Charm sea effects on charmonium physics

F. Knechtli, T. Korzec, M. Peardon, J. A. Urrea-Niño

- Introduction
- Optimal creation operators for charmonium spectroscopy on the lattice
- Charm sea effects in the spectrum and mixing with glueballs
- Conclusions & Outlook

based on `2205.11564`, Phys.Rev.D 106 (2022)

QWG 2022 – The 15th International Workshop on Heavy Quarkonium,
GSI Darmstadt, 30 Sep 2022

FOR 5269

Funded by DFG Deutsche Forschungsgemeinschaft German Research Foundation

Part I

Introduction

Charm sea effects

Consider QCD with quarks q^i, $i = \{u, d, s, c\}$ and Dirac operators $D_i = \not{D}[A_\mu] + m_i$. The expectation value of a physical observable $A[q^i, U]$ is

$$\langle A[q^i, U] \rangle = \frac{1}{Z} \int \mathcal{D}[U] \left(\prod_{j=u,d,s} \det D_j \right) \det D_c \, \tilde{A}[D_i^{-1}, U] \, e^{-S[U]}$$

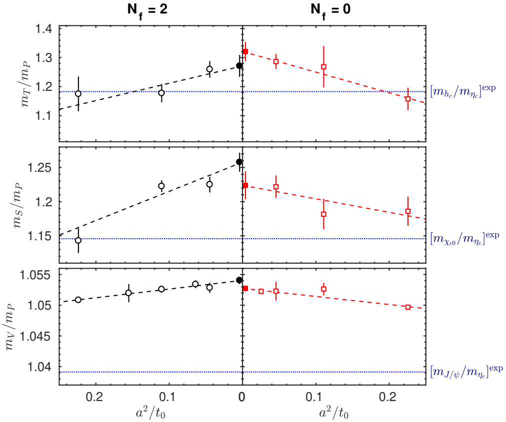
Charm sea effects stems from $\det D_c$

When are charm sea effects relevant?

- Appelquist-Carazzone decoupling theorem : heavy quarks decouple from low energy physics [T. Appelquist, J. Carazzone, Phys.Rev.D 11 (1975)]
- Effective theory [S. Weinberg, Phys.Lett.B 91 (1980)], $1/M^2$ corrections make only 2 permille effects for charm [FK, T. Korzec, B. Leder, G. Moir, Phys.Lett.B 774 (2017)]
- Decoupling applies to binding energies of charmonium ($\ll m_{\mathrm{charm}}$) and decay constants
- In this talk: dynamical charm is essential to compute charm-annihilation effects in charmonium or charmonium–glueball mixing
- Shift of mass of η_c is estimated to $+7.3(1.2)$ MeV [HPQCD Collaboration, Phys.Rev.D 102 (2020)]. Perturbative NRQCD has wrong sign at leading order

Decoupling in charmonium

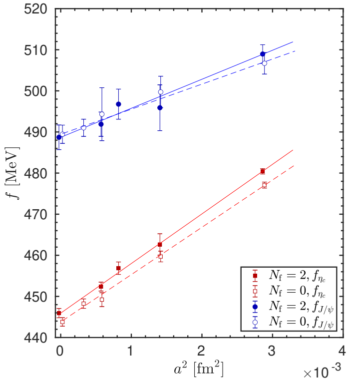
Comparison $N_\mathrm{f} = 2$ charm quarks with pure gauge



Charm sea effects $([m_V/m_P]^{N_\mathrm{f}=2} - [m_V/m_P]^{N_\mathrm{f}=0})/[m_V/m_P]^{N_\mathrm{f}=2} = 0.12(7)\%$
$\Rightarrow$ below 2% for the hyperfine splitting $(m_V - m_P)/m_P$
Difference with exp: no light quarks, charm annihilation, electromagnetism; one charm quark too many [S. Calì, FK, T. Korzec, Eur.Phys.J.C 79 (2019)]

Decoupling in charmonium contd

Decay constans of charmonium (for leptonic decays)



Charm sea effects are barely resolvable (below 1%) despite the great accuracy of continuum extrapolations [S. Calì, K. Eckert, J. Heitger, FK, T. Korzec, Eur.Phys.J.C 81 (2021)] Decoupling of charm works well up to 500 MeV

Part II

Optimal creation operators for charmonium spectroscopy on the lattice

The Distillation Method

Replace $\psi \to VV^\dagger\psi$, where V contains the N_v lowest eigenmodes of the 3D Laplacian operator. [M. Peardon et al. (2009)]

Focus: Meson operator $\bar{\psi}\Gamma\psi$.

Building blocks

- Laplacian eigenvectors $V[t]$
- Perambulators $\tau[t_1, t_2] = V^\dagger[t_1]D^{-1}V[t_2]$ smeared all-to-all propagator
- Elementals $\Phi[t] = V^\dagger[t]\Gamma V[t]$

Meson 2-point functions:

$$C(t) = -\left\langle Tr\left(\Phi[t]\tau[t,0]\bar{\Phi}[0]\tau[0,t]\right)\right\rangle + \text{disconnected piece (isoscalar)}$$

Advantages

- ✓ Perambulators/elementals have manageable sizes.
- ✓ Perambulators are independent from elementals.

Disadvantages

- × N_v scales with 3D physical lattice volume.
- × Many inversions required.

New improvement

Starting point: Quark distillation profile $g(\lambda)$ used via $\psi \to VJV^\dagger\psi$ with $J[t]_{ij} = \delta_{ij} g(\lambda_i[t])$. Modulate contribution from each vector.
For a fixed Γ and energy level e one can build an optimal elemental given by

$$\tilde{\Phi}^{(\Gamma,e)}[t]_{\substack{ij \\ \alpha\beta}} = \tilde{f}^{(\Gamma,e)}(\lambda_i[t], \lambda_j[t]) v_i[t]^\dagger \Gamma_{\alpha\beta} v_j[t]$$

which includes the optimal meson distillation profile given as

$$\tilde{f}^{(\Gamma,e)}(\lambda_i[t], \lambda_j[t]) = \sum_k \eta_k^{(\Gamma,e)} g_k(\lambda_i[t])^* g_k(\lambda_j[t]).$$

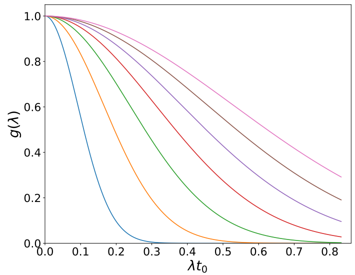
This can be done by solving a Generalized Eigenvalue Problem (GEVP) for different profile functions $g_i(\lambda)$ [F. Knechtli, T. Korzec, M. Peardon, J. A. Urrea-Niño, Phys. Rev. D106 (2022)]

Advantages:

- ✓ $C(t)$ requires **very little** additional cost to build. Elementals required come "for free" from the standard one.
- ✓ $\tilde{f}^{(\Gamma,e)}(\lambda_i[t], \lambda_j[t])$ tells us if N_v is large enough and how to use the N_v eigenvectors **for each Γ and energy state**.

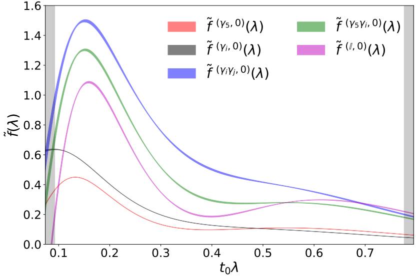
Applying the method

- QCD with $N_f = 2$ at half the physical charm quark mass.
  No light quarks. Clover-improved Wilson fermions.
- 48×24^3 and 96×48^3 lattices with $a \approx 0.0658, 0.049$ fm. Check effectiveness at smaller resolutions and larger volumes.
- Both local and derivative Γ. [J. J. Dudek et al. (2008)]



We start with Gaussian profiles

$$g_i(\lambda) = \exp\left(-\frac{\lambda^2}{2\sigma_i^2}\right)$$



At the end we get optimal profiles

$$\tilde{f}^{(\Gamma,e)}(\lambda_i, \lambda_j) \neq 1$$

($e = 0$ ground, $e = 1$ first excited)

Objects of interest

Meson 2-point functions:

- iso-vector: $C^V_{ab}(t) = -\left\langle Tr\left(\Phi_a[t]\tau[t,0]\bar{\Phi}_b[0]\tau[0,t]\right)\right\rangle$
- iso-scalar: $C^S_{ab}(t) = C^V_{ab}(t) + \left\langle 2\,Tr\left(\Phi_a[t]\tau[t,t]\right)\,Tr\left(\bar{\Phi}_b[0]\tau[0,0]\right)\right\rangle$ Measured exactly.

Glueball-meson 2-point function:

- $C_{MG}(t) = \langle Tr\left(\Phi_a[t]\tau[t,t]\right)\,G[0]\rangle$

Effective masses:

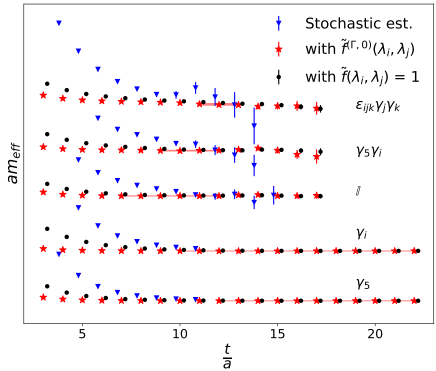
$$C(t)\,w_e(t,t_G) = \rho_e(t,t_G)\,C(t_G)\,w_e(t,t_G) \quad \text{GEVP}$$

$$\rho_e(t,t_G) = 2c_e \mathrm{e}^{-\frac{T}{2}m_e}\cosh\left[\left(\frac{T}{2}-t\right)m_e\right] \quad \text{at large t}$$

Goal of the method: Increase overlap with wanted state and decrease overlaps with unwanted states **without** much additional cost.

Coarse lattice ($L \approx 1.51\ fm$) with $N_v = 200$

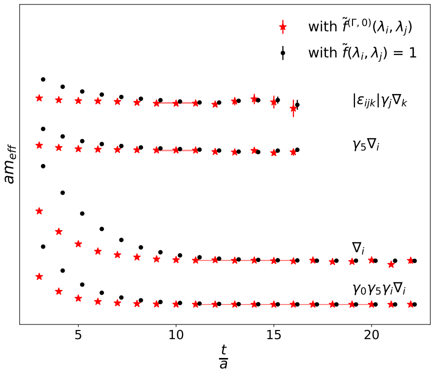
Local iso-vector operators



Fractional overlaps:

- γ_5: 0.9272(3) $\rightarrow$ 0.9858(2)
- γ_i: 0.8743(10) $\rightarrow$ 0.9900(5)
- $\epsilon_{ijk}\gamma_j\gamma_k$: 0.77(7) $\rightarrow$ 0.93(1)

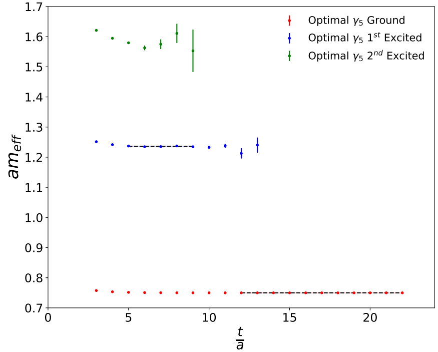
Derivative iso-vector operators



Fractional overlaps:

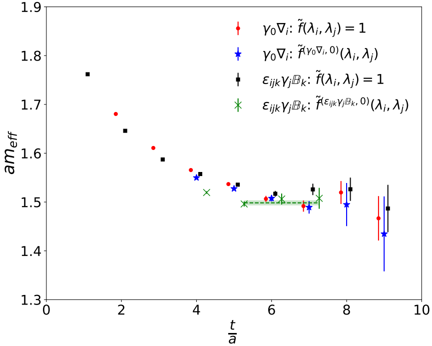
- ∇_i: 0.4758(7) $\rightarrow$ 0.742(2)
- $\gamma_5\nabla_i$: 0.84(1) $\rightarrow$ 0.970(5)
- $\mathbb{Q}_{ijk}\gamma_j\nabla_k$: 0.858(8) $\rightarrow$ 0.981(3)

Excited states



- Inclusion of profiles grants acces to excited states

The spin-exotic 1^{-+}



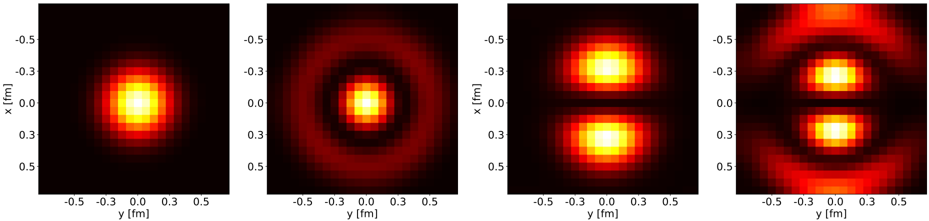
- The $\epsilon_{ijk}\gamma_j\mathbb{B}_k$ operator with the optimal profile has the best overlap with the eigenstate.

Spatial Profiles

Spatial profile can be recovered:

- $\Psi^{(\gamma_5,e)}(\vec{x}) = \frac{1}{N_t}\sum_t ||Tr\left(\gamma_5 V[t]\tilde{\Phi}^{(\gamma_5,e)}[t]V[t]^\dagger\right)\phi_0||^2$
- $\Psi^{(\gamma_5\nabla_1,e)}(\vec{x}) = \frac{1}{N_t}\sum_t ||Tr\left(\gamma_5 V[t]\tilde{\Phi}^{(\gamma_5\nabla_1,e)}[t]V[t]^\dagger\right)\phi_0||^2$

with ϕ_0 a 3D point source. Profiles dictate spatial structure.

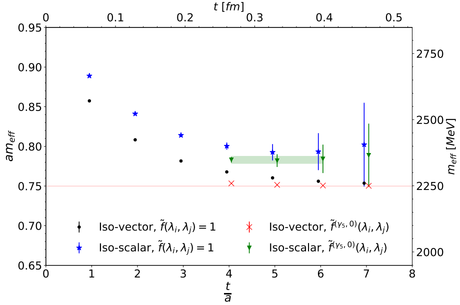


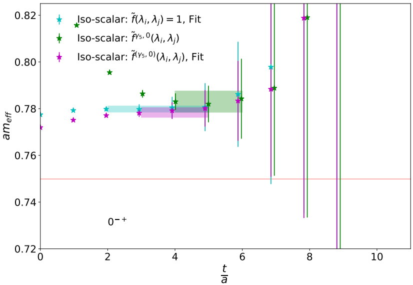
- Spatial behavior of state can be visualized.
- Finite-volume effects can be monitored.

Part III

Charm sea effects in the spectrum and mixing with glueballs

Iso-scalar 0^{-+} (coarse lattice)





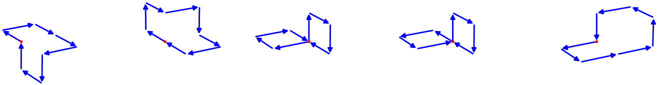
- Optimal profile from iso-vector improves the iso-scalar too.
- Mass splitting is resolved.

- "One Loop Ground state Analysis" [H. Neff, N. Eicker, T. Lippert, J.W. Negele, K. Schilling, Phys.Rev.D 64 (2001)] [K. Jansen, C. Michael, C. Urbach, Eur.Phys.J.C 58 (2008)]

Charmonium-Glueball mixing

To keep in mind:

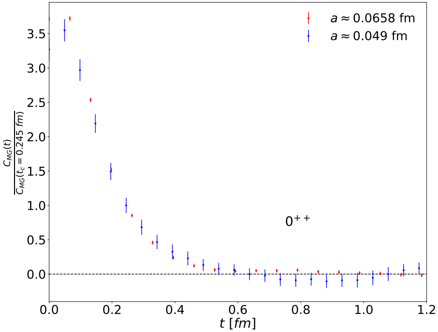
- Iso-scalar meson operators require disconnected pieces in correlation function. Feasable thanks to distillation.
- Glueballs are hard to find in un-quenched QCD. Optimal operators must be found via GEVP
  - Different loop shapes and windings. [C. J. Morningstar & M. Peardon, (1999)] [B. Berg & A. Billoire, (1983)]



  - Different smearing schemes and levels:
    - ⋆ 3D-HYP [A. Hasenfratz & F. Knechtli, (2001)]
    - ⋆ 3D improved APE [B. Lucini et al. (2004)]

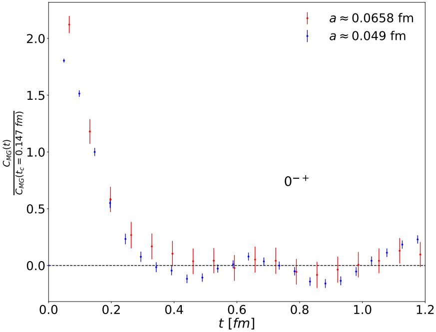
Scalar channel

$0^{++} \to \Gamma = \mathbb{I},\ \tilde{f}(\lambda_i, \lambda_j) = 1$



Pseudo-Scalar channel

$0^{-+} \to \Gamma = \gamma_5,\ \tilde{f}^{(\gamma_5,0)}(\lambda_i, \lambda_j)$



- $C_{MG}(t) = \left\langle Tr\left(\Phi^{(\Gamma)}[t]\tau[t,t]\right) G^{(R^{PC})}(0)\right\rangle$.
- Correlators normalized at fixed time in physical units.
- Noise is dominated by the glueball. Glueballs require more statistics than mesons.

Part IV

Conclusions & Outlook

Conclusions

- Improvement of distillation optimizes overlap with the wanted state
- Test in a model with two quarks in the sea at half the charm quark mass
- Disconnected (charm annihilation) contributions to spectrum can be resolved
- Mixing of charmonium with glueballs is observed

Outlook

- Charmonium spectrum with 3 degenerate light quarks ($m_\pi = 420$ MeV) and one physical charm quark [R. Höllwieser, FK, T. Korzec, Eur.Phys.J.C 80 (2020)]
- Mixing of charmonium with glueballs and light hadrons at heavy "light" quark masses