

# Higgs decay to charmonia via $c$ -quark fragmentation

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# Quarkonium: From the Standard Model to beyond

## Recall the history

- ▶ The “Standard Model” in the 1960s: “up”, “down”, “strange”
- ▶ **November Revolution:** The discovery of  $J/\psi$  in 1974  $\Rightarrow$  “charm”  
Richter and Ting explored th new energy regimes, not just to test the GIM mechanism.

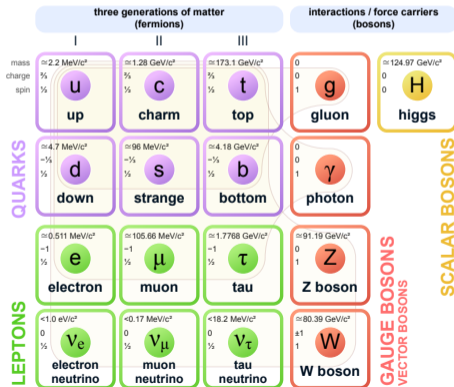
## Nowadays quarkium physics

- ▶ For over 20 years, we have been working the Standard Model better
- ▶ There may also be chance to see the hint of new physics beyond the Standard Model

# Why Higgs?

## A well understood and well tested model

### Standard Model of Elementary Particles

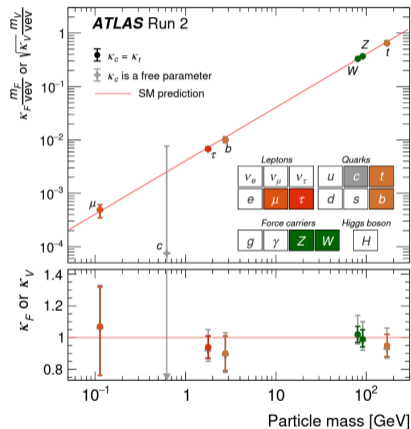


- ▶ Model doesn't make sense without Higgs or something like it
- ▶ The Higgs is a scalar particle whose interactions with other particles are predicted in terms of the Higgs mass
- ▶ It provides masses to all other elementary particles

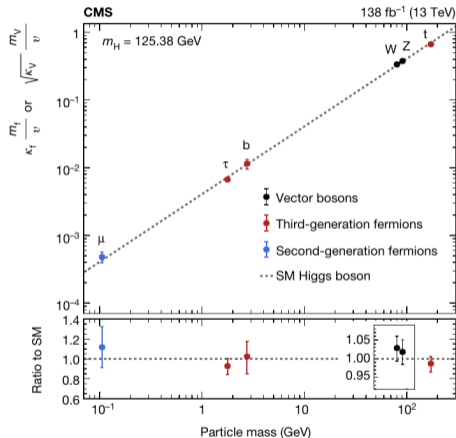
### Higgs physics: A portal to new physics

- ▶ LHC has gone from discovery to precision
- ▶ A telescope to high scale physics
- ▶ Interplay of theory and experiment is important

# Measure the Higgs couplings



[Nature 607 (2022) 52]



[Nature 607 (2022) 60]

Higgs to light fermion couplings are to be measured  $\Rightarrow$  The next task is charm quark

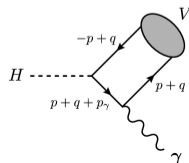
## Measure the Charm-Higgs coupling: current status

### Measuring $Hc\bar{c}$ coupling is not easy

- ▶ Small mass  $\Rightarrow$  Small branching fraction  $\text{BR}(H \rightarrow c\bar{c}) \simeq 2.8\%$
- ▶ Large QCD background at hadron colliders  $\Rightarrow$  Need  $c$ -tagging
- ▶  $c$ -tagging is challenging

### Current experimental searching

- ▶  $\kappa$  framework: For  $y_c^{\text{SM}} = \sqrt{2}m_c/v$ , set  $y_c = \kappa_c y_c^{\text{SM}}$
- ▶  $pp \rightarrow VH(c\bar{c})$ : **Need  $c$ -tagging**
  - ▶ LHC Run 2: ATLAS  $\kappa_c \leq 8.5$  [2201.11428], CMS  $1.1 < |\kappa_c| < 5.5$  [2205.05550]
  - ▶ Future HL-LHC:  $\kappa_c \leq 3$ . [2201.11428]
- ▶ Production of  $c\bar{c}$  bound states via Higgs decay:  $H \rightarrow J/\psi + \gamma$ 
  - ▶ Clean final states  $J/\psi \rightarrow \mu^+\mu^-$ , avoid  $c$ -tagging
  - ▶ The rate is too low:  $BR \sim 10^{-6}$ . [1306.5770, 1407.6695]
  - ▶ Result is less sensitive:  $\kappa_c \leq 100$ . [1807.00802, 1810.10056]



# Higgs decay to charmonia in Non-relativistic QCD (NRQCD)

## Separate the physics into two parts

$$\Gamma = \sum_{\mathbf{N}} \hat{\Gamma}_{\mathbf{N}}(H \rightarrow (Q\bar{Q})[\mathbf{N}] + X) \times \langle \mathcal{O}^h[\mathbf{N}] \rangle,$$

### ▶ Short distance coefficient (SDC):

$$d\hat{\Gamma}_{\mathbf{N}} = \frac{1}{2m_H} \frac{|\mathcal{M}|^2}{\langle \mathcal{O}^{Q\bar{Q}} \rangle} d\Phi_3$$

### ▶ Long distance matrix element (LDME)

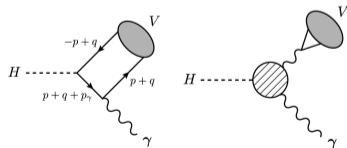
Related to the wave function at origin

$$\langle \mathcal{O}^{J/\psi} [{}^3S_1^{[1]}] \rangle = \frac{3N_c}{2\pi} |R(0)|^2, \quad \langle \mathcal{O}^{\eta_c} [{}^1S_0^{[1]}] \rangle = \frac{N_c}{2\pi} |R(0)|^2,$$

$$\langle \mathcal{O}^{Q\bar{Q}} \rangle = 6N_c, \text{ for } {}^3S_1^{[1]}, \quad \langle \mathcal{O}^{Q\bar{Q}} \rangle = 2N_c, \text{ for } {}^1S_0^{[1]}$$

## Higgs decay to $J/\psi$ and a photon

- ▶  $Hc\bar{c}$  diagram is suppressed  
⇒ Small branching fraction
- ▶ The dominant contribution is from  $H\gamma\gamma$  diagram ⇒ **Less sensitive to  $\kappa_c$**   
 $\Gamma_{H\gamma\gamma^*} \simeq 1.32 \times 10^{-8} \text{ GeV},$   
 $\Gamma_{\text{SM}} \simeq 1.00 \times 10^{-8} \text{ GeV}$  [1306.5770, 1407.6695]



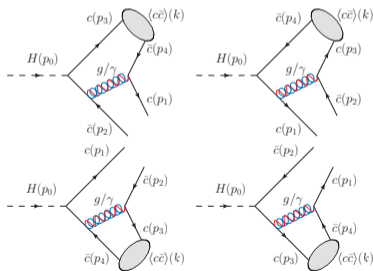
## Our idea: Look for a process with higher rate

$$H \rightarrow c + \bar{c} + J/\psi \text{ (or } \eta_c)$$

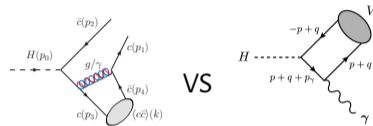
### Main contribution (Color-singlet):

Charm quark fragmentation to charmonia:

$${}^3S_1^{[1]}(J/\psi) \text{ and } {}^1S_0^{[1]}(\eta_c)$$



### Compare with $H \rightarrow J/\psi + \gamma$



- ▶ Enhancement from the quark fragmentation  $\Rightarrow$  **Larger rate**
- ▶ The  $Hc\bar{c}$  channel dominates  $\Rightarrow$  **More sensitive to  $\kappa_c$**

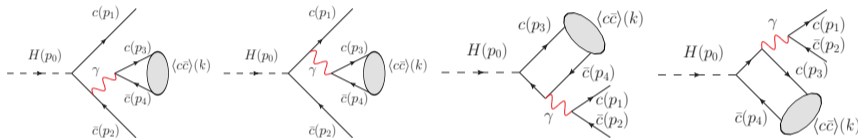
### More to calculate

- ▶ Corrections from QED and EW
- ▶ The color-octet mechanism

## More corrections from QED and EW sector

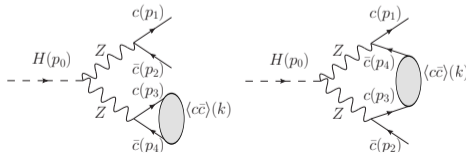
**Pure QED diagrams: sizable correction to  $^3S_1^{[1]}(J/\psi)$  production**

Single photon fragmentation (SPF):  $1/q^2 = 1/m_{J/\psi}^2 \Rightarrow$  **logarithmic enhancement**



**Electroweak correction from the  $HZZ$  diagrams**

One of the  $Z$  can be on shell  $\Rightarrow$  **resonance enhancement**



- Sizable for  $^1S_0^{[1]}(\eta_c)$  due to the larger axial  $Zc\bar{c}$  coupling.



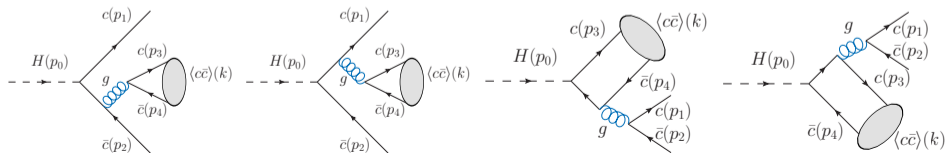
## Charmonium production via color octet states

- ▶ A quarkonium can also be produced through **color-octet**  $Q\bar{Q}$  Fock states
- ▶ New states involved:  ${}^3S_1^{[8]}$ ,  ${}^1S_0^{[8]}$ ,  ${}^3P_J^{[8]}$ , and  ${}^1P_1^{[8]}$
- ▶ The LDMEs  $\langle \mathcal{O}^h [{}^{2S+1}L_J^{\text{color}}] \rangle$  need to be fitted from experimental data

Reference	$\langle \mathcal{O}^{J/\psi} [{}^1S_0^{[8]}] \rangle$	$\langle \mathcal{O}^{J/\psi} [{}^3S_1^{[8]}] \rangle$	$\langle \mathcal{O}^{J/\psi} [{}^3P_0^{[8]}] \rangle / m_c^2$
G. Bodwin,	$(9.9 \pm 2.2) \times 10^{-2}$	$(1.1 \pm 1.0) \times 10^{-2}$	$(4.89 \pm 4.44) \times 10^{-3}$
K.T. Chao,	$(8.9 \pm 0.98) \times 10^{-2}$	$(3.0 \pm 1.2) \times 10^{-3}$	$(5.6 \pm 2.1) \times 10^{-3}$
Y. Feng,	$(5.66 \pm 4.7) \times 10^{-2}$	$(1.77 \pm 0.58) \times 10^{-3}$	$(3.42 \pm 1.02) \times 10^{-3}$

### New diagrams for ${}^3S_1^{[8]}$

Single gluon fragmentation (SGF):  $1/q^2 = 1/m_{J/\psi}^2 \Rightarrow$  **logarithmic enhancement**



# Numerical parameters

## Standard Model parameters

$$\alpha = 1/132.5, \alpha_s(2m_c) = 0.235, m_c^{\text{pole}} = 1.5 \text{ GeV}, m_c(m_H) = 0.694 \text{ GeV}, m_H = 125 \text{ GeV},$$

$$m_W = 80.419 \text{ GeV}, m_Z = 91.188 \text{ GeV}, v = 246.22 \text{ GeV}, y_c^{\text{SM}} = \frac{\sqrt{2}m_c(m_H)}{v} \approx 3.986 \times 10^{-3}.$$

## Choose the color-octet LDMEs

- ▶ Different fitting strategies lead to different LDME values.

Reference	$\langle \mathcal{O}^{J/\psi} [^1S_0^{[8]}] \rangle$	$\langle \mathcal{O}^{J/\psi} [^3S_1^{[8]}] \rangle$	$\langle \mathcal{O}^{J/\psi} [^3P_0^{[8]}] \rangle / m_c^2$
G. Bodwin,	$(9.9 \pm 2.2) \times 10^{-2}$	$(1.1 \pm 1.0) \times 10^{-2}$	$(4.89 \pm 4.44) \times 10^{-3}$
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- ▶ We take Bodwin's LDME fitting from CMS and CDF high  $p_T$  data.
- ▶ Use heavy quark spin symmetry (HQSS) to obtain the LDMEs for  $\eta_c$

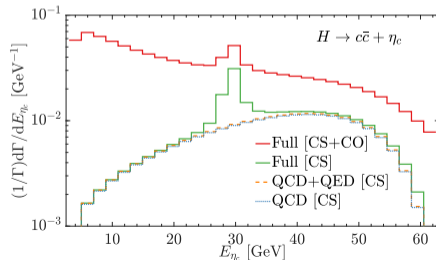
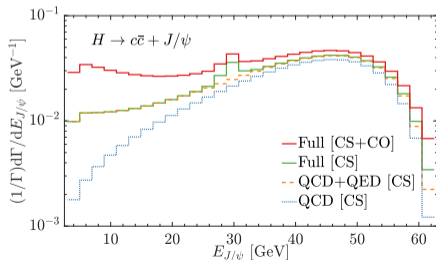
$$\langle \mathcal{O}^{\eta_c} [^1S_0^{[1,8]}] \rangle = \frac{1}{3} \langle \mathcal{O}^{J/\psi} [^3S_1^{[1,8]}] \rangle, \langle \mathcal{O}^{\eta_c} [^3S_1^{[8]}] \rangle = \langle \mathcal{O}^{J/\psi} [^1S_0^{[8]}] \rangle, \langle \mathcal{O}^{\eta_c} [^1P_1^{[8]}] \rangle = 3 \langle \mathcal{O}^{J/\psi} [^3P_0^{[8]}] \rangle,$$

# Standard Model results: The overall picture

## Decay width and branching fraction

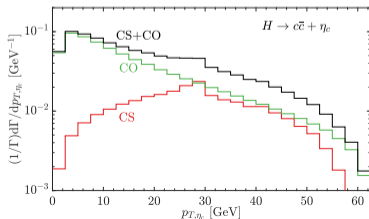
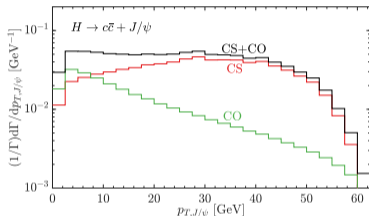
	QCD [CS]	QCD+QED [CS]	Full [CS]	Full [CO]	Full [CS+CO]
$\Gamma(H \rightarrow c\bar{c} + J/\psi)$ (GeV)	$4.8 \times 10^{-8}$	$5.8 \times 10^{-8}$	$6.1 \times 10^{-8}$	$2.2 \times 10^{-8}$	$8.3 \times 10^{-8}$
$\text{BR}(H \rightarrow c\bar{c} + J/\psi)$	$1.2 \times 10^{-5}$	$1.4 \times 10^{-5}$	$1.5 \times 10^{-5}$	$5.3 \times 10^{-6}$	$2.0 \times 10^{-5}$
$\Gamma(H \rightarrow c\bar{c} + \eta_c)$ (GeV)	$4.9 \times 10^{-8}$	$5.1 \times 10^{-8}$	$6.3 \times 10^{-8}$	$1.8 \times 10^{-7}$	$2.4 \times 10^{-7}$
$\text{BR}(H \rightarrow c\bar{c} + \eta_c)$	$1.2 \times 10^{-5}$	$1.2 \times 10^{-5}$	$1.5 \times 10^{-5}$	$4.5 \times 10^{-5}$	$6.0 \times 10^{-5}$

## Charmonium energy distributions

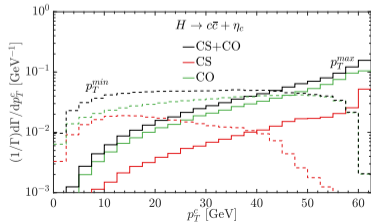
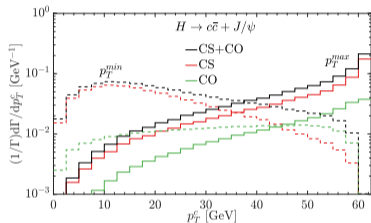


# Standard Model results: Transverse momentum ( $p_T$ ) distributions

## Charmonium $p_T$ distributions



## Free charm quark $p_T$ distributions



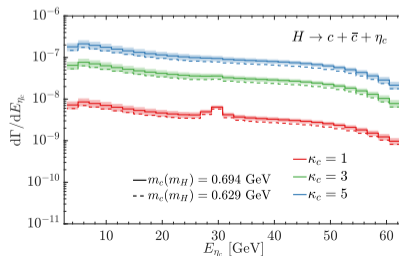
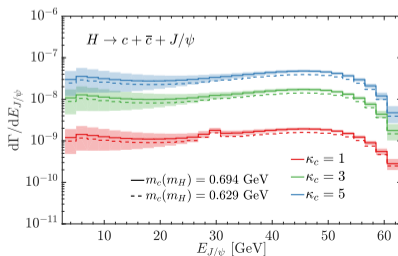
# Color-octet uncertainties from the LDMEs

Color-octet contributions:  ${}^3S_1^{[8]}$  dominates

	${}^3S_1^{[8]}$	${}^1S_0^{[8]}$	${}^1P_1^{[8]}$	${}^3P_J^{[8]}$	Total
$\Gamma(H \rightarrow c\bar{c} + J/\psi)$ (GeV)	$2.0 \times 10^{-8}$	$9.8 \times 10^{-10}$	-	$2.2 \times 10^{-10}$	$2.2 \times 10^{-8}$
$\text{BR}(H \rightarrow c\bar{c} + J/\psi)$	$5.0 \times 10^{-6}$	$2.4 \times 10^{-7}$	-	$5.3 \times 10^{-8}$	$5.3 \times 10^{-6}$
$\Gamma(H \rightarrow c\bar{c} + \eta_c)$ (GeV)	$1.8 \times 10^{-7}$	$3.6 \times 10^{-11}$	$1.0 \times 10^{-10}$	-	$1.8 \times 10^{-7}$
$\text{BR}(H \rightarrow c\bar{c} + \eta_c)$	$4.5 \times 10^{-5}$	$8.9 \times 10^{-9}$	$2.5 \times 10^{-8}$	-	$4.5 \times 10^{-5}$

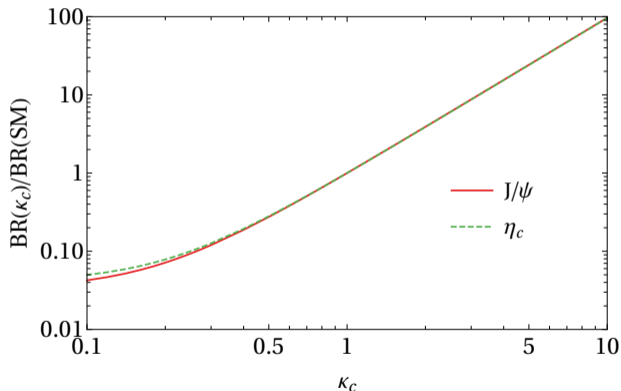
Take the  ${}^3S_1^{[8]}$  LDME for the uncertainty estimation

$$\text{BR}(H \rightarrow c\bar{c} + J/\psi) = (2.0 \pm 0.5) \times 10^{-5}, \quad \text{BR}(H \rightarrow c\bar{c} + \eta_c) = (6.0 \pm 1.0) \times 10^{-5}.$$



## Probe the $Hc\bar{c}$ coupling

Use the  $\kappa$  framework  $y_c = \kappa_c y_c^{\text{SM}}$ ,  $\text{BR} \approx \kappa_c^2 \text{BR}^{\text{SM}}$

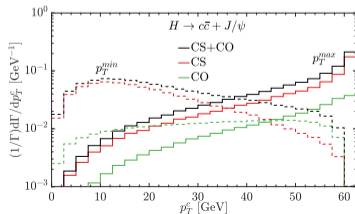


- ▶  $HZZ$  diagrams
- ▶ The  $H \rightarrow g^*g^*/\gamma^*\gamma^* \rightarrow J/\psi + c\bar{c}$  channel

## Some rough analysis

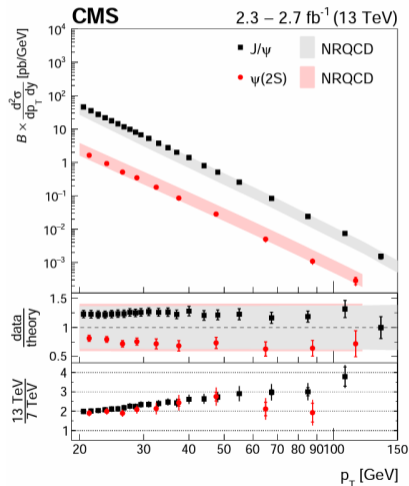
- ▶ Higgs production cross section at LHC  $\sigma_H \sim 50$  pb
- ▶ Expect HL-LHC  $L \sim 3 \text{ ab}^{-1}$  at ATLAS and CMS and  $L \sim 0.3 \text{ ab}^{-1}$  at LHCb
- ▶ Detection efficiency  $\epsilon$  for the final state  $c\bar{c} + \ell^+\ell^-$
- ▶  $\text{BR}(J/\psi \rightarrow \ell^+\ell^-) \sim 12\%$ ,  $\text{BR}(H \rightarrow J/\psi + c\bar{c}) \sim 2 \times 10^{-5}$
- ▶ Event number  $N = L\sigma_H \epsilon \text{BR}(H \rightarrow c\bar{c}\ell^+\ell^-) \approx 12 \kappa_c^2 \times \frac{L}{\text{ab}^{-1}} \times \frac{\epsilon}{10\%}$
- ▶ Considering the statistical error only  $\delta N \sim \sqrt{N}$  gives

$$\Delta\kappa_c \approx 15\% \times \left( \frac{L}{\text{ab}^{-1}} \times \frac{\epsilon}{10\%} \right)^{-1/2}$$



### Detection efficiency $\epsilon$ :

- ▶ Double charm-tagging  $(40\%)^2 \sim 16\%$
- ▶ Kinematic acceptance 50%
- ▶ Assume  $\epsilon \sim 10\% \Rightarrow \Delta\kappa_c \sim 15\%$

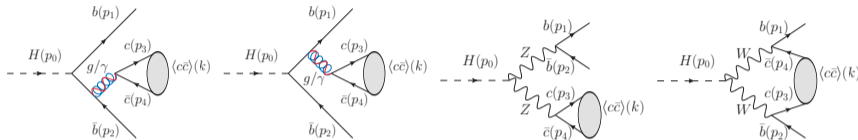
Background from  $pp \rightarrow J/\psi + X$ 

- ▶ Prompt  $J/\psi$  production  
 $\text{BR}(J/\psi \rightarrow \mu^+ \mu^-) \times \sigma(pp \rightarrow J/\psi) \simeq 860 \text{ pb}$   
**Charm-tagging is needed.**
- ▶ Estimate 75000 events for  $pp \rightarrow J/\psi + c\bar{c}$  at a 3 ab<sup>-1</sup> HL-LHC  
 Corresponding to a 25 fb cross section  
**Some kinematic cut may help.**



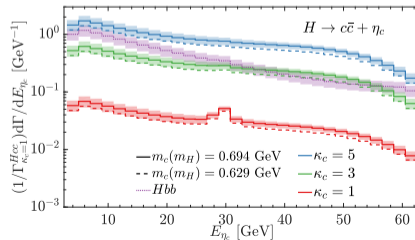
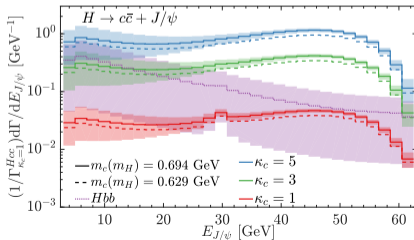
# Background from $H \rightarrow J/\psi + b\bar{b}$

Color-octet contribution dominates



## Charmonium energy distributions

Take the color-octet LDME uncertainty for error estimation



## More realistic discussions

- **If there were no background:**  $\Delta\kappa_c \sim 15\%$
- **However, there is background in the real world:**
  - ▶ Assume 10,000 background events after the selection cuts at the HL-LHC
  - ▶ Assume the detection efficiency  $\epsilon \sim 10\%$
  - ▶ The signal event number is given by

$$N = L\sigma_H \epsilon \text{BR}(H \rightarrow c\bar{c}\ell^+\ell^-) \approx 12 \kappa_c^2 \times \frac{L}{\text{ab}^{-1}} \times \frac{\epsilon}{10\%}$$

- ▶ Sensitivity  $S \simeq N_{\text{signal}}/\sqrt{N_{\text{Background}}}$   
 $\Rightarrow$  It is possible to reach  $2\sigma$  for  $\kappa_c \approx 2.4$ .
- ▶ systematic effect  $N_{\text{signal}}/N_{\text{Background}} = 2\%$  for  $\kappa_c \approx 2.4$ .

## Summary and prospects

- ▶ Quarkonium study can help to understand the SM better and also search for BSM physics.
- ▶ Testing the SM mass generation mechanism helps BSM physics searches.
- ▶ The Yukawa couplings of the 3rd generation fermions are precisely measured  
⇒ Charm quark is the next target.

### **New approach to determine the Charm-Higgs coupling:** $H \rightarrow J/\psi + c\bar{c}$

- ▶ The rate is larger due to the fragmentation enhancements
- ▶ There are both color-singlet and color-octet contributions
- ▶ The QED and EW corrections can be sizable, so need to be included
- ▶ The SM prediction gives  $BR \sim 2 \times 10^{-5}$
- ▶ For a possible  $3 \text{ ab}^{-1}$  HL-LHC, with a 10% final state detection efficiency  $\Rightarrow \Delta\kappa_c \sim 10\%$
- ▶ Assume there are 10,000 background events  $\Rightarrow 2\sigma$  for  $\kappa_c \simeq 2.4$

### **More work in progress:**

- ▶ Background analysis, detector/systematic effects
- ▶ Better LDMEs fittings, higher order calculations/resummation ...

# Standard Model results: Who is contributing?

## Color-octet contributions

	${}^3S_1^{[8]}$	${}^1S_0^{[8]}$	${}^1P_1^{[8]}$	${}^3P_J^{[8]}$	Total
$\Gamma(H \rightarrow c\bar{c} + J/\psi)$ (GeV)	$2.0 \times 10^{-8}$	$9.8 \times 10^{-10}$	-	$2.2 \times 10^{-10}$	$2.2 \times 10^{-8}$
$\text{BR}(H \rightarrow c\bar{c} + J/\psi)$	$5.0 \times 10^{-6}$	$2.4 \times 10^{-7}$	-	$5.3 \times 10^{-8}$	$5.3 \times 10^{-6}$
$\Gamma(H \rightarrow c\bar{c} + \eta_c)$ (GeV)	$1.8 \times 10^{-7}$	$3.6 \times 10^{-11}$	$1.0 \times 10^{-10}$	-	$1.8 \times 10^{-7}$
$\text{BR}(H \rightarrow c\bar{c} + \eta_c)$	$4.5 \times 10^{-5}$	$8.9 \times 10^{-9}$	$2.5 \times 10^{-8}$	-	$4.5 \times 10^{-5}$

## Contributions with respect to QCD

$\hat{\Gamma}_N / \hat{\Gamma}_N^{\text{QCD}}$	${}^1S_0^{[1]}$	${}^3S_1^{[1]}$	${}^1S_0^{[8]}$	${}^3S_1^{[8]}$	${}^1P_1^{[8]}$	${}^3P_0^{[8]}$	${}^3P_1^{[8]}$	${}^3P_2^{[8]}$
QCD	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
QED	$1.1 \times 10^{-4}$	0.077	0.0073	$1.1 \times 10^{-5}$	0.0068	0.0073	0.0073	0.0073
QCD×QED	0.021	0.14	-0.17	0.0012	-0.15	-0.17	-0.17	-0.17
EW	0.24	0.051	0.28	$2.6 \times 10^{-4}$	1.4	0.29	0.33	1.5

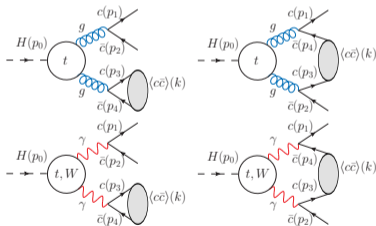
- ▶ QCD is dominant in most of the Fock states
- ▶ SPF brings sizable QED correction to  ${}^3S_1^{[1]}$ , but it is forbidden for  ${}^1S_0^{[1]}$
- ▶ SGF makes  ${}^3S_1^{[8]}$  super large
- ▶ For  ${}^1S_0^{[8]}$  and  ${}^3P_J^{[8]}$ , only quark fragmentation contributions  $\Rightarrow$  QED and QCD differ by a constant
- ▶ EW correction is large since  $Z$  is closed to its mass shell

## Worry about VMD?

$$H \rightarrow J/\psi + c\bar{c}$$

- ▶ Larger decay rate:  $BR \simeq 2 \times 10^{-5}$
- ▶ Sensitive to  $Hc\bar{c}$  coupling: QCD dominates
- ▶ Other diagrams

$$H \rightarrow g^*g^*/\gamma^*\gamma^* \rightarrow J/\psi + c\bar{c}$$



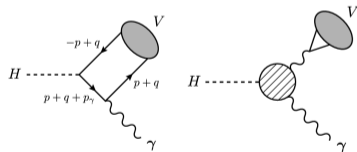
$$BR(g^*g^*) \sim 2.5 \times 10^{-6}, BR(\gamma^*\gamma^*) < 2 \times 10^{-7}$$

● **No need to worry about VMD**

$$H \rightarrow J/\psi + \gamma$$

- ▶ Small decay rate:  $BR \simeq 2.8 \times 10^{-6}$
- ▶ Insensitive to  $Hc\bar{c}$  coupling  
 $\Rightarrow \kappa_c \leq 100$

## VMD dominates



- $\gamma^* \rightarrow J/\psi$  dominates over  $Hc\bar{c}$   
Two orders of magnitude larger.

When is  $y_c$  not related to the charm mass?

### Higgs Effective Field Theory (HEFT)

$SU(2)$  doublets of the global  $SU(2)_{L,R}$  symmetries:

$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix}, \quad Q_R = \begin{pmatrix} U_R \\ D_R \end{pmatrix}, \quad L_L = \begin{pmatrix} \nu_L \\ E_L \end{pmatrix}, \quad L_R = \begin{pmatrix} 0 \\ E_R \end{pmatrix}.$$

Define  $U(x) \equiv \exp(i\sigma_a \pi^a(x)/v)$ , so that the Lagrangian contains

$$\mathcal{L} \supset -\frac{v}{\sqrt{2}} \bar{Q}_L U y_Q(h) Q_R - \frac{v}{\sqrt{2}} \bar{L}_L U y_L(h) L_R + h.c.$$

The functions  $y_Q(h)$  and  $y_L(h)$  control the Yukawa couplings

$$y_Q(h) \equiv \text{diag} \left( \sum_n y_U^{(n)} \frac{h^n}{v^n}, \sum_n y_D^{(n)} \frac{h^n}{v^n} \right), \quad y_L(h) \equiv \text{diag} \left( 0, \sum_n y_\ell^{(n)} \frac{h^n}{v^n} \right) L$$

$n = 0$  is for mass term,  $n = 1$  is for Yukawa coupling.

# Fragmentation formalism

## The decay width is written as a convolution

Define  $z \equiv 2E_\psi/m_H$

$$\frac{d\Gamma}{dz}(H \rightarrow \psi(z)q\bar{q}) = 2C_q \otimes D_q + C_g \otimes D_g, C \otimes D \equiv \int_z^1 C(y)D(z/y) \frac{dy}{y}$$

## Hard coefficient

$$C_q(\mu^2, z) = \Gamma(H \rightarrow q\bar{q})\delta(1-z)$$

$$C_g(\mu^2, z) = \frac{4\alpha_s}{3\pi}\Gamma(H \rightarrow q\bar{q}) \left[ \frac{(z-1)^2 + 1}{z} \log \left( \frac{(1-z)z^2 m_H^2}{\mu^2} \right) - z \right]$$

## Fragmentation functions

$$D_{c \rightarrow J/\psi}^{(1)}(\mu^2, z) = \frac{128\alpha_s^2}{243m_{J/\psi}^3} \frac{z(1-z^2)}{(2-z)^6} (16 - 32z + 72z^2 - 32z^3 + 5z^4) \langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle$$

$$D_{q \rightarrow \psi}^{(8)}(\mu^2, z) = \frac{2\alpha_s^2}{9m_\psi^3} \left[ \frac{(z-1)^2 + 1}{z} \log \left( \frac{\mu^2}{m_\psi^2(1-z)} \right) - z \right] \langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle$$