ON X(3872) COMPOSITION

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Based on:
Esposito, Glioti, ADP, Rattazzi, [in preparation]
Esposito, Maiani, Pilloni, ADP, Riquer *Phys. Rev. D* 105 (2022) 3, L031503
ADP, *Phys. Lett. B* 746 (2015) 248-250

FEATURES OF $X(3872)$

1. $X(3872)$ has a mass almost equal to $m_D + m_{D^*}$
2. It is a narrow state ≈ 1MeV (?)
3. Its strong decays in $J/\psi\,\rho$ and $J/\psi\,\omega$ violate isospin
4. It is produced in prompt hadron collisions with very high cross section and hard p_T cuts
5. It has been found in the X^0 neutral charge state only (for the moment?)

Some interpretations given over the years:
Compact tetraquark, DD^* **hadron molecule (deuson)**, kinematical effect, hadrocharmonium, standard cahrmonium, (Georgi) unparticle

DEUTERONS & `DEUSONS`

Is there a way to tell from data if the deuteron is **elementary** (compact six quarks) or **composite** (a pn molecule)?

The *effective range* from np scattering amplitude is a discriminating observable [Weinberg '65].

For the $X(3872)$ mesonic deuteron there is no way of performing $D\bar{D}^*$ scatterings, but the resonance lineshape is well studied experimentally, and it encodes r_0.

ELEMENTARY AND COMPOSITE DEUTERON

The physical normalized $|d\rangle$ deuteron state is

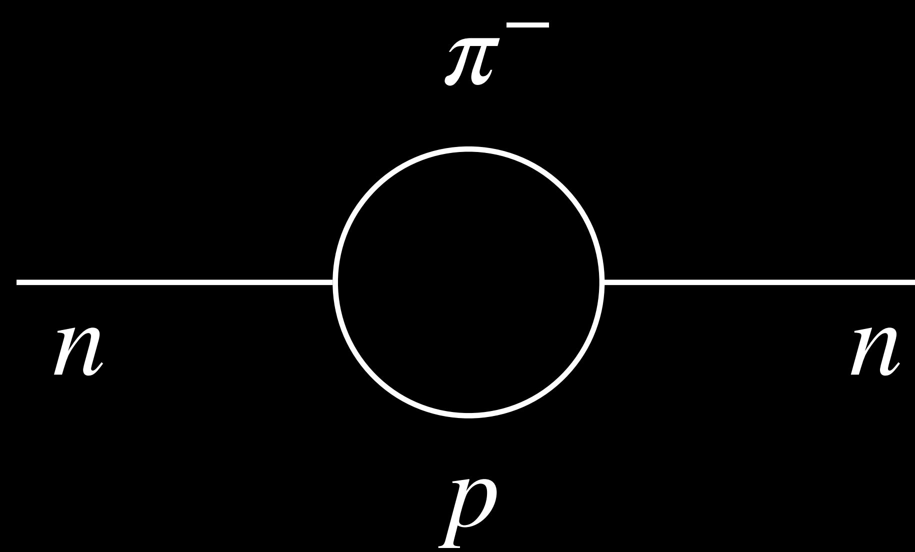
$$|d\rangle = \sqrt{Z}\,|\mathfrak{d}\rangle + \int d\mathbf{k}\, C_{\mathbf{k}}\,|np(\mathbf{k})\rangle$$

with one elementary deuteron state $|\mathfrak{d}\rangle$ — not physical.

$$\int d\mathbf{k}\,|\mathbf{C}_{\mathbf{k}}|^{2} = \mathbf{1} - \mathbf{Z}$$

See Weinberg Phys. Rev. 137, B672 (1965)

THE NEUTRON IN LEE MODEL



$$|n, \mathrm{in}\rangle = \sqrt{Z}\,|n, \mathrm{bare}\rangle + \int_{\mathbf{k}} \Psi_\pi(\mathbf{k})\,|p\,\pi^-(\mathbf{k})\rangle$$

$$\int_{\mathbf{k}} |\Psi_\pi(\mathbf{k})|^2 = 1 - Z$$

See the “Lee-model” (’54) in Henley & Thirring, Elementary Quantum Field Theory, McGraw-Hill
T.D. Lee, Phys. Rev. 95, 1329 (1954)

$|\mathfrak{d}\rangle$ not a physical state — in the interacting theory

$$|d\rangle = \underbrace{\sqrt{Z}\,|\mathfrak{d}\rangle + \sqrt{Z'}\int d\mathbf{k}\, C_{\mathbf{k}}\,|np(\mathbf{k})\rangle}_{\sqrt{\alpha}|D\rangle} + \sqrt{\beta}\,|B\rangle$$

$|D\rangle = |D_{\rm in}\rangle$ like in the Lee Model

$$\alpha^2 + \beta^2 = 1$$

If you have access to Z: $\beta \leq 1 - Z$

at any rate, if $Z \neq 1$ we would get 2 states, orthogonal

$|D\rangle \to |\mathfrak{d}\rangle$ in the limit of free theory

THE MEANING OF Z

The case $Z = 1$ is somewhat special.
The statement made about an elementary deuteron is

We have an elementary $\mathfrak{d}$ for **every** value of Z such that $0 < Z < 1$. If $Z = 0$ we speak of a molecule.

Is it possible then to extract Z from data?

See Weinberg Phys. Rev. 137, B672 (1965)

WEINBERG & DEUTERON (1965)

Weinberg finds, for *shallow bound states*, a relation between Z and the *effective range* r_0 (with no $|B\rangle$)

$$r_0 = -\frac{Z}{1-Z}R + O(\frac{1}{m_\pi})$$

$$R = \frac{1}{\sqrt{2mB}}$$

The “molecule” has $Z = 0$ thus $r_0 = O(1/m_\pi)$.

What is the sign of the unknown corrections?

See Weinberg Phys. Rev. 137, B672 (1965)

A THEOREM ON SHALLOW BOUND STATES IN QM

BETHE ('49), LANDAU-SMORODINSKY ('48)

$$\mathbf{r_0 > 0}$$

(indeed $r_0 = +1.74$ fm for deuteron)

Shallow bound states with purely attractive binding force always give positive r_0

From Weinberg's paper

"An elementary deuteron would have $0 < Z < 1$"

"The true token that the deuteron is composite is an r_0 small and positive rather than large and negative "

"…an elementary deuteron would entail a large and negative r_0**"**

Esposito et al. 2108.11413

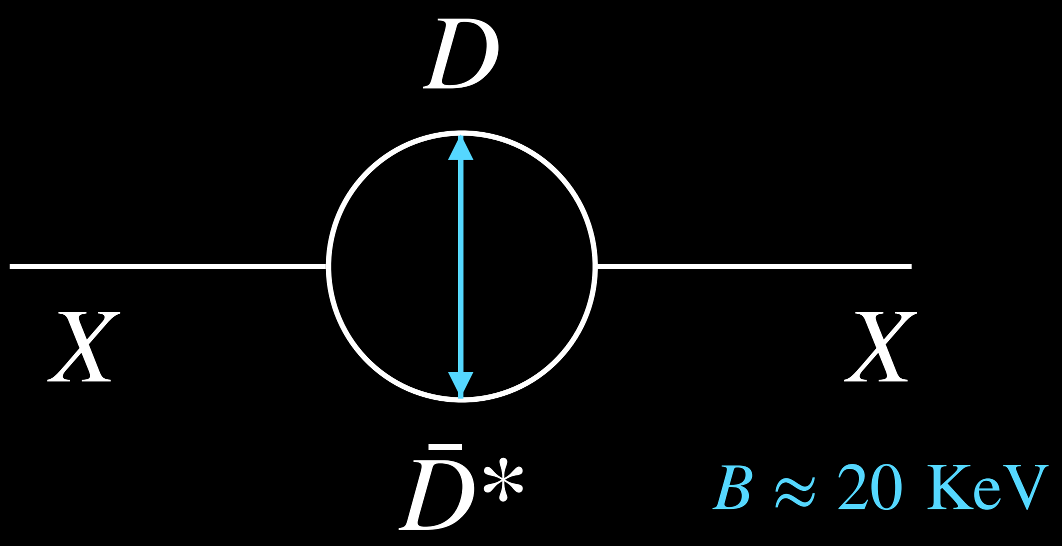
LHCB (2020)

arXiv:2005.13419

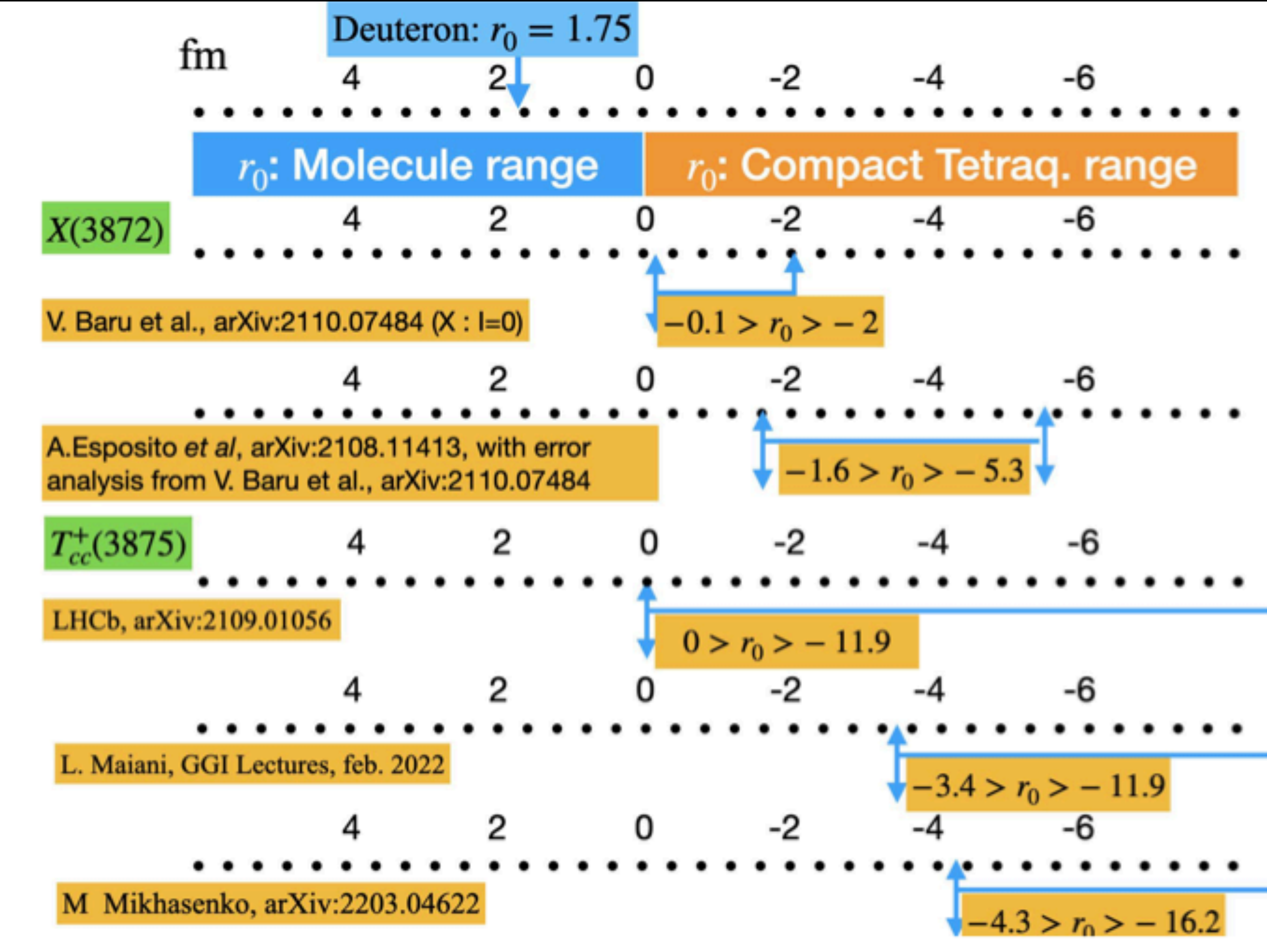
Allows to compute the effective range r_0 for the $X(3872)$. This was dubbed as the "deuson", a $D\bar{D}^*$ mesonic molecule analogue of deuteron: a viable option **iff** $Z = 0$ or $r_0 > 0$ and $O(1/m_\pi)$.

However we find $r_0 = -5.43$ fm and $|r_0| > 1/m_\pi$!

"…an elementary deuteron would entail a large and negative r_0"



Esposito et al. 2108.11413



Deuteron: $r_0 = 1.75$
fm
4 2 0 -2 -4 -6
r_0: Molecule range
r_0: Compact Tetraq. range
$X(3872)$
4 2 0 -2 -4 -6
V. Baru et al., arXiv:2110.07484 (X : I=0)
$-0.1 > r_0 > -2$
4 2 0 -2 -4 -6
A.Esposito *et al*, arXiv:2108.11413, with error analysis from V. Baru et al., arXiv:2110.07484
$-1.6 > r_0 > -5.3$
$T_{cc}^{+}(3875)$
4 2 0 -2 -4 -6
LHCb, arXiv:2109.01056
$0 > r_0 > -11.9$
4 2 0 -2 -4 -6
L. Maiani, GGI Lectures, feb. 2022
$-3.4 > r_0 > -11.9$
4 2 0 -2 -4 -6
M Mikhasenko, arXiv:2203.04622
$-4.3 > r_0 > -16.2$

THIS INTERPRETATION OF RESULTS IS DBATED.

See C. Hanhart et al. 2110.07484 — isospin breaking treatment.

Measuring a `small` Z (even if for X and the new tetraquark that is $14\% \div 30\,\%$) means that the state is essentially a molecule and marginally a compact quark state.

We would rather say that if $Z \neq 0$, then there is an elementary X **bare** field (which comes with its **renormalization constant**).

FEATURES OF $X(3872)$

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2. It is a narrow state ≈ 1MeV (?)
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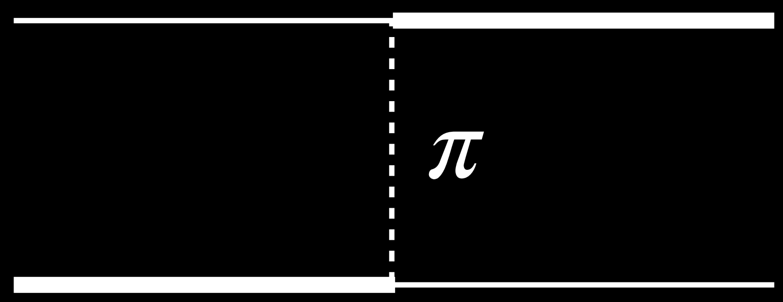
4. It is produced in prompt hadron collisions with very high cross section and hard p_T cuts
5. It has been found in the X^0 neutral charge state only, for the moment (?)
6. $m_{D*} - m_D \simeq m_\pi$

π–EXCHANGE IN X



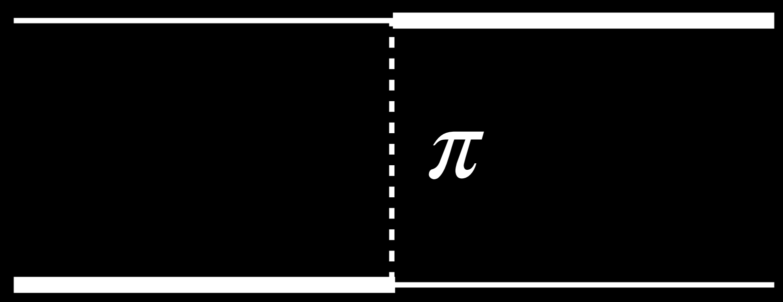
$$\int \frac{q_i q_j \, e^{i\mathbf{q}\cdot\mathbf{r}}}{q^2 + m_\pi^2 - i\epsilon} d^3q = \int \frac{q_i q_j \, e^{i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2 - \mu^2 - i\epsilon} d^3q \simeq \int \frac{q_i q_j \, e^{i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2 - i\epsilon} d^3q = \nabla_i \nabla_j \int \frac{e^{i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2 - i\epsilon} d^3q$$

$$\mu^2 = (m_{D*} - m_D)^2 - m_\pi^2 \approx 40 \text{ MeV}$$

$1/r^3$ potentials do not have bound states

[Esposito, Glioti, ADP, Rattazzi, work in progress]

π–EXCHANGE IN X



$$\int \frac{q_i q_j\, e^{i\mathbf{q}\cdot\mathbf{r}}}{q^2 + m_\pi^2 - i\epsilon}\, d^3q = \int \frac{q_i q_j\, e^{i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2 - \mu^2 - i\epsilon}\, d^3q \simeq \int \frac{q_i q_j\, e^{i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2 - i\epsilon}\, d^3q = \nabla_i \nabla_j \int \frac{e^{i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2 - i\epsilon}\, d^3q$$

$$\mu^2 = (m_{D*} - m_D)^2 - m_\pi^2 \approx 40 \text{ MeV}$$

However the mass of the pion has disappeared.

What is the contribution to r_0 from pion exchange?

[Esposito, Glioti, ADP, Rattazzi, work in progress]

π–EXCHANGE IN X

Given that the potential is the FT of the propagator in the no-recoil approximation

$$\frac{g^2}{2f_\pi^2}\int \frac{q_i q_j\, e^{i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2-\mu^2-i\epsilon}\,\frac{d^3q}{(2\pi)^3} = \frac{g^2}{6f_\pi^2}\left(\delta^3(r)+\mu^2\frac{e^{i\mu r}}{4\pi r}\right)\delta_{ij}$$

where we used in S-wave

$$\langle n_i n_j\rangle = \frac{1}{3}\delta_{ij}$$

to be contracted with polarizations $e_i^{(\alpha)}(p_1)\ \bar{e}_j^{(\beta)}(k_2)$

[Esposito, Glioti, ADP, Rattazzi, work in progress]

π–EXCHANGE IN X

Treat this potential as a perturbation in non-relativistic quantum mechanics.

$$\frac{g^2}{2f_\pi^2}\int \frac{q_i q_j\, e^{i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2-\mu^2-i\epsilon}\,\frac{d^3q}{(2\pi)^3} = \underbrace{\frac{g^2}{6f_\pi^2}}_{\alpha}\left(\delta^3(r)+\mu^2\frac{e^{i\mu r}}{4\pi r}\right)\delta_{ij}$$

The unperturbed potential is a **binding $\delta^3(r)$ potential**, responsible for the **molecular state**.

[Esposito, Glioti, ADP, Rattazzi, work in progress]

DISTORTED WAVE BORN APPROX.

$$V = V_1 + V_2$$

$$A = \frac{1}{k\cot\delta - ik} = \frac{e^{i\delta_1}\sin\delta_1}{k} + \frac{e^{i\delta_2}\sin\delta_2}{k}$$

$$\simeq \frac{1}{-1/a_s - ik} - 4\frac{2m}{4k^2}\int V_2(r)\,\chi_1^2(r)\,dr$$

$r_0 = 0$

here χ_1 are the eigenf. of the (strong) $\delta^3(r)$ potential

[Esposito, Glioti, ADP, Rattazzi, work in progress]

DWBA

r_0 is determined by the k^2 **coefficient** in the expansion around $k = 0$ (and $\alpha = 0$) of this expression

$$\left(\frac{1}{-1/a_s - ik} - 4\frac{2m}{4k^2} \int V_2(r)\, \chi_1^2(r)\, dr \right)^{-1}$$

[Esposito, Glioti, ADP, Rattazzi, work in progress]

r_0 FOR THE COMPLEX POTENTIAL

$$r_0 = \frac{2m\alpha}{\mu^2}\left(\frac{2}{\mu^2 a_s^2} - \frac{8i}{3\mu a_s} - 1\right)$$

[Esposito, Glioti, ADP, Rattazzi, work in progress]

r_0 FOR THE YUKAWA AND COMPLEX POTENTIAL

We are checking that the results obtained in this way correspond *analytically* to what computed in non-relativistic effective field theory by Braaten, He & Jiang (2010.0580) (we also compared to Jansen, Hammer & Jia 1310.6937) — *two loop calculations*.

[Esposito, Glioti, ADP, Rattazzi, work in progress]

r_0 VALUES

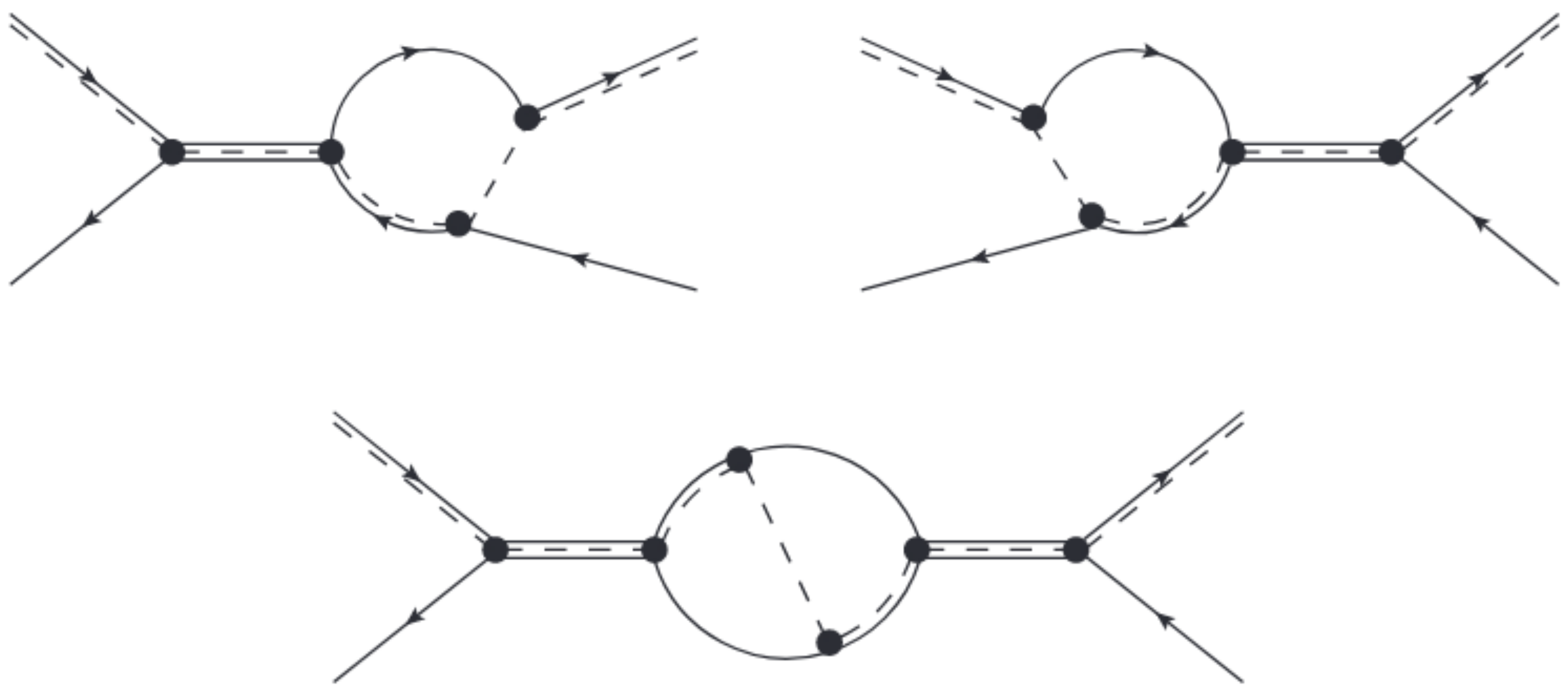
$$-0.20\ \text{fm} \lesssim \ \text{Re}\, r_0 \lesssim -0.15\ \text{fm}$$

$$-0.19\ \text{fm} \lesssim \ \text{Im}\, r_0 \lesssim 0\ \text{fm}$$

[Esposito, Glioti, ADP, Rattazzi, work in progress]

BACKUP

Diagrammi e XEFT



Braaten, Galilean invariant XEFT, Phys. Rev. D 103, 036014 (2021), arXiv:2010.05801 [hep-ph]

ELEMENTARY AND COMPOSITE DEUTERON

The physical normalized $|d\rangle$ deuteron state is

$$\underbrace{|d\rangle}_{\Psi} = \underbrace{\sqrt{Z}\,|\mathfrak{d}\rangle}_{\Psi_Q} + \underbrace{\int d\mathbf{k}\, C_{\mathbf{k}}\, |np(\mathbf{k})\rangle}_{\Psi_P}$$

with one elementary deuteron state $|\mathfrak{d}\rangle$.

$$\int d\mathbf{k}\, |\mathbf{C}_{\mathbf{k}}|^2 = \mathbf{1} - \mathbf{Z}$$

See Weinberg Phys. Rev. 137, B672 (1965)

ELEMENTARY AND COMPOSITE DEUTERON: MIXING

$$\begin{pmatrix} H_0 + V_{np} & H_{PQ} \\ H_{QP} & H_0 \end{pmatrix} \begin{pmatrix} \Psi_P \\ \Psi_Q \end{pmatrix} = M_d \begin{pmatrix} \Psi_P \\ \Psi_Q \end{pmatrix}$$

$Z \to 1 \Rightarrow \Psi_P = 0$ and requires $H_{PQ} = 0$.
The $(Z = 1)$ compact deuteron does not couple to pn

$Z \to 0 \Rightarrow \Psi_Q = 0$ and requires $H_{PQ} = 0$.
The deuteron is a molecule and there
is no compact state to couple to (the "easy" case).

SCATTERING AMPLITUDE

$$f = \frac{1}{k\cot\delta(k) - ik} = \frac{1}{-1/a + \frac{1}{2}r_0k^2 - ik + \ldots}$$

Compares with NR-BW formula

$$f = -\frac{\frac{1}{2}g_{\mathrm{BW}}^2}{E - m_{\mathrm{BW}} + \frac{i}{2}g_{\mathrm{BW}}^2 k}$$

$$g_{\mathrm{BW}}^2 = -\frac{2}{\mu r_0} \quad m_{\mathrm{BW}} = \frac{1}{a\mu r_0} \quad E = \frac{k^2}{2\mu}$$

Esposito et al. 2108.11413

SCATTERING AMPLITUDE

$$f = -\frac{\frac{1}{2}g_{\mathrm{BW}}^2}{E - m_{\mathrm{BW}} + \frac{i}{2}g_{\mathrm{BW}}^2 k}$$

$$g_{\mathrm{BW}}^2 = -\frac{2}{\mu r_0} \quad m_{\mathrm{BW}} = \frac{1}{a\mu r_0} \quad E = \frac{k^2}{2\mu}$$

For $r_0 < 0$ and $m_{BW} \gg \mu g_{BW}^4$, this expression describes an ordinary resonance above threshold with width $\Gamma \simeq g_{BW}^2\sqrt{2\mu m_{BW}}$. *But we are interested in a shallow bound state below threshold. Requires further refinements.*

Esposito et al. 2108.11413

A FORMULA FOR r_0

We find

$$r_0 = -\frac{2}{\mu g_{\text{LHCb}}} - \sqrt{\frac{\mu_+}{2\mu^2\delta}} = -5.34 \text{ fm}$$

Where μ_+ is the reduced mass of the charged open charm pair, μ of the neutral and δ is

$$\delta = m_{D^+} + m_{D^{*-}} - m_{D^0} - m_{\bar{D}^{*0}}$$

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DELTA FUNCTION POTENTIAL (JACKIW 91)

$$\chi_1(r) = \frac{e^{i\delta}\sin(kr+\delta)}{k} - \frac{e^{i\delta}\sin\delta}{k}$$

$$\lim_{r\to 0}\chi_1(r) = \frac{1}{1 - i\tan\delta} = \chi_{1,\text{reg.}}(0)$$

with $\quad \delta = \cot^{-1}\left(-\frac{1}{ka_s}\right)$

We do the subtraction only in the vicinity of $r = 0$: in the previous integral, integrate up to λ, compute r_0 and take the $\lambda \to 0$ limit.

[Esposito, Glioti, ADP, Rattazzi, work in progress]

$|\mathfrak{d}\rangle$ not a physical state — in the interacting theory

$$|d\rangle = \underbrace{\sqrt{Z}\,|\mathfrak{d}\rangle + \sqrt{Z'}\int d\mathbf{k}\, C_{\mathbf{k}}\,|np(\mathbf{k})\rangle}_{\sqrt{\alpha}|D\rangle} + \sqrt{\beta}\,|B\rangle$$

$$\alpha^2 + \beta^2 = 1$$

$$\beta \leq 1 - Z$$

$|D\rangle \to |\mathfrak{d}\rangle$ in the limit of free theory

ELEMENTARY AND COMPOSITE DEUTERON: THE TWO EXTREME CASES

1) $Z = 0$; molecular case.

Include a potential V binding n with p such that

$$(H_0 + V_{np})\,|\,d\rangle = (M_{np} + \frac{\mathbf{k}^2}{2\mu} + V_{np})\,\Psi_P = \underbrace{(M_{np} - B)}_{M_d}\,\Psi_P$$

2) $Z = 1$; would name it fully elementary deuteron

$$H_0\,|\,d\rangle = H_0\,|\,\mathfrak{d}\rangle = M_d\,|\,d\rangle$$

(deuteron at rest)