$T_{cc}^{+}(3875)$ relevant DD^* scattering from $N_f = 2$ lattice QCD [1]

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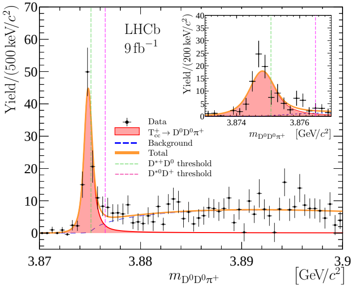
[1] Based on: Physics Letters B 833 (2022) 137391

Outline

- I. Background
- II. Lattice setup
- III. Results & Discussion
- IV. Summary

Background:

1. LHCb reported $\mathrm{J^P} = 1^+ \ \mathrm{T_{cc}^+}$.
   LHCb. *Nat.Phys.*18,751(2022)
2. The study of effective theory for exotic states.
   Review of new hadron states. H.X.Chen,*et al.*arXiv:2204.02649
   M.L.Du,*et al.PRD.*105,014024
3. Vector meson exchange effects.
   A.Feijoo. *et al.PRD.*104.114015
   X.K. Dong. *et al.Comm.Theor.Phys.*(2021)73.125201



Lattice QCD study for doubly charm tetraquarks

1. General class tetraquarks spectrum search from lattice QCD in the charm sector.
   G.Cheung *et al.*(Hadron Spectrum Collab) *JHEP*11(2017)033.
2. Doubly heavy tetraquarks in lattice QCD. Junnarkar *et al.*(2019)*PRD.*99.034507
3. Signature of a doubly charm tetraquark pole in $\mathrm{DD^*}$ scattering on the lattice M. Padmanath and S. Prelovsek *PRL.*129,032002 See the NEXT talk.

Tetraquarks from finite box: Lattice QCD

Finite volume scattering theory

$$\begin{cases} \text{characteristic size: } R_a \ll L \\ \text{temporal size: } L_t \ll L \\ \text{scattering energy: } E_k < m_1 + m_2 \end{cases} \xrightarrow[\text{Lüscher's formulea}]{} \begin{cases} \text{ERE: } a_l, k_l, \\ \text{phase shift: } \delta_l, \\ \text{dynamical pole.} \end{cases}$$

Lüscher's formulism

single channel:[a]

$$p \cot \delta_0(q^2) = \frac{2}{L a_s \sqrt{\pi}} \mathcal{Z}_{00}(1, q^2). \tag{1}$$

multi-channel: [b] [c]

$$\det\left[\delta_{ij}\delta_{JJ'} + i\rho_i t_{ij}^{(J)}(s)\left(\delta_{JJ'} + i\mathcal{M}_{JJ'}^{\vec{P}\Lambda}(p_i L)\right)\right] = 0. \tag{2}$$

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[a] Lüscher *NPB*.354,2-3,(1991)531-578

[b] L.Leskovec and S.Prelovsek*PRD*.(2015)85, 114507

[c] J.J.Dudek *et al*.(Hadron Spectrum Collab.)*PRL*.(2014)113,182001

Lattice setup:

Anisotropic $N_f = 2$ clover gauge ensembles with degenerate u, d quarks:

Table: Parameters of ensembles. arXiv:2205.12541

$L^3 \times T$	β	a_t^{-1}(GeV)	ξ	N_{cfg}	m_π(MeV)	$m_\pi L a_s$	$m_{J/\psi}$(MeV)	N_{vec}
$16^3 \times 128$	2.0	6.894(51)	5.30(30)	6950	348.5(1.0)	3.9	3099(1)	70

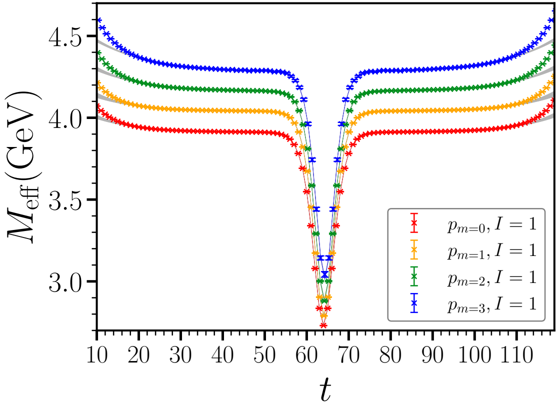
Distillation method: M.Peardon *et al.*(Hadron Spectrum Collab.)(2009)*PRD*80,054506

S-wave Opetrators sets

$$O_{DD^*}(p,t) = \frac{1}{N_{\vec{p}}} \sum_{R\in O} O_D(R\circ\vec{p},t) O_{D^*}(-R\circ\vec{p},t), \tag{3}$$

$$\begin{cases} I=0: & |DD^*\rangle = \frac{1}{\sqrt{2}}\left(|D^0D^{*+}\rangle - |D^+D^{*0}\rangle\right), \\ I=1: & |DD^*\rangle = \frac{1}{\sqrt{2}}\left(|D^0D^{*+}\rangle + |D^+D^{*0}\rangle\right). \end{cases} \tag{4}$$

$E^{(I)}_{DD^*}$ fitting I



1. Momentum modes up to 3,
2. GEVP procedure.
3. jackknife resampling.
4. $E^{(I)}_{DD^*}$ energy level fitting:

$$
\begin{aligned}
& C^{(I)}(p_m, t) \\
= \ & W^{(I)}_1 \cosh\left(E^{(I)}_{DD^*}(p_m)(t-\frac{T}{2})\right) \\
& + W^{(I)}_2 \cosh\left([E_D(p_m) - E_{D^*}(p_m)]\,(t-\frac{T}{2})\right) \\
& + W'^{(I)} \cosh\left(E'(t-\frac{T}{2})\right),
\end{aligned}
\tag{5}
$$

Results I

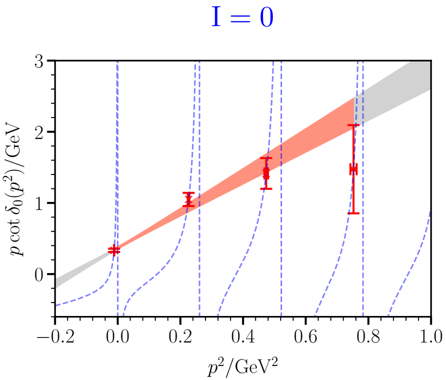
$\vec{p}_m$ modes $E_D(\vec{p}_m) + E_{D^*}(\vec{p}_m)$ (GeV)	$m = 0$ 3.9035(14)	$m = 1$ 4.0338(19)	$m = 2$ 4.1617(29)	$m = 3$ 4.2864(36)
$E^{(I=0)}_{DD^*}(p_m)$ (GeV)	3.8977(14)	4.0166(15)	4.1369(18)	4.2682(28)
$\Delta E^{(I=0)}$ (GeV)	-0.00582(22)	-0.0172(12)	-0.0248(23)	-0.0183(32)
$E^{(I=1)}_{DD^*}(p_m)$(GeV)	3.9120(13)	4.0405(14)	4.1628(16)	4.2836(22)
$\Delta E^{(I=1)}$(GeV)	0.00851(23)	0.0067(12)	0.0011(23)	-0.0028(33)

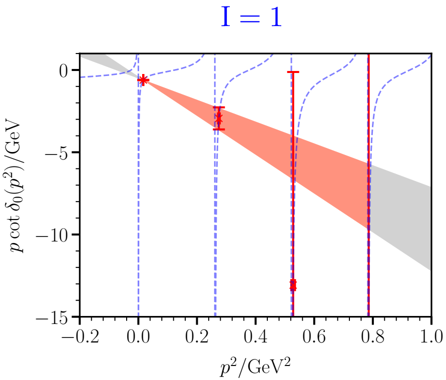
$$p \cot \delta_0(q^2) = \frac{2}{La_s\sqrt{\pi}}\mathcal{Z}_{00}(1, q^2) = \frac{1}{\pi L}\lim_{R\to\infty}\left[\sum_{\vec{n}\in Z_3}^{|\vec{n}|<R}\frac{1}{\vec{n}^2 - q^2} - 4\pi R\right]. \quad (6)$$

Effective range expension

$$p \cot \delta_0(p) = \frac{1}{a_0} + \frac{1}{2}r_0 p^2 + \mathcal{O}(p^4) \quad (7)$$

Results II



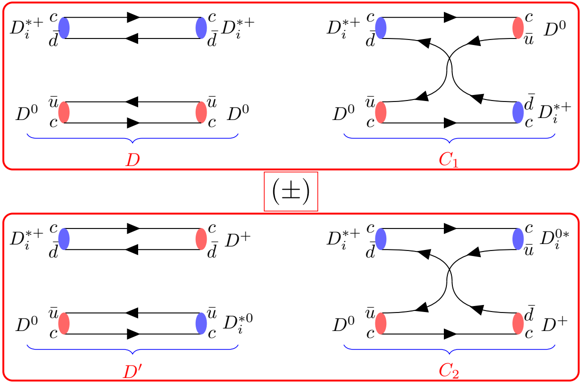


Effective range expension results

$$a_0^{(I=0)} = 0.538(33)\ \text{fm}, \quad r_0^{(I=0)} = 0.99(11)\ \text{fm}, \quad \text{likely attractive ,}$$

$$a_0^{(I=1)} = -0.433(43)\ \text{fm}, \quad r_0^{(I=1)} = -3.6(1.0)\ \text{fm}, \quad \text{likely repulsive .}$$

Discussion: in view of QCD

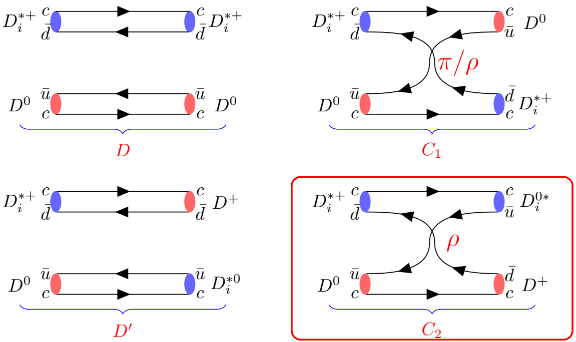


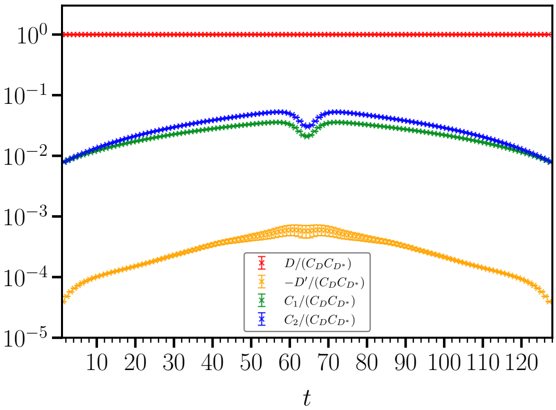
Recall : $|\mathrm{DD}^{*(\mathrm{I})}\rangle = \frac{1}{\sqrt{2}}\left[|\mathrm{D}^0\mathrm{D}^{*+}\rangle + (-)^{(\mathrm{I}+1)}|\mathrm{D}^+\mathrm{D}^{*0}\rangle\right],$

Quark contraction diagram:

$$\mathrm{C}^{(\mathrm{I})}(\mathrm{p},\mathrm{t}) = \mathrm{D} - \mathrm{C}_1 + (-)^{\mathrm{I}+1}(\mathrm{D}' - \mathrm{C}_2)\,. \quad (8)$$

Discussion: in view of hadronic level





Contribution of each term

$$E_{DD^*}^{(I)} \approx \ln \frac{C^{(I)}(p,t)}{C^{(I)}(p,t+1)} \approx E_0 + \epsilon_1 \delta E_1 e^{\delta E_1 t} + (-)^{I+1} \epsilon_2 \delta E_2 e^{\delta E_2 t} \quad (9)$$

Summary

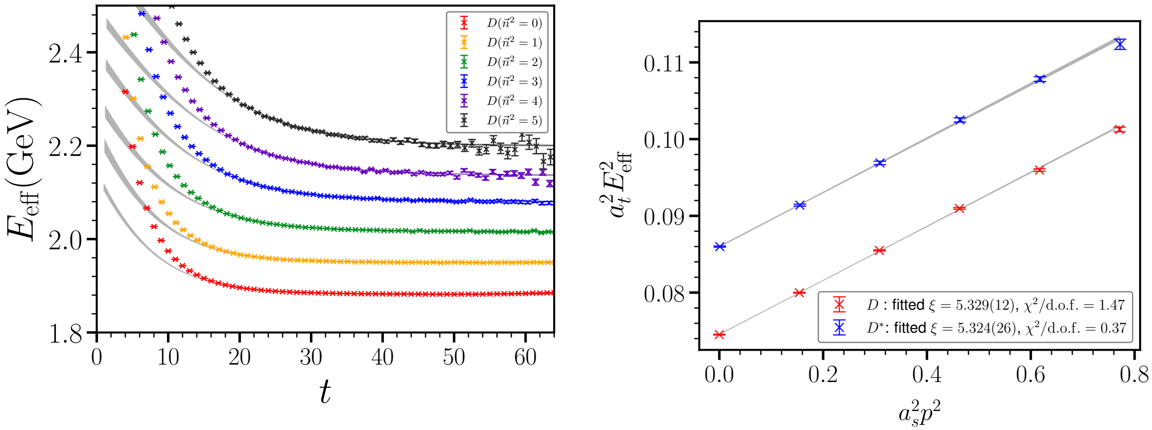
1. We calculate S-wave DD^* scattering in $\mathrm{N_f} = 2$ lattice QCD at $\mathrm{m}_\pi \approx 350$ MeV.

$$a_0^{(I=0)} = 0.538(33)\ \text{fm}, \quad r_0^{(I=0)} = 0.99(11)\ \text{fm}, \quad \text{likely attractive},$$
$$a_0^{(I=1)} = -0.433(43)\ \text{fm}, \quad r_0^{(I=1)} = -3.6(1.0)\ \text{fm}, \quad \text{likely repulsive}.$$

2. We analyse the isospin $\mathrm{I} = 0, 1$ dependence of the DD^* interaction.
3. The charged light vector meson ρ meson exchange may play a crucial role in the formation of $\mathrm{T}_{cc}^{+}(3875)$.

Thank You

Numerical details: D and D^* dispersion relation



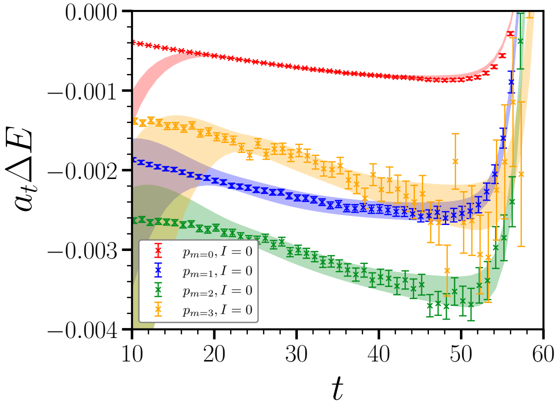
$$\mathrm{C_X}(\vec{\mathrm{p}},\mathrm{t}) = \mathrm{W_1} \cosh\left[-\mathrm{E_X}(\vec{\mathrm{p}})(\frac{\mathrm{T}}{2}-\mathrm{t})\right] + \mathrm{W_2} \cosh\left[-\mathrm{E'_X}(\vec{\mathrm{p}})(\frac{\mathrm{T}}{2}-\mathrm{t})\right], \quad (10)$$

$$\mathrm{E_X^2}(\vec{\mathrm{p}}) = \mathrm{m_X^2} + \frac{1}{\xi^2}|\vec{\mathrm{p}}|^2. \quad (11)$$

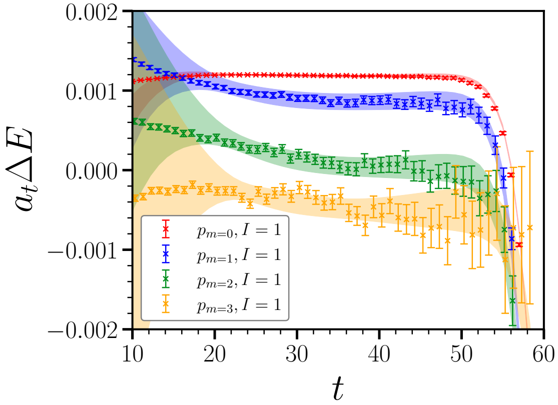
The hyperfine splitting: $\Delta\mathrm{m} = \mathrm{E_{D^*}}(\vec{0}) - \mathrm{E_D}(\vec{0}) = 139.70(57)$ MeV.

Numerical details: $E^{(I)}_{DD^*}$ fitting II– A consistant check

$I = 0$



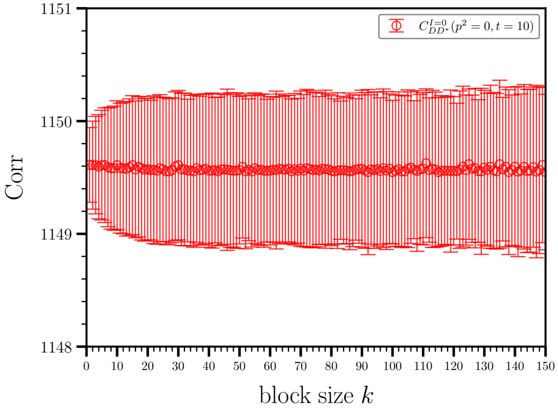
$I = 1$



$$R(p_m, t) \equiv \frac{C^{(I=0)}_{DD^*}(p_m, t)}{C_D(\vec{p}_m, t) C_{D^*}(\vec{p}_m, t)} \sim e^{-\Delta E(p_m)t} \quad (t \gg 1). \tag{12}$$

Numerical details: Blocked jackknife analysis

correlation



effective mass