The static force from the lattice with gradient flow

QwG at GSI 2022

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Outline

Motivation: Static Energy E(r)

- Interested in the QCD static energy of a quark-antiquark pair $E(r)$
- Given by the Wilson loop

$$E(r) = -\lim_{T\to\infty} \frac{\ln\langle \mathrm{Tr}(W_{r\times T})\rangle}{T}, \qquad W_{r\times T} = P\left\{\exp\left(i\oint_{r\times T} dz_\mu g A_\mu\right)\right\}$$

- Can be described by perturbation theory and measured on the lattice
- For $r\Lambda_{\mathrm{QCD}} \ll 1$ both descriptions should agree

→ can be used for precise α_S-running extraction by comparing PT and lattice

Motivation: Issues with E(r)

- Perturbative form of $E(r)$:

$$E(r) = \Lambda_S - \frac{C_F \alpha_S}{r} \left(1 + \#\alpha_S + \#\alpha_S^2 + \#\alpha_S^3 + \#\alpha_S^3 \ln \alpha_S + ...\right)$$

- $E(r)$ is known up to N^3LL
- The perturbative expansion affected by a renormalon ambiguity of order Λ in PT side
- On lattice: Linear UV divergence
- All interesting physics is in the slope

➡ take a derivative of $E(r)$ for the force $F(r) = \partial_r E(r)$

Outline

Setup: Alternative definition of $F(r)$

- Direct measurement of $F(r)$:
  (A. Vairo Mod. Phys. Lett. A 31 (2016) & EPJ Web Conf. 126 (2016), Brambilla et.al.PRD63 (2001))

$$
\begin{aligned}
F(r) &= -\lim_{T\to\infty} \frac{i}{\langle \mathrm{Tr}(W_{r\times T})\rangle} \left\langle \mathrm{Tr}\left(P\left\{ \exp\left(i \oint_{r\times T} dz_\mu g A_\mu \right) \hat{\mathbf{r}}\cdot g\mathbf{E}(\mathbf{r}, t^*) \right\} \right) \right\rangle \\
&= \frac{\langle \mathrm{Tr}\{P W_{r\times T} g E_j(r, t^*)\}\rangle}{\langle \mathrm{Tr}\{P W_{r\times T}\}\rangle}
\end{aligned}
$$

- Chromoelectric field E inserted into Wilson loop
- The insertion location t^* is arbitrary $\rightarrow$ reduce boundary terms and choose $t^* = T/2$
- Can be used to extract α_S without the usual renormalon issues and for scale setting

➡ On the lattice: modifying Wilson loop with a discretized E-field insertion

Setup: Discretiziation of the E-field insertion

- Clover discretization of E:

$$E_i = \frac{1}{2iga^2}\left(\Pi_{i0} - \Pi_{i0}^\dagger\right) \qquad \Pi_{\mu\nu} = \frac{1}{4}\left(P_{\mu,\nu} + P_{\nu,-\mu} + P_{-\mu,-\nu} + P_{-\nu,\mu}\right)$$

- E has finite size on the lattice
- The self energy contribution of E converges slowly to continuum

  (See e.g. Lepage et.al.PRD48 (1993), G. Bali Phys. Rept. 343 (2001), and many others...)

  $\rightarrow$ need renormalization Z_E

We use **Gradient flow** for targeting the renormalization and the signal to noise ratio problems, new scale: **flowtime** τ_F, **flowradius** $\sqrt{8\tau_F}$, **flowtime ratio** τ_F/r^2

Setup: Gradient flow on the lattice

- Gradient flow is originated in lattice, acts as smearing with flowradius $\sqrt{8\tau_F}$
- Renormalizes lattice field strength components by reducing the self-energy contributions
- Gradient flow equation:
  (Martin Lüscher JHEP 08 (2010))

$$\dot{V}_{\tau_F}(x,\mu) = -g_0^2\{\partial_{x,\mu} S_{W/S}(V_{\tau_F})\}V_{\tau_F}(x,\mu) \qquad V_{\tau_F}(x,\mu)|_{\tau_F=0} = U_\mu(x)$$

- Measuring flowed operators on the lattice:

$$\langle O(\tau_F)\rangle = \frac{1}{Z}\int \mathcal{D}[U]e^{-S_E}O(V_{\tau_F})$$

→ Obtain $\lim_{\tau_F\to 0}\langle O(\tau_F)\rangle$ for the physical quantity

Setup: Specific extra motivations

- The statice force is complementary to the static energy extraction from Wilson loops
- We measured the force directly with the E-field insertion in Wilson loops and Polyakov loops first, with multilevel so far
  (Brambilla et. al. Phys.Rev.D 105 (2022))
  which introduces an additional factor Z_E
- One to one comparison to $\partial_r E(r)$ is possible
- Here we address first time the force measurement with gradient flow
- This study is a preparation for similar objects with field insertions needed it NREFTs

→ Use the implications of the force measurement with gradient flow

Setup: Continuum results

- One-loop calculation of the flowed force is known:
  (Hee Sok Chung et. al. JHEP01(2022)184 (2022)), Xiangpeng Wang's talk
- We focus on the $n_f = 0$ result
- Small τ_F exansion:

$$r^2 F(r;\tau_F) \approx r^2 F(r;\tau_F = 0) + \frac{\alpha_S^2 C_F}{4\pi} \underbrace{\left[-12\beta_0 - 6C_A c_L\right]}_{8n_f} \frac{\tau_F}{r^2} \qquad c_L = -\frac{22}{3}$$

→ At small flowtime the force is constant in pure gauge ($n_f = 0$)

Outline

Lattice results: setup and parameters

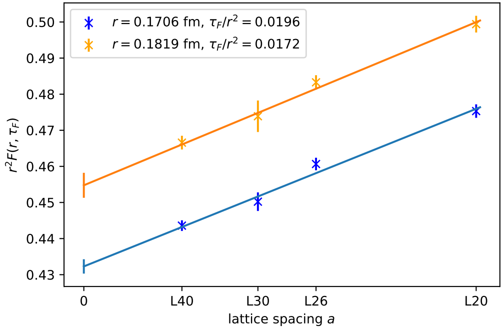
- Parameters:

N_S	N_T	β	a [fm]	N_{conf}	Label
20	40	6.284	0.060	6000	L20
26	52	6.481	0.046	6000	L26
30	60	6.594	0.040	6000	L30
40	80	6.816	0.030	2700	L40

- Pure gauge configuration produced with overrelaxation and heatbath
- Scale setting with
  $\ln(a/r_0) = -1.6804 - 1.7331(\beta - 6) + 0.7849(\beta - 6)^2 - 0.4428(\beta - 6)^3$
  (1S. Necco & R. Sommer. Nucl. Phys. B622 (2002))
- Gradient flow with **fixed** and **adaptive** solver, with **Symanzik** action
  (Bazavov and Chuna 2101.05320 (2021))

Lattice results: Crucial steps

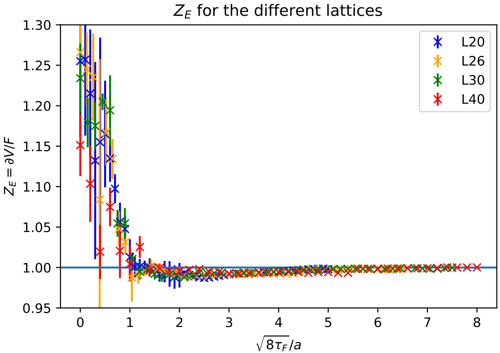
- Extraction of the $\lim_{T\to\infty}$- limit
  (William I. Jay, Ethan T. Neil Phys. Rev. D 103, 114502 (2021))
  at each fixed separation r and fixed flowtime τ_F or flowtime ratio τ_F/r^2
- Working out the continuum limit



Lattice results: Discretization effects

- Nonperturbative determination of Z_E:

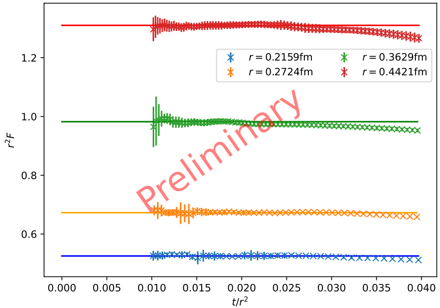
$$Z_E(r) = \frac{\partial_r E(r)}{F(r)}$$

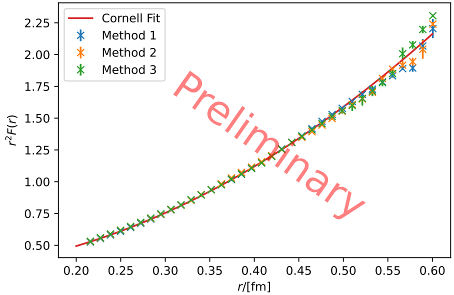


- Z_E has low r-dependence
  (Brambilla et. al. Phys.Rev.D 105 (2022))
- Examine the flowed Z_E
- $Z_E \to 1$ for flowradius $\sqrt{8\tau_F} > a$

➡ Gradient flow reduces discretization effects of field insertions for $\sqrt{8\tau_F} > a$

Lattice results: Continuum results at large r

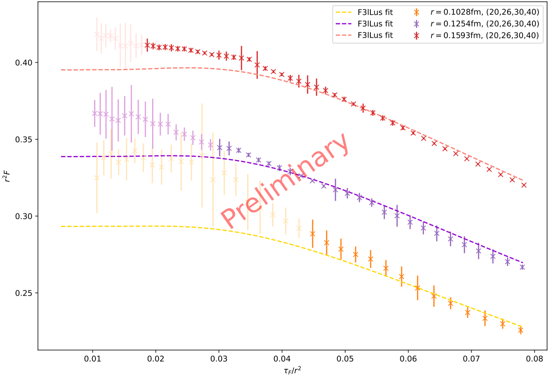




- Cornell fit $r^2F(r) = A + \sigma r^2$
- $\sigma = 5.18 \ldots 5.23$ fm^{-2}, literature: 5.5 fm^{-2}
- $A = 0.2853 \ldots 0.2954$

Lattice results: Continuum results at short r I

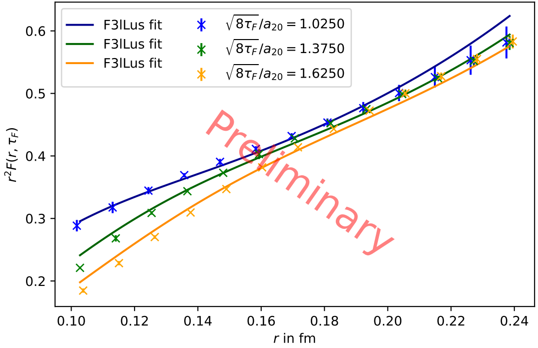
- Select valid points ($Z_E \approx 1$)
- We try a combined fit with Λ_0 as fit parameter:
  $F^{\text{fit}}(\tau_F) = F^{N-l}(0) + Fl(\tau_F)$
  with
  $Fl(\tau_F) = F^{1-l}(\tau_F) - F^{1-l}(\tau_F = 0)$
- Fit results:
  $r = 0.102$ fm: $\Lambda_0 = 0.233$ GeV
  $r = 0.125$ fm: $\Lambda_0 = 0.249$ GeV
  $r = 0.159$ fm: $\Lambda_0 = 0.246$ GeV
  A proper error estimation is still pending...

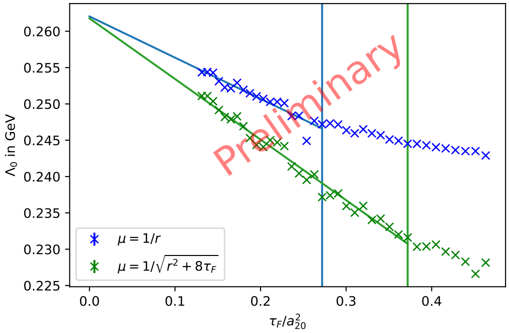


→ Constant r-fit for Λ_0 has small r-dependence, but captures the shape well

Lattice results: Continuum results at (short) r II

- Fit at fixed flowtime along r, the r-range is fixed



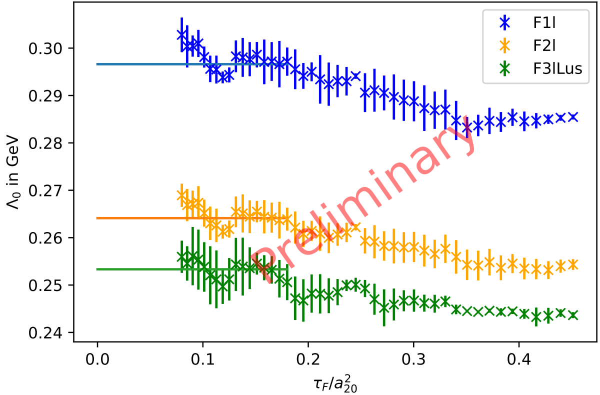


Lattice results: Continuum results at (short) r II

- Fit at fixed ratio along r, but with Bayesian model weighting for different r-ranges

- Λ_0 seems to be constant in a small flowtime window
- Fit results:
  F1l: 0.297 GeV
  F2l: 0.264 GeV
  F3lLus: 0.253 GeV
- Error analysis still pending...

➡ Λ_0-fit is very sensitive to the choice of the r-window



Outline

Conclusion

- Summary ond observations:
  - Gradient flow reduces effectively discretization effects
  - Gradient flow improves qualitatively the signal to noise ratio
  - Good preparation for future applications in NREFTs
- For the future:
  - continuing a proper Λ extraction
  - Go to finer lattices
  - Other operators with field insertions
  - Extend to dynamical fermions

Thank you for your attention!

r_0 scale flow dependence

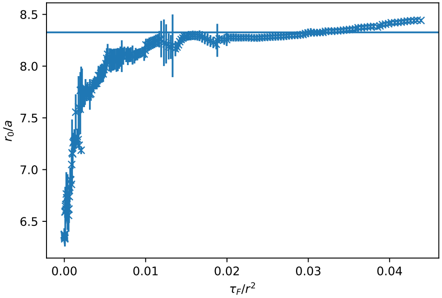


Figure 1 The r_0-scale in lattice units for the L20 lattice. The horizontal line corresponds to the expected value.

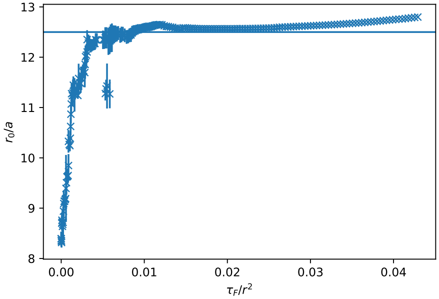


Figure 2 The r_0-scale in lattice units for the L30 lattice. The horizontal line corresponds to the expected value.

Tree level behavior

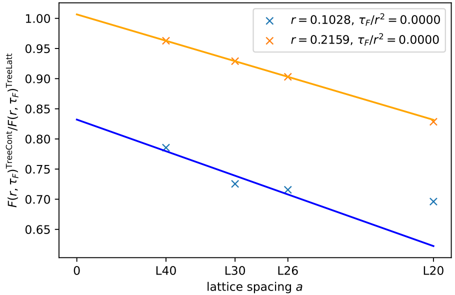


Figure 3 The ratio of the tree level forces in continuum and on the lattice at zero flowtime.

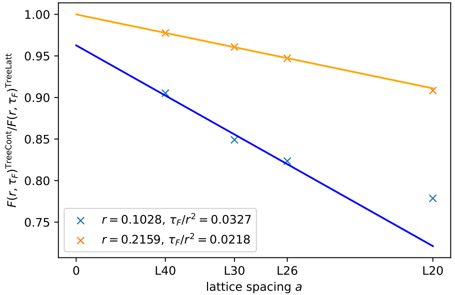


Figure 4 The ratio of the tree level forces in continuum and on the lattice at finite flowtime.