42nd Workshop on High-Energy-Density Physics with laser and Ion beams

Kinetic Alfvén Wave to study solar coronal heating

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Outlines

- Introduction.
- Model equation.
- Simulation method and results.
- Conclusion.

Introduction

- Alfven waves are transverse electromagnetic waves that travel along the magnetic field lines.
- The AWs categorized as inertial AW (IAW) or kinetic AW (KAW), depends on the electron inertial effect or finite thermal effect.
- When the AWs propagate perpendicular to ambient magnetic field in intermediate-β $(m_e/m_i \ll \beta \ll 1)$ plasma, then the AW known as KAW.
- The study of turbulent spectrum in the astrophysical plasma is an old but still it is a huge challenge.

Model equation.

- Let us consider a low frequency KAW propagating in x-z plane and an ambient background magnetic field $\vec{\mathbf{B}}_{\mathbf{0}} = \mathbf{B_0}\hat{\mathbf{z}}$.

- We use two fluid model and apply procedure of linearization in equations.

- The density, velocity and the electromagnetic fields can be written as

$$\mathbf{n} = \mathbf{n_0} + \mathbf{n_1},\ \vec{\mathbf{v}} = \vec{\mathbf{v}}_{\mathbf{0}} + \vec{\mathbf{v}}_{\mathbf{1}},\ \vec{\mathbf{E}} = \vec{\mathbf{E}}_{\mathbf{0}} + \vec{\mathbf{E}}_{\mathbf{1}},\ \ \vec{\mathbf{B}} = \vec{\mathbf{B}}_{\mathbf{0}} + \vec{\mathbf{B}}_{\mathbf{1}} \quad (1)$$

- In the equilibrium plasma, the undisturbed density, velocity and fields are constant and uniform, i.e.

$$\vec{\boldsymbol{\nabla}}\mathbf{n_0} = \vec{\mathbf{v}}_{\mathbf{0}} = \vec{\mathbf{E}}_{\mathbf{0}} = \mathbf{0} \qquad (2)$$

- Equation of motion

$$\frac{\partial \mathbf{v}_{j\perp}}{\partial t} \approx \frac{q_j}{m_j}\mathbf{E}_1 + \frac{q_j}{c m_j}\left(\mathbf{v}_{j1} \times \mathbf{B}_0\right) - \frac{\gamma_j k_B T_j}{n_{j0} m_j}\nabla n_{j1} \qquad (3)$$

- Where $\boldsymbol{\gamma}_j$ is the ratio of the specific heats $\mathbf{c_p/c_v}$, $\mathbf{k_B}$ is the Boltzmann constant.

- Here we consider the ions and electron to be isothermal i.e $\boldsymbol{\gamma}_i = \boldsymbol{\gamma}_e = \mathbf{1}$.

- Continuity equation:

$$\frac{\partial n_{j1}}{\partial t} + n_{j0}\vec{\nabla}.\vec{v}_{j1} \approx 0 \qquad (4)$$

- Faraday's law:

$$\vec{\nabla} \times \vec{E}_1 = -\frac{1}{c}\frac{\partial \vec{B}_1}{\partial t} \qquad (5)$$

From equation (4), and low frequency approximation ($(\boldsymbol{\omega} \ll \boldsymbol{\omega_{ci}})$ and $(\boldsymbol{\omega} \ll \boldsymbol{\omega_{ce}})$) we get

$$(\mathbf{v_{e\perp}}) \approx \frac{\mathbf{c}}{\mathbf{B_0}}\mathbf{E_\perp} \times \hat{\mathbf{z}} - \frac{\mathbf{k_B T_e}}{\mathbf{m_e \omega_{ce} n_0}}\hat{\mathbf{z}} \times \nabla_\perp \mathbf{n_1} \qquad (7)$$

$$\mathbf{v_{i\perp}} \approx \frac{\mathbf{e}}{\mathbf{\omega_{ci} m_i}}\left[\mathbf{E_\perp} - \frac{\mathbf{k_B T_i}}{\mathbf{e n_0}}\nabla_\perp \mathbf{n_1}\right] \times \hat{\mathbf{z}} - \frac{\mathbf{i\omega}}{\mathbf{\omega^2}_{\mathbf{ci}}}\frac{\mathbf{e}}{\mathbf{m_i}}\left[\mathbf{E_\perp} - \frac{\mathbf{k_B T_i}}{\mathbf{e n_0}}\nabla_\perp \mathbf{n_1}\right] \qquad (8)$$

Where $\boldsymbol{\omega_{ci}}, \boldsymbol{\omega_{ce}}$ is the ion and electron cyclotron frequency.

The parallel component of electron velocity is

$$\frac{\partial(\vec{\mathbf{v}}_{\mathbf{e1}})_{\mathbf{z}}}{\partial \mathbf{t}} = \frac{-\mathbf{e E_{1z}}}{\mathbf{m_e}} - \frac{\mathbf{k_B T_e}}{\mathbf{m_e n_0}}\frac{\partial \mathbf{n_1}}{\partial \mathbf{z}} \qquad (9)$$

The current density is given by

$$\vec{\mathbf{J}} \approx \mathbf{e n_0}(\vec{\mathbf{v}}_{\mathbf{i1}} - \vec{\mathbf{v}}_{\mathbf{e1}}) \qquad (10)$$

Now we will drop the subscript "1" while representing the perturbating parts of $\vec{\mathbf{v}}, \vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ except while representing the varying part of $\mathbf{n}$.

➢The y-component of Faraday's law is

$$\frac{1}{c}\frac{\partial B_y}{\partial t} = \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \qquad (11)$$

➢Using the conservation of current density equation, $\vec{\nabla}.\vec{J} = 0$, and writing the parallel and perpendicular components of current density as $\vec{J}_z \approx -en_0\vec{v}_{ez}$ and $\vec{J}_\perp \approx en_0(\vec{v}_{i\perp} - \vec{v}_{e\perp})$, we get

$$\frac{\partial^2 E_x}{\partial x \partial t} = \frac{B_0\omega_{ci}}{c}\left(\frac{\partial v_{ez}}{\partial z}\right) \qquad (12)$$

➢By taking the z-component of Ampere's law and eliminating the parallel component of current density and substituting it in equation (2.12), we get,

$$\frac{\partial E_x}{\partial t} = -\frac{{v_A}^2}{c}\left(1 - \frac{n_1}{n_0}\right)\frac{\partial B_y}{\partial z} \qquad (13)$$

➢Where $v_A(= \sqrt{\frac{{B_0}^2}{4\pi n_0 m_i}}$) is the Alfven speed.

➢We calculate the time derivative of parallel electric field from the Ampere's law. We get

$$\frac{\partial E_z}{\partial t} = -\frac{{v_{te}}^2{\lambda_e}^2}{c}\frac{\partial}{\partial x}\frac{\partial^2 B_y}{\partial z^2} \qquad (14)$$

➢Where $\lambda_e \left(= \sqrt{4\pi n_0 e^2/m_e}\right)$ in the electron inertial length and $v_{te}\left(= \sqrt{\frac{k_B T_e}{m_e}}\right)$ denotes the electron thermal speed.

➢We differentiate equation (11) w.r.t. time and inserting the z-derivative of equation (13) and the x-derivative of equation (14), we get the governing dynamical equation of DAWs.

$$\frac{\partial^2 B_y}{\partial t^2} = {\lambda_e}^2\frac{\partial^4 B_y}{\partial x^2 \partial t^2} - {v_A}^2{\rho_s}^2\frac{\partial^4 B_y}{\partial x^2 \partial z^2} + {v_A}^2\left(1 - \frac{n_1}{n_0}\right)\frac{\partial^2 B_y}{\partial z^2} \quad (15)$$

➢Where $\rho_s (= \lambda_e v_{te}/v_A)$ is the ion acoustic gyroradius.

➢In intermediate-β plasma ($m_e/m_i \ll \beta \ll 1$) regime, the DAW is known as kinetic AW. The governing equation for the KAWs in this β regime reduces to

$$\frac{\partial^2 B_y}{\partial t^2} = -{v_A}^2{\rho_s}^2\frac{\partial^4 B_y}{\partial x^2 \partial z^2} + {v_A}^2\left(1 - \frac{\delta n_s}{n_0}\right)\frac{\partial^2 B_y}{\partial z^2} \qquad (16)$$

➢The dispersion relation of KAWs can be recovered by taking Fourier transform to equation (16) as

$$\frac{\omega^2}{{k_{0z}}^2{v_A}^2} = 1 + {k_{0x}}^2{\rho_s}^2 \qquad (17)$$

➢We consider plane wave solation as

$$B_y = B'_y(x,z)e^{i(k_{0x}x + k_{0z}z - \omega t)} \qquad (18)$$

➢Where $B'_y(x,z,t)$ is amplitude of magnetic field. Now we substitute it in equation (16) satisfied by KAWs to get

$$i\frac{2}{k_{0z}}\frac{\partial {B_y}'}{\partial z} + {\rho_s}^2\frac{{k_{0x}}^2}{{k_{0z}}^2}\frac{\partial^2 {B_y}'}{\partial z^2} + {\rho_s}^2\frac{\partial^2 B'_y}{\partial x^2} + 2ik_{0x}{\rho_s}^2\frac{\partial B'_y}{\partial x} + \frac{n_1}{n_0}B'_y = 0 \qquad (19)$$

➢In an inhomogeneous plasma, a nonlinear magnetic gradient force called ponderomotive force is developed and is given as

$$n_1 \approx n_0\left(e^{\xi|B'_y|^2} - 1\right) \qquad (20)$$

➢Where $\xi = \{[1 - \Delta(1+\delta)v^2{}_A k_{0z}]/16\pi n_0 T_e \omega^2\}, \Delta = \omega^2/\omega^2{}_{ci}$, $\delta = m_e k^2{}_{0x}/m_i k^2{}_{0z}$.

➢Substituting equation (20) in to equation (19) and normalizing the equation we get

$$i\frac{\partial B_y{}'}{\partial z} + 2iK\frac{\partial B_y{}'}{\partial x} + K^2\frac{\partial^2 B_y{}'}{\partial z^2} + \frac{\partial^2 B_y{}'}{\partial x^2} + \frac{1}{2g}\left(e^{2g|B_y{}'|^2} - 1\right)B_y{}' = 0$$

(21)

➢Where $K = k_{0x}\rho_s$ is the dimensionless parameter. The normalizing parameters are $t_n = \frac{2\omega}{v_A{}^2 k_{0z}{}^2}, z_n = 2/k_{0z}, x_n = \rho_s,\ B_N = \left[\{1 - \Delta(1+\delta)\}V_A{}^2 k_{0z}{}^2/16\pi n_0 T_e \omega^2\right]^{-1/2}$.

Numerical Simulation

- We carried out the numerical simulation of equation (21) by taking the following four initial conditions

$$B_y'(x,0) = B_{y0}[1+\varepsilon\, cos(\alpha_x x)] \qquad \textbf{(IC-1)}$$

$$B_y'(x,0) = B_{y0}\left[1+\varepsilon \exp\left(-x^2/{r_{01}}^2\right)\right] \qquad \textbf{(IC-2)}$$

$$B_y'(x,0) = B_{y0}\left[exp\left(-x^2/{r_0}^2\right)+\varepsilon\, cos(\alpha_x x)\right] \qquad \textbf{(IC-3)}$$

$$B_y'(x,0) = B_{y0}\left[\exp\left(-x^2/{r_0}^2\right)+\varepsilon \exp(2\pi i\theta(x))\right] \qquad \textbf{(IC-4)}$$

- Where ε and α_x are the magnitude and wave number of the perturbation and r_{01} is the transverse scale size of perturbation, r_0 is the initial KAW beam width, $\theta(x)$ is the variable which distributed on [0,1].
- Using the finite-difference method with a predictor-corrector scheme with $L_x = 2\pi/\alpha_x$ (periodic length), 2^8 grid point and using small time step size $dz = 5\times 10^{-5}$.
- The simulation parameters are as follows $\varepsilon = 0.1, \alpha_x = 0.5, r_0 = 1.0, r_{01} = 5, L_x = 12.5, g = 0.01, B_{y0} = 1.005$.
- Further we get the parameter values from Helios 2 spacecraft data at 1 AU as $B_0 \approx 1\times 10^{-4}G, n_0 \approx 5\ cm^{-3}, T_e \approx 0.5\times 10^5 K, \beta = 0.121, v_{te} \approx 8.7\times 10^7 cm\ s^{-1}$.

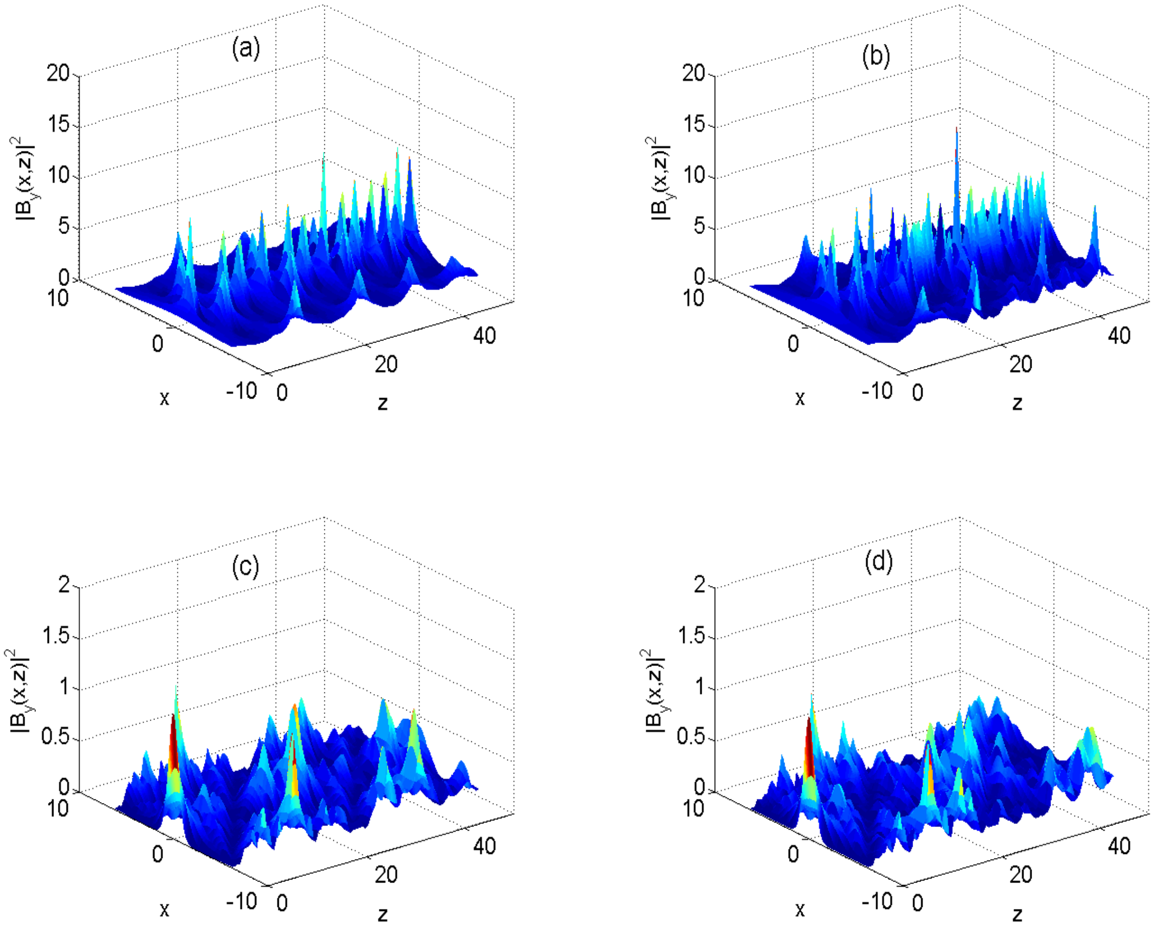

Fig. 4.1 2D-spatial profile of magnetic field intensity for solar wind at 1 AU: (a) IC-1, (b) IC-2, (c) IC-3 and (d) IC-4. Intensity reaches 12.29, 14.17, for IC-1, IC-2 and 0.96 for IC-3 and IC-4.

- Fig. (1) depicted the magnetic field intensity profile of KAW for IC-1, IC-2, IC-3, IC-4.
- For IC-1 and IC-2, the pattern of the filament are almost same and the peak intensity reaches $|B_y|^2 \sim 12.29$ for IC-1 and $|B_y|^2 \sim 14.17$ for IC-2.
- The coherent strictures are breaking at high intensity peak, leading to kinetic scale of KAWs (Ladau damping), thereby heating the plasma particles.

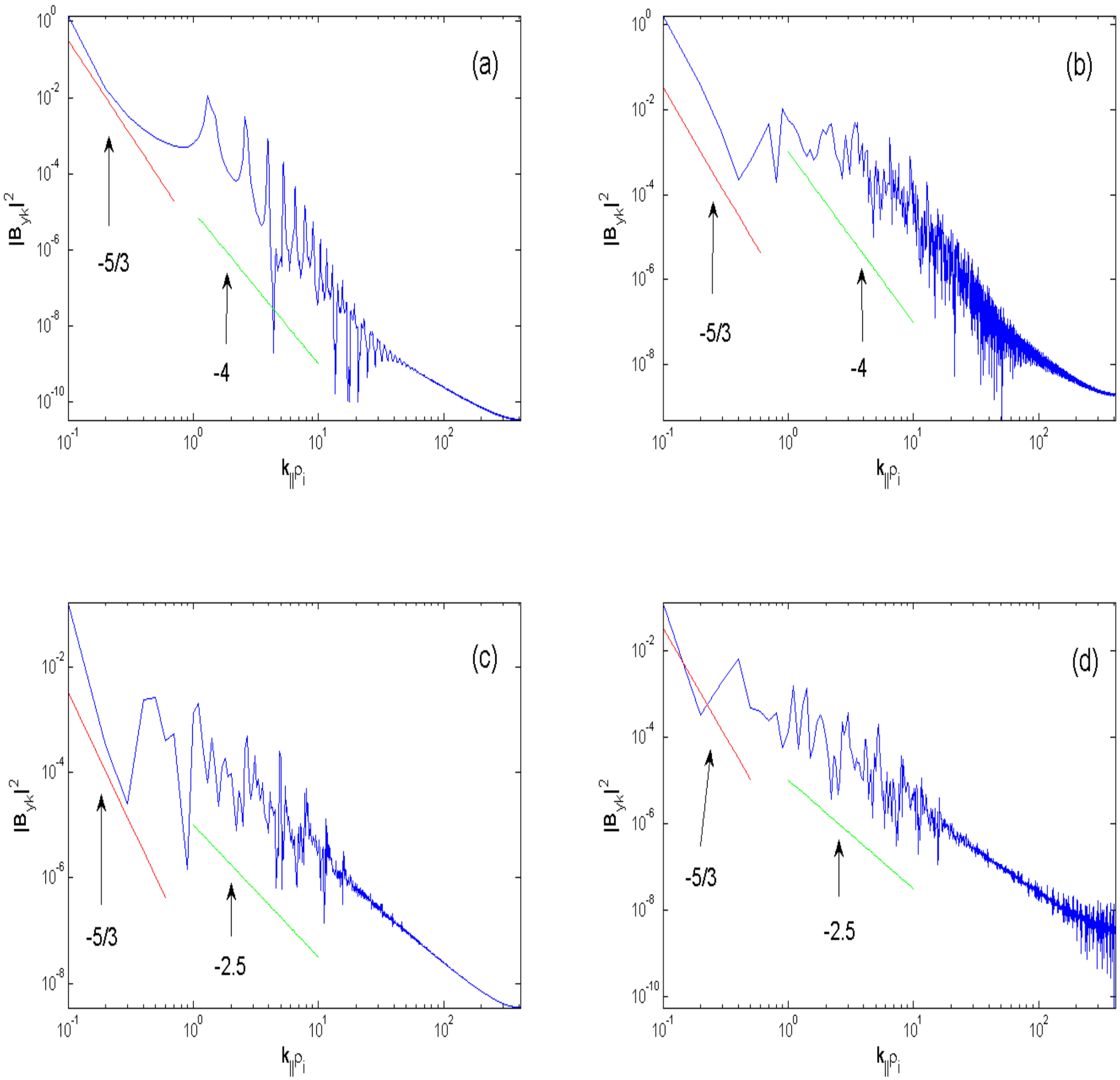

Fig. 2. Magnetic field spectral intensity $|B_{yk}|^2$ versus $k_{\|}\rho_s$ for solar wind at 1AU: (a) IC-1, (b) IC-2, (c) IC-3 and (d) IC-4.

- At all the initial conditions for $k_{\|}\rho_s < 1$ i.e. in the inertial range, the spectral indices are of Kolmogorov scale $k^{-5/3}$ as observed by many authors.
- In figure (2), these slopes are -4,-4,-2.5 and -2.5 for IC-1, IC-2, IC-3 and IC-4.

Conclusion

- The coherent structures of KAWs with high intensity are formed and they are dependent on the types of four different initial condition of simulation.

- It was found that the perturbation present in the magnetic field may lead to forming these coherent structures via taking energy from the pump KAWs.

- In our study we found the spectral index following the Kolmogorov scale of $-5/3$ which is in the inertial range followed by deeper indices varying from -2.5 to -8 in the kinetic dissipation range.

- Hence, the formed KAWs coherent structures and power spectrum dependence on initial conditions explain the particle acceleration and heating of solar corona.

Thank You