Quarkonium in medium

up to NLO in E/T

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Based on

(1) N. Brambilla, M.A. Escobedo, A. Islam, M. Strickland, A. Tiwari, A. Vairo and P. Vander Griend
Heavy quarkonium dynamics at next-to-leading order in the binding energy over temperature
JHEP 08 (2022) 303 `arXiv:2205.10289`

(2) N. Brambilla, M.A. Escobedo, M. Strickland, A. Vairo, P. Vander Griend and J.H. Weber
Bottomonium production in heavy-ion collisions using quantum trajectories: Differential observables and momentum anisotropy
Phys. Rev. D 104 (2021) 094049 `arXiv:2107.06222`

(3) N. Brambilla, M.A. Escobedo, M. Strickland, A. Vairo, P. Vander Griend and J.H. Weber
Bottomonium suppression in an open quantum system using the quantum trajectories method
JHEP 05 (2021) 136 `arXiv:2012.01240`

(4) N. Brambilla, M.A. Escobedo, J. Soto and A. Vairo
Heavy quarkonium suppression in a fireball
Phys. Rev. D 97 (2018) 074009 `arXiv:1711.04515`

(5) N. Brambilla, M.A. Escobedo, J. Soto and A. Vairo
Quarkonium suppression in heavy-ion collisions: an open quantum system approach
Phys. Rev. D 96 (2017) 034021 `arXiv:1612.07248`

Energy scales

Quarkonium in a medium is characterized by several energy scales:

- the scales of a non-relativistic bound state
  (v is the relative heavy-quark velocity; $v \sim \alpha_{\rm s}$ for a Coulombic bound state):
  M (mass),
  Mv (momentum transfer, inverse distance),
  Mv^2 (kinetic energy, binding energy, potential V), ...

- the thermodynamical scales:
  T (temperature), ...

  T stands for a generic inverse correlation length characterizing the medium.
  For definiteness we will assume that the system is locally in thermal equilibrium so that a slowly varying time-dependent temperature can be defined.

The non-relativistic scales are hierarchically ordered: $M \gg Mv \gg Mv^2$

Degrees of freedom (@ energy $\ll Mv$)

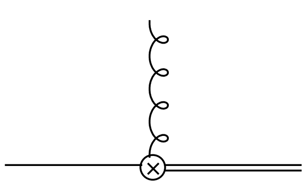
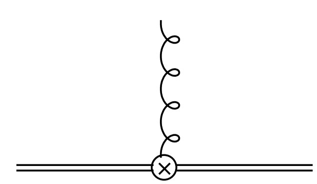
Fields:

- $S^\dagger$ creates a quark-antiquark pair in a color singlet configuration.
- $O^\dagger$ creates a (unbound) quark-antiquark pair in a color octet configuration.
- gluons and light quarks.

Propagators:

- singlet ——— and octet ═══ governed (in a Coulombic system) by the Hamiltonians $h_s = \frac{\mathbf{p}^2}{M} - \frac{4}{3}\frac{\alpha_s}{r} + \ldots$ and $h_o = \frac{\mathbf{p}^2}{M} + \frac{\alpha_s}{6r} + \ldots$, respectively.

Electric-dipole interactions:

-  $= O^\dagger \mathbf{r} \cdot g\mathbf{E}\,\mathrm{S}$  $= O^\dagger \{\mathbf{r} \cdot g\mathbf{E},\, \mathrm{O}\}$

Open quantum system

- System: heavy quarks/quarkonium
- Environment: quark gluon plasma

We may define a (reduced) density matrix for the heavy quark-antiquark pair in a color singlet and octet configuration:

$$\begin{aligned}
\langle \mathbf{r}', \mathbf{R}' | \rho_s(t'; t) | \mathbf{r}, \mathbf{R} \rangle &\equiv \mathrm{Tr}\{\rho_{\mathrm{full}}(\tau_{\mathrm{med}})\, S^\dagger(t, \mathbf{r}, \mathbf{R}) S(t', \mathbf{r}', \mathbf{R}')\} \\
\langle \mathbf{r}', \mathbf{R}' | \rho_o(t'; t) | \mathbf{r}, \mathbf{R} \rangle \frac{\delta^{ab}}{8} &\equiv \mathrm{Tr}\{\rho_{\mathrm{full}}(\tau_{\mathrm{med}})\, O^{a\dagger}(t, \mathbf{r}, \mathbf{R}) O^b(t', \mathbf{r}', \mathbf{R}')\}
\end{aligned}$$

τ_{med} fm is the time formation of the plasma.

The system is in non-equilibrium because through interaction with the environment (quark gluon plasma) singlet and octet quark-antiquark states continuously transform in each other although the number of heavy quarks is conserved: $\mathrm{Tr}\{\rho_s\} + \mathrm{Tr}\{\rho_o\} = 1$.

Expansions

- The density of heavy quarks is much smaller than the one of the light d.o.f.: we expand at first order in the heavy quark-antiquark density.
- We consider T much smaller than the inverse Bohr radius of the quarkonium: we expand up to order r^2 in the multipole expansion.

Evolution equations

A way of writing the evolution equations is

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_{nm} h_{nm} \left(L_i^n \rho L_i^{m\,\dagger} - \frac{1}{2}\{L_i^{m\,\dagger} L_i^n, \rho\} \right)$$

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix} \qquad H = \begin{pmatrix} h_s + \frac{\Sigma_s - \Sigma_s^\dagger}{2i} & 0 \\ 0 & h_o + \frac{\Sigma_o - \Sigma_o^\dagger}{2i} \end{pmatrix}$$

$$\Sigma_s(t) = r^i\, A_i^{so\,\dagger}(t) \qquad \Sigma_o(t) = \frac{r^i\, A_i^{os\,\dagger}(t)}{8} + \frac{5}{16} r^i\, A_i^{oo\,\dagger}(t)$$

$$L_i^0 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} r^i \qquad L_i^1 = \begin{pmatrix} 0 & 0 \\ 0 & \frac{5}{16} A_i^{oo\,\dagger} \end{pmatrix}$$

$$L_i^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} r^i \qquad L_i^3 = \begin{pmatrix} 0 & \frac{1}{8} A_i^{os\,\dagger} \\ A_i^{so\,\dagger} & 0 \end{pmatrix}$$

with $\quad A_i^{so}(t) = \frac{g^2}{6} \int_{\tau_{\rm med}}^{t} dt_2\, e^{ih_s(t_2-t)}\, r^j\, e^{ih_o(t-t_2)}\, \langle E^{a,j}(t_2,\mathbf{0}) E^{a,i}(t,\mathbf{0})\rangle$

Positivity

The matrix h_{nm} is

$$h = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

If h were a positive definite matrix then it would always be possible to redefine the operators L_i^n in such a way that the evolution equation would be of the Lindblad form.

Since, however, h is not a positive definite matrix, the Lindblad theorem does not guarantee that the equations may be brought into a Lindblad form.

A special case is the strongly-coupled case at LO in E/T (similarly at NLO).
There $L_i^1 \propto L_i^0$ and $L_i^3 \propto L_i^2$, which allows to rotate L_i^n in such a way that they are orthogonal to the eigenspace of h with negative eigenvalues, eventually leading to an evolution equation of the Lindblad form.

Time scales

Environment correlation time: $\tau_E \sim \dfrac{1}{T}$

System intrinsic time scale: $\tau_S \sim \dfrac{1}{E}$

System relaxation time: $\tau_R \sim \dfrac{1}{\text{self-energy}} \sim \dfrac{1}{\alpha_s a_0^2 \Lambda^3}$ $\qquad a_0 =$ Bohr radius, $\Lambda = T, E$

- Because we have assumed $1/a_0 \gg \Lambda$, it follows $\tau_R \gg \tau_S, \tau_E$ which, after resummation ($t - \tau_{\rm med} \gg \tau_R$), qualifies the system as Markovian.

- If $T \gg E$ then $\tau_S \gg \tau_E$ which qualifies the motion of the system as quantum Brownian.

○ Akamatsu PRD 91 (2015) 056002

From the evolution equations to the Lindblad equation

Under the Markovian

$$\tau_R \gg \tau_S, \tau_E \quad \text{or} \quad \frac{1}{a_0} \gg E, T$$

and quantum Brownian motion condition

$$\tau_S \gg \tau_E \quad \text{or} \quad T \gg E$$

at (N)LO in E/T the evolution equations can be written in the Lindblad form.

Lindblad equation for a strongly coupled plasma @ LO in E/T

At LO in E/T the Lindblad equation for a strongly coupled plasma reads

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_i (C_i \rho C_i^\dagger - \frac{1}{2}\{C_i^\dagger C_i, \rho\})$$

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2}\gamma \begin{pmatrix} 1 & 0 \\ 0 & \frac{7}{16} \end{pmatrix}$$

$$C_i^0 = \sqrt{\frac{\kappa}{8}}\, r^i \begin{pmatrix} 0 & 1 \\ \sqrt{8} & 0 \end{pmatrix}, \qquad C_i^1 = \sqrt{\frac{5\kappa}{16}}\, r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Thermal width and mass shift

The quantity κ is related to the thermal decay width of the heavy quarkonium.
In particular for $1S$ states, we have (Σ_s = self-energy)

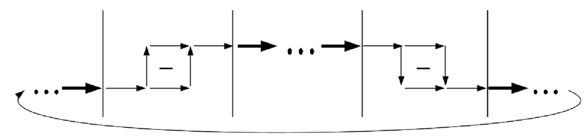
$$\Gamma(1S) = -2\langle \mathrm{Im}\,(-i\Sigma_s)\rangle = 3a_0^2\,\kappa$$

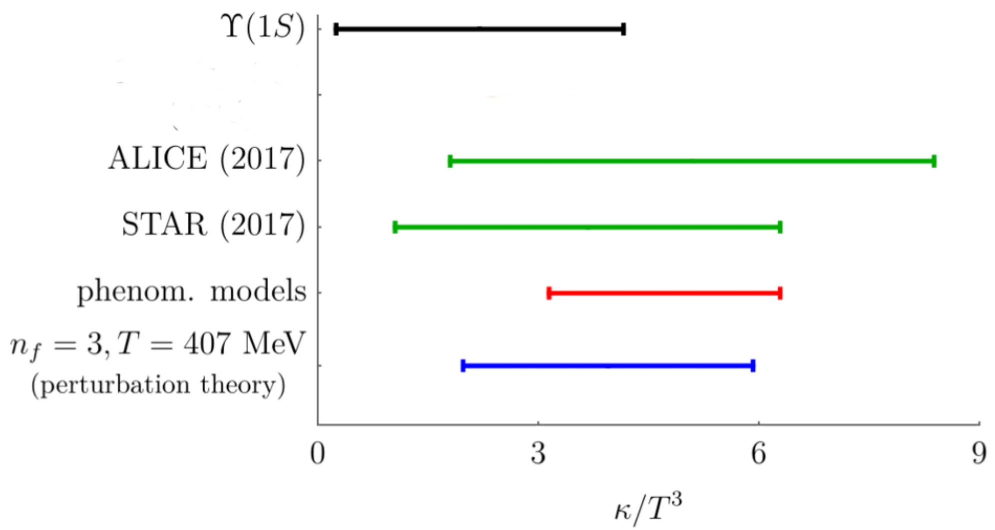
The quantity γ is related to the thermal mass shift of the heavy quarkonium.
In particular for $1S$ states, we have

$$\delta M(1S) = \langle \mathrm{Re}\,(-i\Sigma_s)\rangle = \frac{3}{2}a_0^2\,\gamma$$

κ

$$\kappa = \frac{g^2}{18} \, \mathrm{Re} \int_{-\infty}^{+\infty} ds \, \langle \mathrm{T} \, E^{a,i}(s, \mathbf{0}) \, \phi^{ab}(s, 0) \, E^{b,i}(0, \mathbf{0}) \rangle =$$



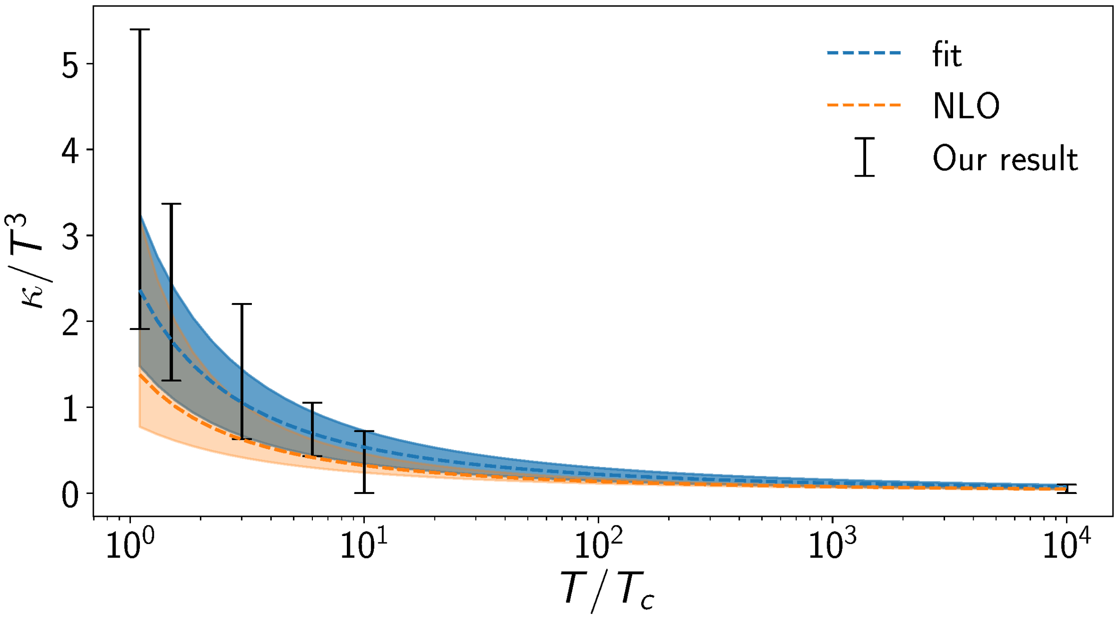


$[\hat{\kappa} \equiv \kappa/T^3]$

- Brambilla Escobedo Vairo Vander Griend PRD 100 (2019) 054025
  from the lattice data of Aarts et al JHEP 11 (2011) 103
  and Kim Petreczky Rothkopf JHEP 11 (2018) 088

Momentum diffusion coefficient

κ is related to the quark momentum diffusion coefficient:



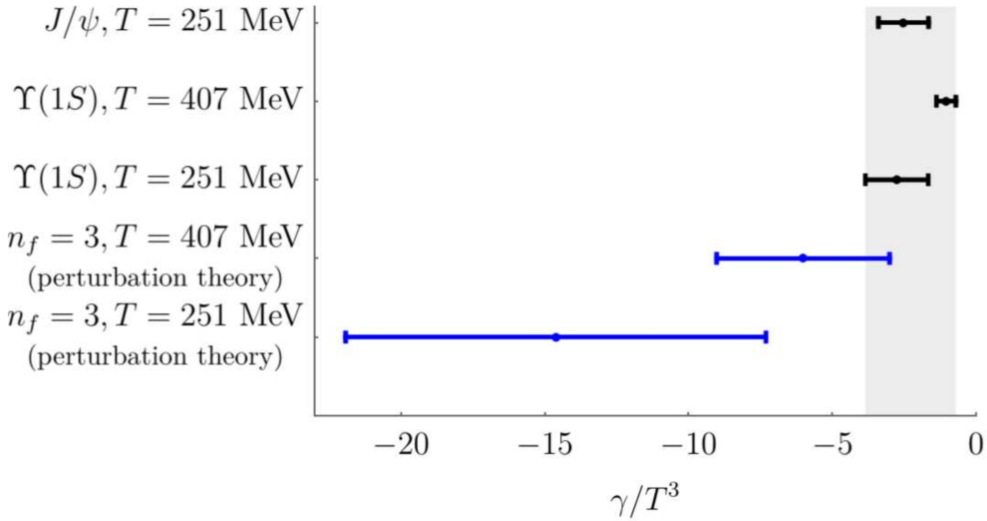
○ `Brambilla Leino Petreczky Vairo PRD 102 (2020) 074503`

The nature of the exact relation is however under investigation.

○ `Eller Ghiglieri Moore PRD 99 (2019) 094042`

γ

$$\gamma = \frac{g^2}{18}\,\mathrm{Im}\int_{-\infty}^{+\infty} ds\,\langle\,\mathrm{T}\,E^{a,i}(s,\mathbf{0})\,\phi^{ab}(s,0)\,E^{b,i}(0,\mathbf{0})\rangle$$



$[\hat{\gamma} \equiv \gamma/T^3]$

- Brambilla Escobedo Vairo Vander Griend PRD 100 (2019) 054025
  from the lattice data of Kim Petreczky Rothkopf JHEP 11 (2018) 088

Evolution set up

- After heavy-ion collision, quark-antiquarks propagate freely up to $\tau_{\mathrm{med}} = 0.6$ fm.
- From τ_{med} fm to the freeze-out time t_F they propagate in medium.
- We assume the medium to be locally in thermal equilibrium.
- We use a 3+1D dissipative relativistic hydrodynamics code that makes use of the quasiparticle anisotropic hydrodynamics (aHydroQP) framework. The code uses a realistic equation of state fit to lattice QCD measurements and is tuned to soft hadronic data collected in 5.02 TeV collisions using smooth optical Glauber initial conditions.
  - `Alqahtani Nopoush Strickland PRC 92 (2015) 054910, 95 (2017) 034906`
    `Alqahtani Nopoush Strickland PPNP 101 (2018) 204`

Quantum trajectories algorithm

The `QTraj` code implements the quantum trajectories algorithm and the waiting time approach as follows.

1. Initialize a wave function $|\psi(t_0)\rangle$ at initial time t_0, which corresponds to the initial quantum state of the particle given by $\rho(t_0) = |\psi(t_0)\rangle\langle\psi(t_0)|$.
2. Generate a random number $0 < r_1 < 1$ and evolve the wave function forward in time with H_{eff} until $||\, e^{-i\int_{t_0}^{t} dt' H_{\text{eff}}(t')}|\psi(t_0)\rangle\, ||^2 \leq r_1$ where $H_{\text{eff}} = H - i\Gamma/2$, $\Gamma = \sum \Gamma_n$ and $\Gamma_n = C_n^\dagger C_n$. Denote the first time step fulfilling the inequality as the jump time t_j. If the jump time is greater than the simulation run time t_F, end the simulation at time t_F; otherwise, proceed to step 3.
3. At time t_j, initiate a quantum jump:
   - (a) If the system is in a singlet configuration, jump to octet. If the system is in an octet configuration, generate a random number $0 < r_2 < 1$ and jump to singlet if $r_2 < 2/7$; otherwise, remain in the octet configuration.
   - (b) Generate a random number $0 < r_3 < 1$; if $r_3 < l/(2l+1)$, take $l \to l-1$; otherwise, take $l \to l+1$.
   - (c) Multiply the wavefunction by r and normalize.
4. Continue from step 2.

- Ba et al CPC 273 (2022) 108266

Jumps and probabilities

The probabilities in step 3 correspond to the branching fractions into a state of different color and/or angular momentum:

$$p_n = \frac{\langle \psi(t)|\Gamma_n|\psi(t)\rangle}{\langle \psi(t)|\Gamma|\psi(t)\rangle}$$

Each evolution of the wave function from time t_0 to t_F is called a quantum trajectory.
In practice, a large number of quantum trajectories must be generated.
As the number of trajectories considered increases, the average converges to the solution of the Lindblad equation.

- Dalibard Castin Molmer PRL 68 (1992) 580
  Daley AP 63 (2014) 77

Simulation set up

We employ a radial lattice of $\texttt{NUM}= 4096$ lattice sites and a radial length of $\texttt{L}= 80\,\mathrm{GeV}^{-1}$, corresponding to a radial lattice spacing of $a \approx 0.0195\,\mathrm{GeV}^{-1}$. The real time integration is discretized with a time step of $\texttt{dt}= 0.001\,\mathrm{GeV}^{-1}$.

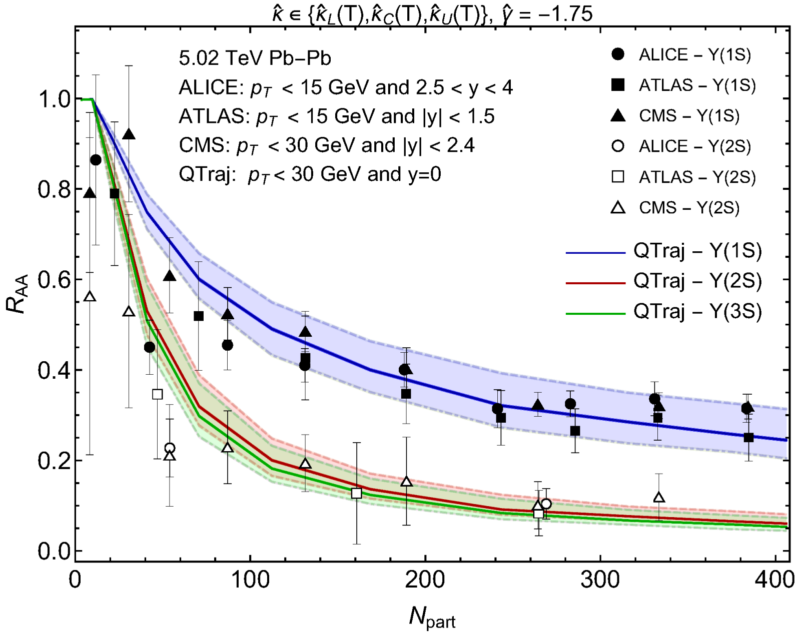
We sample approximately 7 - 9 $\times 10^5$ independent physical trajectories for each choice of κ/T^3 and γ/T^3, with approximately 50-100 quantum trajectories per physical trajectory. To generate each physical trajectory, we sample the bottomonium production point in the transverse plane using the nuclear binary collision overlap profile $N_{AA}^{\text{bin}}(x, y, b)$, the initial transverse momentum of the state p_T from an E_T^{-4} spectrum, and the initial azimuthal angle ϕ of the state's momentum uniformly in $[0, 2\pi)$. We bin the results for the survival probability as a function of centrality, p_T, and ϕ. This allows us to make predictions for differential observables such as R_{AA} as a function of p_T and elliptic flow.

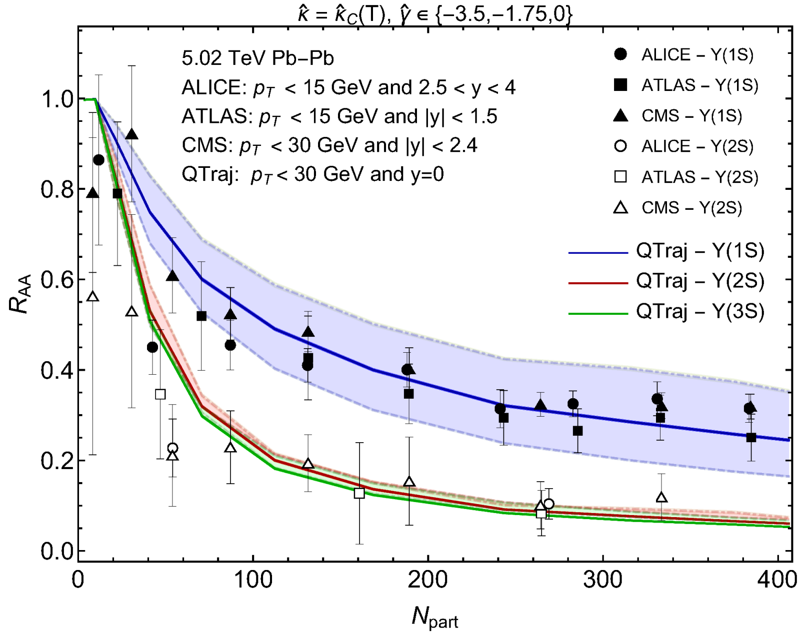
To ensure that the hierarchy of energy scales of the EFT is fulfilled, we evolve the state in the vacuum when the temperature falls below $T_F = 250$ MeV.

Bottomonium nuclear modification factor

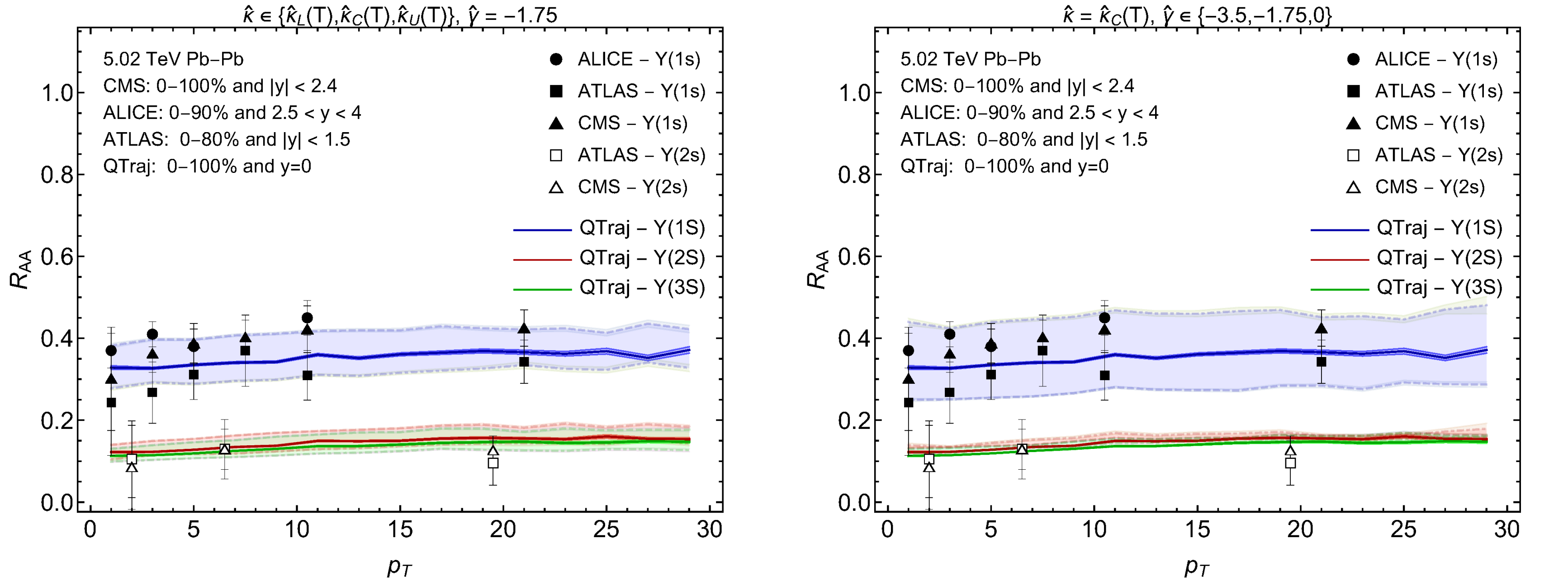
We compute the nuclear modification factor R_{AA} from

$$R_{AA}(nS) = \frac{\langle n, \mathbf{q} | \rho_s(t_F; t_F) | n, \mathbf{q} \rangle}{\langle n, \mathbf{q} | \rho_s(0; 0) | n, \mathbf{q} \rangle}$$

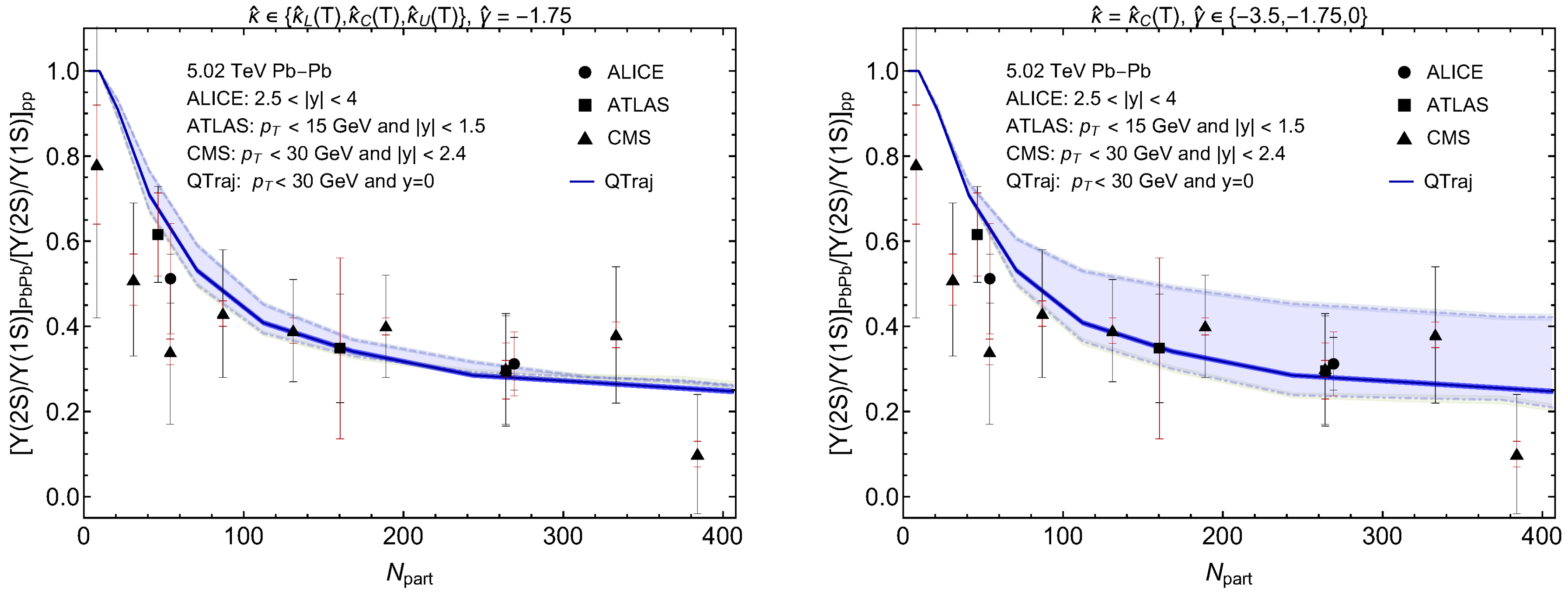




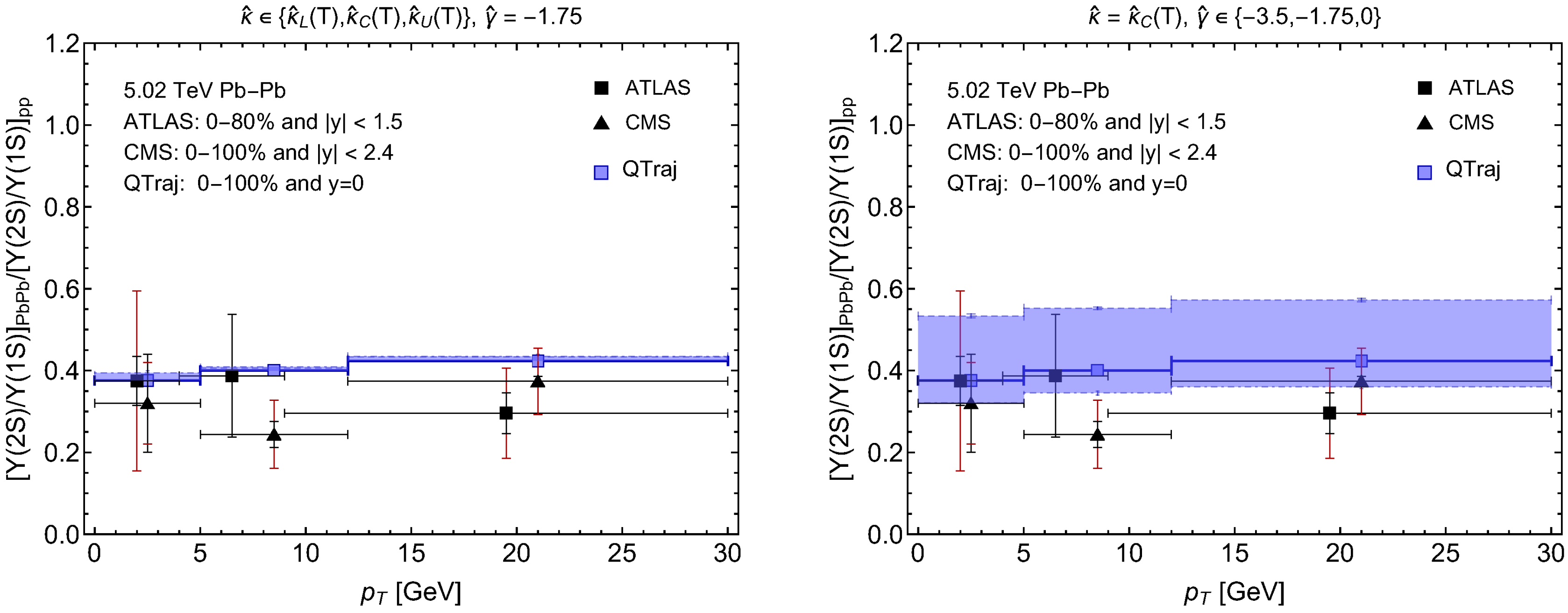
Bottomonium nuclear modification factor vs p_T



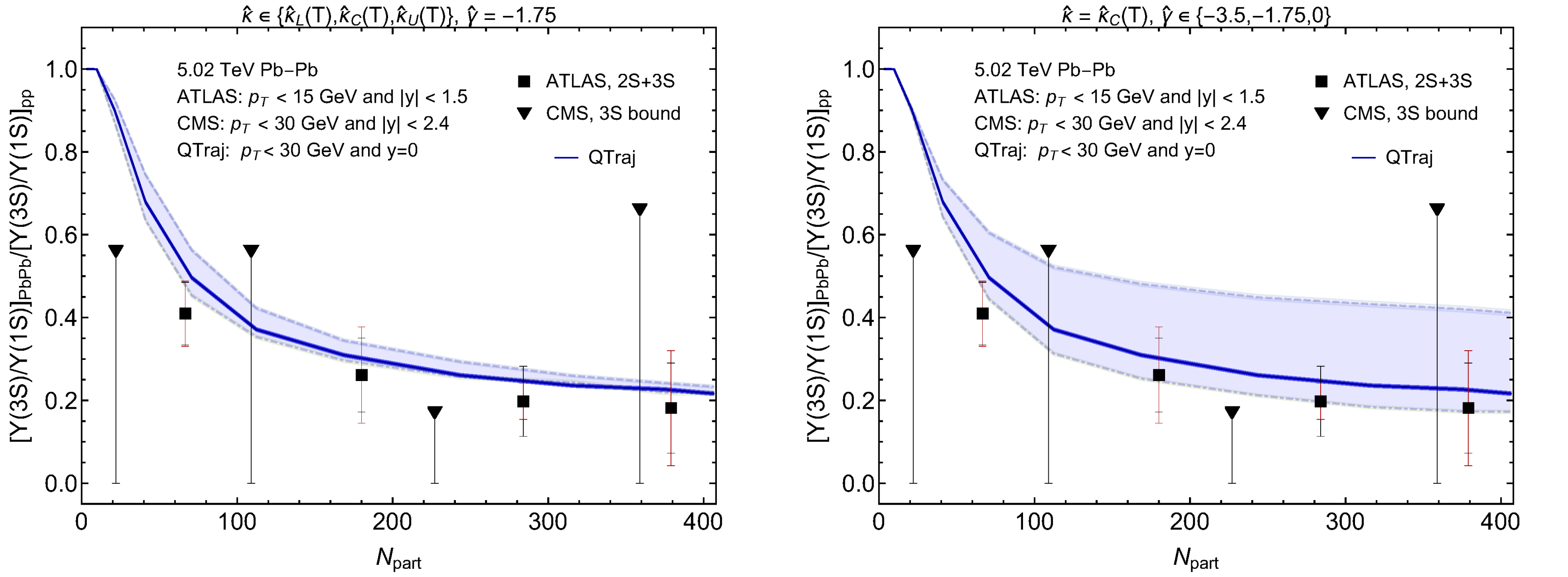
Double ratio $R_{AA}[\Upsilon(2S)]$ to $R_{AA}[\Upsilon(1S)]$



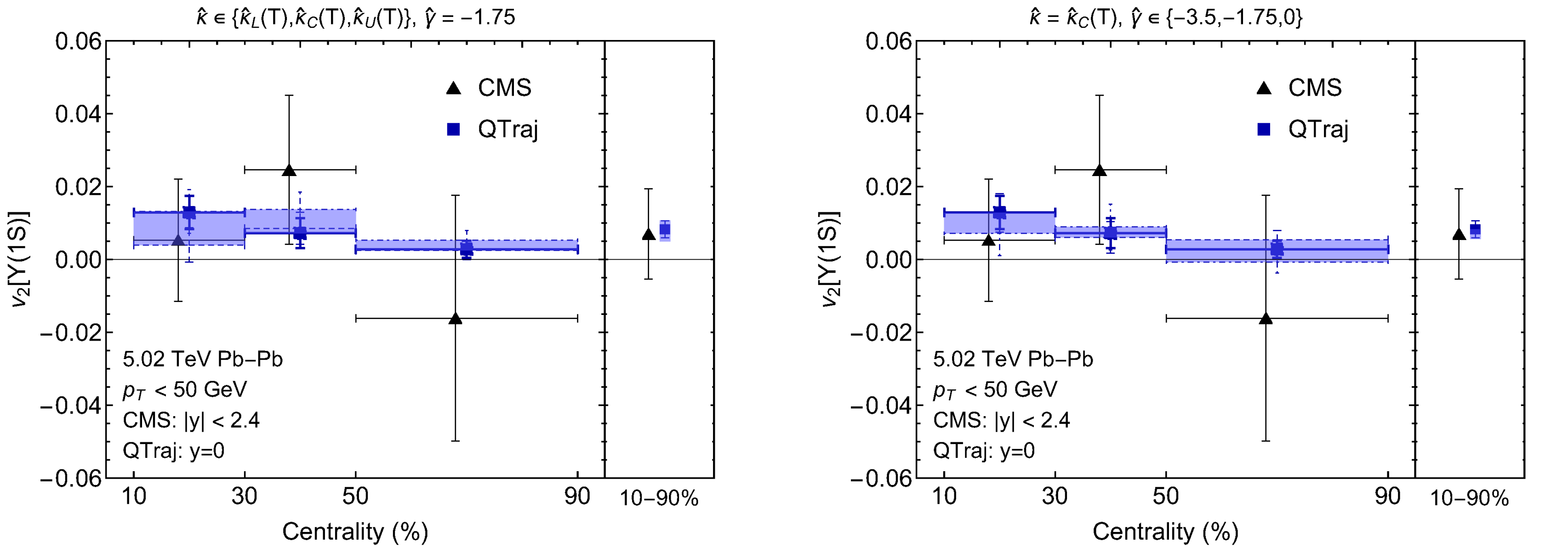
Double ratio $R_{AA}[\Upsilon(2S)]$ to $R_{AA}[\Upsilon(1S)]$ vs p_T



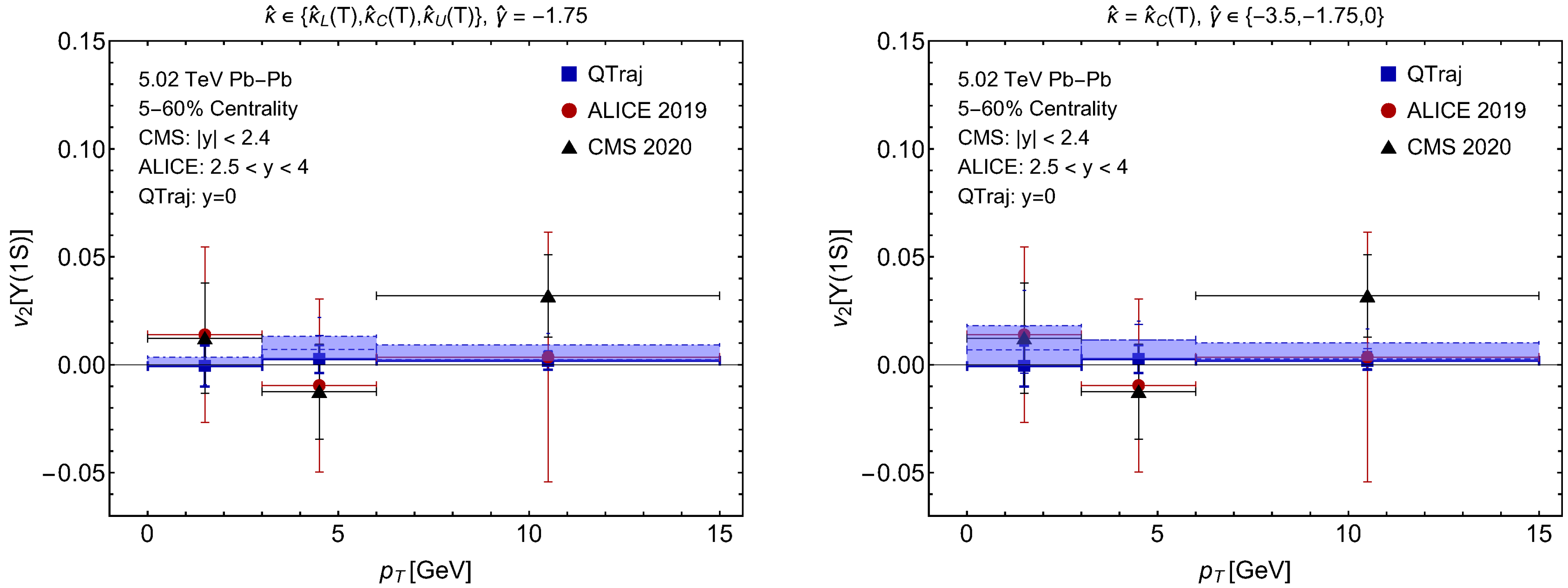
Double ratio $R_{AA}[\Upsilon(3S)]$ to $R_{AA}[\Upsilon(1S)]$



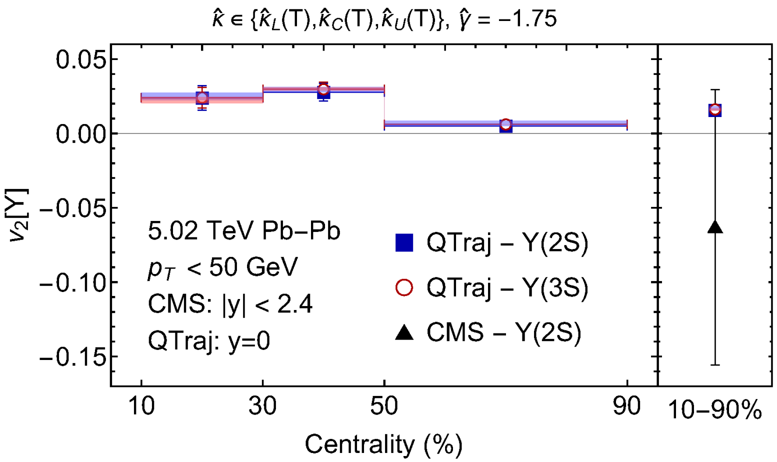
Elliptic flow v_2 of the $\Upsilon(1S)$

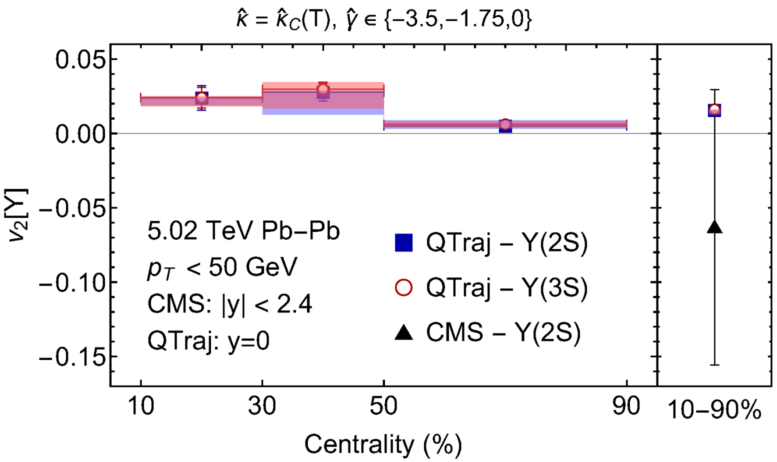


Elliptic flow v_2 of the $\Upsilon(1S)$ vs p_T



Elliptic flow v_2 of the $\Upsilon(2S)$ and $\Upsilon(3S)$





Lindblad equation for a strongly coupled plasma @ NLO in E/T

At NLO in E/T the evolution equation for a strongly coupled plasma is still a Lindblad equation, with H and collaps operators

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & \frac{7}{16} \end{pmatrix} \left(\frac{r^2}{2}\,\gamma + \frac{\kappa}{4MT}\{r_i, p_i\} \right)$$

$$C_i^0 = \sqrt{\frac{\kappa}{8}}\, r^i \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \left(r_i + \frac{ip_i}{2MT} + \frac{3\alpha_{\rm s}}{8T}\hat{r}_i \right)$$

$$+ \sqrt{\kappa} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \left(r_i + \frac{ip_i}{2MT} - \frac{3\alpha_{\rm s}}{8T}\hat{r}_i \right)$$

$$C_i^1 = \sqrt{\frac{5\kappa}{16}}\, r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \left(r_i + \frac{ip_i}{2MT} \right)$$

- The Lindblad equation at NLO in E/T depends on the same transport coefficients κ and γ as the LO one, because we have expanded the chromoelectric correlator around its instantaneous limit

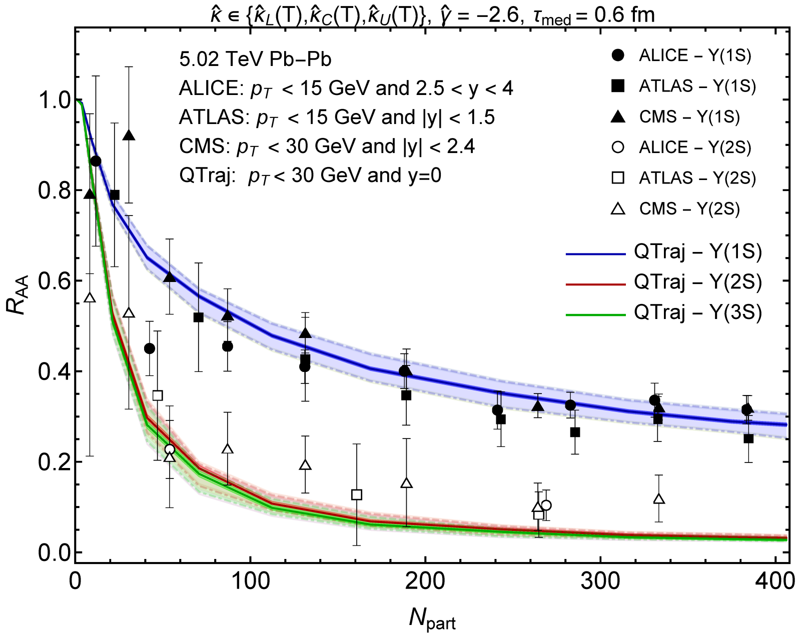
$$i\frac{g^2}{18}\int_0^\infty \mathrm{d}t\, t\left\langle \tilde{E}_i^a(t,\mathbf{0})\tilde{E}_i^a(0,\mathbf{0})\right\rangle \approx \frac{\kappa}{4T}$$

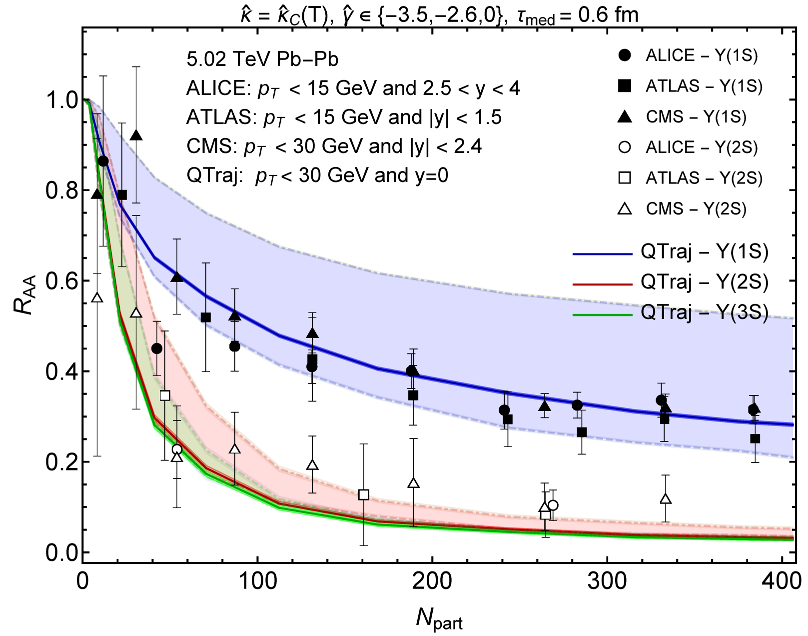
- The Lindblad equation at NLO in E/T allows to extend the in medium evolution from $T_F = 250$ MeV (@ LO) to $T_F = 190$ MeV. The Lindblad equation at NLO in E/T favors either a lower γ, $\hat{\gamma} = -2.6$ vs $\hat{\gamma} = -1.75$ @LO, or an earlier medium formation $\tau_{\mathrm{med}} = 0.25$ fm vs $\tau_{\mathrm{med}} = 0.6$ fm @ LO.
- We evaluate the NLO evolution by dropping the (small) effect of quantum jumps,

$$\frac{d\rho}{dt} \approx -i[H_{\mathrm{eff}}, \rho], \quad \text{i.e.} \quad \sum_i C_i \rho C_i^\dagger \quad \text{is neglected}$$

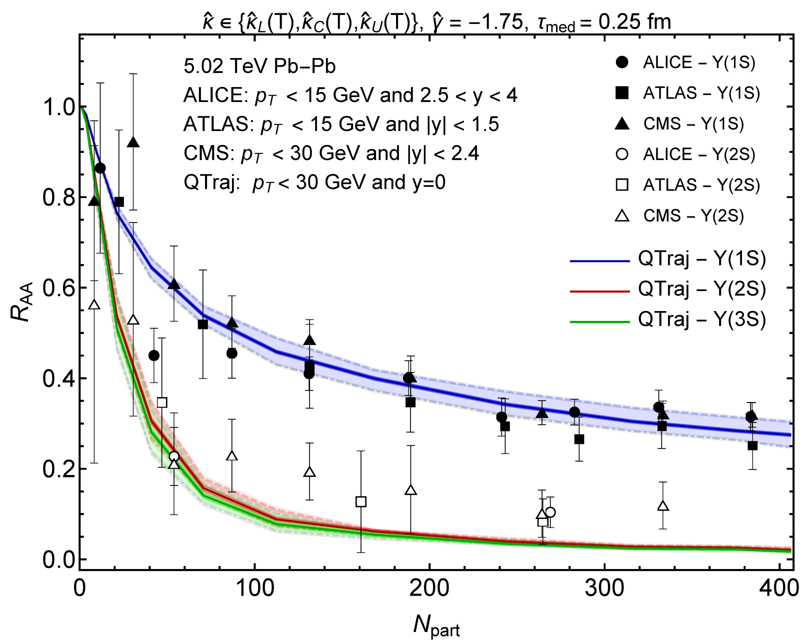
and using an ensemble of 80,000 sampled physical trajectories.

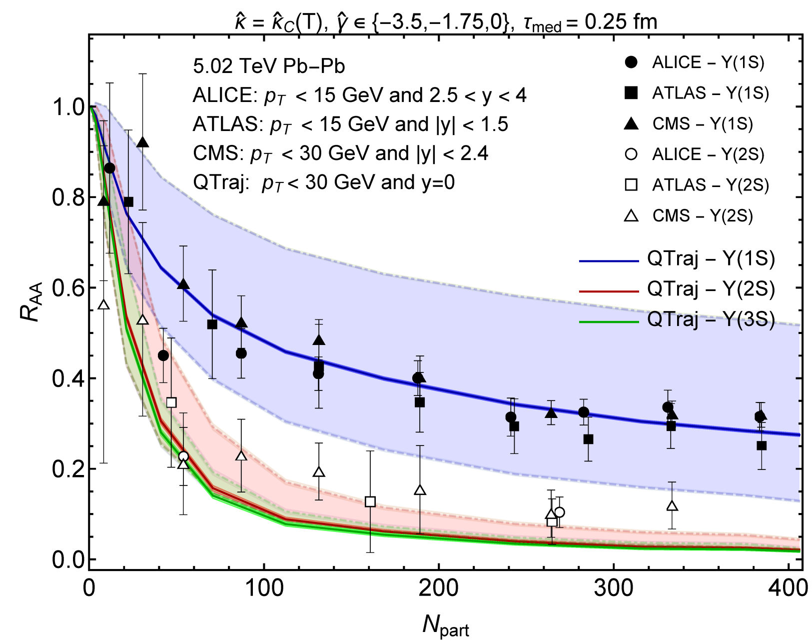
Bottomonium nuclear modification factor @ NLO and $\tau_{\text{med}} = 0.6$ fm



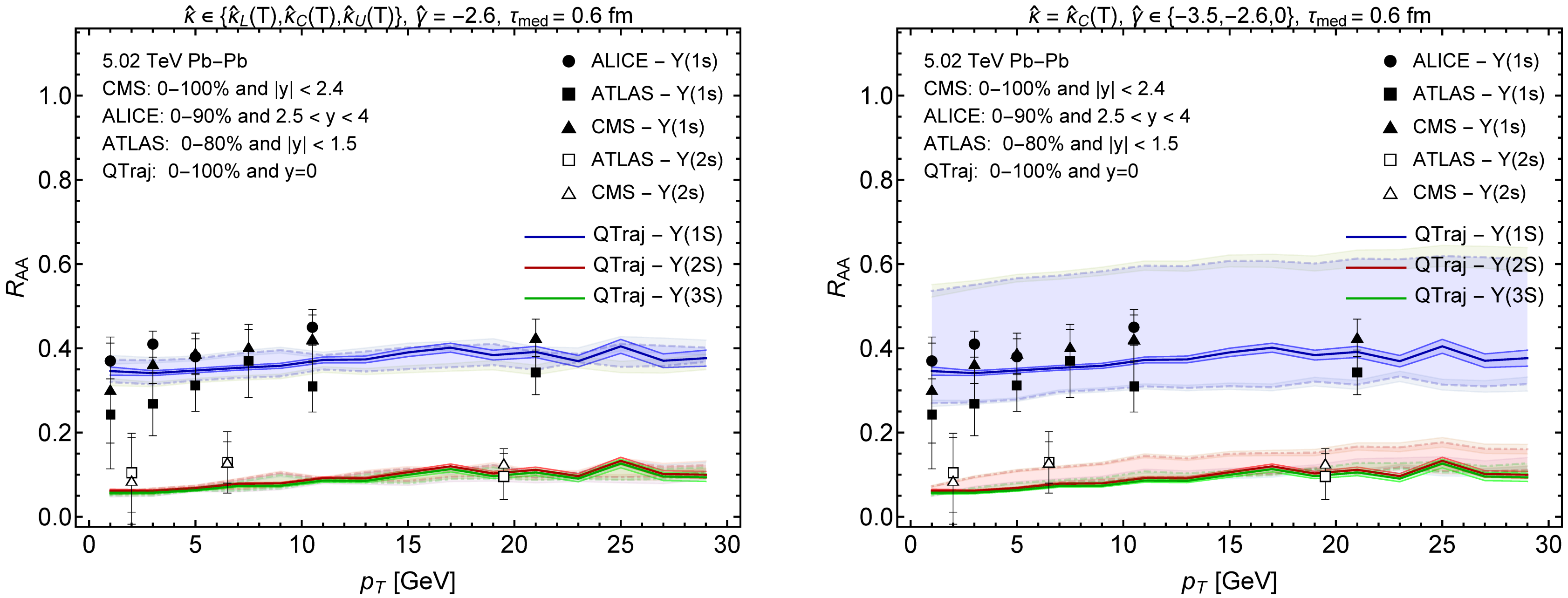


Bottomonium nuclear modification factor @ NLO and $\tau_{\rm med} = 0.25$ fm

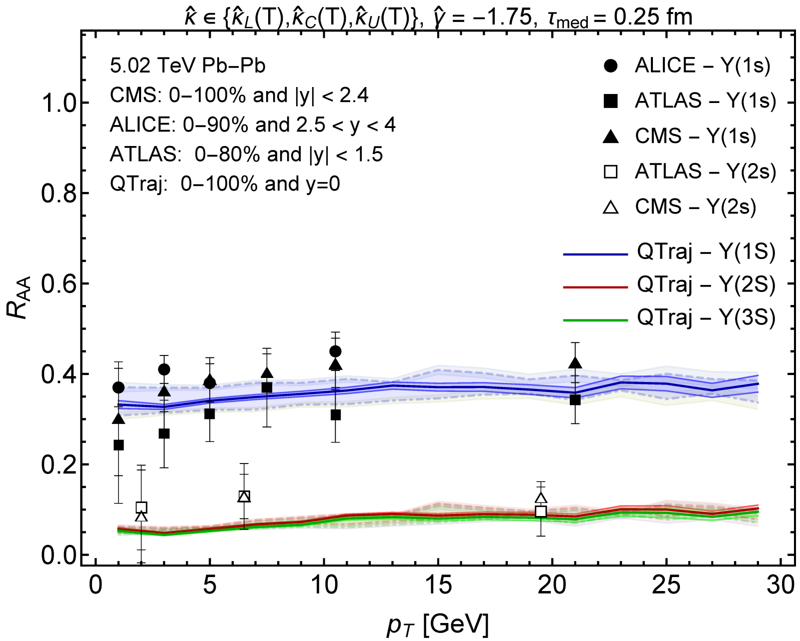


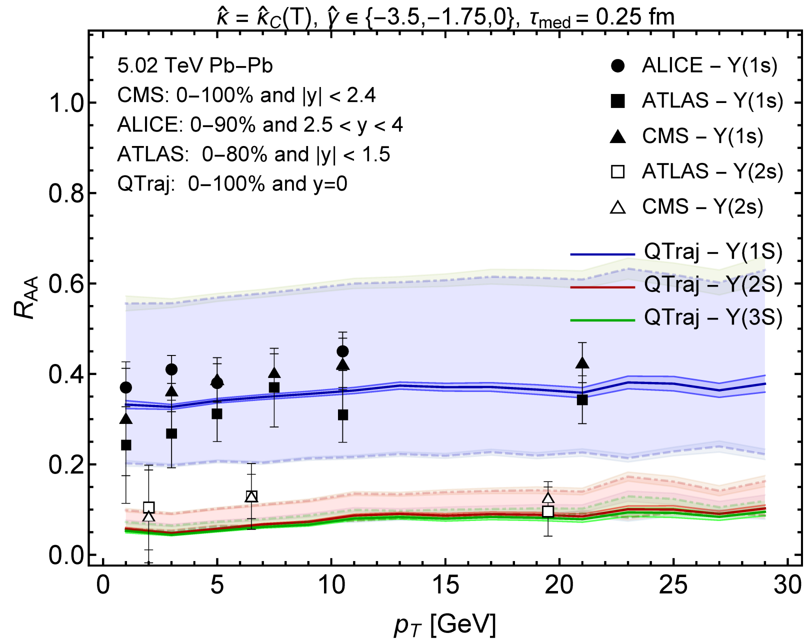


Bottomonium nuclear modification factor vs p_T @ NLO and $\tau_{\rm med} = 0.6$ fm



Bottomonium nuclear modification factor vs p_T @ NLO and $\tau_{\mathrm{med}} = 0.25$ fm





Conclusions

We have shown how the heavy quark-antiquark pair out-of-equilibrium evolution can be treated in the framework of QCD non relativistic EFTs.
With respect to previous determinations:

- the medium may be a strongly-coupled plasma (not necessarily a quark-gluon plasma) whose characteristics are determined by lattice calculations;
- the total number of heavy quarks, i.e., $\mathrm{Tr}\{\rho_s\} + \mathrm{Tr}\{\rho_o\}$, is preserved by the evolution equations;
- the non-abelian nature of QCD is fully accounted for;
- the treatment does not rely on classical approximations.

The evolution equations follow from assuming the inverse size of the quark-antiquark system to be larger than any other scale of the medium and from being accurate at first non-trivial order in the multipole expansion and at first order in the heavy-quark density.

Under some conditions (large time, quasistatic evolution, quantum Brownian motion) the evolution equations are of the Lindblad form. Their numerical solution provides $R_{AA}[\Upsilon(nS)]$ and differential observables in good agreement with LHC data.