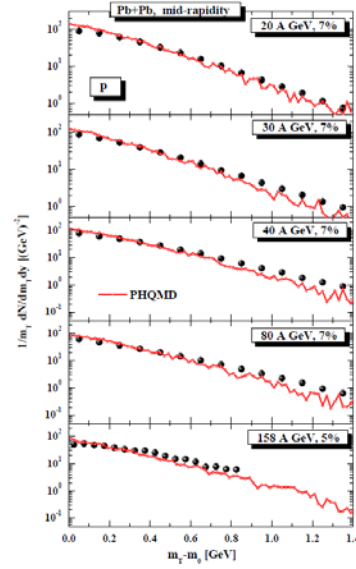
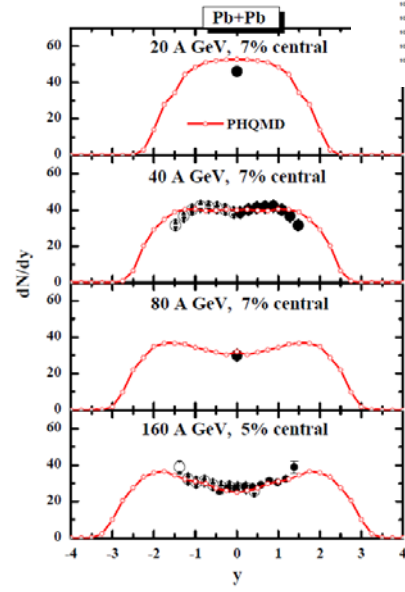
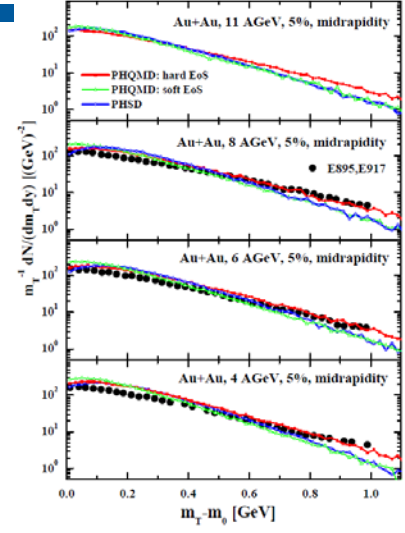
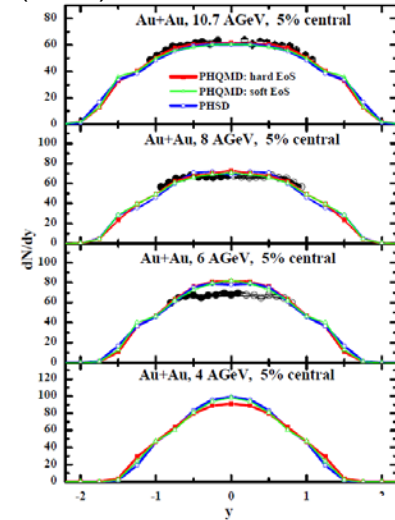
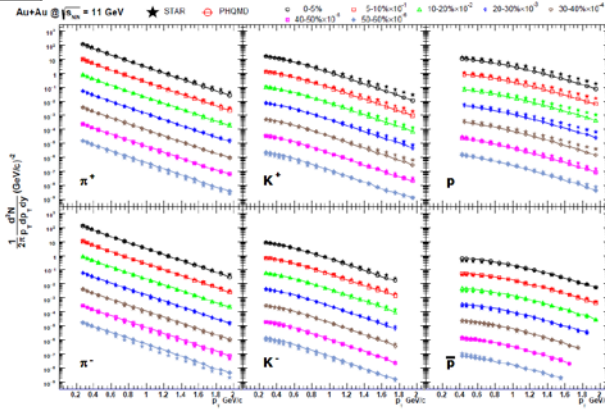
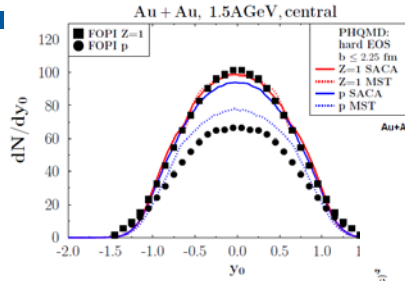
Le Fevre et al. PRC100,034904



Rapidity and p_T spectra of hyper-clusters are reproduced despite of the complicated physics:

- Modeling of Λ production
- Interface between participants and spectators
- Absorption of Λ by spectators

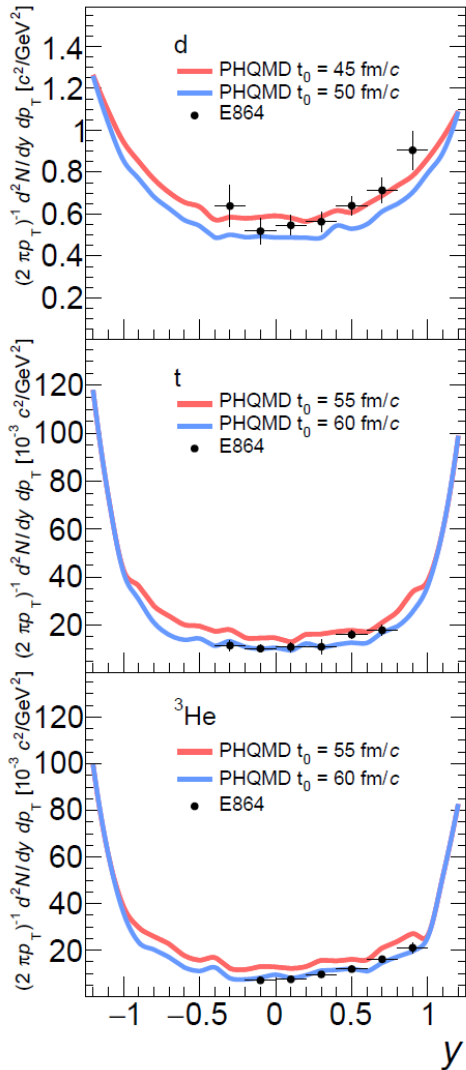
PHQMD: J. Aichelin et al., PRC 101 (2020) 044905



PHQMD provides a good description of hadronic 'bulk' observables from SIS to RHIC energies

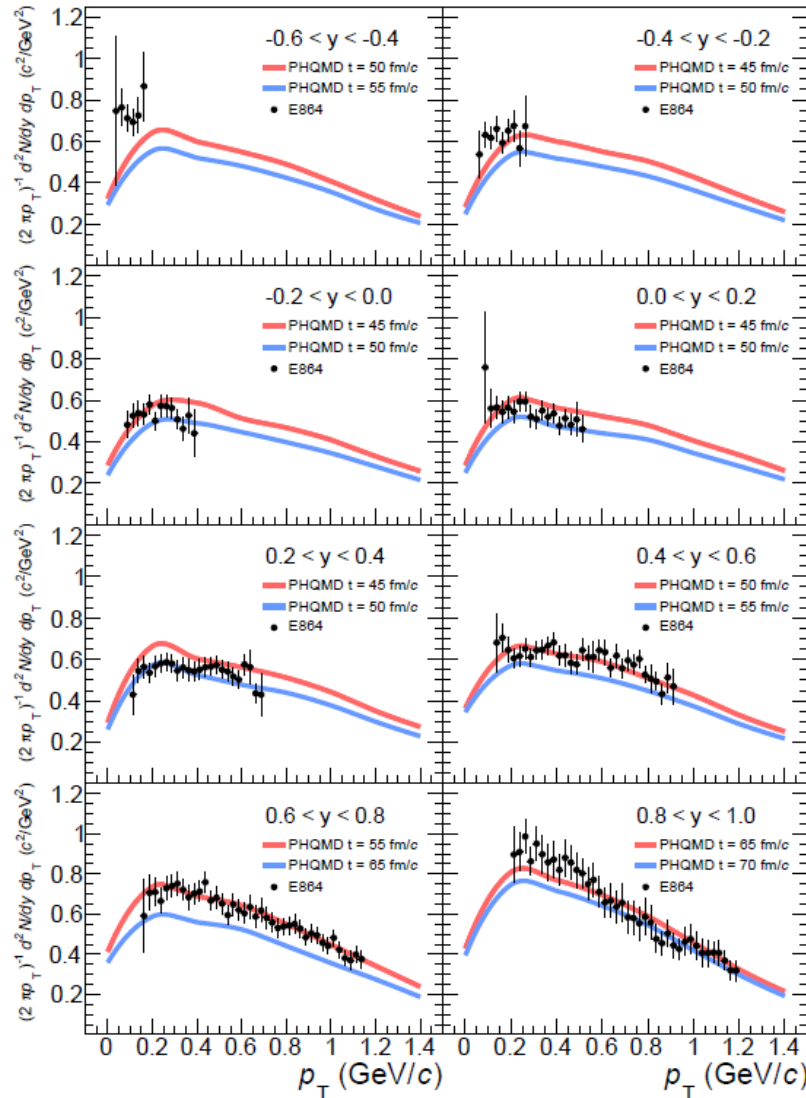
y- distributions of d, t, ^3He

Au+Pb@10.6 AGeV



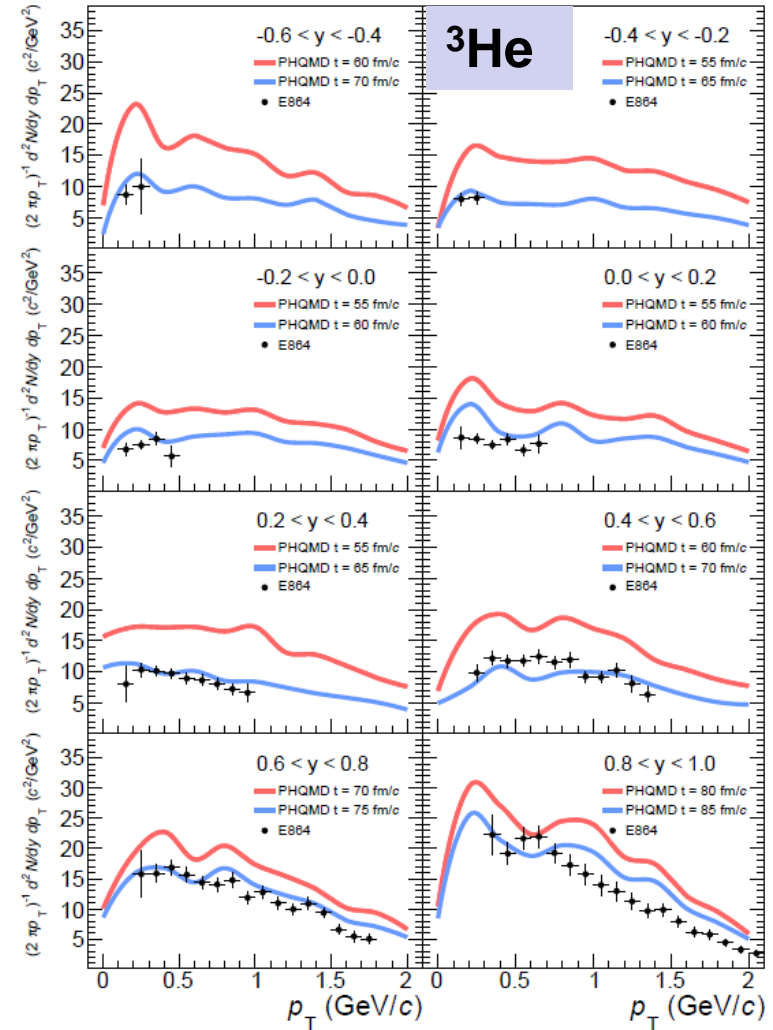
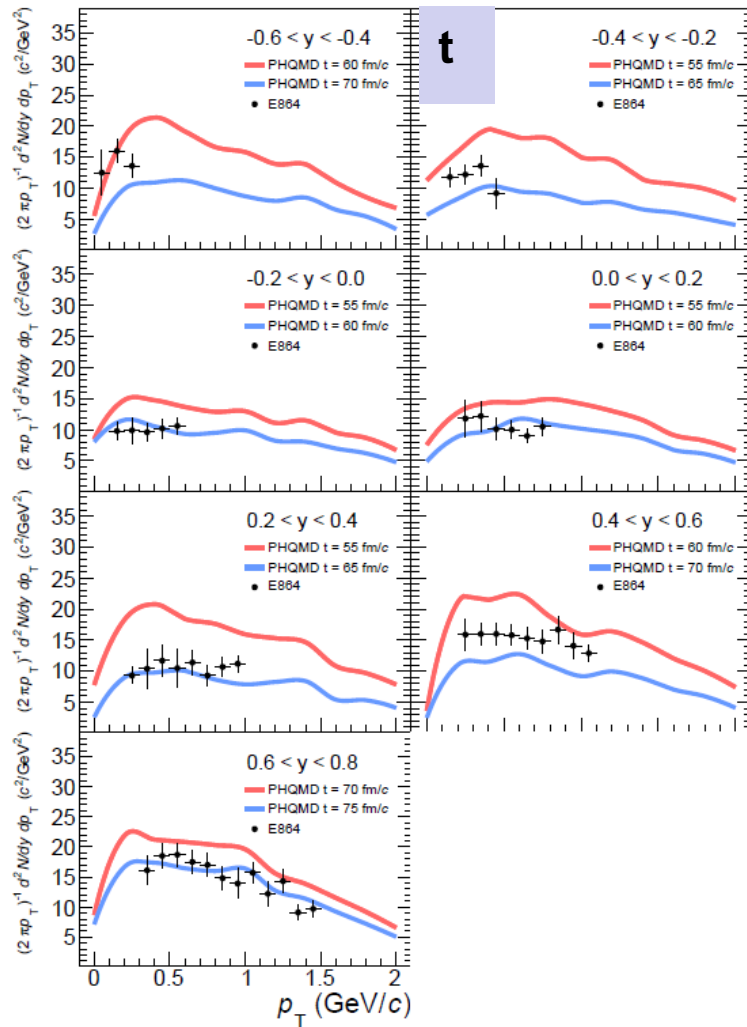
p_T - distribution of deuterons

Au+Pb@10.6 AGeV

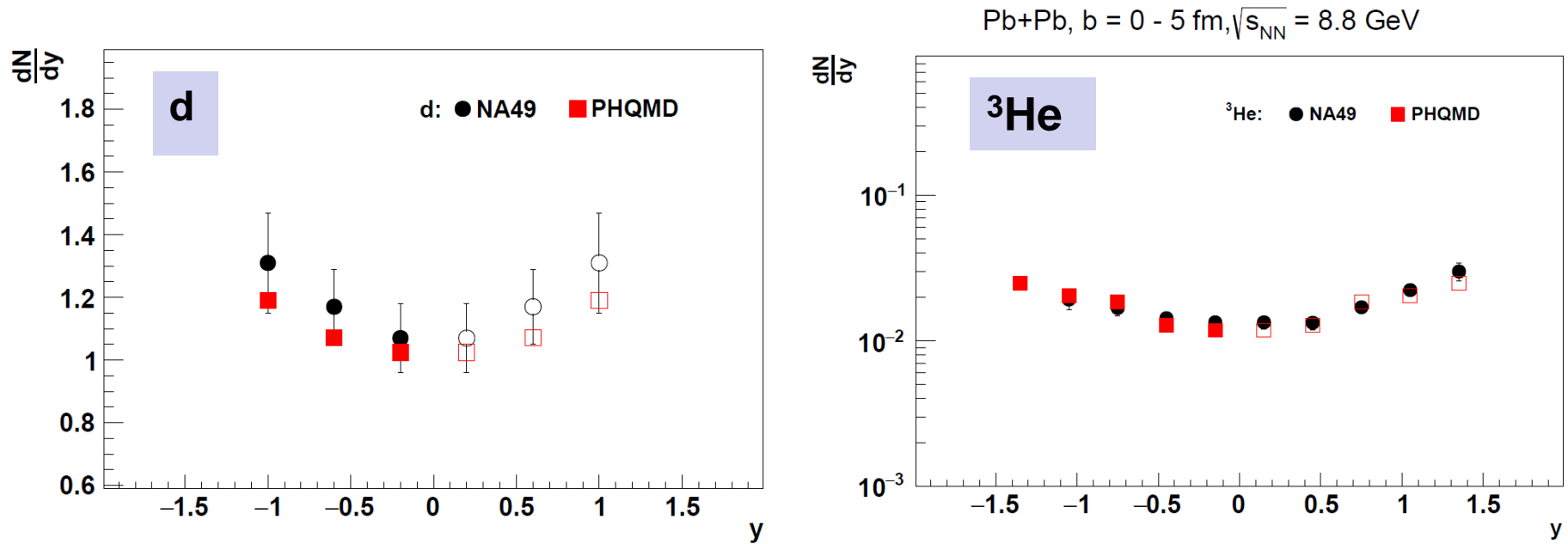


The PHQMD results are taken at $t = t_0 \cosh(y)$, where t_0 is the time at $y=0$

The p_T - distributions of t and ${}^3\text{He}$ from Au+Pb at 10.6 A GeV



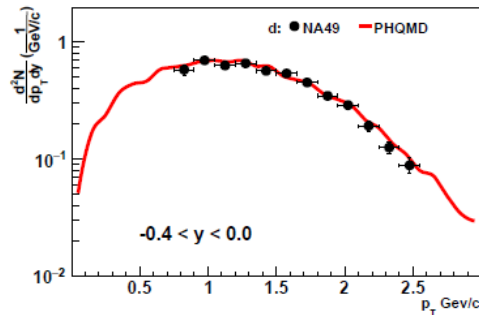
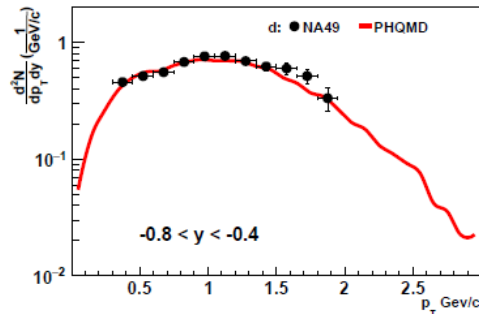
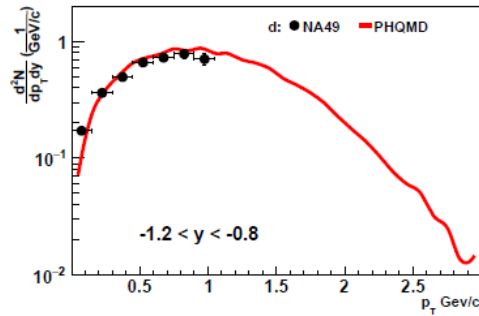
The rapidity distributions of **d** and **³He** from Pb+Pb at **30 A GeV**



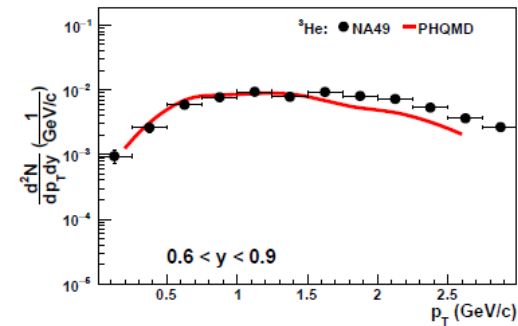
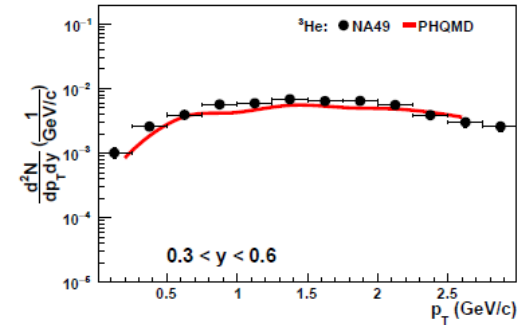
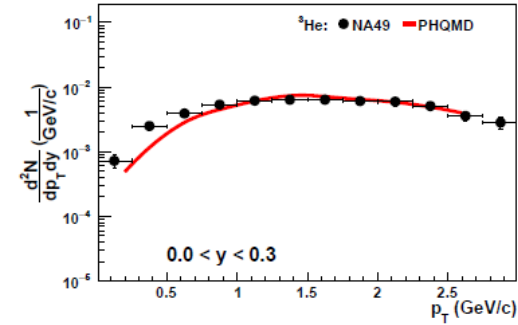
The PHQMD results for d and ³He agree with NA49 data

The p_T - distributions of **d** and ^3He from Pb+Pb at 30 A GeV

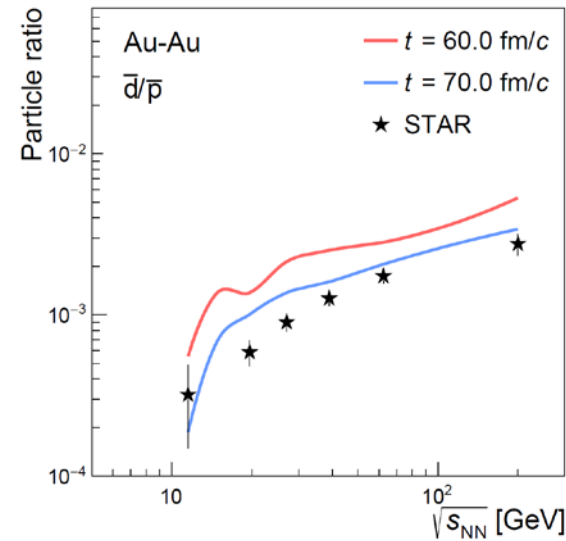
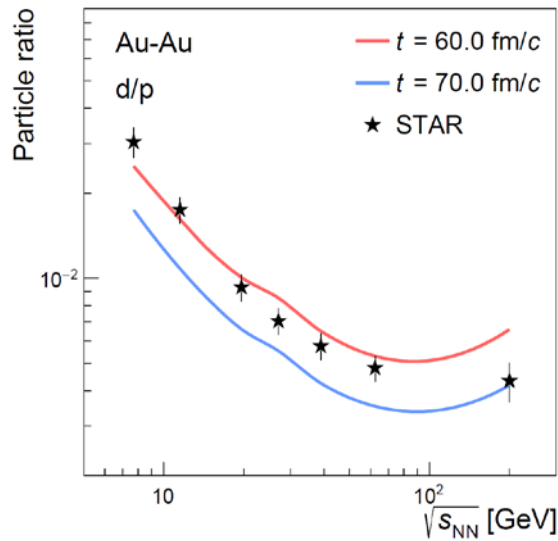
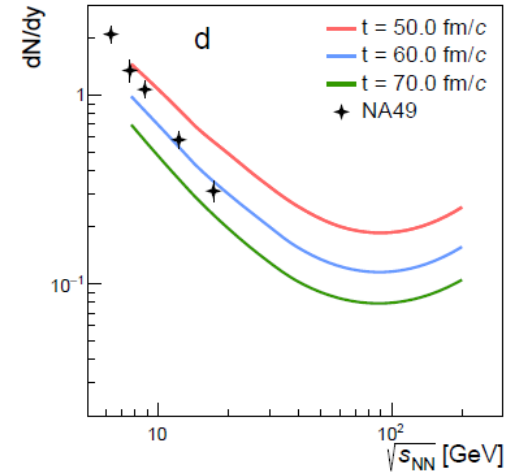
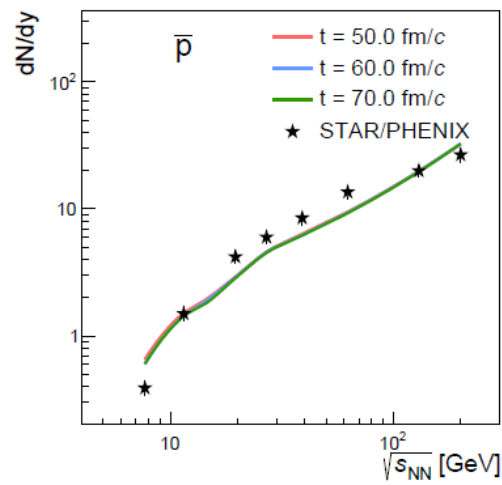
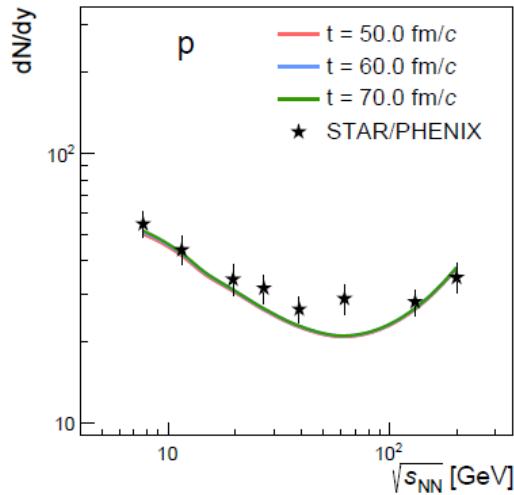
d



^3He

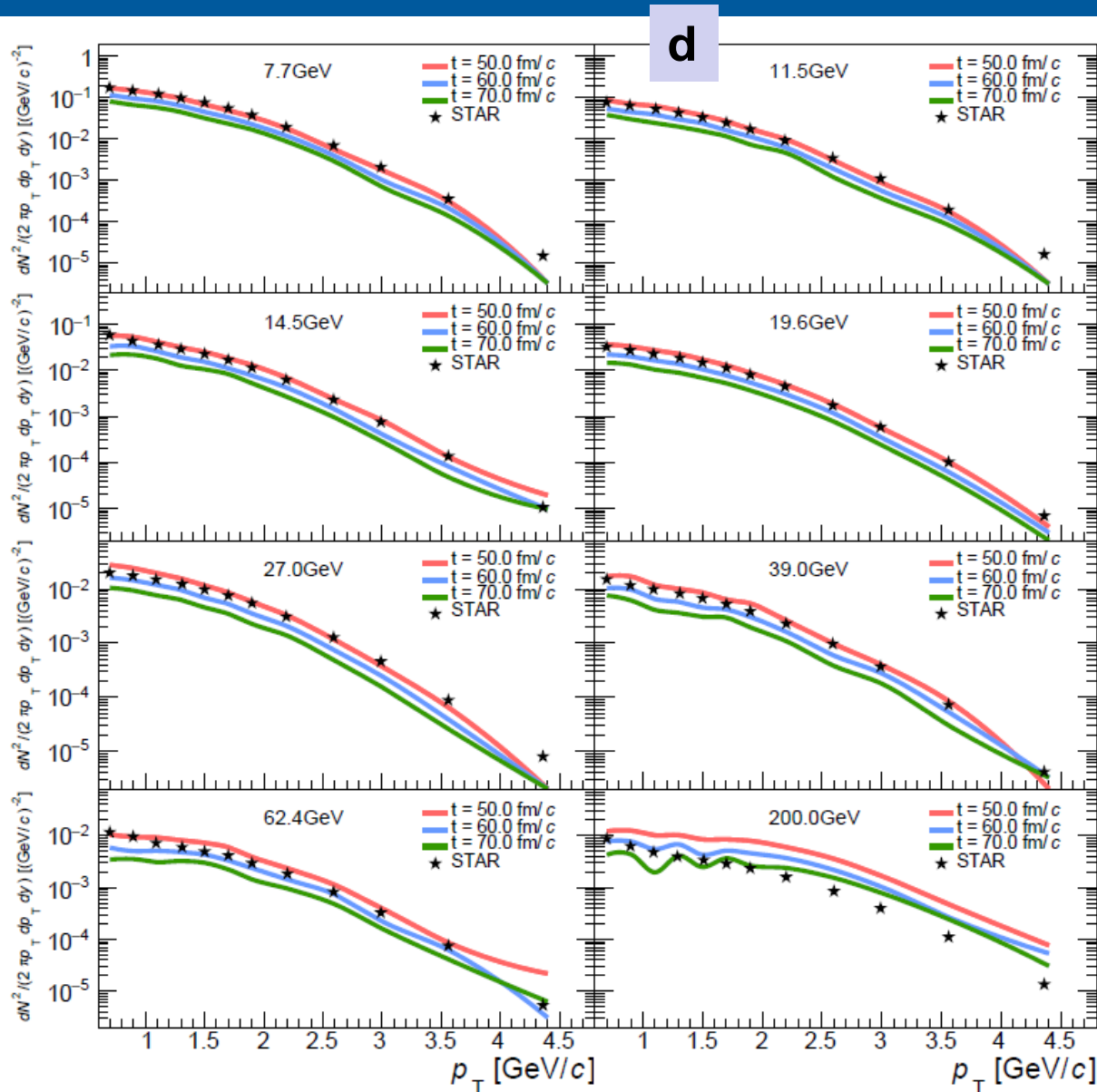


Excitation function of multiplicity of p, \bar{p}, d, \bar{d}



The p, \bar{p} yields at $y \sim 0$ are stable, the d, \bar{d} yields are better described at $t = 60-70 \text{ fm/c}$

Deuteron p_T spectra from 7.7 GeV to 200 GeV

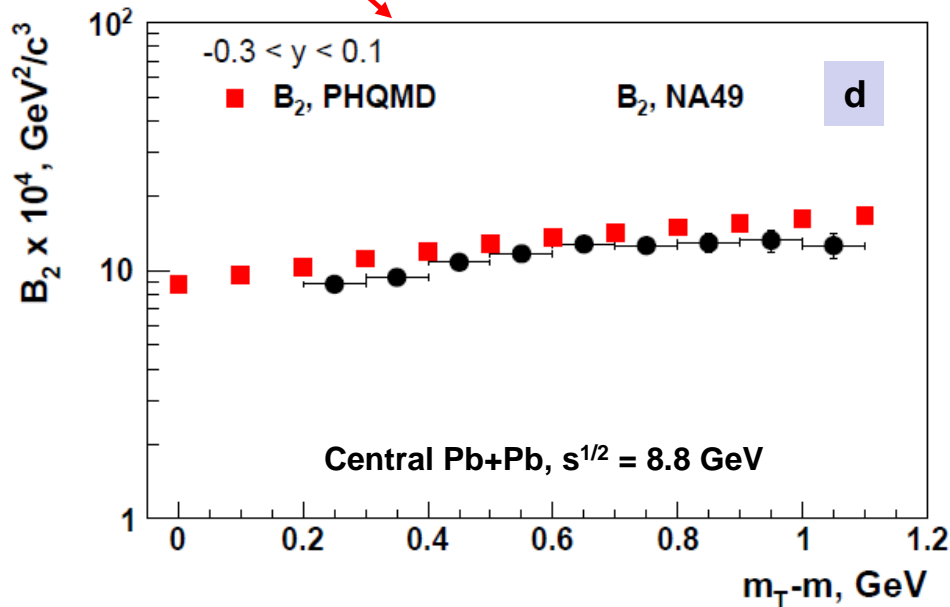


Comparison of the PHQMD results for the **deuteron** p_T -spectra at midrapidity with STAR data

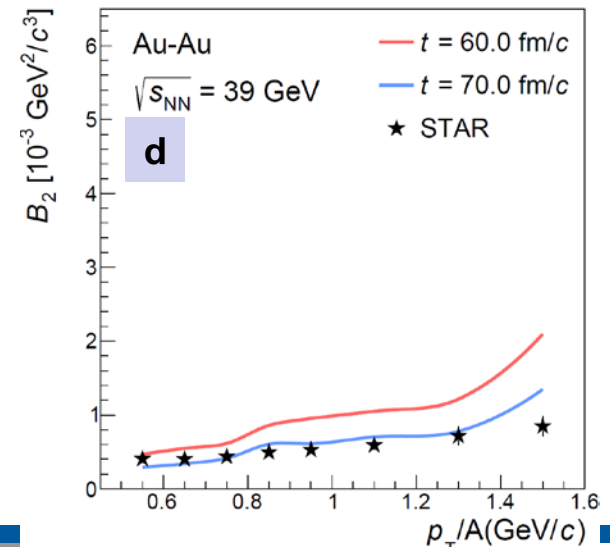
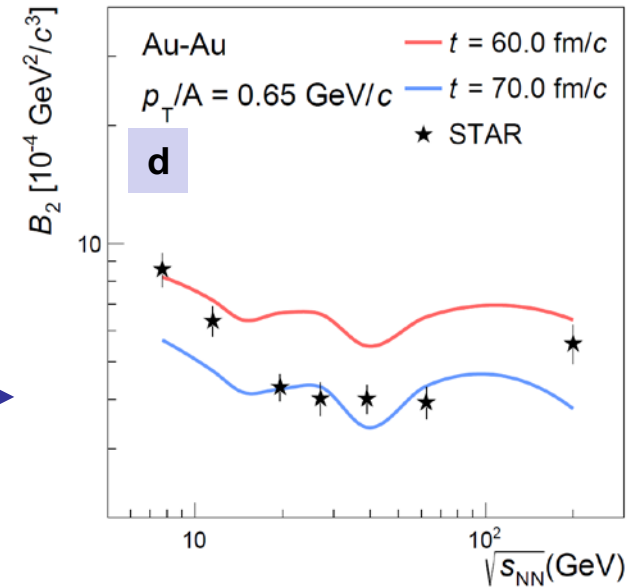
Coalescence parameter B_2 :

$$B_2 = \frac{E_d \frac{d^3 N_d}{d^3 P_d}}{\left(E_p \frac{d^3 N_p}{d^3 p_p} \Big|_{p_p = P_d/2} \right)^2}$$

Comparison of the PHQMD results with **NA49** and **STAR** data

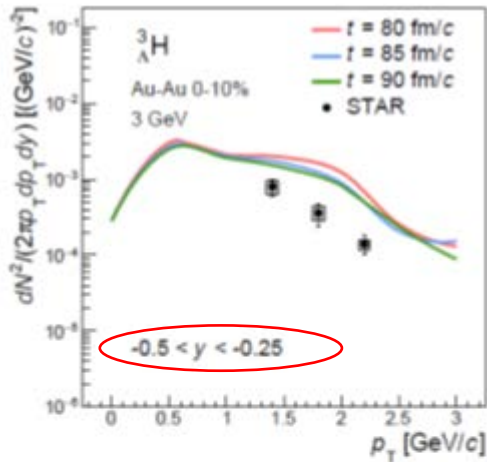


central Au+Au collisions



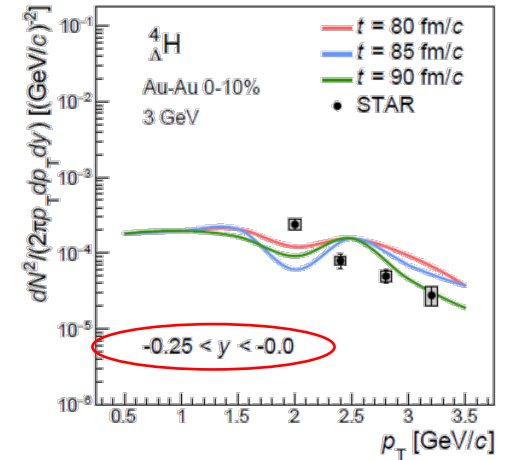
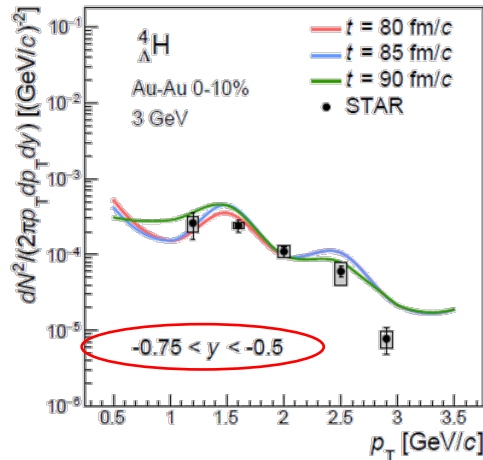
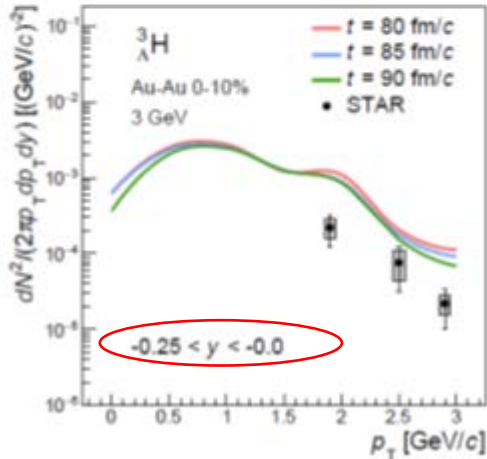
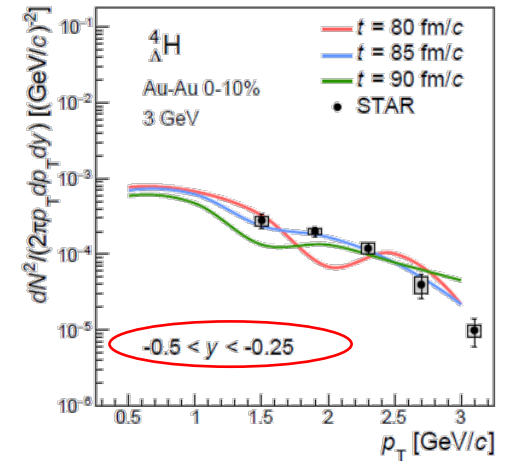
Hypernuclei from PHQMD

The PHQMD comparison with most recent STAR
 fixed target p_T distribution of ${}^3\text{H}_\Lambda$, ${}^4\text{H}_\Lambda$ from Au+Au central collisions at $\sqrt{s} = 3$ GeV
 nucleon-hyperon potential: $V_{N\Lambda} = 2/3 V_{NN}$



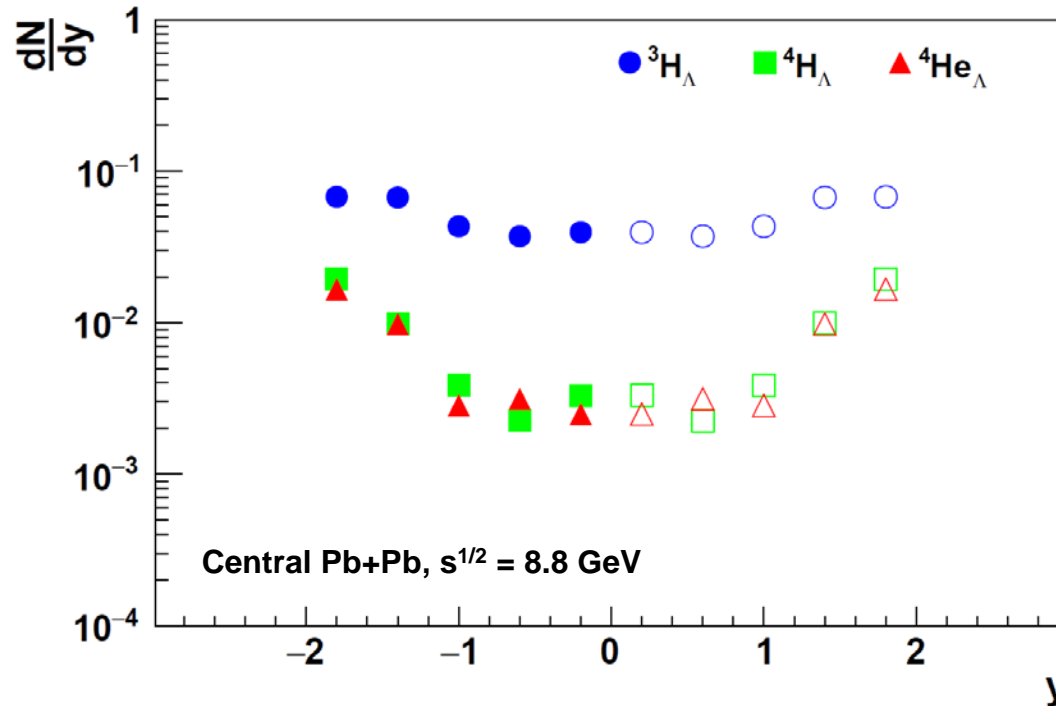
Star data preliminary

Good description in view
 of these very complex
 hypernuclei



The PHQMD predictions of the rapidity distribution of ${}^3\text{H}_\Lambda$, ${}^4\text{H}_\Lambda$ and ${}^4\text{He}_\Lambda$ from Pb+Pb central collisions at **30 A GeV** ($s^{1/2} = 8.8$ GeV)

- nucleon-hyperon potential: $V_{N\Lambda} = 2/3 V_{NN}$

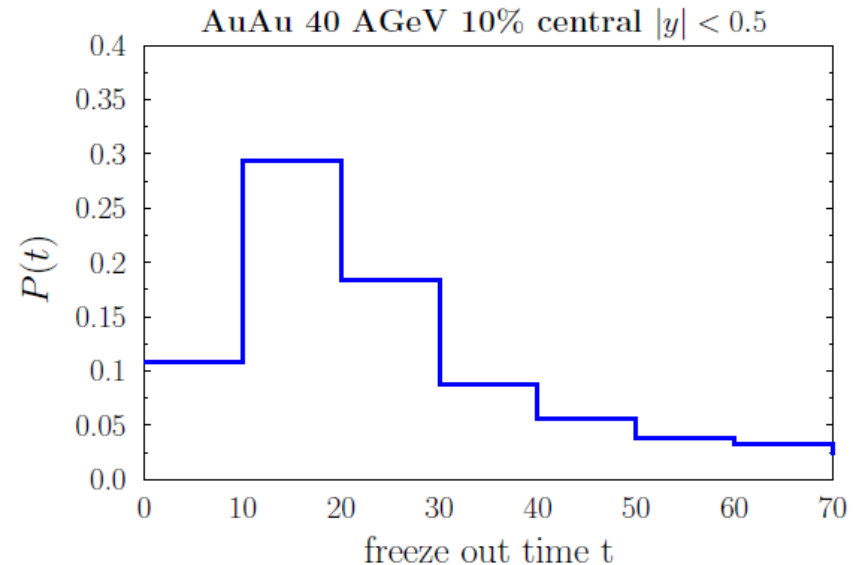
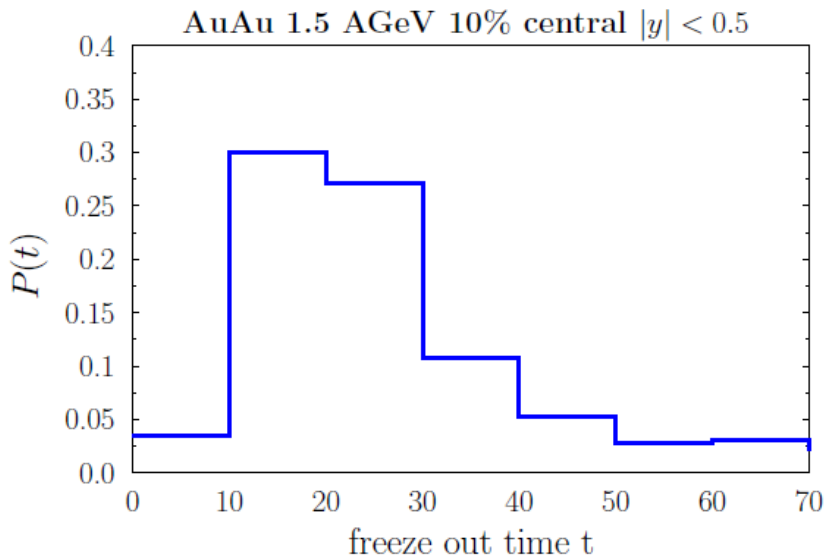


How the clusters are produced
(‘ice in fire’ puzzle)



When does the system freeze out?

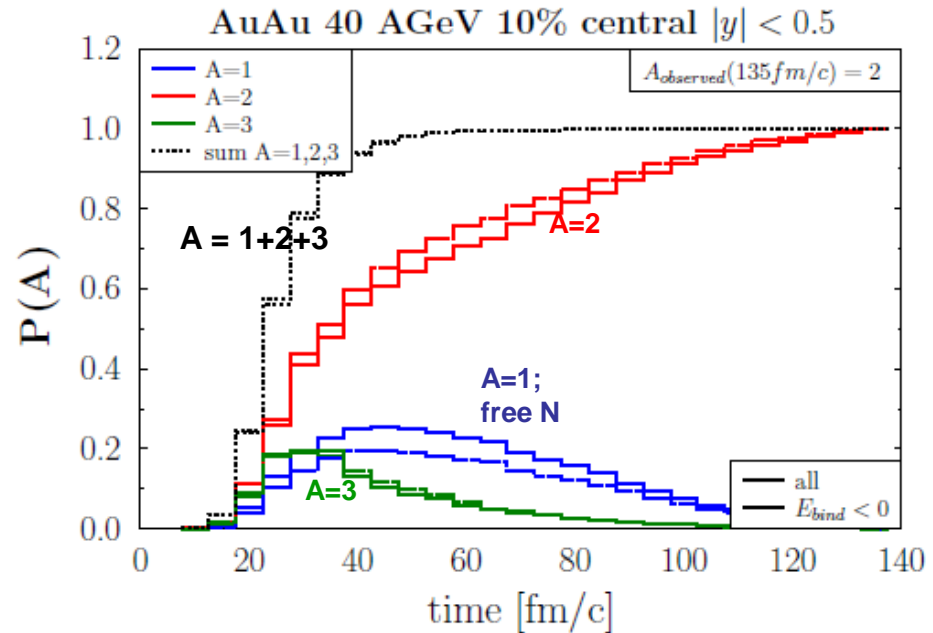
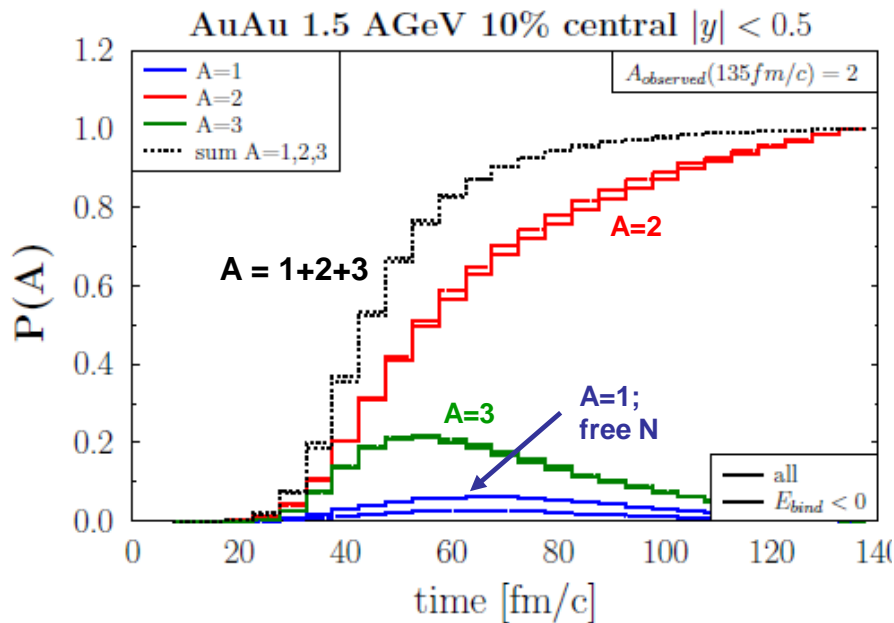
- The normalized distribution of the **freeze-out time of baryons** (nucleons and hyperons) which are finally observed at mid-rapidity $|y| < 0.5$
 - * Here freeze-out time is defined as a **last elastic or inelastic collision**, after that **only potential interaction** between baryons occurs



- ➔ Freeze-out time of baryons in Au+Au at 1.5 AGeV and 40 AGeV:
 - **similar profile** since expansion velocity of mid-rapidity fireball is roughly independent of the beam energy

Where are the clusters formed?

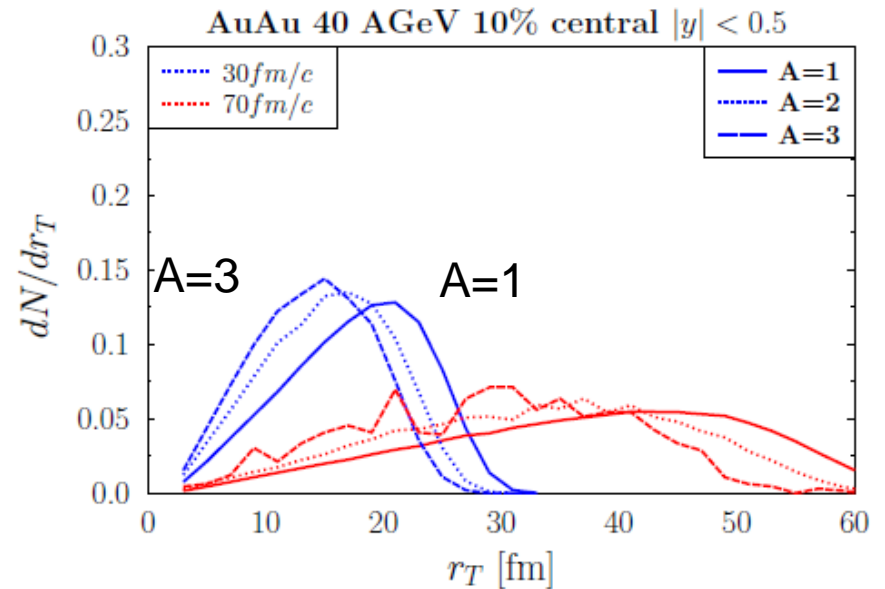
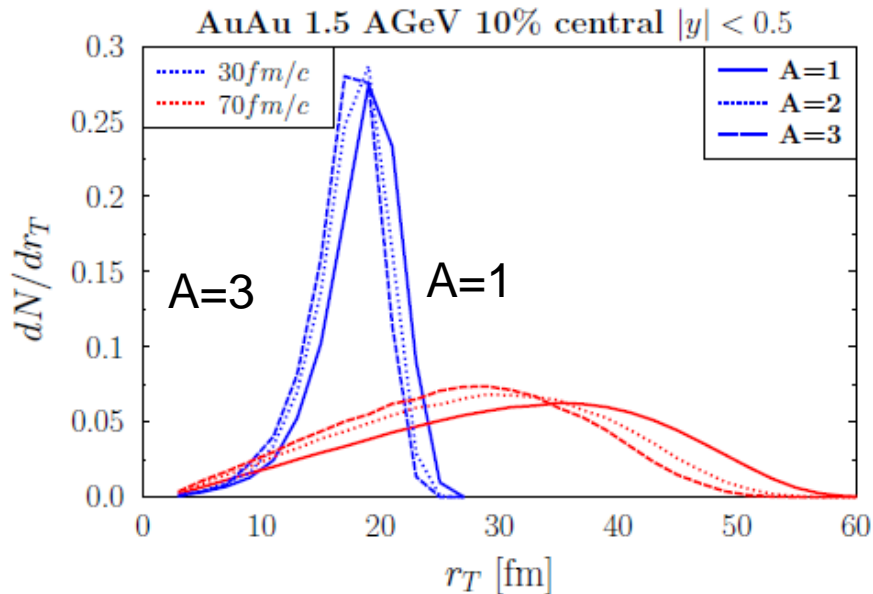
- █ The conditional probability $P(A)$ that the nucleons, which are finally observed in $A=2$ clusters at time 135 fm/c, were at time t the members of $A=1$ (free nucleons), $A=2$ or $A=3$ clusters



➔ Stable clusters (observed at 135 fm/c) are formed shortly after the dynamical freeze-out

Where are the clusters formed?

- ❑ The snapshot (taken at time 30 and 70 fm/c) of the **normalized distribution of the transverse distance r_T of the nucleons to the center of the fireball.**
- ❑ It is shown for $A=1$ (free nucleons) and for the nucleons in $A=2$ and $A=3$ clusters



- ➔ **Transverse distance profile of free nucleons and clusters are different!**
- Clusters are formed behind the “front” of free nucleons of the expanding fireball
- ➔ **Cluster nucleons and free nucleons are not at the same place**
- ➔ **Clusters do not get destroyed by the free hadrons**

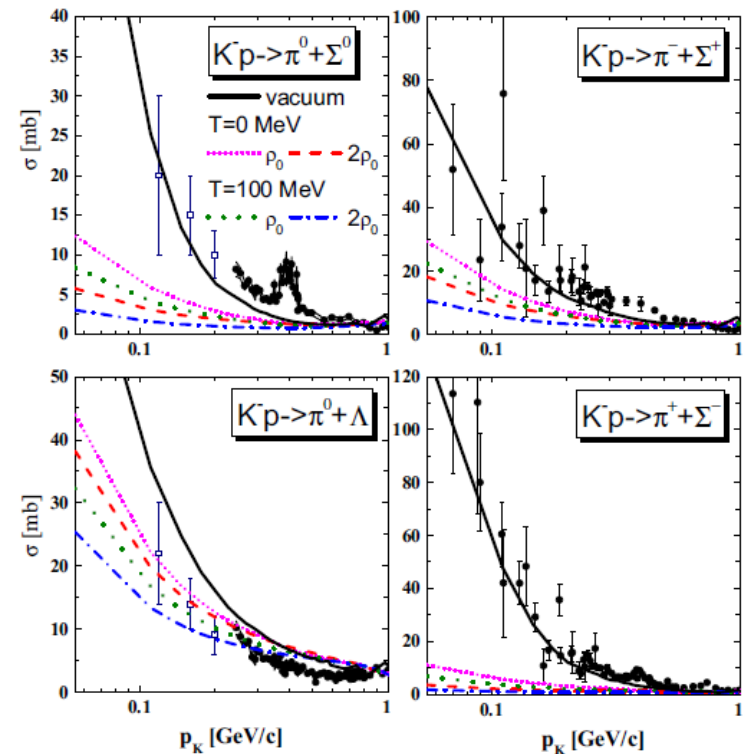
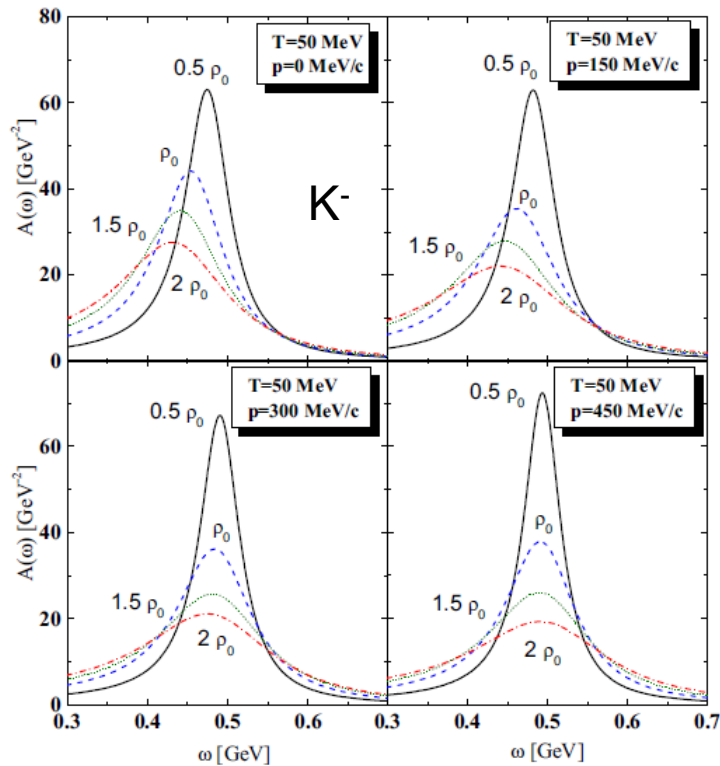
Perspectives

Up to now: $V_{\Lambda N} = 2/3 V_{NN}$; $m_{\Lambda} = 1115$ MeV ; K^{\pm} only density dependent V

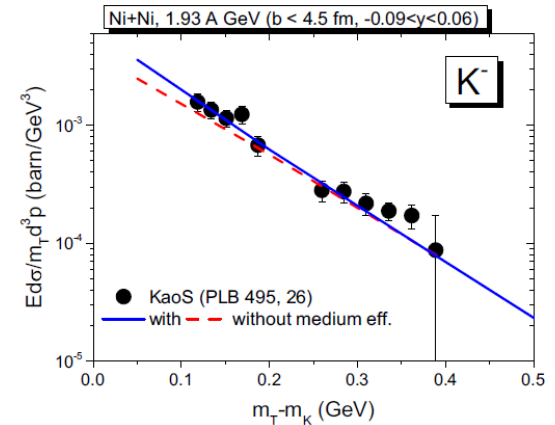
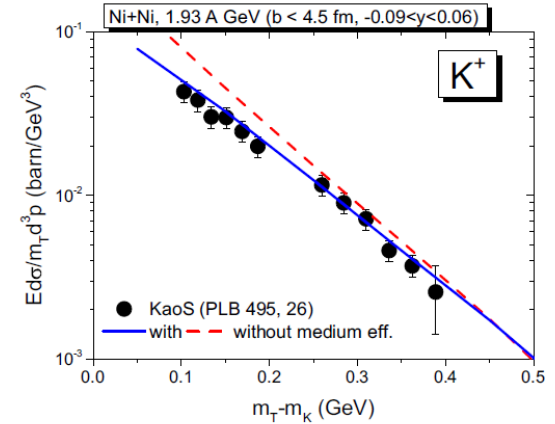
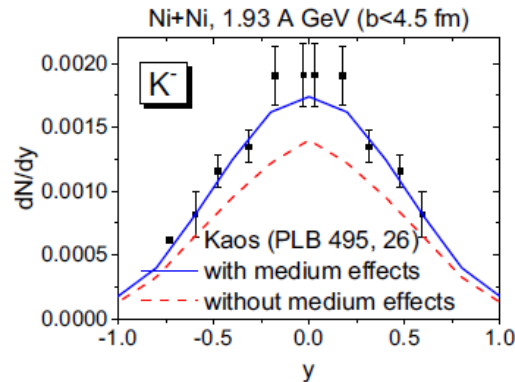
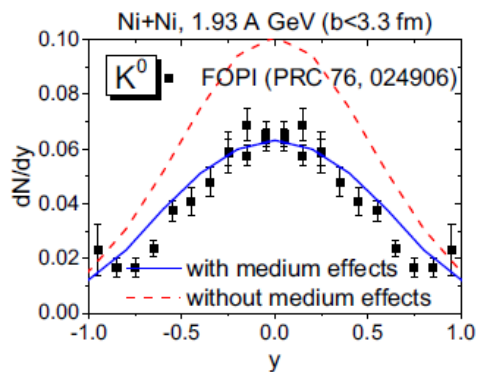
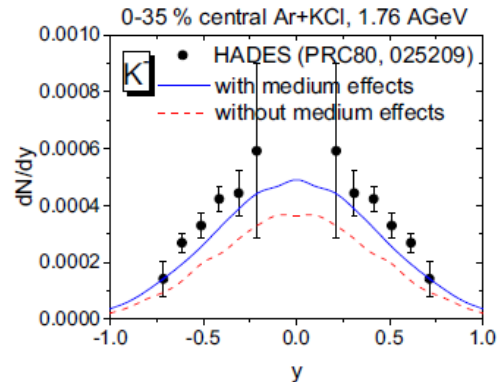
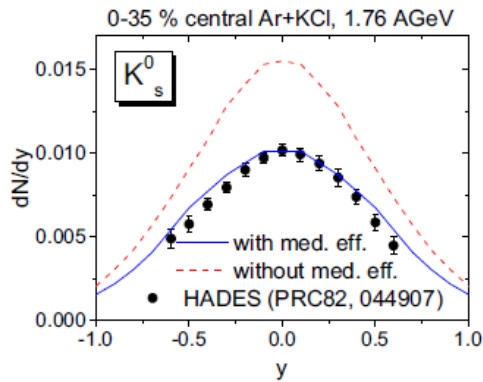
Reality is much more complicated

Coupled channel G-matrix with effective Chiral Lagrangian (PRC103,044901):

Modification of particle properties and cross sections



Both modifies the observables (PHSD)



This has to be integrated in PHQMD for more precise predictions

Conclusions

cluster studies → N-body approach for the dynamics of the nucleons

K^+ yield up to today the most precise probe to measure the nuclear EOS

(Hyper)clusters:

Minimum spanning tree (only applicable for $t \rightarrow \infty$)

Simulated annealing (SACA, FRIGA)

can identify fragments during the HI reaction

→ allow for identifying when and how fragments are formed

not easy to be applied for small clusters

PHQMD and IQMD are quite successful to interpret (hyper)cluster data

Perspectives: More realistic $K \wedge N$ dynamics

hopefully much more (hyper)cluster data

to see whether the predicted production yield is correct

to study hyperclusters in detail

Thank you