QWG 22 - GSI Darmstadt, September 28 2022

Standard Model prediction of the B_c lifetime

Presented by Jason Aebischer

University of Zurich

Outline

1. Motivation
2. Procedure
3. Setup
4. Results
5. Novel way
6. Summary

based on: 2105.02988, 2108.10285 in collaboration with Benjamín Grinstein

Outline

LFU violation in charged currents

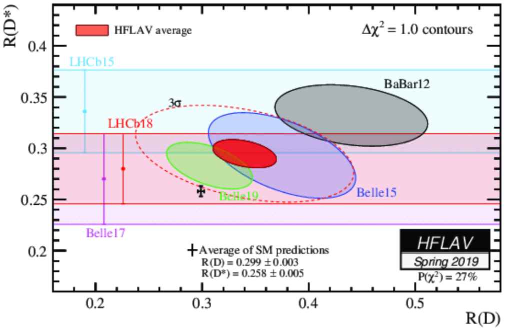
Measurement

R_D and R_{D^*}

BaBar: 1205.5442, 1303.0571, LHCb: 1506.08614, 1708.08856
Belle: 1507.03233, 1607.07923, 1612.00529

$$R_{D^{(*)}} = \frac{BR(B \to D^{(*)}\tau\nu)}{BR(B \to D^{(*)}\ell\nu)}$$

$$\ell \in \{e, \mu\}$$



HFLAV: 1909.12524

NP from τ_{B_c}

$B_c \to \tau\nu_\tau$

Not exceed τ_{B_c}

$Br(B_c \to \tau\nu_\tau)$

Pseudoscalar scenarios constrained

Alonso/Grinstein/Camalich: 1611.06676

Polarization observables

$F_L(D^*)$, τ-polarization

Blanke/Crivellin/de Boer/Kitahara/Moscati: 1811.09603

Blanke/Crivellin//Kitahara/Moscati/Nierste: 1811.09603

Status

Experimental value

$\tau_{B_c} = 0.510(9)\text{ps}$

LHCb: 1401.6932, 1411.6899
CMS:1710.08949

Theoretical predictions

Operator Product Expansion (OPE)

Beneke/Buchalla(BB): hep-ph/9601249
Bigi: hep-ph/9510325
Chang/Chen/Feng/Li: hep-ph/0007162

QCD sum rules

Kiselev/Kovalsky/Likhoded: hep-ph/0002127

Potential Models

Gershtein/Kiselev/Likhoded/Tkabladze: hep-ph/9504319

OPE result from BB

$\tau_{B_c} = 0.52\,\text{ps}, \quad 0.4\,\text{ps} < \tau_{B_c} < 0.7\,\text{ps}$

Beneke/Buchalla(BB): hep-ph/9601249

Outline

EFT approach

Effective Hamiltonian

At μ_W, RGE running

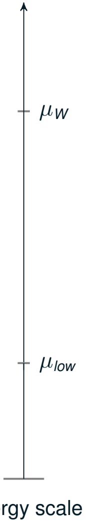
OPE

At μ_{low}

Non-Relativistic QCD (NRQCD)

Integrate out (anti-)quark fields

EFT approach



$$\mathcal{L}_{SM}$$

↓ Matching

$$\mathcal{H}_{\text{eff}} = -\sum_i C_i O_i$$

↓ RGE

OPE ⟶ NRQCD

Optical Theorem

Forward scattering

$\Gamma_{B_c} = \frac{1}{2M_{B_c}} \langle B_c | \mathcal{T} | B_c \rangle$

Transition Operator

$\mathcal{T} = \mathrm{Im}\, i \int d^4x \, T \, \mathcal{H}_{eff}(x) \mathcal{H}_{eff}(0)$

OPE

$\mathcal{T} =$ series of local operators

Contributions

$\bar{b}$-decays

$\bar{b} \to \bar{c}u(\bar{s} + \bar{d}), \bar{c}c(\bar{s} + \bar{d}), \bar{c}\ell\nu$

c-decays

$c \to (s + d)u(\bar{s} + \bar{d}), (s + d)\ell\nu$

Weak Annihilation (WA), Pauli Interference (PI)

1-loop graphs

Expansion of OPE operators

dim 3

$$\bar{Q}Q \to \psi_q^\dagger \left(1 + \frac{1}{2m^2}(\vec{D})^2 + \frac{g_s}{2m^2}\vec{\sigma}\cdot\vec{B} + \frac{g_s}{4m^3}[\vec{D}\cdot\vec{E}] + \frac{25}{64m^4}(\vec{D})^4 + \cdots\right)\psi_q$$

dim 5

$$\bar{Q}\sigma_{\mu\nu}g_s G^{\mu\nu}Q \to \psi_q^\dagger\left(-2g_s\vec{\sigma}\cdot\vec{B} - \frac{g_s}{m}[\vec{D}\cdot\vec{E}] + \cdots\right)\psi_q$$

dim 6

$$(\bar{b}\gamma_\mu P_L b)(\bar{c}\gamma^\mu P_L c) = (\overline{X}_-^{(b)}\gamma_\mu P_L X_-^{(b)})(\overline{\Psi}_+^{(c)}\gamma^\mu P_L \Psi_+^{(c)}) + \cdots$$

Matrix Elements

Gershtein/Kiselev/Likhoded/Tkabladze: hep-ph/9406339

Kinetic Energy

$$\frac{\langle B_c|\psi^\dagger(i\vec{D})^2\psi|B_c\rangle}{2M_{B_c}} = \frac{2m_c m_b}{(m_c+m_b)}T\,, \qquad T = \text{Kinetic energy}$$

Fermi and Darwin

$$\frac{\langle B_c|\psi_b^\dagger g_s\vec{\sigma}\cdot\vec{B}\psi_{\bar{b}}|B_c\rangle}{2M_{B_c}} = -\frac{4}{3}g_s^2\frac{|\Psi^{\mathrm{WF}}(0)|^2}{m_c}\,, \qquad M_{B_c^*} - M_{B_c} = \frac{8}{9}g_s^2\frac{|\Psi^{\mathrm{WF}}(0)|^2}{m_b m_c}$$

$$\frac{\langle B_c|\psi_b^\dagger g_s[\vec{D}\cdot\vec{E}]\psi_{\bar{b}}|B_c\rangle}{2M_{B_c}} = \frac{4}{3}g_s^2|\Psi^{\mathrm{WF}}(0)|^2\,, \qquad f_{B_c}^2 = \frac{12|\Psi^{\mathrm{WF}}(0)|^2}{M_{B_c}}$$

D^4

$$\frac{\langle B_c|\psi^\dagger(i\vec{D})^4\psi|B_c\rangle}{2M_{B_c}} = \frac{4m_c^2 m_b^2}{(m_c+m_b)^2}T^2$$

Outline

Improvements over BB

Mass schemes

$\overline{\text{MS}}$, meson, Upsilon

Spin symmetry

Relates matrix elements

Penguin contributions

Included

Better input values

α_s, f_{B_c}, CKM parameters

Mass schemes

$\overline{\text{MS}}$ scheme

m_b^{OS} and m_c^{OS} in terms of $\overline{m}_b(\mu_b)$ and $\overline{m}_c(\mu_c)$

Meson scheme

m_b^{OS} in terms of m_Υ
m_c^{OS} in terms of m_b^{OS} and $\overline{m}_B - \overline{m}_D$

Upsilon scheme

Like meson scheme
For c decays: m_c^{OS} in terms of Upsilon expansion of $m_{J/\psi}$

Upsilon scheme

Hoang/Ligeti/Monahar: hep-ph/9809423

$\bar{b}$ decays, WA, PI

$$\frac{1}{2} m_\Upsilon = m_b^{OS}\left[1 - \frac{(\alpha_s C_F)^2}{8}\left\{1 + \frac{\alpha_s}{\pi}\left[\left(\ln\left(\frac{\mu}{\alpha_s C_F m_b^{OS}}\right) + \frac{11}{6}\right)\beta_0 - 4\right] + \cdots\right\}\right]$$

$$m_b^{OS} - m_c^{OS} = \overline{m}_B - \overline{m}_D + \frac{1}{2}\lambda_1\left(\frac{1}{m_b^{OS}} - \frac{1}{m_c^{OS}}\right)$$

$\overline{m}_B, \overline{m}_D =$ (iso)spin-averaged masses

c decays

$$\frac{1}{2} m_{J/\psi} = m_c^{OS}\left[1 - \frac{(\alpha_s C_F)^2}{8}\left\{1 + \frac{\alpha_s}{\pi}\left[\left(\ln\left(\frac{\mu}{\alpha_s C_F m_c^{OS}}\right) + \frac{11}{6}\right)\beta_0 - 4\right] + \cdots\right\}\right]$$

strange mass

$m_s = 0$ or $\overline{\text{MS}}$

Outline

Results

Massles strange quark

$$
\begin{aligned}
\Gamma_{B_c}^{\overline{\text{MS}}} &= \left(1.58 \pm 0.40\big|^{\mu} \pm 0.08\big|^{\text{n.p.}} \pm 0.02\big|^{\overline{m}} \pm 0.01\big|^{V_{cb}}\right)\ \text{ps}^{-1} \\
\Gamma_{B_c}^{\text{meson}} &= \left(1.77 \pm 0.25\big|^{\mu} \pm 0.20\big|^{\text{n.p.}} \pm 0.01\big|^{V_{cb}}\right)\ \text{ps}^{-1} \\
\Gamma_{B_c}^{\text{Upsilon}} &= \left(2.51 \pm 0.19\big|^{\mu} \pm 0.21\big|^{\text{n.p.}} \pm 0.01\big|^{V_{cb}}\right)\ \text{ps}^{-1}
\end{aligned}
$$

Massive strange quark

$$
\begin{aligned}
\Gamma_{B_c}^{\overline{\text{MS}}} &= \left(1.51 \pm 0.38\big|^{\mu} \pm 0.08\big|^{\text{n.p.}} \pm 0.02\big|^{\overline{m}} \pm 0.01\big|^{m_s} \pm 0.01\big|^{V_{cb}}\right)\ \text{ps}^{-1} \\
\Gamma_{B_c}^{\text{meson}} &= \left(1.70 \pm 0.24\big|^{\mu} \pm 0.20\big|^{\text{n.p.}} \pm 0.01\big|^{m_s} \pm 0.01\big|^{V_{cb}}\right)\ \text{ps}^{-1} \\
\Gamma_{B_c}^{\text{Upsilon}} &= \left(2.40 \pm 0.19\big|^{\mu} \pm 0.21\big|^{\text{n.p.}} \pm 0.01\big|^{m_s} \pm 0.01\big|^{V_{cb}}\right)\ \text{ps}^{-1}
\end{aligned}
$$

$(\Gamma_{B_c}^{exp} = 1.961 \pm 35\ \text{ps}^{-1})$

Possible Improvements

Higher order in α_s

To reduce μ-dependence

Higher order in v

To reduce n.p. uncertainty

Matrix elements

Lattice calculation

Outline

Novel way to determine Γ_{B_c}: Idea

JA/Grinstein: 2108.10285

Decay rates

Compute parts of $\Gamma(B)$, $\Gamma(D)$, $\Gamma(B_c)$
use $\Gamma^{exp}(B)$, $\Gamma^{exp}(D)$

Taking difference between B, D, B_c

$\Gamma(B) + \Gamma(D) - \Gamma(B_c)$

Various combinations

$(\Gamma_{B^0}, \Gamma_{D^0})$, $(\Gamma_{B^+}, \Gamma_{D^0})$, $(\Gamma_{B^0}, \Gamma_{D^+})$ and $(\Gamma_{B^+}, \Gamma_{D^+})$

Novel way to determine Γ_{B_c}

General width for meson H_Q

$$\Gamma(H_Q) = \Gamma_Q^{(0)} + \Gamma^{n.p.}(H_Q) + \Gamma^{\mathrm{WA+PI}}(H_Q) + \mathcal{O}(\frac{1}{m_Q^4})$$

Taking difference between B, D, B_c

$$\Gamma(B) + \Gamma(D) - \Gamma(B_c) = \Gamma^{n.p.}(B) + \Gamma^{n.p.}(D) - \Gamma^{n.p.}(B_c)$$
$$+\Gamma^{\mathrm{WA+PI}}(B) + \Gamma^{\mathrm{WA+PI}}(D) - \Gamma^{\mathrm{WA+PI}}(B_c)$$

Advantage

quark decay uncertainties drop out

Results

(B^0, D^0) and (B^+, D^0)

$\Gamma_{B_c} = 3.03 \pm 0.54 \text{ ps}^{-1}$

(B^0, D^+) and (B^+, D^+)

$\Gamma_{B_c} = 3.38 \pm 0.98 \text{ ps}^{-1}$

Discrepancy with experiment

$\Gamma_{B_c}^{exp} = 1.961 \pm 35 \text{ ps}^{-1}$

Possible explanations

Underestimated uncertainties

NNLO, $1/m^4$, parametric etc.

Eye graph

Not included in lattice calculation

Quark hadron duality

violated

Outline

Summary

OPE

Agreement with experiment: large scheme dependence

Improvements

NNLO, $1/m^4$, lattice results

Novel way

Discrepancy: underestimation, eye-graph, duality violation?

Spin symmetry

Noether current

$$J^\mu = u^\mu \overline{\Psi}_+ \Psi_+ + \frac{1}{2m} \overline{\Psi}_+ iD_\perp^\mu \Psi_+$$

Relation

$$\frac{1}{2M_{B_c}} \langle B_c(p) | \overline{\Psi}_+ \Gamma \Psi_+ | B_c(p) \rangle = \frac{1}{2} \mathrm{Tr} \left[\Gamma \left(\frac{1 + \not{v}}{2} \right) \right]$$

4quark MEs

$$\langle B_c(q) | (\overline{X}_-^{(b)\alpha} \gamma_\mu P_L X_-^{(b)\beta})(\overline{\Psi}_+^{(c)\beta} \gamma_\nu P_L \Psi_+^{(c)\alpha}) | B_c(q) \rangle = \frac{f_{B_c}^2 B_{B_c}}{4} \left(\frac{1}{2} q^2 g_{\mu\nu} - q_\mu q_\nu \right)$$

Uncertainties

Non-Perturbative (n.p.)

velocity expansion

scale uncertainty

μ dependence

Parametric

V_{cb} etc

Strange quark mass

$m_s \neq 0$

Error estimate

velocity expansion

v^4 terms

parametric, μ depdendence

vary parameter p, compute $\frac{\Delta\Gamma}{\Gamma}\frac{p}{\Delta p}$

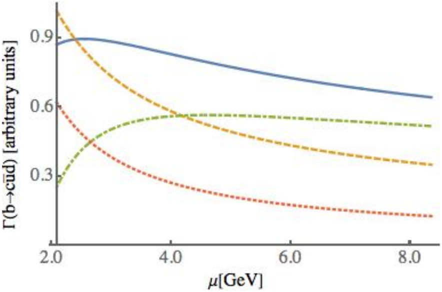
Strange quark mass

keep m_s in charm decays

Error estimate: $\frac{\Delta\Gamma}{\Gamma}\frac{p}{\Delta p}$

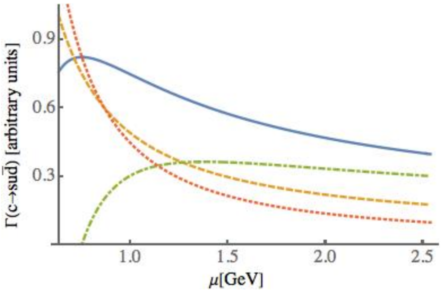
parameter p	$\Delta p/p$	$\overline{\text{MS}}$	meson	Upsilon
$\overline{m}_b(\overline{m}_b)$	0.2%	1.815	–	–
$\overline{m}_c(\overline{m}_c)$	0.3%	2.798	–	–
μ	10%	-0.359	-0.204	-0.112
T	10%	-0.029	-0.034	-0.057
$M_{B_c^*} - M_{B_c}$	6%	0.012	0.015	0.016
B_{B_c}	30%	-0.004	0.021	0.042
B'_{B_c}	30%	0.065	0.060	0.030
λ_1	50%	–	-0.011	0.017
f_{B_c}	1%	0.122	0.164	0.147
V_{cb}	1%	0.644	0.769	0.575

$\mu-$dependence: $b \rightarrow cud$



blue: LO+NLO
orange: LO
green: LO + $\alpha_s \ln(\mu)$
red: $\alpha_s \ln(\mu)$

$\mu-$dependence: $c \to sud$



blue: LO+NLO
orange: LO
green: LO + $\alpha_s \ln(\mu)$
red: $\alpha_s \ln(\mu)$

Partial rates

Mode	BB	$\overline{\text{MS}}$	meson	Upsilon
$\bar{b} \to \bar{c}u(\bar{s}+\bar{d})$	0.310	0.205	0.266	
$\bar{b} \to \bar{c}c(\bar{s}+\bar{d})$	0.137	0.093	0.122	
$\bar{b} \to \bar{c}e\nu$	0.075	0.053	0.066	
$\bar{b} \to \bar{c}\tau\nu$	0.018	0.010	0.015	
$\sum \bar{b} \to \bar{c}$	0.615	0.414	0.535	
$c \to (s+d)u(\bar{d}+\bar{s})$	0.905	0.752	0.770	1.290
$c \to (s+d)e\nu$	0.162	0.161	0.162	0.250
$\sum c \to s$	1.229	1.075	1.095	1.790
WA: $\bar{b}c \to c(\bar{s}+\bar{d})$	0.138	0.079	0.126	0.157
WA: $\bar{b}c \to \tau\nu$	0.056	0.039	0.042	0.042
PI	-0.124	-0.023	-0.024	-0.017
Γ_{B_c} $(\Gamma_{B_c}^{exp} = 1.961(35)\text{ps}^{-1})$	1.914	1.584	1.774	2.506

Branching ratios

Br(process)	$\overline{\text{MS}}$	meson	Upsilon
$b \to cu(d+s)$	13.6	15.7	11.1
$b \to cc(d+s)$	6.2	7.2	5.1
$b \to ce\nu$	3.5	3.9	2.7
$b \to c\tau\nu$	0.6	0.9	0.6
$b \to c$	27.3	31.4	22.2
$c \to su\bar{d}$	41.8	38.0	45.6
$c \to su\bar{s}$	2.1	1.9	2.4
$c \to du\bar{d}$	2.4	2.2	2.6
$c \to se\bar{\nu}$	9.4	8.4	9.2
$c \to de\bar{\nu}$	0.5	0.5	0.5
$c \to s$	66.4	60.1	70.2
$bc \to cs$	3.7	6.0	5.8
$bc \to \tau\nu$	2.6	2.5	1.8

Convergence of expansion

Example: For $\Gamma\big(B \to X_c e\nu\big)$ one finds

$1 - 0.20\epsilon - 0.20\epsilon^2 + \cdots$	(pole mass)
$1 + 0.27\epsilon + 0.09\epsilon^2 + \cdots$	($\overline{\text{MS}}$ scheme)
$1 - 0.10\epsilon - 0.03\epsilon^2 + \cdots$	(meson/Upsilon scheme)

$\epsilon =$ expansion parameter