Three-loop QCD corrections to quarkonium electroweak decays

arXiv:2207.14259 and arXiv:2208.04302

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in collaboration with

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- B_c decay to $l\nu$
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Motivation

Motivation

- Quarkonium leptonic decays are fundamental processes in high energy physics experiments.
- Theoretically, it plays an important role of probing the decay constant, which is a basic nonperturbative parameter.
- Previous works on short distance coefficient calculations:

  Vector quarkonium leptonic decay:

  - Tree level: Van Royen, Weisskopf, Nuovo Cim, 1967.
  - One loop: Barbieri, R. Gatto, et al., PLB1975; Celmaster, PRD1979.
  - Two loops: Czarnecki, Melnikov, PRL1998; Beneke, Signer, Smirnov, PRL1998; Kniehl, Onishchenko, et al., PLB2006; Egner, Fael, et al., PRD2021.
  - Three loops: Marquard, Piclum, et al., NPB2006, PLB2009, PRD2014; Beneke, Kiyo, et al., PRL2014; Egner, Fael, et. al., arXiv:2203.11231; Fael, Lange, et. al., arXiv:2207.00027 without indirect singlet and charm mass effect.

B_c meson:

- One loop: Braaten, Fleming, PRD1995.
- Two loops: Onishchenko, Veretin, EPJC2007; Chen, Qiao, PLB2015.

In our work, we calculate the complete three-loop correction to the Υ decay constant with massive charm and indirect singlet contributions, and new three-loop correction to the B_c decay constant.

Υ decay to e^+e^-

Factorization of the decay constants

Decay width of vector quarkonium

$$\Gamma(V \to l^+ l^-) = \frac{4\pi\alpha^2}{3M_V}|f_V|^2 \quad \alpha \text{ is the fine structure constant}$$

The leptonic decay constants f_V for vector quarkonia $V = J/\psi, \Upsilon$ are given by

$$\langle 0|\mathcal{J}^\mu_{\text{EM}}|V(\epsilon)\rangle = M_V f_V \epsilon^\mu_V, \quad \mathcal{J}^\mu_{\text{EM}} = \sum_f e_f \bar{\Psi}_f \gamma^\mu \Psi_f$$

According to the NRQCD factorization,

$$\langle 0|\mathcal{J}^i_{\text{EM}}|V(\epsilon)\rangle = \sqrt{2M_V} e_Q \mathcal{C}_0 \langle 0|\chi^\dagger \sigma^i \psi|V(\epsilon)\rangle_{\text{NR}} + \mathcal{O}(v^2)$$

The SDCs can be obtained by matching the on-shell quark-antiquark vertex function

$$\boxed{Z_2 Z_v^{-1}\Gamma = \sqrt{2M_V}\mathcal{C}_0(\mu_\Lambda)\tilde{Z}_2\tilde{Z}_v^{-1}(\mu_\Lambda)\tilde{\Gamma} + \mathcal{O}\left(v^2\right)}$$

Method of regions: Beneke, Smirnov, Nucl.Phys.B 1998.

SDCs

$$\gamma \equiv \frac{\mathrm{d}\ln \tilde{Z}}{\mathrm{d}\ln \mu_\Lambda^2} \equiv \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^2 \gamma^{(2)} + \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^3 \gamma^{(3)}(\mu_\Lambda) + \mathcal{O}\left(\alpha_s^4\right).$$

$$\begin{aligned}
&\mathcal{C}_0\left(\frac{\mu_R}{m_Q}, \frac{\mu_\Lambda}{m_Q}, x\right) = \\
&1 + \frac{\alpha_s(\mu_R)}{\pi}\mathcal{C}^{(1)}(x) + \left(\frac{\alpha_s(\mu_R)}{\pi}\right)^2 \left[\mathcal{C}^{(1)}\frac{\beta_0}{4}\ln\frac{\mu_R^2}{m_Q^2} + \gamma^{(2)}\ln\frac{\mu_\Lambda^2}{m_Q^2} + \mathcal{C}^{(2)}(x)\right] \\
&+ \left(\frac{\alpha_s(\mu_R)}{\pi}\right)^3 \left\{\frac{\mathcal{C}^{(1)}}{16}\beta_0^2\ln^2\frac{\mu_R^2}{m_Q^2} + \left[\frac{\mathcal{C}^{(1)}}{16}\beta_1 + \mathcal{C}^{(2)}(x)\frac{\beta_0}{2}\right]\ln\frac{\mu_R^2}{m_Q^2}\right. \\
&+ \gamma^{(2)}\frac{\beta_0}{2}\ln\frac{\mu_\Lambda^2}{m_Q^2}\ln\frac{\mu_R^2}{m_Q^2} + \frac{1}{4}\left[2\frac{\mathrm{d}\gamma^{(3)}(\mu_\Lambda)}{\mathrm{d}\ln\mu_\Lambda^2} - \beta_0\gamma^{(2)}\right]\ln^2\frac{\mu_\Lambda^2}{m_Q^2} \\
&\left. + \left[\mathcal{C}^{(1)}\gamma^{(2)} + \gamma^{(3)}(m_Q)\right]\ln\frac{\mu_\Lambda^2}{m_Q^2} + \mathcal{C}^{(3)}(x)\right\} + \mathcal{O}\left(\alpha_s^4\right).
\end{aligned}$$

$$x = m_M/m_Q,$$

Feynman diagrams

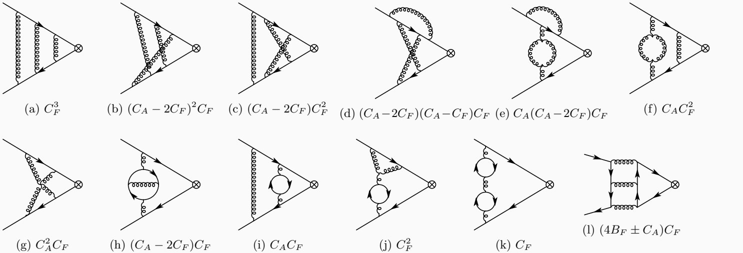


Figure 1: Representative diagrams of the direct channel. The color factor B_F is defined as $\sum_{bc} d^{abc} d^{ebc} = 4B_F \delta^{ae}$ and $B_F = (N_c^2 - 4)/(4N_c)$ for $SU(N_c)$ group.

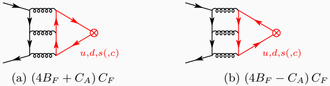


Figure 2: Representative diagrams of the indirect channel.

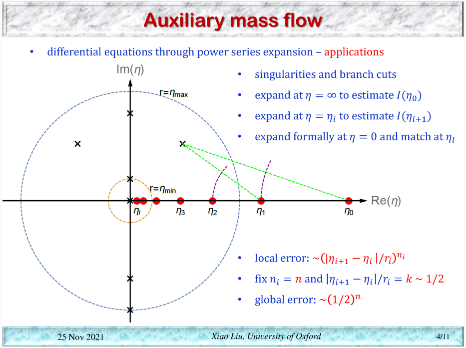
$$\langle 0|\mathcal{J}^i_{\text{EM}}|V(\epsilon)\rangle = \sqrt{2M_V}\,e_Q\left(\mathcal{C}_{\text{dir}} + \sum_{f\neq Q}\mathcal{C}_{\text{ind,f}}\frac{e_f}{e_Q}\right)\langle 0|\chi^\dagger\sigma^i\psi|V(\epsilon)\rangle_{\text{NR}} + \mathcal{O}\left(v^2\right)$$

Tool chain

The more than 300 Feynman diagrams are generated by the packages `QGraf`/`FeynArts` for crosscheck. `FormLink`/`FeynCalc` are then utilized to deal with Dirac and color matrices. After applying partial fraction by `Apart` and IBP reduction by `FIRE`, we get roughly 300 master integrals.

The master integrals are evaluated by the auxiliary mass flow method implemented as the package `AMFlow` [arXiv: 1711.09572, 2201.11669].

See Yan-Qing Ma's slides



SDCs of Υ decay constants

With 4 active flavors in α_s and β_i, terms that are independent of x are given by (For all terms, see arXiv:2207.14259)

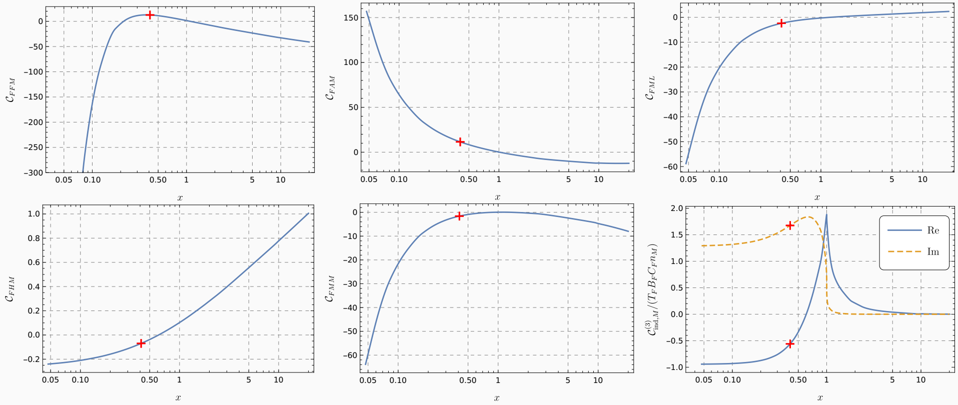
$$
\begin{aligned}
\mathcal{C}_{\text{dir}}^{(3)} =& C_F \big[C_F^2 \mathcal{C}_{FFF} + C_F C_A \mathcal{C}_{FFA} + C_A^2 \mathcal{C}_{FAA} \\
& + T_F n_L \left(C_F \mathcal{C}_{FFL} + C_A \mathcal{C}_{FAL} + T_F n_H \mathcal{C}_{FHL} + T_F n_M \mathcal{C}_{FML}(x) + T_F n_L \mathcal{C}_{FLL} \right) \\
& + T_F n_H \left(C_F \mathcal{C}_{FFH} + C_A \mathcal{C}_{FAH} + T_F n_H \mathcal{C}_{FHH} + T_F n_M \mathcal{C}_{FHM}(x) + B_F \mathcal{C}_{BFH} \right) \\
& + T_F n_M \left(C_F \mathcal{C}_{FFM}(x) + C_A \mathcal{C}_{FAM}(x) + T_F n_M \mathcal{C}_{FMM}(x) \right) \big].
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{C}_{FFF} =& 36.49486245880592537633476189872792031664181, \\
\mathcal{C}_{FFA} =& -188.07784165988071390579994023278476450389105, \\
\mathcal{C}_{FAA} =& -97.734973269918386342345245004574098439887181, \\
\mathcal{C}_{BFH} =& 2.1155782679809064984368222219139443700443356 \\
& + \mathrm{i}\, 0.494212710700672040241218108020160381155220487. \\
& \vdots
\end{aligned}
$$

Here we confirm the results from Egner, Fael, et al. arXiv:2203.11231

SDCs of Υ decay constants (continued)

Terms that are dependent of x. Red crosses are the physical values with three-loop pole masses of quarks $m_Q \equiv m_b = 4.98$ GeV, $m_M \equiv m_c = 2.04$ GeV (`RunDec` from $\overline{\text{MS}}$ masses $m_c(m_c) = 1.28\text{GeV}$, $m_b(m_b) = 4.18\text{GeV}$), $x_{\text{phys}} \approx 0.41$:



Renormalization constant of NRQCD operators and anomalous dimensions

Thanks to the extremely high precision of the results generated by `AMFlow`, one can reconstruct the analytic expressions for the non-renormalized poles of the SDCs to infer the renormalization constant of the NRQCD operator and corresponding anomalous dimension.

$$\tilde{Z} \equiv 1 + \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^2 \tilde{Z}^{(2)} + \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^3 \tilde{Z}^{(3)} + \mathcal{O}\left(\alpha_s^4\right),$$

$$\gamma \equiv \frac{\mathrm{d}\ln \tilde{Z}}{\mathrm{d}\ln \mu_\Lambda^2} \equiv \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^2 \gamma^{(2)} + \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^3 \gamma^{(3)}(\mu_\Lambda) + \mathcal{O}\left(\alpha_s^4\right).$$

Renormalization constant of the NRQCD vector current

$$
\begin{aligned}
\tilde{Z}_v = & 1 + \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^2 \frac{C_F \pi^2}{\epsilon}\left(\frac{1}{12}C_F + \frac{1}{8}C_A\right) + \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^3 C_F\pi^2 \\
& \times \Bigg\{ C_F^2\left[\frac{5}{144\epsilon^2} + \left(\frac{43}{144} - \frac{1}{2}\ln 2 + \frac{5}{48}\ln\frac{\mu_\Lambda^2}{m_Q^2}\right)\frac{1}{\epsilon}\right] \\
& + C_F C_A\left[\frac{1}{864\epsilon^2} + \left(\frac{113}{324} + \frac{1}{4}\ln 2 + \frac{5}{32}\ln\frac{\mu_\Lambda^2}{m_Q^2}\right)\frac{1}{\epsilon}\right] \\
& + C_A^2\left[-\frac{1}{16\epsilon^2} + \left(\frac{2}{27} + \frac{1}{4}\ln 2 + \frac{1}{24}\ln\frac{\mu_\Lambda^2}{m_Q^2}\right)\frac{1}{\epsilon}\right] \\
& + T_F n_L\left[C_F\left(\frac{1}{54\epsilon^2} - \frac{25}{324\epsilon}\right) + C_A\left(\frac{1}{36\epsilon^2} - \frac{37}{432\epsilon}\right)\right] \\
& + T_F n_H \frac{C_F}{60\epsilon} + T_F n_M \frac{1}{\epsilon}\left[\frac{C_F m_Q^2}{60 m_M^2} - \left(\frac{C_F}{18} + \frac{C_A}{12}\right)\ln\frac{\mu_\Lambda^2}{m_M^2}\right]\Bigg\} \\
& + \mathcal{O}\left(\alpha_s^4\right).
\end{aligned}
$$

Anomalous dimensions of the NRQCD vector current

$$\gamma \equiv \frac{\mathrm{d}\ln\tilde{Z}}{\mathrm{d}\ln\mu_\Lambda^2} \equiv \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^2 \gamma^{(2)} + \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^3 \gamma^{(3)}(x\,,\mu_\Lambda) + \mathcal{O}\left(\alpha_s^4\right)$$

$$x = \frac{m_c}{m_b}$$

$$\gamma_v^{(2)} = -3\pi^2 C_F \left(\frac{1}{18} C_F + \frac{1}{12} C_A\right),$$

$$\begin{aligned}\gamma_v^{(3)}(\mu_\Lambda) = &-3\pi^2 C_F \Bigg\{ \left(\frac{43}{144} - \frac{1}{2}\ln 2\right) C_F^2 + \left(\frac{113}{324} + \frac{1}{4}\ln 2\right) C_F C_A + \left(\frac{2}{27} + \frac{1}{4}\ln 2\right) C_A^2 \\ &+ T_F n_L \left(-\frac{25}{324} C_F - \frac{37}{432} C_A\right) + \frac{1}{60} T_F n_H C_F \\ &+ T_F n_M \left[\frac{C_F}{60x^2} + \left(\frac{1}{18} C_F + \frac{1}{12} C_A\right)\ln x^2\right] + \ln\frac{\mu_\Lambda^2}{m_Q^2}\Bigg[\frac{5}{48} C_F^2 + \frac{5}{32} C_F C_A \\ &+ \frac{1}{24} C_A^2 - T_F n_M \left(\frac{1}{18} C_F + \frac{1}{12} C_A\right)\Bigg]\Bigg\}.\end{aligned}$$

The SDC for Υ as an example:

$$\mathcal{C}_0(\mu_R = m_b, \mu_\Lambda = 1.5\ \text{GeV}, x_{\text{phys}}) = 1 - \frac{8}{3}\frac{\alpha_s(m_b)}{\pi} - 11.1686\left(\frac{\alpha_s(m_b)}{\pi}\right)^2$$
$$+\left(\frac{\alpha_s(m_b)}{\pi}\right)^3 [-1702.68 + 91.42 n_L + 0.10 n_H + 19.18 n_M - 0.82 n_L^2 + 0.02 n_H^2 - 0.53 n_M^2$$
$$-0.80 n_L n_M - 0.09 n_L n_H - 0.02 n_M n_H + (0.59 + 0.14i)_{\text{sing},H} + (0.31 - 0.93i)_{\text{ind},M}]$$
$$= 1 - 2.66667\frac{\alpha_s(m_b)}{\pi} - 11.1686\left(\frac{\alpha_s(m_b)}{\pi}\right)^2 - (1418.85 + 0.793514i)\left(\frac{\alpha_s(m_b)}{\pi}\right)^3$$

The LDME is normally estimated by the wave function at the origin:

$$\langle 0|(\chi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}\psi)(\mu_\Lambda)|V(\boldsymbol{\epsilon})\rangle_{\text{NR}} \approx \sqrt{\frac{N_c}{2\pi}} R_V(0)$$

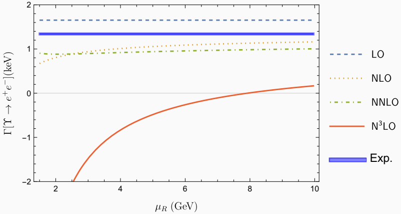
We choose the Buchmüller-Tye potential for the wave function input:

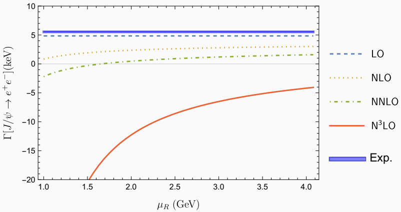
$$|R_\Upsilon(0)|^2 = 6.477\ \text{GeV}^3, \quad |R_{J/\psi}(0)|^2 = 0.810\ \text{GeV}^3.$$

Phenomenology: Leptonic decay width of J/ψ and Υ

$\Gamma(\mathrm{keV})$ \ V	LO	NLO	NNLO	$\mathrm{N^3LO}$ Direct ($m_M = 0$)	$\mathrm{N^3LO}$ Direct ($m_M \neq 0$)	$\mathrm{N^3LO}$ Total	PDG
Υ	1.6529	$1.0556^{+0.1063}_{-0.3838}$	$0.9400^{+0.0647}_{-0.0570}$	$-0.4304^{+0.6238}_{-4.5256}$	$-0.4887^{+0.6551}_{-4.8010}$	$-0.4884^{+0.6549}_{-4.7999}$	1.340 ± 0.018
J/ψ	4.8392	$2.3901^{+0.6318}_{-1.5593}$	$0.5135^{+1.0758}_{-2.7118}$	$-11.8733^{+7.8098}_{-40.0618}$			5.53 ± 0.10

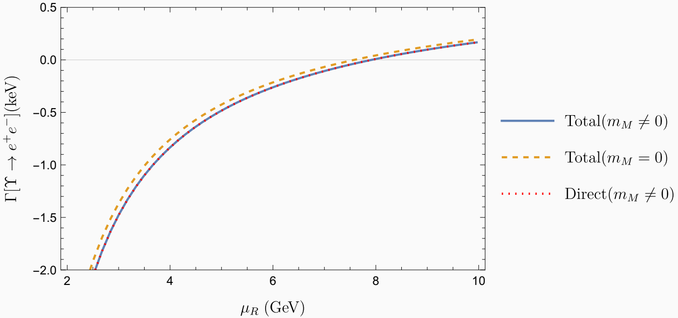
Table 1: Decay width for Υ and J/ψ. The central values of predictions are obtained by setting $\mu_R = m_Q$, while the errors are estimated by varying μ_R from μ_Λ to $2m_Q$. The factorization scale $\mu_\Lambda = 1.5$ GeV for Υ and 1 GeV for J/ψ.





Charm mass and indirect term effect in Υ decay

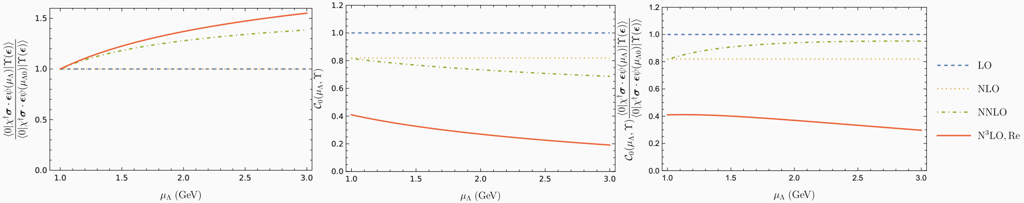
Our complete result is plotted in solid line. The dashed line treats charm quark as massless. The dotted line takes out the contribution from the indirect diagrams. It's shown that indirect channel only has invisible effect on the plot, while charm mass leads to visible small correction.



Factorization scale μ_Λ dependence of the SDC

Fixing the renormalization scale $\mu_R = m_Q$, The factorization scale μ_Λ dependence of the renormalized NRQCD matrix element, the μ_Λ dependence of SDC and their product are plotted.

Υ case:

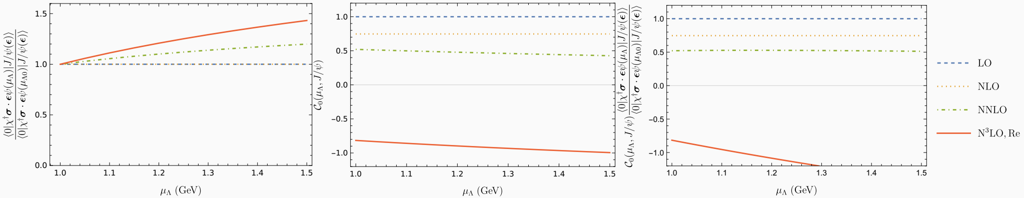


Matrix element | SDC | Product

$$\frac{\mathrm{d}\ln\langle 0|\chi^\dagger\sigma^i\psi|V\rangle}{\mathrm{d}\ln\mu_\Lambda^2} = -\left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^2\gamma^{(2)} - \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^3\gamma^{(3)}(\mu_\Lambda) + \mathcal{O}\left(\alpha_s^4\right)$$

Factorization scale μ_Λ dependence of the SDC

J/ψ case:



Matrix element

SDC

Product

B_c decay to $l\nu$

Factorization of the decay constants

$$\Gamma\left(B_c^+ \to l^+ \nu_l\right) = \frac{1}{8\pi} |V_{bc}|^2 G_F^2 M_{B_c} m_l^2 \left(1 - \frac{m_l^2}{M_{B_c}^2}\right)^2 f_{B_c}^2$$

The leptonic decay constants f_{B_c} for pseudo-scalar quarkonium B_c are given by

$$\langle 0|\bar{b}\gamma^\mu \gamma_5 c|B_c^+(P)\rangle = iP^\mu f_{B_c}$$

According to the NRQCD factorization,

$$\langle 0|\bar{b}\gamma^0 \gamma_5 c|B_c^+\rangle = i\sqrt{2M_{B_c}}\mathcal{C}_0(x)\langle 0|\chi_b^\dagger \psi_c|B_c^+\rangle_{\mathrm{NR}} + \mathcal{O}(v^2), \quad x = \frac{m_c}{m_b}$$

The SDCs can be obtained by matching the on-shell quark-antiquark vertex function

$$\boxed{\sqrt{Z_{2b}Z_{2c}}Z_a^{-1}\Gamma = \sqrt{2M_{B_c}}\mathcal{C}_0(\mu_\Lambda)\sqrt{\tilde{Z}_{2b}\tilde{Z}_{2c}}\tilde{Z}_p^{-1}(\mu_\Lambda)\tilde{\Gamma} + \mathcal{O}\left(v^2\right)}$$

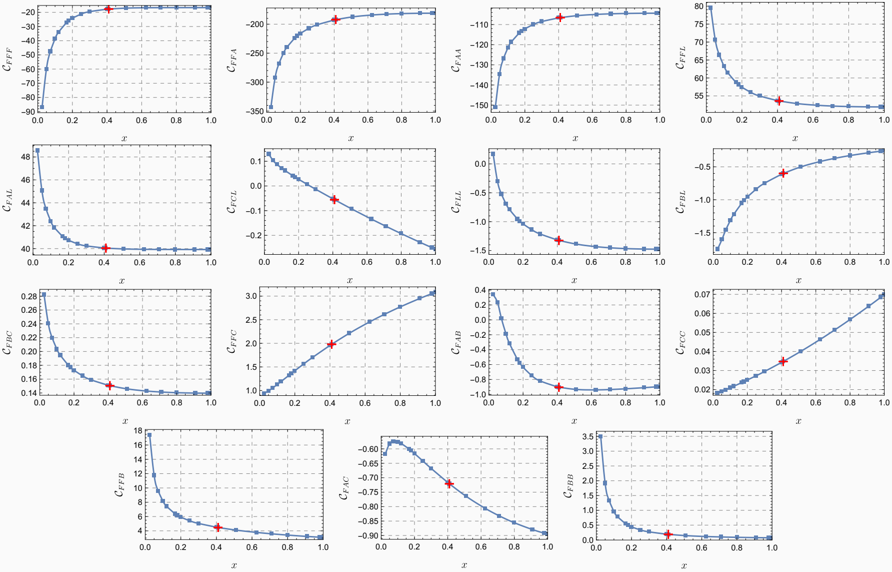
SDCs of B_c decay constant

$$
\begin{aligned}
\mathcal{C}_0(x) = & C_F \big[C_F^2 \mathcal{C}_{FFF} + C_F C_A \mathcal{C}_{FFA} + C_A^2 \mathcal{C}_{FAA} \\
& + T_F n_L \left(C_F \mathcal{C}_{FFL} + C_A \mathcal{C}_{FAL} + T_F n_H \mathcal{C}_{FHL} + T_F n_M \mathcal{C}_{FML} + T_F n_L \mathcal{C}_{FLL} \right) \\
& + T_F n_H \left(C_F \mathcal{C}_{FFH} + C_A \mathcal{C}_{FAH} + T_F n_H \mathcal{C}_{FHH} + T_F n_M \mathcal{C}_{FHM} + B_F \mathcal{C}_{BFH} \right) \\
& + T_F n_M \left(C_F \mathcal{C}_{FFM} + C_A \mathcal{C}_{FAM} + T_F n_M \mathcal{C}_{FMM} \right) \big].
\end{aligned}
$$

$m_b = 4.98$ GeV, $m_c = 2.04$ GeV, with 3 active flavors in α_s and β_i.

$$
\begin{aligned}
\mathcal{C}_{FFF} &= -17.6481252546417535391\,31, & \mathcal{C}_{FFA} &= -192.151798224347908747121, \\
\mathcal{C}_{FAA} &= -106.557000740278858592 42, & \mathcal{C}_{FFL} &= 53.590882380320998839852 8, \\
\mathcal{C}_{FAL} &= 40.0419439556257077283 91, & \mathcal{C}_{FCL} &= -0.59955659588604920607755, \\
\mathcal{C}_{FBL} &= -0.05567360504047408860700, & \mathcal{C}_{FLL} &= -1.32484367522413099859707, \\
\mathcal{C}_{FBC} &= 0.15047037340977620584792, & \mathcal{C}_{FFC} &= 4.46892700776466970199 1, \\
\mathcal{C}_{FAC} &= -0.90391224294954408740 57, & \mathcal{C}_{FCC} &= 0.18738217573423910690057, \\
\mathcal{C}_{FFB} &= 1.97991279879730446941 23, & \mathcal{C}_{FAB} &= -0.72105476302894669430 49, \\
\mathcal{C}_{FBB} &= 0.03474911743391490676344.
\end{aligned}
$$

SDCs of B_c decay constant



Renormalization constant of the NRQCD current for B_c

We implement the Thiele's interpolation formula to reconstruct rational functions and use PSLQ algorithm to speculate transcendental functions.

$$
\begin{aligned}
\tilde{Z}_p =& 1 + \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^2 \frac{\pi^2 C_F}{\epsilon}\left(\frac{3+z}{8(1+z)}C_F + \frac{1}{8}C_A\right) \\
&+ \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^3 \pi^2 C_F \Bigg\{ \frac{1}{\epsilon^2}\left(\frac{-1+6z}{72\,(1+z)}C_F^2 - \frac{5}{48\,(1+z)}C_F C_A - \frac{1}{16}C_A^2\right) \\
&+ \frac{1}{\epsilon}\Bigg[C_F^2\left(\frac{29+38z}{72\,(1+z)} - \frac{7}{12}\ln 2 - \frac{2-3x-22x^2-3x^3+2x^4}{12\,(1-x)\,(1+x)^3}\ln x + \frac{1}{12}\ln(1+z) + \frac{-1+6z}{24\,(1+z)}\ln\frac{\mu_\Lambda^2}{m_b m_c}\right) \\
&+ C_F C_A\left(\frac{93+52z}{216\,(1+z)} + \frac{1}{8}\ln 2 - \frac{5+2x+5x^2}{48\,(1-x)\,(1+x)}\ln x + \frac{1}{8}\ln(1+z) + \frac{18+11z}{48\,(1+z)}\ln\frac{\mu_\Lambda^2}{m_b m_c}\right) \\
&+ C_A^2\left(\frac{2}{27} + \frac{5}{24}\ln 2 + \frac{1}{24}\ln(1+z) + \frac{1}{24}\ln\frac{\mu_\Lambda^2}{m_b m_c}\right)\Bigg] \\
&+ T_f n_L\left[\left(\frac{3+z}{36\,(1+z)}\frac{1}{\epsilon^2} - \frac{15+7z}{108\,(1+z)}\frac{1}{\epsilon}\right)C_F + \left(\frac{1}{36}\frac{1}{\epsilon^2} - \frac{37}{432}\frac{1}{\epsilon}\right)C_A\right] \\
&+ T_f n_{H,b}\left(\frac{1}{15\,(1+1/x)^2}\frac{1}{\epsilon}\right)C_F + T_f n_{H,c}\left(\frac{1}{15\,(1+x)^2}\frac{1}{\epsilon}\right)C_F \Bigg\}
\end{aligned}
$$

where $x = \frac{m_c}{m_b}$, $z = \frac{1}{2}\left(x + \frac{1}{x}\right)$. For corresponding anomalous dimension.

Anomalous dimension of the NRQCD current for B_c

$$\gamma^{(2)}(x) = -\pi^2 C_F \left(\frac{3+z}{4(1+z)} C_F + \frac{1}{4} C_A \right),$$

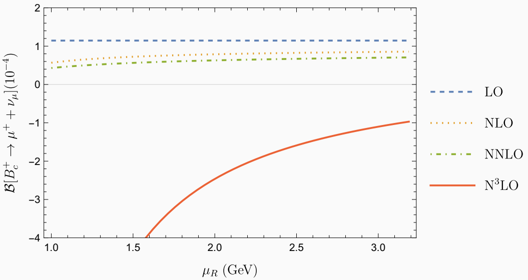
$$\gamma^{(3)}\left(x, \frac{\mu_\Lambda^2}{m_b m_c}\right) = -\pi^2 C_F \Bigg[\left(\frac{29+38z}{24(1+z)} - \frac{7}{4}\ln 2 - \frac{2-3x-22x^2-3x^3+2x^4}{4(1-x)(1+x)^3} \ln x + \frac{1}{4}\ln(1+z) \right.$$
$$\left. + \frac{-1+6z}{8(1+z)} \ln \frac{\mu_\Lambda^2}{m_b m_c} \right) C_F^2 + \left(\frac{93+52z}{72(1+z)} + \frac{3}{8}\ln 2 - \frac{5+2x+5x^2}{16(1-x)(1+x)} \ln x + \frac{3}{8}\ln(1+z) \right.$$
$$\left. + \frac{18+11z}{16(1+z)} \ln \frac{\mu_\Lambda^2}{m_b m_c} \right) C_F C_A + \left(\frac{2}{9} + \frac{5}{8}\ln 2 + \frac{1}{8}\ln(1+z) + \frac{1}{8}\ln \frac{\mu_\Lambda^2}{m_b m_c} \right) C_A^2$$
$$- T_F n_l \left(\frac{15+7z}{36(1+z)} C_F + \frac{37}{144} C_A \right) + T_F n_b \frac{1}{5(1+1/x)^2} C_F + T_F n_c \frac{1}{5(1+x)^2} C_F \Bigg].$$

where $x = \frac{m_c}{m_b}$, $z = \frac{1}{2}\left(x + \frac{1}{x}\right)$.

Leptonic decay width of B_c

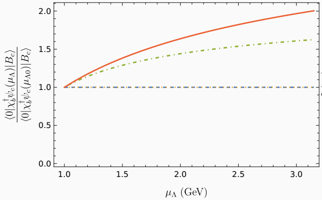
	LO	NLO	NNLO	N^3LO
leptonic width($\times 10^{-7}$ eV)	1.4776	$0.9207^{+0.1848}_{-0.1895}$	$0.7148^{+0.1965}_{-0.1606}$	$-6.2285^{+4.9766}_{-9.7396}$
$\mathcal{B}(B_c \to \mu^+ + \nu_\mu)$ ($\times 10^{-4}$)	1.1449	$0.7134^{+0.1432}_{-0.1468}$	$0.5539^{+0.1522}_{-0.1245}$	$-4.8260^{+3.8560}_{-7.5465}$

Table 2: Branching ratio of the B_c leptonic decay. The uncertainties are given by varying renormalization scale from factorization scale $\mu_\Lambda = 1$ GeV to $m_q = \sqrt{m_b m_c}$. Central value is chosen at $m_r = \frac{m_b m_c}{m_b + m_c}$.

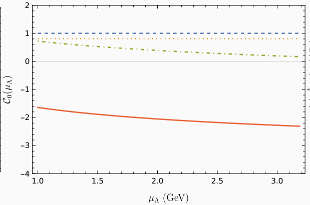


Factorization scale μ_Λ dependence of the SDC

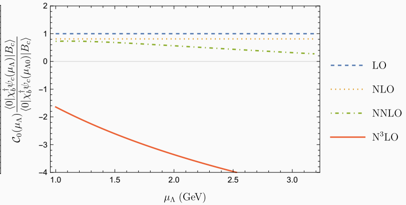
Fixing the renormalization scale $\mu_R = m_r$, The factorization scale μ_Λ dependence of the renormalized NRQCD matrix element, the μ_Λ dependence of SDC and their product are plotted.



Matrix element



SDC



Product

Summary

Summary

- We confirm the known three loop results of SDCs of Υ, J/ψ electromagnetic leptonic decays. Furthermore, our results of indirect and charm mass effect to Υ decay is new. While the former has very minor effect, the latter is noticeable.
- A new term in NRQCD anomalous dimension of vector NRQCD current $\chi^\dagger \boldsymbol{\sigma} \psi$ corresponding to the intermediate flavor of quark is first obtained. For B_c weak decay, the three loop SDC and anomalous dimension of pseudo-scalar NRQCD current $\chi_b^\dagger \psi_c$ are totally new.
- Our three-loop results for Υ and B_c share similar patterns that $\mathrm{N}^3\mathrm{LO}$ corrections drop to negative values and depend heavily on the renormalization scale and spoil the convergence of the perturbative expansion. This phenomenon may draw forth a new puzzle which deserves further research. Our treatment of the phenomenology is different from Beneke, Kiyo, et al., PRL2014.

Thank you for your attention