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Light hadrons in J/ψ Radiative Decay from Lattice QCD

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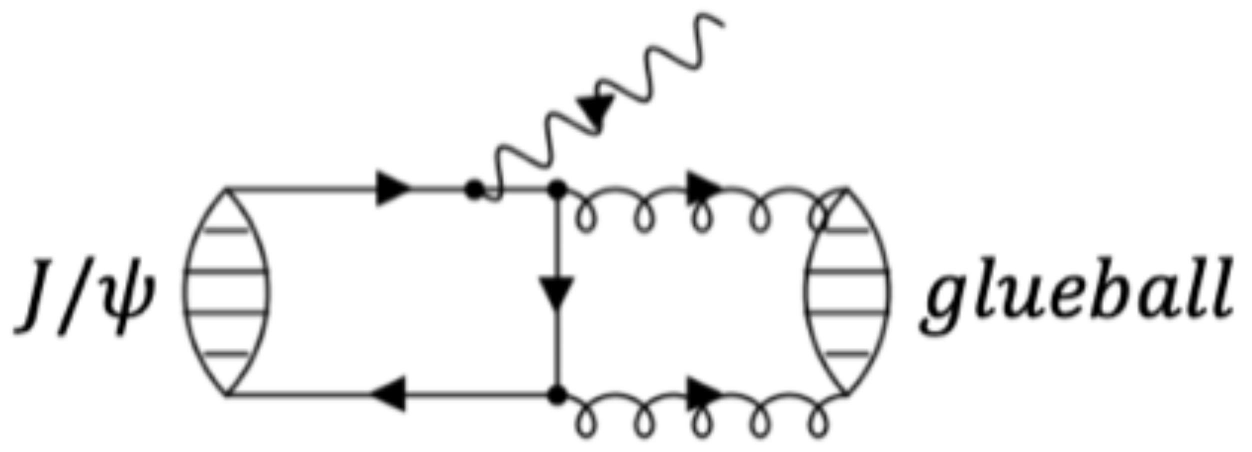
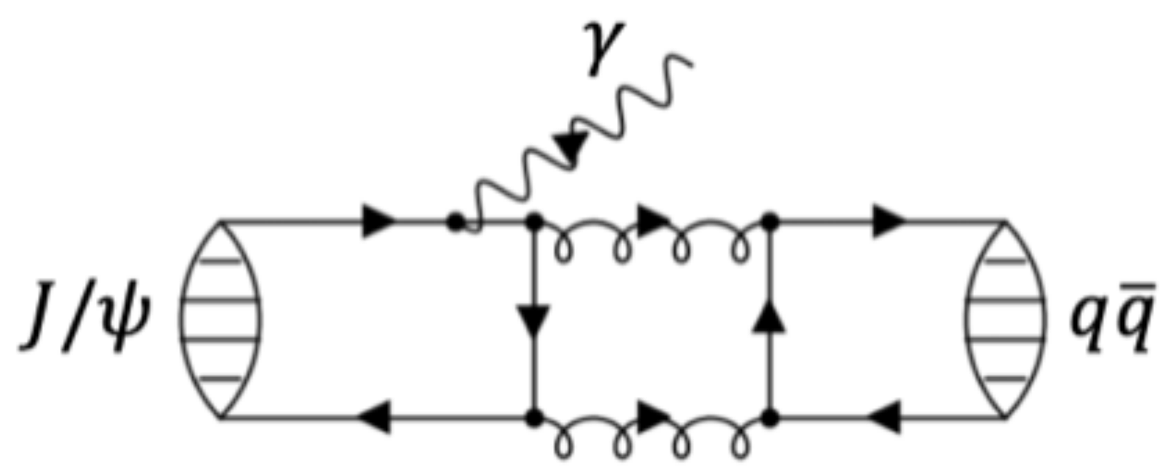
I. Motivation

J/ψ radiative decay

- The radiative decay of J/ψ is a good place to study light hadrans
    - Gluons are abundant in J/ψ decays through $c\bar{c}$ annihilation
    - Gluons are hadronized into final state light hadrons
    - Gluons are flavor singlet (isoscalar)
    - Final state should have $C = +$ due to the C-parity conservation
- J/ψ radiative decay products: $q\bar{q}$ mesons vs. glueballs
    - Naive power counting indicates the $\mathcal{O}(\alpha_s^2)$ suppression of $q\bar{q}$ meson production



$\mathcal{O}(1)$



$\mathcal{O}(\alpha_s^2)$

II. Formalism and lattice setup

Theoretical formalism for the calculation

- Radiative decay width: $\Gamma(i \to \gamma f) = \frac{1}{2J_i + 1} \frac{1}{32\pi^2} \int \mathrm{d}\Omega_q \frac{|\vec{q}|}{M_i^2} \sum_{r_i, r_f, r_\gamma} |\mathscr{M}_{r_i r_f r_\gamma}|^2$
- Transition amplitude: $\mathscr{M}_{r_i r_j r_\gamma} = \epsilon_\mu^*(q, r_\gamma) \langle f(p_f, r_f) | j_{\mathrm{em}}^\mu(0) | i(p_i, r_i) \rangle$
- Multipole decomposition: $\langle f(p_f, r_f) | j_{\mathrm{em}}^\mu(0) | i(p_i, r_i) \rangle = \sum_k \alpha_k^\mu(p_f, r_f, p_i, r_i) F_k(Q^2)$
- Decay width in terms of multipole form factors: $\Gamma(i \to \gamma f) \propto \sum_k |F_k(0)|^2$
- The matrix elements here can be derived from three-point functions on the lattice: $\Gamma_{(3)}^{\alpha\mu\beta}(\vec{p}_f, \vec{q}; t_f, t) = \sum_{\vec{y}} e^{-i\vec{q}\cdot\vec{y}} \langle \Omega | \mathcal{O}_f^\alpha(\vec{p}_f, t_f) j_{\mathrm{em}}^\mu(\vec{y}, t) \mathcal{O}_i^{\beta\dagger}(\vec{p}_i, 0) | \Omega \rangle$
- Here $q = p_i - p_f$, $Q^2 = -q^2$.

$L^3 \times T$	β	a_t^{-1}(GeV)	ξ	m_π(MeV)	$N_{\rm cfg}$
$16^3 \times 128$	2.0	6.894(51)	~ 5.3	348.5(1.0)	6991

Lattice setup

Jiang et al, 2205.12541(hep-lat)

- Anisotropic lattice: compromise of the resolution in the time direction and the computation expenses.
- Large statistics: decay process takes place through disconnected diagrams, and a large statistics is mandatory for good S/N.
- Lattice actions:
  - Tadpole improved Symanzik's gauge action (C. Morningstar, PRD60(1999)034509)
  - Tadpole improved clover fermion action for $N_f = 2$ degenerated u, d sea quarks
- Distillation method (M. Peardon et al. (HSC), PRD80(2009)054506)

III. Partial width of $J/\psi \to \gamma\eta$

J/ψ radiative decay to pseudoscalar

- Why η?
  - Isoscalar η in $N_f = 2$ is the counterpart of the flavor singlet η_1 in $N_f = 3$
  - Isoscalar η is the lightest isoscalar pesudoscalar in $N_f = 2$ and is stable

X	m_X (MeV)	$\mathrm{Br}(J/\psi \to \gamma X)$
η	547.862(17)	$1.108(27) \times 10^{-3}$
$\eta'(958)$	957.78(6)	$5.25(7) \times 10^{-3}$
$\eta(1405/1475)$	1441.9(2.2)	$> 4.88(72) \times 10^{-3}$
$\eta(1760)$	1751(15)	$> 2.11(34) \times 10^{-3}$

Zyla et al., PDG(2020)

- Physical interests
  - The production rates of pseudoscalars are usually large in J/ψ radiative decays
  - The production rate of 0^{-+} glueballs is not large, $\mathrm{Br}(J/\psi \to \gamma G_{0^{-+}}) \approx 2.4(9) \times 10^{-4}$ (L. Gui, et al., PRD100(2019)054511)
  - There may be a mechanism for the large production rates of η

Numerical setup

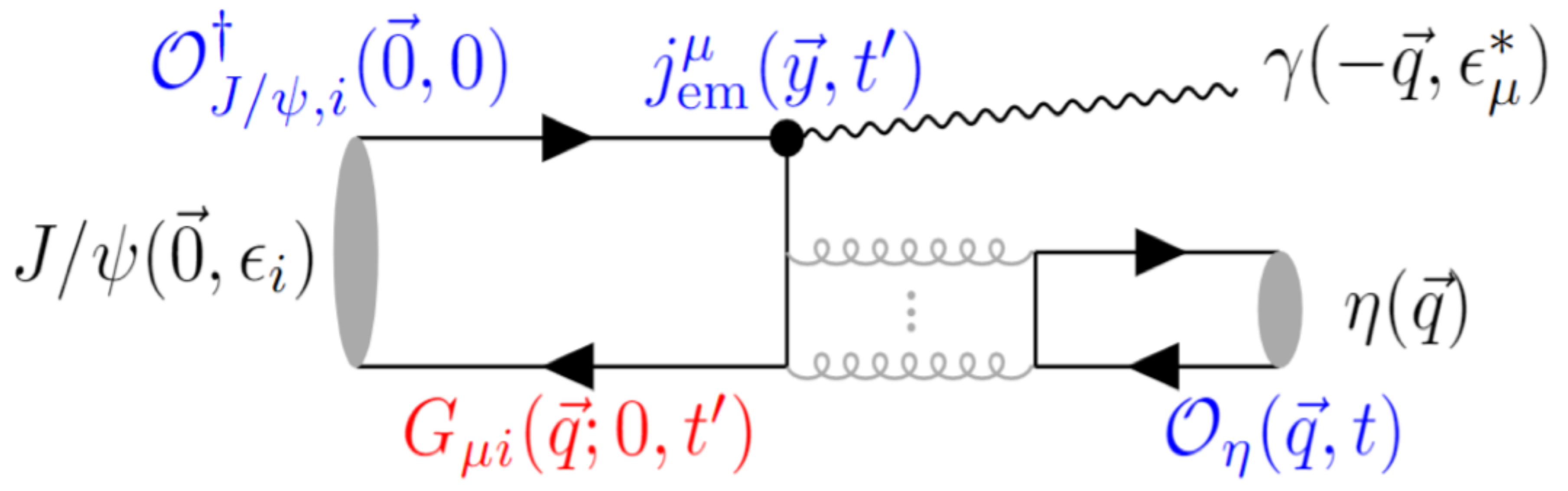
- Kinematic configuration: J/ψ is at rest and η moves with spatial momentum $\overrightarrow{q}$

- Three-point function (loop over T):

  - Can be divided into two parts and they can be calculated separately

  - $$\Gamma^{\mu i}_{(3)}(\overrightarrow{q}, t, t') = \frac{1}{T}\sum_{\tau=0}^{T-1} \langle \mathcal{O}_\eta(\overrightarrow{q}, \tau + t) G_{\mu i}(\overrightarrow{q}, \tau + t', \tau) \rangle$$

  - $$G_{\mu i}(\overrightarrow{q}, \tau + t', \tau) = \sum_{\overrightarrow{y}} e^{-i\overrightarrow{q}\cdot\overrightarrow{y}} j^{\mu}_{\rm em}(\overrightarrow{y}, \tau + t') \mathcal{O}^{i\dagger}_{J/\psi}(\overrightarrow{0}, \tau)$$

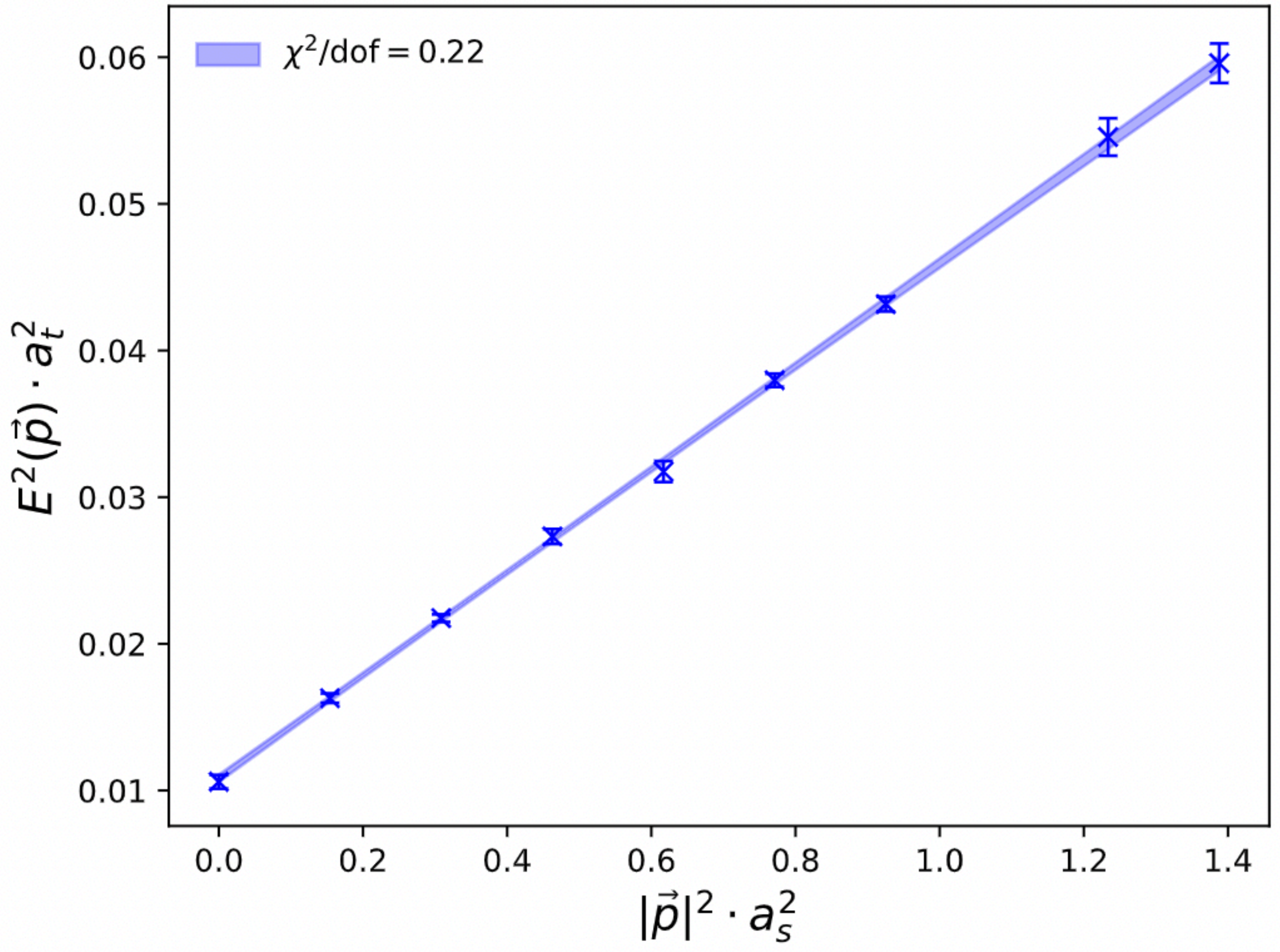


  - Light quark part ($\mathcal{O}_\eta$, a loop of light quark) is calculated by distillation method

  - Charm quark part ($G_{\mu i}$) is calculated by wall source

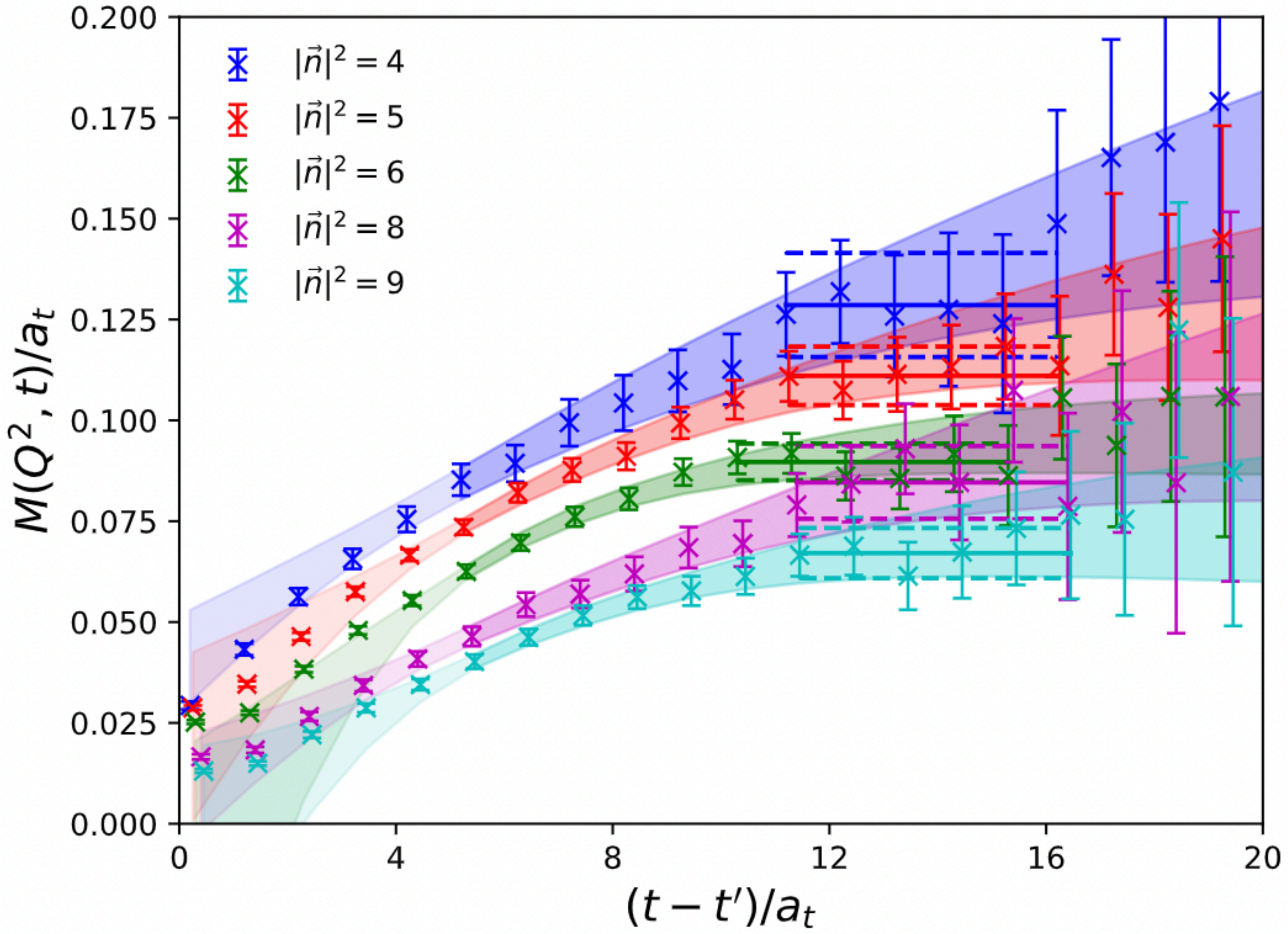
Numerical setup

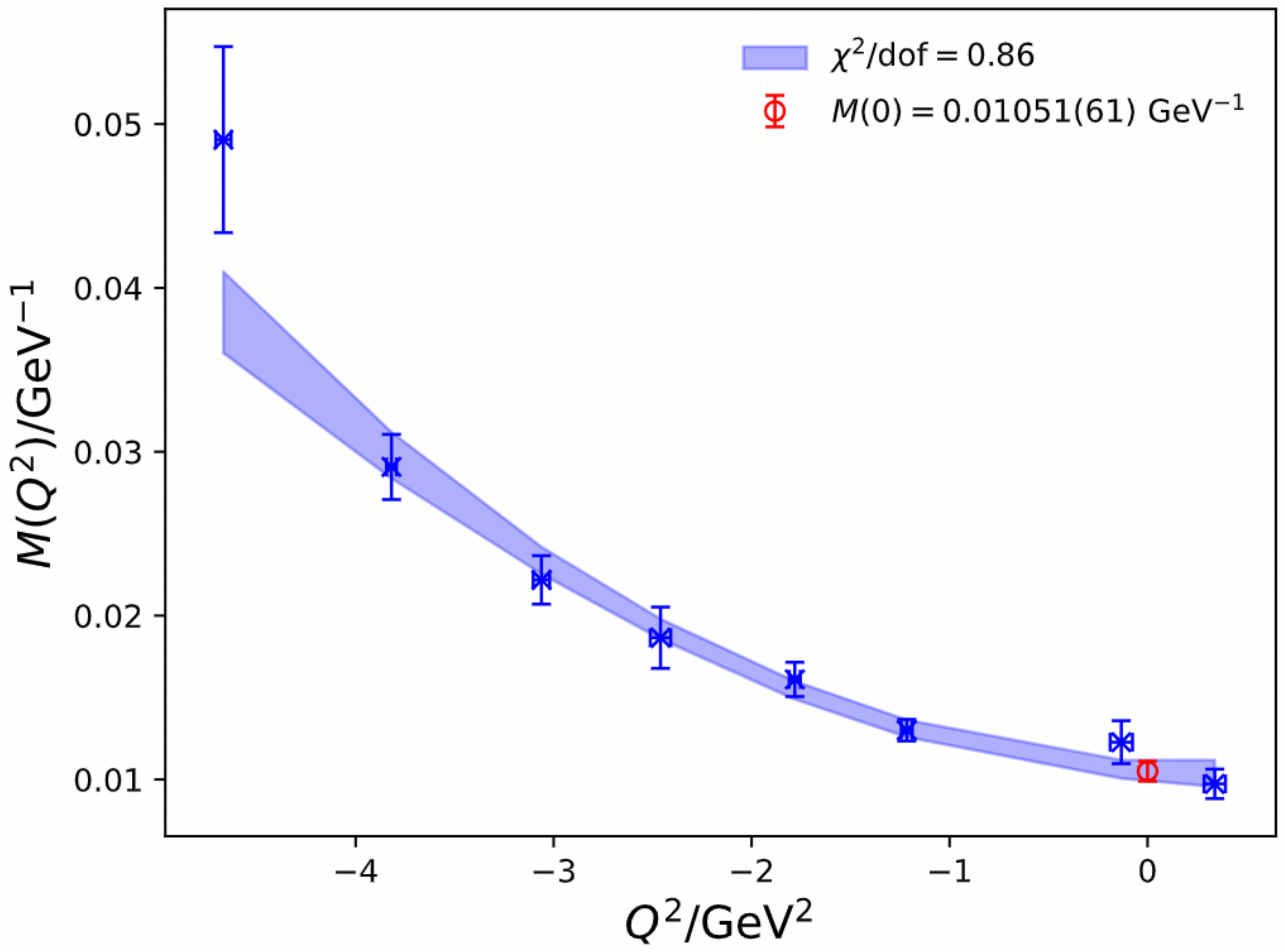
- $\mathcal{O}_\eta$ is defined as $\bar{u}\Gamma u + \bar{d}\Gamma d$, and here Γ is the bilinear operator insertion with $J^{PC} = 0^{-+}$
- Γ with different momenta are obtained from GEVP method, and is the combination of $\gamma_4\gamma_5$, $\gamma_4\gamma_5\gamma_i\nabla_i$ and $\epsilon_{ijk}\gamma_i\nabla_j\nabla_k$.
- Dispersion relation of η:
  - $E_\eta^2 a_t^2 = m_\eta^2 a_t^2 + \frac{1}{\xi^2}|\vec{p}|^2 a_s^2$
  - $\xi = 5.34(4)$, $m_\eta = 717.4(8.4)\mathrm{MeV}$



Lattice result (X. Jiang, et al., arxiv:2206.02724)

- The decay width can be written as $\Gamma(J/\psi \to \gamma\eta) = \frac{4\alpha}{27} |\vec{p}_\gamma|^3 |M(0)|$
- We fixed $t' = 40$ to make sure J/ψ dominate on $\Gamma^{\mu i}_{(3)}$
- $M(Q^2)$ still has $t - t'$ dependency because of the contribution from excited states of η.
- $M(Q^2)$ is obtained from the plateau regions. The value is also tested by $M(Q^2, t-t') = M(Q^2) + Ae^{-\delta m(t-t')}$
- We interpolate $M(Q^2)$ to the on-shell value $M(0)$ by using the polynomial form
  - $M(Q^2) = M(0) + aQ^2 + bQ^4 + \mathcal{O}(Q^6)$
  - $M(0) = 0.01051(61)\mathrm{GeV}^{-1}$



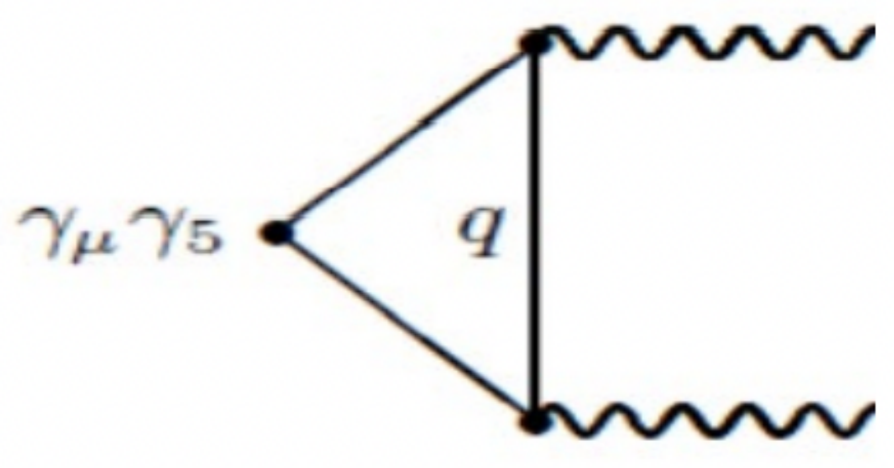


Lattice result

- Branching fraction $J/\psi \to \gamma\eta$ on $N_f = 2$ lattice is predicted to be
  - $\Gamma(J/\psi \to \gamma\eta) = \frac{4\alpha}{27} |\vec{p}_\gamma|^3 |M(0)|^2 = 0.385(45)\mathrm{keV}$
  - $\mathrm{Br}(J/\psi \to \gamma\eta) = 4.16(49) \times 10^{-3}$, with $\Gamma_{\mathrm{total}} = 92.6(1.7)\mathrm{keV}$
- This result is already comparable with experimental result $\mathrm{Br}(J/\psi \to \gamma\eta') = 5.25(7) \times 10^{-3}$.
- The $M(0)$ of $J/\psi \to \gamma\eta$ is close to that of $J/\psi \to \gamma G_{0-+}$ ($0.0090(16)\mathrm{GeV}^{-1}$, Gui, et al., PRD100(2019)054511)
- No clear $\mathcal{O}(\alpha_s^2)$ suppression is shown

$U_A(1)$ anomaly?

- $\partial_\mu j_5^\mu(x) = 2imj_5(x) + \sqrt{N_f}\frac{g^2}{32\pi^2}G^a_{\mu\nu}(x)\tilde{G}^{a,\mu\nu}(x)$
- $j_5^\mu = \frac{1}{\sqrt{N_f}}\sum_i \bar{\psi}_i\gamma^\mu\gamma_5\psi_i,\ j_5 = \frac{1}{\sqrt{N_f}}\sum_i \bar{\psi}_i\gamma_5\psi_i$



- $U_A(1)$ anomaly introduces coupling between the flavor singlet pseudoscalar and gluons
- Lattice study of the $D_s \to \eta/\eta'$ semileptonic decay also indicates the importance of $U_A(1)$ anomaly in such disconnect diagrams (G. Bali et al., PRD91(2015)014503)
- Assuming the $U_A(1)$ anomaly dominance in the production of η
  - Mass of flavor singlet pseudoscalar for $N_f = 3$ case $m_{\eta_1} \approx 936\mathrm{MeV}$ is predicted when applying Witten-Veneziano mechanism (E. Witten, NPB149(1979)285 and G. Veneziano, NPB159(1979)213)
  - For $SU_F(3)$ case: $M_{N_f=3}(0) = \sqrt{3/2}M(0) = 0.129(8)\mathrm{GeV}^{-1}$

Applying the $\eta - \eta'$ mixing

- In experiment:
  - $\mathrm{Br}(J/\psi \to \gamma\eta) = 1.11(3) \times 10^{-3}$, $\mathrm{Br}(J/\psi \to \gamma\eta') = 5.25(7) \times 10^{-3}$
- $\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix}$, $m_\eta = 547\mathrm{MeV}$, $m_{\eta'} = 958\mathrm{MeV}$
- Our prediction using quadratic GMO relation $\theta_{\mathrm{quad}} \approx -11.3^\circ$:
  - $\mathrm{Br}(J/\psi \to \gamma\eta) = 0.256(30) \times 10^{-3}$, $\mathrm{Br}(J/\psi \to \gamma\eta') = 5.21(62) \times 10^{-3}$
- Our prediction using linear GMO relation $\theta_{\mathrm{lin}} \approx -24.5^\circ$ (The result prefer this one):
  - $\mathrm{Br}(J/\psi \to \gamma\eta) = 1.15(14) \times 10^{-3}$, $\mathrm{Br}(J/\psi \to \gamma\eta') = 4.49(53) \times 10^{-3}$

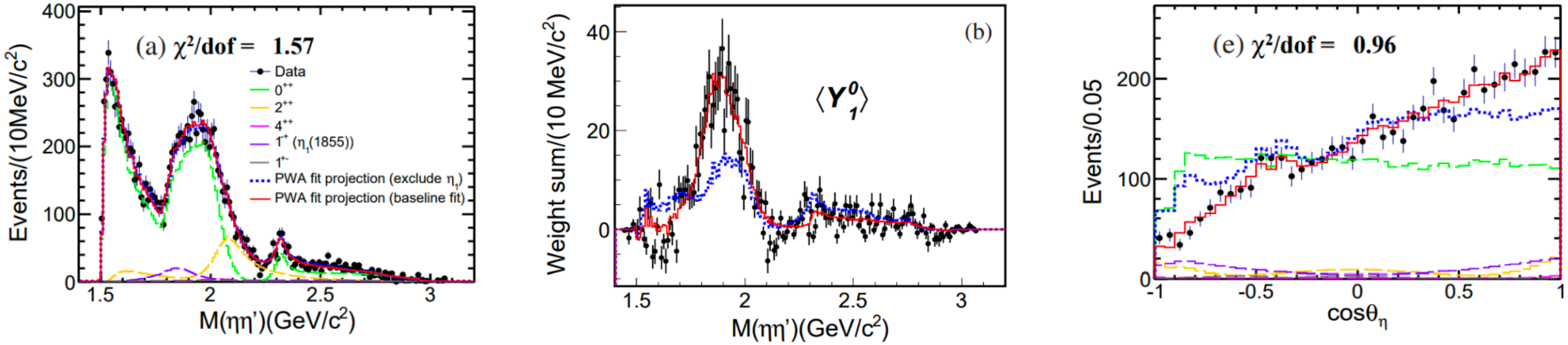
- A recent lattice study of η/η' mass and decay also gives a mixing angle in the gluonic sector around 24° (G. Bali et al., JHEP08(2021)137)

IV. Partial width of $J/\psi \to \gamma\eta_1(1^{-+})$

J/ψ radiative decay to 1^{-+}

- $\eta_1(1855)$ observed by BESIII (BESIII, arxiv:2202.00621)

  - Partial wave analysis of the process $J/\psi \to \gamma\eta\eta'$



  - The first candidate of isoscalar 1^{-+} hybrid

  - $m_{\eta_1} = 1855 \pm 9^{+6}_{-1}$ MeV, $\Gamma_{\eta_1} = 188 \pm 188^{+3}_{-8}$ MeV, $\mathrm{Br}(J/\psi \to \gamma\eta_1 \to \gamma\eta\eta') = 2.70 \pm 0.41^{+0.16}_{-0.35} \times 10^{-6}$

Numerical setup

- Kinetic configuration: η_1 is at rest and J/ψ moves with spatial momentum $\overrightarrow{q}$

- Three-point function (loop over T):

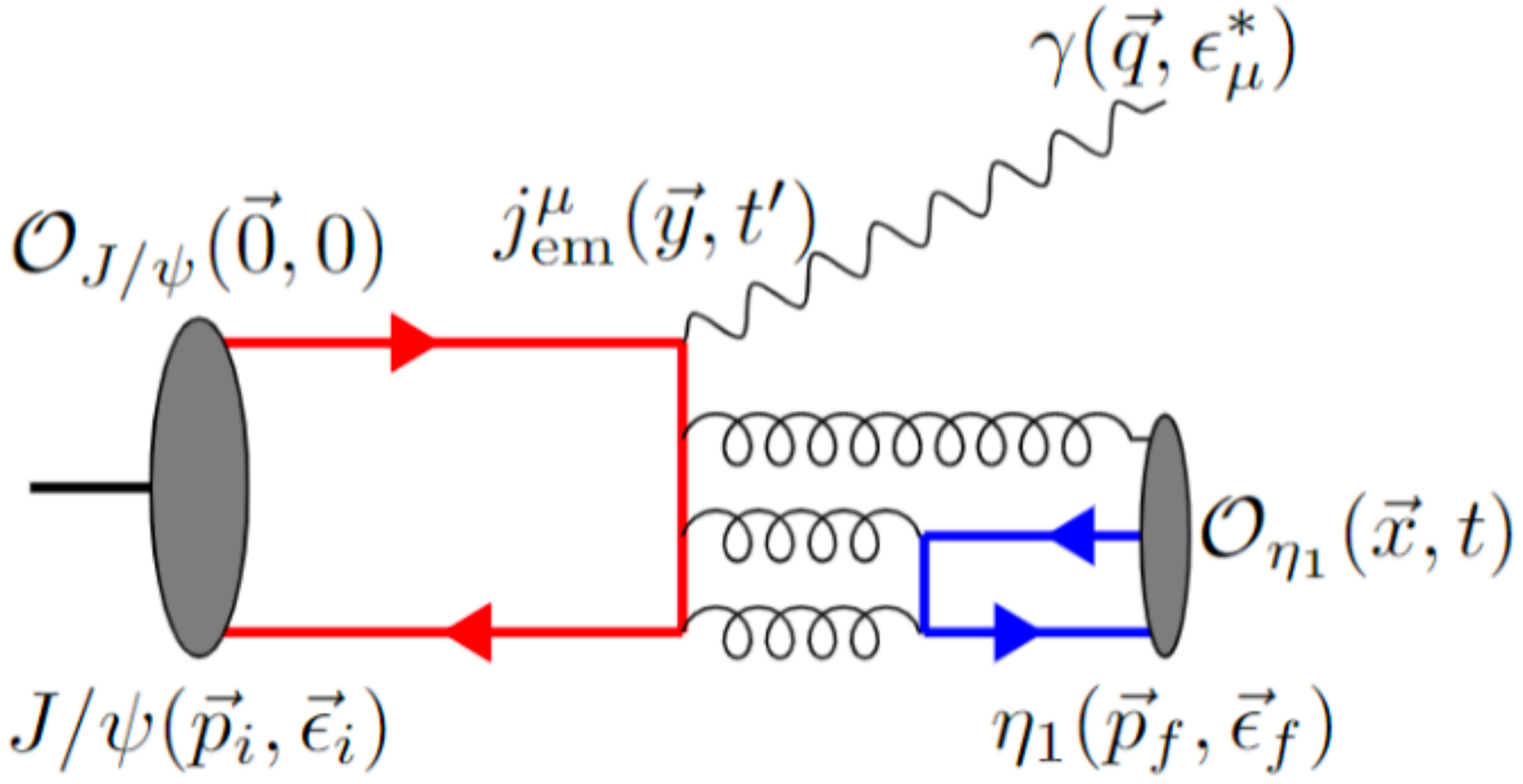
  - $\Gamma^{j\mu i}_{(3)}(\overrightarrow{q}, t, t') = \frac{1}{T}\sum_{\tau=0}^{T-1}\langle \mathcal{O}^{j}_{\eta_1}(\overrightarrow{0}, \tau + t) G_{\mu i}(\overrightarrow{q}, \tau + t', \tau)\rangle$

  - $G_{\mu i}(\overrightarrow{q}, \tau + t', \tau) = \sum_{\overrightarrow{y}} e^{i\overrightarrow{q}\cdot\overrightarrow{y}} j^{\mu}_{\text{em}}(\overrightarrow{y}, \tau + t')\mathcal{O}^{i\dagger}_{J/\psi}(\overrightarrow{q}, \tau)$

  - Light quark part ($\mathcal{O}_{\eta}$, a loop of light quark) is calculated by distillation meth

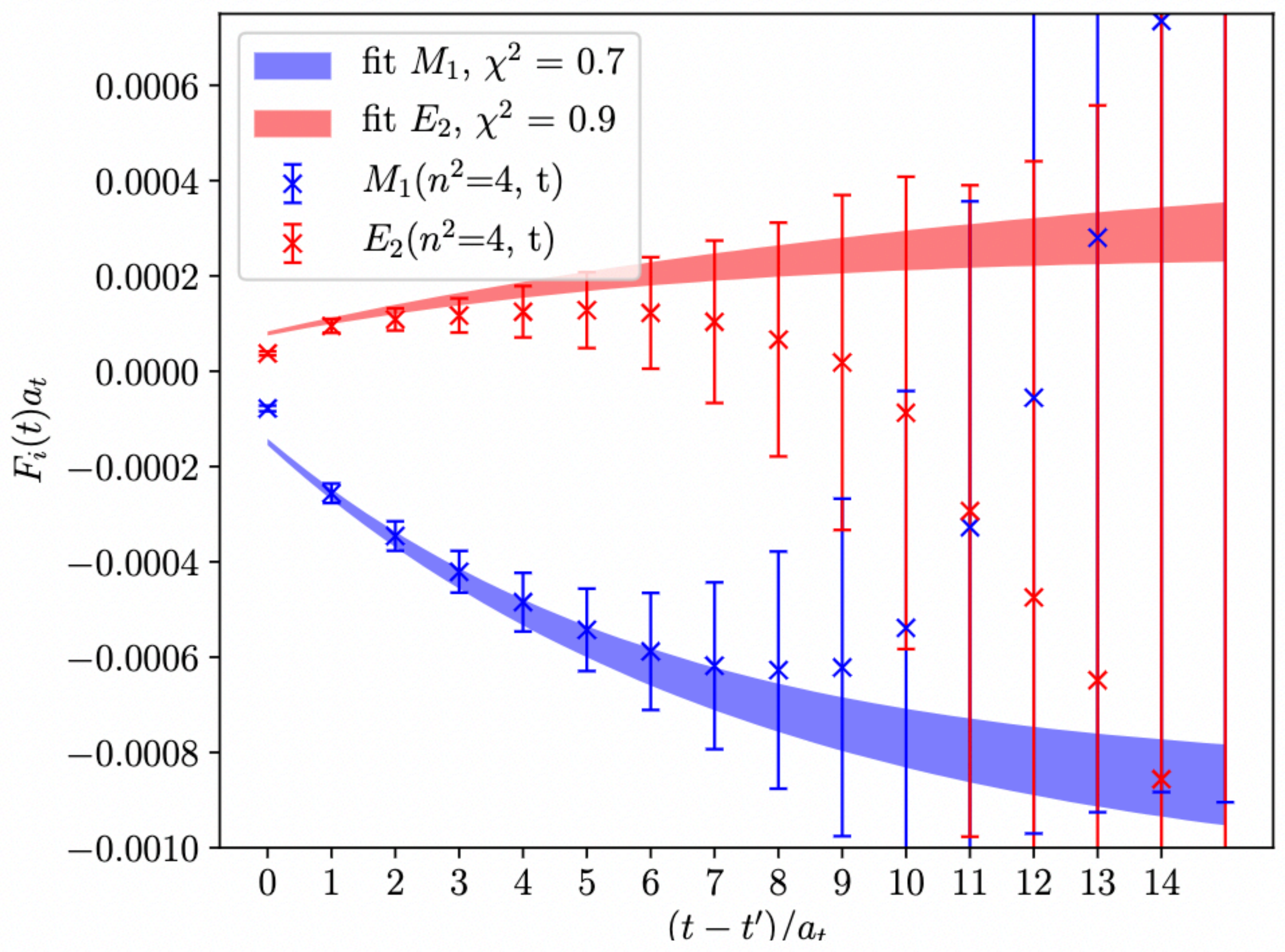
  - Charm quark part ($G_{\mu i}$) is calculated by eigenvector source and momentum sink

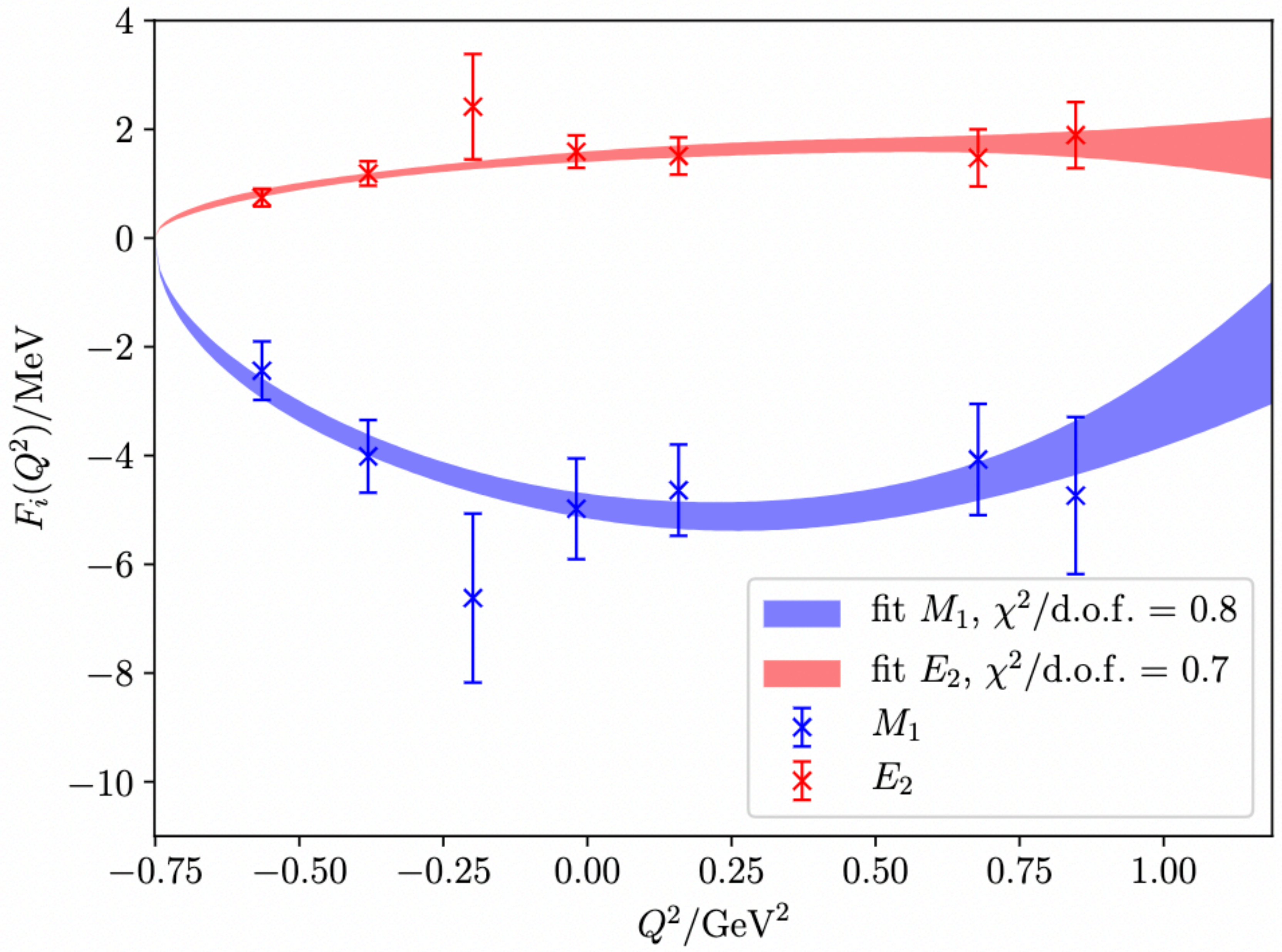
  - $\mathcal{O}_{\eta_1}$ is defined as $\frac{1}{\sqrt{2}}\epsilon^{ijk}(\bar{u}\gamma_j\mathbb{B}_k u + \bar{d}\gamma_j\mathbb{B}_k d)$ where $\mathbb{B}_i = \epsilon_{ijk}\nabla_j\nabla_k$



Lattice result (F. Chen, et al., arxiv:2207.04694)

- $\Gamma(J/\psi \to \gamma\eta_1) = \frac{4\alpha}{27}\frac{|\vec{p}_\gamma|}{2m_\psi^2}\left(|M_1(0)|^2 + |E_2(0)|^2\right)$
- We make the weighted average of the matrix element for $t' \in [20,40]$ to get the larger statistics
- $F_i(Q^2)$ ($M_1(Q^2)$ or $E_2(Q^2)$) still has $t - t'$ dependency, and the fitting formula is $F_i(Q^2, t-t') = F_i(Q^2) + Ae^{-\delta m(t-t')}$
- We interpolate $F_i(Q^2)$ to the on-shell value $F_i(0)$ by using the polynomial form
  - $F_i(Q^2) = \nu(Q^2)\left[a_i + b_i\nu^2(Q^2) + c_i\nu^4(Q^2) + \mathcal{O}(\nu^6)\right]$
  - $\nu(Q^2) = \frac{\sqrt{\Omega(Q^2)}}{m_{J/\psi}m_{\eta_1}}$, $\Omega(Q^2) = \frac{1}{4}\left[m_{J/\psi}^2 + m_{\eta_1}^2 + Q^2\right]\left[m_{J/\psi}^2 - m_{\eta_1}^2 + Q^2\right]$
  - $M_1(0) = -4.96(90)\text{MeV}$, $E_2(0) = 1.41(26)\text{MeV}$





Lattice result

- Branching fraction $J/\psi \to \gamma\eta_1$ on $N_f = 2$ lattice is predicted to be
  - $\Gamma(J/\psi \to \gamma\eta_1) = \frac{4\alpha}{27}\frac{|\vec{p}_\gamma|}{2m_\psi^2}\left(|M_1(0)|^2 + |E_2(0)|^2\right) = 2.29(47)\mathrm{eV}$, $\mathrm{Br}(J/\psi \to \gamma\eta_1) = 2.47(83) \times 10^{-5}$
- The mass dependence of the width is expected to be $\Gamma \propto \frac{|p_\gamma|^3}{m_{\eta_1}^2}$, and our $m_{\eta_1} = 2.230(39)\mathrm{GeV}$ is larger than $m_{\eta_1(1855)}$, we predict
  - $\mathrm{Br}(J/\psi \to \gamma\eta_1(1855)) = 6.2(2.2) \times 10^{-5}$
- And combining the experimental result $\mathrm{Br}(J/\psi \to \gamma\eta_1(1855) \to \gamma\eta\eta') = 2.70 \pm 0.41^{+0.16}_{-0.35} \times 10^{-6}$, we predict
  - $\Gamma(\eta_1(1855) \to \eta\eta') \approx 8.1(3.3)\mathrm{MeV}$
  - Agrees with a phenomenological study (H. Chen, et al., Chin. Phys. Lett. 39 (2022) 051201) $\Gamma(\eta_1(1855) \to \eta\eta') \approx 11\mathrm{MeV}$

V. Summary

- The J/ψ radiative decay is a good place to study the properties of light hadrons.
- It is also an important place to search exotic hadron such as glueballs and hybrids.
- $\mathrm{Br}(J/\psi \to \gamma\eta)$ is calculated on $N_f = 2$ lattice, and the result is in agreement with experimental value. This confirms the $U_A(1)$ anomaly dominance here.
- Lattice QCD is promising for this task.
- $\mathrm{Br}(J/\psi \to \gamma\eta_1)$ is predicted to be $6.1(2.2) \times 10^{-5}$ on $N_f = 2$ lattice.
- The similar studies on scalar and tensor $q\bar{q}$ states is in progress.

Thank you for your attention!

Witten-Veneziano mechanism

- Flavor singlet pseudoscalar meson satisfies: (E. Witten, NPB149(1979)285 and G. Veneziano, NPB159(1979)213)
  - $m_1^2 = \tilde{m}_1^2 + m_0^2,\ m_0^2 = \frac{2N_f}{f_\pi^2}\chi_{\mathrm{top}}$
  - For $SU_{\mathrm{I}}(2)$: $\tilde{m}_1^2 = m_\pi^2$, for $SU_{\mathrm{F}}(3)$: $\tilde{m}_1^2 = \frac{1}{3}(2m_K^2 + m_\pi^2) \approx 0.170\mathrm{GeV}^2$.
  - On this lattice: $m_\pi \approx 348.5(1.0)\mathrm{MeV}$, $m_\eta \approx 714.1(5.8)\mathrm{MeV}$
  - $f_\pi \approx 1.18 f_\pi^{\mathrm{phys}}$ from $N_f = 2$ chiral perturbation theory (D. Zhao et al., 2201.04910(hep-lat))
- Finally we obtain $\chi_{\mathrm{top}}^{1/4} \approx 177\mathrm{MeV}$, and estimate $m_{\eta_1} \approx 936\mathrm{MeV}$ for $SU_{\mathrm{F}}(3)$ situation.