QWG 2022 - The 15th International Workshop on Heavy Quarkonium

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Europe/Berlin timezone

# QCD Factorization for Hadronic Heavy Quarkonium Production at High PT

- Scales for heavy quarkonium production?
- QCD factorization for pT-distribution of heavy quarkonium production
- Both leading power and next-to-leading power contributions are needed
- Factorized QCD calculations can describe existing data from the LHC to Tevatron
- Summary and outlook

**Jianwei Qiu**

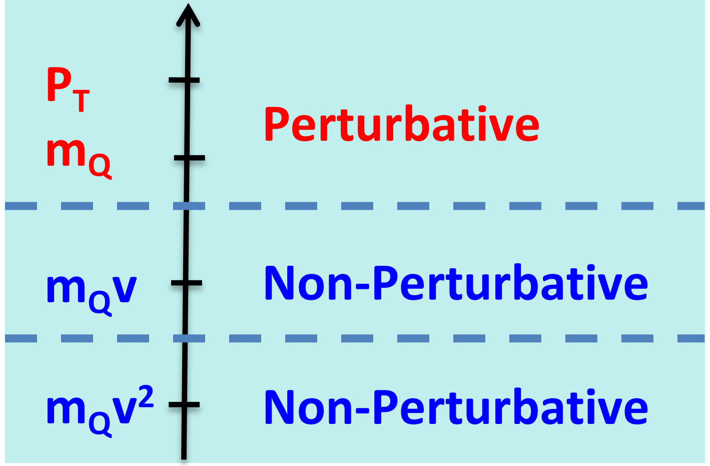
**Jefferson Lab, Theory Center**

*Collaboration with K. Lee, G. Sterman, K. Watanabe, arXiv:2108.00305 [hep-ph] and in preparation*

Jefferson Lab TMD Collaboration U.S. DEPARTMENT OF ENERGY Office of Science JSA

# Scales for heavy quarkonium production at high $p_T$

- **Well-separated momentum scales – effective theory:**




**Hard — Production of $Q\bar{Q}$** **[pQCD]**

*To make this part as reliable as we can!*

**Soft — Relative Momentum** **[NRQCD]**

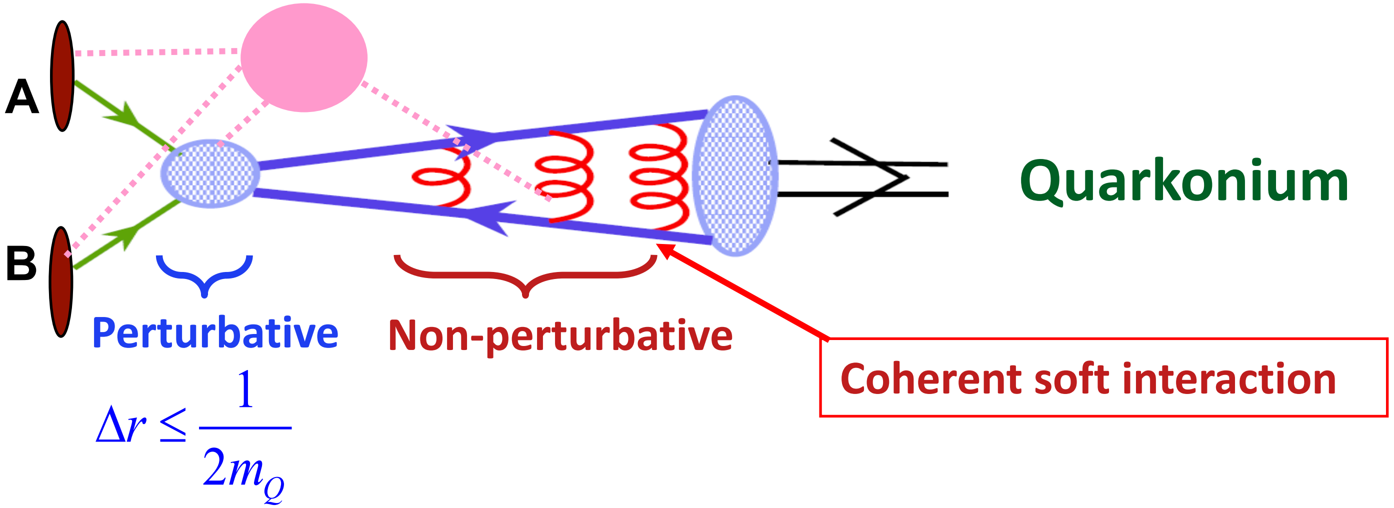
$\Lambda_{QCD}$ $\Lambda_{QCD}$

**Ultrasoft — Binding Energy** **[pNRQCD]**

**Known quarks**

| Flavor | Mass |
|---|---|
| $u$ | 1.5 – 4.5 MeV |
| $d$ | 5.0 – 8.5 MeV |
| $s$ | 80 – 155 MeV |
| $c$ | 1.0 – 1.4 GeV |
| $b$ | 4.0 – 4.5 GeV |
| $t$ | 174.3 ± 5.1 GeV |

- **Basic production mechanism:**




- **QCD Factorization is "expected" to work for the production of heavy quarks**
- **Difficulty: how the heavy quark pair becomes a quarkonium?**

# NRQCD factorization and the “lack” of universality of LDMEs

Bodwin, Braaten, Lepage, PRD, 1995

- **NRQCD factorization:**

$$d\sigma_{A+B\to H+X} = \sum_n d\sigma_{A+B\to Q\bar{Q}(n)+X}\langle\mathcal{O}^H(n)\rangle$$

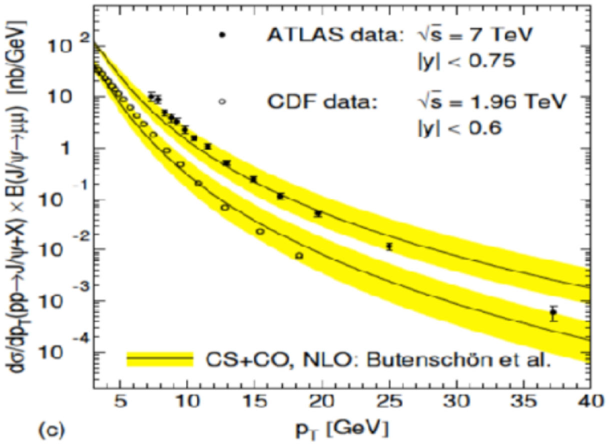
**Expansion in powers of both $\alpha_s$ and $v$ !**

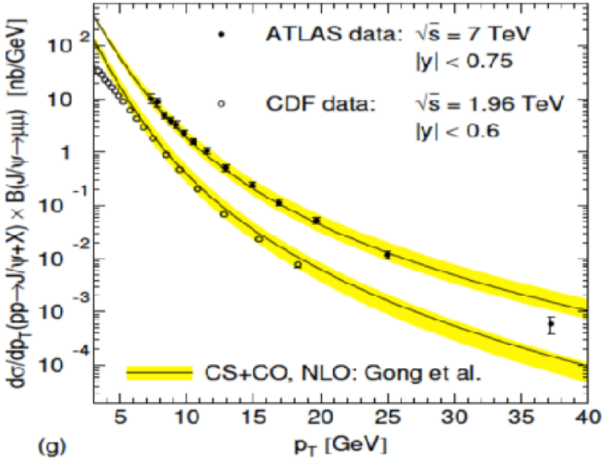
**Hadronization**

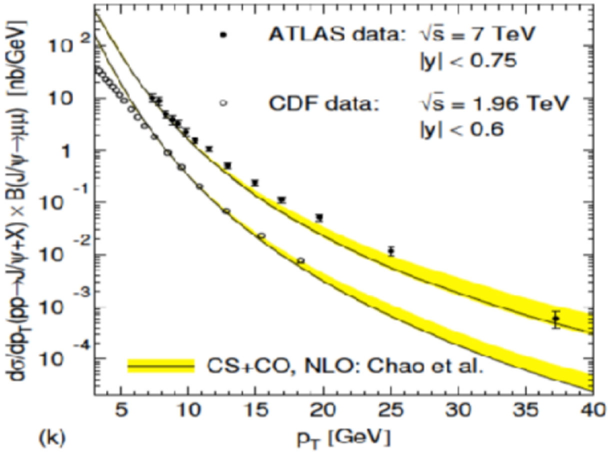
- **4 leading channels in v:**

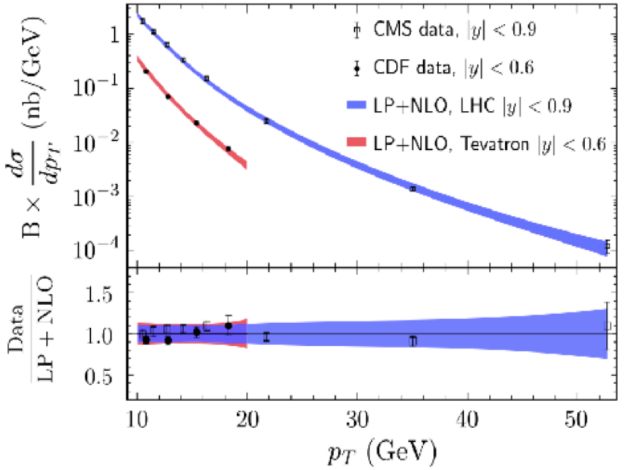
$${}^3S_1^{[1]},\ {}^1S_0^{[8]},\ {}^3S_1^{[8]},\ {}^3P_J^{[8]}$$

- **Phenomenology – full NLO in $\alpha_s$:**




| | $\langle\mathcal{O}(^3S_1^{[1]})\rangle$ GeV$^3$ | $\langle\mathcal{O}(^1S_0^{[8]})\rangle$ $10^{-2}$ GeV$^3$ | $\langle\mathcal{O}(^3S_1^{[8]})\rangle$ $10^{-2}$ GeV$^3$ | $\langle\mathcal{O}(^3P_0^{[8]})\rangle$ $10^{-2}$ GeV$^5$ |
|---|---|---|---|---|
| Set I (Butenschoen *et al.*) | 1.32 | 3.04 | 0.16 | −0.91 |
| Set II (Chao *et al.*) | 1.16 | 8.9 | 0.30 | 1.26 |
| Set III (Gong *et al.*) | 1.16 | 9.7 | −0.46 | −2.14 |
| Set IV (Bodwin *et al.*) | - | 9.9 | 1.1 | 1.1 |

**LDMEs should be universal, however:**

- **Numbers are not the same.**
- **Not even the sign.**

**More work is needed!**


**Fits in NRQCD**

Butenschoen, Kniehl, PRD84, 051501 (2011).
Chao, Ma, Shao, Wang, Zhang, PRL108, 242004 (2012).
Gong, Wan, Wang, Zhang, PRL110, 042002 (2013).
Bodwin, Chung, Kim, Lee, PRL113, 022001 (2014).
…

**Fits in pNRQCD**

Brambilla, Chung, Vairo, Wang, PRD105, no.11, L111503 (2022).

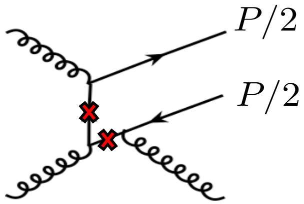
# Heavy quarkonium production at high $p_T$

Kang, Qiu and Sterman, 2011

## ❑ $O(\alpha_s)$ expansion vs. $1/p_T$ expansion:

**CS channel as a case study**

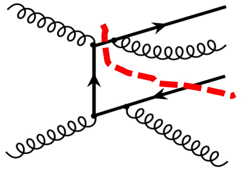
**LO in $\alpha_s$:**

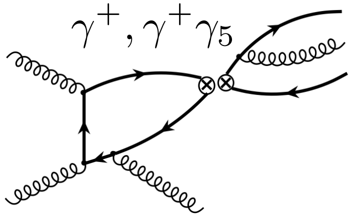



$$\hat{\sigma}^{\mathrm{LO}} \propto \frac{\alpha_s^3(p_T)}{p_T^8}$$

**$P_T$ Power!**

**NLO in $\alpha_s$:**

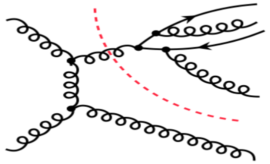





$$\hat{\sigma}^{\mathrm{NLO}} \rightarrow \frac{\alpha_s^3(p_T)}{p_T^6} \otimes \alpha_s(\mu) \log(\mu^2/\mu_0^2)$$

$$\mu_0 \gtrsim 2m_Q$$

**NNLO in $\alpha_s$:**



$$\hat{\sigma}^{\mathrm{NNLP}} \rightarrow \frac{\alpha_s^2(p_T)}{p_T^4} \otimes \alpha_s^3(\mu) \log^m(\mu^2/\mu_0^2)$$

- **When $p_T >> m_Q$, the expansion in powers of $\alpha_s$ is not reliable!**
- **Leading order in $\alpha_s$-expansion =\= leading power in $1/p_T$-expansion!**

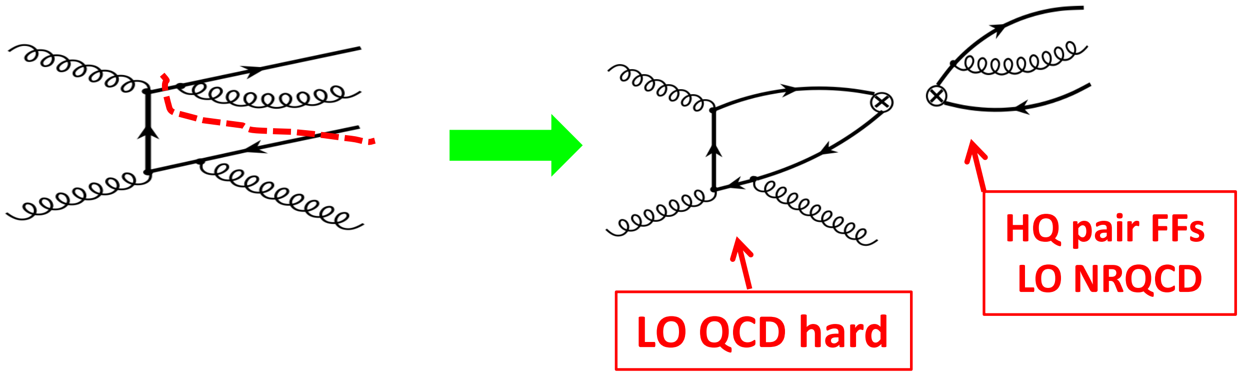
## ❑ PQCD factorization:

- $1/p_T$ expansion first: leading power (LP) & next-to-leading power (NLP) are factorizable!
- $O(\alpha_s)$-expansion: leading order (LO) & next-to-leading order (NLO) are calculated

# QCD factorization + NRQCD factorization

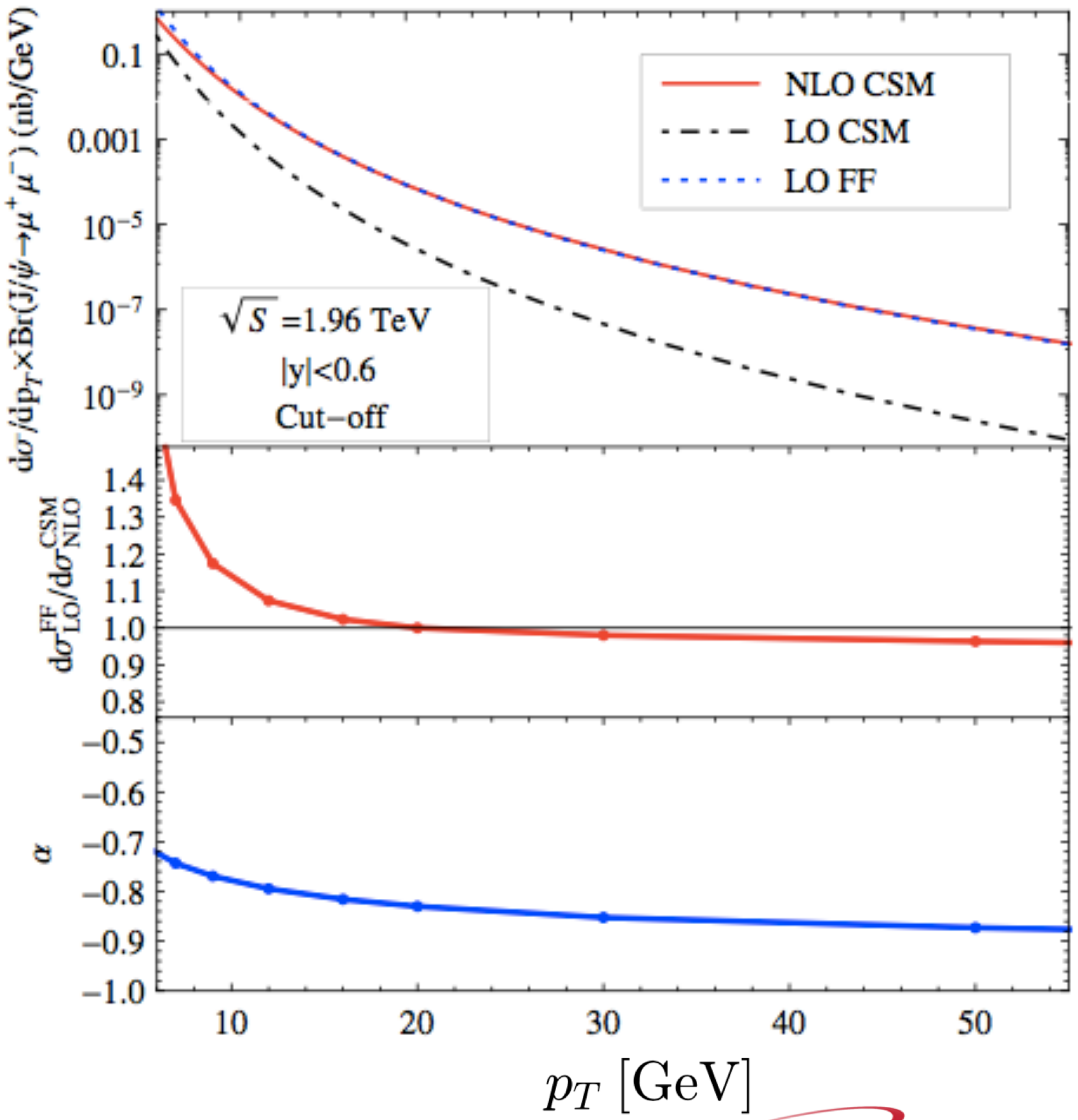
Kang, Qiu and Sterman, 2011

- Color singlet as an example:




$$\sigma^{(\mathrm{NLO})}_{\mathrm{NRQCD}} \propto \left[ d\hat{\sigma}^{A(\mathrm{LO})}_{ab\to[Q\bar{Q}(v8)]} \otimes \mathcal{D}^{(\mathrm{LO})}_{[Q\bar{Q}(v8)]\to J/\psi} + d\hat{\sigma}^{S(\mathrm{LO})}_{ab\to[Q\bar{Q}(a8)]} \otimes \mathcal{D}^{(\mathrm{LO})}_{[Q\bar{Q}(a8)]\to J/\psi} \right]$$

$$v8 = [\gamma^+]^{[8]}$$
$$a8 = [\gamma^+\gamma_5]^{[8]}$$




**Reproduce NLO CSM for $p_T$ > 10 GeV!**

**Cross section + polarization**

***Different kinematics, different approximation,***
***Dominance of different production channels!***

# Heavy quarkonium production at high $p_T$

## ❑ PQCD + NRQCD factorization:

Lee, Qiu, Sterman, Watanabe, 2022

$$E\frac{d\sigma_{hh'\to J/\psi(P)X}}{d^3P} = \sum_{c\bar{c}[n]} \langle O^{J/\psi}_{c\bar{c}[n]}(0)\rangle \sum_{a,b} \int dx_a\, f_{a/h}(x_a,\mu_f^2) \int dx_b\, f_{b/h'}(x_b,\mu_f^2)$$

$$\times\left[E\frac{d\tilde{\sigma}^{\rm Resum}_{ab\to c\bar{c}[n](P)X}}{d^3P} + E\frac{d\tilde{\sigma}^{\rm NRQCD}_{ab\to c\bar{c}[n](P)X}}{d^3P} - E\frac{d\tilde{\sigma}^{\rm Asym}_{ab\to c\bar{c}[n](P)X}}{d^3P}\right]$$

$$c\bar{c}[n] = c\bar{c}[^{2S+1}L_J^{[1,8]}]$$



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# Heavy quarkonium production at high $p_T$

## ❑ PQCD + NRQCD factorization:

Lee, Qiu, Sterman, Watanabe, 2022

$$E\frac{d\sigma_{hh'\to J/\psi(P)X}}{d^3P} = \sum_{c\bar{c}[n]} \langle O^{J/\psi}_{c\bar{c}[n]}(0)\rangle \sum_{a,b} \int dx_a\, f_{a/h}(x_a,\mu_f^2) \int dx_b\, f_{b/h'}(x_b,\mu_f^2)$$

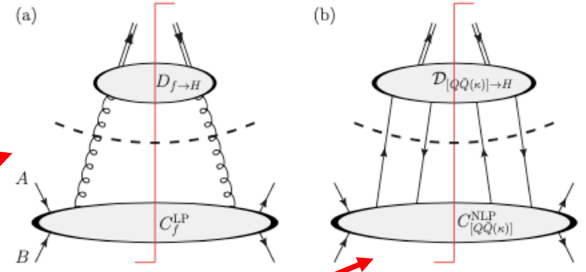
$$\times \left[ E\frac{d\tilde{\sigma}^{\text{Resum}}_{ab\to c\bar{c}[n](P)X}}{d^3P} + E\frac{d\tilde{\sigma}^{\text{NRQCD}}_{ab\to c\bar{c}[n](P)X}}{d^3P} - E\frac{d\tilde{\sigma}^{\text{Asym}}_{ab\to c\bar{c}[n](P)X}}{d^3P} \right]$$

$$c\bar{c}[n] = c\bar{c}[^{2S+1}L_J^{[1,8]}]$$

- **PQCD factorization + NRQCD FFs:** $\kappa = (v,a,t)^{[1,8]}$

$$E\frac{d\tilde{\sigma}^{\text{Resum}}_{ab\to c\bar{c}[n](P)X}}{d^3P} \approx \sum_f \int \frac{dz}{z^2} D_{f\to c\bar{c}[n]}(z,\mu_f^2) E_f \frac{d\hat{\sigma}_{ab\to f(p_f)X}}{d^3p_f}(z, p_f = P/z, \mu_f^2)$$

$$+ \sum_{[c\bar{c}(\kappa)]} \int \frac{dz}{z^2} D_{[c\bar{c}(\kappa)]\to c\bar{c}[n]}(z,\mu_f^2) E_c \frac{d\hat{\sigma}_{ab\to [c\bar{c}(\kappa)](p_c)X}}{d^3p_c}(z, p_c = P/z, \mu_f^2)$$




Kang, Ma, Qiu, Sterman, 2014

- **NRQCD fixed-order:**

$$E\frac{d\tilde{\sigma}^{\text{NRQCD}}_{ab\to c\bar{c}[n](P)X}}{d^3P}$$

- **Asymptotic contribution:**

$$E\frac{d\tilde{\sigma}^{\text{Asym}}_{ab\to c\bar{c}[n](P)X}}{d^3P} = E\frac{d\tilde{\sigma}^{\text{Resum}}_{ab\to c\bar{c}[n](P)X}}{d^3P}\Bigg|_{\text{fixed order}}$$



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# Heavy quarkonium production at high $p_T$

Lee, Qiu, Sterman, Watanabe, 2022

## ❑ PQCD + NRQCD factorization:

$$E\frac{d\sigma_{hh'\to J/\psi(P)X}}{d^3P} = \sum_{c\bar{c}[n]} \langle O^{J/\psi}_{c\bar{c}[n]}(0)\rangle \sum_{a,b} \int dx_a\, f_{a/h}(x_a,\mu_f^2) \int dx_b\, f_{b/h'}(x_b,\mu_f^2) \qquad c\bar{c}[n] = c\bar{c}[^{2S+1}L_J^{[1,8]}]$$

$$\times \left[ E\frac{d\tilde{\sigma}^{\rm Resum}_{ab\to c\bar{c}[n](P)X}}{d^3P} + E\frac{d\tilde{\sigma}^{\rm NRQCD}_{ab\to c\bar{c}[n](P)X}}{d^3P} - E\frac{d\tilde{\sigma}^{\rm Asym}_{ab\to c\bar{c}[n](P)X}}{d^3P} \right]$$

- **PQCD factorization + NRQCD FFs:** $\kappa = (v,a,t)^{[1,8]}$

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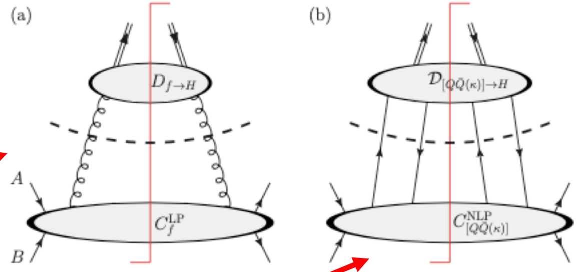
$$+ \sum_{[c\bar{c}(\kappa)]} \int \frac{dz}{z^2} D_{[c\bar{c}(\kappa)]\to c\bar{c}[n]}(z,\mu_f^2) E_c \frac{d\hat{\sigma}_{ab\to [c\bar{c}(\kappa)](p_c)X}}{d^3p_c}(z, p_c = P/z, \mu_f^2)$$




Kang, Ma, Qiu, Sterman, 2014

- **NRQCD fixed-order:**

$$E\frac{d\tilde{\sigma}^{\rm NRQCD}_{ab\to c\bar{c}[n](P)X}}{d^3P}$$

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$$E\frac{d\tilde{\sigma}^{\rm Asym}_{ab\to c\bar{c}[n](P)X}}{d^3P} = E\frac{d\tilde{\sigma}^{\rm Resum}_{ab\to c\bar{c}[n](P)X}}{d^3P}\bigg|_{\rm fixed\ order}$$

**When** $P_T \gg m_c$, $E\frac{d\tilde{\sigma}^{\rm Asym}_{ab\to c\bar{c}[n](P)X}}{d^3P}$ **cancels** $E\frac{d\tilde{\sigma}^{\rm NRQCD}_{ab\to c\bar{c}[n](P)X}}{d^3P}$

**When** $P_T \gtrsim m_c$, $E\frac{d\tilde{\sigma}^{\rm Asym}_{ab\to c\bar{c}[n](P)X}}{d^3P}$ **cancels** $E\frac{d\tilde{\sigma}^{\rm Resum}_{ab\to c\bar{c}[n](P)X}}{d^3P}$

# Renormalization group improvement

## ❑ Renormalization group:

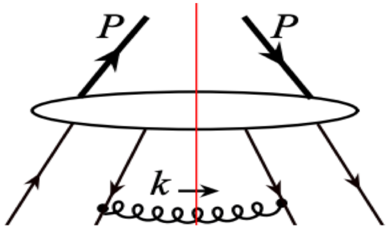
Kang, Ma, Qiu, Sterman, PRD 90, 034006 (2014)

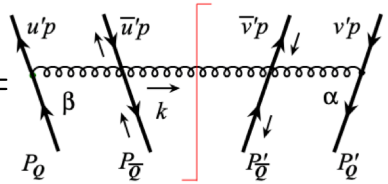
$$\frac{d}{d\ln\mu_f^2}\left[E\frac{d\tilde{\sigma}^{\rm Resum}_{ab\to c\bar{c}[n](P)X}}{d^3P}\right]=0$$

**To be accurate up to the 1st power correction**

## ❑ Modified evolution equations: $H = c\bar{c}[^{2S+1}L_J^{[1,8]}]$

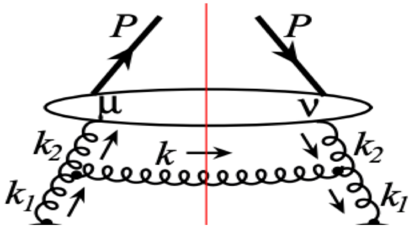
$$\frac{\partial\mathcal{D}_{[Q\bar{Q}(n)]\to H}}{\partial\ln\mu_f^2}=\Gamma_{[Q\bar{Q}(n)]\to[Q\bar{Q}(\kappa)]}\otimes\mathcal{D}_{[Q\bar{Q}(\kappa)]\to H}$$




$\Gamma =$ 

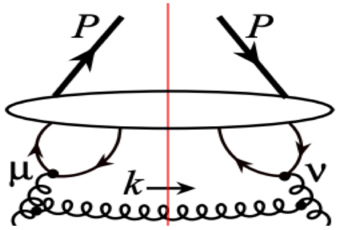

**DGLAP-type: Heavy quark pair produced at the hard scale**

$$\frac{\partial D_{[f]\to H}}{\partial\ln\mu_f^2}=\gamma_{[f]\to[f']}\otimes D_{[f']\to H}$$




**Heavy quark pair produced at the input scale**

$$+\frac{1}{\mu_f^2}\,\overline{\gamma}_{[f]\to[Q\bar{Q}(\kappa)]}\otimes\mathcal{D}_{[Q\bar{Q}(\kappa)]\to H}$$




$\longleftarrow\ \overline{\gamma}_{g\to[Q\bar{Q}]}$

**Heavy quark pair produced between the hard scale and the input scale**

**Modified DGLAP – inhomogeneous evolution**

# Evolution of $c\bar{c}$-fragmentation function in μ, ν space

Lee, Qiu, Sterman, Watanabe, in preparation

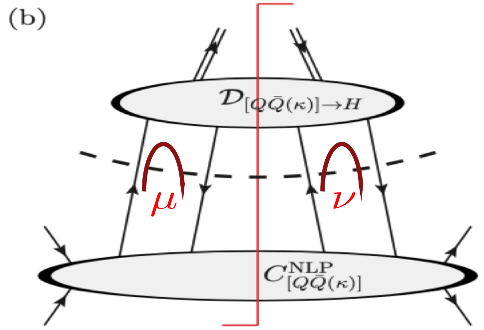
❑ **To justify an approximation at $\mu = \nu = 1/2$ :**

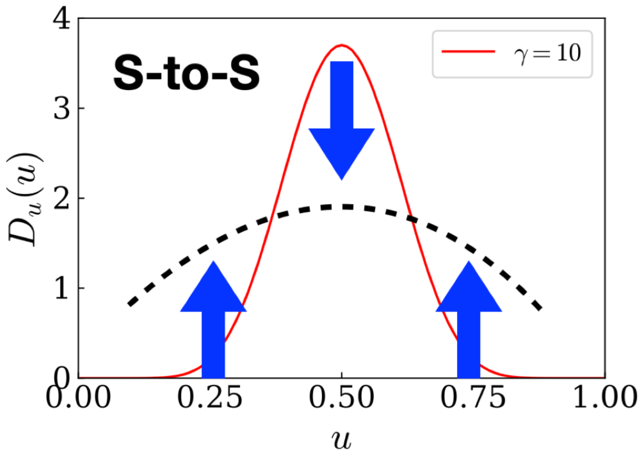
$$D'_{\kappa\to n}(z,u,v) \equiv \frac{2\pi}{\alpha_s}\frac{dD_{\kappa\to n}(z,u,v)}{d\ln\mu^2},$$

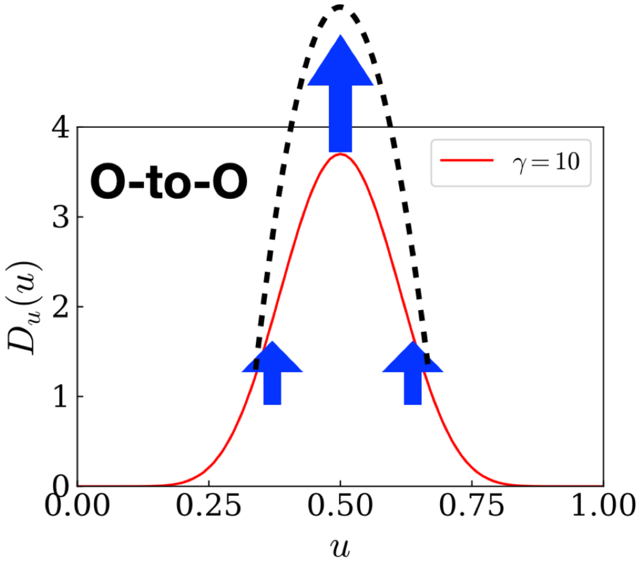
$$D(z,u,v) \to D_z(z)D_u(u)D_v(v),$$

$$D_z(z,\alpha) = \frac{z^\alpha(1-z)^\beta}{B[1+\alpha,1+\beta]},$$

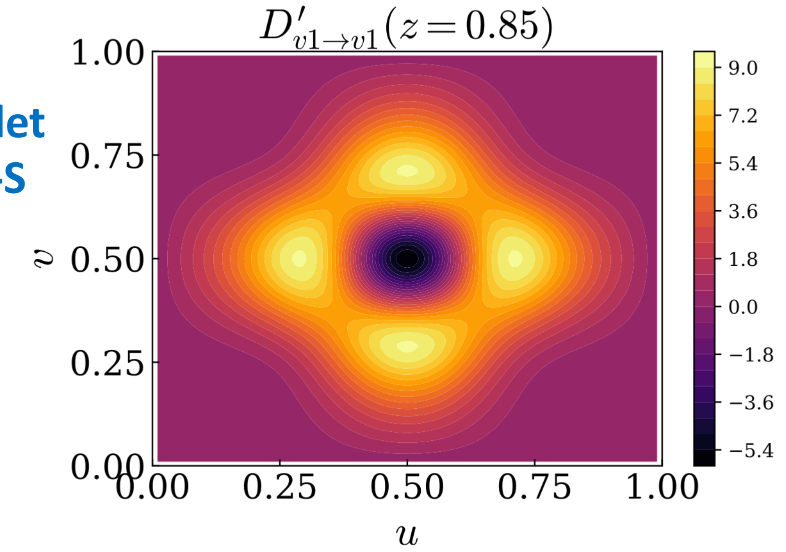
$$D_{u,v}(x,\gamma) = \frac{x^\gamma(1-x)^\gamma}{B[1+\gamma,1+\gamma]},$$



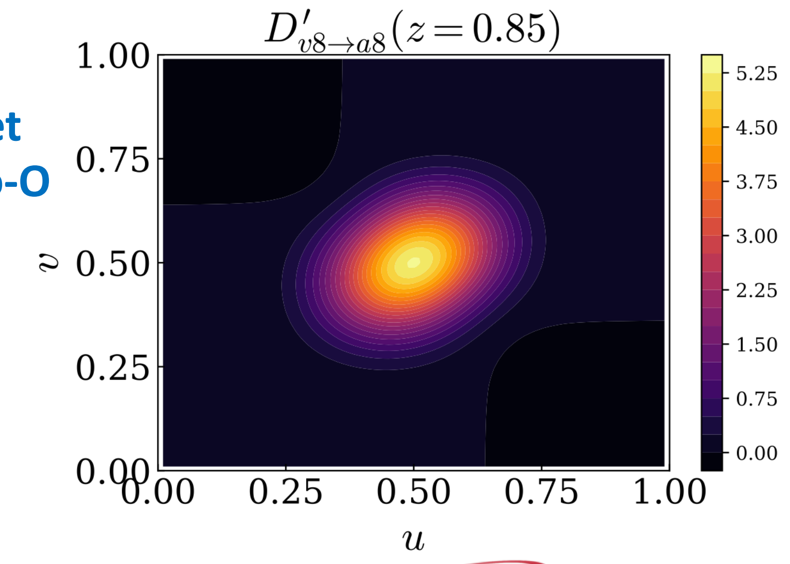




**Diagonal singlet channel: S-to-S**




**Diagonal octet channel: O-to-O**




- **S-to-S DP FFs get broader in -space after evolution.**
- **O-to-O DP FFs become narrower with a large peak around .**
- **Off-diagonal channels: similar to O-to-O.**

# Input fragmentation functions at $\mu_0 \sim \#\, m_c$

## ❑ Input FFs from NRQCD:

Ma, Qiu, Zhang, PRD89 (2014) 094029; ibid. 94030
Lee, Qiu, Steerman, Watanabe, SciPost Phys. Proc. 8, 143 (2022)

$$D_{f\to H}(z;m,\mu_0) = \sum_{[Q\bar{Q}(n)]} \pi\alpha_s \left\{ \hat{d}^{(1)}_{f\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \frac{\alpha_s}{\pi}\hat{d}^{(2)}_{f\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \mathcal{O}(\alpha_s^2) \right\} \frac{\langle \mathcal{O}^H_{[Q\bar{Q}(n)]}(\mu_\Lambda)\rangle}{m^{2L+3}}$$

$$D_{[Q\bar{Q}(\kappa)]\to H}(z;m,\mu_0) = \sum_{[Q\bar{Q}(n)]} \left\{ \hat{d}^{(0)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \frac{\alpha_s}{\pi}\hat{d}^{(1)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \mathcal{O}(\alpha_s^2) \right\} \frac{\langle \mathcal{O}^H_{[Q\bar{Q}(n)]}(\mu_\Lambda)\rangle}{m^{2L+1}}$$

$\mu_0 = \mathcal{O}(2m)$: input scale, $\mu_\Lambda = \mathcal{O}(m)$: NRQCD factorization scale $\qquad \kappa = v^{[c]}, a^{[c]}, t^{[c]}, \quad n = {}^{2S+1}L_J^{[c]}$

Perturbative SDCs $\hat{d}^{(n)}(z)$ of input FFs in $\alpha_s$ and $\nu$ expansion in the NRQCD are reliable only when SDCs << O(1).
SDCs $\hat{d}^{(n)}(z)$ calculated in NRQCD factorization is not reliable as $z \to 1$ for the following terms:

1. $\delta(1-z)$ at LO in $\alpha_s$ expansion
2. $f(z)\ln(1-z)$ with $f(z)$ being a regular function
3. $\dfrac{f(z)}{[1-z]_+}$, $f(z)\left[\dfrac{\ln(1-z)}{1-z}\right]_+$ due to the perturbative cancelation of IR divergences

In our current analysis, we use analytic results if those vanish as $z \to 1$; and for singular or negative input FFs, we model them with proper normalization:

$C_{[Q\bar{Q}(n)]}(\alpha_s)$ : abs. value of the first moment

$$D_{[Q\bar{Q}(n)]}(z) = C_{[Q\bar{Q}(n)]}(\alpha_s)\frac{z^\alpha(1-z)^\beta}{B[1+\alpha,1+\beta]} \qquad (\alpha \gg 1,\ 1 > \beta > 0)$$

→ to be tuned, imitating $\delta$-function at LO

# NLP contribution to single parton fragmentation functions

Lee, Qiu, Steerman, Watanabe 2022

## ❑ Impact of inhomogeneous term:

$$\frac{\partial D_{f\to H}}{\partial \ln \mu^2} = \gamma_{f\to f'} \otimes D_{f'\to H} + \frac{1}{\mu^2}\gamma_{f\to[Q\bar{Q}(\kappa)]} \otimes \mathscr{D}_{[Q\bar{Q}(\kappa)]\to H}$$

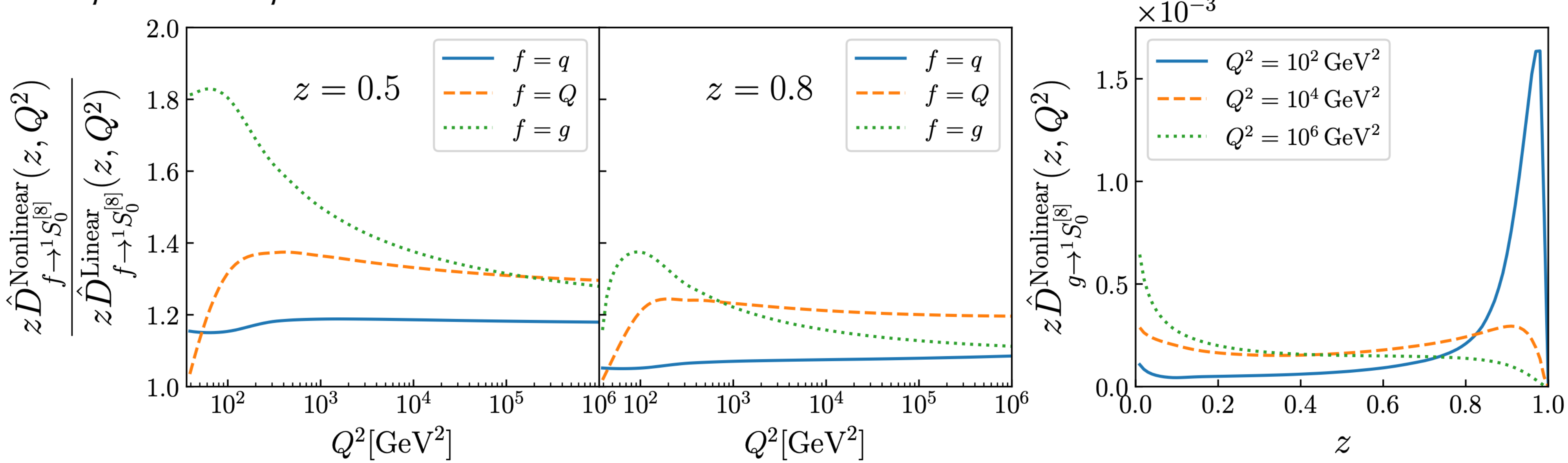
$$\frac{\partial D^{\text{Nonlinear}}_{f\to H}}{\partial \ln \mu^2} \sim \frac{\partial D^{\text{Linear}}_{f\to H}}{\partial \ln \mu^2}$$

$\mu^2 \to \infty$: the slope of $D_{f\to H}$ is the same as LP DGLAP.

$\alpha = 30, \beta = 0.5$

$\mu_0 = 4m_c = 6\,\text{GeV}$

$\mu_\Lambda = m_c = 1.5\,\text{GeV}$




**The inhomogeneous quark pair corrections remain significant even at high $Q^2 \sim \mu^2 \sim pT^2$**

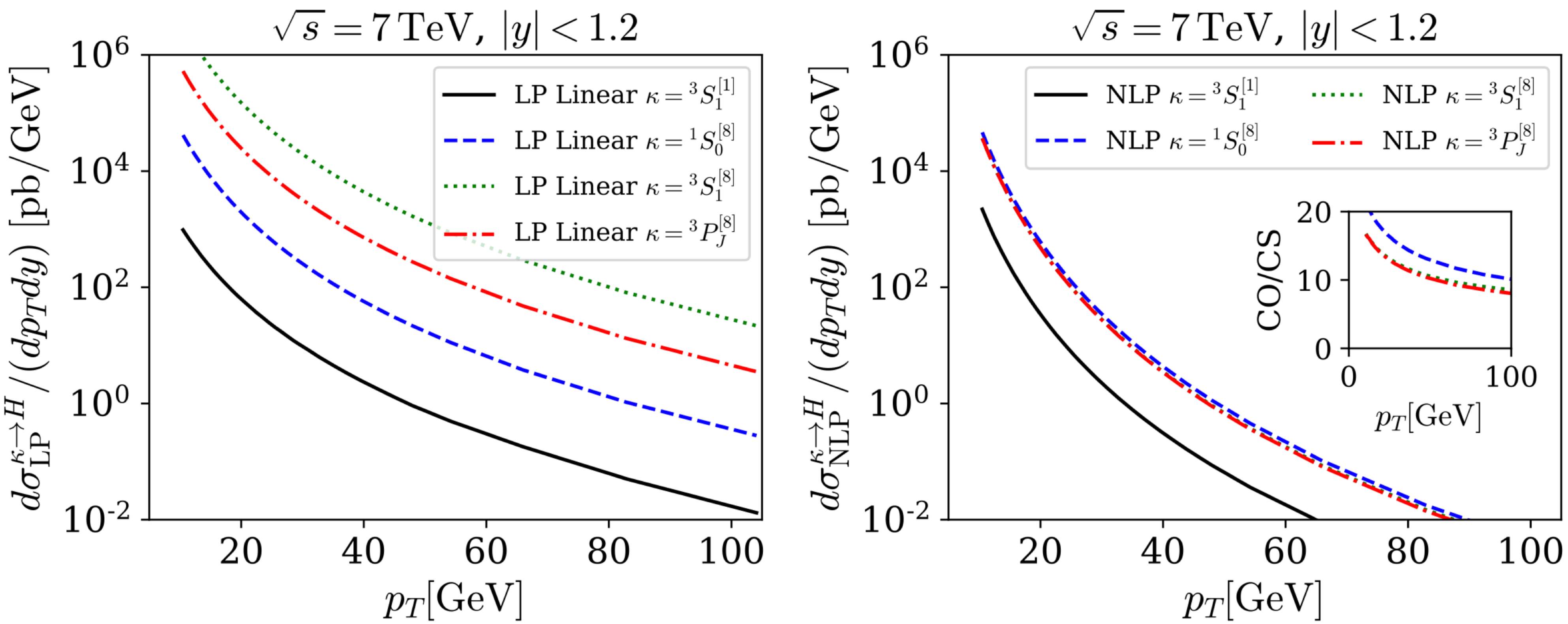
The power corrections effect at low $\mu^2$ does not go away fast: analogous to nonlinear gluon recombination effects to gluon PDF at small-x and large $\mu^2$.

Mueller and Qiu, NPB268, 427 (1986)
Qiu, NPB291, 746 (1987)
Eskola, Honkanen, Kolhinen, Qiu and Salgado, NPB660, 211 (2003)

# J/ψ-production in hadronic collisions

Lee, Qiu, Steerman, Watanabe, 2022

## ❑ Separate LDMEs from pQCD effects:




- Unweighted results: $\langle \mathcal{O}({}^3S_1^{[1]})\rangle/\mathrm{GeV}^3 = \langle \mathcal{O}({}^1S_0^{[8]})\rangle/\mathrm{GeV}^3 = \langle \mathcal{O}({}^3S_1^{[8]})\rangle/\mathrm{GeV}^3 = \langle \mathcal{O}({}^3P_0^{[8]})\rangle/\mathrm{GeV}^5 = 1.$
- $\alpha = 30, \beta = 0.5$ are fixed for both SP and DP FFs.
- ${}^1S_0^{[8]}$ is two orders of magnitude smaller than ${}^3S_1^{[8]}$ at LP.
- Three color octet channels at NLP provide similar contributions, steeply falling with $p_T$.

# J/ψ-production in hadronic collisions

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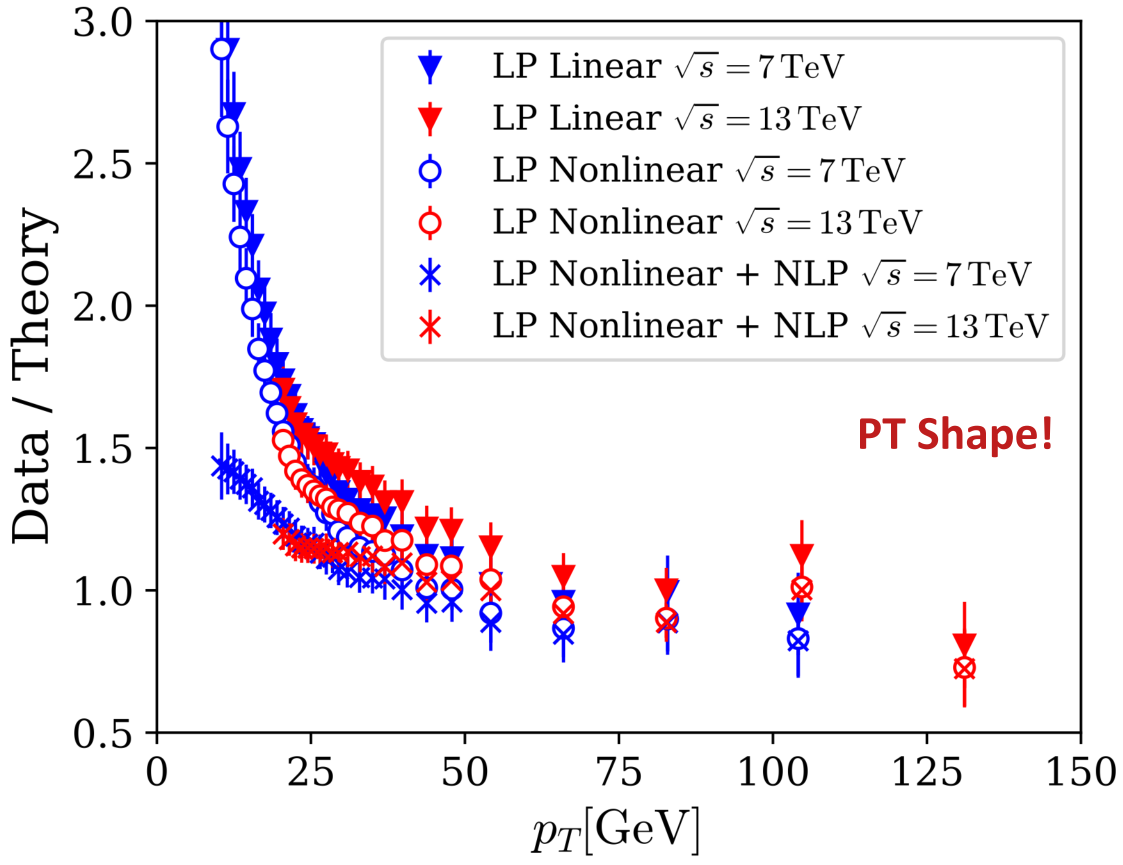
## ❑ Leading power contribution:

- Fitting the LP formalism with the linear evolution eq. to CMS data on high $p_T$ prompt $J/\psi$ at $\sqrt{s} = 7, 13\,\text{TeV}$ in the bin, $|y| < 1.2$.
- # of data points in a fit: 3@7TeV + 4@13TeV = 7 for $p_T \geq 60\,\text{GeV}$.
- Only the ${}^1S_0^{[8]}$ channel is considered, yielding unpolarized $J/\psi$. The other two color octet channels could overshoot data by combining LP and NLP.
- $\langle \mathcal{O}({}^1S_0^{[8]}) \rangle/\text{GeV}^3 = 0.1286 \pm 5.179 \cdot 10^{-3}$ fitted by high $p_T$ data is similar to the one extracted using fixed order NRQCD at NLO. Chao, Ma, Shao, Wang, Zhang, PRL108, 242004 (2012)
- Global data fitting is useful to pin down LDMEs and the shape of input FFs.




The power corrections do not vanish even at the highest $p_T$, giving 10-30% corrections.
At $p_T$ = *30 GeV* and below, the NLP corrections become significant.



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# J/ψ-production in hadronic collisions

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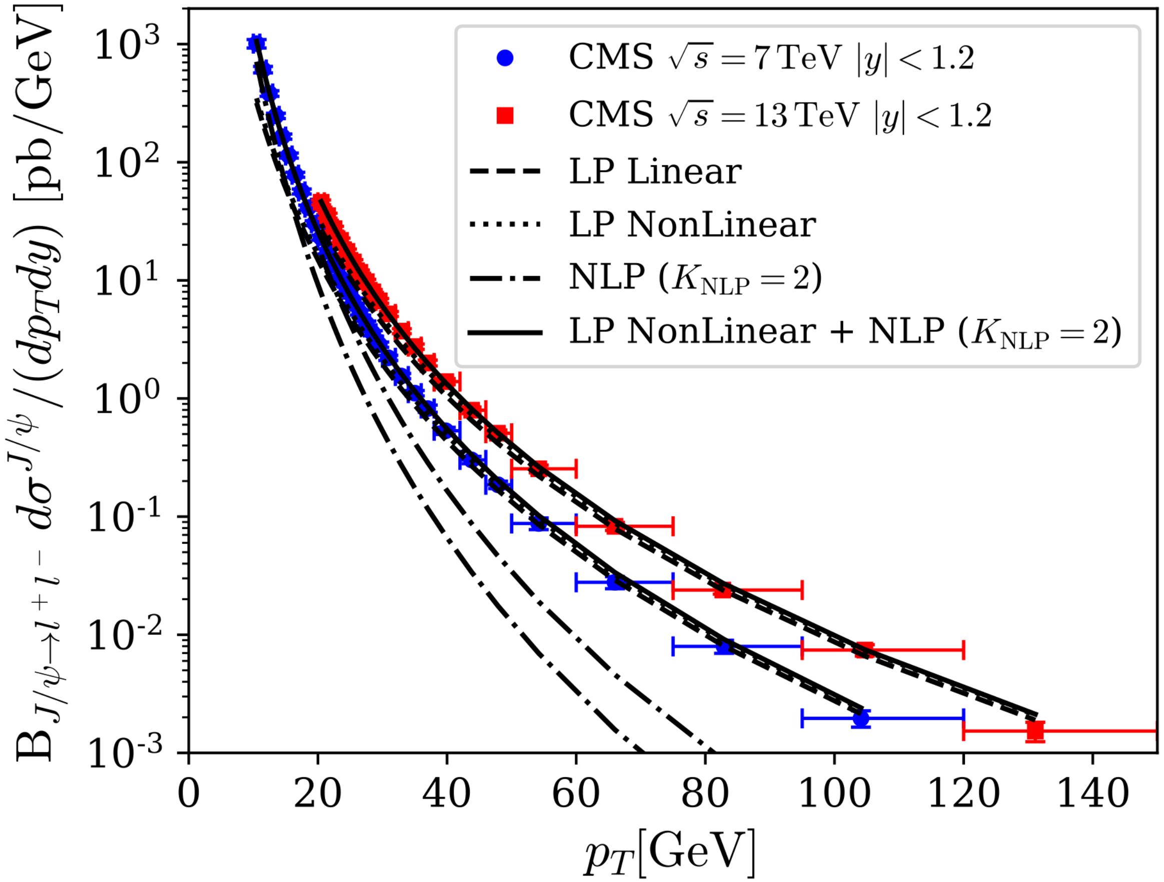
## ❑ LP + NLP contributions:

- Putting $\alpha = 30, \beta = 0.5$ at $\mu_0 = 4m_c$ and $\mu_\Lambda = m_c$, $\langle \mathcal{O}(^1S_0^{[8]}) \rangle/\mathrm{GeV}^3 = 0.1286 \pm 5.179 \cdot 10^{-3}$ is obtained.
- *K*-factor is included to account for higher order corrections of the NLP partonic cross section. We simply fix $K_{\mathrm{NLP}} = 2$.




**Choose two numbers with a smaller set of data**

# J/ψ-production in hadronic collisions

Lee, Qiu, Steerman, Watanabe, 2022

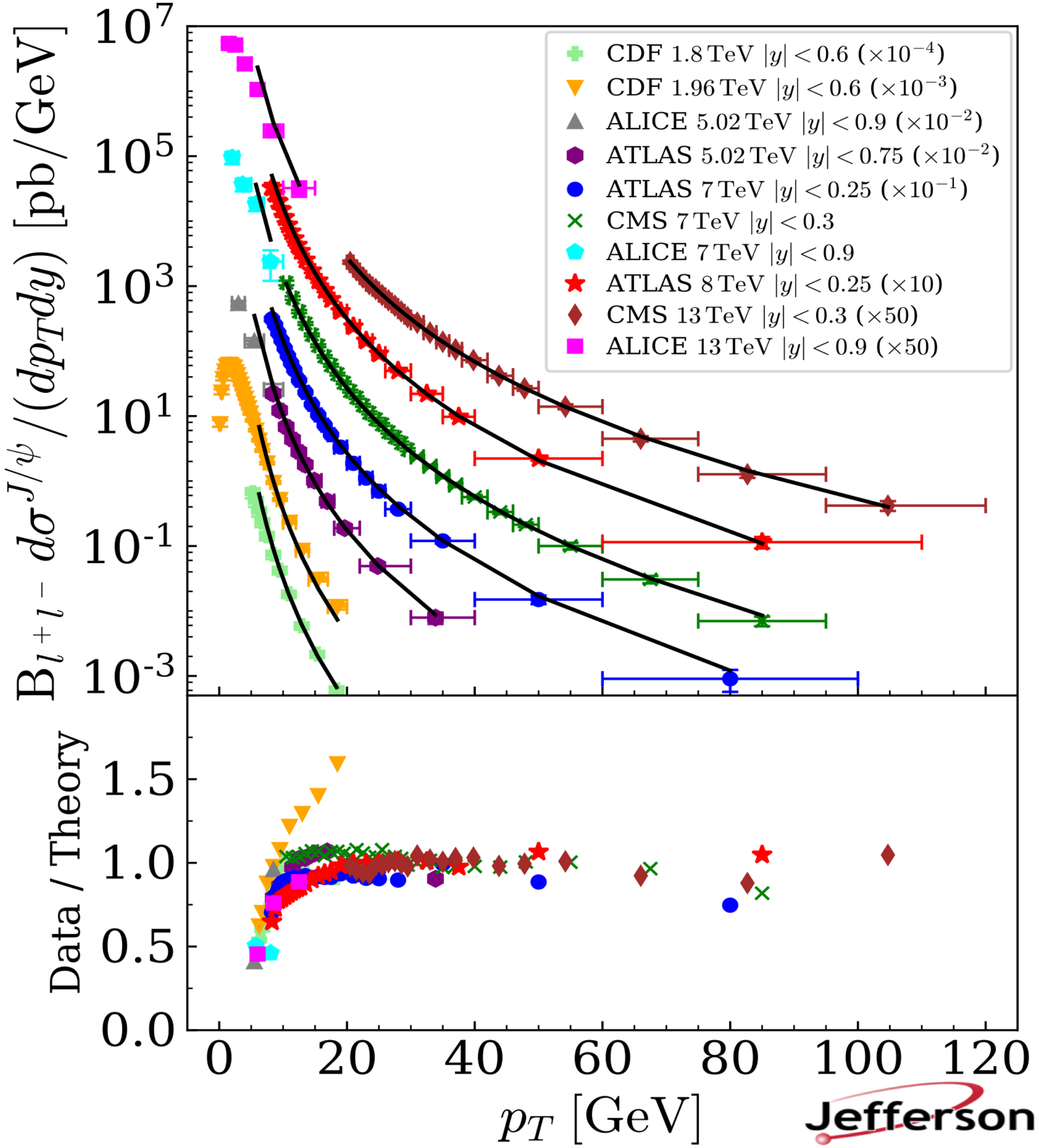
## ❑ Test the consistency:

- Given that the overall normalization factor is fixed, QCD factorization approach describes LHC data on prompt $J/\psi$ production in hadronic collisions.

→ **QCD global data analysis is possible.**

- We could modify $K_{\text{NLP}}$ at Tevatron energies, but $K_{\text{NLP}} = 2$ is fixed here.

**Compare with both the LHC and Tevatron data without changing parameters!**



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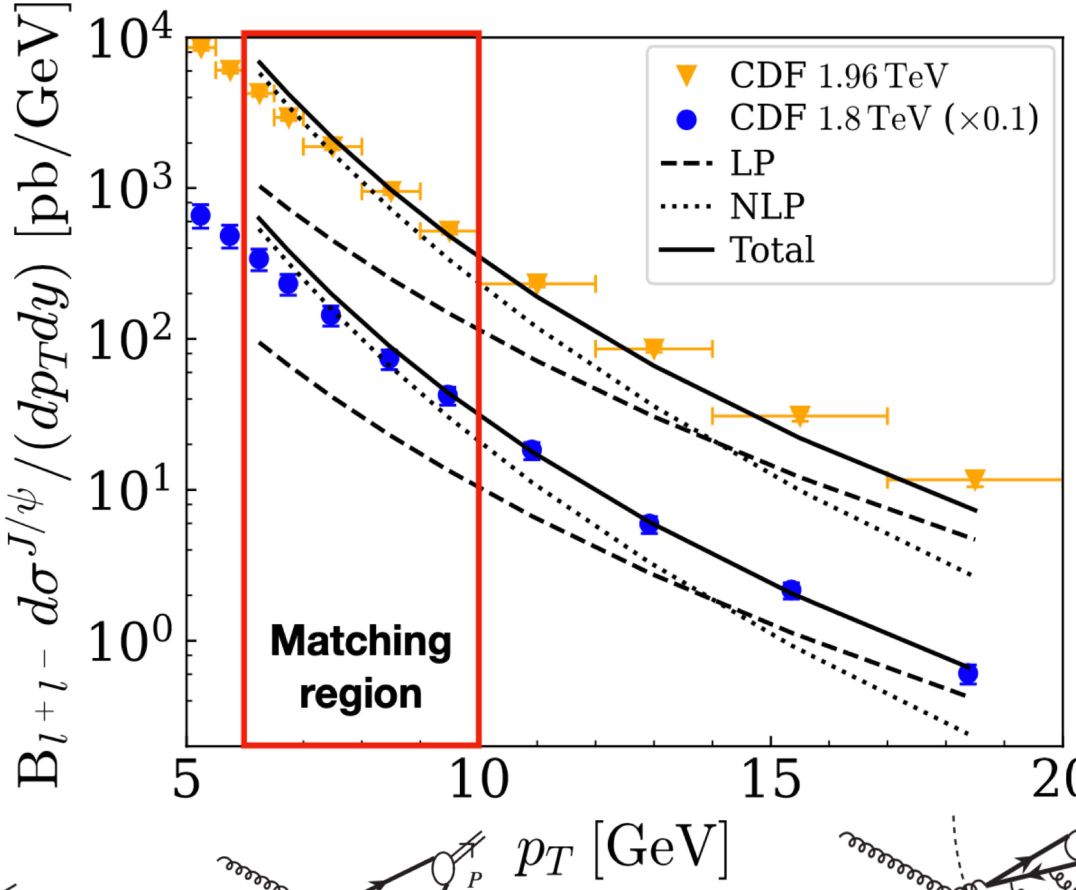
# Matching to fixed-order NRQCD calculation

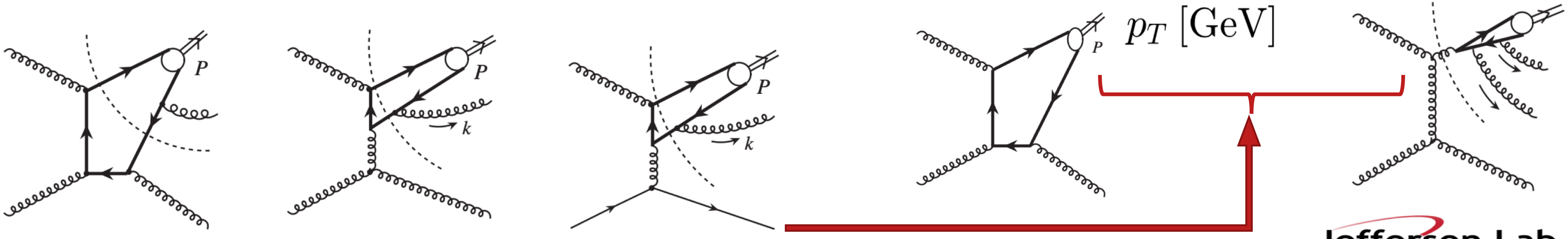
Lee, Qiu, Steerman, Watanabe, 2022

1. $\ln(p_T^2/m^2)$-type logarithmically enhanced contributions start to dominate when $p_T \gtrsim 5\,(\text{or}\,7)\,(2m_c) \sim 15 - 20\,\text{GeV}$, where the LP is significant, power corrections are small.

2. The NLP contribution is important at $p_T = \mathcal{O}(2m_c) \lesssim 10\,\text{GeV}$, where matching between QCD factorization and NRQCD factorization can be made.

3. Further exploration of the shape of the FFs at large-$z$ would help us understand the quarkonium production mechanism.



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# Summary and Outlook

- It has been almost 50 years since the discovery of J/Ψ, but, we are still not completely sure about its production mechanism
- We have studied the QCD factorization for hadronic quarkonium production at high pT
- We demonstrated that the LP contributions are significant for hadronic quarkonium production at high pT while the NLP contributions are sizable at lower pT but different in shape, and both are needed, leading to a smooth matching to fixed-order calculations
- Power corrections to the evolution of LP FFs are important even at high pT, impacting quarkonium polarization
- The initial success of QCD factorization formalism should encourage a global data analysis. There is sufficient room to improve the input FFs
- Matching between the QCD factorization and fixed order NRQCD factorization should enable us to describe quarkonium production not only in hadronic collisions but also in other scattering processes in a broader pT region.

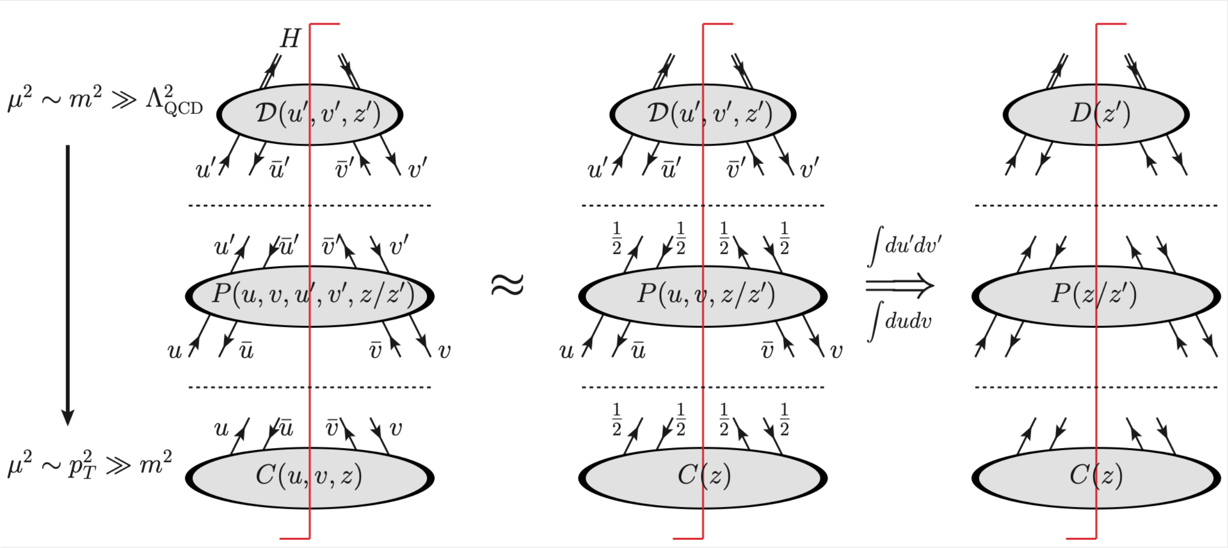
**Thanks!**

# Evolution equations in a simplified situation

Lee, Qiu, Sterman, Watanabe, in preparation

## ❑ Simplified evolution equations:

- The produced heavy quark pair is dominated by its on-shell state at high .
- We may expand the SDCs and evolution kernels on lower virtuality sides at each evolution step around .
- This can be a reasonable approximation suggested by the evolution of DP FFs in μ,ν-space. S-to-S channels are not dominant at high .




$$\frac{d\sigma^H_{\mathrm{NLP}}}{dyd^2p_T} = \int dzdudv\, C_{[Q\bar{Q}]}(p_Q, p_{\bar{Q}}, \mu)\, \mathscr{D}_{[Q\bar{Q}]\to H}(u,v,z,\mu) \approx \int dz\, C_{[Q\bar{Q}]}(\hat{p}^+_Q = \frac{1}{2}p^+_c, \hat{p}^+_{\bar{Q}} = \frac{1}{2}p^+_c, \mu) \underbrace{\int dudv\, \mathscr{D}_{[Q\bar{Q}]\to H}(u,v,z,\mu)}_{\equiv D_{[Q\bar{Q}]\to H}(z,\mu)}$$

$$\frac{\partial D_{[Q\bar{Q}(\kappa)]\to H}(z,\mu)}{\partial \ln \mu^2} \approx \sum_n \int_z^1 \frac{dz'}{z'} \int_0^1 du \int_0^1 dv\, \Gamma_{[Q\bar{Q}(n)]\to[Q\bar{Q}(\kappa)]}\left(u, v, u' = \frac{1}{2}, v' = \frac{1}{2}, \frac{z}{z'}\right) D_{[Q\bar{Q}(\kappa)]\to H}(z',\mu),$$

$$\frac{\partial D_{f\to H}(z,\mu)}{\partial \ln \mu^2} \approx \frac{\alpha_s}{2\pi} \sum_{f'} \int_z^1 \frac{dz'}{z'} P_{f\to f'}(z/z') D_{f'\to H}(z') + \frac{\alpha_s^2(\mu)}{\mu^2} \sum_{[Q\bar{Q}(\kappa)]} \int_z^1 \frac{dz'}{z'} P_{f\to[Q\bar{Q}(\kappa)]}\left(u' = \frac{1}{2}, v' = \frac{1}{2}, \frac{z}{z'}\right) D_{[Q\bar{Q}(\kappa)]\to H}(z',\mu)$$