Codes toward automatic multiloop calculations for quarkonium physics

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Outline of the codes

1. Generate Feynman amplitudes
   - FeynArts / Qgraf
2. Manipulate the amplitudes
   - Final result: linear combinations of FIs with rational coefficients
3. Decompose Feynman integrals to bases
   - Called master integrals
4. Calculate master integrals
5. Calculate phase-space integrals

2. Manipulate the amplitudes

- For color and kinematics
  - Dirac matrix, Lorentz index, color algebra, projection …
- For Feynman integrals
  - Partial fraction, decompose integrals to predefined clusters (families)
- Package: LoopCalc (Since 2009)
  - Currently private, to be released soon
- Alternative packages on the market
  - FeynCalc, FDC, Form, …

3. Integral decomposition

➢ Techniques

- Integration-By-Parts identities
- Laporta algorithm
- Finite field reconstruction
- Block-triangular relations

➢ Package: Blade (Since 2020)

- Using FiniteFlow as a linear-equations solver
- Currently private, to be released soon

➢ Alternative packages on the market

- FIRE, LiteRed, Kira, Reduze, …

4. Master integrals calculation

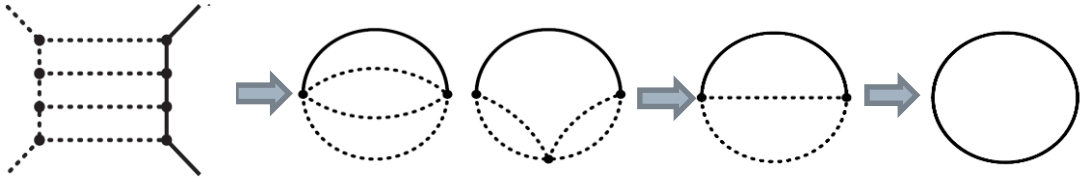
➢ Techniques

- Differential equations; Generalized power expansion; Auxiliary mass flow

➢ Package: AMFlow (Since 2018)

Liu, Ma, 2201.11669

- https://gitlab.com/multiloop-pku/amflow



➢ Already used for many quarkonium processes

- 2202.11615: NNLO for $e^+e^- \to J/\psi + \chi_{cJ}$
- 2205.06124: Two-loop for $\Upsilon \to$double charmonia
- 2206.03848: Two-loop $\eta_{c,b}$ leptonium production and decay
- 2207.14259: Three-loop for leptonic width of vector quarkonium
- 2208.04302: Three-loop for the decay constant of B_c
- 2208.10118: NNLO for $Z \to$ P-wave quarkonium

See also talks by Jia-Yue Zhang and Zhewen Mo

5. Phase space integrals

➢ Techniques

- Relating to loop integrals via reverse unitarity (if no jet)

$$\int \frac{\mathrm{d}^D p}{(2\pi)^D}(2\pi)\delta_+(p^2) = \int \frac{\mathrm{d}^D p}{(2\pi)^D}\left(\frac{\mathrm{i}}{p^2+\mathrm{i}0^+}+\frac{-\mathrm{i}}{p^2-\mathrm{i}0^+}\right)$$

- Then use techniques for loop integrals

➢ Alternative methods: Monte Carlo simulation

- Phase space slicing
- Integrand subtraction (dipole subtraction, antenna, …)

Summary

	Generate amplitudes	Manipulate amplitudes	Integral decomposition	Master integrals calculation	Phase space integrals
Package used	FeynArts or Qgraf	LoopCalc	Blade	AMFlow	Change to loop integrals
Notes	Open source	To be released	To be realsed	Open source	---

➢ The codes are general purpuse

- Valid for any-loop order, for both decays and productions
- Very stable for different kinematics
- Main challenge: integral decomposition is time/resource consuming

➢ Final result

- Piecewise functions in generalized power expansion form
- Small file size, easy to distribute to colleagues, fast for any later using