Exclusive quarkonia decays: endpoint divergencies & color-octet contribution: a lesson from $\chi_{cJ} \to K^*(892)\bar{K}$ decays.

Nikolay Kivel

Technische Universität, München

based on N. Kivel, Eur. Phys. A 54, 2018

Motivation

Experiment: many precise data (BESII, BESIII, BABAR, BELLE, LHC)

Theory: QCD factorisation = EFT's (NRQCD, pNRQCD, SCET)

Although the spectrum of low-lying charmonium states is sufficiently well understood, their hadronic decays are still puzzling.

qualitative prediction $A[\Psi(nS) \to hh] \sim R_{n0}(0) \times a[c\bar{c} \to hh']$

this gives 13% rule $\frac{Br[\Psi(2S) \to e^+e^-]}{Br[J/\Psi \to e^+e^-]} \simeq 0.13 \approx \frac{Br[\psi(2S) \to hh']}{Br[J/\psi \to hh']}$

for some decays this works quite well $\frac{Br[\psi(2S) \to p\bar{p}]}{Br[J/\psi \to p\bar{p}]} = 0.139$

for some not $\frac{Br[\Psi(2S) \to \rho\pi]}{Br[J/\psi \to \rho\pi]} = 0.002 \ll 0.13$ $\frac{Br[\Psi(2S) \to K^{+*}\bar{K}^-]}{Br[J/\psi \to K^{+*}\bar{K}^-}] = 0.006$

well known problem with a long history ...

Chen, Braaten, PRL 80 (1998) Large effect from color-octet mechanism?

$\chi_{cJ} \to K^* \bar{K}$ decays

Branching ratios in units of 10^{-4}

$\chi_{cJ} \to VP$	$K^*(892)^0 \bar{K}^0$+c.c.	$K^*(892)^+ \bar{K}^-$+c.c.
χ_{c1}	10 ± 4	15 ± 7
χ_{c2}	1.3 ± 0.28	1.5 ± 0.22

BESII, PRD 74, 2006

BESIII, PRD 96, 2017

$Br[\chi_{c2} \to K^* \bar{K}^*] = 23 \times 10^{-4}$ $\quad$ $Br[\chi_{c2} \to \pi\pi] = 22.3 \times 10^{-4}$ (for comparison)

The amplitudes are suppressed because of

SU(3)$_f$ breaking $\quad A[\chi_{cJ} \to K^*_{\parallel,\perp} \bar{K}] \sim m_s - m_{u,d}$

QCD helicity selection rule: $\quad A[\chi_{cJ} \to K^*_{\perp} \bar{K}]$

$\chi_{c1} \to K K^*_{\parallel,\perp}$ $\quad$ $A[\chi_{c1} \to \bar{K} K^*] = (\epsilon_\chi \cdot k)\,(e^*_V \cdot k)\,\frac{m_V}{M^2} \mathcal{A}_1^{\parallel} + (\epsilon_{\chi\perp} \cdot e^*_{V\perp})\,\frac{(kP)}{M} \mathcal{A}_1^{\perp}$

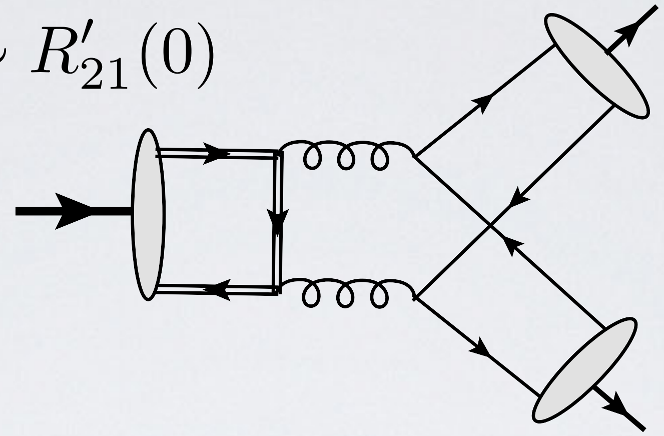
$\chi_{c2} \to K K^*_{\perp}$ $\quad$ $A[\chi_{c2} \to \bar{K} K^*] = \epsilon_\chi^{\mu\nu} k_\nu i\varepsilon_{\mu\alpha\beta\rho} (e^*_V)^\alpha \frac{k^\beta p^\rho}{(kp)} \mathcal{A}_2^{\perp}$

QCD counting: $\quad \mathcal{A}_1^{\parallel} \sim v^4 \left(\frac{\Lambda}{m_c}\right)^2 \quad \mathcal{A}_{1,2}^{\perp} \sim v^4 \left(\frac{\Lambda}{m_c}\right)^3 \quad m_c \gg \Lambda$

Decay amplitude $A[\chi_{c1} \to K^*_\parallel K]$

$A[\chi_{cJ} \to K^*_\parallel \bar{K}]$

$\sim R'_{21}(0)$



$\phi_{K^*_\parallel}$ LCDA, twist-2

ϕ_K LCDA, twist-2

$$A[\chi_{c1} \to K^*_\parallel K] \sim \frac{R'_{21}(0)}{m_c^{5/2}} \frac{f_V^\parallel f_P}{m_c^2} \alpha_s^2 \int_0^1 dx \, \frac{\phi_{K^*_\parallel}(x)}{x(1-x)} \int_0^1 dy \, \frac{\phi_K(y)}{y(1-y)} \frac{y-x}{xy+(1-x)(1-y)}.$$

IR finite expression

$x \to 1-x \; : \; \phi(x) \to \phi(1-x) \;\; \Leftrightarrow \;\; q \leftrightarrow \bar{q}$

SU(3)$_f$ breaking $\quad K(q\bar{s}) \quad \phi(x) \neq \phi(1-x)$ $\qquad \phi(x) - \phi(1-x) \sim m_s - m_q$

Power counting in EFT:

$R'_{21}(0) \sim v^4 \qquad \frac{f_V^\parallel f_P}{m_c^2} \sim \frac{\Lambda^2}{m_c^2} \qquad A[\chi_{c1} \to K^*_\parallel K] \sim v^4 (\Lambda/m_c)^2$

Branching ratio $Br[\chi_{c1} \to K^*_{\parallel} K]$

$$\Gamma[\chi_{c1} \to \bar{K} K^*] = \frac{|\vec{k}|}{8\pi} \frac{2}{3} \frac{k_0^2}{M^2} \left(\cancel{\left|\mathcal{A}_1^{\perp}\right|^2} + \frac{1}{2} \frac{(pk)^2}{M^4} \left|\mathcal{A}_1^{\parallel}\right|^2 \right)$$

Estimate: $Br[\chi_{c1} \to K^*_{\parallel} K] \simeq (0.2\text{–}0.6) \times 10^{-4}$

experiment: $Br[\chi_{c1} \to K^* K] \simeq (10 \pm 4/15 \pm 7) \times 10^{-4}$

This calculation strongly underestimates the data.

Large effect from the subleading amplitude $\chi_{c1} \to K K^*_{\perp}$?

notice that $Br[\chi_{c2} \to K^*_{\perp} K] \simeq (1.3 \pm 0.3/1.5 \pm 0.2) \times 10^{-4}$

is also relatively large

Decay amplitudes $A[\chi_{cJ} \to K_\perp^* K]$

N.K. Eur. Phys. A 54, 2018

$$\mu_P = m_K^2/(m_s + m_q)$$

$$A[\chi_{cJ} \to K K_\perp^*] \sim \frac{R'_{21}(0)}{m_c^{5/2}} \frac{f_V^\perp f_P \mu_P}{m_c^3} \alpha_s^2 \int_0^1 dx \frac{\phi_{K_\perp^*}(x)}{x\bar{x}} \int_0^1 dy\, F(x,y)$$

the integral is IR singular

endpoint region $x \to 1,\ y \to 0$

$$\mu_P \int_{1-\delta}^1 dx \frac{\phi_{K_\perp^*}(x)}{x\bar{x}} \int_0^\delta dy\, F(x,y) \sim (m_s - m_q)\phi'_{K_\perp^*}(1) \int_{1-\delta}^1 dx \int_0^\delta dy \frac{1 + \ln y}{[y + (1-x)]^2}$$

$$\sim (m_s - m_q)\phi'_{K_\perp^*}(1) \int_0^\delta dy \frac{\ln y}{y}$$

double log!

! $\delta \to 1$

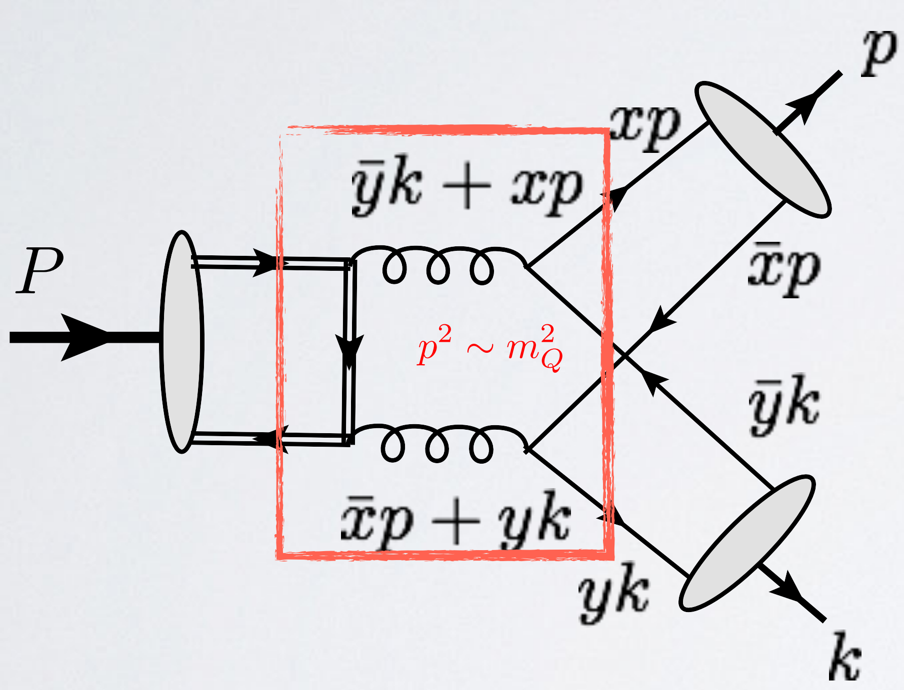
same for region $x \to 0,\ y \to 1$

$\Rightarrow$ the collinear factorisation scheme is not well defined!

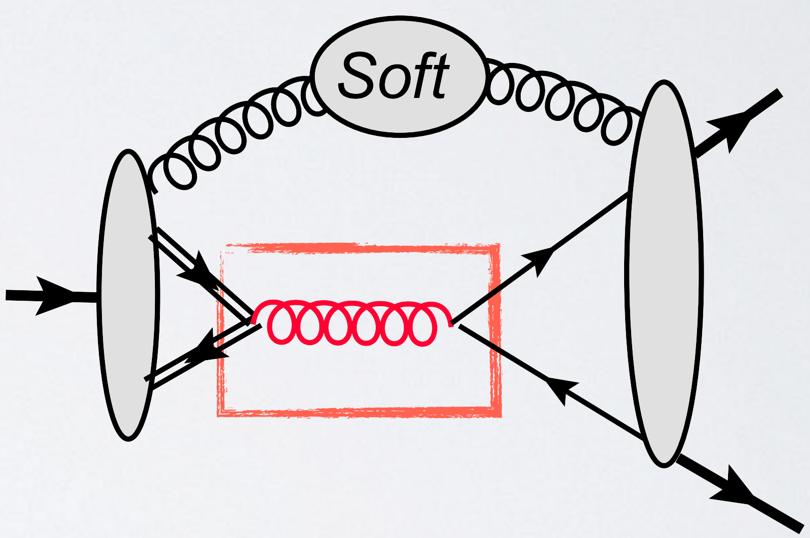
Endpoint domain as colour-octet contribution

$$\mathcal{A}^{\perp}_{1,2} \sim v^4 \left(\frac{\Lambda}{m_c}\right)^3$$

factorisable (colour singlet)



non-factorisable (color-octet)



Factorisation for decay ampplitudes $A[\chi_{cJ} \to K_\perp^* K]$

Proposal for the factorisation

$$A[\chi_{c1} \to K_\perp^* K] = \quad R'_{21}(0) \;\; \phi_{K^*} * \alpha_s^2 T_1 * \phi_K + \langle K_\perp^* K | \alpha_s C_h (\bar{Q}Q)_8 (\bar{q}q)_8 | \chi_{c1} \rangle$$

$$A[\chi_{c2} \to K_\perp^* K] = \quad R'_{21}(0) \;\; \phi_{K^*} * \alpha_s^2 T_2 * \phi_K \;\; + \langle K_\perp^* K | \alpha_s C_h (\bar{Q}Q)_8 (\bar{q}q)_8 | \chi_{c2} \rangle$$

singlet (IR div.) octet (UV div.)

IR div's in color-singlet must match UV-div's in the color-octet

How to verify this ?

Heavy Quark Spin Symmetry: does it works for the soft-overlap matrix elements?

if yes: $\langle K_\perp^* K | \alpha_s C_h (\bar{Q}Q)_8 (\bar{q}q)_8 | \chi_{c1} \rangle \overset{!}{=} \lambda \langle K_\perp^* K | \alpha_s C_h (\bar{Q}Q)_8 (\bar{q}q)_8 | \chi_{c2} \rangle$

⬇

HQS-symmetry breaking corr's

$$A[\chi_{c1} \to K_\perp^* K] = \lambda \, A[\chi_{c2} \to K_\perp^* K] + R'_{21}(0) \, \phi_{K^*} * \alpha_s^2 (T_1 - \lambda T_2) * \phi_K$$

$\phi_{K^*} * \alpha_s^2 (T_1 - \lambda T_2) * \phi_K$ IR-singularities must cancel in the difference!

Theoretical LAB: octet m.e. in the Coulomb limit

Charmonium & hadronic

$m_c \gg m_c v \gg m_c v^2 \sim \Lambda$

ultrasoft d.o.f. are non-perturbative

end-point region $x \sim \Lambda/m_c$

NRQCD + SCET

"nonfactorisable"

$\langle K_\perp^* K | \alpha_s C_h (\bar{C}C)_8 (\bar{q}q)_8 | \chi_{cJ} \rangle$

->

Coulomb limit $m_Q \to \infty$ Λ is fixed

$m_Q \gg m_Q v \gg m_Q v^2 \gg \Lambda$

ultrasoft d.o.f. are perturbative

end-point region $x \sim v^2 \gg \Lambda/m_c$

final hadronic state can be described in terms of LCDAS (collinear factorisation)

"factorisable" because $v^2 \gg \Lambda/m_Q$

$\langle K_\perp^* K | \alpha_s C_h (\bar{Q}Q)_8 (\bar{q}q)_8 | \chi_{QJ} \rangle$

the overlap mechanism of singlet & octet contributions can be studied in pQCD

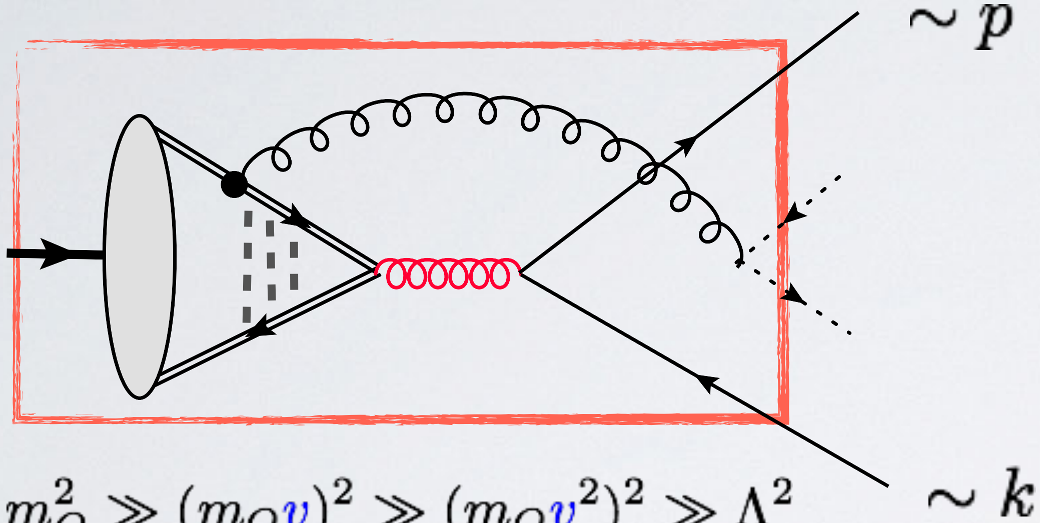
This idea is already used for analysis $B \to K + \text{charmonia}$

Beneke, Vernazza, NPB, 2009

Octet matrix element in the Coulomb limit

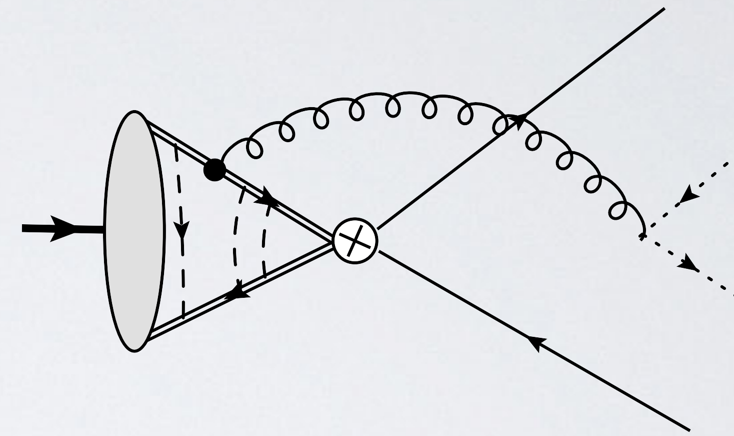
Coulomb: $v^2 \gg \Lambda/m_Q$ $\langle K_\perp^* K|\alpha_s C_h(\bar{Q}Q)_8(\bar{q}q)_8|\chi_{QJ}\rangle \sim v^4(\Lambda/m_Q)^3$

QCD



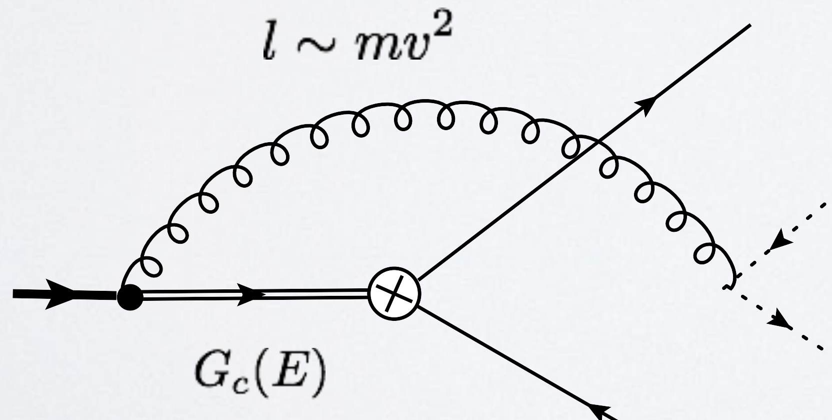
$m_Q^2 \gg (m_Q v)^2 \gg (m_Q v^2)^2 \gg \Lambda^2$

NRQCD



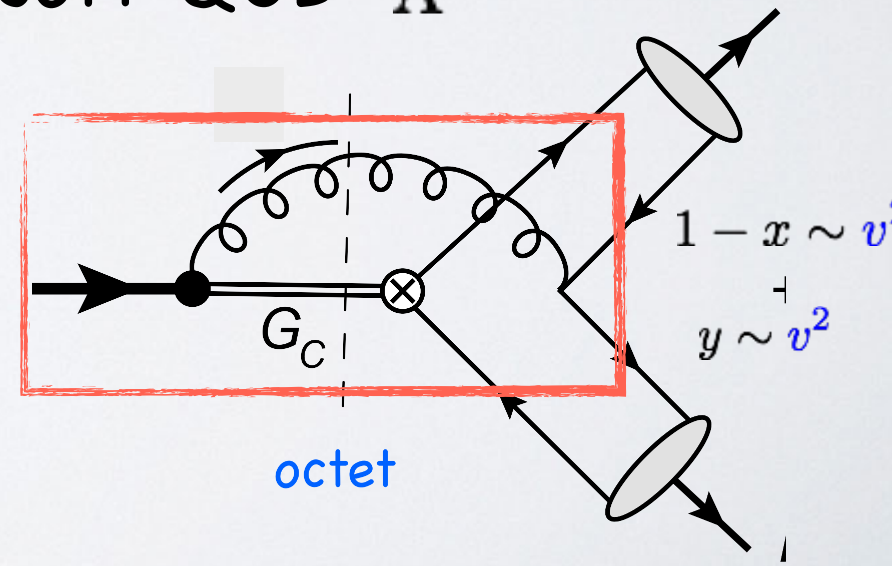
$(m_Q v)^2 \gg (m_Q v^2)^2 \gg \Lambda^2$

pNRQCD

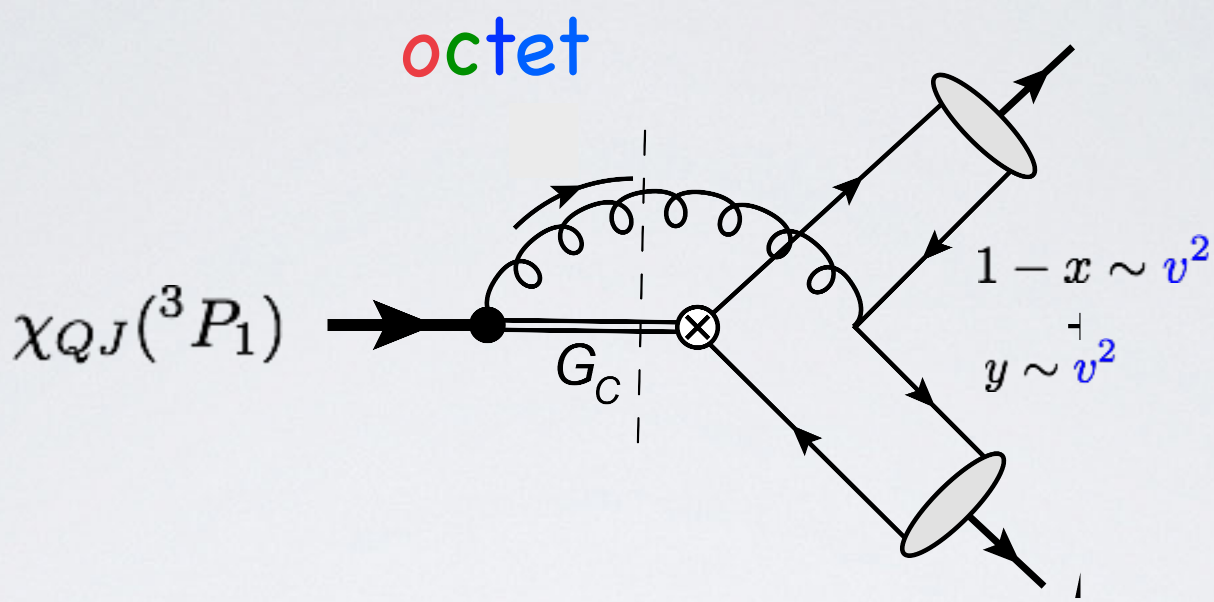


$(m_Q v^2)^2 \gg \Lambda^2$

soft QCD Λ^2



Octet matrix element in the Coulomb limit



$$\mathcal{A}_J^8\big|_{\rm UV} \sim \alpha_s(m_Q v^2)\alpha_s(m_Q)\,\underline{(-1)^J 2^{J/2}} \int d^3\Delta\, \tilde{R}^c_{21}(\Delta)\,\Delta$$

$$\times (m_s - m_q)\phi'_{K^*_\perp}(1) \underline{\int_0^\infty dx \int_0^\infty dy\, \frac{(1+\ln y)}{[E - m_Q(y+x) - \Delta^2/m_Q + i\varepsilon]^2}}$$

$\bar{x} \to x$ $\qquad$ $x \sim y \sim v^2$

IR finite!

Octet matrix element in the Coulomb limit

UV-limit: octet amplitudes in the endpoint region

$$\mathcal{A}_J^8\big|_{\rm UV} \sim \alpha_s(m_Q v^2)\alpha_s(m_Q)\,\underline{(-1)^J 2^{J/2}} \int d^3\Delta\, \tilde{R}^c_{21}(\Delta)\,\Delta \nearrow R'_{21}(0)$$

$$\times (m_s - m_q)\phi'_{K^*_\perp}(1) \int_0^\infty dx \int_0^\infty dy\, \frac{(1+\ln y)}{[\not{E} - m_Q(y+x) - \not{\Delta^2/m_Q} + i\varepsilon]^2}$$

$$x \sim y \to \infty$$

IR-limit: singlet amplitudes in the endpoint region

$$A^0[\chi_{cJ} \to K^*_\perp K] \sim R'_{21}(0)\alpha_s^2\,(m_s - m_q)\phi'_{K^*_\perp}(1) \int_{1-\delta}^1 dx \int_0^\delta dy\, \frac{1+\ln y}{[y+(1-x)]^2}$$

$\Longrightarrow$ UV and IR div's match each other

Phenomenology $\chi_{cJ} \to K^* K$

$$\lambda = -\frac{1}{\sqrt{2}}$$

HQSS breaking corr's ΔA

$$A[\chi_{c1} \to K^*_\perp K] = \lambda\, A[\chi_{c2} \to K^*_\perp K] + R'_{21}(0)\ \phi_{K^*} * \alpha_s^2 (T_1 - \lambda T_2) * \phi_K$$

IR-div's cancel as it must be

$|\mathcal{A}_2^\perp| = (7.0 \pm 1.5) \times 10^{-3}$ can be fixed from data $\chi_{c2} \to K K^*_\perp$

$\Delta A = (-1.6 \pm 0.2) \times 10^{-3}$ HQSS breaking corr.'s, real

Neglecting HQSS breaking corr's

$$A[\chi_{c1} \to K^*_\perp K] \simeq -\frac{1}{\sqrt{2}} A[\chi_{c2} \to K^*_\perp K]$$

then $R_{\rm th} = \dfrac{\Gamma[\chi_{c2} \to K^* K]}{\Gamma[\chi_{c1} \to K^* K]} \simeq 0.55$ $R_{exp} = 0.30 \pm 0.13$

BESIII, PRD 96, 2017

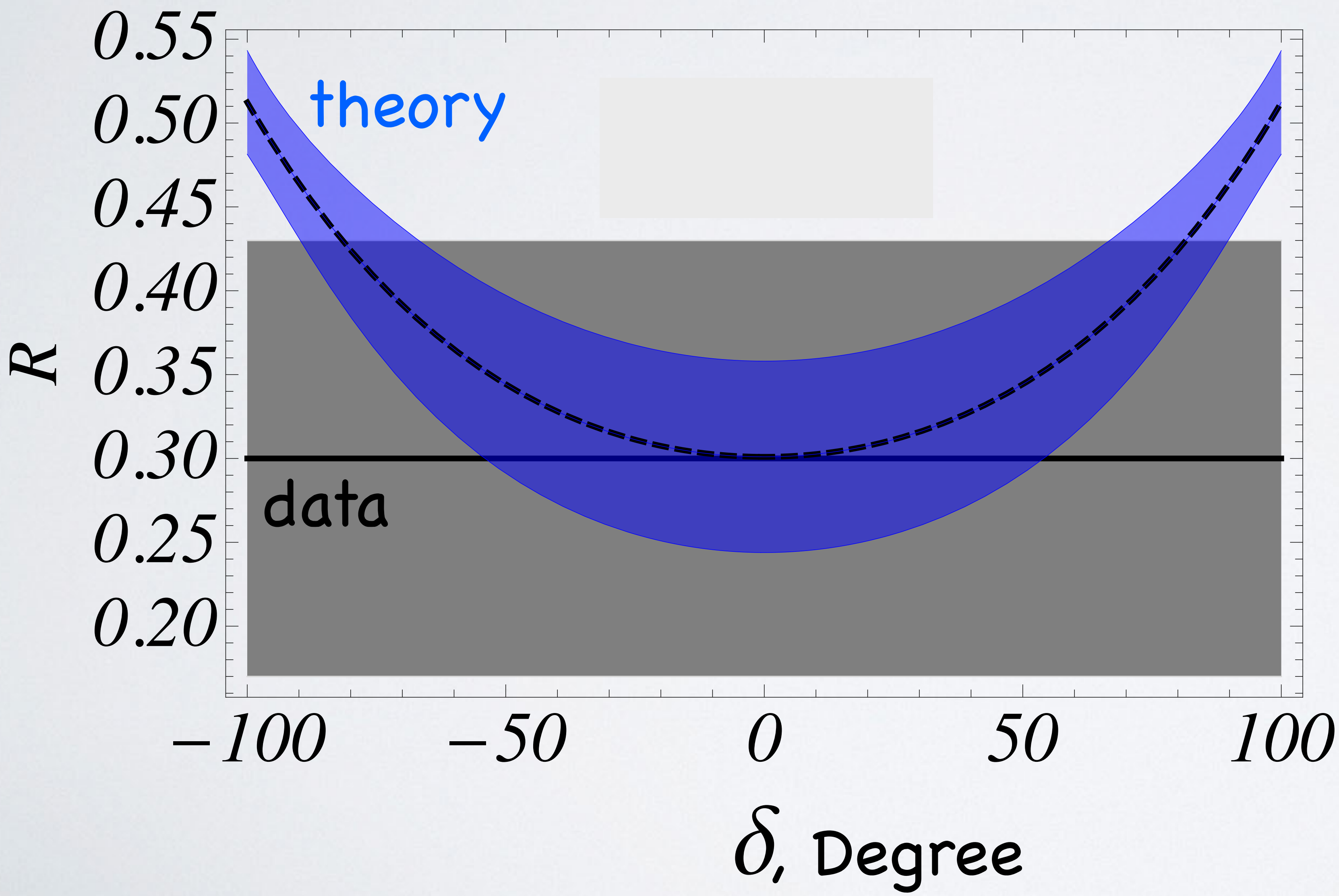
BESII, PRD 74, 2006

Phenomenology $\chi_{cJ} \to K^* K$

$\mathcal{A}_2^{\perp} = |\mathcal{A}_2^{\perp}| e^{i\delta}$ $\quad |\mathcal{A}_2^{\perp}| = (7.0 \pm 1.5) \times 10^{-3}$ $\quad \Delta A = (-1.6 \pm 0.2) \times 10^{-3}$ real

unknown phase

$$R_{\rm th} = \frac{\Gamma[\chi_{c2} \to K^* K]}{\Gamma[\chi_{c1} \to K^* K]} \simeq \frac{0.55}{1 + 0.65 \cos\delta + 0.10}$$



The interference of $A_2^{\perp}$ and ΔA is numerically important

However large exp. errors don't allow one to get a reliable estimate for the phase δ

Conclusions

The octet contribution naturally arises if the singlet one is IR-divergent

In exclusive amplitudes these IR div.'s are the endpoint collinear singularities (collinear factorisation is violated by soft overlap)

The octet m.e.'s are not factorisable but HQSS can relate them that is helpful for a phenomenological analysis

Obviously, octet contribution is more sensitive to the long distance part of the quarkonia wave function and can be considered as a potential dynamical source violating of the "13% rule".

Phenomenology indicates that the octet contribution gives probably dominant contribution in $\chi_{cJ} \to K^* \bar{K}$

Can it help to understand $\rho\pi$-puzzle? The work is in progress ...

Thank you!

Additional slides

Endpoint domain as colour-octet contribution

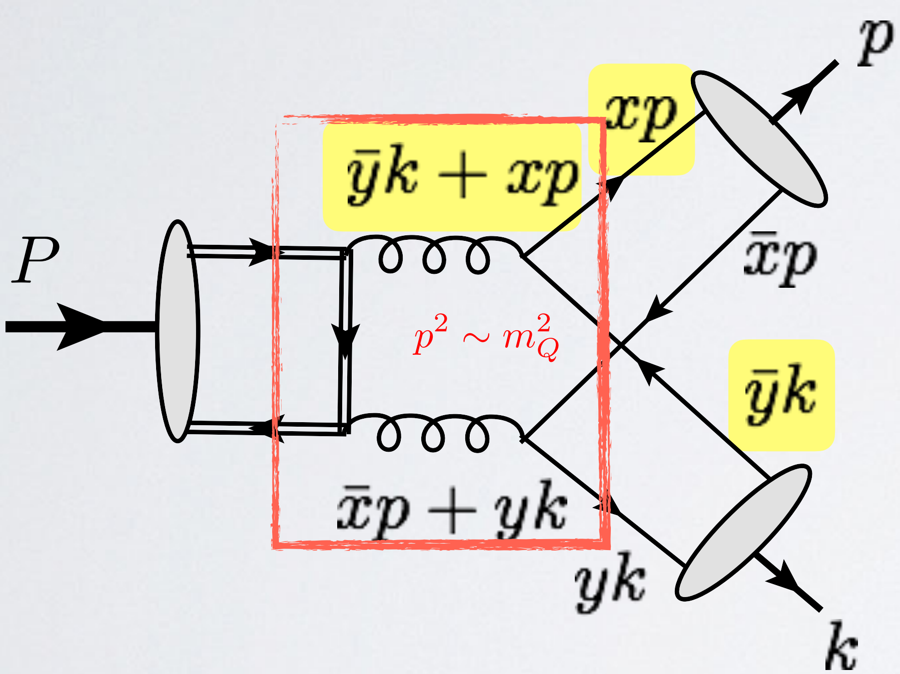
collinear region

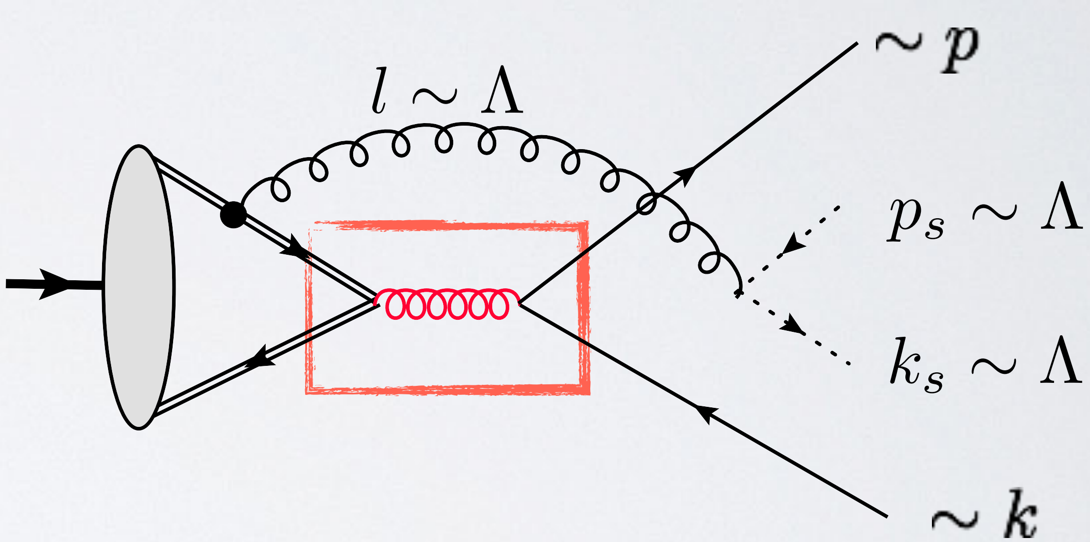
$0 \ll x \ll 1, 0 \ll y \ll 1$

$\bar{x} \equiv 1 - x$

endpoint region $x \to 0,\, y \to 1$

in reality $x \sim 1 - y \sim \Lambda/m_c$





usoft exchange between all sectors!

$$\mathcal{A}_{1,2}^{\perp} \sim v^4 \left(\frac{\Lambda}{m_c}\right)^3$$

NRQCD $l \sim m_Q v^2 \quad \Rightarrow \quad v^2 \sim \Lambda/m_Q$

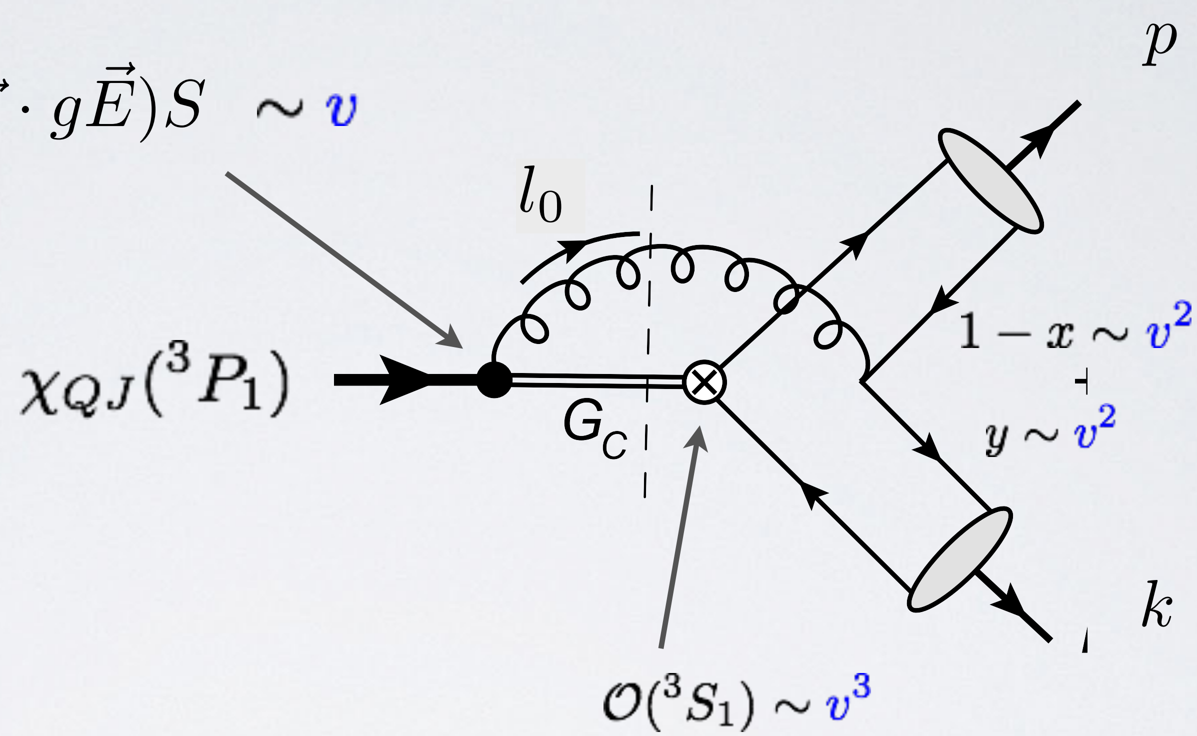
consistent power counting

Octet matrix element in the Coulomb limit

octet $\langle K^*_\perp K | \alpha_s C_h (\bar{Q}Q)_8 (\bar{q}q)_8 | \chi_{QJ} \rangle \sim v^4 (\Lambda/m_Q)^3$

$$\mathcal{L}_{int} = \int d\vec{r}\, O^\dagger (\vec{r}\cdot g\vec{E}) S \sim v$$

does not depend on HQ spin!



$l_0 = \bar{x}p_0 + yk_0$
$= m_Q(\bar{x} + y)$

$$\langle K^*_\perp K | \ldots | \chi_{QJ} \rangle \sim \alpha_s(m_Q v^2)\alpha_s(m_Q) \int d^3\Delta\, \tilde{R}^c_{21}(\Delta)\, \mathrm{tr}\,[\mathcal{P}_J \gamma_\alpha]\, V_{us}(\Delta)$$

$$\times \int_{1-\delta}^{1} dx \int_0^\delta dy \int d^3\Delta' G_c(\vec{\Delta}, \vec{\Delta}', E - l_0,) \quad [(m_s - m_q)(1 + \ln y)\phi'_{K^*}(1)]$$

$\delta \to \infty \qquad y \sim v^2 \qquad 1 - x \sim v^2$

Octet matrix element in the Coulomb limit

$$\langle K_\perp^* K|\ldots|\chi_{QJ}\rangle \sim \alpha_s(m_Q v^2)\alpha_s(m_Q)\int d^3\Delta\ \tilde{R}^c_{21}(\Delta)\,\underline{\mathrm{tr}\left[\mathcal{P}_J\gamma_\alpha\right]}\,V_{us}(\Delta)$$

$$\times\int_{1-\delta}^{1}dx\int_0^{\delta}dy\int d^3\Delta' G_c(\vec{\Delta},\vec{\Delta}',E-l_0,)\ \ \left[(m_s-m_q)(1+\ln y)\phi'_{K^*}(1)\right]$$

$\xrightarrow{\text{UV-limit}}$ $\delta\to\infty \quad y\sim v^2 \quad 1-x\sim v^2 \quad l_0=m_Q(\bar{x}+y)$

The Green function

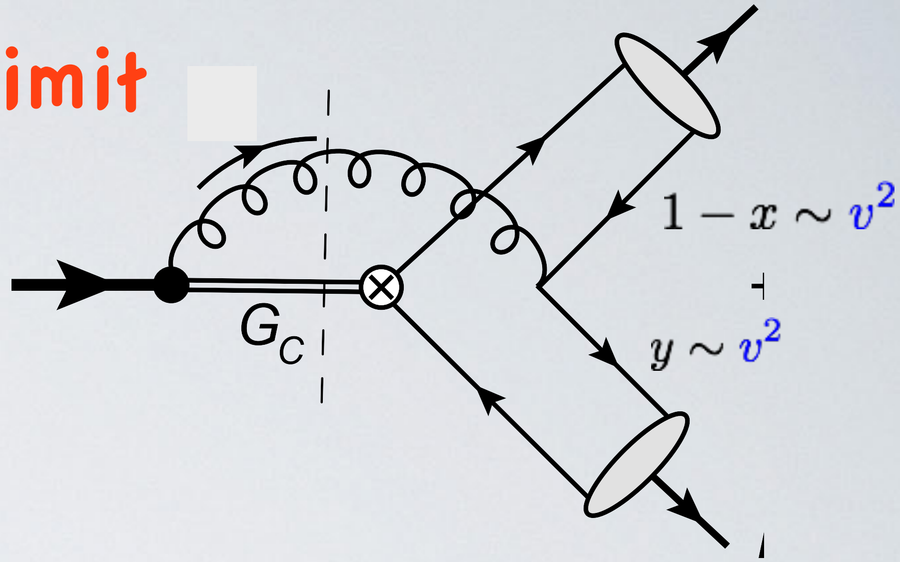
$$G_c(\vec{\Delta},\vec{\Delta}',E-l_0,)=-(2\pi)^3\frac{\delta(\vec{\Delta}-\vec{\Delta}')}{E-l_0-\vec{\Delta}^2/m_Q}+\underline{G_c^{(int)}(\vec{\Delta},\vec{\Delta}',E-l_0)}$$

gives UV-regular terms only

Obvious simplification

$$V_{us}(\Delta)\int d^3\Delta' G_c(\vec{\Delta},\vec{\Delta}',E-l_0,)\sim\frac{1}{[E-m_Q(\bar{x}+y)-\vec{\Delta}^2/m_Q]^2}$$

Octet matrix element in the Coulomb limit



UV finite Im phase from the in pole G_C

$$\mathcal{A}^8_J\big|_{\rm UV} \sim \int d^3\Delta \tilde{R}^c_{21}(\Delta) \int_0^\infty dx \int_0^\infty dy \frac{1+\ln y}{[E-\Delta^2/m_Q - m_Q(x+y)+i\varepsilon]^2}$$

$$\sim -i\pi \int d^3\Delta \tilde{R}^c_{21}(\Delta)(1+\ln[E-\Delta^2/m_Q])\underline{\theta(E>\Delta^2/m_Q)}$$

Im part is UV-finite as it must be

Im part is sensitive to the region $\vec{\Delta}^2 < m_Q E$

$$\tilde{R}^c_{21}(\Delta) = R'_{21}(0)\frac{16\pi\gamma_B\Delta}{(\Delta^2+\gamma_B^2/4)^3} \qquad \gamma_B = \frac{1}{2}m_Q\alpha_s C_F$$